Demographic Structure and Macroeconomic Trends[*]

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Abstract

We analyse both empirically and theoretically the effects of changes in demographic structure on the macroeconomy, looking particular at their impact to medium-term trends. Our empirical exercise examines the impact of the proportion of the population in each age group, on growth, savings, investment, hours, interest rates and inflation using a panel VAR estimated from data for 20 OECD economies for the period 1970-2007. This flexible dynamic structure with interactions among the main variables allows us to estimate both the direct impact of demographic structure and their feedback effects. Our estimates confirm the importance of age structure, with young and old dependants having a negative impact on most macroeconomic variables while workers contribute positively. Our theoretical framework incorporates demographic heterogeneity and endogenous productivity, allowing us to study the medium-term interaction of demographic changes and savings, investment, and innovation decisions. Theoretical simulations incorporating the changes in demographic structure experienced by many OECD countries in the past decades replicate well our empirical findings. The current trend of population aging and reduced fertility, expected to continue in the next decades, is found to be a strong force in reducing output growth and real interest rates across OECD countries.

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1. Introduction

The slow recovery after the great recession and the disappointingly small growth rate of productivity in the last decade has fostered the debate on the medium to long-run prospects of developed economies. This debate has centred on two main topics: the production of new ideas and the structural characteristics that can be important in shaping future economic conditions. Disagreement seems to be the norm as regards the production of new ideas with Gordon (2012, 2014) presenting a more pessimistic view while, amongst others, Fernald and Jones (2014) and Brynjolfsson and McAfee (2011) are more optimistic. The importance and impact of structural characteristics are more widely accepted. Gordon (2012, 2014) and Fernald and Jones (2014), looking particularly at the U.S., stress the importance of education attainment and demography. Demographic changes, in particular their effect on labour supply as a result of demographic transitions, are often mentioned as one of the 'headwinds' of the observed slowdown in macroeconomic performance in advanced economies. Although important, this narrow interpretation may restrict the impact of demographic changes on the macroeconomy. In this paper we take a more general view, arguing that changes in the demographic structure, defined as the variations in proportions of the population in each age group from year to year, matters for macroeconomic activity and may also be related to the production of ideas.

The demographic structure may affect the long and short term macroeconomic conditions through several channels. Different age groups (i) have different savings behaviour, according to the life-cycle hypothesis; (ii) have different productivity levels, according to the age profile of wages; (iii) work different amounts, the very young and very old tend not to work, with implications for labour input; (iv) contribute differently to the innovation process, with young and middle age workers contributing the most; and (v) provide different investment opportunities, as firms target their different needs. Thus, demographic structure changes can reasonably expected to influence real interest rates, inflation and real output in the long and short term either directly or via their effects on expectations on the future course of key variables.

As Figure 1 illustrates, the demographic age profiles in OECD economies are changing. The average proportion of the population aged 60+ across our sample is projected to increase from 16% in 1970 to 29% in 2030, with most of the corresponding decline experienced in the $0-19$ group. Though the proportion of the population in the "working age" group $(20-59)$ is similar in the two years at 50% and 48% respectively, it initially increased to around 56% in 2003 before starting to decline again. Given the scale of the age profile shifts observed in most developed economies and the relevance of increasing our understanding of the link between the economy's structural features and its future prospects, this paper investigates both empirically and theoretically the effects of changes in demographic structure on the macroeconomy, looking particular at their impact on medium-term trends.

In the first part of the paper we present empirical evidence on the short and long term relevance of demographic structure for the macroeconomy. While the theoretical literature and most economic commentary on policy strongly emphasise the importance of demographic structure, the econometric evidence for its importance is less compelling. There are a number of reasons for this. Changes in demographic structure are low frequency phenomena, difficult to distinguish from the other low frequency trends that dominate economic
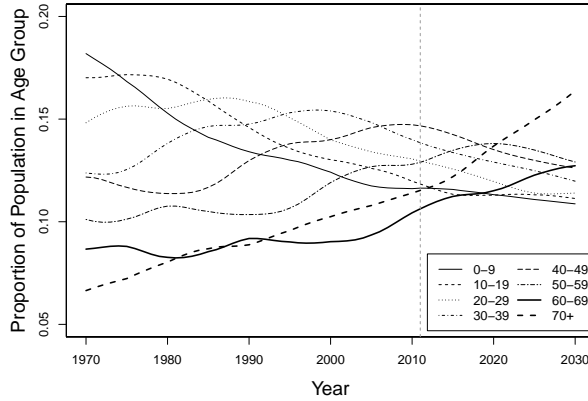
(Source: United Nations, 2011)

Figure 1: (Unweighted) Sample Mean Proportions in each Age Group by Year

time series. The vector of proportions in each age group is inevitably highly collinear, making precise estimation of the effect of each age group difficult. Hence it is common to impose very strong restrictions on the effect of the age structure, for instance through the use a single variable, the dependency ratio. Estimation of the coefficients of low frequency collinear determinants will be inevitably sensitive to the exact specification of the equations and the estimation method used. Endogeneity is a serious problem because although the proportions in each age group are plausibly exogenous (the high birth rate that produced the baby-boomers after 1945 is unlikely to be influenced by growth rates 30 years later) the other variables in the system are likely to be responding to the low frequency demographic impacts, reducing the marginal contribution of the demographic variables. Finally, general equilibrium effects are likely to be important, as other variables adjust. In particular, crucial intervening variables in the transmission of demographic structure to growth and savings are years in education; the age, sex and skill specific labour force participation rates and pension wealth. Although there are difficult measurement issues associated with each of these factors, all seem to have shown large variations over our sample.

With those concerns in mind our empirical analysis utilises a large panel of OECD countries, over the period 1970-2007 for most countries, and incorporating as much detail on the demographic structure as data availability allows. In our benchmark model, we ask how much of the variation of long-run growth in these countries can be explained by the evolution of their demographic structure, represented by share of age groups $(0 - 9, \ 10 - 19, \ldots, 70+)$ in total population, allowing for the interactions between the main macroeconomic variables of interest, and controlling for oil prices and population growth. We employ a panel VAR technique to uncover long-run association between real output, investment, savings, hours worked, nominal short term interest rates, and price inflation, and the slowly changing demographic profile. We also provide an extension to our benchmark model that recognises the importance of innovation activities for capital

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and labour productivity and their impact on the macroeconomy.

We find that the changing age profile across OECD countries has economically and statistically significant impacts on all key macroeconomic variables and that when we allow for the indirect effects of the changing age profile on the variables of interest we find that the long-term impacts are even stronger. Crucially, we find that the changing age profile impact roughly follows a life-cycle pattern; that is, dependant cohorts tend to have a negative impact on all real macroeconomic variables including real returns and add positive inflationary pressures in the long-run. We also test for the robustness of our results to the use of time effects, to the exclusion of individual countries and structural breaks. We find that the results are robust to time effects and exclusion of individual countries. However, while real output, investment, savings and hours worked do not suffer from structural breaks, inflation may do so in the early 1990's.

We then use the estimates to investigate the impact of the baby-boomers entering the labour market in 1970's and approaching retirement in late 2000's in the individual countries analysed. For the in sample period of 2000-2007 we find that changes in age profile would have contributed to a significant reduction in hours worked, with Japan being the country most significantly affected. Our model also suggests that, ceteris paribus, the changing age profile will have significant negative impact on real output growth in the 2010-19 decade in our sample of countries. When compared to 2000-09 decade the decline in average annual real output growth will range from 0.62% in Japan to 1.33 % in the U.S. We also find that the inclusion of patent applications as a proxy for innovative activities does not alter our benchmark results for the macroeconomic short and long term dynamics. However we find evidence of demographic structure effects on innovative activities, with older workers (in particular 50-59 age group) having a strong negative impact on total number of patent applications. In general, innovation, which can also be considered a measure of productivity gains, is positively affected by young and middle aged cohorts and negatively affected by dependants and retirees. Finally, we use the United Nations (UN) population predictions to measure the impact of the expected population changes on output growth and real interest rates until 2030. For most countries the decrease in working-age population and increase in proportion of retirees expected for the next 20 years would result in a strong decrease in trend output growth and significantly lower real rates of interest.

In the second part of the paper we develop a theoretical model to match the observed life cycle characteristics we found in the data and use it to study the main mechanisms through which demographic changes affect the macroeconomy. We set-up an economic environment incorporating (i) life cycle properties as in Gertler (1999), although we extend to allow for three generations of the population (dependant young, workers and retirees) and introduce investment in human capital and (ii) endogenous productivity and medium-term dynamics as in Comin and Gertler (2006), and thus can study the long-term interaction of demographic changes and savings, investment and innovation decisions. Our model highlights three channels through which age profiles affect the macroeconomy. Firstly, changes in fertility and availability of resources of workers affect investment in human capital. Secondly, aging affects the saving decision of workers. Finally, reflecting our empirical findings we assume the share of young workers impacts the innovation process positively and, as a result, a change in the demographic profile that skews the distribution of the population to the right, leads to a decline in innovation activity.

We are able to replicate most of our empirical findings at the theoretical level. We find that a relative increase in the share of young dependants and retirees decrease output growth and investment while an increase in workers does the opposite. A permanent increase in longevity (increase in life expectancy) leads to increased growth rates in the short-term as the decrease in the marginal propensity to consume of workers leads to lower real interest rate and an increase in innovative activity. However, as the share of young workers decrease, productivity in innovation decreases leading to permanently lower output growth and investment. Finally, we use the UN population predictions to feed into the model the expected changes in population dynamics for different countries in our samples, matching the prediction exercise done with the empirical model. Although our theoretical model only incorporates three age groups (relative to the 8 groups in the benchmark estimation) it does well in capturing the estimated impact of changes in demographic structure on output growth and real interest rates for different countries. Increases in average age and reduced fertility is found to be a strong force reducing output growth and real rates across OECD countries.

Related Literature

Our work is related to a large empirical literature on the effects of demography, in particular the age structure of the population, on macroeconomic variables, which arise through life cycle influences on savings and the differences in productivity, arising from the fact that different age groups have different participation rates and different human capital.

Several studies that look at the effect of demography on the macroeconomy, measure the changes in age structure either as the proportion of the population of working age (or the dependency ratios) or by life expectancy. Higgins and Williamson (1997) study the dependency hypothesis for Asia and argue that the significant increase in the Asian saving rates can be explained by the significant decline in youth dependency ratios that is associated with increased investment and reduced foreign capital dependency. Higgins (1998) examines the relationship between age-distribution, savings investment and thus the current account for a panel of countries, using 5 year averages for the variables. He also uses a low order polynomial function for the coefficients of 15 age distribution shares. He shows that demographic effects, i.e. increases in both youth and old-age dependency ratios, can explain different levels of decline in savings and investments and increase in capital imports. Acemoglu and Johnson (2007) study a panel of 75 countries. They argue that increase in life expectancy due to advancements in medicine against infectious diseases led to a significant increase in population, as birth rates did not decline sufficiently to compensate for the increase in life expectancy. They argue that the increases in life expectancy (and the associated increases in population) appear to have reduced income per capita. Bloom, Canning, Fink, and Finlay (2007) find that inclusion of life expectancy and the initial working-age share improves per capita income growth forecast performance for the period of 1980-2000 for a panel of 67 economies. (see also Bloom, Canning, and Fink (2010)and references therein.) Finally, Gómez and Hérnandez de Cos (2008) find that the proportions of 'mature' (15-64 year olds) and 'prime age' (34-54 year olds) people in the population can explain more than half of global growth since 1960, and that 'maturation' is also responsible for the continuing divergence of rich and poor countries as age structure

in the former has improved more dramatically than in the latter. [1]

A number of other studies, like ours, focuses on a more granular representation of the age structure. Fair and Dominguez (1991) examine the effect of demographics on various US macro variables. They find that the impact of US age distribution on consumption, money demand, housing investment and labour force participation is highly significant. Lindh and Malmberg (1999) consider age structure in a transitional growth regression on a panel of 5-year periods in OECD countries. They find that growth of GDP per worker is strongly influenced by the age structure, with 50-64 year olds having a positive influence and the 65-plus age group a negative one. Feyrer (2007) considers the age structure of the workforce, rather than the population as a whole, and its impact on productivity and hence output. He also finds a strong demographic effect, with the 40-50 year age-group having the most positive impact. Our approach differs from these in at least two crucial ways: first, we consider one-year periods rather than 5-year ones, and can hence adopt a panel time-series approach to estimation. Second, we allow for interaction effects between key macro-variables by estimating a VAR rather than an individual equation.[2]

On the theoretical side, the framework developed here incorporates demographic heterogeneity, building on Gertler (1999), Blanchard (1985) and Yaari (1965)[3] and endogenous productivity models, following Comin and Gertler (2006) and Romer (1990). Our work is also related to the recently re-popularised argument by Hansen (1939) on whether mature economies are experiencing a long lasting stagnation due to permanently low demand. Most of this literature currently focuses on the effects of aggregate demand externalities in periods of financial deleveraging that may lead to prolonged periods of lower real rates of return after the global financial crisis in 2008. Eggertsson and Mehrotra (2014) provide an OLG analysis where demand may be constrained by debt limits on young generation which leads to a decline in steady state real rates. Jimeno (2014) extends this model to show that (exogenously) lower population and productivity growth amplify this mechanism. By linking demographic changes and low real interest rates and future output growth, our results provide further indication that OECD economies are more likely to experience episodes where aggregate demand externalities may lead to stagnation in the following decades.

The paper is organised as follows. Section 2 presents the data and the econometric framework used. Section 3 presents the panel VAR estimates for the benchmark model, the individual country analysis and provides a series of robustness tests. Section 4 presents the results for the panel VAR estimates when a measure of innovation activities is also included. Section 5 introduces the theoretical framework while the simulation results are

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[1]Other interesting studies that focus on effects of demography are Jaimovich and Siu (2009) and Park (2010). The first examines the impact of demography on business cycle volatility in the G7 countries. The young and old have more volatile hours and employment than the prime-age workforce and thus an increasing share of prime-age workforce may have contributed to the great moderation. The second examines the impact of age distribution on stock market price-earnings ratios in the US, using a Fourier flexible form, rather than a polynomial.

[2]Allowing for those interaction generates a methodology that potentially captures general equilibrium effects. Miles (1999) has a careful discussion of the advantages and disadvantages of the use of different types of evidence to assess the impact of demographic change and argues for the use of calibrated general equilibrium models.

[3]See Ferrero (2010), Carvalho and Ferrero (2013) and Sterk and Tenreyro (2013) for other contributions that incorporate demographic heterogeneity in a similar way.

presented in section 6. Finally, Section 7 concludes.

2. Data and econometric model

The annual dataset covers the period 1970-2007. The demographic data was obtained from the United Nations (2011). The annual data on savings and investment rates were calculated from Nominal GDP, Investment and Savings series obtained from the OECD (2010), which also supplied the data on hours worked. Annual data on policy rates and the Consumer Price Index (CPI) were obtained from the IMF (2010). Per-capita GDP growth rates were calculated from per-capita real GDP obtained from Penn World Tables (Heston, Summers, and Aten (2009)).

The twenty countries covered by the data are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. For some countries data is not available over the whole period, so the panel is unbalanced. Data on hours are only available for Austria from 1995-2007, for Greece from 1983-2007 and for Portugal from 1986-2007. Savings and investment rates for Switzerland are only available from 1990-2007. All other countries have full datasets.[4]

We have data for countries, $i = 1, 2, \ldots, N$, for years $t = 1, 2, ..., T$. For data on age structure Park (2010) uses age by year, and restricts the shape of their effect, but given the lack of data for many countries we use age by decade. With only 8 demographic proportions and a fairly large panel we chose not to restrict the age coefficients. Denote the share of age group $j = 1, 2, ..8$ $(0 - 9, \ 10 - 19, \ldots, 70+)$ in total population by w_{jit} and suppose the effect on the variable of interest, say x_{it}, takes the form

$$x_{it} = \alpha + \sum_{j=1}^{8} \delta_j w_{ji,t} + u_{it}.$$

Since $\sum_{j=1}^{8} w_{jit} = 1$, there is exact collinearity if all the demographic shares are included. To deal with this, we restrict the coefficients to sum to 0, use $(w_{ji,t} - w_{8i,t})$ as explanatory variables and recover the coefficient of the oldest age group from $\delta_8 = -\sum_{j=1}^{7} \delta_j$. We denote the 7 element vector of $(w_{ji,t} - w_{8i,t})$ as W_{it}.

We estimate two sets of models. In the first set, the six endogenous variables of the system are the growth rate of the real GDP, g_{it}, the share of investment in GDP, I_{it}, the share of personal savings in GDP, S_{it}, the logarithms of hours worked H_{it}, the real short interest rate, R_{it} and the rate of inflation π_{it}. We denote the vector of these six variables as $Y_{it} = (g_{it}, I_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$. Demographic shares, W_{it} and two lags of the logarithm of the real oil prices are exogenous variables in our system of equations. Crucially, we also control for a measure of population growth (both current and one lagged) for each country in the sample, as we are essentially interested in the macroeconomic dynamics induced by the composition of the demography rather than the impact of an increase or

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[4]Though it would also be desirable to include Germany and Turkey as mature OECD economies, we exclude Germany due to reunification and Turkey due to incomplete demographic data. However, we include predictions for Germany in the tables.

decrease in the population.[5] In our second set of estimations we analyse the link between demographic structure and innovation, incorporating a proxy for $R\&D$ activities. As such we include residential patent applications $(R\&D^{PA})$ as recorded by the OECD, utilizing a vector of seven variables given by $Y_{it} = (g_{it}, I_{it}, S_{it}, H_{it}, rr_{it}, R\&D^{PA}, \pi_{it})$.

There are likely to be complicated dynamic interactions between the six economic variables and there is relatively little literature suggesting an appropriate model for panel data. For instance Bond, Leblebiciolu, and Schiantarelli (2010) consider the relationship between g_{it} and I_{it} in detail, but one may also expect interaction with the other variables because of other theoretical linkages. Ideally one would like to estimate an identified structural system between these six variables allowing for expectations. Suppose, ignoring oil prices and population growth, that such a structural system took the form

$$\Phi_0 Y_t = \Phi_1 E_t(Y_{t+1}) + \Phi_2 Y_{t-1} + \Gamma W_t + \varepsilon_t. \tag{1}$$

Then there is a unique and stationary solution if all the eigenvalues of A and $(I - \Phi_1 A)^{-1}\Phi_1$ lie strictly inside the unit circle, where A solves the quadratic matrix equation

$$\Phi_1 A^2 - \Phi_0 A + \Phi_2 = 0. \tag{2}$$

In that case the system is given by

$$Y_t = A Y_{t-1} + \Phi_0^{-1}\Gamma W_t + \Phi_0^{-1}\varepsilon_t. \tag{3}$$

Identifying the structural system is likely to be difficult. If there are m endogenous variables, identifying (1) requires $2m^2$ identifying restrictions (see the discussion in Koop, Pesaran, and Smith, 2011; Komunjer and Ng, 2011). Therefore we estimate the solution or reduced form of such a structural system and assume that conditional on the exogenous variables, it can be written as a VAR like (3). Notice that since A will be a complicated function of all the structural parameters, as (2) makes clear, it may be difficult to interpret the coefficients. However, our objective is primarily to provide predictions of the long-run effect of the demographic variables and the same predictions would be obtained from any just identified structural model such as (3). Over-identifying restrictions, if available and correct, would increase the efficiency of the estimation, but given that we have a large panel that seems a secondary consideration.

Additionally, we allow for intercept heterogeneity through a_i but assume slope homogeneity and estimate a one way fixed effect augmented panel VAR(2) of the form:

$$Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + D W_{it} + u_{it},$$

plus two lags of the oil price and population growth. D is the 6×7 matrix of coefficients of the demographic variables. Our estimate of the effect of the demographic variables is

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[5]In a dynamic stochastic general equilibrium setting, savings (hence consumption) should be subject to both substitution and wealth (income) effects. In our savings analysis we include short term rates and inflation to capture intertemporal consumption preferences. We also experimented with a specification with two measures of wealth (financial and housing) to capture the wealth effects. The data for this was taken from Slacalek (2009) and was only available for a sub-sample of the data we use. On the sub-sample, the Schwarz Bayesian information criterion indicated that the specification excluding wealth gives a better fit, therefore the main analysis is performed on the full range of data and excludes wealth.

then the marginal effect after having controlled for lagged Y_{it}, the oil price and population growth. Implicitly we are assuming either that all the variables are stationary or that a flexible unrestricted VAR will capture stationary combinations by differencing or cointegrating linear combinations.[6]

Slope heterogeneity is undoubtedly important and it can have unfortunate consequences in dynamic panels. Pesaran and Smith (1995) show that it biases the coefficient of the lagged dependent variable towards one and the coefficient of the exogenous variable towards zero, though these two biases may offset each other in the calculation of the long-run effects, the focus of our interest. However, we adopt a fixed effect estimator which imposes slope homogeneity across countries, partly because we are estimating 21 slope parameters and partly because the demographic variables show very low frequency variation relative to annual time-series and the elements are highly correlated. Thus heterogeneous estimates based on relatively few degrees of freedom may be poorly determined and likely to produce outliers. We found this to be the case when we experimented with VARs for each country. In addition, Baltagi and Griffin (1997) and Baltagi, Griffin, and Xiong (2000) show that the homogeneous estimators tend to have better forecasting properties. As a result, since our main aim is to predict the variables conditional on demographics, the homogeneous estimators may provide better predictors of this demographic contribution.

The long-run moving equilibrium for system is then given by

$$Y_{it}^* = (I - A_1 - A_2)^{-1} a_i + (I - A_1 - A_2)^{-1} DW_{it},$$

where the effect of the demographic variables is given by $(I - A_1 - A_2)^{-1} D$, which reflects both the direct effect of demographics on each variable and the feedback between the endogenous variables. This allows, for instance, the effects of demography on savings to influence growth through the effect of savings on growth. We can isolate the long-run contribution of demography to each variable in each country by obtaining

$$Y_{it}^D = (I - A_1 - A_2)^{-1} DW_{it}. \tag{4}$$

This is the demographic attractor for the economic variables at any moment in time. It is important to distinguish between our long-run estimate and a long-run steady state. Our estimates provide a long run forecast for the economic variables conditional on a particular vector of demographic shares after the completion of the endogenous adjustment of the economic variables. However, as time passes the demographic structure might evolve towards a steady state demographic distribution. We do not model this process and thus are not providing an estimate of the effects of this convergence process of current demographic structure to its steady state. In summary, we examine the movements of elements of this vector, Y_{it}^D, over time to indicate the low frequency contribution of demographics to the evolution of a particular variable of interest in a particular country.

3. Panel VAR estimates-Benchmark model

We chose between possible specifications on the basis of the Schwarz Bayesian information criterion, SBC. On that basis, a one way fixed effect model with country intercepts

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[6]Phillips and Moon (1999) and Coakley, Fuertes, and Smith (2006) suggest that spurious regression may be less of a problem in panels. Also see a discussion of this issue with respect to the investment share in Bond, Leblebiciolu, and Schiantarelli (2010).

was preferred for every equation to a two way fixed effect model with country and year intercepts, but without the oil price. This suggests that cross-section dependence or common trends is not a major problem with the model, but we investigate the robustness of our results to this below. A VAR(1) and a VAR(2) had almost identical SBCs. We used a VAR(2) to allow for more flexible dynamics and to deal with potential non-stationarity. Full estimates are given in an appendix Tables 16 and 17, together with HAC robust standard errors.

We report below, in Table 1, the $A_1 + A_2$ matrix, where each row represents an equation in the panel VAR representation. We note that hours worked, investment, savings and real rates are highly persistent and real output and inflation rate are moderately so. There is evidence that all our endogenous variables are Granger causal for some other variables in the system, except in the case of savings which does not have a significant influence on any other variable.[7] Therefore, we seem to capture well the dynamic interactions between the main economic variables.

	g_{t-1}	I_{t-1}	S_{t-1}	H_{t-1}	rr_{t-1}	π_{t-1}
g	0.24	-0.18	0.01	-0.01	-0.26	-0.28
I	0.17	0.76	0.01	0.01	-0.10	-0.10
S	-0.12	-0.10	0.77	-0.01	-0.10	-0.07
H	0.22	-0.05	0.01	0.92	-0.13	-0.11
rr	-0.19	-0.18	-0.10	0.05	0.90	0.24
π	0.36	0.21	0.05	-0.02	-0.16	0.55

Table 1: Sum of VAR coefficients $A_1 + A_2$

Table 2 gives the D matrix of short term demographic impacts on the six variables. As expected the individual coefficients are not well determined because of collinearity, but the hypothesis that the coefficients of the demographic variables are all zero is strongly rejected for all equations (see tables 16 and 17 in the Appendix). Generally the results look plausible, meaning dependent population as represented by the 0-9, 10-19 and 70+ have in general a negative impact on real output, investment, savings, hours worked and real rates while working population (20 - 60 groups) generally have a positive impact.[8] Younger and older generations appear to have positive impact on inflation whereas working age groups impact inflation negatively.

Table 3 gives the $(I - A_1 - A_2)^{-1} D$ matrix. First, allowing for the dynamics and interactions makes a strong difference, the long-run effects are much larger. Second, we find the impact of demographics on savings and interest rates gives support to the life cycle hypothesis. Savings increase when the share of workers approaching retirement increase and decrease substantially then the share of retirees increase. Moreover, when the share of dependents (old and young) increase, interest rate tend to decrease indicating the marginal propensity to consume out of income from workers is decreasing. Life cycles effects are also observed for hours worked. The effect on hours and savings are particularly marked as

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[7]Perhaps the most surprising feature is that lagged investment has a negative effect on growth, though as there is a strong positive contemporaneous correlation between the growth and investment residuals (See Appendix). For OECD countries Bond, Leblebiciolu, and Schiantarelli (2010) found a small positive effect in the bivariate relationship.

[8]The 30-39 and 40-49 groups have negative effect on growth but estimates are quite close to zero.

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.06	0.25	0.18	-0.03	-0.03	0.02	-0.07	-0.25
I	-0.03	0.04	0.08	-0.03	-0.06	0.03	0.18	-0.20
S	-0.10	0.17	0.02	0.11	0.08	0.19	0.01	-0.49
H	-0.10	-0.02	0.07	0.14	-0.03	0.08	0.05	-0.20
rr	-0.33	-0.08	0.14	0.29	0.21	0.16	0.01	-0.39
π	0.50	0.13	-0.16	-0.46	-0.30	-0.07	0.18	0.19

Table 2: Short-Run Demographic Impact

these are highly persistent. Investment is negatively affected by young and old dependants and strongly positively affected by mature workers (30 - 49). One surprising finding is the slight positive long term contribution of 70+ group on growth while as expected 0-9 and 60-69 age groups negatively contribute to long term growth. Larger dependent age groups generally lead to a long term decline in hours worked, real rates, savings and investment and higher inflation, whereas a larger proportion in working age groups impact inflation negatively. Finally, the matrix of correlations between the residuals of each equation of the VAR (presented in the Appendix) shows a very strong contemporaneous correlations between the residuals of some of the equations, perhaps reflecting business cycle effects.

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

Table 3: Long-Run Demographic Impact

3.1. Individual country counterfactual and prediction analysis

We use our benchmark long-run estimates to perform two distinct individual country analyses. Firstly, we look at the effect of the end of the demographic dividend associated with the baby-boomers, who were approaching retirement towards the end of our sample period. As such, we provide a counterfactual analysis that measures the contribution of the change in demographic structure between 2000 and 2007 to changes in the six macroeconomic variables of interest for the countries with available data. This is calculated using equation (4) and the long-run estimates from the one way fixed effect model. Table 4 shows the results.
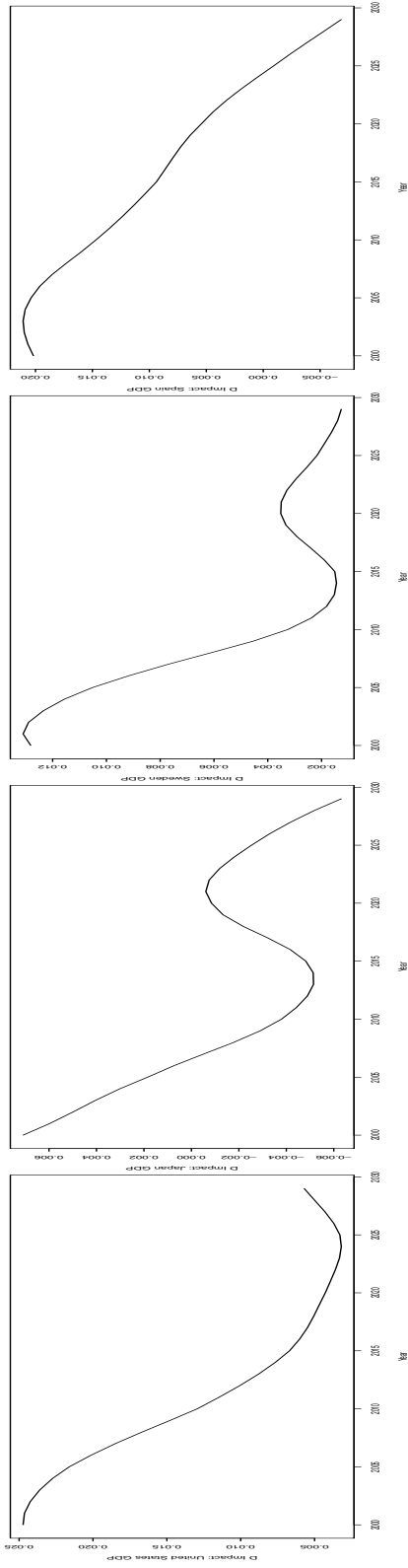
The estimated impact of demographic changes on GDP varies across countries, however a decline in the 2000-2007 growth is a common feature across all countries in our sample. Given our model real GDP growth in 2007 would have been 1.09% *less* for Japan as compared to 2000 and 1.18% in the US. In general, as compared to the year 2000, growth rates would have been significantly affected by the changes in the age profile, as all variables would have been depressed including the hours worked. There is a clear negative impact

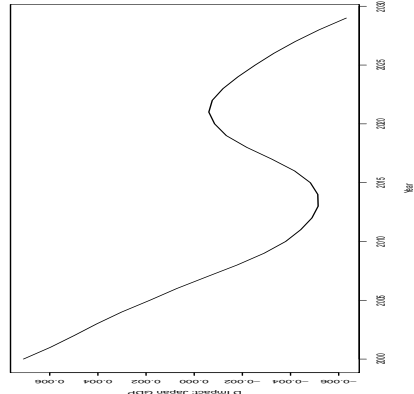|	g_{t-1}	I_{t-1}	S_{t-1}	H_{t-1}	rr_{t-1}	π_{t-1}
Australia	-0.78	0.41	-1.16	-3.05	0.77	0.25
Austria	-0.45	-0.69	-1.56	-7.27	-0.33	0.70
Belgium	-0.47	-1.83	-1.28	-5.57	-1.13	1.74
Canada	-1.11	-0.02	-1.20	-5.69	0.91	0.94
Denmark	-0.88	-0.04	-0.17	-8.71	-1.21	3.29
Finland	-1.33	-0.50	-4.14	-8.58	-0.81	2.57
France	-0.90	-1.54	-2.19	-6.29	-0.72	1.64
Germany	0.04	-3.62	-5.35	-9.89	-3.01	1.75
Greece	-0.19	-3.40	-2.90	-0.03	-1.92	-2.06
Iceland	-0.65	0.68	0.40	-0.14	1.61	-1.09
Ireland	-1.05	-0.59	-0.82	1.01	1.58	-3.00
Italy	-0.27	-3.26	-2.34	-5.69	-2.91	0.67
Japan	-1.09	-3.64	-8.66	-7.24	-4.59	0.80
Netherlands	-1.10	-1.37	-1.30	-10.98	-1.32	3.43
New Zealand	-0.73	0.13	-1.04	-5.04	0.52	0.77
Norway	-0.79	1.56	2.33	-3.25	1.41	0.82
Portugal	-0.47	-2.12	-1.44	0.58	-0.77	-2.37
Spain	-0.55	-2.36	0.25	0.14	-0.42	-2.30
Sweden	-0.96	0.98	-0.93	-5.43	0.58	1.05
Switzerland	-0.38	-0.19	-1.32	-5.24	-0.29	0.43
United Kingdom	-0.45	-0.04	-1.56	-5.53	0.03	0.71
United States	-1.18	-0.02	-0.54	-6.15	1.06	1.71

Table 4: Difference in Predicted Impact of Demographic Factors between 2000 and 2007

of demographic changes on inflation in Japan and in some peripheral European countries such as Greece, Ireland and Portugal between 2000 and 20007. The estimated demographic impact on real rates is mixed. Our model predicts a decline in real rates in most countries including Japan and Germany, while there seems to be a positive effect on real interest effect in other countries including the US.
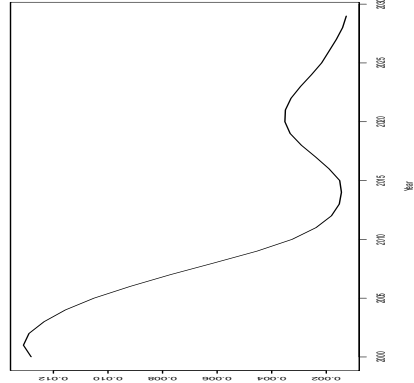
Secondly, we utilize the predicted future demographic structure as provided by the UN World Population Prospects (2010) and feed into our reduced form model to project the effect of changes in demographic structure expected for each country in our sample on our macroeconomic variables in the next decades. Table 18 provides forecasts of the impact of demographic structure on average annual per-capita GDP growth over the 2010-2019 period, and compares it to that over 2000-2009. It suggests that in all countries in our sample, as well as Germany, the impact of demographic factors over this decade would put downward pressure on GDP growth. The magnitude of this pressure is highly economically significant: for the US, for example, it is -1.33% and for Japan -0.62%. Figure 2 shows the predicted path of output growth and real interest rate for a subset of our countries (the prediction for the other countries is shown in the Appendix). As it can be seen, demographic changes are expected to contributed to significantly reduced trend output growth and real interest rate in many OECD countries in the next decades; in some cases we observe negative real interest rates and output growth rates.
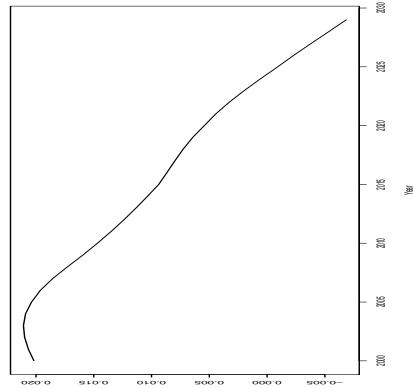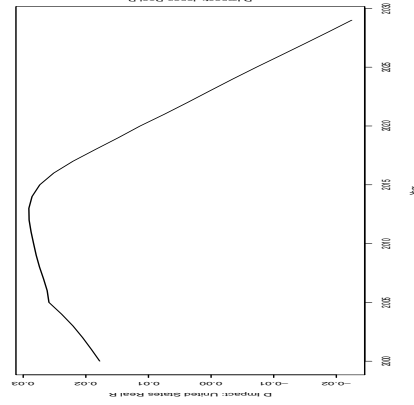
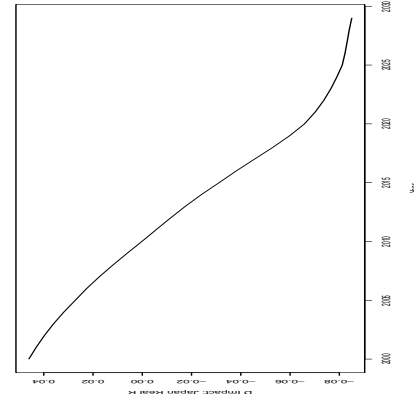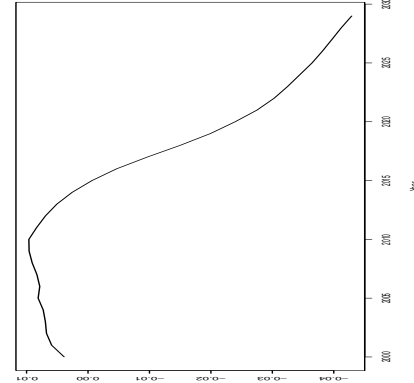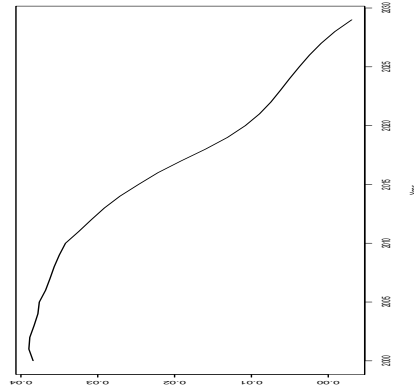Figure 2: Impact of Predicted Future Demographic Structure

	2000-2009	2010-2019	Change
Australia	2.02	1.05	-0.97
Austria	1.58	0.89	-0.69
Belgium	1.37	0.35	-1.02
Canada	2.21	0.58	-1.63
Denmark	0.85	0.30	-0.56
Finland	1.00	-0.39	-1.39
France	1.49	0.33	-1.16
Germany	0.88	0.27	-0.60
Greece	1.42	0.67	-0.76
Iceland	2.39	1.21	-1.18
Ireland	2.38	1.26	-1.12
Italy	0.96	0.39	-0.57
Japan	0.22	-0.39	-0.62
Netherlands	1.46	0.30	-1.16
New Zealand	2.09	0.95	-1.14
Norway	1.58	0.67	-0.91
Portugal	1.40	0.75	-0.65
Spain	1.96	1.02	-0.94
Sweden	1.01	0.22	-0.78
Switzerland	1.56	0.71	-0.85
United Kingdom	1.42	0.70	-0.73
United States	2.12	0.79	-1.33

Table 5: Average Predicted Impact on GDP Growth by Country

3.2. Three generations

In order to capture common characteristics across age groups and more general life cycle effects, we also present our results for broader segments of the society according to their age. To this end, we reclassify demographic groups and estimate for three demographic groups at any given time. In particular, we bundle together age groups 0-9 and 10-19 as young dependants, age groups 20-29, 30-39, 40-49 and 50-59 as workers and 60-69, 70+ as older workers and retirees. Of course, this way of classifying age groups is somewhat imprecise. Given the official retirement age in most OECD countries is around 65, there are some in that age group who should actually be in the category of workers. Similarly, as there are several young people who are already in the workforce after compulsory schooling. However, given that in our theoretical model, due to parsimony, we assume three heterogenous groups (youngsters, workers and retirees), this additional estimation provides a closer link between theory and empirics. We report below the long term demographic effects for these age groups (β's). We observe that there is a strong long term negative impact of the oldest age groups on all the variables except inflation. Young dependants have a significant positive impact on savings and inflation and a negative impact on hours worked and real rates. The proportion of working age has a positive impact on all variables except inflation.

	β_1	β_2	β_3
g_{t-1}	0.02	0.12	-0.14
I_{t-1}	0.03	0.17	-0.20
S_{t-1}	0.28	0.31	-0.59
H_{t-1}	-0.64	1.53	-0.89
rr_{t-1}	-0.11	0.32	-0.20
π_{t-1}	0.68	-0.85	0.17

Table 6: Long-Run Demographic Impact

3.3. Robustness

Robustness to the use of time effects

As mentioned above the model chosen using SBC assumes one-way fixed effects and includes oil prices as a measure of technology shocks across countries. One potential drawback of this approach concerns trends: if there are shared, cross-country, factors driving the trend in the dependent variable as well as the demographic variables, this trend may be wrongly attributed to the demographic variables in the one-way, country, fixed effect model. A two-way effects model avoids this issue by removing any common cross-country factors from all variables prior to estimation.

Table 7 shows the long-term impact of demographic variables under a two-way fixed effects model. Comparison with Table 3 reveals that long term demographic effects are generally robust to the chosen effect. The only sign reversal occurs in the case of 70+ groups on real output but the impact seems to be rather small. We conclude that the impact of demographic variables on the macroeconomy identified by the model is not merely a spurious correlation.

	Benchmark								Three Generations		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	β_1	β_2	β_3
g_{t-1}	-0.16	0.18	0.09	0.11	0.14	-0.04	-0.29	-0.03	0.01	0.13	-0.14
I_{t-1}	-0.71	-0.12	0.27	0.31	0.42	0.15	0.42	-0.75	-0.26	0.36	-0.10
S_{t-1}	0.09	0.78	0.01	0.39	0.25	0.22	0.12	-1.87	0.63	0.20	-0.83
H_{t-1}	-2.13	-0.14	0.50	2.62	1.04	0.37	-0.90	-1.36	-0.91	1.76	-0.85
rr_{t-1}	-0.19	-0.08	0.41	0.07	-0.12	0.50	0.17	-0.76	-0.02	0.15	-0.13
π_{t-1}	0.54	0.28	-0.56	-0.83	0.00	-0.03	0.02	0.57	0.44	-0.59	0.15

Table 7: Long-Run Demographic Impact (2-way effects)

Robustness to exclusion of individual countries

We test the robustness with respect to the selected countries by re-estimating the model on a dataset with each country excluded in turn. The results are very stable with respect to these exclusions, as are the tests as to whether the demographic variables are significant in each equation.

Structural Change

We also test for potential structural change by estimating the model on sub-periods of the entire dataset, and selecting the preferred model using the SBC. A single model over the whole period was preferred over models with structural breaks in any given year for the first four equations in the VAR - growth, investment, savings and hours worked. For the last two equations, interest rates and inflation, models with breaks in 1992 and 1989 respectively were optimal under the SBC.

Estimating the model over two subsets spanning 1970-1990 and 1990-2007 respectively yields results that differ from the full-period estimation as well as each other, indicating the possible presence of structural instability. The ranges of the demographic variables for the two periods are also somewhat different, however, and the second period has a vastly reduced variation in interest rates since the euro member countries in our sample shared a common rate for much of the period.

4. Panel VAR Estimation - Introducing Innovation variables

Feyrer (2007) examines the link between productivity and demographic structure and finds strong and robust relationship between these two. In two other papers (Feyrer (2008), Feyrer (forthcoming)) he suggests two potential channels through which age structure can affect productivity: innovation and adoption of ideas through managerial and entrepreneurial activity. [9]

In this section, in order to account for possible dynamic interactions between demographic structure and innovation which in turn will affect technological progress, we re-estimate the model including an additional variable that proxies for $R\&D$ activity.[10] To this end, we utilize World Development Indictors of the World Bank on residential patent applications ($R\&D^{PA}$) in log difference form.[11]

Table 8 (left panel) gives the $(I - A_1 - A_2)^{-1} D$ matrix of long term demographic impacts with seven endogenous variables, while the right panel show the results when we estimate the model over three demographic groups. First, we note that allowing for the dynamics and interactions leads to large long-run effects generally in line with the benchmark model. Second, the results for the 3 generations case indicate that young dependants and older generations contribute negatively to variations in patent applications whereas the workers (20-60) contribute positively. Finally, in line with the evidence in Jones (2005) we find a strong positive effect of mature workers (40 - 49) but a negative effect of older workers (50-59 age group) on $R\&D^{PA}$, identifying the potential asymmetric effect of different working-age groups on innovation.

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[9]He shows that in the US innovators' median age is stable around 48 over the 1975-95 sample period whereas median age of managers who adopt ideas are lower around the age of 40 and the managerial median age is affected by the entry of the babyboom generation into the workforce over the years. He argues that changes in the supply of workers may have an impact on the innovation rate. By contrast, entry of babyboomers into the workforce may have resulted in a lowering of the managerial quality due to inexperience and contributed to the US productivity slowdown in early 1970's.

[10]We also estimate the model with trademark applications ($R\&D^{TM}$). Results are similar to inclusion of $R\&D^{PA}$ and available upon request from authors.

[11]Note that the data for residential patent applications for Australia and Italy are incomplete, therefore we exclude these countries in our estimations. We also interpolate residential patent applications data for Japan for the years 1981 and 1982 as there seemed to be anomaly in their data for these two years.

	Benchmark								Three Generations		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	β_1	β_2	β_3
y_{t-1}	-0.13	0.15	0.08	0.12	0.08	-0.06	-0.30	0.06	0.02	0.11	-0.13
I_{t-1}	-0.59	0.09	0.40	0.42	0.03	0.03	0.34	-0.72	0.01	0.17	-0.18
S_{t-1}	-0.24	0.58	-0.22	0.36	0.42	0.78	-0.11	-1.57	0.27	0.34	-0.61
H_{t-1}	-1.60	-0.20	0.43	2.43	0.63	0.48	-0.88	-1.29	-0.58	1.55	-0.97
rr_{t-1}	-0.38	-0.57	0.54	0.34	0.20	0.52	0.41	-1.05	-0.16	0.34	-0.19
$R\&D^{PA}_{t-1}$	0.50	-0.56	0.02	0.05	0.70	-1.32	0.17	0.44	-0.16	0.22	-0.06
π_{t-1}	0.87	0.80	-0.35	-0.95	-0.66	-0.27	0.11	0.45	0.68	-0.87	0.20

Table 8: Long-Run Demographic Impact

5. Theoretical Model

In this section we propose a model that accounts for the main empirical findings presented here and use it to perform different simulations studying the effect of demographic changes. Given that we are interested in those effects after the completion of the endogenous adjustments of the economic variables, our modelling frameworks focuses on demographic heterogeneity and medium-run dynamics. As such, we set-up an economic environment incorporating life cycle properties as in Gertler (1999) and endogenous productivity and medium-term dynamics as in Comin and Gertler (2006). The economy consists of three main structures: a production sector, an innovation sector and households. The production sector comprises a final good producer, whose factors are differentiated goods (inputs), and input producers, whose production process employs capital, labour and a composite of intermediate goods. The number of input producers is endogenously determined, hence entry and exit is permitted. The composite of intermediate goods aggregates an endogenous set of product varieties, defined by the innovation process. Product innovation consists of two joint processes. Product creation (prototypes) or R&D and product adoption, in which prototypes are made ready to be used in the production process. Individuals, who supply labour, accumulate assets and consume, exhibit life-cycle behaviour, albeit of a simple form. Individuals face three stages of life: young/dependant, worker and retiree. Finally, there is a zero expected profit financial intermediary to facilitate the allocation of assets between the household and the production and innovation sectors.

5.1. Production

The final good producer combines inputs from N_t^f firms, denoted by superscript j. Total output is thus is given by

$$Y_{c,t} = \left[\int_0^{N_t^f} (Y_{c,t}^j)^{(1/\mu_t)} dj \right]^{\mu_t}, \qquad (5)$$

where μ_t denotes the mark-up of input firms. We assume $\mu_t = \mu(N_t^f)$, $\mu'(\cdot) < 0$ and that profits of intermediate good firms $\Pi(\mu_t, Y_{c,t}^j)$ must equate operating costs given by

$\Omega\tilde{\Psi}_t$, where $\tilde{\Psi}_t$ is a scaling factor defined to ensure we obtain a balanced growth path (see below).

Each firm j produces a specialised good using capital (K_t^j), labour (L_t^j) and an intermediate composite good (M_t^j). Production is given by

$$Y_{c,t}^j = \left[(U_t^j K_t^j)^\alpha (\xi_t L_t^j)^{(1-\alpha)}\right]^{(1-\gamma_I)} \left[M_t^j\right]^{\gamma_I}, \tag{6}$$

where U_t^j is the utilisation rate, γ_I the intermediate good share, $\xi_t L_t$ denotes the effective labour units employed in production and α the capital share of added value. The intermediate composite good used by firm j aggregates A_t specialised goods such that

$$M_t^j = \left[\int_0^{A_t} (M_t^{ji})^{(1/\vartheta)} di\right]^\vartheta. \tag{7}$$

Each producer i acquires the right to market the good via the creation and adoption process. Total costs of production for firm j are then given by

$$TC = W_t \xi_t L_t^j + (r_t^k + \delta(U_t)) K_t^j + P_t^M M_t^j$$

Where W_t is the wage, r_t^k is the rent of capital, $\delta(U_t)$ is the capital depreciation rate, with $\delta'(\cdot) > 0$, and P_t^M is the price of the intermediate composite good.

5.2. R&D and Adoption

The creation of intermediate good varieties is divided into two stages: R&D and conversion/adoption.

R&D

Let Z_t^p be the stock of invented goods for an innovator p. Then at every period an innovator spends S_t^p to add new goods to this stock. Each unit spent produces φ_t new goods. Thus, Z_{t+1}^p is given by

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p,$$

where ϕ is the implied product survival rate. In Comin and Gertler (2006) the productivity of new inventions φ_t is assumed to be given by $\varphi_t^{CG} = \chi Z_t [\tilde{\Psi}_t^\rho (S_t)^{1-\rho}]^{-1}$, where χ is a scale parameter. Thus, it depends on the aggregate stock of invented goods (Z_t), so there is a positive spillover as in Romer (1990), and on a congestion externality via the factor[12] $[\tilde{\Psi}_t^\rho (S_t)^{1-\rho}]^{-1}$, as such, the R&D elasticity of new technology creation in equilibrium is ρ. However, as Kremer (1993) discusses if each person's chance of being lucky or smart enough to inventing something is independent of population, then the number of individuals working relative to total population will be important to determine the growth rate of invented goods in an economy. Moreover, Jones (2010) and Feyrer (2008) analyse the

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[12]As a way to ensure that the growth rate of new intermediate product is stationary, they also assume that the congestion effect depends positively on the scaling factor $\tilde{\Psi}_t$. Thus, everything else equal the marginal gain from R&D declines as the economy evolves.

age profile of inventors/innovators and show that young and middle-aged workers contribute the most to the pace of the innovation process. Finally, our estimation results also suggest that age groups of young/middle age workers (20-30, 30-40 and 40-50) contribute positively to patent applications while older workers (50-60) contribute negatively[13]. As such, innovation does not seem to be independent of the demographic structure and particularly the proportion of young and middle aged workers seems to correlate positively with innovation.

In order to incorporate the importance of the ratio of workers in the innovation process we assume the productivity of innovation is given by $\varphi_t \equiv (\Gamma_t^{yw})^{\rho_{yw}} \chi Z_t [\tilde{\Psi}_t^\rho (S_t)^{1-\rho}]^{-1}$, where Γ_t^{yw} is a measure of the stock of workers relative to the rest of the population and ρ_{yw} controls the importance of workers to the aggregate productivity of innovation. If $\rho_{yw} = 0$, the innovation process is equivalent to the one assumed in Comin and Gertler (2006). We present the definition of Γ_t^{yw} when we discuss the population dynamics below.

Based on that the flow of the stock of invented products (33a) now becomes

$$Z_{t+1}^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi Z_t [(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} S_t^p + \phi Z_t^p, \tag{8}$$

We assume that innovators borrow S_t^p from the financial intermediary. Define J_t as the value of an invented intermediary good. Then, innovator p will invest S_t^p until the marginal cost equates the expected gain. Thus,

$$\phi E[J_{t+1}] = \frac{R_{t+1}}{\varphi_t}. \tag{9}$$

Where R_{t+1} is the interest rate. The realised profits of an innovator is

$$\Pi_t^{RD} = \phi J_t (Z_t^p - \phi Z_{t-1}^p) - S_{t-1} R_t. \tag{10}$$

Adoption

Let $A_t^q \subset Z_t^q$ denoted the stock of converted goods ready to be marketed to firms. Adopters (q) obtain the rights of technology from innovators and make an investment expenditure (intensity) of Ξ_t to transform Z_t^q into A_t^q. This conversion process is successful with probability λ_t. We assume $\lambda_t = \lambda \left(\frac{A_t^q}{\tilde{\Psi}_t} \Xi_t \right)$ and $\lambda'(\cdot) > 0$, thus more intensity yields more adoptions. If unsuccessful the good remain in its invented form (prototype). A converted good can be marketed at every period to firms, thus its value, denoted V_t is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}, \tag{11}$$

where $\Pi_{m,t}$ is the profit from selling an intermediate good to input firms. We can now determine the value of a unadopted product (J_t). That is

$$J_t = \max_{\Xi_t} -\Xi_t + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}]. \tag{12}$$

---

[13]Liang, Wang, and Lazear (2014), although looking at entrepreneurship and not directly at R&D production shows that a high proportion of old workers prevents young workers gaining the necessary knowledge to start up a new business, thus reducing entrepreneurship.

The stock of unadopted goods at period t is given by $(Z_t^q - A_t^q)$. Thus, the flow of adopted goods for an adopter q is

$$A_{t+1}^q = \lambda_t \phi(Z_t^q - A_t^q) + \phi A_t^q. \tag{13}$$

The expenditure in consumption goods of adopters, financed by borrowing, is given by

$$\Xi_t(Z_t^q - A_t^q). \tag{14}$$

That way the profit of an adopter q is

$$\Pi_t^A = \int_0^{A_t^q} \Pi_{m,t} - \phi J_t(Z_t^p - \phi Z_{t-1}^p) - R_t \Xi_{t-1}(Z_{t-1}^q - A_{t-1}^q). \tag{15}$$

5.3. Household Sector

There are a continuum of agents of mass N_t. Individuals are born as dependents (young) and remain so from period t to period $t+1$ with probability ω^y and become a worker otherwise. Workers (w) at time t remain so in period $t+1$ with probability ω^r and retire otherwise. Once retired (r) the individual survives from period t to $t+1$ with probability $\gamma_{t,t+1}$. Let N_t^r be the mass of retirees, N_t^w the the mass of workers, and N_t^y the the mass of young. Furthermore, we assume $\tilde{n}_{t,t+1}N_t^y$ dependents are born at period t. As a result, population dynamics are such that[14]

$$
\begin{align}
N_{t+1}^y &= \tilde{n}_{t,t+1}N_t^y + \omega^y N_t^y = (\tilde{n}_{t,t+1} + \omega^y)N_t^y = n_{t,t+1}N_t^y, \tag{16} \\
N_{t+1}^w &= (1-\omega^y)N_t^y + \omega^r N_t^w, \tag{17} \\
N_{t+1}^r &= (1-\omega^r)N_t^w + \gamma_{t,t+1}N_t^r, \tag{18} \\
&\quad \text{define } \zeta_t^r = N_t^r/N_t^w \text{ and } \zeta_t^y = N_t^y/N_t^w \text{ then} \tag{19} \\
n_{t,t+1} &= \frac{\zeta_{t+1}^y}{\zeta_t^y}(\omega^r + \zeta_t^y(1-\omega^y)) \tag{20} \\
\zeta_{t+1}^r &= ((1-\omega^r) + \gamma_{t,t+1}\zeta_t^r)(\omega^r + (1-\omega^y)\zeta_t^y)^{-1} \text{ and} \tag{21} \\
\frac{N_{t+1}}{N_t} &= n_{t,t+1}(1 + 1/\zeta_t^y + \zeta_t^r/\zeta_t^y)^{-1} + (\omega^r + (1-\omega^y)\zeta_t^y)(1 + \zeta_t^r + \zeta_t^y)^{-1} \\
&\quad + \left(\frac{1-\omega^r}{\zeta_t^r} + \gamma_{t,t+1}\right)(1 + 1/\zeta_t^r + \zeta_t^y/\zeta_t^r)^{-1}. \tag{22}
\end{align}
$$

We define the measure of the stock of workers (Γ_t^{yw}), which influence the innovation process, to be equal to

$$\Gamma_t^{yw} \equiv (1-\omega^y)\frac{N_t^y}{N_t} + (1-\lambda^y)\Gamma_{t-1}^{yw} = (1-\omega^y)\frac{\zeta_t^y}{1 + \zeta_t^y + \zeta_t^r} + (1-\lambda^y)\Gamma_{t-1}^{yw}, \tag{23}$$

where $0 < \lambda^y \leqslant 1$ denotes how much the previous stock of young that became workers before t are important for the measure of that stock at the current period. If $\lambda^y = 1$ the stock is made only of the ratio of young that just entered their working life and if $\lambda^y < 1$

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[14]Also note that $N_{t+1}^w = N_t^w(\omega^r + (1-\omega^y)\zeta_t^y)$ and $N_{t+1}^r = N_t^r\left(\frac{1-\omega^r}{\zeta_t^r} + \gamma_{t,t+1}\right)$

then at time t the stock of young is augmented by the ratio of young that entered in their working life at time $t - h$ with the decaying weight of $(1 - \lambda^y)^h$. As such, the stock of workers that contribute to innovation is particularly sensitive to the stock of young dependents that become workers (young workers) at each period, and less sensitive to more experienced workers, reflecting the empirical evidence (see Jones (2010) and Feyrer (2008)).

We assume the society ('social planner') collects transfers from workers that are then used to sustain the young and finance their educational investment. This expenditure will increase the effective labour units that will be supplied by the young when they become workers. In order to define the amount of investment in education at each period society determines the social cost of obtaining resources from current period workers, which decreases their consumption at t, and the benefits of higher effective labour supply, which leads to higher workers' consumption in the following periods. The 'social planner' then sets the educational investment to offset its marginal cost and benefit (see the Appendix: Theoretical Model for details). The young are thus passive in our model. Workers and retirees, on the other hand, decide their consumption to maximise welfare subject to a budget constraint.

As in Gertler (1999), we make two key assumptions to simplify the model. An individual faces two idiosyncratic risks during her lifetime: loss of wage income at retirement and time of death. The impact of uncertainty about time of death is eliminated by introducing a perfect annuity market allowing retirees to insure against this type of risk. That way, retirees turn their wealth over to perfectly competitive financial intermediaries which invest the proceeds and pays back a return of $R_t/\gamma_{t-1,t}$ for surviving retirees. The higher return than the market is financed by the asset holdings of retirees who did not survive.

The uncertainty about employment tenure is assume not to affect workers since they are risk-neutral. In order to also incorporate a motive for consumption smoothing we assume individual preferences belong to the recursive non-expected utility family. Thus, for $z = \{w, r\}$ we assume agent j selects consumption and asset holdings to maximise

$$V_t^{jz} = \left\{ (C^{jz})^{\rho_U} + \beta_{t,t+1}^z (E_t[V_{t+1}^j \mid z]^{\rho_U}) \right\}^{1/\rho_U} \tag{24}$$

subject to

$$C_t^{jz} + FA_{t+1}^{jz} = R_t^z FA_t^{jz} + W_t \xi_t^j I^z + d_t^z - \tau_t^{jz} I^z \tag{25}$$

where $\beta_{t,t+1}^z$ is the discount factor, which is equal to β for workers and $\beta\gamma_{t,t+1}$ for retirees, R_t^z is the return on assets, which is equal to R_t for workers and $R_t/\gamma_{t-1,t}$ for retirees, W_t is the wage, ξ_t^j is the effective unit of labour supplied by worker j, and I^z is an indicator function that takes the value one when $z = w$ and zero otherwise, thus we assume retiree do not work and workers' labour supply is fixed[15], FA_t^{jz} are the assets acquired from the financial intermediary and d_t^z is the dividend from the financial intermediary. Finally, τ_t^{jz} is the transfer a worker j makes to society for the expenditure on the young with the total transfer at time t given by $\tau_t = \int_0^{N_t^w} \tau_t^{jz}$.

Let ξ_t be the average effective units across workers at period t, or the current level of labour productivity or labour skill in the society. Each young who becomes a worker at the end of period t will provide ξ_{t+1}^y effective units. We assume

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[15]The framework can be extended to incorporate variable labour supply. See Gertler (1999) for details.

$$\xi_{t+1}^y = \rho_E \xi_t + \frac{\chi_E}{2}\left(\frac{I_t^y}{\xi_t}\right)^2 \xi_t \tag{26}$$

Where $\rho_E < 1$ and denotes the obsolescence of labour skills and I_t^y is the total effective expenditure on the young and is defined as the ratio between total funds and their labour cost.

$$I_t^y = \frac{\tau_t}{W_t N_t^w} \tag{27}$$

Based on the population dynamics we can now determine the evolution of workers effective labour units, that is

$$\begin{aligned}
\xi_{t+1} &= \omega_r \frac{N_t^w}{N_{t+1}^w}\xi_t + (1-\omega^y)\frac{N_t^y}{N_{t+1}^w}\xi_{t+1}^y \\
&= (\omega^r + (1-\omega^y)\zeta_t^y)^{-1}(\omega_r\xi_t + (1-\omega^y)\zeta_t^y\xi_{t+1}^y)
\end{aligned} \tag{28}$$

5.4. Financial Intermediary

The financial intermediary sells assets to the households (FA_t^w, FA_t^r), holds the capital (K_t) and rents it to firms and lends funds (B_{t+1}) to innovators and adopters to finance their expenditure (given by S_t and $\Xi_t(Z_t - A_t)$, respectively). Finally, we assume it owns the innovators and adopters enterprises, receiving their dividends at the end of the period. Thus, financial intermediary profits are

$$\Pi_t^F = [r_t^k + 1]K_t + R_t B_t - R_t(FA_t^w + FA_t^r) - K_{t+1} - B_{t+1} + FA_{t+1}^w + FA_{t+1}^r + \sum_x (\Pi_t^{RD} + \Pi_t^A), \tag{29}$$

where $B_{t+1} = S_t + \Xi_t(Z_t - A_t)$ and $FA_t = FA_t^w + FA_t^r$.

5.5. Equilibrium

The symmetric equilibrium is a sequence of endogenous predetermined variables $\{FA_{t+1}^z, K_{t+1}, A_{t+1}, Z_{t+1}, FA_{t+1}, B_{t+1}, \xi_{t+1}\}$ and a sequence of endogenous variables $\{C_t^z, H_t^w, T_t^w, d_t^z, D_t^z, K_{t+1}, L_t, Y_t, \Xi_t, \mu_t, N_t^f, S_t, V_t, J_t, \lambda_t, \Pi_t^{RD}, \Pi_t^A, Y_t, C_t, L_t, U_t, r_t^k, \delta_t, R_t, \Pi_t^F, W_t, P_t^M, \varepsilon_t, \tau_t, I_t^y, \varsigma_t\}$ for $z = \{w, r\}$ obtained such that:

a. Workers and retirees, maximize utility subject to their budget constraint and investment in education is such that society's marginal cost and benefit is equated;

b. Input and final firms maximize profits, and firm entry occurs until profits are equal to operating costs;

c. Innovators and adopters maximise their gains;

d. The financial intermediary selects assets to maximize profits, and their profits are shared amongst retirees and workers according to their share of assets;

e. Consumption goods, capital, labour and asset markets clear;

given the initial values of all the predetermined variables $\{FA_t^z, K_t, A_t, Z_t, \xi_t, FA_t, B_t\}$ and given the sequence of exogenous predetermined variables $\{N_t^y, N_t^w, N_t^r, N_t, \zeta_t^y, \zeta_t^r\}$ specified by the population dynamics, stock of young workers and effective labour unit evolution conditions, given below.

$$N_{t+1}^y = n_{t,t+1} N_t^y, \tag{30a}$$

$$N_{t+1}^w = N_t^w(\omega^r + (1 - \omega^y)\zeta_t^y) \tag{30b}$$

$$N_{t+1}^r = N_t^r \left(\frac{1 - \omega^r}{\zeta_t^r} + \gamma_{t,t+1} \right) \tag{30c}$$

$$n_{t,t+1} = \frac{\zeta_{t+1}^y}{\zeta_t^y} (\omega^r + \zeta_t^y(1 - \omega^y)) \tag{30d}$$

$$\zeta_{t+1}^r = ((1 - \omega^r) + \gamma_{t,t+1}\zeta_t^r)(\omega^r + (1 - \omega^y)\zeta_t^y)^{-1} \text{ and} \tag{30e}$$

$$\frac{N_{t+1}}{N_t} = n_{t,t+1}(1 + 1/\zeta_t^y + \zeta_t^r/\zeta_t^y)^{-1} + (\omega^r + (1 - \omega^y)\zeta_t^y)(1 + \zeta_t^r + \zeta_t^y)^{-1}$$

$$+ \left(\frac{1 - \omega^r}{\zeta_t^r} + \gamma_{t,t+1} \right)(1 + 1/\zeta_t^r + \zeta_t^y/\zeta_t^r)^{-1} \tag{30f}$$

$$\Gamma_t^{yw} = (1 - \omega^y)\frac{\zeta_t^y}{1 + \zeta_t^y + \zeta_t^r} + (1 - \lambda^y)\Gamma_{t-1}^{yw} \tag{30g}$$

$$\xi_{t+1} = (\omega^r + (1 - \omega^y)\zeta_t^y)^{-1}(\omega^r\xi_t + (1 - \omega^y)\zeta_t^y\xi_{t+1}^y) \tag{30h}$$

$$\xi_{t+1}^y = \rho_E\xi_t + \frac{\chi_E}{2}\left(\frac{I_t^y}{\xi_t} \right)^2 \xi_t. \tag{30i}$$

The equilibrium conditions that ensure *a.* are:

$$H_t^w = W_t\xi_t L_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}}H_{t+1}^w\frac{N_t^w}{N_{t+1}^w} \tag{31a}$$

$$T_t^w = \tau_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}}T_{t+1}^w\frac{N_t^w}{N_{t+1}^w} \tag{31b}$$

$$D_t^w = d_t^w + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}}D_{t+1}^w\frac{N_t^w}{N_{t+1}^w} + \frac{(1 - \omega^r)\varepsilon_{t+1}^{(\rho_U - 1)/\rho_U}}{R_{t+1}\mathfrak{Z}_{t,t+1}}D_{t+1}^r\frac{N_t^w}{N_{t+1}^r} \tag{31c}$$

$$D_t^r = d_t^r + \frac{\gamma_{t,t+1}}{R_{t+1}}D_{t+1}^r\frac{N_t^r}{N_{t+1}^r} \tag{31d}$$

$$C_t^w = \varsigma_t[R_tFA_t^w + H_t^w + D_t^w - T_t^w] \tag{31e}$$

$$C_t^r = \varepsilon_t\varsigma_t[R_tFA_t^r + D_t^r] \tag{31f}$$

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}}\frac{(\beta R_{t+1}\mathfrak{Z}_{t+1})^{1/(1-\rho_U)}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \tag{31g}$$

$$1 - \varepsilon_t\varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\gamma_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}} \tag{31h}$$

$$\tau_t = W_tN_t^wI_t^y \tag{31i}$$

$$\varsigma_t^{-1/\rho_U} = \varsigma_{t+1}^{-1/\rho_U}\beta(1 - \omega^y)\zeta_t^y\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{31j}$$

where $\mathfrak{Z}_{t+1} = \omega^r + (1 - \omega^r)\varepsilon_{t+1}^{(\rho_U - 1)/\rho_U}$, H_t^w is the present value of gains from human capital, T_t^w is the present value of transfers, D_t^z is the present value of dividends for $z = \{w, r\}$, ς_t the marginal propensity of consumption of workers and $\varepsilon_t \varsigma_t$ the one for retirees. The first four equations define the value of the stock of human capital, the present value of transfers and the present value of the profits of financial intermediaries. Following that we have the two consumption rules and the dynamics of the marginal propensities of consumption. Finally, the last two conditions determine total transfers and investment in labour skills.

The equilibrium conditions that ensure $b.$ are:

$$(1 - \alpha)(1 - \gamma_I)Y_{c,t} = \mu_t W_t \xi_t L_t \tag{32a}$$

$$\alpha(1 - \gamma_I)Y_{c,t} = \mu_t[r_t^k + \delta_t]K_t \tag{32b}$$

$$\alpha(1 - \gamma_I)Y_{c,t} = \mu_t \delta_t'(U_t)K_t U_t \tag{32c}$$

$$\mu_t M_t P_t^M = \gamma_I Y_{c,t} \tag{32d}$$

$$P_t^M = \vartheta A_t^{1-\vartheta} \tag{32e}$$

$$Y_{c,t} = (N_t^f)^{\mu_t - 1} \left[(U_t \frac{K_t}{\xi_t L_t})^\alpha (\xi_t L_t) \right]^{(1-\gamma_I)} [M_t]^{\gamma_I} \tag{32f}$$

$$\frac{\mu_t - 1}{\mu_t} Y_{c,t}(N_t^f)^{-\mu_t} = \Omega \tilde{\Psi}_t \tag{32g}$$

$$\mu_t = \mu(N_t^f) \tag{32h}$$

$$\delta_t = \delta(U_t) \tag{32i}$$

The first three equations jointly determine the equilibrium wage, the rent of capital and the utilisation rate. The following two equations determine the intermediate good composite and their price. The final four equations determine output, the number of firms (through entry condition), the mark-up and the depreciation rate.

The equilibrium conditions that ensure $c.$ are:

$$\frac{Z_{t+1}}{Z_t} = (\Gamma_t^{yw})^{\rho_{yw}} \chi \left(\frac{S_t}{\tilde{\Psi}_t} \right)^\rho + \phi \frac{Z_{t+1}}{Z_t} = \chi \left(\frac{S_t}{\tilde{\Psi}_t} \right)^\rho + \phi \tag{33a}$$

$$\frac{A_{t+1}}{A_t} = \lambda \left(\frac{A_t \Xi_t}{\tilde{\Psi}_t} \right) \phi[Z_t/A_t - 1] + \phi \tag{33b}$$

$$S_t = R_{t+1}^{-1} \phi E_t J_{t+1}(Z_{t+1} - \phi Z_t) \tag{33c}$$

$$\Xi_t = \epsilon_\lambda \lambda_t R_{t+1}^{-1} \phi[V_{t+1} - J_{t+1}] \tag{33d}$$

$$J_t = -\Xi_t + (R_{t+1})^{-1} \phi E_t[\lambda_t V_{t+1} + (1 - \lambda_t)J_{t+1}] \tag{33e}$$

$$V_t = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t A_t} + (R_{t+1})^{-1} \phi E_t V_{t+1} \tag{33f}$$

$$\lambda_t = \lambda \left(\frac{A_t \Xi_t}{\tilde{\Psi}_t} \right) \tag{33g}$$

$$\Pi_t^{RD} = \phi J_t(Z_t - \phi Z_{t-1}) - S_{t-1}R_t \tag{33h}$$

$$\Pi_t^A = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t} - \phi J_t(Z_t - \phi Z_{t-1}) - \Xi_{t-1}(Z_{t-1} - A_{t-1})R_t \tag{33i}$$

The first two equations determine the stock of invented and adopted goods. The third equation determines the intensity of innovation efforts while the last six jointly determine the expenditure on adoption, its probability of success, the value of an invented and an adopted good, and finally the profits of inventors and adopters.

The equilibrium conditions that ensure d. are:

$$E_t[r_{t+1}^k + 1] = R_{t+1} \tag{34a}$$

$$d_t^z = \Pi_t^F \frac{FA_t^z}{FA_t} \text{ for } z = r, w \tag{34b}$$

$$\Pi_t^F = [r_t^k + 1]K_t + R_t B_t - R_t(FA_t) - K_{t+1} - B_{t+1} + FA_{t+1} + \Pi_t^{RD} + \Pi_t^A \tag{34c}$$

$$B_{t+1} = S_t + \Xi_t(Z_t - A_t) \tag{34d}$$

The first equation is the arbitrage condition, the second determines how profits are shared across household types and the last two determine profits and total loans.

The equilibrium conditions that ensure e. are:

$$L_t = N_t^w \tag{35a}$$

$$K_{t+1} = K_t(1 - \delta(U_t)) + I_t \tag{35b}$$

$$Y_t = Y_{c,t} - A_t^{1-\vartheta}M_t - \Omega\tilde{\Psi}_t \tag{35c}$$

$$Y_t = C_t + I_t + S_t + \Xi_t(Z_t - A_t) + \tau_t \tag{35d}$$

$$C_t = C_t^w + C_t^r \tag{35e}$$

$$FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \tag{35f}$$

$$FA_{t+1}^r = R_t FA_t^r + d_t^r - C_t^r + (1 - \omega^r)(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{35g}$$

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r \tag{35h}$$

The first equation equates labour supply and demand and the second gives the dynamics of the capital stock, the following two define added value output from supply and demand sides. The condition that ensures aggregate consumption is a sum of consumption across household types follows. Finally, asset market flows and clearing condition are given. Also note that $FA_{t+1}^w = \omega^r(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t)$. Details of all equilibrium conditions are provided in Appendix A.

Finally, we must define $\tilde{\Psi}_t$ such that a balance growth path obtains. Comin and Gertler (2006) selects the current value of capital stock. Given that in their model the price of capital is determined at time t, $\tilde{\Psi}_t$ fluctuates accordingly ensuring uniqueness. Given that we simplify our model to consider only one sector, the price of capital is constant and the value of the capital stock is also constant at t, invalidating this choice of scaling factor. We thus select the current value of adopted goods as our scaling factor. Thus,

$$\tilde{\Psi}_t \equiv V_t A_t \tag{36}$$

5.6. Calibration and Steady State

All quantity variables of our model grow as a result of three main drivers, the exogenously given rate of growth of population (n), the growth of the effective labour force (ξ_t) and due to the endogenous process of invention and adoption of new intermediate

goods (A_t), which increases the productivity of the other factors of production (capital and labour). It is convenient therefore to normalise certain variables relative to final goods output (which is used as the *numeraire*), obtaining then a system of equations that provide a stationary steady state given the set of parameters. The de-trended system of equations is shown in the appendix, which the definition of the new variables (all in lower case) all depicted (e.g. for aggregate consumption we have $c = \frac{C}{Y_{c,t}}$).

We now discuss the parameters values selected to simulate our model economy. The standard parameters present in most macro models are shown first. Given our emphasis on medium-run dynamics, one period in the model is set to one year. We thus set the discount factor β equal to 0.96. Capital share (α) as usual is set to 0.33. We set depreciation (δ) to 0.08, capital utilisation (U) to 80% and the elasticity of the change in the depreciation rate with respect to utilisation to 0.33. The share of intermediate goods (γ_I) is set to 0.5 (all those parameters choices are in line with Comin and Gertler (2006)). Based on evidence in Basu and Fernald (1997) we set mark-up in the consumption sector (μ) to 1.1. Finally, following Gertler (1999) we set the elasticity of substitution ($1/(1 - \rho_U)$) equal to 0.25.

We next come to the parameters that govern the innovation process. We follow Comin and Gertler (2006) closely. We set obsolescence and productivity in innovation such that growth rate of output per working age person is 0.024 and share of research expenditures in total GDP is 0.012[16]. That way, $\phi = 0.97$, $\chi = 94.42$. The mark-up for specialised intermediate goods is set to 1.6. The elasticity of intermediate goods with respect to R&D (ρ) is set to 0.9. Average adoption time is set to 10 years thus $\lambda = 0.1$. The elasticity of this rate to increasing intensity (ϵ_λ) is set to 0.9. The price mark-up elasticity to entry (ϵ_μ) is set to 1.

Finally, we set the parameters that govern population dynamics. We initially assume individuals are young on average from age 0 till 20, thus setting probability of becoming a worker ($1 - \omega^y$) equal to 0.05. Individuals work from age 21 to 65, thus setting the probability of retirement ($1-\omega^r$) equal to 0.023, and then live in retirement on average from 66 until 75, thus setting γ equal to 0.9. That implies the ratio of young to workers is 48%, the ratio of retirees to workers is 20% and retirees hold around 16% of the assets. Finally, assume workers remain part of the pool that influences invention with probability $1 - \lambda_y = 2/3$ and that $\rho_{yw} = 0.9$. These two last parameters directly link demographic structure and innovation, hence we verify how their variation affects our main results.

6. Results

We perform three sets of simulations to assess the impact of different demographic structures on the medium-run macroeconomy dynamics. The first simulation titled *baby-boomers* analyses the effect of increasing fertility holding longevity constant. The second set of simulations, titled *aging* looks at the effects of increasing longevity by increasing

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[16]Note that as opposed to here, in Comin and Gertler (2006) there are two sectors. Thus to obtain our measure we combine the total expenditure in both sectors in their calibration.

γ and firstly leaving population growth constant (hence fertility must reduce otherwise population naturally increases) and secondly holding fertility constant and thus allowing population to grow during the adjustment process. Finally, the third set of simulations, titled *prediction*, attempt to match the change in the demographic structure predicted for a selected number of countries in our sample during the next two decades and measure their impact on growth and real interest rates.

Simulation: Baby-boomers

In the first simulation results, presented in Figure 3, we analyse the effect of increasing fertility[17] for the first 10 periods, reducing back to the benchmark level after that. We can then analyse how the changes in age structure affect the economy through time, first with an increase in dependents, then an increase in workers and finally retirees. Initially the increase in fertility leads to a decrease in growth and investment. A high proportion of dependents is a cost to society, reducing the resources available for workers, and thus reducing savings and investment. Moreover, current workers also expect the growth rate to increase in the future when those youngsters join the labour force and accordingly increase their marginal propensity to consume, reducing savings further. As a result, during the fertility boom period, technological gains (g^A) and output growth are below their steady state level. The model therefore matches well the empirical results that show that 0-20 share of population has a negative impact on investment, savings and output growth.

As youngsters become workers (note this happens at every period in the model since a proportion of ω_y dependents become workers) and fertility decreases the share of youngsters decrease (see period 10 to 20) while the share of workers increase (thus, the share of retirees decreases). Society is then benefiting from the demographic dividends of the previous increase in fertility. As the proportion of young workers increase (Γ_{yw}), innovation increases and the growth rate of technology (or varieties) increases sharply, peaking 25 to 30 years after the fertility burst. This increase in growth is accompanied by both, an increase in investment and consumption. Finally, workers marginal propensity to consume continues to increase, leading to higher real interest rates. Hence, the increasing share of workers leads to higher growth, investment and real rates, matching the empirical estimates. Finally, as the proportion of dependents does not change significantly (30 to 40 years after the increase in fertility) and the proportion of workers decrease (thus the stock of young workers is reduced), innovation, technological gains and output growth decrease. At this point the share of retirees is increasing, consumption of retirees (who benefited from greater asset accumulation during the higher growth period) also increases. Contrary to retirees, workers are forced to increase their savings relative to the previous generation reducing real rates. Lower investment and innovation implies that as the share of retirees increase in the finals stages of the adjustment output growth rates deviations (relative to the steady state level) become slightly negative. Overall the model matches well the main empirical findings.

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[17]Instead of shocking fertility directly, we alter the replacement rate, which we obtain by calculating the ratio between total birth ($\tilde{n}N_y$) and the proportion of childbearing women in the economy. This proportion is given by $20 * 0.4 * \frac{N_t^w \times 45}{20}$, assuming 40% of workers are woman that bear child and childbearing years are the first 20 years of workers life.
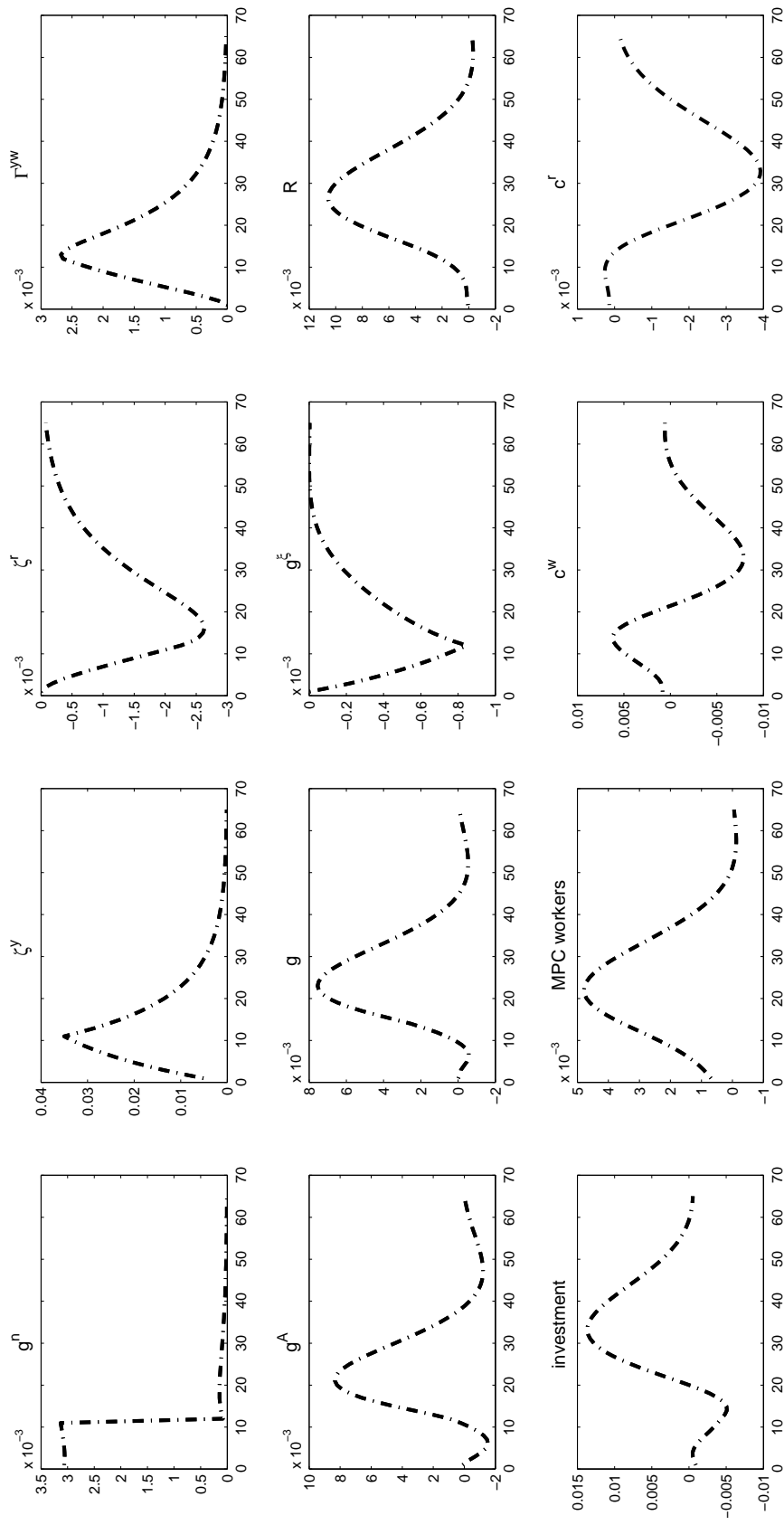
Figure 3: Simulation: *baby-boomers*

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Simulation: Aging

Most economies during the period used in our estimation experienced a constant increase in life expectancy. That has resulted in a significant increase in the share of retirees in the population. In this set of simulations we smoothly increase the parameter γ such that the average retiree lives an additional 10 years, increasing societies' average age.[18] We considered two cases. The first holds population growth (g^n) constant. As longevity increases, ceteris paribus, population also increases. Thus, in order to keep population constant, fertility must decrease during the adjustment process. Note that in our estimations demographic structure matters although we controlled for population growth, hence, by keeping population growth constant this simulation allows us to analyse the impact of shifting demographic structures due to aging as in the estimation. In the second case, we increase longevity but keep fertility constant, thus in the second case, although population is growing we also obtain a shift in demographic structure such that share of retirees in the population increases. Results are displayed in Figure 4.

This set of simulations allow us to highlight the three main mechanisms through which demography impacts the economy. First, as longevity increases current workers are expected to live longer and thus have to accordingly adjust their savings, increasing asset accumulation during their working life. Workers consumption therefore falls leading to a decrease in real rates. Those additional funds are allocated to investment in capital and innovation. Capital accumulation and technological gains increase, pushing the growth rate of output up. Therefore, life cycle consumption adjustment, our first mechanism, leads to an increase in growth rates. Note that our model cannot generate a paradox of thrift such that greater desire to save decrease aggregate demand sufficiently to reduce resources such that no additional savings is done. As a result additional resources always flow to the innovation sector increasing growth. Altering the aggregate demand features of the model may generate stronger negative effects on growth due to lower consumption. A second aspect of aggregate demand that is left out of our model which may also alter this mechanism is consumption demand composition. Aguiar and Hurst (2013) show that there is substantial heterogeneity across consumption goods over the life cycle profile with respect to the mean and the cross-household variance in expenditures. They also show that the decline in nondurable expenditure after the age of 45 is mainly due to a reduction in food, nondurable transportation, and clothing/personal care categories. Both of these aggregate demand factors may decrease the positive response of growth we obtain in our model due life-cycle consumption adjustments.

The second mechanism occurs through the adjustment of human capital accumulation due to the decrease in fertility (that only materialises when aging occurs under constant population growth - in this case the ratio of dependants to workers decrease). As workers must increase saving for retirement, the total investment in the education of young decreases. However, as the ratio of dependants decrease, the per capita investment in education increases, leading to a growth in human capital (g^ξ). As expected that pushes the growth rate up. When fertility is kept constant, the decrease in workers resources lead to a small decrease in the growth of human capital.

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[18]We set $\Delta\gamma_t = 0.9\Delta\gamma_{t-1} + 0.005$, thus γ increases at a decreasing rate from 0.9 to 0.95 in roughly 50 years.
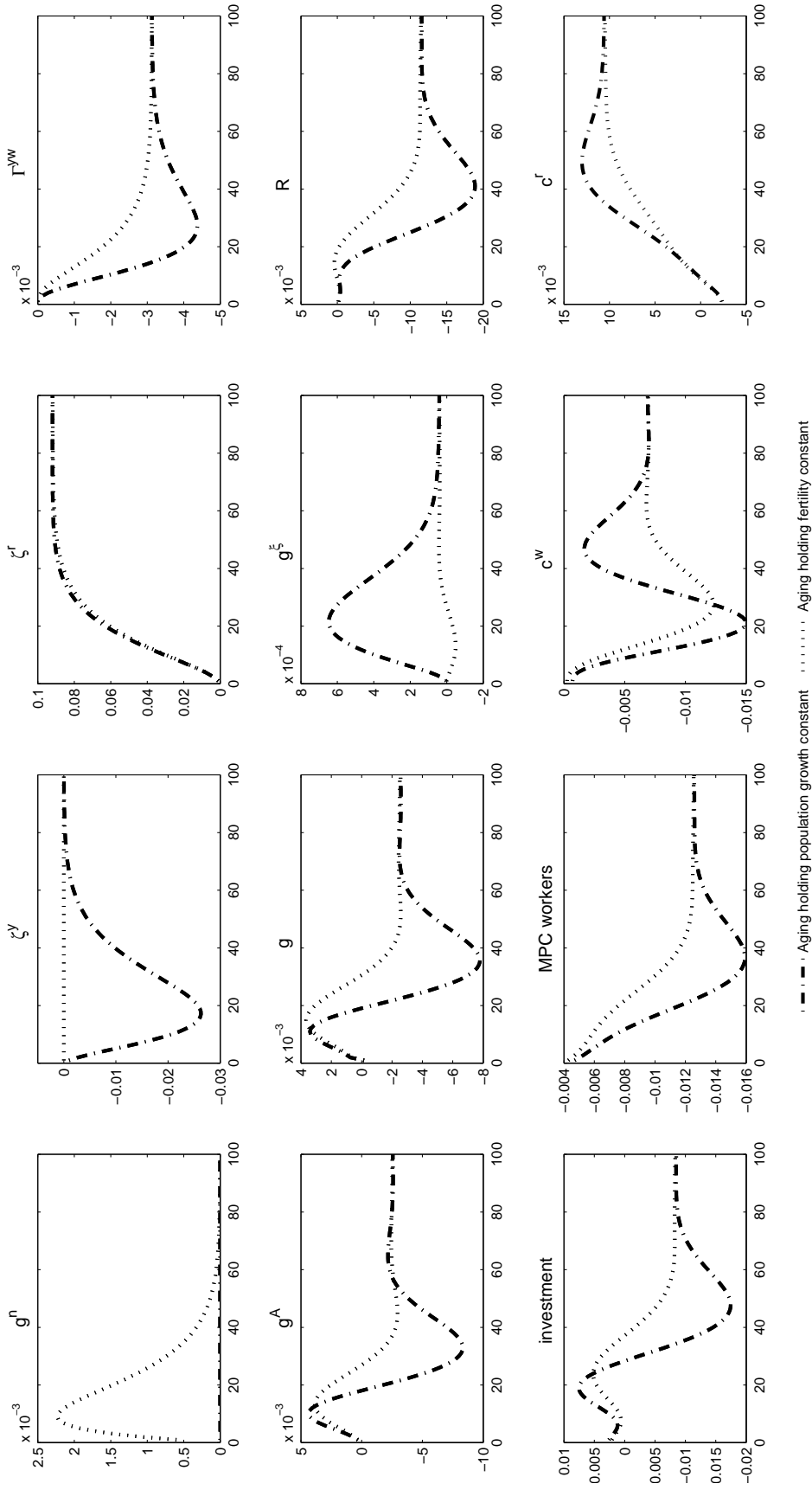
Figure 4: Simulation: *aging*

Finally, the third mechanism goes through the invention process. As our estimation results point out, as well as results from the literature of demographics and productivity and innovation (see Jones (2010) and Feyrer (2008)) and demographics and entrepreneurship (see Liang, Wang, and Lazear (2014)), young and middle age workers are relatively more important in the innovation process relative to other age groups. We account for that feature by assuming the stock of young workers relative to the total population impacts the productivity of the innovation process. Due to the aging of society this relative stock decreases, leading to a lower rate of invention and technological gains. This process is particularly strong when longevity is coupled with decreased fertility as we observed in most of the economies in the OECD.[19] If we shutdown this mechanism by setting $\rho_{yw} = 0$ thus productivity of innovation investment is independent of the demographic structure, growth rates in long-run increase relative to its steady state level. Therefore, the life-cycle consumption channel (when the paradox of thrift is not present), by increasing the saving rate, lowering the real rates, and thus increasing investment in innovation, have a permanent and positive impact on the growth rate of technology (the results of the simulation setting $\rho_{yw} = 0$ and $\rho_{yw} = 0, 5$ (recall that $\rho_{yw} = 0, 9$ in the benchmark model) are presented in the Appendix).

Note that the first mechanism, occurring through adjustments in consumption and savings as a result of life cycle changes is strongly supported by our estimations. Population aging has been found to impact negatively interest rates. Our model indicates that this movement in interest rates is a result of workers lowering their marginal propensity to consume. Second, the third mechanism is also supported by our estimation results. Aging leads to lower patent application and thus to potentially lower contribution of innovation to growth. Moreover, as modelled here, this positive association between innovation and growth is stronger for populations with a relatively younger worker population. Therefore, our theoretical model matches well the macroeconomic impacts of demographic changes but also incorporates the main channels that our empirical results give support to.

The Appendix depicts two additional robustness analysis. The first one alters λ_y, which determines the persistence of the effects of the stock of workers on innovation. Increasing λ_y decreases the amplitude of the fluctuations of the demographic changes but the main qualitative results are unchanged. The second alters the flow of the stock of workers. In the benchmark case, all youngsters who become workers influence innovation in the current period. In the alternative specification we consider that youngster who became workers 10 years before influence innovation in the current period (implicitly the alternative specification assumes innovation activity of an individual would peak after she acquires some work experience). In this specification $\Gamma_t^{yw} \equiv (1 - \omega^y) \frac{N_{t-10}^y}{N_t} + (1 - \lambda^y) \Gamma_{t-1}^{yw}$. The simulation of the alternative specification shows a smaller response for the first 10 years, with a similar shaped response relative to the benchmark case occurring after that. Essentially, the macroeconomic effects of the demographic changes are delayed due to the assumed delayed effect of those on the innovation efforts.

Simulation: Prediction

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[19]Note that in the long-run, fertility is equal to its steady state level in both cases, only γ changes permanently. Thus, in both simulations in Figure 4, long-run growth decreases by the same magnitude.

Period	Δs_w	Δs_r	g^n
2000-2005	0.5%	0.5%	1.053
2005-2011	-1.3%	2.0%	1.056
2011-2016	-1.4%	1.9%	1.043
2016-2021	-2.1%	2.2%	1.040
2021-2026	-1.3%	1.7%	1.037
2026-2031	-0.3%	0.8%	1.033

Table 9: *Prediction* Data Input: United States

In the final set of simulations we employ our model to analyse the effect of the predicted changes in the demographic structure on output growth and real interest rates for the next two decades in a subset of the countries in our sample, matching the prediction exercise done with the estimated model. We start by selecting three measures of expected population dynamics to feed into the model. The first is expected population growth (g^n). The second is the percentage point change in the share of workers (following our empirical results we calculate that by obtaining the combined population with ages between 20 and 60 years old and dividing it to total population, denoting it Δs_w) and finally the third is the share of retirees (population with ages 60 and over divided by total population, denoted Δs_r). In order to match these three measures $\{g^n, \Delta s_w, \Delta s_r\}$[20] we implicitly select three structural parameters, the fertility rate $\tilde{n}$, the longevity parameter γ and the probability a dependent become a worker ω_y.

As in the estimation exercise we use actual population data from 2000 till 2010 and United Nations predictions from 2011 till 2031. In the prediction exercise in the empirical section we use the long-run estimates to obtain the impact of demographic structure on to the main macroeconomic variables. As such we select the average change of our three empirical measures of population dynamics for 5 year intervals such that some degree of endogenous feedback due to changes in demographic variables are capture in the theoretical simulation. As an example Table 9 shows the population dynamic measures we use for the six subperiods from 2000 till 2031 for the US.[21] That implies an agent in the U.S. at time t = 2000 gets to know that the yearly changes in population dynamics for the period 2005-20011 will be such that in those five years population will growth 5.6 percent, the ratio of workers will decrease by 1.3 percentage points and the ratio of retirees will increase by 2 percentage points. We do not calibrate the steady state of the model to match any of the countries in the sample - for all countries the initial point is the steady state of our model as discussed in the calibration section. Hence, we only focus on how the predicted changes in demographic structure and population growth impact the changes (or deviations from steady state) of the macroeconomic variable in the model.

[20]The share of workers in the population is given by $\frac{1}{1+\zeta_y+\zeta_r}$ and the share of retirees is given by $\frac{\zeta_r}{1+\zeta_y+\zeta_r}$, by setting those shares we are essentially selecting ζ_y and ζ_r, the young and retirees dependency ratios.
[21]In order to use the first period of the prediction (2011) we stretch one subperiod to 6 years (2005-2011).
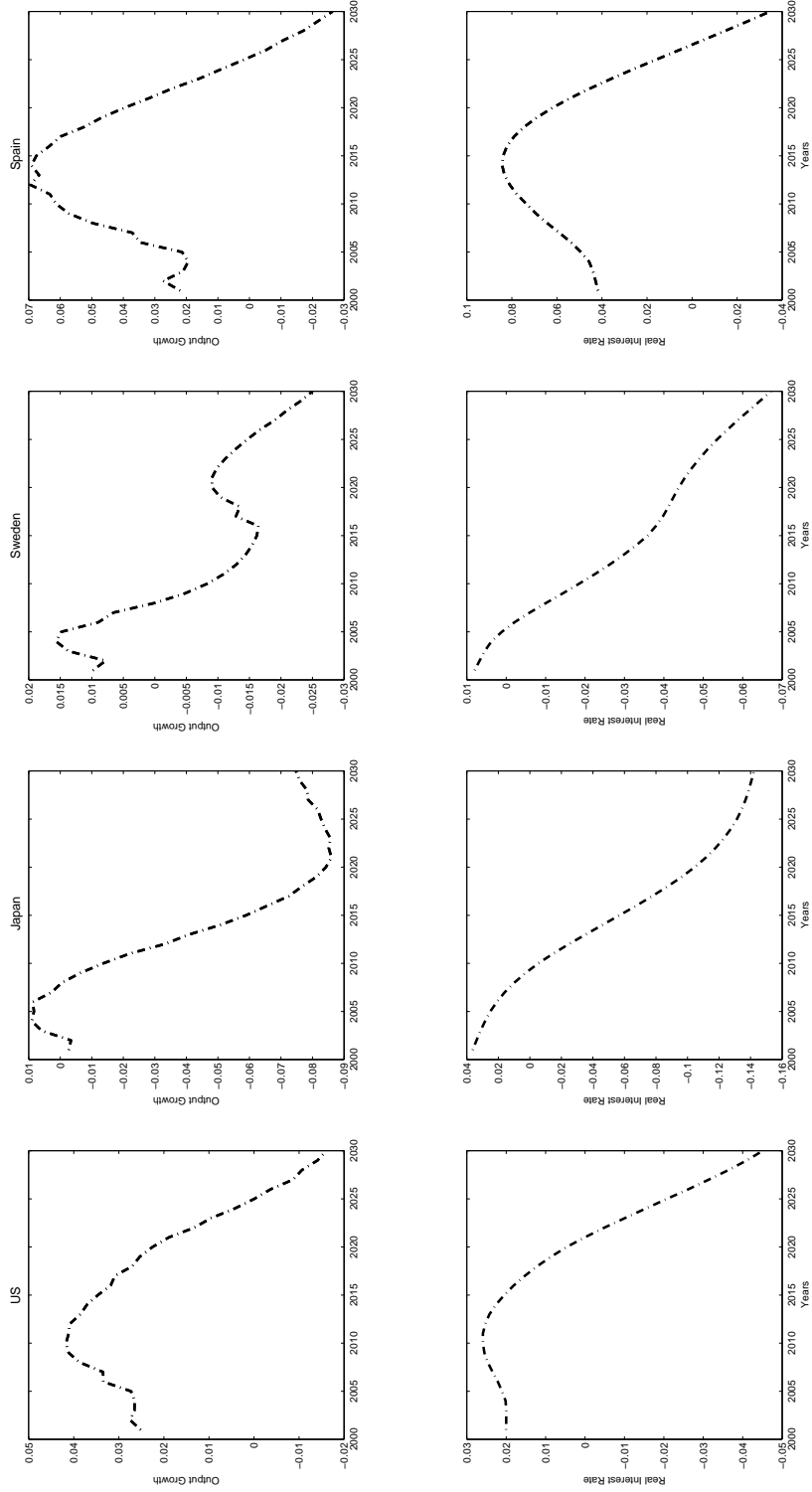
Figure 5: Simulation: *prediction*

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Figure 13 shows the results for U.S., Japan, Sweden and Spain, matching Figure 2 in section 3. Our model does a fairly good job in matching the predicted path of real rates and output growth for the countries in our sample (in the Appendix we show the estimation and theoretical simulation based on the UN predictions for four additional countries). The model does particularly well in matching the drop in real rates and growth expected for most of the countries during the 2010-2030 period, which occurs due to increase in aging and the drop in labour force as fertility is reduced. Moreover, when fertility is expected to increase, offsetting the impact of aging and leading to a recovery in output, as in the case of Sweden in the next decade, the model is also able to capture the reversal in the trend.

However, demographic structure within the working population may also change, resulting in a relatively older labour force and generally leading to a drop in trend output growth. This is in fact the case for many countries in the first decade of this century. Our model, due to the simplification of including only three age groups, cannot capture such effects. This lack of more detailed age profile is the reason the theoretical model still predicts an increase in the trend growth at the beginning of the 2000's while the empirical counterpart shows a decrease in the trend (this discrepancy is most significant for Spain, but also occurs for the US, Canada and France for instance). Nonetheless, when the aging process affects the 60+ share, the theoretical model captures its effect and delivers a drop in trend output growth. Finally, we observe that the model delivers stronger effects on output and interest rates relative to the estimation results. We therefore run another set of simulation reducing ρ_{yw} from 0.9 to 0.5. Results are shown in the Appendix. The qualitative impact of demographic changes remain the same but the drop in output and real rates are smaller and closer to the empirical effects estimated.

7. Conclusions

We start by presenting a parsimonious econometric model that aims to capture the impact of the demographic changes that currently affect nearly all developed economies on key macroeconomic variables of interest. The use of a panel VAR in six main macroeconomic variables, for 20 OECD countries over the period 1970-2007 allows us to obtain estimates of the long-run impact of demographic structure on the economy. Our results indicate that the age profile of the population has both economically and statistically significant impacts on output growth, investment, savings, hours worked, real interest rates and inflation. The magnitude of the long term impact is large. Demographic factors are predicted to depress average annual GDP growth over the current decade, 2010-2019, by 0.86% in our sample of OECD countries, with the strongest predicted negative impact in the US at 1.22%. We also provide evidence of the link between demographic structure and innovation activity. We find that patent application is positively affected by young and middle aged cohorts and negatively affected by dependants and retirees. We generally find our empirical results to be robust to time effects and exclusion of individual countries.

Based on the empirical findings and the importance of considering the effects of demographic changes after all interactions between macroeconomic variables are allowed for, including their effect on innovation, we develop a theoretical model that incorporates life cycle properties and endogenous productivity. Our model highlights three main channels by which demographics affects the macroeconomy: i) through life cycle consumption

decisions, ii) through incentives that alter human capital accumulation process and iii) through the influence of young workers on the innovation process. Our model is able to replicate most of our empirical findings, with the third channel being particular important to generate reduced long-term output growth due to aging. Our empirical and theoretical results indicate that the current trend of population aging and reduced fertility, expected to continue in the next decades, may contribute to reduced output growth and real interest rates across OECD economies.

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Appendix: Data

This provides a description of the data used in the empirical study.

- World Population Prospects: The 2010 Revision File 1A; Total population (both sexes combined) by five-year age group, major area, region and country, annually for 1950-2010 (thousands): United Nations, Population Division.

- Residential Patent Applications (annual): World Bank (2014), World Development Indicators.

- Trademark Applications (annual): World Bank (2014), World Development Indicators.

- Central Bank Discount Rates (annual): International Financial Statistics/IMF.

- Consumer Price Index (annual): International Financial Statistics/IMF.

- Households Savings Rate (annual): National Accounts, OECD.

- Hours worked (annual): Productivity Statistics, OECD.

- Gross Domestic Product (annual): National Accounts, OECD.

- Gross Fixed Capital Formation (annual): National Accounts, OECD.

- GDP per capita (annual): Penn World Tables.

Appendix: Estimation

This Appendix provides additional results on the estimations discussed in the main body of the paper.

Benchmark Panel VAR

	g	I	S	H	rr	π
g	1.00	0.55	0.44	0.45	-0.13	0.27
I	0.55	1.00	0.02	0.31	-0.10	0.19
S	0.44	0.02	1.00	0.26	-0.06	0.05
H	0.45	0.31	0.26	1.00	-0.02	0.15
rr	-0.13	-0.10	-0.06	-0.02	1.00	-0.77
π	0.27	0.19	0.05	0.15	-0.77	1.00

Table 10: Residual Correlation Matrix - Benchmark

	Estimate	Std. Error		Estimate	Std. Error		Estimate	Std. Error	
g_{t-1}	0.25	0.05	*	0.13	0.03	*	-0.07	0.07	
I_{t-1}	-0.27	0.12	*	0.92	0.06	*	0.06	0.09	
S_{t-1}	0.09	0.06		0.06	0.03		0.99	0.05	*
H_{t-1}	0.07	0.06		-0.00	0.03		0.03	0.04	
rr_{t-1}	-0.22	0.07	*	-0.09	0.02	*	-0.06	0.04	
π_{t-1}	-0.28	0.07	*	-0.08	0.02	*	-0.05	0.03	
g_{t-2}	-0.01	0.04		0.04	0.02		-0.05	0.04	
I_{t-2}	0.09	0.11		-0.16	0.05	*	-0.16	0.08	*
S_{t-2}	-0.07	0.06		-0.05	0.03		-0.21	0.06	*
H_{t-2}	-0.08	0.07		0.02	0.03		-0.04	0.04	
rr_{t-2}	-0.04	0.06		-0.01	0.02		-0.04	0.04	
π_{t-2}	-0.00	0.04		-0.02	0.01		-0.03	0.03	
$POIL_{t-1}$	-0.02	0.00	*	0.00	0.00		-0.01	0.00	*
$POIL_{t-2}$	0.02	0.00	*	0.00	0.00		0.00	0.00	
$popGrowth$	2.74	1.06	*	0.51	0.50		1.58	0.74	*
$popGrowth_{t-1}$	-2.22	0.99	*	0.29	0.50		-1.17	0.79	
δ_1	-0.06	0.08		-0.03	0.06		-0.10	0.06	
δ_2	0.25	0.11	*	0.04	0.05		0.17	0.05	*
δ_3	0.18	0.06	*	0.08	0.03	*	0.02	0.06	
δ_4	-0.03	0.07		-0.03	0.05		0.11	0.07	
δ_5	-0.03	0.09		-0.06	0.05		0.08	0.07	
δ_6	0.02	0.06		0.03	0.04		0.19	0.10	
δ_7	-0.07	0.13		0.18	0.09	*	0.01	0.10	
R^2	0.29			0.88			0.82		
$Pr(\delta_j = 0)$	0.00			0.01			0.00		
obs	665			665			665		

Table 11: Results for Growth, Investment and Savings - Benchmark

	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.20	0.03 *	-0.12	0.11	0.24	0.06 *
I_{t-1}	0.02	0.09	0.28	0.12 *	-0.40	0.19 *
S_{t-1}	0.06	0.04	-0.02	0.13	0.02	0.10
H_{t-1}	1.15	0.05 *	0.11	0.07	0.15	0.07 *
rr_{t-1}	-0.14	0.03 *	0.51	0.19 *	-0.12	0.06 *
π_{t-1}	-0.13	0.02 *	0.10	0.18	0.41	0.05 *
g_{t-2}	0.02	0.03	-0.07	0.06	0.12	0.07
I_{t-2}	-0.07	0.08	-0.46	0.18 *	0.61	0.30 *
S_{t-2}	-0.05	0.04	-0.08	0.08	0.03	0.06
H_{t-2}	-0.23	0.05 *	-0.06	0.09	-0.18	0.09
rr_{t-2}	0.01	0.03	0.39	0.19 *	-0.03	0.04
π_{t-2}	0.02	0.04	0.14	0.17	0.15	0.05 *
$POIL_{t-1}$	-0.01	0.00 *	0.01	0.01	-0.02	0.01 *
$POIL_{t-2}$	0.01	0.00 *	-0.00	0.01	0.02	0.01 *
$popGrowth$	-0.25	0.97	-2.44	1.72	3.65	1.84 *
$popGrowth_{t-1}$	0.56	0.91	1.77	1.70	-2.84	1.63
δ_1	-0.10	0.06	-0.33	0.12 *	0.50	0.11 *
δ_2	-0.02	0.08	-0.08	0.14	0.13	0.08
δ_3	0.07	0.06	0.14	0.11	-0.16	0.09
δ_4	0.14	0.07 *	0.29	0.10 *	-0.46	0.15 *
δ_5	-0.03	0.06	0.21	0.07 *	-0.30	0.11 *
δ_6	0.08	0.08	0.16	0.21	-0.07	0.15
δ_7	0.05	0.09	0.01	0.23	0.18	0.22
R^2	0.93		0.66		0.75	
$\Pr(\delta_j = 0)$	0.00		0.01		0.00	
obs	665		665		665	

Table 12: Results for Hours, rr and pi - Benchmark

Estimations with Residential Patent Applications

	g_{t-1}	I_{t-1}	S_{t-1}	H_{t-1}	rr_{t-1}	$R\&D^{PA}_{t-1}$	π_{t-1}
g	0.22	-0.20	0.02	0.01	-0.28	0.01	-0.30
I	0.16	0.75	0.00	0.02	-0.11	-0.01	-0.11
S	-0.12	-0.11	0.78	-0.00	-0.10	-0.00	-0.08
H	0.23	-0.06	0.01	0.92	-0.14	0.00	-0.11
rr	-0.16	-0.22	-0.10	0.06	0.90	0.03	0.23
$R\&D^{PA}$	0.29	0.05	-0.18	-0.15	-0.12	-0.28	0.22
π	0.35	0.23	0.05	-0.03	-0.15	-0.02	0.56

Table: Sum of VAR coefficients $A_1 + A_2$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.06	0.21	0.19	-0.03	-0.09	0.01	-0.01	-0.23
I	-0.04	0.02	0.10	-0.03	-0.06	0.02	0.21	-0.22
S	-0.11	0.16	0.03	0.11	0.08	0.20	0.02	-0.50
H	-0.09	-0.04	0.08	0.13	-0.02	0.09	0.09	-0.24
rr	-0.33	-0.11	0.19	0.26	0.18	0.20	0.08	-0.46
$R\&D^{PA}$	0.19	-0.93	0.14	0.68	1.21	-1.34	0.16	-0.11
π	0.48	0.14	-0.17	-0.45	-0.28	-0.08	0.12	0.24

Table: Short-Run Demographic Impact

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.15	0.17	0.08	0.13	0.10	-0.08	-0.26	0.02
I_{t-1}	-0.68	-0.14	0.28	0.38	0.34	0.09	0.50	-0.77
S_{t-1}	0.03	0.83	0.05	0.39	0.25	0.28	0.05	-1.89
H_{t-1}	-1.80	-0.26	0.30	2.59	1.16	0.13	-0.61	-1.51
rr_{t-1}	-0.17	-0.18	0.58	0.06	-0.17	0.50	0.33	-0.95
$R\&D^{PA}_{t-1}$	0.55	-0.69	0.25	-0.21	0.99	-1.75	0.59	0.28
π_{t-1}	0.42	0.43	-0.67	-0.80	-0.03	0.02	-0.19	0.81

Table: Long-Run Demographic Impact (2-way effects)

Table 13: Additional Results: Panel VAR incorporating Patent Application

	β_1	β_2	β_3
g_{t-1}	-0.00	0.12	-0.12
I_{t-1}	-0.29	0.35	-0.06
S_{t-1}	0.63	0.21	-0.84
H_{t-1}	-0.80	1.74	-0.94
rr_{t-1}	-0.06	0.17	-0.10
$R\&D^{PA}_{t-1}$	-0.34	0.30	0.05
π_{t-1}	0.43	-0.61	0.18

Table 14: Long-Run Demographic Impact (2-way effects)

	g_{t-1}	I_{t-1}	S_{t-1}	H_{t-1}	rr_{t-1}	$R\&D^{PA}_{t-1}$	π_{t-1}
Australia	-0.76	0.51	-1.22	-3.17	1.31	-1.17	-0.00
Austria	-0.57	-0.92	-1.50	-7.15	0.19	0.95	0.24
Belgium	-0.47	-2.00	-1.31	-5.70	-1.06	-1.42	1.68
Canada	-1.14	-0.04	-1.18	-6.13	1.65	-3.66	0.62
Denmark	-0.84	0.07	-0.33	-7.63	-1.63	1.10	3.26
Finland	-1.20	-0.32	-4.40	-8.74	-0.38	-0.33	2.39
France	-0.85	-1.56	-2.35	-6.28	-0.33	-0.86	1.41
Germany	-0.07	-4.20	-5.24	-11.07	-2.76	0.28	1.64
Greece	-0.07	-3.47	-3.07	-0.07	-2.20	1.25	-1.83
Iceland	-0.68	0.72	0.49	-0.49	2.10	-3.35	-1.24
Ireland	-0.95	-0.31	-1.12	1.96	2.54	2.11	-3.62
Italy	-0.20	-3.40	-2.53	-5.03	-3.50	2.82	0.83
Japan	-0.68	-3.26	-9.32	-6.87	-5.36	6.03	1.29
Netherlands	-1.10	-1.46	-1.42	-10.41	-1.26	-0.57	3.20
New Zealand	-0.78	0.09	-1.01	-5.22	1.06	-1.54	0.47
Norway	-0.84	1.69	2.34	-2.40	1.57	-1.21	0.55
Portugal	-0.36	-2.05	-1.61	0.90	-0.96	0.74	-2.24
Spain	-0.47	-2.30	0.01	1.42	-0.44	2.43	-2.49
Sweden	-0.93	1.22	-1.14	-4.46	1.09	2.58	0.55
Switzerland	-0.44	-0.30	-1.28	-5.16	-0.05	0.75	0.19
United Kingdom	-0.52	-0.12	-1.58	-5.33	0.60	1.52	0.26
United States	-1.21	-0.01	-0.58	-6.22	1.93	-3.11	1.23

Table 15: Difference in Predicted Impact of Demographic Factors between 2000 and 2007

	Estimate	Std. Error		Estimate	Std. Error		Estimate	Std. Error	
g_{t-1}	0.24	0.05	*	0.12	0.03	*	-0.06	0.07	
I_{t-1}	-0.27	0.13	*	0.92	0.07	*	0.05	0.09	
S_{t-1}	0.08	0.06		0.06	0.03		0.98	0.05	*
H_{t-1}	0.08	0.07		0.01	0.03		0.05	0.04	
rr_{t-1}	-0.22	0.07	*	-0.09	0.02	*	-0.05	0.03	
$R\&D^{PA}_{t-1}$	0.00	0.01		-0.00	0.00	*	0.00	0.01	
π_{t-1}	-0.28	0.07	*	-0.08	0.02	*	-0.05	0.03	
g_{t-2}	-0.02	0.04		0.04	0.02		-0.06	0.04	
I_{t-2}	0.07	0.12		-0.17	0.05	*	-0.15	0.09	
S_{t-2}	-0.07	0.06		-0.05	0.03		-0.20	0.06	*
H_{t-2}	-0.08	0.08		0.01	0.03		-0.05	0.04	
rr_{t-2}	-0.06	0.06		-0.02	0.02		-0.05	0.03	
$R\&D^{PA}_{t-2}$	0.00	0.01		-0.00	0.00	*	-0.00	0.00	
π_{t-2}	-0.01	0.04		-0.02	0.01		-0.03	0.02	
$POIL_{t-1}$	-0.02	0.01	*	0.00	0.00		-0.01	0.00	*
$POIL_{t-2}$	0.02	0.00	*	0.00	0.00		0.00	0.00	
$popGrowth$	2.97	1.01	*	0.36	0.49		1.36	0.74	
$popGrowth_{t-1}$	-2.51	1.00	*	0.40	0.50		-1.02	0.82	
δ_1	-0.06	0.08		-0.04	0.07		-0.11	0.06	
δ_2	0.21	0.12		0.02	0.05		0.16	0.06	*
δ_3	0.19	0.07	*	0.10	0.04	*	0.03	0.06	
δ_4	-0.03	0.08		-0.03	0.05		0.11	0.08	
δ_5	-0.09	0.08		-0.06	0.06		0.08	0.07	
δ_6	0.01	0.06		0.02	0.05		0.20	0.11	
δ_7	-0.01	0.15		0.21	0.10	*	0.02	0.11	
R^2	0.29			0.88			0.82		
$\Pr(\delta_j = 0)$	0.00			0.00			0.00		
obs	623			623			623		

Table 16: Results for Growth, Investment and Savings

	Estimate	Std. Error		Estimate	Std. Error		Estimate	Std. Error		Estimate	Std. Error	
g_{t-1}	0.21	0.03	*	-0.10	0.13		0.11	0.65		0.23	0.06	*
I_{t-1}	-0.02	0.08		0.26	0.12	*	0.49	0.81		-0.40	0.15	*
S_{t-1}	0.05	0.04		-0.03	0.15		-1.08	0.49	*	0.03	0.12	
H_{t-1}	1.16	0.06	*	0.08	0.08		0.47	0.27		0.17	0.07	*
rr_{t-1}	-0.14	0.02	*	0.47	0.19	*	-0.77	0.22	*	-0.10	0.07	
$R\&D^{PA}_{t-1}$	0.01	0.00		0.03	0.01	*	-0.19	0.03	*	-0.03	0.01	*
π_{t-1}	-0.13	0.02	*	0.07	0.19		-0.35	0.23		0.42	0.05	*
g_{t-2}	0.02	0.04		-0.06	0.08		0.18	0.17		0.11	0.09	
I_{t-2}	-0.04	0.08		-0.48	0.16	*	-0.44	0.64		0.63	0.27	*
S_{t-2}	-0.04	0.04		-0.08	0.09		0.90	0.49		0.02	0.07	
H_{t-2}	-0.23	0.06	*	-0.02	0.07		-0.61	0.29	*	-0.20	0.08	*
rr_{t-2}	-0.00	0.03		0.42	0.19	*	0.64	0.24	*	-0.06	0.04	
$R\&D^{PA}_{t-2}$	-0.00	0.01		-0.00	0.03		-0.09	0.04	*	0.00	0.02	
π_{t-2}	0.02	0.04		0.16	0.17		0.58	0.11	*	0.14	0.05	*
$POIL_{t-1}$	-0.01	0.00	*	0.01	0.01	*	0.02	0.02		-0.02	0.01	*
$POIL_{t-2}$	0.01	0.00	*	-0.00	0.01		0.02	0.03		0.02	0.01	*
$popGrowth$	-0.22	0.99		-2.48	2.33		-6.65	10.85		3.54	2.03	
$popGrowth_{t-1}$	0.47	0.93		1.63	2.18		6.96	11.94		-2.68	1.78	
δ_1	-0.09	0.07		-0.33	0.13	*	0.19	0.81		0.48	0.11	*
δ_2	-0.04	0.08		-0.11	0.16		-0.93	0.59		0.14	0.09	
δ_3	0.08	0.07		0.19	0.14		0.14	0.64		-0.17	0.10	
δ_4	0.13	0.07		0.26	0.10	*	0.68	0.47		-0.45	0.15	*
δ_5	-0.02	0.07		0.18	0.08	*	1.21	0.56	*	-0.28	0.11	*
δ_6	0.09	0.09		0.20	0.20		-1.34	1.10		-0.08	0.14	
δ_7	0.09	0.09		0.08	0.25		0.16	1.09		0.12	0.25	
R^2	0.93			0.66			0.11			0.75		
$\Pr(\delta_j=0)$	0.00			0.09			0.00			0.02		
obs	623			623			620			623		

Table 17: Results for Hours, Interest Rate, R&D and Inflation

	2000-2009	2010-2019	Change
Australia	1.75	0.86	-0.88
Austria	1.45	0.75	-0.70
Belgium	1.26	0.30	-0.96
Canada	1.86	0.37	-1.49
Denmark	0.71	0.20	-0.51
Finland	0.78	-0.36	-1.14
France	1.37	0.30	-1.07
Germany	0.80	0.26	-0.55
Greece	1.36	0.68	-0.68
Iceland	2.07	0.98	-1.09
Ireland	2.07	1.08	-0.99
Italy	0.99	0.43	-0.56
Japan	0.24	-0.11	-0.34
Netherlands	1.24	0.14	-1.10
New Zealand	1.81	0.73	-1.08
Norway	1.44	0.51	-0.92
Portugal	1.30	0.73	-0.56
Spain	1.87	0.97	-0.90
Sweden	0.95	0.23	-0.71
Switzerland	1.41	0.58	-0.83
United Kingdom	1.31	0.60	-0.70
United States	1.80	0.58	-1.22

Table 18: Average Predicted Impact on GDP Growth by Country

Appendix: Theoretical Model

This appendix shows how the equilibrium conditions are determined.

We start by looking at the factor markets with the final and input firms decisions.

Production Sector

Firms in consumption and capital producing sectors maximise profits selecting capital, its utilisation, labour and intermediate goods demand.

Labour allocation is such that

$$(1 - \alpha)(1 - \gamma_I)Y_{c,t} = \mu_t W_t \xi_t L_t, \tag{37}$$

$$\tag{38}$$

Capital stock and utilisation are such that

$$\alpha(1 - \gamma_I)Y_{c,t} = \mu_t[r_t^k + \delta(U_t)]K_t, \tag{39}$$
$$\alpha(1 - \gamma_I)Y_{c,t} = \mu_t\delta'(U_t)K_tU_t. \tag{40}$$

Where I_t is the investment in capital made by the financial intermediary, who holds all production and R& D assets. Intermediate goods are set such that

$$\mu_t M_t P_t^M = \gamma_I Y_{c,t} \tag{41}$$

where P_t^M is the price of intermediate goods.

In order to obtain this price one can minimise total cost of intermediary goods $\int_0^A \tilde{P}^M M^i di$ subject to (7) to obtain

$$P_t^M = \vartheta A_t^{1-\vartheta} \tag{42}$$

Combining (5) and (6) and defining total labour supply as $L_t \equiv \int_0^{N_t^f} L_t^j dj$ and total intermediate composite demand as $M_t \equiv \int_0^{N_t^f} M_t^j dj$, then[22]

$$Y_t = (N_t^f)^{\mu_t-1}\left[(U_t\frac{K_t}{\xi_tL_t})^\alpha(\xi_tL_t)\right]^{(1-\gamma_I)} [M_t]^{\gamma_I} \text{ for } x = c, k. \tag{43}$$

Due to free entry the number of final good firms is such that their profits are equal to the operating costs. Using (5) total output per firm is given by $Y_t(N_t^f)^{-\mu_t}$, while their mark-up is given by $\frac{\mu_t-1}{\mu_t}$, thus

$$\frac{\mu_t - 1}{\mu_t}Y_{c,t}(N_t^f)^{-\mu_t} = \Omega\tilde{\Psi}_t \tag{44}$$

Finally, let Y_t denote aggregate value added output. Y_t is equal to the total output net intermediate goods and operating costs. Thus, using (42)[23],

$$Y_t = Y_{c,t} - A_t^{1-\vartheta}M_t - \Omega\tilde{\Psi}_t. \tag{45}$$

On the expenditure side, output must be equal to consumption, investment and costs of R&D and adoption. Thus,

$$Y_t = C_t + I_t + S_t + \Xi_t(Z_t - A_t) + \tau_t. \tag{46}$$

Innovation Process

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[22]Note that all firms select the same capital labour ratio.

[23]In order to net out intermediate goods one has to compute total expenditure on intermediate goods ($\int_0^A \tilde{P}^M M^i di$) minus the markup on intermediate goods ($\int_0^A (\tilde{P}^M - 1)M^i di$).

From conditions (8) and (13) one can easily determine the flow of the stock of invented (prototypes) and adopted goods, which are given by

$$\frac{Z_{t+1}}{Z_t} = \chi \left(\frac{S_t}{\tilde{\Psi}_t} \right)^{\rho} + \phi, \text{ and} \tag{47}$$

$$\frac{A_{t+1}}{A_t} = \lambda \left(\frac{A_t \Xi_t}{\tilde{\Psi}_t} \right) \phi[Z_t/A_t - 1] + \phi \tag{48}$$

Investment in R&D (S_t) is determined by (9), which using (8) becomes

$$S_t = R_{t+1}^{-1} \phi E_t J_{t+1}(Z_{t+1} - \phi Z_t). \tag{49}$$

Profits are given by the total gain in seeling the right to goods invented as a result of the previous period investment $S_{x,t-1}$ to adopters minus the cost of borrowing for that investment. Thus,

$$\Pi_t^{RD} = \phi J_t(Z_t - \phi Z_{t-1}) + S_{t-1} R_t$$

Thus, in perfect foresight equilibrium $\Pi_t^{RD} = 0$.

Investment in adoption (Ξ_t) is determined by solving (12). We thus obtain the following condition

$$\frac{A_t}{K_t} \lambda' R_t^{-1} \phi[V_{t+1} - J_{t+1}] = 1 \tag{50}$$

where $\frac{A_t}{K_t} \lambda' = \frac{\partial \lambda \left(\frac{A_t}{\tilde{\Psi}_t} \right)}{\partial \Xi_t \Xi_t}$. Assuming the elasticity of λ_t to changes in Ξ_t is constant, thus $\epsilon_\lambda = \frac{\lambda'}{\lambda_t} \frac{A_t \Xi_t}{K_t}$, then we obtain

$$\Xi_t = \epsilon_\lambda \lambda_t R_t^{-1} \phi[V_{t+1} - J_{t+1}] \tag{51}$$

Finally, the value of an invented good and an adopted good are given by

$$J_t = -\Xi_t + (R_{t+1})^{-1} \phi E_t[\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}], \text{ and} \tag{52}$$

$$V_t = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t A_t} + (R_{t+1})^{-1} \phi E_t V_{t+1} \tag{53}$$

$$\tag{54}$$

where $\lambda_t = \lambda \left(\frac{A_t \Xi_t}{\tilde{\Psi}_t} \right)$ and $\Pi_{m,t} = (1 - 1/\vartheta) P_t^M M_t = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t A_t}$.

Profits for adopters are given by the gain from marketing specialised intermediated goods net the amount paid to inventors to gain access to new goods and the expenditures on loans to pay for adoption intensity.

$$\Pi_t^A = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t} - J_t(Z_t - \phi Z_{t-1}) - \Xi_{t-1}(Z_{t-1} - A_{t-1}) R_t$$

Household Sector

Retiree j decision problem is

$$\max V_t^{jr} = \left\{ (C_t^{jr})^{\rho_U} + \beta\gamma_{t,t+1}([V_{t+1}^{jr}]^{\rho_U}) \right\}^{1/\rho_U} \tag{55}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\gamma_{t-1,t}} FA_t^{jr} + d_t^{jr}. \tag{56}$$

The first order condition and envelop theorem are

$$(C_t^{jr})^{\rho_U-1} = \beta\gamma_{t,t+1}\frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jr}}(V_{t+1}^{jr})^{\rho_U-1}, \tag{57}$$

$$\frac{\partial V_t^{jr}}{\partial FA_t^{jr}} = (V_{t+1}^{jr})^{1-\rho_U}(C_t^{jr})^{\rho_U-1}\frac{R_t}{\gamma_{t-1,t}}. \tag{58}$$

Combining these conditions above gives the Euler equation

$$C_{t+1}^{jr} = (\beta R_{t+1})^{1/(1-\rho_U)}C_t^{jr} \tag{59}$$

Conjecture that retirees consume a fraction of all assets (including financial assets, profits from financial intermediaries), such that

$$C_t^{jr} = \varepsilon_t\varsigma_t\left[\frac{R_t}{\gamma_{t-1,t}}FA_t^{rj} + D_t^{rj}\right]. \tag{60}$$

Combining these and the budget constraint gives

$$FA_{t+1}^{jr} = \frac{R_t}{\gamma_{t-1,t}}FA_t^{jr}(1 - \varepsilon_t\varsigma_t) + d_t^{jr} - \varepsilon_t\varsigma_t(D_t^{rj}).$$

Using the condition above the Euler equation and the solution for consumption gives

$$(\beta R_{t+1})^{1/(1-\rho_U)}\varepsilon_t\varsigma_t[\frac{R_t}{\gamma_{t-1,t}}FA_t^{rj} + D_t^{rj}] = \tag{61}$$

$$\varepsilon_{t+1}\varsigma_{t+1}\left[\frac{R_{t+1}}{\gamma_{t,t+1}}\left(\frac{R_t}{\gamma_{t-1,t}}FA_t^{jr}(1 - \varepsilon_t\varsigma_t) + d_t^{jr} - \varepsilon_t\varsigma_t D_t^{rj}\right) + D_{t+1}^{jr}\right].$$

Collecting terms we have that

$$1 - \varepsilon_t\varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\gamma_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}}, \tag{62}$$

$$D_t^{jr} = d_t^{jr} + \frac{\gamma_{t,t+1}}{R_{t+1}}D_{t+1}^{jr}. \tag{63}$$

One can also show that $V_t^{jr} = (\varepsilon_t \varsigma_t)^{-1/\rho_U} C_t^{jr}$.

Worker j decision problem is

$$\max V_t^{jw} = \left\{ (C_t^{jw})^{\rho_U} + \beta[\omega^r V_{t+1}^{jw} + (1-\omega^r)V_{t+1}^{jr}]^{\rho_U} \right\}^{1/\rho_U} \tag{64}$$

subject to

$$C_t^{jw} + FA_{t+1}^{jw} = R_t FA_t^{jw} + W_t \xi_t + d_t^{jw} - \tau_t^{jw}. \tag{65}$$

First order conditions and envelop theorem are

$$(C_t^{jw})^{\rho_U - 1} = \beta[\omega^r V_{t+1}^{jw} + (1-\omega^r)V_{t+1}^{jr}]^{\rho_U - 1} \left[\omega^r \frac{\partial V_{t+1}^{jw}}{\partial FA_{t+1}^{jw}} + (1-\omega^r)\frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jw}} \right],$$

$$\frac{\partial V_t^{jw}}{\partial FA_t^{jw}} = (V_{t+1}^{jw})^{1-\rho_U}(C_t^{jw})^{\rho_U - 1}R_t, \text{ and} \tag{66}$$

$$\frac{\partial V_t^{jr}}{\partial FA_t^{jw}} = \frac{\partial V_t^{jr}}{\partial FA_t^{jr}}\frac{\partial FA_t^{jr}}{\partial FA_t^{jw}} = \frac{\partial V_t^{jr}}{\partial FA_t^{jr}}\frac{1}{\gamma_{t-1,t}} = (V_{t+1}^{jr})^{1-\rho_U}(C_t^{jr})^{\rho_U - 1}R_t. \tag{67}$$

$\frac{\partial FA_t^{jr}}{\partial FA_t^{jw}} = \frac{1}{\gamma_{t-1,t}}$ since as individuals are risk neutral with respect to labour income they select the same asset profile independent of their worker/retiree status, adjusting only for expected return due to probability of death.

Combining these conditions above, and using the conjecture that $V_t^{jw} = (\varsigma_t)^{-1/\rho_U}C_t^{jw}$, gives the Euler equation

$$C_t^{jw} = \left((\beta R_{t+1} \mathfrak{Z}_{t+1})^{1/(1-\rho_U)} \right)^{-1} [\omega^r C_{t+1}^{jw} + (1-\omega^r)\varepsilon_{t+1}^{\frac{-1}{\rho_U}}C_{t+1}^{jr}] \tag{68}$$

$$\text{where } \mathfrak{Z}_{t+1} = (\omega^r + (1-\omega^r)\varepsilon_{t+1}^{(\rho_U - 1)/\rho_U}).$$

Conjecture that retirees consume a fraction of all assets (including financial assets, human capital and profits from financial intermediaries), such that

$$C_t^{jw} = \varsigma_t[R_t FA_t^{jw} + H_t^{jw} + D_t^{jw} - T_t^{jw}]. \tag{69}$$

Following the same procedure as before we have that

$$\varsigma_t[R_t FA_t^{jw} + H_t^{jw} + D_t^{jw}](\beta R_{t+1}\mathfrak{Z}_{t+1})^{1/(1-\rho_U)} = \tag{70}$$

$$\omega^r \varsigma_{t+1}\left[R_{t+1}\left(R_t FA_t^{jw}(1-\varsigma_t) + W_t\xi_t + d_t^{jw} - \tau_t^{jw} - \varsigma_t(H_t^{jw} + D_t^{jw} - T_t^{jw})\right) + H_{t+1}^{jw} + D_{t+1}^{jw} - T_{t+1}^{jw}\right] +$$

$$\varepsilon_{t+1}^{\frac{-1}{\rho_U}}(1-\omega^r)\varepsilon_{t+1}\varsigma_{t+1}\left[R_{t+1}\left(R_t FA_t^{jw}(1-\varsigma_t) + W_t\xi_t + d_t^{jw} - \tau_t^{jw} - \varsigma_t(H_t^{jw} + D_t^{jw} - T_t^{jw})\right) + D_{t+1}^{jr}\right].$$

Collecting terms and simplifying we have that

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1}\mathfrak{Z}_{t+1})^{1/(1-\rho_U)}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \tag{71}$$

$$H_t^{jw} = W_t \xi_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^{jw} \tag{72}$$

$$T_t^{jw} = \tau_t^{jw} + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}} T_{t+1}^{jw} \text{ and} \tag{73}$$

$$D_t^{jw} = d_t^{jw} + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^{jr}. \tag{74}$$

Aggregation across households

Assume that for any variable X_t^{jz} we have that $X_t^z = \int_0^{N_t^z} X_t^{jz}$ for $z = \{w, r\}$, then

$$L_t = N_t^w, \tag{75}$$

$$H_t^w = W_t \xi_t L_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{76}$$

$$T_t^w = \tau_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}} T_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{77}$$

$$D_t^w = d_t^w + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^w \frac{N_t^w}{N_{t+1}^w} + \frac{(1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{Z}_{t,t+1}} D_{t+1}^r \frac{N_t^w}{N_{t+1}^r}, \tag{78}$$

$$C_t^w = \varsigma_t[R_t F A_t^w + H_t^w + D_t^w - T_t^w], \tag{79}$$

$$D_t^r = d_t^r + \frac{\gamma_{t,t+1}}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r}, \tag{80}$$

$$C_t^r = \varepsilon_t \varsigma_t[R_t F A_t^r + D_t^r]. \tag{81}$$

Note that $\gamma_{t,t+1}$ is not shown in the last equation due to the perfect annuity market for retirees, allowing for the redistribution of assets of retirees who died at the end of the period.

Decision of Investment in Labour Skill

The marginal cost of increasing lump-sum taxes for worker j today to finance higher investment in young's education is given by

$$MC_t^{Ej} = -\frac{\partial V_t^{wj}}{\partial \tau_t^{wj}} = \frac{\partial V_t^{wj}}{\partial C_t^{wj}} = \varsigma_t^{-1/\rho_U} \tag{82}$$

The marginal benefit of increasing lump-sum taxes at time t for a young h who becomes

a worker next period is

$$MB_t^{Eh} = \beta(1-\omega^y)\frac{\partial V_{t+1}^{wh}}{\partial \tau_t^{wj}} = \beta(1-\omega^y)\frac{\partial V_{t+1}^{wh}}{\partial \xi_{t+1}^y}\frac{\partial \xi_{t+1}^y}{\partial I_t^y}\frac{\partial I_t^y}{\partial \tau_t}\frac{\partial \tau_t}{\partial \tau_t^{wj}} \tag{83}$$

$$= \beta(1-\omega^y)\varsigma_{t+1}^{-1/\rho_U}\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{84}$$

Adding costs across all workers and benefits across all young at time t gives the condition that determines I_t^y. That is

$$\varsigma_t^{-1/\rho_U} = \beta(1-\omega^y)\varsigma_{t+1}^{-1/\rho_U}\zeta_t^y\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{85}$$

Financial Intermediary

Due to standard arbitrage arguments all assets must pay same expected return thus

$$E_t\left[r_{t+1}^k + 1\right] = R_t. \tag{86}$$

The flow of capital is then given by

$$K_{t+1} = K_t(1-\delta(U_t)) + I_t. \tag{87}$$

Also note that under a perfect foresight solution this equality holds without expectations, $\Pi_t^F = 0$ and thus $d_t^r = d_t^w = 0$. If $\Pi_t^F \neq 0$, then we assume profits are divided based on the ratio of assets thus $d_t^r = \Pi_t^F\frac{FA_t^r}{FA_t^r + FA_t^w}$ and $d_t^w = \Pi_t^F\frac{FA_t^w}{FA_t^r + FA_t^w}$.

Asset Markets

Asset Market clearing implies

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \tag{88}$$

Finally, the flow of assets are given by

$$FA_{t+1}^r = R_t FA_t^r + d_t^r - C_t^r + (1-\omega^r)(R_t FA_t^w + W_t\xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{89}$$

$$FA_{t+1}^w = \omega^r(R_t FA_t^w + W_t\xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{90}$$

Detrending equilibrium conditions

Note that $\bar{x}$ denote the steady state of variable x_t.

$$h_t^w = w_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}}\frac{g_{t+1}h_{t+1}^w}{g_{t+1}^w} \text{ where } h_t^w = \frac{H_t^w}{Y_{c,t}}, w_t = \frac{W_t\xi_t L_t}{Y_{c,t}}, g_{t+1} = \frac{Y_{c,t+1}}{Y_{c,t}}, g_{t+1}^w = \frac{N_{t+1}^w}{N_t^w} \tag{91a}$$

$$\tilde{T}_t^w = \tilde{\tau}_t + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}}\frac{g_{t+1}\tilde{T}_{t+1}^w}{g_{t+1}^w} \text{ where } \tilde{T}_t^w = \frac{T_t^w}{Y_{c,t}}, \tilde{\tau}_t = \frac{\tau_t}{Y_{c,t}} \tag{91b}$$

$$\tilde{D}_t^r = \tilde{d}_t^r + \frac{\gamma_{t,t+1}}{R_{t+1}}g_{t+1}\frac{\tilde{D}_{t+1}^r\zeta_t^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^r = \frac{D_t^r}{Y_{c,t}}, \tilde{d}_t^r = \frac{d_t^r}{Y_{c,t}} \tag{91c}$$

$$\tilde{D}_t^w = \tilde{d}_t^w + \frac{\omega^r}{R_{t+1}\mathfrak{Z}_{t,t+1}}\frac{g_{t+1}\tilde{D}_{t+1}^w}{g_{t+1}^w} + \frac{(1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{Z}_{t,t+1}}\frac{g_{t+1}\tilde{D}_{t+1}^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^w = \frac{D_t^w}{Y_{c,t}}, \tilde{d}_t^w = \frac{d_t^w}{Y_{c,t}} \tag{91d}$$

$$c_t^w = \varsigma_t[R_t\frac{fa_t^w}{g_t} + h_t^w + \tilde{D}_t^w - \tilde{T}_t^w] \text{ where } fa_t^w = \frac{FA_t^w}{Y_{c,t-1}}, c_t^w = \frac{C_t^w}{Y_{c,t}} \tag{91e}$$

$$c_t^r = \varepsilon_t\varsigma_t[R_t\frac{fa_t^r}{g_t} + \tilde{D}_t^r] \text{ where } fa_t^r = \frac{FA_t^r}{Y_{c,t-1}}, c_t^r = \frac{C_t^w}{Y_{c,t}} \tag{91f}$$

$$1 - \varepsilon_t\varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\gamma_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}} \tag{91g}$$

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}}\frac{(\beta R_{t+1}\mathfrak{Z}_{t+1})^{\frac{1}{(1-\rho_U)}}}{R_{t+1}\mathfrak{Z}_{t,t+1}} \tag{91h}$$

$$\mathfrak{Z}_{t+1} = (\omega^r + (1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}) \tag{91i}$$

$$g_{t+1}^w = \omega^r + (1-\omega^y)\zeta_t^y \tag{91j}$$

$$n_{t,t+1} = \frac{\zeta_{t+1}^y}{\zeta_t^y}(\omega^r + \zeta_t^y(1-\omega^y)) \tag{91k}$$

$$\zeta_{t+1}^r = ((1-\omega^r) + \gamma_{t,t+1}\zeta_t^r)(\omega^r + (1-\omega^y)\zeta_t^y)^{-1} \text{ and} \tag{91l}$$

$$g_{t+1}^n = (n_{t,t+1}\zeta_t^y) + (\omega^r + (1-\omega^y)\zeta_t^y) + ((1-\omega^r) + \gamma_{t,t+1}\zeta_t^r)(1 + \zeta_t^r + \zeta_t^y)^{-1} \text{ where } g_{t+1}^n = \frac{N_{t+1}}{N_t} \tag{91m}$$

$$g_{t+1}^\xi = (g_{t+1}^w)^{-1}(\omega^r + (1-\omega^y)\zeta_t^y\left(\rho_E + \frac{\chi_E}{2}(i_t^y)^2\right) \text{ where } g_{t+1}^\xi = \frac{\xi_{t+1}}{\xi_t}, i_t^y = \frac{I_t^y}{\xi_t} \tag{91n}$$

$$\tilde{\tau}_t = i_t^y w_t \tag{91o}$$

$$\varsigma_t^{-1/\rho_U} = \varsigma_{t+1}^{-1/\rho_U}\beta(1-\omega^y)\zeta_t^y\chi_E i_t^y\frac{w_{t+1}g_{t+1}}{w_t g_{t+1}^w} \tag{91p}$$

$$(1 - \alpha)(1 - \gamma_I) = \mu_t w_t \tag{92a}$$

$$\alpha(1 - \gamma_I) = \mu_t[r_t^k + \delta(U_t)]k_t/g_t \text{ where } k_t = \frac{K_t}{Y_{c,t-1}} \tag{92b}$$

$$\alpha(1 - \gamma_I) = \mu_t \delta_t' k_t U_t/g_t \tag{92c}$$

$$g_t = \frac{\mu_t}{\mu_{t-1}} g_t^M (g_t^A)^{1-\vartheta} \text{ where } g_t^M = \frac{M_t}{M_{t-1}}, g_t^A = \frac{A_t}{A_{t-1}} \tag{92d}$$

$$g_t = \frac{(N_t^f)^{\mu_t-1}}{(N_{t-1}^f)^{\mu_{t-1}-1}} \left(\frac{U_t k_t}{U_{t-1} k_{t-1}} g_{t-1}\right)^{\alpha(1-\gamma_I)} \left(g_t^\xi g_t^w\right)^{(1-\alpha)(1-\gamma_I)} (g_t^M)^{\gamma_I} \tag{92e}$$

$$\frac{\mu_t - 1}{\mu_t}(N_t^f)^{-\mu_t} = b\Psi_t \text{ where } \Psi_t = \frac{\tilde{\Psi}_t}{Y_{c,t}} \tag{92f}$$

$$\mu_t = \mu(N_t^f) \approx \bar{\mu}\left(1 + \frac{\epsilon_\mu}{\bar{N}^f}(N_t^f - \bar{N}^f)\right) \text{ where } \epsilon_\mu \text{ is the elasticity of } \mu(\cdot) \tag{92g}$$

$$\delta_t = \bar{\delta} + \delta_t'(U_t - \bar{U}) \tag{92h}$$

$$\delta_t' = \bar{\delta}' + \delta''(U_t - \bar{U}) \tag{92i}$$

$$\frac{z_{a,t+1}}{z_{a,t}} g_{t+1}^A = \chi\left(\frac{s_t}{\Psi_t}\right)^\rho + \phi \text{ where } z_{a,t} = \frac{Z_t}{A_t}, s_t = \frac{S_t}{Y_{c,t}} \tag{93a}$$

$$g_{t+1}^A = \lambda_t \phi[z_{a,t} - 1] + \phi \tag{93b}$$

$$s_t = g_{t+1} R_{t+1}^{-1} \phi j_{t+1} \left(1 - \phi \frac{z_{a,t}}{z_{a,t+1} g_{t+1}^A}\right) \text{ where } j_t = \frac{J_t Z_t}{Y_t} \tag{93c}$$

$$v_t = \frac{(1 - 1/\vartheta)\gamma_I}{\mu_t} + (R_{t+1})^{-1}\phi\frac{g_{t+1}}{g_{t+1}^A} v_{t+1} \text{ where } v_t = \frac{V_t A_t}{Y_t} \tag{93d}$$

$$\varpi_t = \epsilon_\lambda \lambda_t R_{t+1}^{-1} \phi \frac{z_{a,t} g_{t+1}}{g_{t+1}^A} \left[v_{t+1} - \frac{j_{t+1}}{z_{at+1}}\right] \text{ where } \varpi_t = \frac{\Xi_t Z_t}{Y_t} \tag{93e}$$

$$j_t = -\varpi_t + (R_{t+1})^{-1}\phi\frac{z_{a,t} g_{t+1}}{g_{t+1}^A}\left[\lambda_t v_{t+1} + (1 - \lambda_t)\frac{j_{t+1}}{z_{a,t+1}}\right] \tag{93f}$$

$$\lambda_t = \lambda\left(\frac{\varpi_t}{z_{a,t}\Psi_t}\right) \approx \bar{\lambda}\left(1 + \epsilon_\lambda\left(\frac{\varpi_t - \bar{\bar{\varpi}}}{\bar{\varpi}} - \frac{z_{a,t} - \bar{z}_a}{\bar{z}_a} - \frac{\Psi_t - \bar{\bar{\Psi}}}{\bar{\Psi}}\right)\right) \tag{93g}$$

$$\pi_t^A = \frac{(1 - 1/\vartheta)\gamma_I}{\mu_t} - \phi j_t\left(1 - \phi\frac{z_{a,t-1}}{z_{a,t} g_t^A}\right) - R_t \varpi_{t-1}(1 - 1/z_{a,t-1})/g_t \tag{93h}$$

$$\pi_t^{RD} = \phi j_t\left(1 - \phi\frac{z_{a,t-1}}{z_{a,t} g_t^A}\right) - R_t s_{t-1}/g_t \tag{93i}$$

where ϵ_λ is the elasticity of $\lambda(\cdot)$

$$r^k_{t+1} + 1 = R_{t+1} \tag{94a}$$

$$\tilde{d}^r_t = \pi^F_t \frac{fa^r_t}{fa_t} \text{ where } \pi^F_t = \frac{\Pi^F_t}{Y_{c,t}} \tag{94b}$$

$$\tilde{d}^w_t = \pi^F_t \frac{fa^w_t}{fa_t} \tag{94c}$$

$$b_{t+1} = s_t + \varpi_t(1 - 1/z_{a,t}) \text{ where } b_{t+1} = \frac{B_{t+1}}{Y_{c,t}} \tag{94d}$$

$$\pi^F_t = (Rk_t + 1)\frac{k_t}{g_t} + \frac{R_t}{g_t}b_t - \frac{R_t}{g_t}(fa_t) - k_{t+1} - b_{t+1} + (fa_{t+1}) + \pi^A_t + \pi^{RD}_t \tag{94e}$$

$$k_{t+1} = (1 - \delta(U_t))\frac{k_t}{g_t} + i_t \text{ where } i_t = \frac{I_t}{Y_{c,t}} \tag{95a}$$

$$y_t = (1 - \gamma_I/(\vartheta\mu_t)) - b\Psi_t \text{ where } y_t = \frac{Y_t}{Y_{c,t}} \tag{95b}$$

$$y_t = c_t + i_t + s_t + \varpi_t(1 - 1/z_{a,t}) + \tilde{\tau}_t \text{ where } c_t = \frac{C_t}{Y_{c,t}} \tag{95c}$$

$$c_t = c^w_t + c^r_t \tag{95d}$$

$$fa^w_{t+1} + fa^r_{t+1} = k_{t+1} + b_{t+1} \tag{95e}$$

$$fa^r_{t+1} = \frac{R_t}{g_t}fa^r_t + \tilde{d}^r_t - c^r_t + (1 - \omega^r)\left(\frac{R_t}{g_t}fa^w_t + w_t + \tilde{d}^w_t - c^w_t - \tilde{\tau}_t\right) \tag{95f}$$

$$fa_{t+1} = fa^w_{t+1} + fa^r_{t+1} \tag{95g}$$

$$\Psi_t = v_t \tag{95h}$$

$$fa^w_{t+1} = \omega^r\left(\frac{R_t}{g_t}fa^w_t + w_t + \tilde{d}^w_t - c^w_t - \tilde{\tau}_t\right)$$
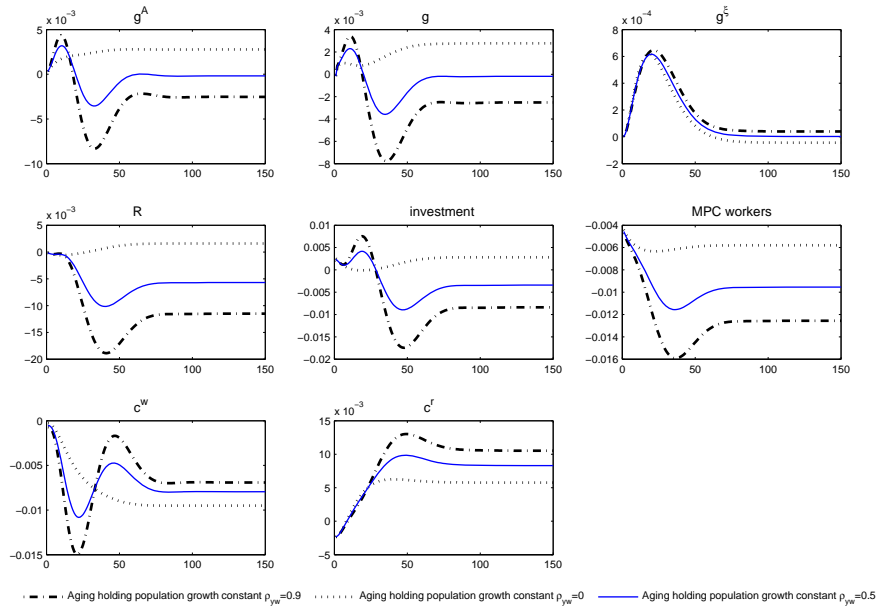
Figure 6: Simulation: benchmark *aging* versus different ρ_{yw}
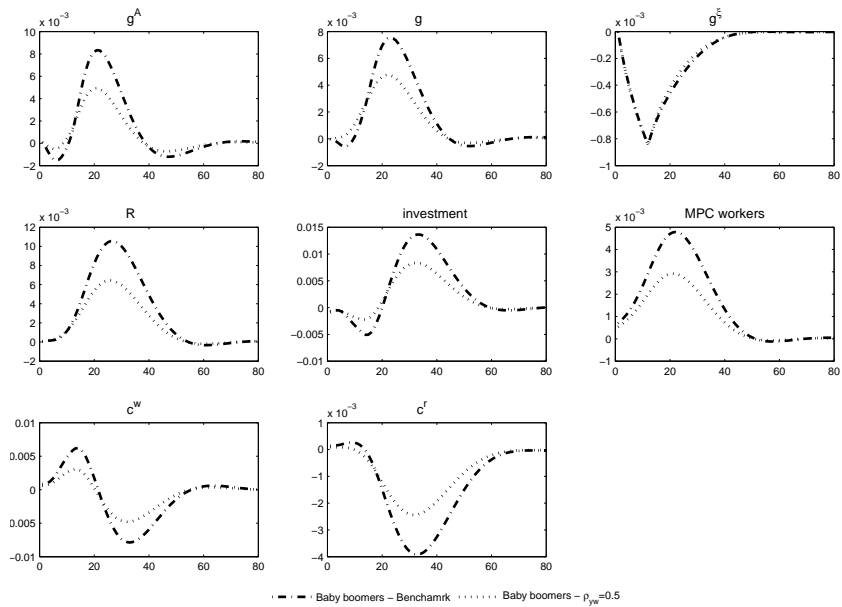
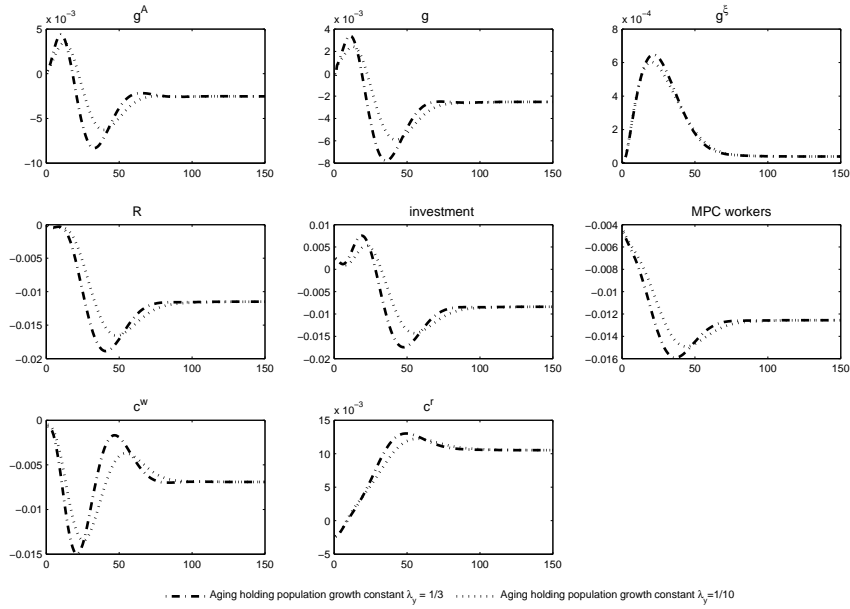Figure 7: Simulation: benchmark *Baby-boomers* versus $\rho_{yw} = 0.5$

Figure 8: Simulation: benchmark *aging* versus different $\lambda_y = 1/10$
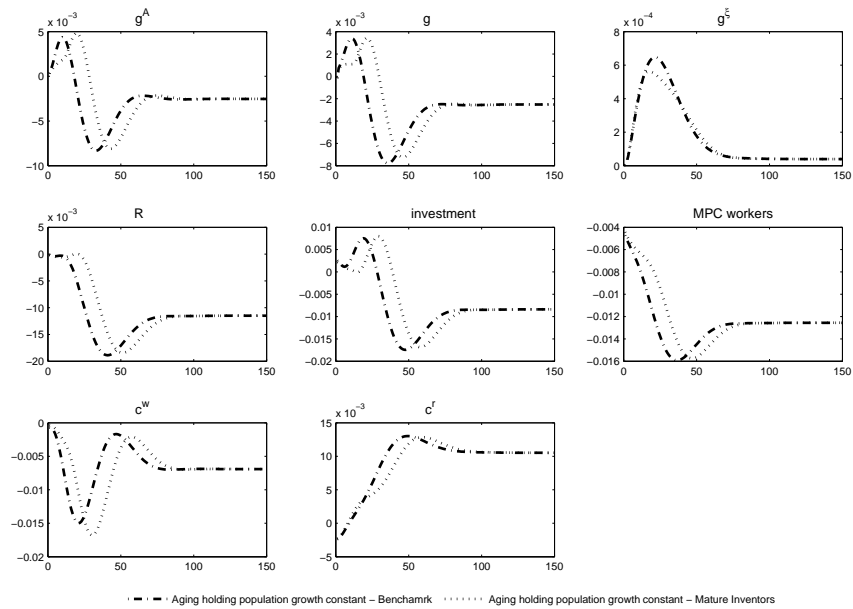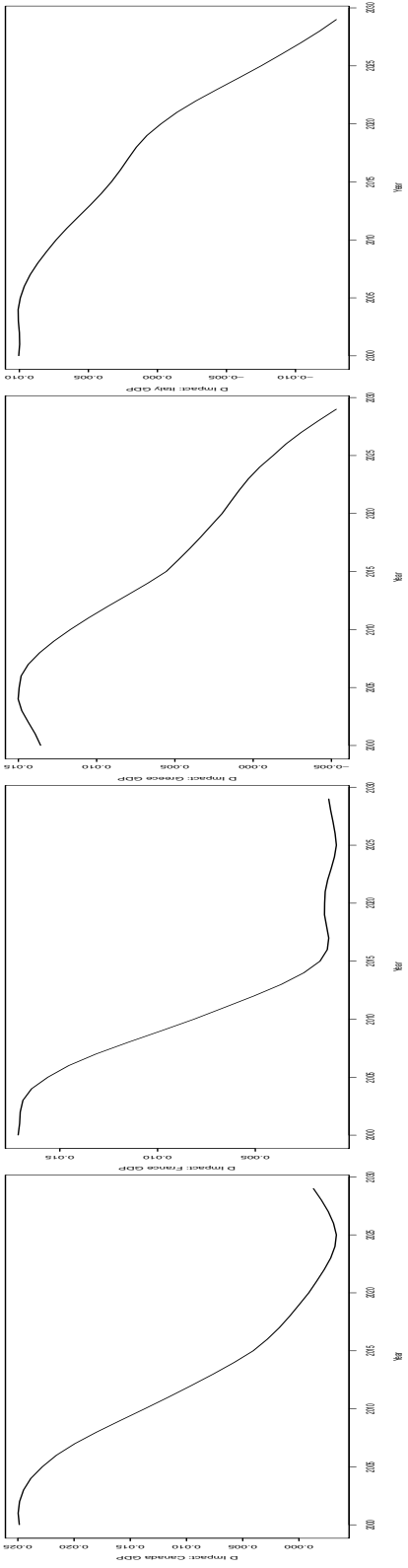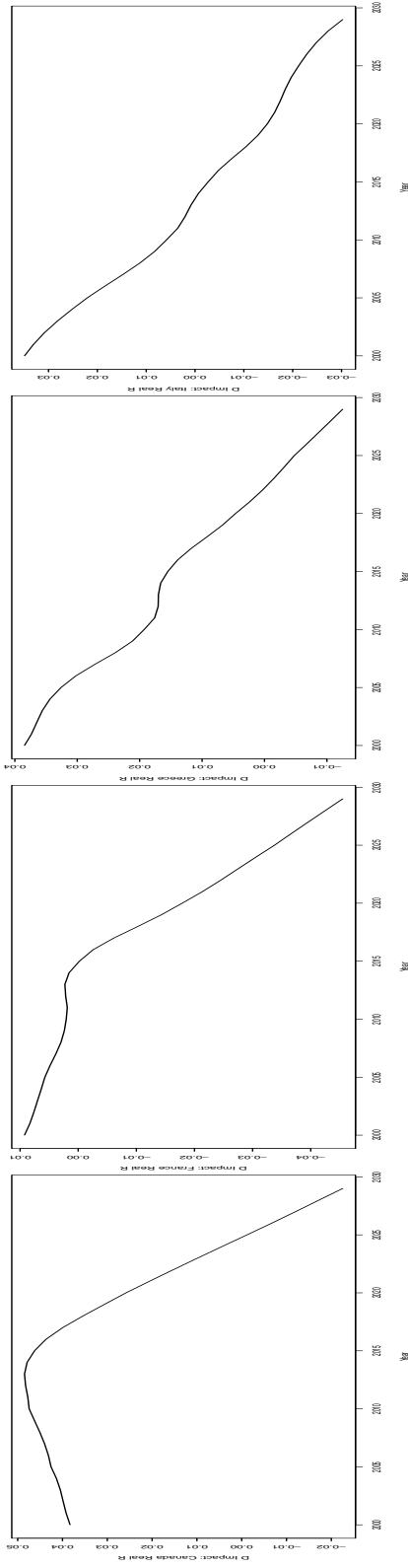

Figure 9: Simulation: benchmark *aging* versus Delayed flow of inventors

(a) Canada GDP   (b) France GDP   (c) Greece GDP   (d) Italy GDP

(e) Canada Real Rates   (f) France Real Rates   (g) Greece Real Rates   (h) Italy Real Rates

Figure 10: Impact of Predicted Future Demographic Structure - Estimation
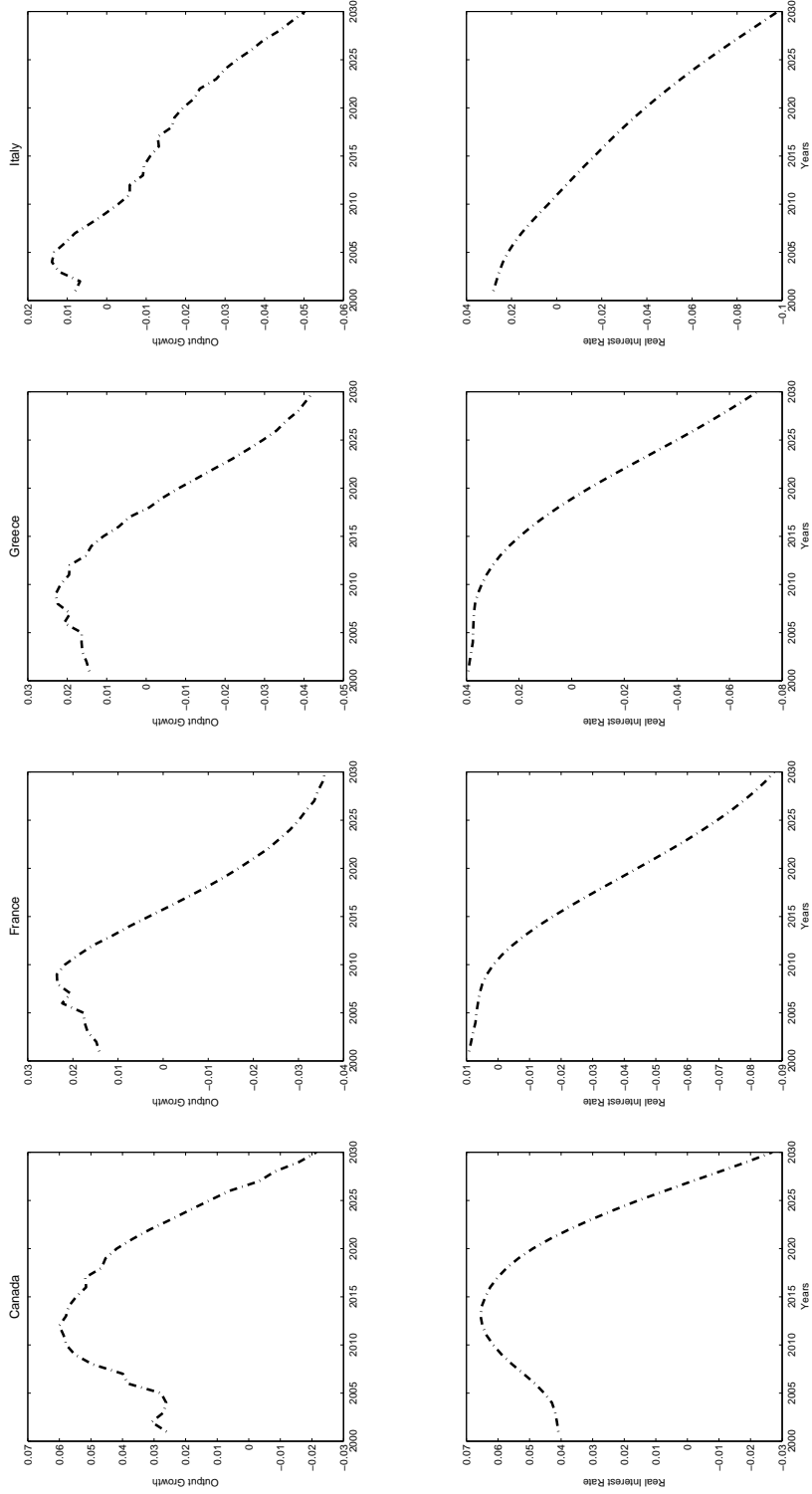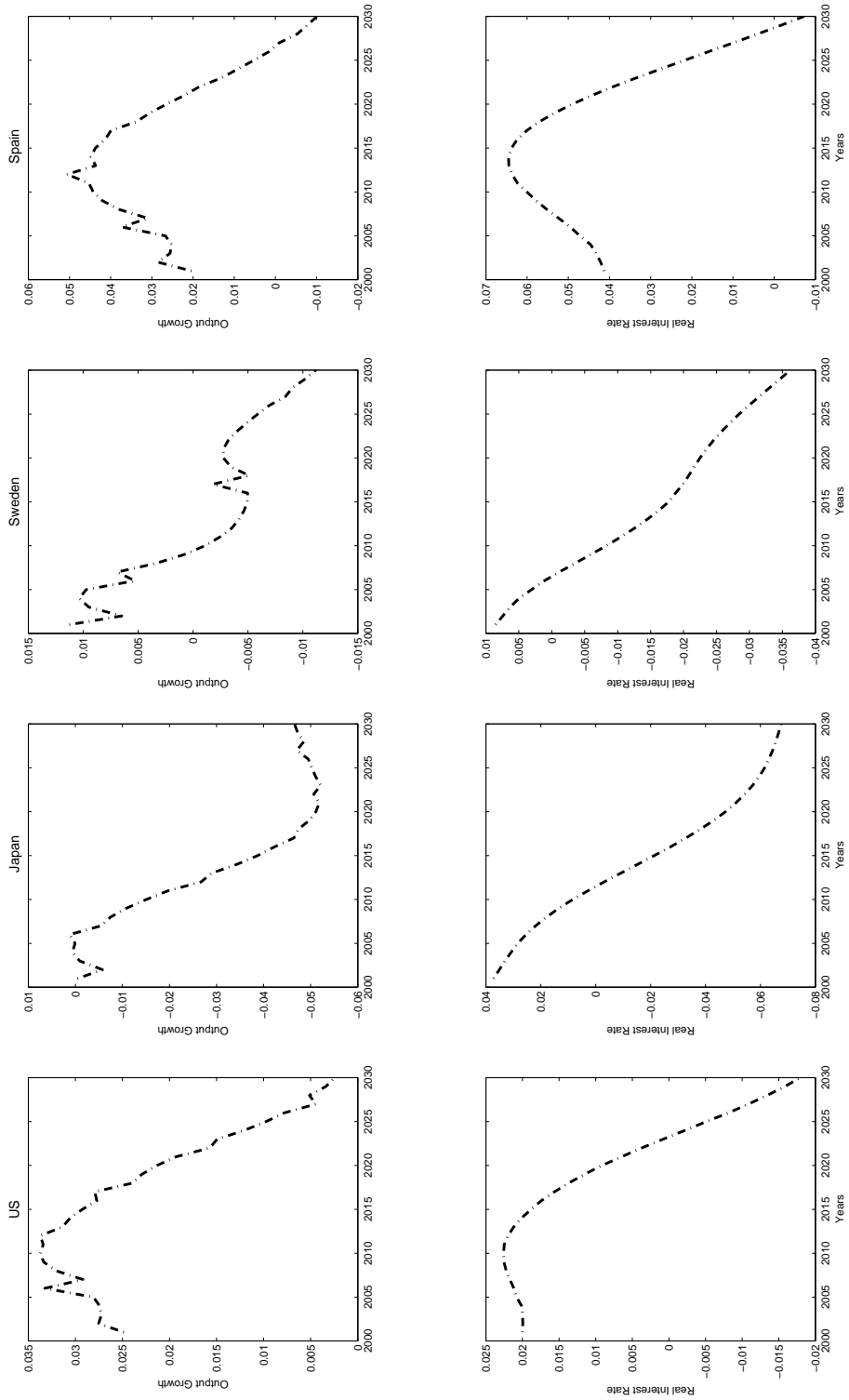
Figure 11: Simulation: *prediction* - Additional Countries

Figure 12: Simulation: *prediction* - Lower ρ_{yw}
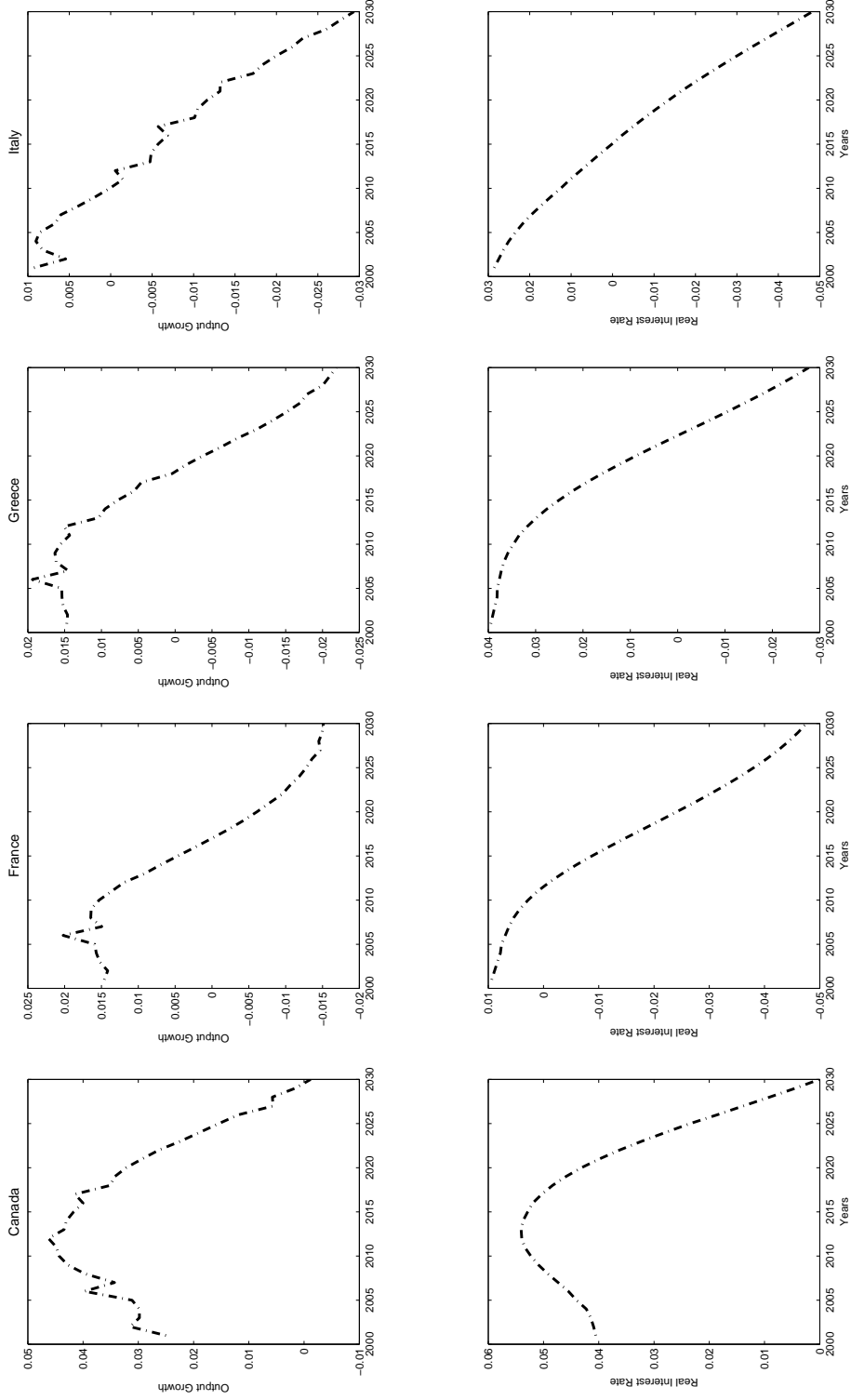
Figure 13: Simulation: *prediction* Additional Countries - Lower ρ_{yw}