Demographic Structure and Macroeconomic Trends

Yunus Aksoy (Birkbeck, London) Henrique S. Basso (Banco de España)
Tobias Grasl (Birkbeck, London) Ron Smith (Birkbeck, London)

6th Joint BoC/ECB Conference

8th June 2015

Medium-run outlook?

- Slow recovery after great recession and disappointedly low growth of productivity in the last decade have fostered a debate on medium-run prospects of develop economies.
- Debate (Gordon (2012, 2014)) has centred around
  - Future impact of innovation
  - Structural characteristics - demographics, education, inequality and debt overhang

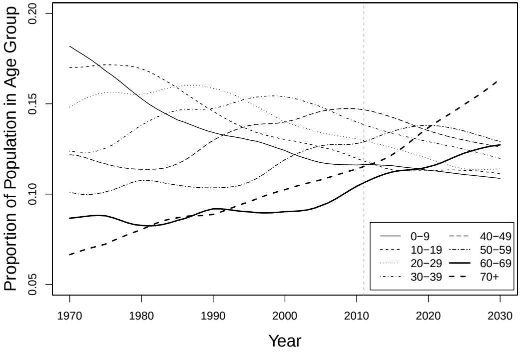


Figure: Demographic Structure in (sample) OECD countries

Population aged 60+ 16% in 1970 to 29% in 2030.
Working age group (20 – 59) 50% in 1970, 56% in 2003, 48% in 2030

Demographics, Labour Supply and Population Growth

- Normally demographics is generally linked to lower population growth and lower labour supply.
- A more general view: demographic structure, defined as the proportion of the population in each age group, may have an impact on economic performance. Different age groups
  - may have different savings behaviour, according to the life-cycle hypothesis;
  - may have different contributions to productivity gains, following the age profile of wages;
  - may contribute differently to the innovation process, with young and middle age workers contributing the most;
  - may generate different investment opportunities, as firms target their different needs.

This paper

Propose a framework to formally assess the impact of demographics in develop economies both empirically and theoretically

- Empirically: Assess the effect of changes in the demographic structure on medium term macroeconomic dynamics.

  Question 1 - Does demographic structure affect the trend of growth, investment, saving, real rates? How about innovation (R&D)?

- Theoretically: Build a model that incorporates both demographic heterogeneity and endogenous productivity to account for the empirical facts and analyse the channels through which demographics affect the macroeconomy.

  Question 2 - What does the theory has to say about the links between demographic structure and macroeconomic trends? Can a model account for the observed empirical patterns?

Methodology - Estimation

- Estimate a Panel VAR with intercept heterogeneity but slope homogeneity given by (we additionally control for population growth and oil prices (2 lags) which as demographics are assumed exogenous)

$$Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + controls + u_{it},$$

  W_{it} denote the matrix with the shares of the 7 first age group minus the last $j = 1, ..8$ $(0-9,\ 10-19, \ldots, 70+)$ in total population. Adjustment is done to avoid collinearity thus we restrict the coefficients of age groups to sum to 0.
  D is the 6×7 matrix of coefficients of the demographic variables.
  Endogenous variables - $Y_{it} = (g_{it}, I_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$

- Dataset covers the period 1970-2007. The twenty countries covered by the data are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

Methodology - Impact of Demographic Structure on Macroeconomy

- Demographic Structure is a slow moving variable. We are looking for not only its direct impact on each variable but the overall impact of demographics on the system after the feedback effects are accounted for, exploring the dynamic properties of the macroeconomic variables (system). We thus concentrate on the long-run contribution of demographics by looking at the demographic attractor

$$Y_{it}^{D} = (I - A_1 - A_2)^{-1} DW_{it}. \tag{1}$$

- Important to distinguish between steady state effect and long-run effect.

Estimation - Results

$$Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + u_{it}$$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

Table: Long-Run Demographic Impact - Matrix - $(I - A_1 - A_2)^{-1} D$

See D, A_1 and A_2

Estimation - Results

Matrix - $(I - A_1 - A_2)^{-1} D$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

Table: Long-Run Demographic Impact

Estimation - Results

Matrix - $(I - A_1 - A_2)^{-1} D$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

Table: Long-Run Demographic Impact

Estimation - Results

Matrix - $(I - A_1 - A_2)^{-1} D$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

Table: Long-Run Demographic Impact

Estimation - Results

Matrix - $(I - A_1 - A_2)^{-1} D$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

Table: Long-Run Demographic Impact

Estimation - Results

Matrix - $(I - A_1 - A_2)^{-1} D$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g_{t-1}	-0.14	0.16	0.11	0.10	0.11	-0.04	-0.32	0.01
I_{t-1}	-0.58	0.13	0.41	0.36	0.06	0.07	0.26	-0.70
S_{t-1}	-0.16	0.53	-0.26	0.36	0.39	0.72	-0.05	-1.53
H_{t-1}	-1.86	-0.13	0.66	2.44	0.47	0.59	-1.11	-1.05
rr_{t-1}	-0.43	-0.30	0.35	0.39	0.17	0.44	0.28	-0.91
π_{t-1}	0.96	0.65	-0.28	-1.01	-0.59	-0.26	0.22	0.32

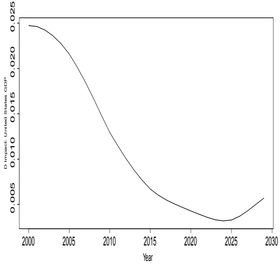
Table: Long-Run Demographic Impact

Estimation - 3 Generations Case

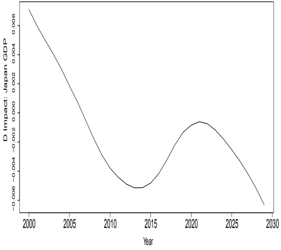
	β_1	β_2	β_3
g_{t-1}	0.02	0.12	-0.14
I_{t-1}	0.03	0.17	-0.20
S_{t-1}	0.28	0.31	-0.59
H_{t-1}	-0.64	1.53	-0.89
rr_{t-1}	-0.11	0.32	-0.20
π_{t-1}	0.68	-0.85	0.17

Table: Long-Run Demographic Impact

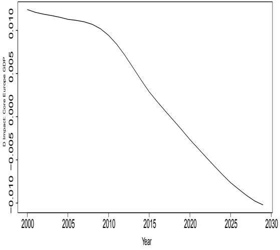
Robustness



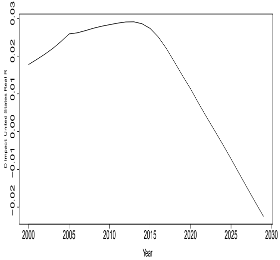
(a) US GDP



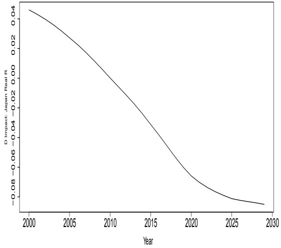
(b) Japan GDP



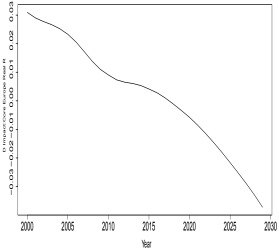
(c) Core Europe GDP



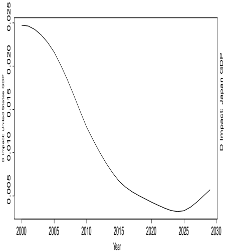
(d) US Real Rates



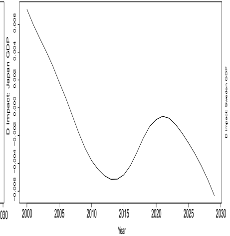
(e) Japan Real Rates



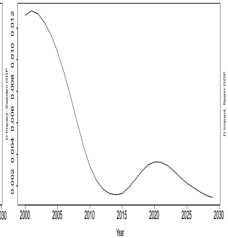
(f) Core Europe Real Rates



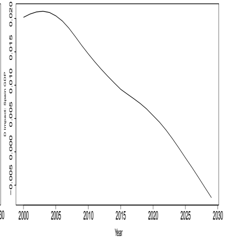
(a) US GDP



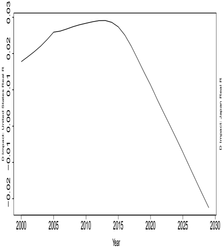
(b) Japan GDP



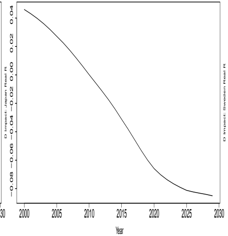
(c) Sweden GDP



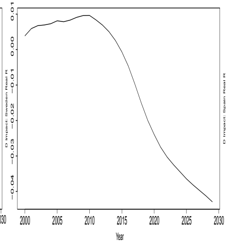
(d) Spain GDP



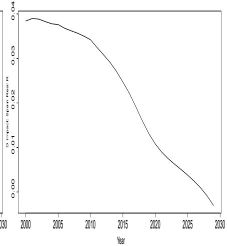
(e) US Real Rates



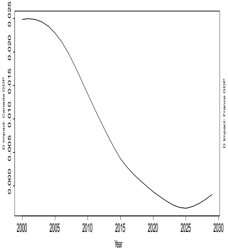
(f) Japan Real Rates



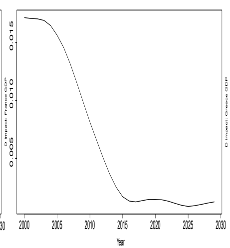
(g) Sweden Real Rates



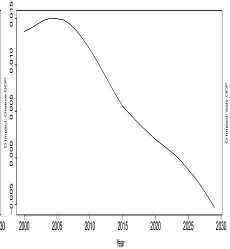
(h) Spain Real Rates



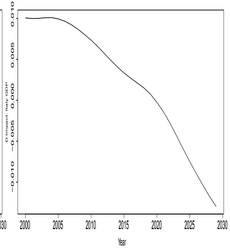
(a) Canada GDP



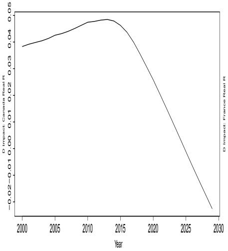
(b) France GDP



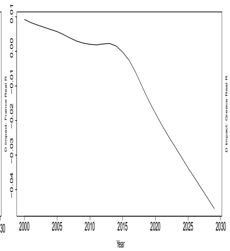
(c) Greece GDP



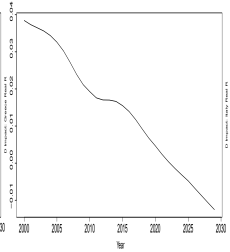
(d) Italy GDP



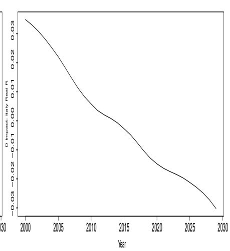
(e) Canada Real Rates



(f) France Real Rates

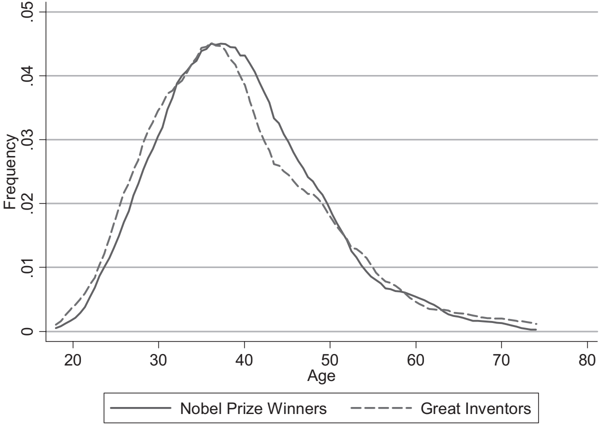


(g) Greece Real Rates



(h) Italy Real Rates

Link between demographics and innovation - Great Inventions



Note: Data are pooled across time.

Figure: Age Distribution of Great Inventions - Source Jones (2010)

Link between demographics and innovation - Patents

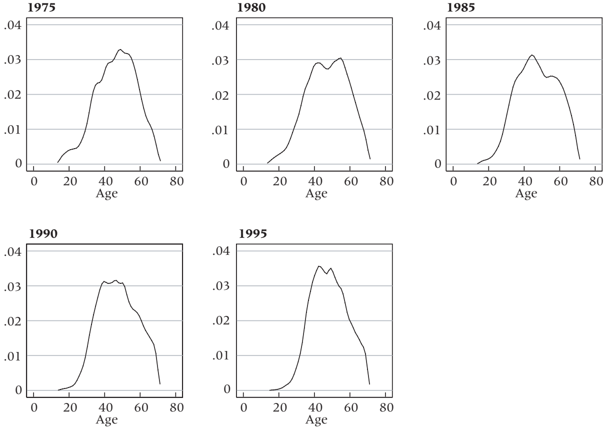


Figure: Distribution by single years of age of US inventors granted patents, 1975-95 - Source Jones (2005)

Estimation - Including Patent Applications

	Benchmark								Three Generations		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	β_1	β_2	β_3
y	-0.13	0.15	0.08	0.12	0.08	-0.06	-0.30	0.06	0.02	0.11	-0.13
I	-0.59	0.09	0.40	0.42	0.03	0.03	0.34	-0.72	0.01	0.17	-0.18
S	-0.24	0.58	-0.22	0.36	0.42	0.78	-0.11	-1.57	0.27	0.34	-0.61
H	-1.60	-0.20	0.43	2.43	0.63	0.48	-0.88	-1.29	-0.58	1.55	-0.97
rr	-0.38	-0.57	0.54	0.34	0.20	0.52	0.41	-1.05	-0.16	0.34	-0.19
PA	0.50	-0.56	0.02	0.05	0.70	-1.32	0.17	0.44	-0.16	0.22	-0.06
π	0.87	0.80	-0.35	-0.95	-0.66	-0.27	0.11	0.45	0.68	-0.87	0.20

Table: Long-Run Demographic Impact

Summary - Empirical Results

- Demographic Structure, after controlling for population growth, has significant effects on macroeconomic variables
- Savings, investment, hours worked, interest rate and output are negatively impacted when dependent group shares (young and old) increase and are positively affected when middle aged shares increase.
- When a measure of innovation is included, we confirm the asymmetry between young/mature workers and workers close to retirement, and that economies with higher share of workers innovate more.
- Using population predictions for the next 20 years we show that demographic changes are a strong force in reducing trend growth and real rates in most OECD economies. Particularly problematic for Southern European countries.

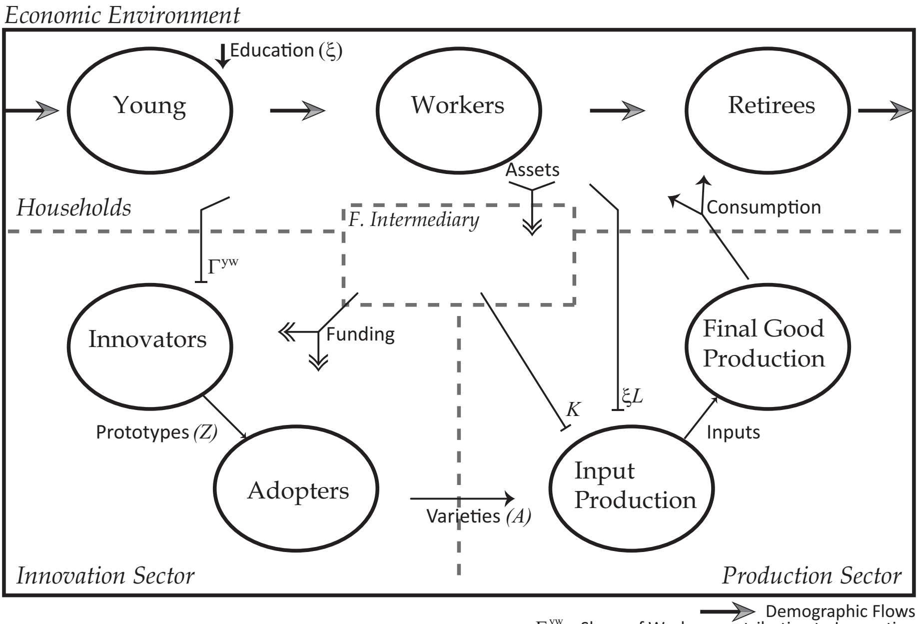
Overview

Two key features:

- demographic heterogeneity - closest framework is Gertler (1999) who develops a model a la Blanchard (1985) and Yaari (1965). We modify it to include young dependents and introduce human capital accumulation.
- endogenous productivity - closest framework is Comin and Gertler (2006) who develops a real business cycle model adding invention of new varieties a la Romer (1990). We simplify the framework to consider only a one sector economy.

Economic Environment

- The economy consists of three sectors: a production sector, an innovation sector and households.
  - The production sector comprises a final good producer and input producers.
  - Innovation sector consists of two joint processes. Product creation (prototypes) or R&D and product adoption, in which prototypes are made ready to be used in the production process.
  - Individuals, who supply labour, accumulate assets and consume, exhibit life-cycle behaviour, albeit of a simple form. Individuals face three stages of life: young/dependant, worker and retiree.
  - A financial intermediary is used to aggregate assets (capital and lending) of households to simplify exposition.



Γ^{yw} = Share of Workers contributing to Innovation

Model - Key Features

Model Detail

- Z_t^p be the stock of invented goods (prototypes) and Γ_t^{yw} = share of workers that contribute to innovation. Thus,

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi [(\tilde{\Psi}_t)^{\rho} (S_t)^{1-\rho}]^{-1} Z_t S_t^p + \phi Z_t^p$$

- Value of an Adopted Product (V_t) is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$$

- Aggregate consumption functions are:

$$C_t^w = \varsigma_t [R_t FA_t^w + H_t^w + D_t^w - T_t^w]$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r]$$

- Population (N_t) grows at rate n_t
  Young (N_t^y) becomes worker with probability $1 - \omega_y$
  Workers (N_t^w) retire with probability $1 - \omega_r$
  Once retired (N_t^r) individual survives with probability γ
- Share of Retirees over Workers, $\zeta_t^r = N_t^r / N_t^w$, and Share of young dependants over workers, $\zeta_t^y = N_t^y / N_t^w$.

Equilibrium

The symmetric equilibrium is a sequence of allocations and prices obtained such that:

a. Workers and retirees, maximize utility subject to their budget constraint and investment in education is such that society's marginal cost and benefit is equated;
b. Input and final firms maximize profits, and firm entry occurs until profits are equal to operating costs;
c. Innovators and adopters maximise their gains;
d. The financial intermediary selects assets to maximize profits, and their profits are shared amongst retirees and workers according to their share of assets;
e. Consumption goods, capital, labour and asset markets clear;

Simulation

Use the parameters of Gertler (1999) (for households and population dynamics) and Comin and Gertler (2006) (firms and innovation) ▸ Parameters. Show results for different ρ_{yw} (importance of workers for innovation) and λ_y (persistency of stock of workers/age for innovation).

Perform three simulation exercises (perfect foresight)

- titled *baby-boomers* analyses the effect of increasing fertility holding longevity constant.
- titled *aging* looks at the effects of increasing longevity by increasing γ permanently.
- titled *prediction*, attempt to match the change in the demographic structure predicted for a selected number of countries in our sample during the next two decades and measure their impact on growth and real interest rates.

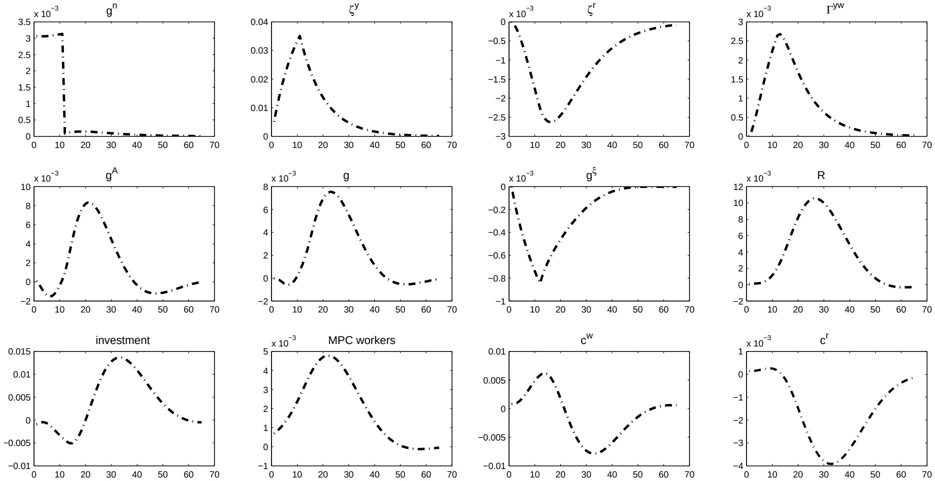


Figure: Simulation: *baby-boomers*

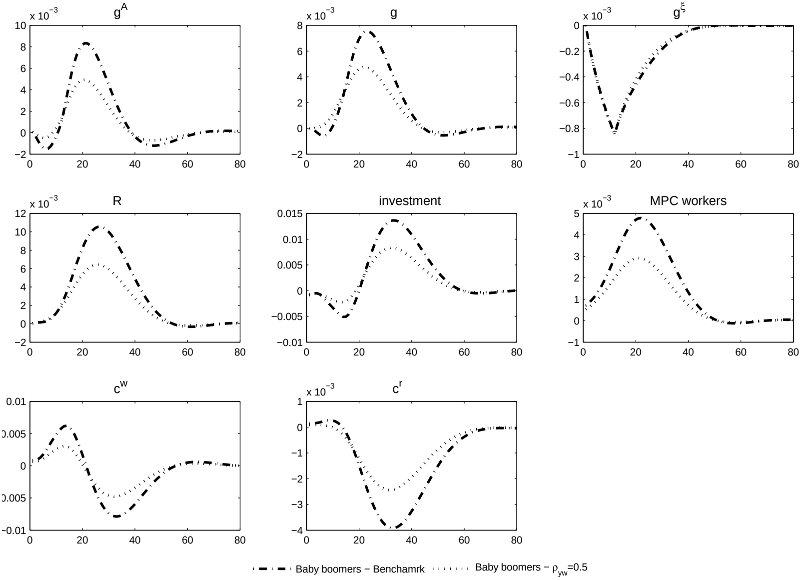


Figure: Simulation: benchmark *Baby-boomers* versus $\rho_{yw} = 0.5$

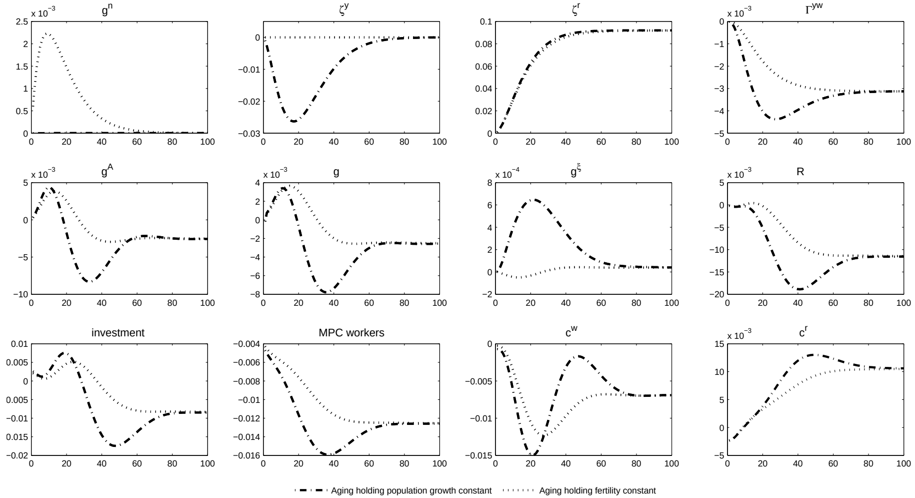


Figure: Simulation: *aging*

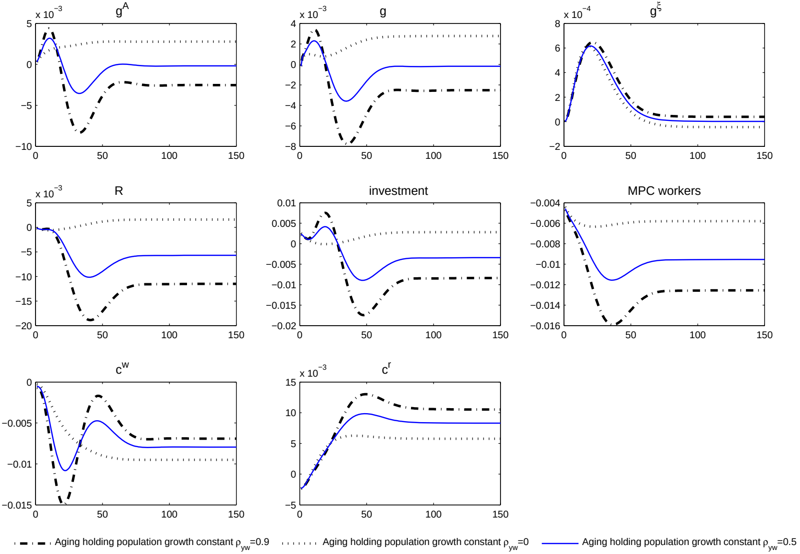


Figure: Simulation: benchmark *aging* versus different ρ_{yw}

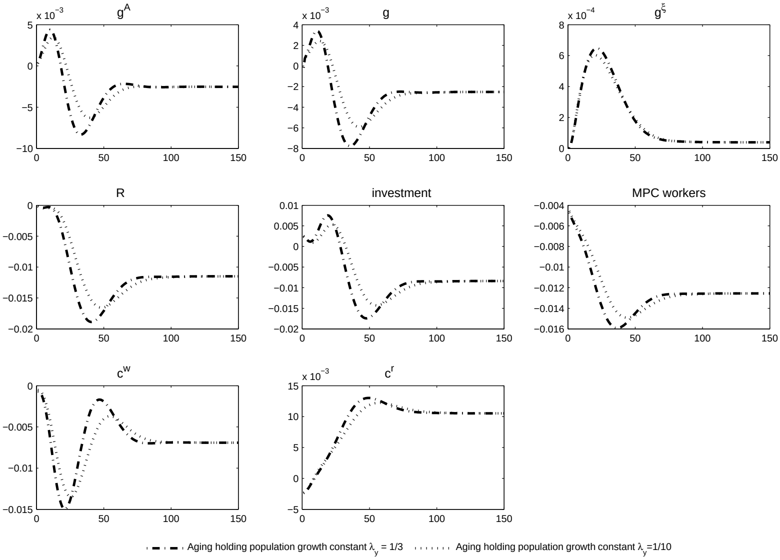


Figure: Simulation: benchmark *aging* versus different $\lambda_y = 1/10$

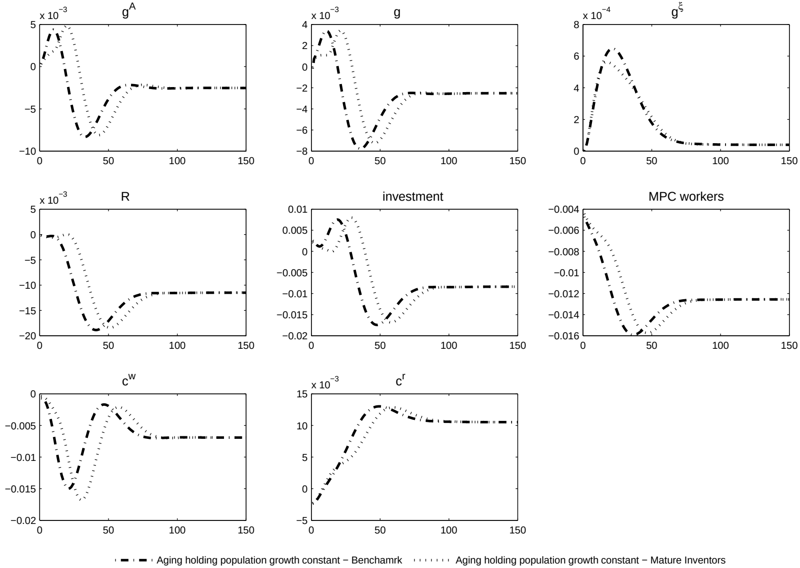


Figure: Simulation: benchmark *aging* versus Delayed flow of inventors

Simulation - Prediction

Period	Δs_w	Δs_r	g^n
2000-2005	0.5%	0.5%	1.053
2005-2011	-1.3%	2.0%	1.056
2011-2016	-1.4%	1.9%	1.043
2016-2021	-2.1%	2.2%	1.040
2021-2026	-1.3%	1.7%	1.037
2026-2031	-0.3%	0.8%	1.033

Table: *Prediction* Data Input: United States

- We match three measures $\{g^n, \Delta s_w, \Delta s_r\}$, namely, population growth, the share of workers and the share of retirees.
- Recall the share of workers in the population is given by $\frac{1}{1+\zeta_y+\zeta_r}$ and the share of retirees is given by $\frac{\zeta_r}{1+\zeta_y+\zeta_r}$, thus by setting those shares we are essentially selecting ζ_y and ζ_r, the young and retirees dependency ratios.
- By implicitly select three structural parameters, the fertility rate $\tilde{n}$, the longevity parameter γ and the probability a dependent become a worker ω_y.

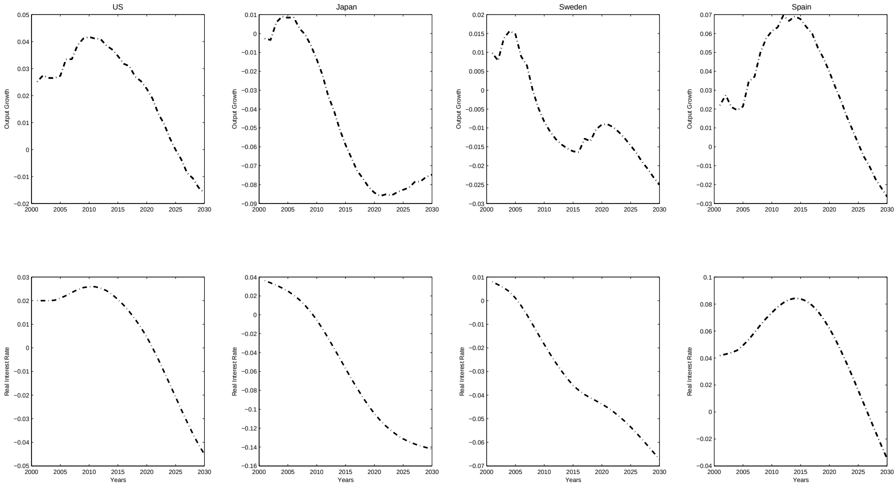
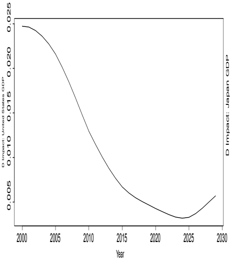
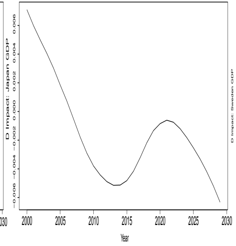


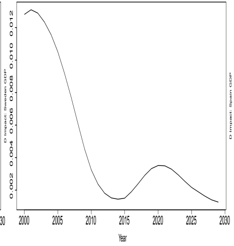
Figure: Simulation: *prediction*



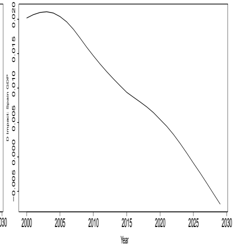
(a) US GDP



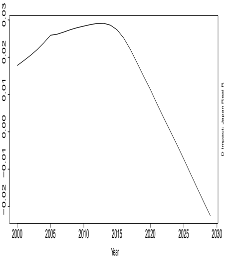
(b) Japan GDP



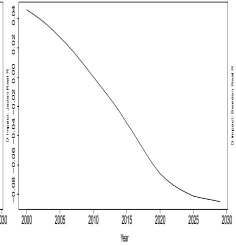
(c) Sweden GDP



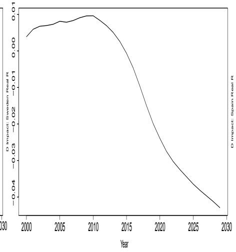
(d) Spain GDP



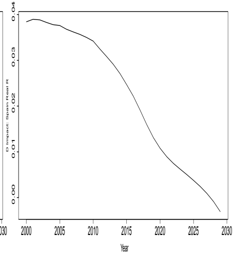
(e) US Real Rates



(f) Japan Real Rates



(g) Sweden Real Rates



(h) Spain Real Rates

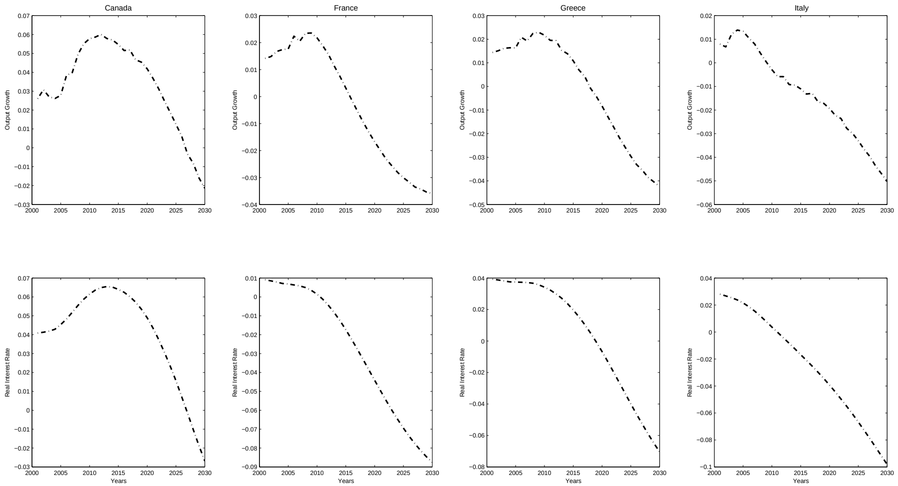
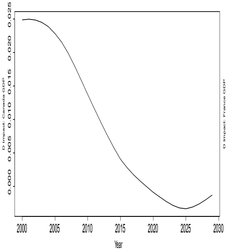
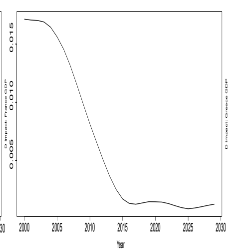


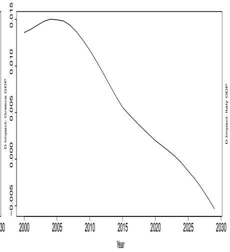
Figure: Simulation: *prediction* - Additional Countries



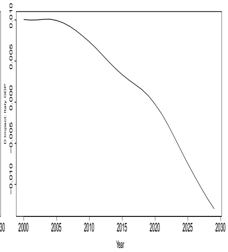
(a) Canada GDP



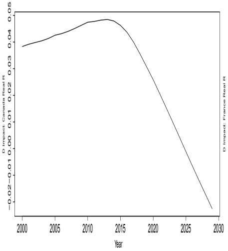
(b) France GDP



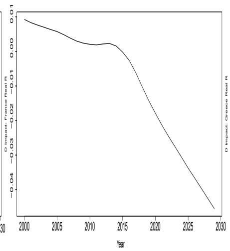
(c) Greece GDP



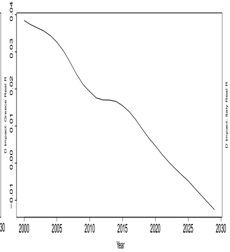
(d) Italy GDP



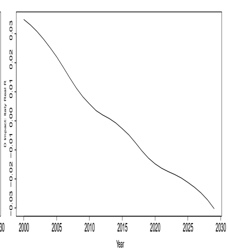
(e) Canada Real Rates



(f) France Real Rates



(g) Greece Real Rates



(h) Italy Real Rates

Conclusions

- Utilize a new empirical methodology to measure the effect of demographic structure on macroeconomic trends. Short-term impact on macro variables so demographic variables can be considered exogenous. Use properties of the dynamic system to obtain long-run impact and show age profile impacts macroeconomic trends.
- Build a model with demographic heterogeneity and endogenous productivity that matches well the empirical findings. Key channel is the link between innovation and demographics, which is supported by our evidence and evidence in Jones (2005).
- Population aging and reduced fertility expected in the next decades imply strong reduction on the trend of growth and real rates across most OECD economies, but particularly in Europe.

References I

BLANCHARD, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223–47.

COMIN, D., AND M. GERTLER (2006): "Medium-Term Business Cycles," *American Economic Review*, 96(3), 523–551.

GERTLER, M. (1999): "Government debt and social security in a life-cycle economy," *Carnegie-Rochester Conference Series on Public Policy*, 50(1), 61–110.

GORDON, R. J. (2012): "Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds," Working Paper 18315, National Bureau of Economic Research.

——— (2014): "The Demise of U.S. Economic Growth: Restatement, Rebuttal, and Reflections," Working Paper 19895, National Bureau of Economic Research.

JONES, B. F. (2005): "Age and Great Invention," NBER Working Paper 11359, NBER.

References II

——— (2010): "Age and Great Invention," *The Review of Economics and Statistics*, 92(1), 1–14.

PARK, C. (2010): "How does changing age distribution impact stock prices? a nonparametric approach," *Journal of Applied Econometrics*, 25(7), 1155–1178.

ROMER, P. M. (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98(5), S71–102.

YAARI, M. E. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, 32(2), 137–150.

Methodology - Demographic Structure

- How granular should demographic structure be? Due to lack of data for all periods for some countries we use data by 10 yrs of cohorts and thus do not to restrict age shape effects (as in Park (2010))
- Denote the share of age group $j = 1,..8$ $(0-9,\ 10-19, \ldots, 70+)$ in total population by w_{jit}. The effect on the variable of interest, say x_{it}, where i denote country and t denotes year, takes the form

$$x_{it} = \alpha + \sum_{j=1}^{8} \delta_j w_{ji,t} + u_{it}.$$

- $\sum_{j=1}^{8} w_{jit} = 1 \Rightarrow$ exact collinearity
  To deal with this, we restrict the coefficients to sum to 0, use $(w_{ji,t} - w_{8i,t})$ as explanatory variables and recover the coefficient of the oldest age group.
- We denote the 7 element vector of $(w_{ji,t} - w_{8i,t})$ as W_{it}.

Methodology - Dynamic System

- Endogenous variables $Y_{it} = (g_{it}, I_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$
- Ideal - Estimate an identified structural system allowing for expectations

$$\Phi_0 Y_t = \Phi_1 E_t(Y_{t+1}) + \Phi_2 Y_{t-1} + \Gamma W_t + \varepsilon_t. \tag{2}$$

- We can only estimate reduced form, where A solves $\Phi_1 A^2 - \Phi_0 A + \Phi_2 = 0$.

$$Y_t = AY_{t-1} + \Phi_0^{-1}\Gamma W_t + \Phi_0^{-1}\varepsilon_t. \tag{3}$$

- Given we want to analyse impact of W_t, we do not need to take a stand on link between A and Φ_0, Φ_1, Φ_2.

Back

Estimation - Results I

$$Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + u_{it}$$

	g_{t-1}	I_{t-1}	S_{t-1}	H_{t-1}	rr_{t-1}	π_{t-1}
g	0.24	-0.18	0.01	-0.01	-0.26	-0.28
I	0.17	0.76	0.01	0.01	-0.10	-0.10
S	-0.12	-0.10	0.77	-0.01	-0.10	-0.07
H	0.22	-0.05	0.01	0.92	-0.13	-0.11
rr	-0.19	-0.18	-0.10	0.05	0.90	0.24
π	0.36	0.21	0.05	-0.02	-0.16	0.55

Table: Sum of VAR coefficients $A_1 + A_2$

- There is evidence that all our endogenous variables are Granger causal for some other variables in the system, except in the case of savings which does not have a significant influence on any other variable
- Only surprising feature lagged investment has a negative effect on growth, though there is a strong positive contemporaneous correlation between the growth and investment residuals.

Estimation - Results II

$$Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + u_{it}$$

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.06	0.25*	0.18*	-0.03	-0.03	0.02	-0.07	-0.25*
I	-0.03	0.04	0.08*	-0.03	-0.06	0.03	0.18*	-0.20*
S	-0.10*	0.17*	0.02	0.11	0.08	0.19*	0.01	-0.49*
H	-0.10*	-0.02	0.07	0.14*	-0.03	0.08	0.05	-0.20*
rr	-0.33*	-0.08	0.14	0.29*	0.21*	0.16	0.01	-0.39*
π	0.50*	0.13	-0.16	-0.46*	-0.30*	-0.07	0.18	0.19*

Table: Short-Run Demographic Impact - Matrix D

Back

Robustness - 2-way effects

- Benchmark model is a one-way fixed effects model and includes oil prices as a the only variable that affects all countries.
- So if there are shared, cross-country factors driving the trend in dependent variables as well as demographic variables, this trend may be wrongly attributed to the demographic variables.
- A two-way effects model avoids this issue by removing any common cross-country factors from all variables prior to estimation.

	Benchmark								Three Generations		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	β_1	β_2	β_3
g	-0.16	0.18	0.09	0.11	0.14	-0.04	-0.29	-0.03	0.01	0.13	-0.14
I	-0.71	-0.12	0.27	0.31	0.42	0.15	0.42	-0.75	-0.26	0.36	-0.10
S	0.09	0.78	0.01	0.39	0.25	0.22	0.12	-1.87	0.63	0.20	-0.83
H	-2.13	-0.14	0.50	2.62	1.04	0.37	-0.90	-1.36	-0.91	1.76	-0.85
rr	-0.19	-0.08	0.41	0.07	-0.12	0.50	0.17	-0.76	-0.02	0.15	-0.13
π	0.54	0.28	-0.56	-0.83	0.00	-0.03	0.02	0.57	0.44	-0.59	0.15

Table: Long-Run Demographic Impact (2-way effects)

Back

Production

- Final Good Producers -

$$Y_{c,t} = \left[\int_0^{N_t^f} (Y_{c,t}^j)^{(1/\mu_t)} dj \right]^{\mu_t}$$

- where N_t^f is the number of firms in input sectors and $\mu_t = \mu(N_t^f)$, $\mu'(\cdot) < 0$. So variable mark-up and fact that firms must pay operating costs control entry and exit.
- Production of input firm j

$$Y_{c,t}^j = \left[(U_t^j K_t^j)^\alpha (\xi_t L_t^j)^{(1-\alpha)} \right]^{(1-\gamma_I)} \left[M_t^j \right]^{\gamma_I}$$

- Intermediate composite good

$$M_t^j = \left[\int_0^{A_t} (M_t^{ji})^{(1/\vartheta)} di \right]^\vartheta$$

where each producer i acquires the right to market the good via the creation and adoption process. Thus A_t is determined by innovation sector.

Innovation: R&D

- Let Z_t^p be the stock of invented goods (prototypes) at the beginning of time t. Inventor p spends S_t^p to add new prototypes to her stock. Productivity of innovation spending is given by φ_t.

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi [(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} Z_t S_t^p + \phi Z_t^p$$

- $\phi =$ implied product survival rate
  $\rho =$ elasticity of new technology creation
  $\Gamma_t^{yw} =$ share of workers that contribute to innovation
  $\rho_{yw} =$ Importance of workers for innovation process.

- Innovators borrow S_t^p from the household. Define J_t as the value of an invented intermediary good. Then

$$\phi E[J_{t+1}] = \frac{R_{t+1}}{\varphi_t}$$

Innovation: Adoption

- Let $A_t^q \subset Z_t^q$ denote the stock of converted goods marketed to firms. Adopter q invest (intensity) Ξ_t to transform Z_t^q into A_t^q. Conversion process is successful with probability $\lambda_t = \lambda\left(\frac{A_t^q}{\tilde{\Psi}_t}\Xi_t\right)$ with $\lambda' > 0$ Flow of converted goods

$$A_{t+1}^q = \lambda_t \phi (Z_t^q - A_t^q) + \phi A_t^q$$

- A converted good can be marketed at every period to firms, thus its value, denoted V_t is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$$

  where $\Pi_{m,t}$ is the profit from selling an intermediate good to input firms.
- The value of a unadopted product (J_t) is

$$J_t = \max_{\Xi_t} -\Xi_t + (R_{t+1})^{-1} \phi E_t[\lambda_t V_{t+1} + (1-\lambda_t) J_{t+1}]$$

Household Sector: Population Dynamics

- Population N_t
  Young (N_t^y) becomes worker with probability $1-\omega_y$
  Workers (N_t^w) retire with probability $1-\omega_r$
  Once retired (N_t^r) individual survives with probability γ.

$$
\begin{aligned}
N_{t+1}^y &= \tilde{n}_{t,t+1}N_t^y + \omega^y N_t^y = (\tilde{n}_{t,t+1}+\omega^y)N_t^y = n_{t,t+1}N_t^y, \\
N_{t+1}^w &= (1-\omega^y)N_t^y + \omega^r N_t^w, \\
N_{t+1}^r &= (1-\omega^r)N_t^w + \gamma_{t,t+1}N_t^r \\
&\quad \text{define } \zeta_t^r = N_t^r/N_t^w \text{ and } \zeta_t^y = N_t^y/N_t^w.
\end{aligned}
$$

- Stock of workers that contribute to innovation

$$
\Gamma_t^{yw} \equiv (1-\omega^y)\frac{N_t^y}{N_t} + (1-\lambda^y)\Gamma_{t-1}^{yw} = (1-\omega^y)\frac{\zeta_t^y}{1+\zeta_t^y+\zeta_t^r} + (1-\lambda^y)\Gamma_{t-1}^{yw},
$$

$\lambda^y < 1$ augments the stock of young workers just entered work! Worker's age matters for innovation.

Household Sector: Human Capital

- Let ξ_t be the average effective units across workers at period t.
  Let $I_t^y = \frac{\tau_t}{W_t N_t^w}$ be the total effective expenditure society makes on the education of the young, financed by transfer τ_t from workers.
  Each young who becomes a worker at the end of period t will provide ξ_{t+1}^y effective units.

$$\xi_{t+1}^y = \rho_E \xi_t + \frac{\chi_E}{2} \left(\frac{I_t^y}{\xi_t} \right)^2 \xi_t$$

- The evolution of workers effective labour units

$$\xi_{t+1} = \omega_r \frac{N_t^w}{N_{t+1}^w} \xi_t + (1 - \omega^y) \frac{N_t^y}{N_{t+1}^w} \xi_{t+1}^y$$

Household Sector: Consumption and Labour

- Retirees are assumed not to work. Two key assumptions to offset impact of risk of death (perfect annuity market) and retirement (risk neutrality) on households decision. Gertler (1999)
- Thus, for $z = \{w, r\}$ we assume agent j selects consumption and asset holdings to maximise

$$V_t^{jz} = \left\{ (C^{jz})^{\rho_U} + \beta_{t,t+1}^{z} (E_t[V_{t+1}^{j} \mid z]^{\rho_U}) \right\}^{1/\rho_U}$$

subject to

$$C_t^{jz} + FA_{t+1}^{jz} = R_t^z FA_t^{jz} + W_t \xi_t^j l^z + d_t^z - \tau_t^{jz} l^z$$

- Aggregate consumption functions are:

$$C_t^w = \varsigma_t [R_t FA_t^w + H_t^w + D_t^w - T_t^w]$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r]$$

Back

Growth

Three drivers of growth:

a. exogenous growth of population, n_t
b. endogenous growth rate of effective labour force, ξ
c. endogenous innovation/adoption of new intermediate goods, A_t that affects K_t, L_t

Parameters

Standard

$\beta = 0.96$ $\alpha = 0.33$ $\delta = 0.08$

$U == 80\%$ $\gamma_I = 0.5$ $\mu = 1.1$

$(1/(1-\rho_U)) = 0.25$

Innovation

obsolescence: (1-ϕ)=0.03

productivity in innovation:$\chi = 94.42$

elasticity of intermediate goods w.r.t R&D $\rho = 0.9$

ave. adoption time $\lambda = 0.1$

elasticity of adoption time to intensity $\epsilon_\lambda = 0.9$

Population

$(1-\omega^y) = 0.05$ $\frac{N^y}{N^w} = 48\%$

$(1-\omega^r) = 0.023$ $\frac{N^r}{N^w} = 20\%$

10 yrs in retirement $\gamma = 0.9$

Population and Inovation

ratio of workers influencing innovation $(1-\lambda_y) = \frac{2}{3}$

importance of worker to innovation productivity $\rho_{yw} = .9$

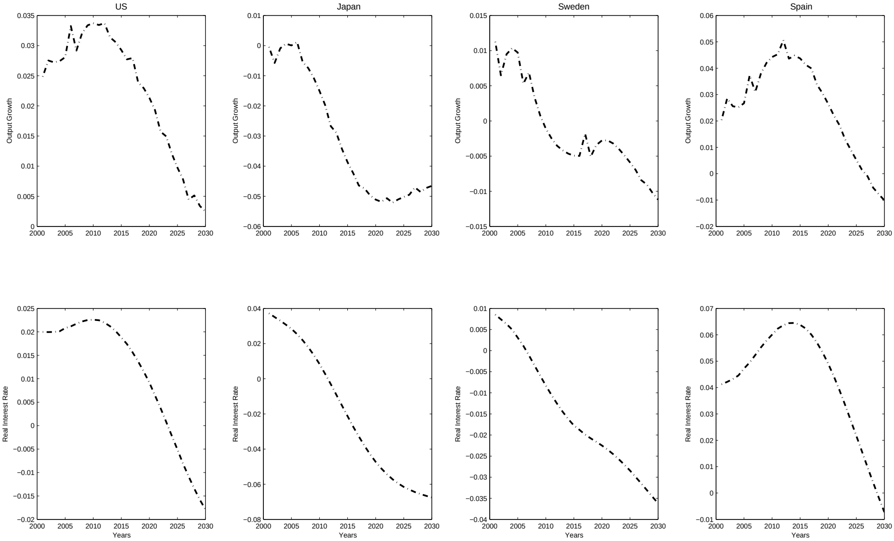


Figure: Simulation: *prediction* - Lower ρ_{yw}

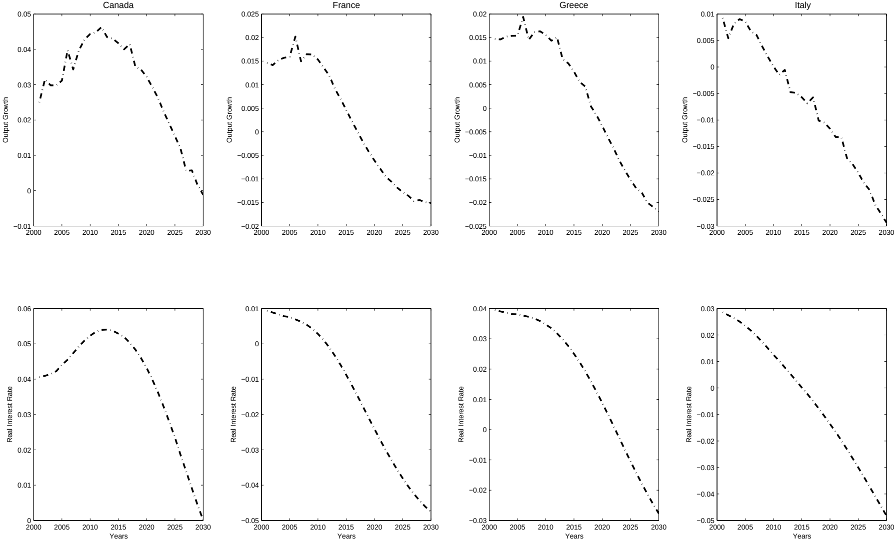


Figure: Simulation: *prediction* Additional Countries - Lower ρ_{yw}