Demographic Structure and Macroeconomic Trends

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Motivation

Medium-run outlook?

- Slow recovery after great recession and disappointedly low growth of productivity in the last decade have fostered a debate on medium-run prospects of develop economies.

- Debate (Gordon (2012, 2014)) has centred around
  - Future impact of innovation
  - Structural characteristics - demographics, education, inequality and debt overhang
Motivation

[Figure: Demographic Structure in (sample) OECD countries]

Population aged 60+ 16% in 1970 to 29% in 2030.
Working age group (20 – 59) 50% in 1970, 56% in 2003, 48% in 2030
Motivation

Demographics, Labour Supply and Population Growth

- Normally demographics is generally linked to lower population growth and lower labour supply.

- A more general view: demographic structure, defined as the proportion of the population in each age group, may have an impact on economic performance.

  - Different age groups
    - may have different savings behaviour, according to the life-cycle hypothesis;
    - may have different contributions to productivity gains, following the age profile of wages;
    - may contribute differently to the innovation process, with young and middle age workers contributing the most;
    - may generate different investment opportunities, as firms target their different needs.
This paper

Propose a framework to formally assess the impact of demographics in developing economies both empirically and theoretically

▶ Empirically: Assess the effect of changes in the demographic structure on medium term macroeconomic dynamics.

Question 1 - Does demographic structure affect the trend of growth, investment, saving, real rates? How about innovation (R&D)?

▶ Theoretically: Build a model that incorporates both demographic heterogeneity and endogenous productivity to account for the empirical facts and analyse the channels through which demographics affect the macroeconomy.

Question 2 - What does the theory have to say about the links between demographic structure and macroeconomic trends? Can a model account for the observed empirical patterns?
Empirical Analysis

Methodology - Estimation

- Estimate a Panel VAR with intercept heterogeneity but slope homogeneity given by (we additionally control for population growth and oil prices (2 lags) which as demographics are assumed exogenous)

\[ Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + D W_{it} + \text{controls} + u_{it}, \]

\( W_{it} \) denote the matrix with the shares of the 7 first age group minus the last \( j = 1, \ldots, 8 \) (0−9, 10−19, \ldots, 70+) in total population. Adjustment is done to avoid collinearity thus we restrict the coefficients of age groups to sum to 0.

\( D \) is the 6 × 7 matrix of coefficients of the demographic variables.

Endogenous variables - \( Y_{it} = (g_{it}, l_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})' \)

- Dataset covers the period 1970-2007. The twenty countries covered by the data are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.
Methodology - Impact of Demographic Structure on Macroeconomy

Demographic Structure is a slow moving variable. We are looking for not only its direct impact on each variable but the overall impact of demographics on the system after the feedback effects are accounted for, exploring the dynamic properties of the macroeconomic variables (system). We thus concentrate on the long-run contribution of demographics by looking at the demographic attractor

$$Y_{it}^D = (I - A_1 - A_2)^{-1} DW_{it}. \tag{1}$$

Important to distinguish between steady state effect and long-run effect.
Estimation - Results

\[ Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + u_{it} \]

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Table: Long-Run Demographic Impact - Matrix - \((I - A_1 - A_2)^{-1} D\)

See \( D, A_1 \) and \( A_2 \)
Estimation - Results

Matrix - \((I - A_1 - A_2)^{-1} D\)

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**Table:** Long-Run Demographic Impact
## Empirical Analysis

### Estimation - Results

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**Estimation - Results**

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**Table:** Long-Run Demographic Impact
## Estimation - 3 Generations Case

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**Table:** Long-Run Demographic Impact
Empirical Analysis

(a) US GDP

(b) Japan GDP

(c) Core Europe GDP

(d) US Real Rates

(e) Japan Real Rates

(f) Core Europe Real Rates
Empirical Analysis

(a) US GDP  
(b) Japan GDP  
(c) Sweden GDP  
(d) Spain GDP

(e) US Real Rates  
(f) Japan Real Rates  
(g) Sweden Real Rates  
(h) Spain Real Rates
Empirical Analysis

(a) Canada GDP
(b) France GDP
(c) Greece GDP
(d) Italy GDP

(e) Canada Real Rates
(f) France Real Rates
(g) Greece Real Rates
(h) Italy Real Rates
Link between demographics and innovation - Great Inventions

Figure: Age Distribution of Great Inventions - Source Jones (2010)

Note: Data are pooled across time.
Link between demographics and innovation - Patents

Figure: Distribution by single years of age of US inventors granted patents, 1975-95 - Source Jones (2005)
Empirical Analysis

Estimation - Including Patent Applications

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</tbody>
</table>

Table: Long-Run Demographic Impact
Summary - Empirical Results

- Demographic Structure, after controlling for population growth, has significant effects on macroeconomic variables.

- Savings, investment, hours worked, interest rate and output are negatively impacted when dependent group shares (young and old) increase and are positively affected when middle aged shares increase.

- When a measure of innovation is included, we confirm the asymmetry between young/mature workers and workers close to retirement, and that economies with higher share of workers innovate more.

- Using population predictions for the next 20 years we show that demographic changes are a strong force in reducing trend growth and real rates in most OECD economies. Particularly problematic for Southern European countries.
Overview

Two key features:

- demographic heterogeneity - closest framework is Gertler (1999) who develops a model a la Blanchard (1985) and Yaari (1965). We modify it to include young dependents and introduce human capital accumulation.

- endogenous productivity - closest framework is Comin and Gertler (2006) who develops a real business cycle model adding invention of new varieties a la Romer (1990). We simplify the framework to consider only a one sector economy.
The economy consists of three sectors: a production sector, an innovation sector and households.

- The production sector comprises a final good producer and input producers.
- Innovation sector consists of two joint processes. Product creation (prototypes) or R&D and product adoption, in which prototypes are made ready to be used in the production process.
- Individuals, who supply labour, accumulate assets and consume, exhibit life-cycle behaviour, albeit of a simple form. Individuals face three stages of life: young/dependent, worker and retiree.
- A financial intermediary is used to aggregate assets (capital and lending) of households to simplify exposition.
**Model - Key Features**

- $Z^p_t$ be the stock of invented goods (prototypes) and $\Gamma^{yw}_t = \text{share of workers that contribute to innovation. Thus,}$

$$Z^p_{t+1} = \varphi_t S^p_t + \phi Z^p_t = (\Gamma^{yw}_t)^{\rho_{yw}} \chi[(\tilde{\Psi}_t)^\rho (S_t)^{1-\rho}]^{-1} Z_t S^p_t + \phi Z^p_t$$

- Value of an Adopted Product ($V_t$) is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$$

- Aggregate consumption functions are:

$$C^w_t = \varsigma_t [R_t F_A^w + H^w_t + D^w_t - T^w_t]$$

$$C^r_t = \varepsilon_t \varsigma_t [R_t F_A^r + D^r_t]$$

- Population ($N_t$) grows at rate $n_t$
- Young ($N^y_t$) becomes worker with probability $1 - \omega_y$
- Workers ($N^w_t$) retire with probability $1 - \omega_r$
- Once retired ($N^r_t$) individual survives with probability $\gamma$

- Share of Retirees over Workers, $\zeta^r_t = N^r_t / N^w_t$, and Share of young dependants over workers, $\zeta^y_t = N^y_t / N^w_t$. 
Equilibrium

The symmetric equilibrium is a sequence of allocations and prices obtained such that:

a. Workers and retirees, maximize utility subject to their budget constraint and investment in education is such that society’s marginal cost and benefit is equated;

b. Input and final firms maximize profits, and firm entry occurs until profits are equal to operating costs;

c. Innovators and adopters maximise their gains;

d. The financial intermediary selects assets to maximize profits, and their profits are shared amongst retirees and workers according to their share of assets;

e. Consumption goods, capital, labour and asset markets clear;
Simulation

Use the parameters of Gertler (1999) (for households and population dynamics) and Comin and Gertler (2006) (firms and innovation). Show results for different $\rho_{yw}$ (importance of workers for innovation) and $\lambda_y$ (persistency of stock of workers/age for innovation).

Perform three simulation exercises (perfect foresight)

- titled baby-boomers analyses the effect of increasing fertility holding longevity constant.
- titled aging looks at the effects of increasing longevity by increasing $\gamma$ permanently.
- titled prediction, attempt to match the change in the demographic structure predicted for a selected number of countries in our sample during the next two decades and measure their impact on growth and real interest rates.
Figure: Simulation: baby-boomers
Figure: Simulation: benchmark Baby-boomers versus $\rho_{yw} = 0.5$
Theoretical Model

Figure: Simulation: *aging*
Theoretical Model

Figure: Simulation: benchmark aging versus different $\rho_{yw}$
Figure: Simulation: benchmark aging versus different $\lambda_y = 1/10$
Figure: Simulation: benchmark aging versus Delayed flow of inventors
Simulation - Prediction

<table>
<thead>
<tr>
<th>Period</th>
<th>$\Delta s_w$</th>
<th>$\Delta s_r$</th>
<th>$g^n$</th>
</tr>
</thead>
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<tr>
<td>2000-2005</td>
<td>0.5%</td>
<td>0.5%</td>
<td>1.053</td>
</tr>
<tr>
<td>2005-2011</td>
<td>-1.3%</td>
<td>2.0%</td>
<td>1.056</td>
</tr>
<tr>
<td>2011-2016</td>
<td>-1.4%</td>
<td>1.9%</td>
<td>1.043</td>
</tr>
<tr>
<td>2016-2021</td>
<td>-2.1%</td>
<td>2.2%</td>
<td>1.040</td>
</tr>
<tr>
<td>2021-2026</td>
<td>-1.3%</td>
<td>1.7%</td>
<td>1.037</td>
</tr>
<tr>
<td>2026-2031</td>
<td>-0.3%</td>
<td>0.8%</td>
<td>1.033</td>
</tr>
</tbody>
</table>

Table: Prediction Data Input: United States

- We match three measures $\{g^n, \Delta s_w, \Delta s_r\}$, namely, population growth, the share of workers and the share of retirees.
- Recall the share of workers in the population is given by $\frac{1}{1+\zeta_y+\zeta_r}$ and the share of retirees is given by $\frac{\zeta_r}{1+\zeta_y+\zeta_r}$, thus by setting those shares we are essentially selecting $\zeta_y$ and $\zeta_r$, the young and retirees dependency ratios.
- By implicitly select three structural parameters, the fertility rate $\tilde{n}$, the longevity parameter $\gamma$ and the probability a dependent become a worker $\omega_y$. 

Figure: Simulation: prediction
Theoretical Model

(a) US GDP
(b) Japan GDP
(c) Sweden GDP
(d) Spain GDP
(e) US Real Rates
(f) Japan Real Rates
(g) Sweden Real Rates
(h) Spain Real Rates
Figure: Simulation: *prediction* - Additional Countries
Conclusions

(a) Canada GDP
(b) France GDP
(c) Greece GDP
(d) Italy GDP

(e) Canada Real Rates
(f) France Real Rates
(g) Greece Real Rates
(h) Italy Real Rates
Conclusions

- Utilize a new empirical methodology to measure the effect of demographic structure on macroeconomic trends. Short-term impact on macro variables so demographic variables can be considered exogenous. Use properties of the dynamic system to obtain long-run impact and show age profile impacts macroeconomic trends.

- Build a model with demographic heterogeneity and endogenous productivity that matches well the empirical findings. Key channel is the link between innovation and demographics, which is supported by our evidence and evidence in Jones (2005).

- Population aging and reduced fertility expected in the next decades imply strong reduction on the trend of growth and real rates across most OECD economies, but particularly in Europe.
References I


References II


Methodology - Demographic Structure

- How granular should demographic structure be? Due to lack of data for all periods for some countries we use data by 10 yrs of cohorts and thus do not to restrict age shape effects (as in Park (2010))

- Denote the share of age group $j = 1, \ldots, 8 \ (0 - 9, \ 10 - 19, \ldots, 70+)$ in total population by $w_{jit}$. The effect on the variable of interest, say $x_{it}$, where $i$ denote country and $t$ denotes year, takes the form

$$x_{it} = \alpha + \sum_{j=1}^{8} \delta_j w_{ji,t} + u_{it}.$$ 

- $\sum_{j=1}^{8} w_{jit} = 1 \Rightarrow$ exact collinearity

  To deal with this, we restrict the coefficients to sum to 0, use $(w_{ji,t} - w_{8i,t})$ as explanatory variables and recover the coefficient of the oldest age group.

- We denote the 7 element vector of $(w_{ji,t} - w_{8i,t})$ as $W_{it}$. 
Methodology - Dynamic System

- Endogenous variables $Y_{it} = (g_{it}, l_{it}, S_{it}, H_{it}, r_{rit}, \pi_{it})'$

- Ideal - Estimate an identified structural system allowing for expectations

\[ \Phi_0 Y_t = \Phi_1 E_t(Y_{t+1}) + \Phi_2 Y_{t-1} + \Gamma W_t + \varepsilon_t. \]  \hspace{1cm} (2)

- We can only estimate reduced form, where $A$ solves $\Phi_1 A^2 - \Phi_0 A + \Phi_2 = 0$.

\[ Y_t = A Y_{t-1} + \Phi_0^{-1} \Gamma W_t + \Phi_0^{-1} \varepsilon_t. \]  \hspace{1cm} (3)

- Given we want to analyse impact of $W_t$, we do not need to take a stand on link between $A$ and $\Phi_0, \Phi_1, \Phi_2$. 

Back
Estimation - Results I

\[ Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + u_{it} \]

<table>
<thead>
<tr>
<th></th>
<th>( g_{t-1} )</th>
<th>( l_{t-1} )</th>
<th>( S_{t-1} )</th>
<th>( H_{t-1} )</th>
<th>( rr_{t-1} )</th>
<th>( \pi_{t-1} )</th>
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<tr>
<td>( g )</td>
<td>0.24</td>
<td>-0.18</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.26</td>
<td>-0.28</td>
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<tr>
<td>( l )</td>
<td>0.17</td>
<td>0.76</td>
<td>0.01</td>
<td>0.01</td>
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<td>-0.10</td>
</tr>
<tr>
<td>( S )</td>
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<td>0.77</td>
<td>-0.01</td>
<td>-0.10</td>
<td>-0.07</td>
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<tr>
<td>( H )</td>
<td>0.22</td>
<td>-0.05</td>
<td>0.01</td>
<td>0.92</td>
<td>-0.13</td>
<td>-0.11</td>
</tr>
<tr>
<td>( rr )</td>
<td>-0.19</td>
<td>-0.18</td>
<td>-0.10</td>
<td>0.05</td>
<td>0.90</td>
<td>0.24</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.36</td>
<td>0.21</td>
<td>0.05</td>
<td>-0.02</td>
<td>-0.16</td>
<td>0.55</td>
</tr>
</tbody>
</table>

**Table**: Sum of VAR coefficients \( A_1 + A_2 \)

- There is evidence that all our endogenous variables are Granger causal for some other variables in the system, except in the case of savings which does not have a significant influence on any other variable.
- Only surprising feature lagged investment has a negative effect on growth, though there is a strong positive contemporaneous correlation between the growth and investment residuals.
Estimation - Results II

\[ Y_{it} = a_i + A_1 Y_{i,t-1} + A_2 Y_{i,t-2} + DW_{it} + u_{it} \]

<table>
<thead>
<tr>
<th></th>
<th>( \delta_1 )</th>
<th>( \delta_2 )</th>
<th>( \delta_3 )</th>
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<th>( \delta_5 )</th>
<th>( \delta_6 )</th>
<th>( \delta_7 )</th>
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<td>-0.06</td>
<td>0.25*</td>
<td>0.18*</td>
<td>-0.03</td>
<td>-0.03</td>
<td>0.02</td>
<td>-0.07</td>
<td>-0.25*</td>
</tr>
<tr>
<td>( l )</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.08*</td>
<td>-0.03</td>
<td>-0.06</td>
<td>0.03</td>
<td>0.18*</td>
<td>-0.20*</td>
</tr>
<tr>
<td>( S )</td>
<td>-0.10*</td>
<td>0.17*</td>
<td>0.02</td>
<td>0.11</td>
<td>0.08</td>
<td>0.19*</td>
<td>0.01</td>
<td>-0.49*</td>
</tr>
<tr>
<td>( H )</td>
<td>-0.10*</td>
<td>-0.02</td>
<td>0.07</td>
<td>0.14*</td>
<td>-0.03</td>
<td>0.08</td>
<td>0.05</td>
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<tr>
<td>( rr )</td>
<td>-0.33*</td>
<td>-0.08</td>
<td>0.14</td>
<td>0.29*</td>
<td>0.21*</td>
<td>0.16</td>
<td>0.01</td>
<td>-0.39*</td>
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<tr>
<td>( \pi )</td>
<td>0.50*</td>
<td>0.13</td>
<td>-0.16</td>
<td>-0.46*</td>
<td>-0.30*</td>
<td>-0.07</td>
<td>0.18</td>
<td>0.19*</td>
</tr>
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**Table:** Short-Run Demographic Impact - Matrix \( D \)
Robustness - 2-way effects

- Benchmark model is a one-way fixed effects model and includes oil prices as the only variable that affects all countries.

- So if there are shared, cross-country factors driving the trend in dependent variables as well as demographic variables, this trend may be wrongly attributed to the demographic variables.

- A two-way effects model avoids this issue by removing any common cross-country factors from all variables prior to estimation.

Table: Long-Run Demographic Impact (2-way effects)
Production

- Final Good Producers -

\[ Y_{c,t} = \left[ \int_0^{N_t^f} (Y_{c,t}^{j})^{(1/\mu_t)} dj \right]^\mu_t \]

- where \( N_t^f \) is the number of firms in input sectors and \( \mu_t = \mu(N_t^f) \), \( \mu'(\cdot) < 0 \). So variable mark-up and fact that firms must pay operating costs control entry and exit.

- Production of input firm \( j \)

\[ Y_{c,t}^j = \left[ (U_t^j K_t^j)^\alpha (\xi_t L_t^j)^{(1-\alpha)} \right]^{(1-\gamma_t)} \left[ M_t^j \right]^{\gamma_t} \]

- Intermediate composite good

\[ M_t^j = \left[ \int_0^{A_t} (M_t^{ji})^{(1/\varphi)} di \right]^{\varphi} \]

where each producer \( i \) acquires the right to market the good via the creation and adoption process. Thus \( A_t \) is determined by innovation sector.
Innovation: R&D

- Let $Z_t^p$ be the stock of invented goods (prototypes) at the beginning of time $t$. Inventor $p$ spends $S_t^p$ to add new prototypes to her stock. Productivity of innovation spending is given by $\varphi_t$.

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi[(\tilde{\psi}_t)^\rho (S_t)^{1-\rho}]^{-1} Z_t S_t^p + \phi Z_t^p$$

- $\phi$ = implied product survival rate
- $\rho$ = elasticity of new technology creation
- $\Gamma_t^{yw}$ = share of workers that contribute to innovation
- $\rho_{yw}$ = Importance of workers for innovation process.

- Innovators borrow $S_t^p$ from the household. Define $J_t$ as the value of an invented intermediary good. Then

$$\phi E[J_{t+1}] = \frac{R_{t+1}}{\varphi_t}$$
Innovation: Adoption

- Let $A^q_t \subset Z^q_t$ denote the stock of converted goods marketed to firms. Adopter $q$ invest (intensity) $\Xi_t$ to transform $Z^q_t$ into $A^q_t$. Conversion process is successful with probability $\lambda_t = \lambda \left( \frac{A^q_t}{\Xi^t} \right)$ with $\lambda' > 0$. Flow of converted goods

$$A^q_{t+1} = \lambda_t \phi(Z^q_t - A^q_t) + \phi A^q_t$$

- A converted good can be marketed at every period to firms, thus its value, denoted $V_t$ is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$$

where $\Pi_{m,t}$ is the profit from selling an intermediate good to input firms.

- The value of a unadopted product ($J_t$) is

$$J_t = \max_{\Xi_t} \Xi_t + (R_{t+1})^{-1} \phi E_t[\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}]$$
Household Sector: Population Dynamics

- Population $N_t$
  
  Young ($N_t^y$) becomes worker with probability $1 - \omega_y$
  
  Workers ($N_t^w$) retire with probability $1 - \omega_r$
  
  Once retired ($N_t^r$) individual survives with probability $\gamma$.

\[
N_{t+1}^y = \tilde{n}_{t+1} N_t^y + \omega^y N_t^r = (\tilde{n}_{t+1} + \omega^y) N_t^y = n_{t+1} N_t^y,
\]

\[
N_{t+1}^w = (1 - \omega^y) N_t^y + \omega^r N_t^w,
\]

\[
N_{t+1}^r = (1 - \omega^r) N_t^w + \gamma_{t+1} N_t^r
\]

define $\zeta_t^r = N_t^r / N_t^w$ and $\zeta_t^y = N_t^y / N_t^w$.

- Stock of workers that contribute to innovation

\[
\Gamma_t^{yw} \equiv (1 - \omega^y) \frac{N_t^y}{N_t} + (1 - \lambda^y)\Gamma_{t-1}^{yw} = (1 - \omega^y) \frac{\zeta_t^y}{1 + \zeta_t^y + \zeta_t^r} + (1 - \lambda^y)\Gamma_{t-1}^{yw},
\]

$\lambda^y < 1$ augments the stock of young workers just entered work! Worker’s age matters for innovation.
Household Sector: Human Capital

- Let $\xi_t$ be the average effective units across workers at period $t$. Let $I_t^y = \tau_t \frac{W_t N_t^w}{N_t}$ be the total effective expenditure society makes on the education of the young, financed by transfer $\tau_t$ from workers. Each young who becomes a worker at the end of period $t$ will provide $\xi_{t+1}^y$ effective units.

\[
\xi_{t+1}^y = \rho E \xi_t + \frac{\chi E}{2} \left( \frac{I_t^y}{\xi_t} \right)^2 \xi_t
\]

- The evolution of workers effective labour units

\[
\xi_{t+1} = \omega_r \frac{N_t^w}{N_{t+1}^w} \xi_t + (1 - \omega^y) \frac{N_t^y}{N_{t+1}^w} \xi_{t+1}^y
\]
Household Sector: Consumption and Labour

- Retirees are assumed not to work. Two key assumptions to offset impact of risk of death (perfect annuity market) and retirement (risk neutrality) on households decision. Gertler (1999)

- Thus, for $z = \{w, r\}$ we assume agent $j$ selects consumption and asset holdings to maximise

$$V_{jt}^{jz} = \left\{ (C_{jt}^{jz})^{\rho u} + \beta_{t,t+1}^{z} (E_{t}[V_{t+1}^{j} | z])^{\rho u} \right\}^{1/\rho u}$$

subject to

$$C_{jt}^{jz} + FA_{t+1}^{jz} = R_{t}^{z} FA_{t}^{jz} + W_{t} \xi_{jt} I^{z} + d_{t}^{z} - \tau_{t}^{jz} I^{z}$$

- Aggregate consumption functions are:

$$C_{t}^{w} = \varsigma_{t} [R_{t} FA_{t}^{w} + H_{t}^{w} + D_{t}^{w} - T_{t}^{w}]$$

$$C_{t}^{r} = \varepsilon_{t} \varsigma_{t} [R_{t} FA_{t}^{r} + D_{t}^{r}]$$
Growth

Three drivers of growth:

a. exogenous growth of population, $n_t$

b. endogenous growth rate of effective labour force, $\xi$

c. endogenous innovation/adoption of new intermediate goods, $A_t$

that affects $K_t, L_t$
Parameters

**Standard**
- $\beta = 0.96$  
- $\alpha = 0.33$  
- $\delta = 0.08$  
- $U = 80\%$  
- $\gamma_i = 0.5$  
- $\mu = 1.1$  
- $(1/ (1 - \rho_U)) = 0.25$

**Innovation**
- obsolescence: $(1-\phi)=0.03$
- productivity in innovation: $\chi = 94.42$
- elasticity of intermediate goods w.r.t R&D $\rho = 0.9$
- ave. adoption time $\lambda = 0.1$
- elasticity of adoption time to intensity $\epsilon_\lambda = 0.9$

**Population**
- $(1 - \omega^y) = 0.05$ \(\frac{N^y}{N^w} = 48\%\)
- $(1 - \omega^r) = 0.023$ \(\frac{N^r}{N^w} = 20\%\)
- 10 yrs in retirement $\gamma = 0.9$

**Population and Innovation**
- ratio of workers influencing innovation $(1 - \lambda_y) = \frac{2}{3}$
- importance of worker to innovation productivity $\rho_{yw} = .9$
Figure: Simulation: prediction - Lower $\rho_{yw}$
Figure: Simulation: prediction Additional Countries - Lower $\rho_{yw}$