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by Josef Schroth

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# Abstract

This paper studies an economy where agents can spend resources on consuming a private good and on funding a public good. There is asymmetric information regarding agents' relative preference for private versus public good consumption. I show how private good consumption should be coordinated across agents within each period to ensure efficient contributions to fund the public good. If agents contributed similar amounts in the past, then coordination takes the form of positively correlated contributions in the current period. If an agent contributed more in the past, then coordination prescribes state-contingent socially wasteful private good consumption in the current period for that agent.

*JEL classification: E62, H21, H23, H77, D82, D86*
*Bank classification: Fiscal policy; Financial stability; Financial system regulation and policies*



# Résumé

Dans cette étude, l'auteur examine une économie où les agents peuvent dépenser des ressources pour consommer un bien privé et financer un bien public. L'information au sujet de la préférence relative des agents à l'égard de la consommation de biens privés ou de biens publics est asymétrique. L'auteur montre comment, pour assurer l'efficience des contributions au financement du bien public, la consommation des biens privés doit être coordonnée entre les agents à l'intérieur de chaque période. Si les agents ont contribué des sommes similaires au bien public dans le passé, la coordination prend la forme d'une corrélation positive entre les contributions des agents au cours de la période actuelle. Si un agent a fourni une contribution plus élevée par le passé, la coordination implique que, pendant la période actuelle, l'utilité sociale de ses dépenses de consommation privée sera subordonnée à l'état du monde.

*Classification JEL : E62, H21, H23, H77, D82, D86*
*Classification de la Banque : Politique budgétaire; Stabilité financière; Réglementation et politiques relatives au système financier*

## Non-Technical Summary

Common-pool problems pose a challenge to policy-makers. For example, regions in a federal state could request federal tax revenue to pay for local expenditures. Another example would be leniency in regulatory oversight by local financial regulators that rely on (implicit) bailout funds from a central authority in case of a local financial crisis. In these examples, policy-makers may rely on information provided by those they wish to help. Because of common-pool problems, however, information is likely misstated in order to obtain help from policy-makers beyond what would be socially beneficial. This paper addresses policy challenges due to common-pool problems and informational asymmetries. To that end, I build a model where agents can spend resources on private consumption and on contributing to a public good, and where each agent knows best his or her own benefit from private consumption. Each agent therefore has an incentive to overstate his or her need to spend on private consumption in order to free ride on other agents' contributions to the public good. I show that an optimal risk-sharing arrangement requires agents' spending on private consumption to be positively correlated even if their preferences are not. Immediate reciprocity in private consumption helps each agent to better internalize the effect of his or her private consumption on public good provision. Further, agents who contributed relatively more to the public good in the past are allowed to engage in state-contingent socially wasteful spending on private consumption in the future. Intensifying immediate reciprocity over time in this way helps agents to transfer utility among each other over time. The model has a number of policy implications. For example, regions in a federal state may not be required to repay federal funds requested in the past. Another example is that past leniency by some local financial regulators may lead to financial instability in the future.

# 1 Introduction

Contributions toward the provision of public goods may be insufficiently low due to common-pool problems. This is especially true when there is asymmetric information regarding the need for potential contributors to spend funds on private consumption instead. For example, a regional government may claim an inability to raise sufficient tax revenue to meet its obligation toward the federal budget (Bordignon et al., 2001; Sanguinetti and Tommasi, 2004).[1] Inefficiencies in the provision of public goods due to asymmetric information can be mitigated by coordinating private good consumption across agents within a period. This paper shows how within-period coordination of private consumption should optimally vary over time. Such optimal variation has implications for public good provision over time, and for whether the discretion to spend resources on private consumption of agents who contributed relatively less to the public good in the past should be limited.

I build a model where agents can spend resources on private good consumption and on contributing to funding a public good. The first key assumption in the model is that agents experience shocks regarding their preference for private relative to public good consumption. Agents can thus benefit from coming to a risk-sharing agreement where an agent with currently a high relative preference for private good consumption contributes relatively less to funding the public good. The second key assumption is that there is asymmetric information regarding agents' preferences for private relative to public good consumption. Any agreement between agents must thus address challenges due to incentives to overstate preferences for private consumption in an

[1]Refraining from an activity that causes a negative externality could also be interpreted as contributing to providing a public good (Alesina et al., 2005). For example, the public good could be interpreted as free trade and private consumption could be interpreted as import tariffs that cause negative externalities on trading partners, as in Amador and Bagwell (2012, 2013).

attempt to free ride on another agent's contributions toward the public good.

The ability of agents to free ride is reduced when private good consumption is coordinated among agents within each period.[2] When agents contributed similar amounts toward the public good in the past, then optimal coordination takes the form of positively correlated private consumption, and positively correlated contributions toward the public good. Spending on private consumption is never socially wasteful in that case. The main contribution of this paper is to show that when an agent contributed relatively more toward the public good in the past, then optimal coordination prescribes state-contingent socially wasteful private consumption by that agent.

In the literature on optimal risk sharing, an agent's discretion to adjust spending based on its private information often decreases as the agent becomes more indebted (Thomas and Worrall, 1990; Atkeson and Lucas, 1992; Taub, 1994). When agents share risk in the presence of a public good, then discretion of debtors to spend on private consumption need not be limited. Creditors will instead police debtors within the period with state-contingent wasteful spending.[3] Specifically, the optimal risk-sharing arrangement prescribes that part of the public good be converted into private good consumption of creditors whenever debtors claim a high need for private good consumption. In states where only debtors have high need for private good consumption, and not creditors, such a conversion of the public good into private consumption of creditors is socially wasteful.

There are two implications for the optimal design of tax policies in fiscal unions

[2]Similarly, in Roberts (1985), firms facing oligopolistic competition can partially overcome inefficiencies due to asymmetric information about production costs by coordinating output within the period (see also Goltsman and Pavlov, 2014).

[3]Creditor refers to an agent who contributed relatively more to the public good in the past, and debtor refers to an agent who contributed relatively less.

where members share risk and also enjoy a public good such as national defense, universal health care or a low federal deficit. I give conditions in terms of model parameters for these implications to hold. The first implication is that public good provision will be lower in expectation if members contributed different amounts to the public good in the past. In that sense, past disagreement about the usefulness of the public good among members of a fiscal union may reduce current public good provision. The second implication is that debtor members, which contributed relatively less to the public good in the past, do not experience a decrease in their discretion to spend on private consumption. In that sense, there may not be a need to repay debt for members of a fiscal union.

The paper is organized as follows. The remainder of this section discusses the related literature. Section 2 presents the model, section 3 characterizes optimal risk sharing, section 4 provides a numerical example and applications, and section 5 concludes.

## 1.1 Related literature

The aggregate amount of resources available to divide among agents is predetermined at the beginning of each period in many models of risk sharing (Thomas and Worrall, 1990; Atkeson and Lucas, 1992; Taub, 1994; Espino and Sanchez, 2010). In this paper, the aggregate budget available for agents' private good consumption can be adjusted instantaneously by altering public good provision. While the aggregate budget can be divided freely among agents, the utility cost associated with expanding it, in the form of lower public good provision, is always shared equally among agents. Coordinating private good consumption within the period is worthwhile in my model because agents cannot be excluded from enjoying the public good.

In the context of the public economics literature, an agent's private consumption can be interpreted as local public good consumption, and informational asymmetries concern the benefit of local public good expenditure by members of a fiscal union (Oates, 1972). In that literature, coordination of private consumption does not, however, play the same incentivizing role as in my paper, due to either availability of transferable utility (Lockwood, 1999; Cornes and Silva, 2002; Huber and Runkel, 2008) or a large number of agents (Bucovetsky et al., 1998). The empirical literature finds that federal governments tend to offer significant insurance to federated states, although states obtain relatively more insurance via private capital markets (Bayoumi and Masson, 1995; Asdrubali et al., 1996).

Recently, policy-makers have become increasingly concerned about deficit bias in particular among regional governments (Velasco, 2000; Cooper et al., 2008; Krogstrup and Wyplosz, 2010). While the literature suggests that deficit bias is due to a weak federal authority, my model shows that even a federal authority with full enforcement power may not require federated states to repay their debt.

# 2 Model

*Time, agents, endowments, uncertainty and information:*

There are two time periods, $t = 1, 2$; two agents, $j = 1, 2$; and a non-storable consumption good. In every period, with certainty, each agent receives endowment $w > 0$. In each period, agent $j = 1, 2$ experiences a preference shock $s_{j,t} \in \{s_L, s_H\}$, with equal probability. Let $S = \{s_L, s_H\} \times \{s_L, s_H\}$. Note that preference shocks are independent across agents and over time. It is assumed that only agent $j$ can observe $s_{j,t}$. Let $s^t$ denote the history of realizations of preference shocks at time $t = 1, 2$ such that $s^t \in S^t$.

The function $\theta : \{s_L, s_H\} \rightarrow \{\theta_L, \theta_H\}$ maps the preference shock into a preference parameter $\theta(s_k) = \theta_k$, $k = L, H$. It is assumed that the expected value of the preference parameter is $\mu < 1$ and that $\frac{1}{2} < \theta_L < \mu < 1 < \theta_H$. Denote the variance of preference parameters by $\sigma^2$ such that $\theta_L = \mu - \sigma$ and $\theta_H = \mu + \sigma$.

*Preferences and consumption:*

Let $c_j = \{c_{j,t}(s^t)\}_{s^t \in S^{2t},\, t=1,2}$ denote non-negative private good consumption paths of agent $j$ and let $\hat{c} = \{\hat{c}_t(s^t)\}_{s^t \in S^{2t},\, t=1,2}$ denote non-negative public good consumption paths of agent $j$. Note that public good consumption is equal across agents due to non-excludability. Agent $j$ ranks consumption paths $\{c_j, \hat{c}\}$ according to the welfare criterion

$$W\left(\{c_j, \hat{c}\}\right) = \sum_{t=1,2} \delta^{t-1} \frac{1}{4^t} \sum_{s^t \in S^t} \left[\theta(s_{j,t}) u\left(c_{j,t}(s^t)\right) + \hat{c}_t(s^t)\right], \tag{1}$$

where $u(c) = \min\{c, w\}$. The parameter $\delta > 0$ is a discount factor. The preference parameters $\theta(s_{j,t})$ determine how much an agent values private consumption relative to public consumption. The assumption that agent $j$'s welfare criterion is not linear in $c_{j,t}(s^t)$ for a given $\theta(s_{j,t})$ – note that marginal utility drops to zero at $w$ – is crucial for the main results in this paper to hold.[4] In each state $s^t$, the aggregate endowment available for both private and public good consumption is $2w$. It is assumed that one unit of the private good can be transformed into one-half units of the public good,

$$\hat{c}_t(s^t) = \frac{1}{2}\left[2w - c_{1,t}(s^t) - c_{2,t}(s^t)\right]. \tag{2}$$

[4] Section 4 considers the case where the function $u$ is strictly concave and obtains the main results of the paper numerically. The case $u(c) = \min\{c, w\}$ is close enough to the linear case to ensure analytic tractability while still delivering all main results.

Up to a constant, welfare of agents one and two can then be written, respectively, as

$$\sum_{t=1,2} \delta^{t-1} \frac{1}{4^t} \sum_{s^t \in S^t} \left[ \left( \theta(s_{1,t}) - \frac{1}{2} \right) c_{1,t}(s^t) - \frac{1}{2} c_{2,t}(s^t) \right], \tag{3}$$

$$\sum_{t=1,2} \delta^{t-1} \frac{1}{4^t} \sum_{s^t \in S^t} \left[ \left( \theta(s_{2,t}) - \frac{1}{2} \right) c_{2,t}(s^t) - \frac{1}{2} c_{1,t}(s^t) \right],$$

where $c_{j,t}(s^t) \in [0, w]$ for all $j = 1, 2$, $s^t \in S^t$, $t = 1, 2$. Joint welfare of the two agents is given by

$$\Omega \equiv \sum_{t=1,2} \delta^{t-1} \frac{1}{4^t} \sum_{s^t \in S^t} \left[ (\theta(s_{1,t}) - 1)\, c_{1,t}(s^t) + (\theta(s_{2,t}) - 1)\, c_{2,t}(s^t) \right], \tag{4}$$

where $c_{j,t}(s^t) \in [0, w]$ for all $j = 1, 2$, $s^t \in S^t$, $t = 1, 2$.

An agent that increases private consumption by one marginal unit internalizes a decrease in its public good consumption by one-half marginal units, but does not internalize the decrease in public good consumption by one-half marginal units for the other agent. In other words, the private marginal benefit of private good consumption of agent $j$ in period $t$ is $\theta(s_{j,t}) - 1/2$, see equation (3), while the social marginal benefit is $\theta(s_{j,t}) - 1$, see equation (4).

It is worth pointing out that the focus on a closed economy (no outside lender, non-storable consumption good) is without loss of generality. The reason is that the marginal utility of private good consumption of an agent who consumes the average endowment in the form of private goods is sufficiently low relative to the agent's marginal benefit of consuming the public good. Agents thus have sufficient resources to insure each other in each period and to transfer utility over time.

## 2.1 First-best allocation

First-best is defined as the allocation of private consumption that maximizes joint welfare $\Omega$ subject to $c_{j,t}(s^t) \in [0, w]$ for $j = 1, 2$, $s^t \in S^t$, $t = 1, 2$.

**Lemma 1.** *The first-best allocation is given by*

$$c^{FB}_{j,t}(s^t) = c^{FB}(s_{j,t}) = \begin{cases} 0, & \text{if } \theta(s_{j,t}) = \theta_L; \\ w, & \text{if } \theta(s_{j,t}) = \theta_H, \end{cases} \qquad s^t \in S^t,\ t = 1, 2\,, j = 1, 2,$$

*and yields joint welfare of* $\Omega^{FB} = (1+\delta)(\theta_H - 1)w$.

*Proof.* From the expression of $\Omega$ in equation (4), it can be seen that private consumption should be as low as possible whenever $\theta(s_{j,t}) < 1$, and as high as possible, up to $w$, whenever $\theta(s_{j,t}) > 1$. Recall that $\theta_L < 1 < \theta_H$. □

The first-best allocation is time-independent and private consumption of agent $j$ depends only on agent $j$'s preference shock. Agents are perfectly insured against shocks that affect their relative preference for private good consumption. That is, in the absence of informational asymmetry, there is no need to coordinate private good consumption.

**Corollary 1.** *The first-best allocation is not incentive compatible.*

*Proof.* The social marginal benefit of private consumption of agent $j$ by equation (4) is given by $\theta(s_{j,t}) - 1$, which is positive only for $\theta(s_{j,t}) = \theta_H$. However, agent $j$'s marginal benefit of private consumption by equation (3) is given by $\theta(s_{j,t}) - 1/2$, which is always positive. An agent thus has an incentive to always claim having received the high preference shock. □

All else constant, an agent prefers its private consumption to be as high as possible irrespective of its preference shock. In section 3, coordination of private consumption will be essential precisely because it removes the notion of 'all else constant.' This paper shows how coordination of private good consumption depends on past realizations of preference shocks.

# 3 Optimal coordination of private good consumption

In this section I focus on a principal-agent problem where agents truthfully report their respective preference shocks (revelation principle) and joint welfare $\Omega$ is to be maximized. Private consumption of each agent is then a function of (truthful) reports of preference shocks by both agents, $c_{j,t} : S^t \rightarrow [0, w]$ for $j = 1, 2$ and $t = 1, 2$. Each period, both agents report their respective preference shock at the same time.

## 3.1 Static case without coordination

Suppose that private consumption of agent $j$ can only depend on agent $j$'s current preference shock, $c_{j,t}(s^t) = c_{j,t}(s_{j,t})$. Then agent $j$ would report the preference shock that yields it the highest level of private consumption, since the private marginal benefit of consuming the private good is always strictly positive (up to an amount $w$). Private good consumption is then constant at some $\bar{c} \in [0, w]$.[5] Expected period welfare per agent, net of its endowment $w$, is given by $(\mu - 1)\bar{c}$. Since $\mu < 1$, the value $\bar{c}$ that yields the highest period welfare is zero. That is, agents should receive no discretion at all with respect to private good spending and should be required to always contribute their full endowment $w$ toward the public good. For the remainder

[5] $\bar{c}$ could also be interpreted as an upper bound on private consumption (see Melumad and Shibano, 1991 and Amador and Bagwell, 2013, and also Athey et al., 2005 and Amador et al., 2006).

of this paper, it is assumed that each agent must at least enjoy expected period welfare of zero at the beginning of each period.

**Assumption 1** (Individual rationality)**.** *At the beginning of each period, before preference shocks are observed, each agent must enjoy expected period welfare, net of endowment $w$, of at least zero.*

## 3.2 Static case with coordination

Suppose now that private consumption of agent $j$ can depend on current preference shocks of both agents, but not on past realizations of preference shocks, $c_{j,t}(s^t) = c_{j,t}(s_{1,t}, s_{2,t})$. Lemma 2 shows how private consumption optimally depends on reported preference shocks in this case.

**Lemma 2.** *Suppose private consumption can only depend on current but not past preference shocks. Then, joint welfare is maximized by private consumption given by*

$$\begin{aligned}
c_{1,t}(s_H, s_H) &= c_{2,t}(s_H, s_H) = w, \\
c_{1,t}(s_H, s_L) &= c_{2,t}(s_L, s_H) = \frac{1-\theta_L}{\theta_L} w, \\
c_{1,t}(s_L, \cdot) &= c_{2,t}(\cdot, s_L) = 0.
\end{aligned}$$

*Proof.* See Appendix A.1. □

Agents are fully insured against common shocks but are only partially insured when experiencing different shocks. This improves upon the case in section 3.1, which offered no insurance at all. Note that there is no socially wasteful private consumption, $c_{1,t}(s_L, \cdot) = c_{2,t}(\cdot, s_L) = 0$. It follows directly from Lemma 2 that period welfare, net of the endowment $w$, for each region under the optimal static fiscal policy, is given by

$$v_0 = \frac{1}{4} \sum_{s_1, s_2} (\theta(s_1) - 1)\, c_{j,t}(s_1, s_2) = \frac{\theta_H - 1}{4\theta_L} w, \tag{5}$$

which is strictly larger than period welfare of zero obtained in the static case without coordination (section 3.1). The reason for this improvement is that coordinating private consumption within the period can relax incentive compatibility constraints.

To see this, consider an agent with preference shock $s_L$ who reports preference shock $s_H$ instead. Such an agent now receives higher private consumption, on average, which increases the agent's welfare payoff by $\frac{1}{2}\left(\theta_L - \frac{1}{2}\right)\frac{1}{\theta_L} w > 0$. But private consumption of the other agent is now also higher, on average, which decreases the welfare payoff by $\frac{1}{2}\frac{1}{2}\left(1 - \frac{1-\theta_L}{\theta_L}\right) w = \frac{1}{2}\left(\theta_L - \frac{1}{2}\right)\frac{1}{\theta_L} w$. Thus, the agent cannot achieve a net increase in its welfare payoff by overstating its need for private consumption. What keeps an agent from overstating its preference shock is the expected increase in the other agent's private consumption and the associated reduction in public good consumption. Agents cannot free ride on each other's contributions toward the public good if private good consumption is coordinated in this way.

## 3.3 Dynamic case with coordination

This section allows each agent's private consumption to depend on both agents' current as well as past preference shocks. Private consumption can thus be coordinated within each period and agents can transfer utility intertemporally. The main result of the paper, which is derived in this section, shows how within-period coordination changes when agents transfer utility intertemporally. Policy implications are discussed in section 4.

### 3.3.1 Second period

It is useful to first characterize the set of feasible second-period welfare pairs that can be delivered to agents. Let $v \in [0, \bar{v}]$ be second-period welfare to be delivered to region two. Note that the lower bound on $v$ is due to Assumption 1, and $\bar{v}$ will be defined below. For a given $v$, let $c_j(s_1, s_2)$ be private consumption of agent $j = 1, 2$ when agent one reports preference shock $s_1$ and agent two reports preference shock $s_2$. Agent two enjoys second-period welfare of at least $v$ whenever the following promise-keeping constraint holds:

$$\frac{1}{4} \sum_{(s_1,s_2)\in S} \left[ \theta(s_2) c_2(s_1, s_2) - \frac{1}{2} c_1(s_1, s_2) - \frac{1}{2} c_2(s_1, s_2) \right] \geq v. \tag{6}$$

Agents will report preference shocks truthfully whenever the following incentive compatibility constraints hold:[6]

$$\frac{1}{2} \sum_{s_2\in\{s_L,s_H\}} \left[ \theta(s_L) c_1(s_L, s_2) - \frac{1}{2} c_1(s_L, s_2) - \frac{1}{2} c_2(s_L, s_2) \right]$$
$$\geq \frac{1}{2} \sum_{s_2\in\{s_L,s_H\}} \left[ \theta(s_L) c_1(s_H, s_2) - \frac{1}{2} c_1(s_H, s_2) - \frac{1}{2} c_2(s_H, s_2) \right], \tag{8}$$

$$\frac{1}{2} \sum_{s_1\in\{s_L,s_H\}} \left[ \theta(s_L) c_2(s_1, s_L) - \frac{1}{2} c_1(s_1, s_L) - \frac{1}{2} c_2(s_1, s_L) \right]$$
$$\geq \frac{1}{2} \sum_{s_1\in\{s_L,s_H\}} \left[ \theta(s_L) c_2(s_1, s_H) - \frac{1}{2} c_1(s_1, s_H) - \frac{1}{2} c_2(s_1, s_H) \right]. \tag{9}$$

[6] When conditions (8) and (9) are satisfied, agents will have no incentive to understate preference shocks as long as the following monotonicity condition holds:

$$\sum_{s_2\in\{s_L,s_H\}} [c_1(s_H, s_2) - c_1(s_L, s_2)] \geq 0, \text{ and } \sum_{s_1\in\{s_L,s_H\}} [c_2(s_1, s_H) - c_2(s_1, s_L)] \geq 0. \tag{7}$$

Condition (7) requires that the function $c_j$ increases in $s_j$ in expectation, for $j = 1, 2$. However, the condition is satisfied at an optimum and can be ignored.

Let $P(v)$ be the highest second-period welfare that can be delivered to agent one given the promise $v$ to agent two. That is, $P(v)$ is defined as

$$P(v) = \max_{\{c_j\}_{j=1,2}} \frac{1}{4} \sum_{(s_1,s_2)\in S} \left[ \left( \theta(s_1) - \frac{1}{2} \right) c_1(s_1,s_2) - \frac{1}{2} c_2(s_1,s_2) \right], \tag{10}$$

subject to (6), (8), (9) and $c_j(s_1,s_2) \in [0,w]$ for all $(s_1,s_2) \in S$. Then the graph of $P$, $\{(v_1,v_2) : v_2 \in [0,\bar{v}],\ v_1 = P(v_2)\}$, is the Pareto frontier in period two. Note that $P(v)$ is decreasing by the promise-keeping constraint (6) and define $\bar{v} = P(0)$. In the case where both agents enjoy the same second-period welfare, $P(v_0) = v_0$, the allocation is given by Lemma 2 and $v_0$ is given by equation (5). Depending on the parameters, there are two cases to consider for how private consumption of agents is affected when $v \neq v_0$. Assumption 2 selects the case for which there is greater benefit from exercising discretion with respect to private consumption.[7]

**Assumption 2** (Discretion matters). *Discretion with respect to spending on private consumption matters in the sense that $\sigma > \sqrt{\mu(1-\mu)}$.*

When $v \neq v_0$, the question arises as to how we can make an agent better off along the Pareto frontier. Since agents are ex ante identical, it is sufficient to characterize the case $v < v_0$ (or $P(v) > v_0$) where region one obtains relatively higher second-period welfare along the Pareto frontier. Lemma 3 shows how this is optimally achieved.

**Lemma 3.** *Suppose $v < v_0$, such that agent one is better off along the Pareto frontier. Then, relative to the allocation at $v_0$,*

1. *$c_1(s_H, s_L)$ is strictly higher,*

[7]The condition can be written as $\theta_H(\theta_H - 1) > \theta_L(1-\theta_L)$ and thus holds whenever $\theta_H$ or $\theta_L$ is large. A larger value of $\theta_H$ implies that it is more socially beneficial to have an agent with high preference parameter spend resources on private consumption. For a given $\theta_H$, a larger $\theta_L$ implies that private consumption of an agent with low preference parameter is less socially costly.

2. *$c_2(s_L, s_H)$ is unchanged for $3\sigma - \mu \leq 0$, and strictly lower otherwise,*

3. *$c_1(s_L, s_H)$ is strictly higher, $c_2(s_H, s_L)$ is unchanged,*

4. *private consumption of both agents is unchanged when both agents experience the same preference shock.*

*Proof.* See Appendix A.1. □

When $v < v_0$, then agent one is better off along the Pareto frontier. Lemma 3 shows how the allocation of private consumption changes relative to the symmetric case $v = v_0$ characterized in Lemma 2. When both agents experience the same preference shock, then private consumption is first-best, just as in the symmetric case $v = v_0$. Agent one is made better off, on average, by being allowed higher private consumption $c_1(s_H, s_L)$ in the state where only agent one has the high preference shock. Depending on the parameters, agent two may not have to decrease its private consumption in any state, i.e. $c_2(s_L, s_H)$ may be unchanged. Both the increased discretion in private consumption spending for agent one and the fact that agent two may not be required to decrease its discretion in private consumption spending work toward weakening incentives to report preference shocks truthfully. Incentives are maintained along the Pareto frontier via state-contingent socially wasteful private consumption by agent one. In particular, agent one has strictly positive private consumption $c_1(s_L, s_H)$ in the state where only agent two has the high preference shock. Such consumption is socially wasteful because $\theta_L - 1 < 0$, but it has the benefit of supporting discretionary spending on the private good for both agents. In contrast, $c_1(s_L, s_L)$ remains at zero, since the consumption of agent one in this state contributes less to maintaining incentives.

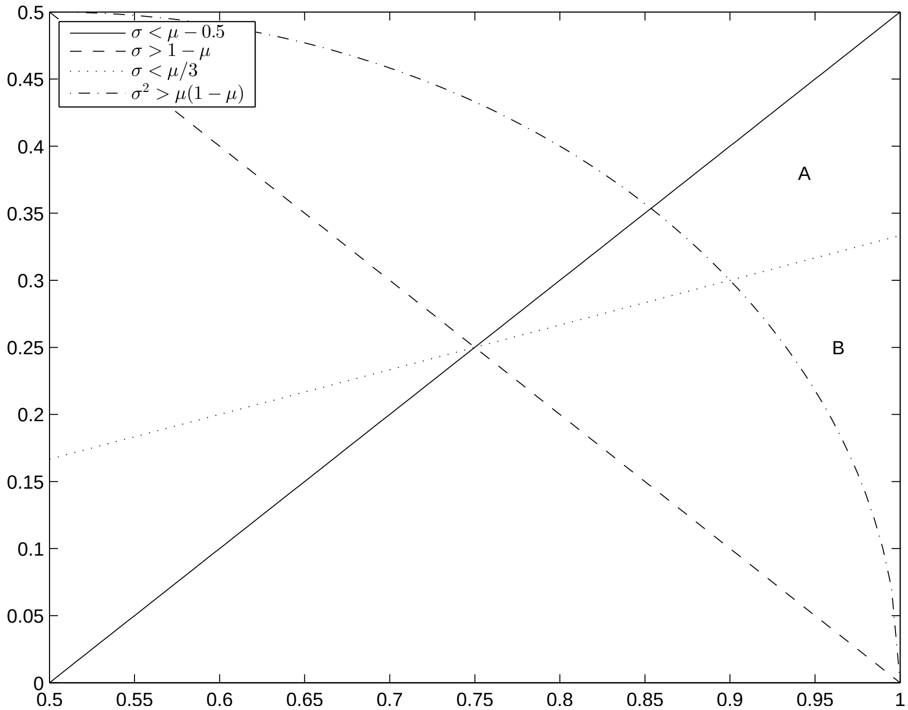



**Figure 1:** Restrictions on model parameters $\mu$ and $\sigma$ in $(\mu, \sigma)$ space. Parameter pairs in A and B satisfy all restrictions imposed by $\mu < 1$, $\theta_L - \frac{1}{2} > 0$, $\theta_H - 1 > 0$, and Assumption 2. The line $\sigma = \frac{\mu}{3}$ divides the region of permissible parameter pairs into two subregions, A and B.

**Lemma 4.** *If $3\sigma - \mu < 0$, then expected public good consumption is strictly lower when $v \neq v_0$ compared to the case $v = v_0$.*

*Proof.* See Appendix A.1. □

The use of state-contingent wasteful spending along the Pareto frontier reduces the need for the agent with lower second-period welfare to decrease its discretion in private consumption spending. Lemma 4 shows that, as a result, expected public good spending may decrease relative to the symmetric case $v = v_0$. If $3\sigma - \mu < 0$ (region B in Figure 1), then a transfer of second-period welfare among agents is associated with reduced contributions toward the public good, on average.

### 3.3.2 First period

Agents can be assigned different second-period welfare $v_1$ and $v_2$ in the first period in a way that encourages them to truthfully reveal their preference shocks, as is standard in the risk-sharing literature. That is, second-period welfare can be made contingent on reports of preference shocks in the first period, $v_j : S \to \mathbb{R}_+$. The Pareto frontier derived in the previous section allows us to express the set of feasible pairs of second-period welfare as

$$\mathcal{P} = \left\{ (v_1, v_2) \in \mathbb{R}_+^2 : v_1 \leq P(v_2) \right\}. \tag{11}$$

Let $c_j$ denote private consumption of agent $j$ in the first period, $c_j : S \to [0, w]$. The problem of a principal that wishes to maximize agents' joint welfare $\Omega$ is as follows:

$$\max_{\{c_j, v_j\}_{j=1,2}} \frac{1}{4} \sum_{(s_1,s_2)\in S} \left[ (\theta(s_1) - 1)c_1(s_1, s_2) + (\theta(s_2) - 1)c_2(s_1, s_2) + \delta(v_1(s_1, s_2) + v_2(s_1, s_2)) \right], \tag{12}$$

subject to incentive compatibility

$$\begin{aligned} &\frac{1}{2} \sum_{s_2\in\{s_L,s_H\}} \left[ \theta(s_L)c_1(s_L, s_2) - \frac{1}{2}c_1(s_L, s_2) - \frac{1}{2}c_2(s_L, s_2) + \delta v_1(s_L, s_2) \right] \\ &\qquad \geq \frac{1}{2} \sum_{s_2\in\{s_L,s_H\}} \left[ \theta(s_L)c_1(s_H, s_2) - \frac{1}{2}c_1(s_H, s_2) - \frac{1}{2}c_2(s_H, s_2) + \delta v_1(s_H, s_2) \right], \end{aligned} \tag{13}$$

$$\begin{aligned} &\frac{1}{2} \sum_{s_1\in\{s_L,s_H\}} \left[ \theta(s_L)c_2(s_1, s_L) - \frac{1}{2}c_1(s_1, s_L) - \frac{1}{2}c_2(s_1, s_L) + \delta v_2(s_1, s_L) \right] \\ &\qquad \geq \frac{1}{2} \sum_{s_1\in\{s_L,s_H\}} \left[ \theta(s_L)c_2(s_1, s_H) - \frac{1}{2}c_1(s_1, s_H) - \frac{1}{2}c_2(s_1, s_H) + \delta v_2(s_1, s_H) \right], \end{aligned} \tag{14}$$

and feasibility $c_j(s_1, s_2) \in [0, w]$, $(v_1(s_1, s_2), v_2(s_1, s_2)) \in \mathcal{P}$ for all $(s_1, s_2) \in S$. Lemma 5 verifies that variation in second-period welfare is in fact used in the first period to make private consumption more responsive to preference shocks.

**Lemma 5.** *The optimal allocation of first-period private consumption and second-period welfare has the following characteristics:*

1. *When agents have the same preference shock in the first period, then both agents receive second-period welfare of $v_0$. Private consumption is first-best as in the case in Lemma 2.*

2. *When agents have different preference shocks in the first period, then second-period welfare is varied along the Pareto frontier, i.e. $\theta(s_i) < \theta(s_j)$ implies $v_i > v_j = P(v_i)$. Agents enjoy more discretion with respect to private good consumption compared to the case in Lemma 2.*

*Proof.* See Appendix A.1. □

When both agents experience the same preference shock, then private consumption given by Lemma 2 already delivers the first-best. Variation in second-period welfare is not beneficial in this case such that both agents receive second-period welfare of $v_0$. In the case where agents experience different preference shocks, it is beneficial to vary second-period welfare in order to improve upon partial insurance provided by the allocation in Lemma 2. Together, Lemmas 3 and 5 yield the main result of the paper. Proposition 1 shows how a past disagreement regarding the desirability of the public consumption good, i.e. a history of different preference shocks across agents, affects current coordination of private good consumption across agents.

**Proposition 1.** *Let $s_1, s_2 \in S$ be preference shocks in the first period. If $s_1 = s_2$, then optimal private consumption in the second period is as given in Lemma 2. If $s_1 \neq s_2$ such that $\theta(s_i) < \theta(s_j)$, then in the second period, compared to the case $s_1 = s_2$,*

1. *agent $i$ receives strictly higher private consumption in the state of the world where only agent $i$ has a high preference shock,*

2. *agent $j$'s private consumption in the state of the world where only agent $j$ has a high preference shock is unchanged if $3\sigma - \mu \leq 0$ and strictly lower otherwise,*

3. *agent $i$ engages in state-contingent socially wasteful private good consumption in the state of the world where only agent $j$ has a high preference shock.*

*Proof.* We know from Lemma 5 that second-period welfare is $v_1 = v_2 = v_0$ whenever $s_1 = s_2$ in the first period. But then second-period private consumption is given in Lemma 2. We know from Lemma 5 that $\theta(s_i) < \theta(s_j)$ implies second-period welfare of $v_i > v_j$ along the Pareto frontier. Then the implications for second-period private consumption follow from Lemma 3. $\square$

The proposition shows that inefficiencies due to a past disagreement regarding the desirability of the public good, i.e. costly (in terms of joint welfare) movements along the Pareto frontier away from symmetric second-period welfare, can be mitigated by coordinating private good consumption more tightly. Coordination is tightened to expand outward the set of feasible second-period welfare pairs and hence to make intertemporal utility transfers cheaper. This is why intertemporal and intratemporal margins for incentive provision should interact at an optimum.

An agent is rewarded for contributing relatively more toward the public good in the first period by allowing it to increase its private consumption by more whenever it experiences a high preference shock in the second period (i.e. it has increased discretion in period two). Period-two incentive compatibility is maintained by allowing that agent to discipline the respective other agent via wasteful state-contingent private consumption. An agent oversees delivery of its higher second-period welfare by disciplining the respective other agent. In that sense, wasteful spending is not a direct punishment of another agent for its past behavior, but rather a means to facilitate intertemporal utility transfers. The model thus gives an example of how short-lived institutions (i.e. one agent disciplining the other) can arise endogenously after certain histories within a long-lived relationship (i.e. the ex-ante optimal risk-sharing arrangement).

The tightening of fiscal policy coordination, in the sense of state-contingent wasteful spending, following past disagreement is the result of the interaction of two channels for incentive provision that have been studied extensively, albeit separately, in the contracting literature. This paper, on the other hand, focuses on the interaction of intratemporal margins (for example Roberts, 1985) and intertemporal margins (for example Taub, 1994) for incentive provision. For instance, many dynamic contracting problems have a solution that features (ex-post) inefficiently high consumption of sufficiently wealthy lenders as a reward for past frugality (e.g. Thomas and Worrall, 1990, Atkeson and Lucas, 1992, Iovino and Golosov, 2013). However, in this paper, such inefficiently high consumption is employed only in certain states as a means to provide additional incentives via immediate reciprocity.

**Corollary 2.** *Suppose $3\sigma - \mu < 0$. Let $s_1, s_2$ be preference shocks in the first period. If $\theta(s_i) < \theta(s_j)$, i.e. agent $i$ contributed relatively more toward the public good in the first period, then in period two, compared to the case where $s_1 = s_2$,*

1. *expected public good provision is strictly lower,*
2. *private consumption of agent $j$ is unchanged.*

*Proof.* The result follows from Lemmas 3 and 4 together with Lemma 5. □

The corollary shows that current public good provision may be lower, on average, when agents disagreed on the relative desirability of the public good in the past. Agents may thus contribute less to the public good, on average, whenever they did not have the same relative preference for the public good in the past. The corollary also shows that an agent that contributes relatively less toward the public good, and instead enjoys relatively higher private consumption, may not be required to reduce its private consumption at all in the following period. In that sense, a debtor agent may not have to repay by contributing more toward the public good in the future. The creditor agent, who contributes relatively more toward the public good, is rewarded in the following period with increased discretion to spend on private consumption.

## 4 Numerical example and discussion

In the numerical example, I allow for an infinite horizon and for a strictly concave private consumption payoff function. In particular, the welfare criterion of agent $i = 1, 2$ is given by

$$E\left\{\sum_{t=0}^{\infty}\delta^t\left[\theta(s_{i,t})u(c_i(s^t)) - \frac{1}{2}\left(c_1(s^t) + c_2(s^t)\right)\right]\right\},$$

where $u$ is both strictly increasing, continuously differentiable, and strictly concave, and $E$ denotes expectation over preference shocks. There are two different preference shocks with $Prob(s_{i,t} = s_k) = \frac{1}{2}$ for $k = L, H$ and the associated preference parameters are $0 < \theta_L < \theta_H$.

The implications in Proposition 1 and Corollary 2 give rise to four measures of interest, for given histories of preference shocks. First, define

$$\chi_1(s^{t-1}) = \frac{1}{2} \sum_{s_2 \in \{s_L, s_H\}} \left[ c_{1,t}\left((s_H, s_2), s^{t-1}\right) - c_{1,t}\left((s_L, s_2), s^{t-1}\right)\right] \tag{15}$$

as a measure of spending discretion with respect to private consumption for agent one when the history of preference shocks is $s^{t-1}$. Second, define

$$\zeta_1(s^{t-1}) = \frac{1}{2} \sum_{s_1 \in \{s_L, s_H\}} \left[ c_{1,t}\left((s_1, s_H), s^{t-1}\right) - c_{1,t}\left((s_1, s_L), s^{t-1}\right)\right] \tag{16}$$

as a measure of within-period, or immediate, reciprocity for agent one. Third, let

$$\xi_1(s^{t-1}) = \frac{1}{4} \sum_{(s_1, s_2) \in S} \left[ c_{1,t}\left((s_1, s_2), s^{t-1}\right) - c^{FB}(s_1)\right] \tag{17}$$

be a measure of socially wasteful spending on private consumption by agent one, where $c^{FB}$ denotes first-best private consumption. Note that first-best private consumption is independent over time and across agents and is given by $u'(c^{FB}(s_1)) = 1$, where $u'$ denotes the first derivative of $u$. Fourth, expected spending on private consumption by agent one is given by

$$\kappa_1(s^{t-1}) = \frac{1}{4} \sum_{(s_1, s_2) \in S} c_{1,t}\left((s_1, s_2), s^{t-1}\right). \tag{18}$$

The measures for agent two are defined accordingly. The continuation welfare enjoyed by agent two following history $s^{t-1}$ is given by

$$v(s^{t-1}) = E\left\{ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ \theta(s_{2,\tau}) u(c_2(s^\tau)) - \frac{1}{2}\left(c_1(s^\tau) + c_2(s^\tau)\right)\right] \middle| s^{t-1} \right\}, \tag{19}$$

where $E\left\{\cdot | s^{t-1}\right\}$ denotes expectation over preference shocks conditional on $s^{t-1}$. Figure 2 shows the defined four measures as functions of $v(s^{t-1})$ for the case where $\theta_L = \frac{1}{2}$, $\theta_H = 1$, $\delta = 0.6$ and $u(c) = 2\sqrt{c}$.[8] Note that $v(s^{t-1})$ is a sufficient statistic for $s^{t-1}$. Private consumption is the same for both agents if $v(s^{t-1}) = v_0 = 1.54$.

If $v(s^{t-1}) < v_0$, then agent one is better off, while agent two is better off whenever $v(s^{t-1}) > v_0$. It can be seen from Figure 2 that the agent that is better off enjoys more discretion to respond to a high relative preference for private consumption. Such a creditor agent also engages in more within-period reciprocity and has higher private consumption, on average. The debtor agent, that is worse off, faces decreased discretion and, on average, lower private consumption. However, the optimal risk-sharing arrangement puts more emphasis on rewarding the creditor than on punishing the debtor, such that the increase in creditor private consumption outweighs the decrease in debtor private consumption. As a result, total private consumption is higher when $v(s^{t-1}) \neq v_0$ compared to the case $v(s^{t-1}) = v_0$, and public good provision is consequently lower.

Public good provision is lower, on average, when $v(s^{t-1}) \neq v_0$, i.e. when agents are separated into debtor and creditor. But the public good is not converted into private good consumption of the creditor in a uniform fashion. It is instead beneficial to convert more of the public good into the private good, for consumption by the creditor, in states where the debtor experiences a high preference shock. Such increased within-period reciprocity by the creditor strengthens the incentives of both agents. The increase in creditor discretion is thus supported, while the need to decrease debtor discretion is reduced.

[8]It is shown in a companion paper that the Pareto frontier of the infinite-horizon risk-sharing problem is self-generating (Abreu et al., 1990) for preferences of this kind. Here, I use this fact and present some numerical results. For the purpose of this section only, let $v_0$ denote symmetric welfare on the Pareto frontier of the infinite-horizon problem.

## 4.1 Policy applications

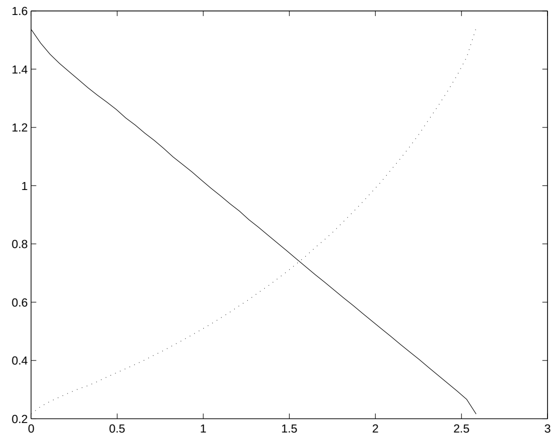
Consider the case of two countries which privately benefit non-linearly from imposing import tariffs but which are hurt by import tariffs imposed by the respective other country. Amador and Bagwell (2012) show that a tariff cap can be optimal in a static setting where coordination is not possible (see also section 3.1). The preceding discussion guides the design of dynamic tariff rules that take the possibility of tariff coordination into account. In particular, a country that set relatively lower tariffs in the past should be allowed to increase its tariffs in immediate response to a tariff increase by the respective other country. A country that had relatively higher tariffs in the past need not experience a limitation in its discretion to set tariffs in the future. Free trade may be restricted, in the sense of higher average tariffs, in the future whenever countries imposed different tariffs in the past.

Members of the European Union (EU) enjoy private benefits such as spending on local public services, pensions or tax relief. However, high spending may cause negative externalities on other members; for example, via implicit bailout promises or a reduction in the ability to contribute to EU-wide joint projects. The Stability and Growth Pact envisions an upper bound on individual member budget deficits to alleviate concerns of excessive externalities imposed on other members via deficit spending (see section 3.1). The analysis in this paper informs the design of dynamic fiscal rules that allow for coordination of taxation within the EU. In particular, a member that had relatively lower deficits in the past should be given more fiscal discretion, and be allowed to increase its deficit in immediate response to higher deficits by other members. A member that had a relatively higher deficit in the past need not experience a decrease in its fiscal discretion in the future (see also Sørensen and Yosha, 1998). Average EU-wide deficits may be higher in the future whenever EU members had different levels of deficits in the past.

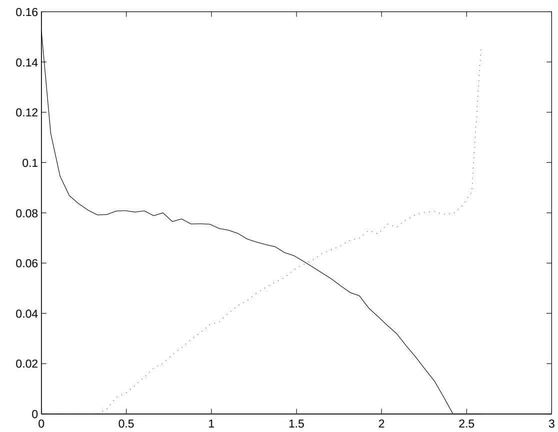
# 5 Conclusion

This paper studies risk sharing among two agents who can contribute funds to a public good but experience shocks affecting their relative preference for public versus private consumption. There is asymmetric information regarding agents' preference shocks such that an agent may attempt to misrepresent its shock in order to free ride on the respective other agent's contributions toward the public good.

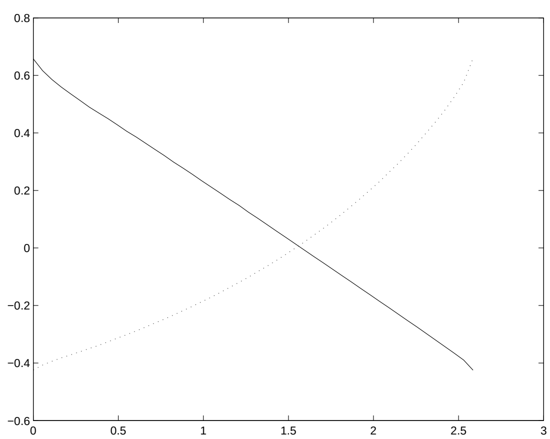
The central insight from the analysis of an optimal risk-sharing arrangement for agents in this economy is that emphasis should be put on rewarding creditor agents rather than on punishing debtor agents. Creditors are rewarded with increased discretion to spend on the private consumption good while, depending on parameters, debtors may not have to experience decreased discretion in private good spending at all. Incentives are maintained by allowing creditors to engage in state-contingent socially wasteful spending on private consumption.



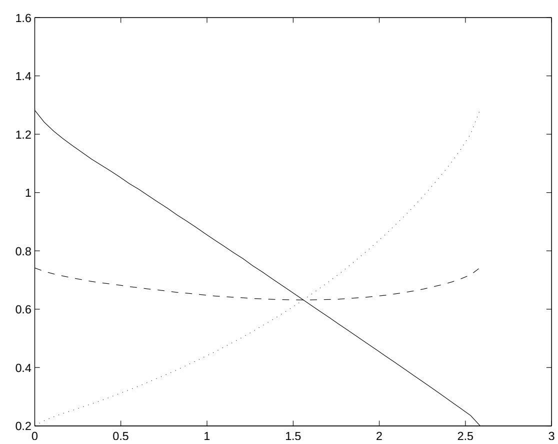

**(a)** Discretion $\chi$




**(b)** Within-period reciprocity $\zeta$




**(c)** Socially wasteful spending $\xi$




**(d)** Private good consumption $\kappa$

**Figure 2:** The solid line refers to agent one and the dotted line refers to agent two. The dashed line in Figure 2d shows expected private consumption by both agents, $\kappa_1 + \kappa_2$.

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# A Appendix

## A.1 Proofs

*Proof of Lemma 1.* This is immediate from the assumption that $\theta_L < 1 < \theta_H$. □

*Proof of Lemma 2.* Since agents have symmetric endowments and preferences, and private consumption is static, it follows that $c_1(s_1, s_2) = c_2(s_2, s_1)$. The problem can then be written as

$$\max_{c_1(s_1,s_2)\in[0,w]} 2(1+\delta)\frac{1}{4}\left[(\theta_L - 1)(c_1(s_L, s_L) + c_1(s_L, s_H)) + (\theta_H - 1)(c_1(s_H, s_L) + c_1(s_H, s_H))\right],$$

subject to incentive compatibility

$$\left(\theta_L - \frac{1}{2}\right)(c_1(s_L, s_L) + c_1(s_L, s_H)) - \frac{1}{2}(c_1(s_L, s_L) + c_1(s_H, s_L))$$
$$\geq \left(\theta_L - \frac{1}{2}\right)(c_1(s_H, s_L) + c_1(s_H, s_H)) - \frac{1}{2}(c_1(s_L, s_H) + c_1(s_H, s_H)).$$

Since this is a linear program, it is sufficient to verify that its first-order conditions are satisfied by the allocation proposed in the lemma. Letting $\psi > 0$ denote the multiplier on the incentive compatibility constraint, these conditions are

$$\begin{aligned}
c_1(s_L, s_L): &\quad \theta_L - 1 + \left(\theta_L - \frac{1}{2}\right)\psi - \frac{1}{2}\psi = -(1-\theta_L)(1+\psi) < 0,\\
c_1(s_L, s_H): &\quad \theta_L - 1 + \left(\theta_L - \frac{1}{2}\right)\psi + \frac{1}{2}\psi < 0,\\
c_1(s_H, s_L): &\quad \theta_H - 1 - \left(\theta_L - \frac{1}{2}\right)\psi - \frac{1}{2}\psi = 0,\\
c_1(s_H, s_H): &\quad \theta_H - 1 - \left(\theta_L - \frac{1}{2}\right)\psi + \frac{1}{2}\psi = \theta_H - 1 + (1-\theta_L)\psi > 0,\\
&\quad \psi \cdot \left[-\frac{1}{2}\frac{1-\theta_L}{\theta_L}w - \left(\left(\theta_L - \frac{1}{2}\right)\left(\frac{1-\theta_L}{\theta_L}w + w\right) - \frac{1}{2}w\right)\right] = 0.
\end{aligned}$$

The first, fourth and fifth conditions clearly hold. To see that the second holds given the third, note that the third condition can be solved for $\psi = \frac{\theta_H - 1}{\theta_L}$ such that

$$\theta_L - 1 + \left(\theta_L - \frac{1}{2}\right)\psi + \frac{1}{2}\psi = \theta_L - 1 + \theta_L\psi = \theta_L - 1 + \theta_H - 1 = 2(\mu - 1) < 0,$$

since $\mu < 1$. □

*Proof of Lemma 3.* To make notation simpler, denote $c_i^{jk} = c_i(s_j, s_k)$ for $j, k \in \{L, H\}$. For use throughout this appendix, let us write out first-order conditions for private consumption (for given $v$).

$$c_1^{LL}: \qquad \theta_L - \frac{1}{2} - \frac{1}{2}\tau + \left(\theta_L - \frac{1}{2}\right)\psi_1 - \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{20}$$

$$c_1^{LH}: \qquad \theta_L - \frac{1}{2} - \frac{1}{2}\tau + \left(\theta_L - \frac{1}{2}\right)\psi_1 + \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{21}$$

$$c_1^{HL}: \qquad \theta_H - \frac{1}{2} - \frac{1}{2}\tau - \left(\theta_L - \frac{1}{2}\right)\psi_1 - \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{22}$$

$$c_1^{HH}: \qquad \theta_H - \frac{1}{2} - \frac{1}{2}\tau - \left(\theta_L - \frac{1}{2}\right)\psi_1 + \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{23}$$

$$c_2^{LL}: \qquad -\frac{1}{2} + \left(\theta_L - \frac{1}{2}\right)\tau - \frac{1}{2}\psi_1 + \left(\theta_L - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{24}$$

$$c_2^{LH}: \qquad -\frac{1}{2} + \left(\theta_H - \frac{1}{2}\right)\tau - \frac{1}{2}\psi_1 - \left(\theta_L - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{25}$$

$$c_2^{HL}: \qquad -\frac{1}{2} + \left(\theta_L - \frac{1}{2}\right)\tau + \frac{1}{2}\psi_1 + \left(\theta_L - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{26}$$

$$c_2^{HH}: \qquad -\frac{1}{2} + \left(\theta_H - \frac{1}{2}\right)\tau + \frac{1}{2}\psi_1 - \left(\theta_L - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{27}$$

where $\tau$ is the Lagrange multiplier on the promise-keeping constraint for agent two and $\psi_i$ is the multiplier on the incentive compatibility constraint for agent $i = 1, 2$. We will compute the optimal allocation for $v \in [\underline{v}, v_0]$, where the case $v \in [v_0, \bar{v}]$ follows from symmetry. $P$ will be piece-wise linear with kinks $\underline{v}_j$, $j = 0, 1, 2$, and $\underline{v}_2 < \underline{v}_1 < v_0$. Below we will guess and verify optimal private consumption that attains $P(v)$ for each $v \in [\underline{v}, v_0]$.

For $v \in [\underline{v}_1, v_0]$ we have that $c_1^{LH}, c_1^{HL}, c_2^{LH}$ take interior values such that (21), (22) and (25) hold with equality. The remaining first-order conditions yield corner solutions $c_1^{LL} = c_2^{LL} = c_2^{HL} = 0$ and $c_1^{HH} = c_2^{HH} = w$. Both incentive compatibility constraints bind such that multipliers are given by

$$\tau = 2\mu - 1, \quad \psi_1 = \frac{\sigma^2 - \mu(1-\mu)}{(\mu - \sigma)(1 - \mu + \sigma)}, \quad \psi_2 = \frac{(2\mu - 1)(\sigma^2 + \mu(1-\mu))}{(\mu - \sigma)(1 - \mu + \sigma)}.$$

Note that $\psi_1$ is strictly positive, since discretion matters (by Assumption 2). To verify that $c_2^{HL} = 0$, note that the left-hand side of (26), whenever the left-hand side of (21) is zero, can be written as $\theta_L(\tau-1)-(1-\theta_L)(\psi_2-\psi_1)<0$.

The interior policies for a given $v \in [\underline{v}_1, v_0]$ are

$$c_1^{LH}=4(v_0-v), \quad c_1^{HL}=\frac{1-\mu+\sigma}{\mu-\sigma}w+4(v_0-v), \quad c_2^{LH}=\frac{1-\mu+\sigma}{\mu-\sigma}w.$$

We have $c_1^{HL}=w$ at $v=\underline{v}_1$ where

$$\underline{v}_1=\frac{3\sigma-\mu}{4(\mu-\sigma)}w.$$

Note that $\underline{v}_1$ can be either positive or negative. In the former case, we are interested in $v \in [0, \underline{v}_1)$ as well. Then $c_1^{LH}, c_2^{LH}$ take interior values such that (21) and (25) hold with equality. The remaining first-order conditions yield corner solutions $c_1^{LL}=c_2^{LL}=c_2^{HL}=0$ and $c_1^{HL}=c_1^{HH}=c_2^{HH}=w$. Only the second agent's incentive compatibility constraint binds such that multipliers are given by

$$\tau=2\mu-1-\frac{\sigma^2-\mu(1-\mu)}{\sigma}, \quad \psi_1=0, \quad \psi_2=\frac{\sigma^2+\mu(1-\mu)}{\sigma},$$

where it is easy to verify that $\tau \in (0,1)$. We have $c_2^{HL}=0$ for the same reason as above, since again $\psi_2-\psi_1>0$. To see that $c_1^{HL}=w$, note that the left-hand side of (22), whenever the left-hand side of (21) is zero, can be written as $\theta_H-\theta_L-\psi_2=(\sigma^2-\mu(1-\mu))/\sigma>0$.

The interior policies for a given $v \in [\underline{v}_2, \underline{v}_1]$ are

$$c_1^{LH}=(2(\mu-\sigma)-1)\,\frac{4v+w}{2\sigma}, \quad c_2^{LH}=\frac{4v+(1-2\sigma)w}{2\sigma}.$$

Note that both are decreasing as $v$ decreases – however, $c_1^{LH}>0$ throughout, such that agent one will still engage in state-contingent socially wasteful private consumption. We have $c_2^{LH}=0$ at $v=\underline{v}_2$, but this value does not satisfy Assumption 1, since

$$\underline{v}_2=-\frac{1-2\sigma}{4}w<0.$$

Thus, as $v$ decreases in $[0, v_0]$, we have $c_1^{HL}$ increasing, $c_1^{LH} > 0$, and $c_2^{LH}$ non-increasing, whenever discretion matters (in the sense of Assumption 2). □

*Proof of Lemma 4.* When $3\sigma - \mu < 0$, then $\underline{v}_1$ in the proof of Lemma 3 is strictly negative. By Assumption 2 it follows that second-period welfare of any agent must be strictly larger than $\underline{v}_1$. But then private consumption is uniformly higher when $v \neq v_0$ compared to the case where $v = v_0$, and strictly higher in states where agents receive different preference shocks. It follows that expected private consumption is strictly higher and thus expected public good consumption strictly lower when $v \neq v_0$ compared to the case where $v = v_0$. □

*Proof of Lemma 5.* To see that the constrained optimization problem is convex, note that the objective is linear and that the non-linear constraints can be written as

$$v_1(s_1, s_2) - P(v_2(s_1, s_2)) \leq 0, \quad (s_1, s_2) \in S^2,$$

where the left-hand side is convex whenever $P$ is concave in $v_2$. To see that $P$ is concave, note that we can use the results from the proof of Lemma 3 to define $P$ as

$$P(v) = \begin{cases} v_0 - \tau_0(v - v_0) & \text{if } v \in [\underline{v}_1, v_0] \\ P(\underline{v}_1) - \tau_1(v - \underline{v}_1) & \text{if } v \in [\underline{v}_2, \underline{v}_1), \end{cases}$$

where

$$\tau_0 = 2\mu - 1, \ \tau_1 = 2\mu - 1 - \frac{\sigma^2 - \mu(1-\mu)}{\sigma}, \ \underline{v}_1 = \frac{3\sigma - \mu}{4(\mu - \sigma)} w, \text{ and, } \underline{v}_2 = -\frac{1 - 2\sigma}{4} w.$$

Due to symmetry in the first period, the first-order conditions for private consumption look almost the same as in the proof of Lemma 2:

$$\begin{aligned} c_1(s_L, s_L): &\quad -(1-\theta_L)(1+\psi) < 0, \\ c_1(s_L, s_H): &\quad \theta_L - 1 + \theta_L \psi < 0, \\ c_1(s_H, s_L): &\quad \theta_H - 1 - \theta_L \psi \geq 0, \\ c_1(s_H, s_H): &\quad \theta_H - 1 + (1-\theta_L)\psi > 0, \end{aligned}$$

where $\psi = 0$ if (13) is slack (or equivalently if (14) is slack). Note that $\psi$ is the Lagrange multiplier on either incentive compatibility constraint. The second line is implied by the third, since $\mu < 1$. If the third line would not hold, then $c_1(s_H, s_L) = 0$ such that $\psi = 0$ (but then $c_1(s_H, s_L)$ should be increased, a contradiction). Hence we know that $\psi \leq \frac{\theta_H - 1}{\theta_L}$.

Ignoring feasibility, the first-order effect of $v_j(s_L, s_L)$ is $\frac{\delta}{4}(1+\psi)$, which is positive. But then $v_1(s_L, s_L) = P(v_2(s_L, s_L)) = v_0$. Similarly, ignoring feasibility, the first-order effect of $v_j(s_H, s_H)$ is $\frac{\delta}{4}(1-\psi)$, which is positive, since $\psi < 1$. But then $v_1(s_H, s_H) = P(v_1(s_H, s_H)) = v_0$ as well. Simplifying notation slightly, we can write (13) as

$$c_1^{HL} = \min\left\{w, \frac{1-\theta_L}{\theta_L} w + \frac{\delta}{\theta_L}\left(v_1^{LH} - v_1^{HL}\right)\right\}.$$

Ignoring feasibility, the first-order effects of $v_1^{LH}$ and $v_1^{HL}$ are $\frac{\delta}{4}(1+\psi)$ and $\frac{\delta}{4}(1-\psi)$, respectively. Both are positive such that, using symmetry, $v_1^{LH} = P\left(v_2^{LH}\right) = P\left(v_1^{HL}\right)$ and $v_1^{HL} = P\left(v_2^{HL}\right) = P\left(v_1^{LH}\right)$. Hence we know that second-period welfare pairs are always on the Pareto frontier.

Finally, we need to show that $\Delta_v \equiv v_1^{LH} - v_1^{HL} > 0$ at the optimum. At $v_1^{HL} = v_1^{LH} = v_0$, suppose we reduce $v_1^{HL}$ by a small $dv > 0$. Then $\Delta_v = \left(-P'\left(v_1^{HL}\right) + 1\right) dv$. The effect on first-period welfare per agent is

$$\Delta_W = \frac{1}{4}\left[(\theta_H - 1)\frac{\delta}{\theta_L}\Delta_v + \delta\left(-P'\left(v_1^{HL}\right) - 1\right) dv\right] = \frac{\delta}{4\theta_L}\left[-(2\mu - 1)P'\left(v_1^{HL}\right) - (1 - 2\sigma)\right] dv.$$

For $dv < v_1^{HL} - \underline{v}_1$ we have $-P'\left(v_1^{HL}\right) = \tau_0 = 2\mu - 1$ such that $\Delta_W = \frac{\delta}{2\theta_L}\left(\sigma - 2\mu(1-\mu)\right) dv > \frac{\delta}{2\theta_L}\sigma(1-2\sigma)$, which is positive. Then $\Delta_v > 0$ at an optimum such that agents' second-period welfare will be varied to increase discretion in period one ($c_1^{HL}$ is higher compared to the allocation in Lemma 2). $\square$