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by Josef Schroth

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## Abstract

I build a model of optimal managerial compensation where managers each have a privately observed propensity to manipulate short-term stock prices. It is shown that this informational asymmetry reverses some of the conventional wisdom about the relationship between reliance on short-term pay and propensity to manipulate. The optimal compensation scheme features a negative relationship between pay duration and manager manipulation activity, reconciling theory with recent empirical findings (Gopalan et al., 2014). Further, the model predicts that managers who spend more resources manipulating short-term stock prices also put more effort into generating long-term firm value.

*JEL classification: D82, G14, G30, M12*

*Bank classification: Labour markets; Economic models; Recent economic and financial developments*

## Résumé

L'auteur construit un modèle de rémunération optimale des gestionnaires où chaque gérant se voit assigner une propension à manipuler les prix courants des actions de son entreprise et observe soi-même cette propension. L'étude montre que l'asymétrie d'information due à cette auto-observation entraîne des constats qui vont à l'encontre d'une partie des idées communément admises sur la relation entre la propension à manipuler et l'importance de la rémunération de court terme. Le régime de rémunération optimale présente une relation négative entre l'horizon de la rémunération et la propension à la manipulation des gérants, et concilie ainsi la théorie et les découvertes empiriques récentes (Gopalan et autres, 2014). Le modèle prévoit en outre que les gestionnaires qui dépensent plus de ressources à la manipulation des prix courants des actions consacrent aussi davantage d'efforts en vue d'augmenter la plus-value de leur entreprise sur le long terme.

*Classification JEL : D82, G14, G30, M12*

*Classification de la Banque : Marchés du travail; Modèles économiques; Évolution économique et financière récente*

## Non-Technical Summary

Firms spend substantial amounts to compensate their top management, particularly in the financial sector. There is an ongoing and intense debate on whether observed pay practices maximize firm owner wealth or primarily benefit managers. There is concern that managers may attempt to manipulate performance measures in order to increase the value of their equity grants. Recent empirical evidence finds that managers who manipulate more are also awarded equity incentives which focus more on the short term. Does this evidence imply that compensation contracts are designed by captured boards with the intention of allowing managers to increase their realized pay by manipulating performance measures? If so, regulatory interventions may be needed to support firms' corporate governance. I build a model of optimal managerial compensation where managers know better, compared to firm owners, how likely they are to engage in manipulation activities. Managers who understate their propensity to manipulate the firm stock price can boost the value of their equity grant by surprising market participants with stronger-than-anticipated manipulation activity. To discourage managers with high manipulation propensity from misleading market participants in this way, firm owners distort downward short-term equity incentives of managers with low manipulation propensity. The optimal compensation contract, which maximizes the wealth of firm owners, therefore has the property that managers who manipulate more receive relatively stronger short-term equity incentives. As a result, an observed negative relationship between pay duration and manipulation activity is consistent with optimal contracting and need not imply a failure of corporate governance with respect to the setting of managerial compensation.

# 1 Introduction

Equity pay constitutes an important part of managerial compensation (Hall and Liebman, 1998; Morgan and Poulsen, 2001; Murphy and Jensen, 2011; Murphy, 2012). The possibility that managers may attempt to manipulate stock prices in the short run (for example, via the timing of news, Aboody and Kasznik, 2000) therefore poses a challenge to the design of firm-value maximizing compensation schemes. Basic agency theory predicts that managers who are expected to manipulate stock prices more should be awarded fewer short-term equity incentives, and more long-term incentives instead. However, empirical evidence finds a positive relationship between different measures of manipulation activity and short-term equity incentives (Cheng and Warfield, 2005; Bergstresser and Philippon, 2006; Burns and Kedia, 2006; Efendi, Srivastava, and Swanson, 2007; Cheng, Luo, and Yue, 2013; Gopalan, Milbourn, Song, and Thakor, 2014). This paper argues that theory can be reconciled with the empirical evidence if we assume that managers know better than firm owners how likely they are to manipulate firm stock prices.

I build a model of optimal managerial compensation where managers can influence market participants' expectations about firm value. Such influence is a way for managers to manipulate the firm's short-term stock price but does not create any firm value. Managers cannot manipulate long-term stock prices; however, basing compensation only on long-term stock prices imposes a high cost in terms of risk on managers. An optimal compensation scheme balances short-term and long-term incentives in order to optimally trade off costs due to risk against costs due to wasteful manipulation.

There are two key assumptions in my model. First, managers observe privately their respective propensity to manipulate firm short-term stock prices. Second, man-

agers are hired by long-term firm owners who do not actively trade firm stock such that, as a result, firm owners reveal the manager compensation contract to market participants.<sup>1</sup> Under these two assumptions, in an equilibrium, market participants correctly discount manager exaggeration of firm value, short-term stock prices are unbiased, and managers earn informational rents. Since firms offer information rents to separate managers according to their unobservable manipulation propensities, the model generates cross-sectional predictions that are not driven by observable manager or firm characteristics. In fact, the empirical literature finds links between managerial compensation and manipulation activity that do not seem to be driven by observable manager or firm characteristics. This paper shows that the empirical links can be potentially explained by heterogeneity across managers with respect to unobservable manipulation propensities. Specifically, it is shown that firms do not find it worthwhile to induce managers with low manipulation propensity to put high effort into increasing firm value even though a manager's manipulation propensity is independent of his or her capability to increase firm value in the model. As a result, managers with high propensity to manipulate receive, in addition to information rents, higher-powered incentives overall, and in particular stronger short-term relative to long-term incentives. The model thus predicts, consistent with the evidence in Gopalan et al. (2014), that pay duration should be negatively correlated with manipulation activity conditional on observables. Since managers who receive stronger short-term incentives also put more effort into increasing long-term firm value, the net effect of a shorter pay duration on firm value is ambiguous in the model.

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<sup>1</sup>The second assumption rules out an insider-trading motive for firm owners and implies that firm owners prefer to pass on information about the compensation contract to market participants. In practice, in the United States, market participants can obtain information about the compensation awarded to the top five managers, including the CEO, from proxy statements that companies file with the Securities and Exchange Commission.

The channel that generates the negative relationship between equity-incentive duration and manipulation activity in the model operates via firm owners' desire to keep manager information rents low. Managers receive information rents to prevent them from understating their respective manipulation propensity, since that would allow them to surprise market participants with higher-than-expected manipulation. Such a surprise in turn would boost short-term stock prices and increase the value of managers' short-term incentive pay. Managers with high manipulation propensity therefore receive information rents that increase with the amount of short-term incentives given to managers with low-manipulation propensity. Since high-manipulation propensity managers cannot be prevented from manipulation, firms find it too costly, in terms of information rents, to provide undistorted short-term incentives to low manipulation propensity managers. The presence of managers with high manipulation propensity implies that managers with low manipulation propensity, but the same entrepreneurial talent, receive distorted incentives that induce lower effort toward increasing firm value.

Peng and Röell (2014) also derive an optimal compensation contract in a model where managers privately observe their respective propensity to manipulate the short-term stock price. However, in their model, compensation contracts are signed before managers observe their respective manipulation propensity, while I assume that contracts are signed after managers observe their manipulation propensity. As a result, the model in Peng and Röell (2014) does not imply any relationship, conditional on observables, between manipulation activity and reliance on short-term incentives, since firms offer the same contract to all managers.

The paper is organized as follows. The remainder of this section relates to the literature. Section 2 presents the model. Section 3 derives the formula for the stock

price. Section 4 characterizes the optimal linear compensation contract and discusses empirical predictions. Section 5 concludes.

## 1.1 Related literature

This paper builds on the Holmstrom and Tirole (1993) model of market monitoring by giving managers privately observed propensities to distort signals that market participants receive about future firm value. The long-run investor in the firm induces managers to reveal their respective propensity to distort the firm's stock price and passes on that information to market participants in order to achieve efficient market monitoring.<sup>2</sup> As a result, market participants filter out any exaggeration in equilibrium, and the demand schedule for the firm's stock is exactly the same as the one in Holmstrom and Tirole (1993). However, since a manager's actions are unobserved, the manager cannot commit not to attempt distorting the stock price according to his or her propensity, similar to the kind of dilemma studied in Stein (1989). In Fischer and Verrecchia (2000), managers can also manipulate stock prices, but they assume that compensation is exogenously given and that, similar to Peng and Röell (2014), market participants are uncertain about the manager's incentive to manipulate the stock price.<sup>3</sup> Kedia and Philippon (2009) and Benmelech, Kandel, and Veronesi (2010) use a stronger notion of manipulation that involves exaggeration of firm performance as well as subsequent suboptimal investment in order to conceal such exaggeration.

These papers also differ in focus and result: while Kedia and Philippon (2009) fo-

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<sup>2</sup>The long-run investor can commit not to trade the firm's stock during the time when managers can influence its price. Bolton, Scheinkman, and Xiong (2006a,b) build models where this is not the case.

<sup>3</sup>Morse, Nanda, and Seru (2011) argue that uncertainty over the compensation contract may be a way for powerful managers to extract rents (see also Yermack, 1997). Core and Guay (2002) develop a method to estimate equity incentives based on information contained in proxy statements or annual reports.

cus on manager insider trading and take the manager's incentive pay as exogenously given, the optimal compensation scheme in Benmelech, Kandel, and Veronesi (2010) does not induce manipulation in equilibrium. Goldman and Slezak (2006) consider the case where managers have known manipulation costs. They derive empirical implications with respect to the costs of manipulation that help explain low observed pay-for-performance elasticities (Murphy, 1999).

Beyer, Guttman, and Marinovic (2012) study the case where the manager's propensity to generate firm value is private information and where managers also have a publicly known propensity to manipulate earnings reports.<sup>4</sup> The optimal compensation scheme in their model implies a positive correlation between pay duration and manipulation activity when conditioning for observables (other than manipulation propensity). A version of my model with publicly known manipulation propensities also generates this result. However, when allowing for manipulation propensities to be private information, my model generates the opposite cross-sectional prediction (again conditioning on observables, see Gopalan et al., 2014): individual managers who manipulate more also obtain relatively more short-term incentives.

## 2 Model

### *Agents*

Consider the case of a publicly traded firm that is being run by a manager who does not own any stock initially. Stock is held by inside owners, liquidity traders, a speculator and a market maker. It is assumed that while inside owners and market participants are risk neutral, the manager is risk averse with constant absolute risk aversion

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<sup>4</sup>Healy (1985) finds that managers exercise discretion with respect to accounting procedures based on the nature of their compensation contract (see also Crocker and Slemrod, 2007).

$r > 0$ . The manager can provide effort  $e$ , and manipulation  $m$ . However, it is assumed that firm owners do not observe manager actions  $e$  and  $m$ . Let  $i$  denote manager gross compensation, or total pay. Then the manager's net certain equivalent (assume a zero outside option) is

$$u(i, e, m) = E(i) - \frac{r}{2} \text{Var}(i) - c(e, m), \quad (2.1)$$

where  $c(e, m) = \frac{1}{2}e^2 + \frac{1}{2\gamma}m^2$  is the cost of activities  $e$  and  $m$ . The parameter  $\gamma > 0$  denotes the manager's manipulation propensity.

### *Timing and technology*

The model has one period, which consists of three parts. In the first part, the firm is established and inside owners contract with the manager. In the second part, the speculator observes his or her private signal  $s$  and trades the firm's stock taking liquidity trader demand as given. Inside owners are not trading stock at this interim stage.<sup>5</sup> In the third part, firm value is realized, and the manager is compensated. Firm liquidation value is determined by manager effort and two independent shocks,  $\theta$  and  $\epsilon$ ,

$$\pi = e + \theta + \epsilon, \quad (2.2)$$

with  $\theta \sim N(0, \sigma_\theta^2)$ ,  $\epsilon \sim N(0, \sigma_\epsilon^2)$ , and  $\sigma_\theta^2, \sigma_\epsilon^2 > 0$ .

### *Information*

At the beginning of the period, managers privately observe their respective manipulation propensity  $\gamma$ . The inside owner knows that  $\gamma$  can take one of two values,  $\gamma_L > 0$  and  $\gamma_H > \gamma_L$ , with  $\text{Prob}(\gamma = \gamma_L) = \rho \in (0, 1/2)$ .<sup>6</sup> The inside owner will communicate

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<sup>5</sup>One could think about inside owners colluding with the manager, and trading against liquidity traders as well as speculators. Assuming that inside traders do not trade at the interim stock price allows us to restrict ourselves to the case where inside owners focus on the firm's long-term value rather than on the interim stock price.

<sup>6</sup>This distributional assumption is sufficient to generate most of the results of interest. Section 4.3

the details of the compensation contract to the speculator and the market maker (recall that the inside owner is not trading the stock at the interim stage).<sup>7</sup> For example,  $\gamma$  is high if the manager is good at conveying a biased interpretation of news about the firm without becoming legally liable in any way. In that sense,  $\{\gamma_L, \gamma_H\}$  and  $\rho$  can be thought of as given by the regulatory and technological environment, affecting all firms equally.

In the second part of the period, the speculator obtains a signal about future firm value from the manager. The signal  $s$  is given by

$$s = e + m + \theta + \eta, \tag{2.3}$$

where  $\eta \sim N(0, \sigma_\eta^2)$ ,  $\sigma_\eta^2 > 0$ , is noise and  $m$  is an additional bias that the manager attaches to the signal.<sup>8</sup> Note that manipulation  $m$  is tolerated and may even be expected (see Lemma 1 below). Managers use their discretion to affect market participants' expectation of future firm performance, for example via accruals (Bergstresser and Philippon, 2006; Gopalan et al., 2014) or by varying the precision of management forecasts (Cheng et al., 2013). The notion of manipulation used in this paper follows Yablon and Hill (2000) and differs from types of misrepresentation of firm fundamentals that could be characterized as fraud and lead to legal sanctions, and sometimes well-publicized scandals, when detected.<sup>9</sup>

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provides a more general statement of the results.

<sup>7</sup>It is assumed that the inside owner is not compensating liquidity traders for losses from trading with speculators and takes liquidity trader behavior as given. Holmstrom and Tirole (1993) model explicitly the inside owner's choice of liquidity trader activity.

<sup>8</sup>In Peng and Röell (2014), managers also have a propensity to manipulate signals. Their timing assumption is such that managers privately observe their cost of manipulation after signing a compensation contract. Below, in contrast, compensation contracts are allowed to depend on the manager's report about his or her privately observed manipulation propensity.

<sup>9</sup>The empirical literature finds conflicting evidence regarding the relationship between fraud and short-term equity incentives. While Johnson et al. (2009) find a positive correlation, as in the literature on non-fraud manipulation and equity incentives, Erickson et al. (2006) find no significant relationship.

### *Compensation contract*

Compensation can depend on the interim stock price, realized firm liquidation value and announced manager manipulation propensity. It is assumed that inside owners are restricting compensation schemes to be linear in interim stock price  $P$  and liquidation value  $\pi$ , but possibly non-linear in announced manipulation propensity.<sup>10</sup> In particular, realized gross compensation is

$$i = a_1\pi + a_2P + a_3, \quad (2.4)$$

where  $P$  is the interim stock price,  $a_1$  denotes long-term equity incentives,  $a_2$  denotes short-term equity incentives and  $a_3$  denotes cash compensation. Both  $a_1$  and  $a_2$  refer to stock price (or equity) related pay, while  $a_2$  denotes the component of stock price related pay, which is more short term (compared to  $a_1$ ). Holmstrom and Tirole (1993) and Peng and Röell (2014) also distinguish between short- and long-term equity pay in this fashion.<sup>11</sup> A compensation contract is denoted by  $(a_1(\hat{\gamma}), a_2(\hat{\gamma}), a_3(\hat{\gamma}))$ , where  $\hat{\gamma}$  denotes the manipulation propensity that the manager announces. Following the revelation principle, there is no loss of generality by assuming that the inside owner offers a different contract for each announced  $\hat{\gamma}$  in a way that induces truthful reporting,  $\hat{\gamma} = \gamma$ , and also reveals  $\hat{\gamma}$  to market participants. (Recall that the inside owner

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In these papers firms that are considered fraudulent are under investigation by the Securities and Exchange Commission.

<sup>10</sup>Note that assuming linearity may limit the model's propensity to be successfully calibrated and taken to the data (Haubrich and Popova, 1998; Baker and Hall, 2004). Deriving quantitative implications is beyond the scope of this paper. However, there has been a recent discussion about making compensation contracts in practice more linear in the sense of exposing managers to more downside risk (Murphy and Jensen, 2011; Chen et al., 2015). In the linear model that I use there is no exogenous limit on downside risk.

<sup>11</sup>The dividend right  $a_1$  entitles the manager to a fraction of the liquidation value of the firm, similar to (long-term) shares in Holmstrom and Tirole (1993). The stock appreciation right  $a_2$  is a cash payment that is a linear function of the interim stock price, exactly as in Holmstrom and Tirole (1993). In Chaigneau (2014) market participants continuously receive signals about the value of the firm such that the optimal compensation contract can depend on a continuum of stock prices in his paper.

does not trade the firm's stock at the interim price  $P$  and that liquidity traders are not compensated by the inside owner. As a result, the inside owner is only interested in maximizing expected firm value net of manager compensation. Bolton et al. (2006a,b) develop models where the inside owner may have an incentive to encourage the manager to increase the interim stock price.)

### 3 Interim stock price

Let  $(\hat{e}, \hat{m})$  be the expected equilibrium effort and manipulation levels, respectively, when the announced manipulation propensity is  $\hat{\gamma}$ . Also, let the exogenous demand of liquidity traders be given by  $y \sim N(0, \sigma_y^2)$ ,  $\sigma_y^2 > 0$ . Suppose speculator's demand when propensity  $\hat{\gamma}$  is announced is linear in its signal,

$$\hat{x}(s) = \hat{\xi}_1 + \hat{\xi}_2 s. \quad (3.1)$$

Verification of this linear demand rule and computation of the equilibrium interim stock price are very similar to the analysis in Holmstrom and Tirole (1993), and yield the exact same speculator demand and equilibrium stock price.

**Proposition 1.** 1. *The speculator's trading rule is characterized by*

$$\hat{\xi}_1 = -(\hat{e} + \hat{m}) \frac{\sigma_y}{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}}$$

$$\hat{\xi}_2 = \frac{\sigma_y}{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}}.$$

2. The equilibrium price is given by

$$P = \hat{e} + \frac{\sigma_\theta^2(\theta + \eta)}{2(\sigma_\theta^2 + \sigma_\eta^2)} + \frac{\sigma_\theta^2}{2(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}} \frac{y}{\sigma_y}.$$

*Proof.* The steps are the same as in Holmstrom and Tirole (1993), the only exception being that the mean of the signal is  $\hat{e} + \hat{m}$  rather than  $\hat{e}$ . However, the mean of the signal drops out of the expression for the price and speculator demand is exactly as in Holmstrom and Tirole (1993).  $\square$

Note that the speculator's demand does not depend on the announced manipulation propensity since

$$\hat{x}(s) = \hat{\xi}_1 + \hat{\xi}_2 s = \hat{\beta}(\theta + \eta) = \frac{\sigma_y}{(\sigma_\theta^2 + \sigma_\eta^2)^{1/2}}(\theta + \eta) \text{ for all } \hat{e}, \hat{m}.$$

The reason is that the speculator and market maker are equally informed about the manager's manipulation propensity. This is the case since the compensation contract induces separation and is communicated to market participants. In equilibrium, managers will provide as much manipulation as expected by market participants such that the ex-ante expectation of the interim stock price equals expected firm value,

$$E(P) = E(\pi) = \hat{e},$$

where  $E$  is the expectation operator when market participants believe that the manager reports his or her manipulation propensity truthfully and the manager indeed reports it truthfully. Note that there will be a strictly positive amount of manipulation in equilibrium, since managers cannot credibly commit not to use manipulation in order to distort the speculator's signal. This is similar to the kind of dilemma studied in

Stein (1989).

To better understand the manager's dilemma and incentive problem, consider the following argument out of equilibrium. Let  $e = e(\gamma, \hat{\gamma})$  and  $m = m(\gamma, \hat{\gamma})$  be the actual manager choices when the true manipulation propensity is  $\gamma$  but the announcement is  $\hat{\gamma}$ . Let  $\hat{e} = e(\hat{\gamma}, \hat{\gamma})$  and  $\hat{m} = m(\hat{\gamma}, \hat{\gamma})$  be the manager choices when the true propensity is  $\hat{\gamma}$ . Then at the beginning of the period, the expected interim stock price from the viewpoint of the manager with true manipulation propensity  $\gamma$  and announced propensity  $\hat{\gamma}$  is

$$\hat{E}(P) = \hat{e} + \frac{\psi}{2}(e + m) - \frac{\psi}{2}(\hat{e} + \hat{m}), \quad (3.2)$$

where  $\psi = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\eta}^2}$  is the speculator's signal-to-noise ratio, and  $\hat{E}$  denotes the expectation of the manager when market participants believe manipulation propensity is  $\hat{\gamma}$  while actual manipulation propensity is  $\gamma$ . For given  $\hat{e}$  and  $\hat{m}$  the manager always has an incentive to not only provide effort  $e$  but also manipulation  $m$  to increase the interim stock price. While the manager always has an incentive to ex-post manipulate the speculator's signal, the manager also has an incentive to understate his or her manipulation propensity in order to achieve an overvaluation of the firm's stock at the interim stage via  $m > \hat{m}$ . The inside owner must thus offer a compensation contract that discourages the manager from increasing the value of short-term pay by understating his or her manipulation propensity and surprising market participants with higher-than-expected manipulation  $m > \hat{m}$ .

The waste of resources due to manipulation as well as the inside owner's concern with misrepresentation of manipulation propensities make compensation that is based on the interim (or short-term) stock price  $P$  expensive relative to compensation based on realized liquidation value  $\pi$ . The following section shows how these additional

costs of short-term incentives affect the optimal compensation scheme.

## 4 Compensation contract

The inside owner's payoff is given by expected firm profit less expected compensation paid to the manager. With compensation contracts that induce manager separation, expressions (2.2) and (2.4) can be used to write the inside owner's objective function as

$$\begin{aligned} \Pi = & \rho \{e(\gamma_L) [1 - a_1(\gamma_L) - a_2(\gamma_L)] - a_3(\gamma_L)\} \\ & + (1 - \rho) \{e(\gamma_H) [1 - a_1(\gamma_H) - a_2(\gamma_H)] - a_3(\gamma_H)\}. \end{aligned} \quad (4.1)$$

Below we will suppress the dependence of the contract  $(a_1, a_2, a_3)$ , manipulation choice  $m$ , and effort level  $e$  on manipulation ability  $\gamma$  wherever possible, in order to make notation less cumbersome. The inside owner understands that, since  $e$  and  $m$  are unobserved, manager effort must coincide with the manager's individually rational choice given the chosen contract  $(a_1, a_2, a_3)$ .

**Lemma 1.** *For a given manipulation ability  $\gamma$ , and contract  $(a_1, a_2, a_3)$ , the manager chooses effort and manipulation levels*

$$\begin{aligned} e &= \left( a_1 + \frac{\psi}{2} a_2 \right), \\ m &= \frac{\psi}{2} a_2 \gamma. \end{aligned}$$

*Proof.* See Appendix A.1. □

Before proceeding with analyzing the optimal contract when managers privately

observe their respective manipulation ability,  $\gamma$ , the following section focuses on the case where firms can observe  $\gamma$ .

#### 4.1 Optimal compensation contract when $\gamma$ is publicly observable

For the purpose of this section only, we assume that the firm can observe manager manipulation ability  $\gamma$ . The firm then sets fixed cash pay to compensate managers for the cost of effort, manipulation, as well as risk. For a given  $\gamma$ , a firm then chooses short-term and long-term equity incentives in order to maximize its net profit,

$$\Pi(\gamma) = \max_{a_1, a_2} e - \frac{r}{2} \text{Var}(i) - c(e, m),$$

subject to effort and manipulation given by Lemma 1 (manager effort and manipulation are still unobserved).

**Proposition 2.** *The optimal linear compensation contract when  $\gamma$  is observable by the firm is characterized by equity incentives such that*

1. long-term incentives  $a_1(\gamma)$  are increasing in  $\gamma$ ,
2. short-term incentives  $a_2(\gamma)$  are decreasing in  $\gamma$ .

*Proof.* See Appendix A.1. □

Proposition 2 says that when firms observe how well managers can manipulate, then managers with higher manipulation propensity receive less short-term pay. Intuitively, firms award fewer short-term equity incentives to managers whose manipulation activities respond more strongly to short-term equity incentives. Note that managers are risk averse, which makes it worthwhile for firms to offer positive short-term incentives even if  $\gamma > 0$ , since managers require less cash compensation if they

suffer less risk.<sup>12</sup> Firms will therefore always balance incentives between the short and long run in order to insure managers partially against bad luck, but will provide incentives that are distorted more toward the long term when they observe a manager with higher short-term manipulation propensity  $\gamma$ . Lemma 2 shows that managers with higher  $\gamma$  receive lower-powered incentives, provide lower effort and generate fewer firm profits. This is again intuitive: since manipulation is wasteful and costly, firms want to discourage managers with high propensity to manipulate from manipulating excessively. But this distorts the balance of incentives and makes it too costly, in terms of risk borne by managers, for the firm to induce high effort.

**Lemma 2.** *When manager manipulation propensity  $\gamma$  is observable by the firm, then the optimal linear compensation scheme has the property that*

1. *induced effort  $e(\gamma) = a_1(\gamma) + \frac{\psi}{2}a_2(\gamma)$  is decreasing in  $\gamma$ ,*
2. *induced manipulation  $m(\gamma) = \frac{\psi}{2}a_2(\gamma)\gamma$  is increasing in  $\gamma$ ,*
3. *firm profits  $\Pi(\gamma) = [1 - a_1(\gamma) - a_2(\gamma)]e(\gamma) - a_3(\gamma)$  are decreasing in  $\gamma$ .*

*Proof.* See Appendix A.1. □

The empirical prediction produced by Proposition 2 and Lemma 2 that higher short-term pay should be associated with lower manipulation activity is intuitive: a firm finds it optimal to award fewer short-term equity incentives whenever it observes a manager whose manipulation activity responds more strongly to short-term incentives. However, the prediction contradicts empirical evidence (Cheng and Warfield,

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<sup>12</sup>Consider the case of vanishing noise  $\sigma_\epsilon^2$ ; then, the signal that market participants receive,  $s$ , see equation (2.3), is merely a noisy signal of observed firm liquidation value  $\pi$ , see equation (2.2). In that case, it will be optimal to set short-term incentives to zero. Formally, it can be seen from the proof of Proposition 2 that  $a_2 = 0$  as  $\sigma_\epsilon^2$  goes to zero. This also holds when  $\gamma$  is privately observed by managers, which can be seen by setting  $\sigma_\epsilon^2 = 0$  in the expressions for short-term incentives in the proof of Proposition 3.

2005; Bergstresser and Philippon, 2006; Burns and Kedia, 2006; Efendi, Srivastava, and Swanson, 2007; Cheng, Luo, and Yue, 2013; Gopalan, Milbourn, Song, and Thakor, 2014). Section 4.2 shows that the effects of manipulation propensity on short-term pay, long-term pay, and induced effort described in Proposition 2 and Lemma 2 are reversed when managers observe their respective manipulation propensity privately. The presence of an informational asymmetry between firms and managers regarding manager manipulation propensities can thus help to reconcile agency theory with the observed empirical evidence.

## 4.2 Optimal compensation contract when $\gamma$ is privately observed

A major concern in the model, when manipulation propensity  $\gamma$  is privately observed by managers, is that managers may use their private information in order to mislead market participants in a way that increases the expected value of short-term stock-price related pay.<sup>13</sup> To see this formally, note that we can use Lemma 1 in equation (2.1) in order to write the utility of a manager with true manipulation propensity  $\gamma$  as

$$\begin{aligned}
u(\gamma, \hat{\gamma}) := & \left( \hat{a}_1 + \frac{\psi}{2} \hat{a}_2 \right) (\hat{a}_1 + \hat{a}_2) + \left( \frac{\psi}{2} \hat{a}_2 \right)^2 (\gamma - \hat{\gamma}) + \hat{a}_3 \\
& - \frac{1}{2} \left( \hat{a}_1 + \frac{\psi}{2} \hat{a}_2 \right)^2 - \frac{1}{2} \left( \frac{\psi}{2} \hat{a}_2 \right)^2 \gamma - \frac{r}{2} \left[ \hat{a}_1^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + \psi \sigma_\theta^2 \hat{a}_2 \left( \hat{a}_1 + \frac{1}{2} \hat{a}_2 \right) \right],
\end{aligned} \tag{4.2}$$

where  $(\hat{a}_1, \hat{a}_2, \hat{a}_3)$  denotes the contract that the firm offers when the firm and market participants expect the manager's manipulation propensity to be  $\hat{\gamma}$ . Note that the

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<sup>13</sup>Morse et al. (2011) argue that some managers ('powerful CEOs') may have an opportunity to reduce transparency surrounding their compensation contract, which may help them in realizing higher pay. One way to think about this, in the context of this model, would be the case of a manager with high  $\gamma$  who can convince the firm to not disclose  $\gamma$  to market participants. Peng and Röell (2014) build a model where firms offer the same compensation contract to all managers and where a manager's manipulation propensity is not disclosed to market participants.

second term is positive if the manager understates his or her propensity to manipulate the market participants' signal and surprises market participants with a stronger-than-expected signal  $s$ , thereby increasing the value of his or her short-term pay. (This formalizes the intuition gained from equation (3.2).) In case of a separating contract, this term is always zero, since managers announce  $\hat{\gamma} = \gamma$ , and market participants fully anticipate manager manipulation of the signal. The analysis proceeds by focusing on separating contracts without loss of generality (revelation principle).

The optimal linear compensation scheme is required to satisfy the incentive compatibility conditions

$$u(\gamma_H, \gamma_H) \geq u(\gamma_H, \gamma_L), \quad u(\gamma_L, \gamma_L) \geq u(\gamma_L, \gamma_H), \quad (4.3)$$

as well as manager individual rationality conditions  $u(\gamma, \gamma) \geq 0$  for  $\gamma \in \{\gamma_L, \gamma_H\}$  (assume an outside option of zero for managers). Lemma 3 shows which of these conditions will be binding under an optimal linear compensation scheme.

**Lemma 3.** *The optimal linear compensation scheme offers two contracts, indexed by  $\gamma \in \{\gamma_L, \gamma_H\}$ , such that*

1. *managers with low manipulation propensity choose the contract indexed by  $\gamma_L$  and managers with high manipulation propensity choose the contract indexed by  $\gamma_H$ ,*
2.  *$u(\gamma_H, \gamma_H) = u(\gamma_H, \gamma_L)$ , i.e. the incentive compatibility condition of managers with high manipulation propensity binds,*
3.  *$u(\gamma_L, \gamma_L) \geq u(\gamma_L, \gamma_H)$  if and only if  $a_2(\gamma_H) \geq a_2(\gamma_L)$ , i.e. managers with high manipulation propensity receive higher short-term incentives,*
4.  *$u(\gamma_H, \gamma_H) > u(\gamma_L, \gamma_L) = 0$ , i.e. managers with low manipulation propensity receive*

their outside option of zero, while managers with high manipulation propensity receive an information rent.

Using the lemma, the information rent is obtained as

$$u(\gamma_H, \gamma_H) = u(\gamma_H, \gamma_L) = u(\gamma_L, \gamma_L) + \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L) = \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L). \quad (4.4)$$

The cash components of the optimal linear compensation scheme can be obtained by setting  $u(\gamma_L, \gamma_L) = 0$  and  $u(\gamma_H, \gamma_H) = \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L)$ , and solving for  $a_3(\gamma_L)$  and  $a_3(\gamma_H)$ , respectively, as follows:

$$\begin{aligned} a_3(\gamma_L) &= \frac{1}{2} \left( a_1(\gamma_L) + \frac{\psi}{2} a_2(\gamma_L) \right)^2 + \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 \gamma_L \\ &\quad + \frac{r}{2} \left[ a_1(\gamma_L)^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + \psi \sigma_\theta^2 a_2(\gamma_L) \left( a_1(\gamma_L) + \frac{1}{2} a_2(\gamma_L) \right) \right] \\ &\quad - \left( a_1(\gamma_L) + \frac{\psi}{2} a_2(\gamma_L) \right) (a_1(\gamma_L) + a_2(\gamma_L)), \\ a_3(\gamma_H) &= \frac{1}{2} \left( a_1(\gamma_H) + \frac{\psi}{2} a_2(\gamma_H) \right)^2 + \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_H)^2 \gamma_H \\ &\quad + \frac{r}{2} \left[ a_1(\gamma_H)^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + \psi \sigma_\theta^2 a_2(\gamma_H) \left( a_1(\gamma_H) + \frac{1}{2} a_2(\gamma_H) \right) \right] \\ &\quad - \left( a_1(\gamma_H) + \frac{\psi}{2} a_2(\gamma_H) \right) (a_1(\gamma_H) + a_2(\gamma_H)) + \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L). \end{aligned}$$

Using these expressions for  $a_3(\gamma_L)$  and  $a_3(\gamma_H)$ , equation (4.5) gives expected firm profit under the optimal linear compensation scheme as expected effort less expected

effort cost, manipulation cost, risk cost, and information rents:

$$\begin{aligned}
\Pi &= \rho \{e(\gamma_L) [1 - a_1(\gamma_L) - a_2(\gamma_L)] - a_3(\gamma_L)\} + (1 - \rho) \{e(\gamma_H) [1 - a_1(\gamma_H) - a_2(\gamma_H)] - a_3(\gamma_H)\} \\
&= \rho \left\{ \left( a_1(\gamma_L) + \frac{\psi}{2} a_2(\gamma_L) \right) - \frac{1}{2} \left( a_1(\gamma_L) + \frac{\psi}{2} a_2(\gamma_L) \right)^2 - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 \gamma_L \right. \\
&\quad \left. - \frac{r}{2} \left[ a_1(\gamma_L)^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + \psi \sigma_\theta^2 a_2(\gamma_L) \left( a_1(\gamma_L) + \frac{1}{2} a_2(\gamma_L) \right) \right] \right\} \\
&\quad + (1 - \rho) \left\{ \left( a_1(\gamma_H) + \frac{\psi}{2} a_2(\gamma_H) \right) - \frac{1}{2} \left( a_1(\gamma_H) + \frac{\psi}{2} a_2(\gamma_H) \right)^2 - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_H)^2 \gamma_H \right. \\
&\quad \left. - \frac{r}{2} \left[ a_1(\gamma_H)^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + \psi \sigma_\theta^2 a_2(\gamma_H) \left( a_1(\gamma_H) + \frac{1}{2} a_2(\gamma_H) \right) \right] - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L) \right\}.
\end{aligned} \tag{4.5}$$

Maximization of expected firm profit as given by equation (4.5) with respect to short-term and long-term pay components for the respective types of managers subject to the monotonicity condition  $a_2(\gamma_H) \geq a_2(\gamma_L)$  yields the optimal linear compensation scheme.

**Proposition 3.** *The optimal linear compensation scheme has the property that*

1.  $a_1(\gamma_H) < a_1(\gamma_L)$ , i.e. managers with high manipulation propensity obtain weaker long-term incentives,
2.  $a_2(\gamma_H) > a_2(\gamma_L)$ , i.e. managers with high manipulation propensity obtain stronger short-term incentives.

*Proof.* See Appendix A.1. □

The proposition shows that the contract chosen by managers with high propensity to manipulate the short-term stock price puts more emphasis on the short-term stock price. But then managers with a shorter pay duration manipulate more (see Lemma 1).

This result is the opposite of what the contract under publicly observable manipulation abilities prescribes (see Proposition 2 and Lemma 2). Hence, when we extend the basic model in section 4.1 by assuming that firms cannot observe manager manipulation propensities, agency theory can be reconciled with recent empirical findings (Cheng and Warfield, 2005; Bergstresser and Philippon, 2006; Burns and Kedia, 2006; Efendi et al., 2007; Cheng et al., 2013; Gopalan et al., 2014).

What explains this reversal under informational asymmetry regarding managers' propensity to manipulate the short-term signal that market participants receive about the liquidation value of the firm? When firm owners cannot ex-ante observe how well a manager can manipulate, they need to offer an information rent to managers who identify themselves as having high propensity to manipulate the short-term performance measure. The reason is that such managers could choose to identify as having low manipulation propensity and then surprise market participants with a more strongly than expected manipulated short-term signal, which would increase the value of such managers' short-term incentive pay. Hence, managers who identify as having high propensity to manipulate the short-term signal must be offered an information rent to induce them to forgo gains from understating their manipulation propensity. The purpose of the monotonicity condition  $a_2(\gamma_H) \geq a_2(\gamma_L)$ , then, is to discourage managers with low manipulation propensity from overstating their manipulation propensity in order to enjoy the information rent as well. Intuitively, the loss to such managers from disappointing market participants with a weaker-than-expected short-term signal, which is a loss that is increasing in  $a_2(\gamma_H)$ , must at least offset the benefit from the obtained information rent, which is a benefit that is increasing in  $a_2(\gamma_L)$ . Therefore, the monotonicity condition requires that  $a_2(\gamma_H)$  should not be too low relative to  $a_2(\gamma_L)$ . The reason firms set  $a_2(\gamma_L)$  lower than required by the monotonicity condition

is that distorting  $a_2(\gamma_L)$  downward lowers the information rent enjoyed by managers with high propensity to manipulate.

Proposition 3 has a number of implications that are summarized in Lemma 4. First, since the information rent depends only on  $a_2(\gamma_L)$  and not on any components of the contract chosen by managers with high propensity to manipulate, such managers receive short-term and long-term incentives that are undistorted. Second, since only the incentives of managers with low propensity to manipulate are being distorted, such managers provide lower effort. Third, contracts indexed by  $\gamma_H$  lead to relatively higher firm profits whenever  $\rho$  is not too high. The reason is that firms find it more worthwhile to reduce the information rent by distorting the incentive of managers with low propensity to manipulate when there are fewer such managers (i.e. when  $\rho$  is low).

**Lemma 4.** *The optimal linear compensation scheme has the property that*

1. *the variable pay components of the contract indexed by  $\gamma_H$  are undistorted in the sense that they are the same as in Proposition 2 for  $\gamma = \gamma_H$ ,*
2. *managers with high manipulation propensity are induced to provide higher effort, i.e.  $e(\gamma_H) > e(\gamma_L)$ ,*
3. *managers with high manipulation propensity generate higher firm profit when there are few managers with low manipulation propensity, i.e.  $\Pi(\gamma_H) > \Pi(\gamma_L)$  if  $\rho \in (0, \hat{\rho})$  for some  $\hat{\rho} \in (0, 1/2)$ , and  $\Pi(\gamma_H) \leq \Pi(\gamma_L)$  otherwise.*

*Proof.* See Appendix A.1. □

Managers with high propensity to manipulate the interim signal to stock market participants receive relatively stronger short-term stock price incentives. As a result,

those managers also spend more resources on wasteful manipulation (see Lemma 1). They also provide more effort, and they generate higher firm profit if information rents are not too high. Intuitively, since firms cannot prevent high-manipulation propensity managers from manipulating, it is too costly for firms to provide undistorted short-term incentives to low-manipulation propensity managers. The presence of managers with high manipulation propensity implies that managers with low manipulation propensity, but the same entrepreneurial talent, receive distorted incentives that induce lower effort.

*Empirical predictions:*

It is predicted that an optimal compensation scheme would imply a positive correlation between the relative strength of short-term incentives and manipulation activity. This prediction depends crucially on the assumption that managers know better than firm owners their respective manipulation propensity (see section 4.1). An informational asymmetry between managers and firm owners with respect to manager manipulation propensities is thus a way of reconciling agency theory with the empirical findings in Cheng and Warfield (2005), Bergstresser and Philippon (2006), Burns and Kedia (2006), Efendi et al. (2007), Cheng et al. (2013), and Gopalan et al. (2014). The paper shows that the observed positive relation between manipulation activity and short-term incentives need not imply a failure of corporate governance with respect to the design of managerial compensation in practice.

It is further predicted that managers who receive shorter-duration pay, and engage in more manipulation of the short-term stock price, put more effort into increasing the long-term value of the firm. However, firm profit, net of manager compensation, may be lower if manager (information) rents are high enough. These are additional predictions which can be used to further empirically explore the channel proposed in

this paper.<sup>14</sup>

#### 4.2.1 Further comparative statics

The variance of manager manipulation propensities has a first-order effect on information rents and thus on the way equity incentives are distorted by the informational asymmetry between firms and managers. Observable manager or firm characteristics may lead to different priors regarding the distribution, and in particular the variance, of manager manipulation propensities.<sup>15</sup> This section shows how the optimal compensation scheme responds to changes in prior uncertainty about manipulation propensities.

**Lemma 5.** *Suppose prior uncertainty about manipulation propensities decreases, while the average manipulation propensity is unchanged; i.e.  $\Delta\gamma = \gamma_H - \gamma_L$  decreases, while  $E(\gamma) = \rho\gamma_L + (1 - \rho)\gamma_H$  stays constant. Then the effect on the optimal compensation scheme is such that*

1. *short-term incentives increase and long-term incentives decrease for each manager,*
2. *each manager provides higher effort,*

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<sup>14</sup>The prediction that manipulation and effort are positively correlated is related to the models of Kedia and Philippon (2009) and Benmelech et al. (2010), in which manipulation is upheld temporarily by an increase in real economic activity at the firm.

Since managers who receive higher short-term equity pay also receive more equity pay in total, i.e. the sum of short- and long-term incentives is larger for those managers, the model also predicts that total equity incentives and effort (firm value before compensation) are positively correlated. Mehran (1995) finds a positive relationship between total equity incentives and firm profit (firm value net of compensation). However, in my model, firm profit  $\Pi$  can either be positively or negatively related to manipulation propensity  $\gamma$  and total equity incentives, depending on whether there are few or many managers with low manipulation propensity  $\gamma = \gamma_L$ . This result of my model is related to the empirical evidence in Palia (2001), who does not find a statistically significant relationship between equity incentives and firm profit.

<sup>15</sup>For example, there may be greater prior uncertainty about manipulation propensities in the case of younger managers compared to the case of older managers. Older managers may have an observable history, which can be used to obtain more accurate prior information about their likely reporting behavior (Bamber et al., 2010; Brochet and Welch, 2011).

3. *average firm profits generated across managers increase,*

4. *average compensation across managers increases.*

*Proof.* See Appendix A.1. □

For a decrease in uncertainty about manager manipulation propensities, firms are predicted to award more short-term incentives, shortening the duration of pay and inducing higher effort from managers. For example, older managers, managers with longer tenure and managers employed by firms with shorter-duration projects are predicted to receive pay that has shorter duration. This is consistent with findings in Gopalan et al. (2014). These comparative statics are similar to those obtained in Peng and Röell (2014); however, in their model lower manipulation uncertainty would lead to a lower absolute, but higher relative (to long-term incentives), amount of short-term incentives awarded.<sup>16</sup>

### 4.3 Results for a more general distribution of $\gamma$

This section shows that the results in section 4.2 hold for more general distributional assumptions regarding manager manipulation propensities, and also derives some additional results. For the purpose of this section, it is assumed that a manager's manipulation propensity  $\gamma$  is the realization of a continuous random variable with domain  $[0, \infty)$  and probability density function (pdf)  $\phi$ . It is also assumed that  $\phi$  has a unique global maximum, its mode, but no further local maxima.<sup>17</sup> Let  $\overset{\circ}{\gamma}$  denote the mode of the distribution of  $\gamma$ .

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<sup>16</sup>The reason is that, in their model, lower manipulation uncertainty translates into more-effective short-term incentives as the stock price becomes more responsive, while in my model the amount of information contained in stock prices does not depend on manipulation uncertainty in equilibrium.

<sup>17</sup>Examples would be the family of gamma distributions with  $\phi(0)$  defined appropriately. We could also accommodate the case where  $\gamma$  is uniform on  $[0, M]$ , with  $M < \infty$  and the 'unique global maximum' of  $\phi$ , or mode, defined as zero.

**Proposition 4.** *The optimal linear compensation scheme when  $\gamma$  has pdf  $\phi$  is such that for some  $\bar{\gamma} \leq \overset{\circ}{\gamma}$ ,*

1. *the amount of short-term incentives  $a_2$  is increasing in manipulation propensity  $\gamma$  for all  $\gamma \geq 0$ , and strictly increasing for  $\gamma < \bar{\gamma}$ ,*
2. *the amount of long-term incentives  $a_1$  is decreasing in  $\gamma$  for all  $\gamma \geq 0$ , and strictly decreasing for  $\gamma < \bar{\gamma}$ ,*
3. *induced effort  $e$  is increasing in  $\gamma$  for all  $\gamma \geq 0$ , and strictly increasing for  $\gamma < \bar{\gamma}$ ,*
4. *manipulation  $m$  is strictly increasing in  $\gamma$  for all  $\gamma \geq 0$ ,*
5. *manager information rents are strictly increasing in  $\gamma$  for all  $\gamma \geq 0$ , and strictly increasing and strictly convex for  $\gamma < \bar{\gamma}$ .*

*Proof.* See Appendix A.2. □

The intuition is similar to that in section 4.2. Since firms need to pay information rents and information rents depend more on short-term incentives granted to managers with low manipulation propensity, firms will find it beneficial to distort short-term incentives of those managers downward. For large enough  $\gamma > \bar{\gamma}$ , however, it ceases to be beneficial for the firm to distort short-term incentives for managers with manipulation propensity below  $\gamma$  such that the monotonicity condition binds eventually.<sup>18</sup>

The empirical predictions in section 4.2 continue to hold: manipulation activity and pay duration are negatively related across managers, and managers who manipulate more also put (weakly) more effort into increasing firm value. A new prediction when

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<sup>18</sup>See also the discussion preceding Lemma 8 in Appendix A.1.

$\gamma$  has pdf  $\phi$  is that the amount of pay that managers receive eventually ceases to induce higher effort. When  $\gamma > \bar{\gamma}$ , manager total compensation, via information rents, is strictly increasing in manipulation propensity  $\gamma$ , but the effort provided remains unchanged at  $e(\bar{\gamma})$ . As a result, the model predicts that firms that pay their managers the most do not pay for performance at the margin.<sup>19</sup>

#### 4.3.1 Further comparative statics for the more general distribution of $\gamma$

This section develops a version of Lemma 5 for the case where  $\gamma$  has pdf  $\phi(\gamma) = \lambda \exp\{-\lambda\gamma\}$ . It is thus assumed that manipulation propensities are distributed exponentially with mean and standard deviation  $\frac{1}{\lambda}$ . Prior uncertainty about manipulation propensities can be reduced by increasing  $\lambda$ . The parameter  $\lambda$  can be interpreted as capturing observable manager and firm characteristics or features of the institutional environment, such as effective disclosure regulations, that limit manager manipulation or make it more predictable. Lemma 6 shows comparative statics with respect to  $\lambda$ .<sup>20</sup>

**Lemma 6.** *Suppose prior uncertainty about manipulation propensities decreases in the case where  $\gamma$  has pdf  $\phi(\gamma) = \lambda \exp\{-\lambda\gamma\}$ , i.e.  $\lambda$  increases. Then the effect on the optimal compensation scheme is such that*

1. *short-term incentives increase and long-term incentives decrease for each manager,*
2. *each manager provides higher effort,*
3. *average firm profits generated across managers increase,*

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<sup>19</sup>Highly-paid managers receive higher cash pay but do not create any additional value for firm owners such that firm profit eventually decreases as cash compensation increases. This result of my model is related to empirical work by Mehran (1995), who finds a negative correlation between cash compensation and firm profit.

<sup>20</sup>This notion of manipulation uncertainty is somewhat closer to the one in Peng and Röell (2014), who use the variance of the logarithm of manipulation propensities. In contrast to the notion of manipulation uncertainty used in section 4.2.1, these measures depend both on the mean and the variance of manipulation propensities.

4. *average manipulation across managers decreases.*

*Proof.* See Appendix A.1. □

A low value for  $\lambda$  implies that managers on average find it easier to introduce a bias into the short-term signal that market participants receive about long-term firm performance. One reason could be that stock market participants find it more difficult to obtain reliable information by themselves, for example due to high R&D expenditure or high expected growth, and thus rely more on guidance by managers. Lemma 6 predicts that in such a case firm owners prefer to rely more on long-term rather than short-term incentives. This prediction is the same as in Peng and Röell (2014) and is in line with the existing empirical literature (Bizjak, Brickley, and Coles, 1993; Cadman, Rusticus, and Sunder, 2013).

In addition, Lemma 6 shows that cross-sectional variation in manipulation uncertainty would produce a positive cross-sectional relationship between manipulation activity and the duration of equity incentives. For instance, a firm facing higher manipulation uncertainty would award fewer short-term incentives and its manager would on average manipulate more.<sup>21</sup> On the other hand, Proposition 4 shows that pay duration and manipulation activity are negatively related conditional on the observable characteristic  $\lambda$ .

**Lemma 7.** *There are thresholds  $0 < \bar{\lambda}_2 < \bar{\lambda}_1 < \infty$  such that*

1. *average information rents are strictly increasing in  $\lambda$  on  $[0, \bar{\lambda}_1]$ , and strictly decreasing in  $\lambda$  on  $(\bar{\lambda}_1, \infty)$ ,*

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<sup>21</sup>In Peng and Röell (2014), a firm that faces higher manipulation uncertainty would award relatively more short-term incentives and its manager would on average manipulate less. This would again produce a positive relationship between equity pay duration and manipulation activity as a result of variation in manipulation uncertainty.

2. *average information rents relative to average firm profits are strictly increasing in  $\lambda$  on  $[0, \bar{\lambda}_2]$ , and strictly decreasing in  $\lambda$  on  $(\bar{\lambda}_2, \infty)$ .*

*Proof.* See Appendix A.1. □

Lemma 7 shows that a reduction in manipulation uncertainty, i.e. an increase in  $\lambda$ , can make managers better off on average provided that manipulation uncertainty was not too low initially. Furthermore, average manager information rents may increase faster than average firm profits. A reduction in manipulation uncertainty, which strengthens performance monitoring by giving market participants more precise ex-ante information about manager manipulation propensities, may benefit managers even more than it benefits firm owners.

## 5 Conclusion

When managers receive relatively strong short-term equity incentives, they may be induced to engage in myopic and wasteful activities, with the goal to increase short-term stock prices at the cost of reducing long-term firm value. However, basing compensation exclusively on long-term performance measures exposes managers to inefficiently high levels of risk. Firms therefore often attempt to find a balance between short-term and long-term incentives. The concern that firms may have recently relied too much on short-term equity incentives is underlined by recent empirical work, which finds a negative relationship between pay duration and manipulation activity.

This paper shows that a negative relationship between pay duration and manipulation activity is obtained within an optimal managerial compensation scheme when managers know better than firm owners their respective propensity to manipulate the short-term stock price. Firm profit is highest in the model when firms offer contracts

that differ in the relative strength of short-term equity incentives and that also carry different expectations of market participants about manager manipulation. Specifically, the firm's informational disadvantage is efficiently mitigated when managers who choose a contract with relatively stronger short-term incentives are expected by market participants to manipulate the firm stock price more. On the one hand, managers with low manipulation propensity will find it too costly, given market expectations, to choose a contract with stronger short-term incentives. On the other hand, awarding fewer short-term incentives to managers with low manipulation propensity allows firms to reduce information rents enjoyed by managers with high manipulation propensity.

The analysis highlights the importance of market expectations of manager manipulation activity for efficiently overcoming informational asymmetries between firms and managers regarding the propensity of managers to manipulate the firm's stock price. The paper contributes to the recent debate on the appropriateness of observed compensation practices by showing that an observed negative relationship between equity pay duration and manipulation activity need not imply a failure of corporate governance with respect to the setting of CEO pay.

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# A Appendix

## A.1 Proofs of main results

*Proof of Lemma 1.* After revealing manipulation propensity  $\gamma$  and receiving compensation contract  $(a_1, a_2, a_3)$ , the manager chooses effort  $e$  and manipulation  $m$  to maximize utility,

$$\max_{e,m} E(i) - \frac{r}{2} \text{Var}(i) - c(e, m),$$

where

$$E(i) = a_1 E(\pi) + a_2 E(P) + a_3 = a_1 e + a_2 \left[ \hat{e} + \frac{\psi}{2}(e + m) - \frac{\psi}{2}(\hat{e} + \hat{m}) \right] + a_3,$$

with  $\hat{e}$  and  $\hat{m}$  denoting effort and manipulation levels expected by market participants, and where  $\text{Var}(i)$  does not depend on efforts chosen by linearity of the contract. The manager's effort and manipulation choice is thus given as the solution to the following simple maximization problem:

$$\max_{e,m} \left( a_1 + \frac{\psi}{2} \right) e + \frac{\psi}{2} a_2 m - \frac{1}{2} e^2 - \frac{1}{2} \frac{m^2}{\gamma}.$$

□

*Proof of Proposition 2.* The firm's objective can be written as

$$\begin{aligned} \Pi(\gamma) = e - \frac{r}{2} \text{Var}(i) - c(e, m) &= a_1 + \frac{\psi}{2} a_2 - \frac{1}{2} \left( a_1 + \frac{\psi}{2} a_2 \right)^2 - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2^2 \gamma \\ &\quad - \frac{r}{2} \left[ a_1^2 (\sigma_\theta^2 + \sigma_\varepsilon^2) + \psi \sigma_\theta^2 a_2 \left( a_1 + \frac{1}{2} a_2 \right) \right]. \end{aligned}$$

Maximizing this expression with respect to  $a_1$  and  $a_2$  yields

$$a_1 = \frac{1}{r\sigma_\epsilon^2 + \left(1 + \frac{\psi}{2}F(\gamma)\right)(1 + r\sigma_\theta^2)},$$

$$a_2 = F(\gamma)a_1,$$

$$F(\gamma) = \frac{r\sigma_\epsilon^2}{\frac{\psi}{2}\gamma + r\sigma_\theta^2\left(1 - \frac{\psi}{2}\right)}.$$

We can verify that long-term incentives  $a_1$  are increasing and short-term incentives  $a_2$  are decreasing in  $\gamma$ .  $\square$

*Proof of Lemma 2.* We can write effort as

$$e(\gamma) = a_1(\gamma) + \frac{\psi}{2}a_2(\gamma) = \left(1 + \frac{\psi}{2}F(\gamma)\right)a_1(\gamma) = \frac{1}{\frac{r\sigma_\epsilon^2}{1 + \frac{\psi}{2}F(\gamma)} + 1 + r\sigma_\theta^2},$$

which is increasing in  $F(\gamma)$  and hence decreasing in  $\gamma$ . To show that manipulation is increasing in  $\gamma$  it is straightforward to check that

$$\frac{dm(\gamma)}{d\gamma} = \frac{\psi}{2}a_2 \left[1 - \frac{\psi}{2}a_2\gamma \frac{1 + r(\sigma_\theta^2 + \sigma_\epsilon^2)}{r\sigma_\epsilon^2}\right] > 0.$$

To show that firm profits are decreasing in  $\gamma$  as well, we show that they are proportional to effort. In fact, we verify

$$\frac{1}{2}e = \Pi(\gamma) = a_1 + \frac{\psi}{2}a_2 - \frac{1}{2}\left(a_1 + \frac{\psi}{2}a_2\right)^2 - \frac{1}{2}\left(\frac{\psi}{2}\right)^2 a_2^2\gamma - \frac{r}{2}\left[a_1^2(\sigma_\theta^2 + \sigma_\epsilon^2) + \psi\sigma_\theta^2 a_2(a_1 + \frac{1}{2}a_2)\right].$$

Rearranging terms yields

$$a_1\left(1 + \frac{\psi}{2}F(\gamma)\right) = a_1^2\left(1 + \frac{\psi}{2}F(\gamma)\right)^2 + \left(\frac{\psi}{2}\right)^2 a_1^2 F(\gamma)^2 \gamma + r a_1^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + r \psi \sigma_\theta^2 a_1^2 F(\gamma) \left(1 + \frac{1}{2}F(\gamma)\right),$$

while dividing by  $a_1^2$  yields

$$\begin{aligned} & \left(1 + \frac{\psi}{2}F(\gamma)\right) \left[r\sigma_\epsilon^2 + \left(1 + \frac{\psi}{2}F(\gamma)\right) (1 + r\sigma_\theta^2)\right] \\ &= \left(1 + \frac{\psi}{2}F(\gamma)\right)^2 + \left(\frac{\psi}{2}\right)^2 F(\gamma)^2\gamma + r(\sigma_\theta^2 + \sigma_\epsilon^2) + r\psi\sigma_\theta^2 F(\gamma) \left(1 + \frac{1}{2}F(\gamma)\right). \end{aligned}$$

This can be simplified, which after collecting terms yields

$$r\sigma_\epsilon^2 = F(\gamma) \left[\frac{\psi}{2}\gamma + \left(1 - \frac{\psi}{2}\right) r\sigma_\theta^2\right] \Leftrightarrow F(\gamma) = F(\gamma),$$

verifying the claim that  $\Pi(\gamma) = \frac{1}{2}e(\gamma)$ . □

*Proof of Lemma 3.* The first part holds by assumption since, using the revelation principle, we can restrict attention to separating contracts. Note that

$$u(\gamma_H, \gamma_L) = u(\gamma_L, \gamma_L) + \frac{1}{2} \left(\frac{\psi}{2}\right)^2 a_2(\gamma_L)^2(\gamma_H - \gamma_L) \leq u(\gamma_H, \gamma_H),$$

where the inequality follows from the incentive compatibility requirement for managers with high manipulation propensity. But then  $u(\gamma_H, \gamma_H) > u(\gamma_L, \gamma_L) \geq 0$ . Suppose  $u(\gamma_H, \gamma_H) > u(\gamma_H, \gamma_L)$ ; then, the firm can increase its profit, and relax the incentive compatibility condition of managers with low manipulation propensity, by reducing cash pay for managers with high manipulation propensity. Hence  $u(\gamma_H, \gamma_H) = u(\gamma_H, \gamma_L)$ , which proves the second part of the lemma.

Note that we can write the incentive compatibility condition for managers with low ma-

nipulation propensity as

$$\begin{aligned}
u(\gamma_L, \gamma_L) \geq u(\gamma_L, \gamma_H) &= u(\gamma_H, \gamma_H) - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_H)^2 (\gamma_H - \gamma_L) \\
&= u(\gamma_H, \gamma_L) - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma_H)^2 (\gamma_H - \gamma_L) \\
&= u(\gamma_L, \gamma_L) - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 [a_2(\gamma_H)^2 - a_2(\gamma_L)^2] (\gamma_H - \gamma_L) \\
&\Leftrightarrow a_2(\gamma_H)^2 - a_2(\gamma_L)^2 \geq 0.
\end{aligned}$$

Hence  $u(\gamma_L, \gamma_L) \geq u(\gamma_L, \gamma_H)$  if and only if  $a_2(\gamma_H) \geq a_2(\gamma_L)$ , which proves the third part of the lemma. The proof of the fourth part of the lemma is completed by noting that nothing prevents the firm from setting  $u(\gamma_L, \gamma_L)$  as low as possible, i.e. to  $u(\gamma_L, \gamma_L) = 0$ .  $\square$

*Proof of Proposition 3.* Straightforward unconstrained maximization of firm profits as given by equation (4.5) yields the variable pay components

$$a_1(\gamma_j) = \frac{1}{r\sigma_\epsilon^2 + \left(1 + \frac{\psi}{2}F_j\right)(1 + r\sigma_\theta^2)} \quad \text{and} \quad a_2(\gamma_j) = F_j a_1(\gamma_j), \quad j = L, H,$$

where the relative weights on short-term incentives  $F_j$  are given by

$$F_L = \frac{r\sigma_\epsilon^2}{\frac{\psi}{2}\gamma_L + r\sigma_\theta^2 \left(1 - \frac{\psi}{2}\right) + \frac{1-\rho}{\rho} \frac{\psi}{2}(\gamma_H - \gamma_L)}, \quad \text{and} \quad F_H = \frac{r\sigma_\epsilon^2}{\frac{\psi}{2}\gamma_H + r\sigma_\theta^2 \left(1 - \frac{\psi}{2}\right)}.$$

To verify monotonicity note that

$$a_2(\gamma_H) \geq a_2(\gamma_L) \Leftrightarrow F_H \geq F_L \Leftrightarrow \rho \leq \frac{1}{2}.$$

This holds with strict inequality, since  $\rho < \frac{1}{2}$ . As a result, we have that the optimal linear compensation scheme features  $a_1(\gamma_H) < a_1(\gamma_L)$  and  $a_2(\gamma_H) > a_2(\gamma_L)$ . The case  $\rho \geq \frac{1}{2}$  is examined in Lemma 8 and constitutes a special case, as the generalization in section 4.3 shows.

From Lemma 1 we see that manipulation is higher for managers with  $\gamma_H$ .  $\square$

If there are too many managers with low manipulation propensity,  $\rho \geq \frac{1}{2}$ , then the firm does not find it worthwhile to distort their short-term incentives downward. But it cannot offer lower short-term incentives to managers with high propensity to manipulate, either, since then managers with low propensity would imitate them. Lemma 8 shows that the firm offers the same equity incentives to all managers in that case.<sup>22</sup> Note that managers with high manipulation propensity will still receive information rents such that cash payments differ across managers,  $a_3(\gamma_H) > a_3(\gamma_L)$ . In that case, managers who are paid more will not put more effort into increasing firm value.

**Lemma 8.** *If  $\rho \geq \frac{1}{2}$ , then the optimal linear compensation scheme offers the same equity incentives to all managers. That is,  $a_1(\gamma_L) = a_1(\gamma_H) = a_1$  and  $a_2(\gamma_L) = a_2(\gamma_H) = a_2$ , where*

$$a_1 = \frac{1}{(1 + r\sigma_\theta^2) \left(1 + \frac{\psi}{2}\tilde{F}\right) + r\sigma_\epsilon^2}, \quad \text{and} \quad a_2 = \tilde{F}a_1,$$

with

$$\tilde{F} = \frac{r\sigma_\epsilon^2}{\frac{\psi}{2} [\rho\gamma_L + (1 - \rho)\gamma_H] + r\sigma_\theta^2 \left(1 - \frac{\psi}{2}\right) + (1 - \rho)\frac{\psi}{2}(\gamma_H - \gamma_L)}.$$

*Proof of Lemma 8.* We know from the proof of Proposition 3 that the monotonicity condition binds for  $\rho \geq \frac{1}{2}$ . Setting  $a_2(\gamma_L) = a_2(\gamma_H) = a_2$  in equation (4.5) and taking the first-order conditions with respect to  $a_1(\gamma_L)$ ,  $a_1(\gamma_H)$ , and  $a_2$  yields the result.  $\square$

*Proof of Lemma 4.* The variable pay components of the contract indexed by  $\gamma_H$  are derived in the proof of Proposition 3 and they are the same as the variable pay components in Proposition 2

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<sup>22</sup>Section 4.3 shows that a sufficient condition for the optimality of a compensation scheme that gives the same equity incentives to all managers is that the (probability) mass of managers has a unique peak at the (greatest) lower bound of the support of manipulation propensities (if the support is not closed, we need to take the limit toward the lower bound). If there is a unique interior peak, then short-term pay is increasing in manipulation propensity up to a manipulation propensity less than the peak and remains constant thereafter. Hence, the case where all managers receive the same equity incentives is a special case.

for  $\gamma = \gamma_H$ . This proves the first part of Lemma 4.

Effort provided by a manager who chooses the contract indexed by  $\gamma_j$  is given by

$$e(\gamma_j) = a_1(\gamma_j) + \frac{\psi}{2}a_2(\gamma_j) = \left(1 + \frac{\psi}{2}F_j\right) a_1(\gamma_j) = \frac{1}{\frac{r\sigma_\varepsilon^2}{1 + \frac{\psi}{2}F_j} + 1 + r\sigma_\theta^2},$$

for  $j = L, H$ . Since  $F_H > F_L$ , we have  $e(\gamma_H) > e(\gamma_L)$ , proving the second part of Lemma 4. Note that manipulation is increasing in short-term incentives and manipulation propensity. Since managers with higher manipulation propensity also receive higher short-term incentives, they provide more manipulation, proving the third result.

To prove the last result we first need to verify that

$$\begin{aligned}\frac{1}{2}e(\gamma_L) &= \Pi(\gamma_L) - \frac{1}{2}\left(\frac{\psi}{2}\right)^2 \frac{1-\rho}{\rho} a_2(\gamma_L)^2 (\gamma_H - \gamma_L), \\ \frac{1}{2}e(\gamma_H) &= \Pi(\gamma_H) + \frac{1}{2}\left(\frac{\psi}{2}\right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L),\end{aligned}$$

which can be done following the steps in the proof of Lemma 2 (i.e., by dividing both sides of the first equation by  $a_1(\gamma_L)$  and both sides of the second equation by  $a_1(\gamma_H)$ , and rearranging).

Define the function  $g(\rho) = 2[\Pi(\gamma_H) - \Pi(\gamma_L)]$ ; then,

$$g(\rho) = e(\gamma_H) - e(\gamma_L) - \frac{1}{\rho}\left(\frac{\psi}{2}\right)^2 a_2(\gamma_L)^2 (\gamma_H - \gamma_L).$$

Note that  $g$  is continuous in  $\rho$  on  $(0, 1/2)$ , that  $\lim_{\rho \rightarrow 0} \frac{a_2(\gamma_L)^2}{\rho} = 0$ , and that  $\lim_{\rho \rightarrow 1/2} e(\gamma_L) = e(\gamma_H)$  such that  $g(0) > 0$  and  $g(1/2) < 0$ . Hence by the intermediate value theorem, we have  $g(\hat{\rho}) = 0$  for some  $\hat{\rho} \in (0, 1/2)$ . This yields  $\Pi(\gamma_H) > \Pi(\gamma_L)$  for  $\rho \in (0, \hat{\rho})$  and  $\Pi(\gamma_H) \leq \Pi(\gamma_L)$  for  $\rho \in [\hat{\rho}, 1/2)$ .

□

*Proof of Lemma 5.* When we change  $\Delta\gamma$  marginally,  $d\Delta\gamma = d\gamma_H - d\gamma_L$ , while keeping the mean of  $\gamma$  fixed,  $\rho d\gamma_L + (1 - \rho)d\gamma_H = 0$ , then  $\frac{d\gamma_L}{d\Delta\gamma} = -(1 - \rho) < 0$  and  $\frac{d\gamma_H}{d\Delta\gamma} = \rho > 0$ . Using the

notation of Proposition 3, we have that

$$\frac{dF_j}{d\Delta\gamma} < 0, \quad \text{and thus} \quad \frac{da_2(\gamma_j)}{d\Delta\gamma} < 0 \quad \text{for } j = L, H.$$

From the proof of Lemma 4 we see that  $e(\gamma_j)$  is increasing in  $F_j$  such that

$$\frac{de(\gamma_j)}{d\Delta\gamma} < 0, \quad \text{for } j = L, H.$$

The proof of Lemma 4 also shows that average firm profits are equal to half of average effort,

$$\rho\Pi(\gamma_L) + (1 - \rho)\Pi(\gamma_H) = \frac{1}{2} \{ \rho e(\gamma_L) + (1 - \rho)e(\gamma_H) \},$$

such that average firm profits are increasing as  $\Delta\gamma$  decreases. Since compensation to managers is the only cost to firms, we have that average compensation is equal to half of average effort and thus also increases as  $\Delta\gamma$  decreases.  $\square$

*Proof of Lemma 6.* Note that when  $\gamma$  is exponentially distributed, the mode of  $\phi$  is at zero such that by Definition 1 (see Appendix A.2) we have  $\bar{\gamma} = 0$ . It was shown in the proof to Proposition 4 that the optimal linear compensation scheme then offers the same equity incentives

$$a_1 = \frac{1}{(1 + r\sigma_\theta^2) \left[ 1 + \frac{\psi}{2}\Theta \right] + r\sigma_\epsilon^2},$$

$$a_2 = \Theta a_1,$$

$$\Theta = \frac{r\sigma_\epsilon^2}{\frac{\psi}{\lambda} + \left( 1 - \frac{\psi}{2} \right) r\sigma_\theta^2},$$

to all managers. Note that  $\Theta$  is strictly increasing in  $\lambda$ . Hence,  $a_1$  is strictly decreasing and  $a_2$

strictly increasing in  $\lambda$ . Managerial effort is given by

$$e = a_1 \left( 1 + \frac{\psi}{2} \Theta \right) = \frac{1}{1 + r\sigma_\theta^2 + \frac{r\sigma_\psi^2}{1 + \frac{\psi}{2} \Theta}},$$

which is strictly increasing in  $\lambda$ . Average manipulation is

$$E(m) = \frac{\psi}{2} a_2 \frac{1}{\lambda} = \frac{\psi}{2} \frac{r\sigma_\epsilon^2}{\psi + \left(1 - \frac{\psi}{2}\right) r\sigma_\theta^2 \lambda} a_1,$$

which is decreasing in  $\lambda$ . To show that firm profit is proportional to gross compensation, it is sufficient to verify that  $e = 2\Pi$ , which can be done following the same steps as in the proof of Lemma 2. Note that profit is the difference between effort and gross compensation  $E(i)$  by definition, see (A.4). Note that

$$\Pi = E(\pi - i) = e - E(i) = 2\Pi - E(i) \Rightarrow E(i) = \Pi.$$

Hence gross compensation equals profit and both are proportional to effort, and hence strictly increasing in  $\lambda$ .  $\square$

*Proof of Lemma 7.* Define the following cut-off values:

$$\bar{\lambda}_1 = \psi \left[ \left(1 - \frac{\psi}{2}\right) r\sigma_\theta^2 + \frac{\frac{\psi}{2} r\sigma_\epsilon^2}{1 + \frac{r\sigma_\epsilon^2}{1 + r\sigma_\theta^2}} \right]^{-1},$$

$$\bar{\lambda}_2 = \left[ \frac{\psi \bar{\lambda}_1}{\left(1 - \frac{\psi}{2}\right) r\sigma_\theta^2 + \frac{\psi}{2} r\sigma_\epsilon^2} \right]^{\frac{1}{2}}.$$

Note that average managerial information rents are  $E(v(\gamma)) = \left(\frac{\psi}{2}\right)^2 a_2^2 \frac{1}{\lambda}$ , see the proof of Proposition 4 for the definition of  $v(\gamma)$ , since  $E(\gamma) = \frac{1}{\lambda}$ . Then it is straightforward to verify that for  $\lambda < \bar{\lambda}_1$  we have  $\frac{dE(v)}{d\lambda} > 0$ , and for  $\lambda < \bar{\lambda}_2$  we have  $\frac{d}{d\lambda} \frac{E(v)}{e} > 0$ . Recall that  $\Pi = \frac{1}{2}e$ .  $\square$

## A.2 Proof of Proposition 4

A contract that induces separation must satisfy the following incentive compatibility conditions:

$$u(\gamma, \gamma) \geq u(\gamma, \hat{\gamma}), \quad \forall \gamma, \forall \hat{\gamma}.$$

Let  $X$  be the space of continuous piecewise differentiable functions on  $R_+$ . I will restrict attention to contracts that feature  $a_1, a_2, a_3 \in X$ . A necessary condition for incentive compatibility is that the manager has no incentive to announce a propensity level that differs marginally from his or her true propensity level, or

$$\left. \frac{\partial u(\gamma, \hat{\gamma})}{\partial \hat{\gamma}} \right|_{\hat{\gamma}=\gamma} = 0. \quad (\text{A.1})$$

Another necessary condition is the concavity requirement

$$\left. \frac{\partial^2 u(\gamma, \hat{\gamma})}{\partial \hat{\gamma}^2} \right|_{\hat{\gamma}=\gamma} \leq 0, \quad (\text{A.2})$$

which is equivalent to requiring the amount of short-term incentives to be non-decreasing in the announced manipulation propensity, as Lemma 9 shows.

**Lemma 9.** *The second-order necessary condition for separation (A.2) is equivalent to the monotonicity condition  $a_2'(\gamma) = \frac{\partial a_2(\gamma)}{\partial \gamma} \geq 0$ .*

*Proof.* Let  $u_1$  and  $u_2$  denote derivatives of  $u$  with respect to the first and second argument, respectively. Further, let  $u_{11} = \frac{\partial u_1}{\partial \gamma}$ ,  $u_2 = \frac{\partial u_2}{\partial \hat{\gamma}}$ , and  $u_{21} = u_{12} = \frac{\partial u_1}{\partial \hat{\gamma}}$ . The first-order condition (A.1) must hold for all  $\gamma$  such that  $u_2(\gamma, \gamma) = 0$  is an identity. But then  $u_{21}(\gamma, \gamma) + u_{22}(\gamma, \gamma) = 0$  and we can write the second-order condition (A.2) as

$$u_{22}(\gamma, \gamma) = -u_{21}(\gamma, \gamma) = -u_{12}(\gamma, \gamma) = -\left(\frac{\psi}{2}\right)^2 a_2(\gamma) a_2'(\gamma) \leq 0 \Rightarrow a_2'(\gamma) \geq 0.$$

□

Lemma 9 implies that we can restrict attention to contracts where the amount of short-term incentives is non-decreasing in the announced manipulation propensity,  $a_2 \in \Omega := \{x \in X : x \text{ non-decreasing}\}$ . Assuming an outside option of zero for the manager and restricting attention to  $a_2 \in \Omega$ , the expression for manager utility (4.2) can be written as

$$v(\gamma) := u(\gamma, \gamma) = u(0, 0) + \int_0^\gamma (u_1(z, z) + u_2(z, z)) dz = \int_0^\gamma u_1(z, z) dz = \frac{1}{2} \left( \frac{\psi}{2} \right)^2 \int_0^\gamma a_2(z)^2 dz. \quad (\text{A.3})$$

$v(\gamma)$  is the information rent that a manager with manipulation propensity  $\gamma$  is earning. The expression for manager rents can be used in (4.2) to solve for the cash component  $a_3$  such that the inside owner objective function, after integrating by parts, can be written as

$$\begin{aligned} \Pi = \int_0^\infty \left\{ \left( a_1 + \frac{\psi}{2} a_2 \right) - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2^2 \gamma - \frac{1}{2} \left( a_1 + \frac{\psi}{2} a_2 \right)^2 \right. \\ \left. - \frac{r}{2} \left[ a_1^2 (\sigma_\theta^2 + \sigma_\epsilon^2) + \psi \sigma_\theta^2 a_2 \left( a_1 + \frac{1}{2} a_2 \right) \right] \right\} \phi(\gamma) d\gamma - \frac{1}{2} \left( \frac{\psi}{2} \right)^2 \int_0^\infty a_2^2 (1 - \Phi(\gamma)) d\gamma, \quad (\text{A.4}) \end{aligned}$$

where  $\Phi$  denotes the cumulative distribution function of  $\gamma$ . The inside owner finds the optimal linear compensation scheme by maximizing (A.4) with respect to  $a_1, a_2 \in X$  subject to  $a_2 \in \Omega$ .

**Lemma 10.** *Define the functions*

$$\begin{aligned} \Gamma_0(\gamma) &= \frac{1 - \Phi(\gamma) + \gamma \phi(\gamma)}{\phi(\gamma)}, \\ \Gamma(\gamma) &= \frac{\int_\gamma^\infty \Gamma_0(z) \phi(z) dz}{1 - \Phi(\gamma)}. \end{aligned}$$

If  $\hat{\gamma} > 0$ , then

1.  $\Gamma_0(\gamma)$  is strictly decreasing for  $\gamma < \hat{\gamma}$ , strictly increasing for  $\gamma > \hat{\gamma}$ , and has a global minimum at  $\hat{\gamma}$ ,
2.  $\Gamma(\gamma) - \Gamma_0(\gamma) = 0$  has a unique solution in  $(0, \hat{\gamma})$ .

*Proof.* Assume  $\overset{\circ}{\gamma} > 0$ ; then, the first derivative of  $\Gamma_0$  is given by

$$\Gamma'_0(\gamma) = -\frac{\phi'(\gamma)}{(\phi(\gamma))^2}(1 - \Phi(\gamma)),$$

which is negative below the mode of  $\gamma$  and positive above. The first derivative of  $\Gamma$  is given by

$$\Gamma'(\gamma) = \frac{\phi(\gamma)}{1 - \Phi(\gamma)}(\Gamma(\gamma) - \Gamma_0(\gamma)),$$

such that any solution  $\bar{\gamma}$  to the equation  $\Gamma(\gamma) - \Gamma_0(\gamma) = 0$  is also a stationary point of  $\Gamma$ .

Integration by parts yields  $\Gamma(0) = 2 \int_0^\infty z\phi(z)dz = 2\alpha\beta$ , while

$$\lim_{\gamma \searrow 0} \Gamma_0(\gamma) = \lim_{\gamma \searrow 0} \frac{1}{\phi(\gamma)} = \infty.$$

Hence  $\Gamma_0(\gamma) > \Gamma(\gamma)$  for all  $\gamma$  low enough. Note also that since  $\Gamma_0$  has a global minimum at  $\overset{\circ}{\gamma}$ , we have  $\Gamma_0(\overset{\circ}{\gamma}) < \Gamma(\overset{\circ}{\gamma})$  such that a solution  $\bar{\gamma} < \overset{\circ}{\gamma}$  exists by continuity of  $\Gamma_0$  and  $\Gamma$ . To show uniqueness, we first rule out that  $\bar{\gamma}$  is a tangency point of  $\Gamma$ . If it were a tangency point, then  $\Gamma'_0(\bar{\gamma}) = \Gamma'(\bar{\gamma})$ . But any solution  $\bar{\gamma}$  is also a stationary point of  $\Gamma$ , contradicting the fact that  $\Gamma'_0(\gamma) = 0$  only at  $\gamma = \overset{\circ}{\gamma} > \bar{\gamma}$ . Since  $\bar{\gamma}$  is not a tangency point of  $\Gamma$ , we have that  $\Gamma_0$  crosses  $\Gamma$  from above (suppose  $\bar{\gamma}$  is the smallest solution). However,  $\Gamma$  is strictly increasing for any  $\gamma$  slightly below  $\bar{\gamma}$ , while  $\Gamma_0$  is strictly decreasing on  $[\bar{\gamma}, \overset{\circ}{\gamma})$ . But  $\Gamma - \Gamma_0$  keeps increasing on  $[\bar{\gamma}, \overset{\circ}{\gamma})$  and hence the solution  $\bar{\gamma}$  is unique.

□

**Definition 1.** If  $\overset{\circ}{\gamma} > 0$ , let  $\bar{\gamma}$  denote the unique solution to  $\Gamma(\gamma) - \Gamma_0(\gamma) = 0$ ; otherwise, define  $\bar{\gamma} = 0$ .

Intuitively, the firm's inside owners care about whether a manager's manipulation propensity is below or above the mode of the population, since this will determine the trade-off between distorting incentives for that particular manager and reducing information rents for all managers with higher manipulation propensity (recall equation (A.3)).

We next show that the optimal linear compensation scheme when  $\gamma$  has pdf  $\phi$  is given by

$$a_1(\gamma) = \frac{1}{(1 + r\sigma_\theta^2) \left[ 1 + \frac{\psi}{2}\Theta(\gamma) \right] + r\sigma_\varepsilon^2},$$

$$a_2(\gamma) = \Theta(\gamma)a_1(\gamma),$$

$$\Theta(\gamma) = \begin{cases} \frac{r\sigma_\varepsilon^2}{\frac{\psi}{2}\Gamma_0(\gamma) + (1 - \frac{\psi}{2})r\sigma_\theta^2}, & \text{if } \gamma \in [0, \bar{\gamma}); \\ \frac{r\sigma_\varepsilon^2}{\frac{\psi}{2}\Gamma(\bar{\gamma}) + (1 - \frac{\psi}{2})r\sigma_\theta^2}, & \text{else,} \end{cases}$$

where  $\bar{\gamma}$  is as given by Definition 1, and  $\Gamma_0, \Gamma$  are as defined in Lemma 10.

The inside owner objective function  $\Pi$  is clearly concave. Hence first-order conditions will be sufficient to characterize the optimal linear scheme. Also note that  $\Pi$  is Frechet differentiable in  $a_1, a_2$ . The first-order condition for  $a_1$  follows from theorem 1 in section 7.4 in Luenberger (1997) and is given by

$$\int_0^\infty \left[ 1 - \left( a_1(\gamma) + \frac{\psi}{2}a_2(\gamma) \right) (1 + r\sigma_\theta^2) - r\sigma_\varepsilon^2 a_1(\gamma) \right] \phi(\gamma)h_1(\gamma)d\gamma = 0, \quad (\text{A.5})$$

for all  $h_1 \in X$ . Since the term in brackets is in  $X$  as well, it follows that

$$1 - \left( a_1(\gamma) + \frac{\psi}{2}a_2(\gamma) \right) (1 + r\sigma_\theta^2) - r\sigma_\varepsilon^2 a_1(\gamma) = 0,$$

for all  $\gamma \geq 0$  by Lemma 1 in section 7.5 in Luenberger (1997). Using this and Lemma 1 in section 8.7 in Luenberger (1997), we can write the first-order condition for  $a_2$  as

$$\int_0^\infty \left\{ r \left[ a_1(\gamma)\sigma_\varepsilon^2 - \sigma_\theta^2 \left( 1 - \frac{\psi}{2} \right) a_2(\gamma) \right] - \frac{\psi}{2}\Gamma_0(\gamma)a_2(\gamma) \right\} \phi(\gamma)h_2(\gamma)d\gamma \leq 0, \quad (\text{A.6})$$

for all  $h_2 \in \Omega$ , with equality for  $h_2 = a_2$ . Consider  $a_2$  increasing on  $[0, \bar{\gamma}]$  for some  $\bar{\gamma} \leq \bar{\gamma}$  and constant thereafter. We will verify that this satisfies (A.6). For  $\gamma < \bar{\gamma}$ , pointwise optimization

yields

$$\begin{aligned} a_1(\gamma) &= \frac{1}{(1+r\sigma_\theta^2) \left[1 + \frac{\varphi}{2}\Theta(\gamma)\right] + r\sigma_\varepsilon^2}, \\ a_2(\gamma) &= \Theta(\gamma)a_1(\gamma), \\ \Theta(\gamma) &= \frac{r\sigma_\varepsilon^2}{\frac{\varphi}{2}\Gamma_0(\gamma) + \left(1 - \frac{\varphi}{2}\right)r\sigma_\theta^2}, \end{aligned}$$

which yields  $a_2$  non-decreasing for  $\gamma \leq \tilde{\gamma}$ . For  $\gamma \geq \tilde{\gamma}$ , equation (A.6) yields

$$\begin{aligned} a_1(\gamma) &= \frac{1}{(1+r\sigma_\theta^2) \left[1 + \frac{\varphi}{2}\Theta(\gamma)\right] + r\sigma_\varepsilon^2}, \\ a_2(\gamma) &= \Theta(\gamma)a_1(\gamma), \\ \Theta(\gamma) &= \frac{r\sigma_\varepsilon^2}{\frac{\varphi}{2}\Gamma(\tilde{\gamma}) + \left(1 - \frac{\varphi}{2}\right)r\sigma_\theta^2}. \end{aligned}$$

Imposing continuity of  $a_1, a_2$  yields  $\tilde{\gamma} = \bar{\gamma}$ , as given by Definition 1. It remains to check whether

$$\int_{\tilde{\gamma}}^{\infty} \left\{ r \left[ a_1(\gamma)\sigma_\varepsilon^2 - \sigma_\theta^2 \left( 1 - \frac{\psi}{2} \right) a_2(\gamma) \right] - \frac{\psi}{2}\Gamma_0(\gamma)a_2(\gamma) \right\} \phi(\gamma)h_2(\gamma)d\gamma \leq 0, \quad (\text{A.7})$$

for all  $h_2 \in \Omega$ . Note that (A.7) can be written as

$$\int_{\tilde{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)h_2(\gamma)d\gamma \leq 0,$$

for all  $h_2 \in \Omega$ , where  $Z(\gamma) = \Gamma(\tilde{\gamma}) - \Gamma_0(\gamma)$ . Note that  $Z$  has two roots: one at  $\tilde{\gamma}$ , and we call the larger one  $\hat{\gamma}$ . Since the condition must hold with equality at  $h_2$  constant at  $a_2(\tilde{\gamma})$ , we have  $\int_{\tilde{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)d\gamma = 0$ . Now suppose that  $\int_{\tilde{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)h_2(\gamma)d\gamma > 0$  for some  $h_2 \in \Omega$ , and define the function  $\hat{h}_2(\gamma) = h_2(\gamma)$  for  $\gamma \leq \hat{\gamma}$ , and  $\hat{h}_2(\gamma) = h_2(\hat{\gamma})$  else. Then,

$$\int_{\tilde{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)\hat{h}_2(\gamma)d\gamma \geq \int_{\tilde{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)h_2(\gamma)d\gamma > 0.$$

But since  $Z(\gamma) > 0$  for all  $\gamma \in (\bar{\gamma}, \hat{\gamma})$ , it follows that

$$0 = \int_{\bar{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)\hat{h}_2(\hat{\gamma})d\gamma \geq \int_{\bar{\gamma}}^{\infty} Z(\gamma)\phi(\gamma)\hat{h}_2(\gamma)d\gamma > 0,$$

which is a contradiction. Hence the proposed function  $a_2$  is non-decreasing and solves the first-order condition (A.6).

Note that we can write manager effort as

$$e(\gamma) = \left( a_1(\gamma) + \frac{\psi}{2}a_2(\gamma) \right) = \frac{1}{(1 + r\sigma_\theta^2) + \frac{r\sigma_\varepsilon^2}{1 + \frac{\psi}{2}\Theta(\gamma)}}, \quad (\text{A.8})$$

which is increasing in  $\gamma$  (strictly for  $\gamma < \bar{\gamma}$ ). Manager rents are given by  $v(\gamma) = \frac{1}{2} \left( \frac{\psi}{2} \right)^2 \int_0^\gamma a_2(z)^2 dz$ . The first derivative is given by  $v'(\gamma) = \frac{1}{2} \left( \frac{\psi}{2} \right)^2 a_2(\gamma)^2$  and is always strictly positive. The second derivative is given by  $v''(\gamma) = \left( \frac{\psi}{2} \right)^2 a_2(\gamma)a_2'(\gamma)$ , which is positive and strictly so for  $\gamma < \bar{\gamma}$ .

□