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Abstract

We build an otherwise-standard business cycle model with housework, calibrated consistently with data on time use, in order to discipline consumption-hours complementarity and relate its strength to the size of fiscal multipliers. We show that if substitutability between home and market goods is calibrated on the empirically relevant range, consumption-hours complementarity is large and the model generates fiscal multipliers that agree with the evidence. Hence, our analysis supports the relevance of consumption-hours complementarity for fiscal multipliers. However, we also find that explicitly modeling the home sector is more appealing than restricting to the consumption-leisure margin and/or to the preferences proposed by Greenwood, Hercowitz and Huffman (1988). A housework model can imply substantial complementarity, without low wealth effects contradicting the microeconomic evidence.

JEL classification: E24, E32, E52, E62
Bank classification: Fiscal policy; Business fluctuations and cycles

Résumé

Nous construisons un modèle de cycle économique standard auquel nous intégrons les travaux ménagers, et que nous étalonnons en fonction de données sur l'emploi du temps, afin d'assujettir aux données la complémentarité entre la consommation et les heures travaillées et d'établir le lien entre le degré de cette complémentarité et la taille des multiplicateurs budgétaires. Nous montrons que, si la substituabilité entre les produits maison et les produits marchands est étalonnée sur la fourchette de données pertinente du point de vue empirique, la complémentarité consommation-heures travaillées est importante et le modèle génère des multiplicateurs budgétaires qui concordent avec les données. Ainsi, l'importance de la complémentarité consommation-heures travaillées pour les multiplicateurs budgétaires est étayée par notre analyse. Par ailleurs, nous constatons également qu'il est plus avantageux de modéliser explicitement la production domestique que de s'en tenir à l'arbitrage consommation-loisirs ou aux préférences proposées par Greenwood, Hercowitz et Huffman (1988). Un modèle de travaux ménagers peut générer une complémentarité considérable, sans que de faibles effets de richesse viennent contredire les observations microéconomiques.

Classification JEL : E24, E32, E52, E62
Classification de la Banque : Politique budgétaire; Cycles et fluctuations économiques

1 Introduction

The propagation of exogenous changes in public consumption to macroeconomic variables is at the center of a controversial and ongoing debate. Standard theories of the business cycle have had difficulty spanning the entire range of estimates for fiscal multipliers, which vary considerably across studies, depending on the assumptions used to identify fiscal shocks.[1] Various theories have been proposed to reconcile theoretical predictions with the evidence. Recent contributions, such as Nakamura and Steinsson (2014), Bilbiie (2011), Hall (2009a) and Monacelli and Perotti (2008, 2010), have emphasized the importance of complementarity between consumption and hours worked. Direct evidence on complementarity is rather scant and its relevance is motivated by the observation that consumption falls upon retirement, as in Aguiar and Hurst (2005). In this paper, we propose a model of housework, calibrated consistently with evidence on time use, in order to discipline consumption-hours complementarity and relate its strength to the size of fiscal multipliers.

On top of the non-negligible size of the home sector, both in terms of time and capital, as stressed, for example, in Benhabib, Rogerson and Wright (1991), our emphasis on home production is motivated by recent contributions pointing to a great deal of substitutability between housework and market work. Aguiar and Hurst (2007) have pointed out that substitutability between housework and market work is important over the life cycle. At business cycle frequencies, home production is estimated to absorb about 30 percent of foregone market work (Aguiar, Hurst and Karabarbounis (2013)). Also, the literature on home production has made available estimates about the substitutability between home and market goods. Such estimates might be informative in assessing the quantitative relevance of consumption-hours complementarity for fiscal multipliers.

We follow Benhabib, Rogerson and Wright (1991) and we build an otherwise-standard business cycle model with nominal price rigidities, where the household can employ time and capital to produce a good that is non-tradable on the market. We calibrate the model to match the size of the home sector in the United States. Then, we map the elasticity of substitution between home and market goods into a measure of complementarity as well as into fiscal multipliers of output, hours worked and market consumption. We find that if substitutability between home and market goods is calibrated on the empirically relevant range, between 2 and 4, consumption-hours complementarity is large and the model spans the whole range of estimates, agreeing with the evidence from vector autoregressions (VARs). In particular, for the midpoint value of substitutability, the output multiplier is larger than 1, the consumption multiplier is mildly positive and it amounts to 0.13 percent.

Our analysis has interesting implications for the theoretical literature on fiscal multipliers. To begin with, we find that evidence on home production supports

[1] See Blanchard and Perotti (2002), Ramey (2011), Burnside, Eichenbaum and Fisher (2004), Mountford and Uhlig (2009), Pappa (2009), Uhlig (2010), Perotti (2008), Caldara and Kamps (2012) and Zubairy (2014) as a representative, though not exhaustive, sample of the empirical literature.

the relevance of consumption-hours complementarity. However, our model of housework delivers further insights. As Benhabib, Rogerson and Wright (1990, 1991) showed, for any utility function specified in a housework model, there exists a reduced-form utility function that delivers the same market outcomes in a model that abstracts from housework. Therefore, to the extent that some evidence on home production is available, this class of models can be used to discriminate among alternative theories that advocate particular preferences to rationalize macroeconomic data. We compare our model with an alternative one that assumes away home production and embeds preferences commonly used in the literature: those proposed by King, Plosser and Rebelo (1988) (KPR) – the most widely used in macroeconomics – and the ones proposed by Greenwood, Hercowitz and Huffman (1988) (GHH) – which have recently become increasingly popular. A model with KPR preferences can generate fiscal multipliers comparable with the ones in our model by assuming an elasticity of intertemporal substitution that is implausibly low. For example, to obtain a consumption multiplier of 0.13 percent, one would need to calibrate the risk-aversion parameter to 5. Since our model is closer to the microeconomic evidence than a model with KPR preferences, our reduced-form preferences must be more general than a KPR defined over market variables, even though we assume KPR preferences in the "structural" model. We conclude that invoking substitutability between market and home goods by explicitly modeling the home sector might be more appealing than restricting to the consumption-leisure margin in a model with KPR preferences. As far as GHH preferences are concerned, they can generate substantial degrees of complementarity by ruling out wealth effects, which, however, seem to be sizeable, according to microeconomic evidence (Imbens, Rubin and Sacerdote (2001)). We measure the strength of the wealth effect in our model and we find that it is substantial. Hence, our analysis is less supportive of theories building on GHH preferences: a housework model can imply empirically relevant degrees of complementarity, without assuming an implausibly low wealth effect. In this respect, our findings are consistent with and support the results by Eusepi and Preston (2009) and Furlanetto and Seneca (2014).

In our model, there are two key features that affect fiscal multipliers. First, after a positive fiscal shock, aggregate demand is boosted, the price markup falls because of nominal rigidity, labor demand shifts outward and the real wage increases, *ceteris paribus*. Hence, at times when the government is spending, it is particularly attractive to work on the market and consume market goods. Although we focus on price stickiness, one could replace it with any alternative theory that yields countercyclical markups, conditional on the fiscal shock. This channel counteracts the negative wealth effect that depresses market consumption and detains the expansion of aggregate economic activity. Second, the size of the home sector and the substitutability between home and market goods affect the incentive to reallocate resources to the market sector when government consumption increases. Equivalently, if home and market goods are good substitutes, market goods and hours worked on the market are complements. When complementarity is strong enough, the outward shift of labor demand outbalances the negative wealth effect, market consumption increases and the output multiplier is larger than one.

Our paper relates to two large strands of macroeconomics. On the one hand, the seminal contributions by Benhabib, Rogerson and Wright (1991) and Greenwood and Hercowitz (1991) have spurred a rich literature. For instance, McGrattan, Rogerson and Wright (1997) introduce housework in a real business cycle model with fiscal policy. Campbell and Ludvigson (2001) further discuss the implications of modeling home production in business cycle models. Canova and Ubide (1998) and Karabarbounis (2014) show that home production is helpful in addressing open-economy puzzles. Aruoba, Davis and Wright (2012) discuss the relevance of housework for monetary policy. On the other hand, many theories have been proposed to rationalize estimated fiscal multipliers. Galí, López-Salido and Vallés (2007) first modeled hand-to-mouth consumers to generate sizeable demand effects, making consumption respond to current income. Corsetti, Meier and Müller (2012) explain a positive private consumption response with spending reversals: current higher government expenditure implies permanently lower future expenditure, so as to keep constant long-run government debt. Finally, Ravn, Schmitt-Grohé and Uribe (2012) focus on deep habits. In this case, an increase in domestic aggregate demand provides an incentive for firms to lower markups shifting the labor demand curve outward, which is similar to our case of sticky prices.

The rest of the paper is organized as follows: Section 2 presents the model and Section 3 its baseline parametrization. Section 4 computes impulse responses, inspects the mechanism and, after summarizing the implications of our findings for the literature, it performs extensive robustness analysis. Section 5 concludes.

2 The Model

We consider an otherwise-standard New Keynesian model, where households can combine time and capital to produce non-tradable home goods. As in Benhabib, Rogerson and Wright (1991) and McGrattan, Rogerson and Wright (1997),[2] households enjoy leisure and consumption of a composite index, which aggregates market and home goods. The fiscal authority buys market goods and subsidizes production so as to offset the steady-state distortion due to firms' market power. Expenditures are financed by levying lump-sum taxes.[3] Finally, the central bank is in charge of setting the nominal interest rate.

[2]Differently from Greenwood and Hercowitz (1991), we allow households to substitute leisure with time spent working either at home or on the market.

[3]In Appendix B, we show that our findings continue to hold in the case of distortionary taxation. However, we always retain the assumption that distortionary taxes do not respond to transitory government expenditure shocks, since we follow the empirical literature in focusing on deficit spending.

2.1 Policy-makers

In the economy, there are infinitely many varieties of market goods indexed by $i \in [0, 1]$. The fiscal authority buys each variety, $G_t(i)$, at its market price, $P_t(i)$. We define aggregate government expenditure, G_t, as a composite index:

$$G_t = \left[\int_0^1 \left(G_t(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.1}$$

where $\varepsilon > 1$ is the elasticity of substitution across varieties and $log(G_t)$ exogenously evolves according to a first-order autoregressive process, with mean equal to $log(G)$ and persistence ρ_g. We assume that the government chooses quantities $G_t(i)$ in order to minimize total expenditure, $\int_0^1 P_t(i)G_t(i)\ di$, given G_t. Hence, the condition

$$G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t \tag{2.2}$$

pins down public consumption of each variety, i, where

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon}\ di \right]^{\frac{1}{1-\varepsilon}}. \tag{2.3}$$

The central bank decides on the nominal interest rate by following a Taylor-type rule:

$$(1 + R_t) = (1 + R_{t-1})^{\rho_m} \left(\beta^{-1} \Pi_t^{\Phi_\pi} \tilde{y_t}^{\Phi_y}\right)^{1-\rho_m} \left(\frac{\tilde{y_t}}{\tilde{y_{t-1}}}\right)^{\Phi_{dy}}. \tag{2.4}$$

$\Pi_t \equiv (P_t/P_{t-1})$ and $\tilde{y_t}$ denote inflation and market production in deviation from the flexible-price equilibrium, respectively. ρ_m, Φ_π, Φ_y and Φ_{dy} are parameters chosen by the monetary authority. Among others, this rule has been considered by Smets and Wouters (2007).[4]

2.2 Households

Households can buy market goods, which can be either allocated to consumption, $C_{m,t}(i)$, or stored for investment purposes, $I_t(i)$. We define aggregate market consumption and investment as

$$C_{m,t} = \left[\int_0^1 \left(C_{m,t}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad I_t = \left[\int_0^1 \left(I_t(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.5}$$

The evolution of capital over time is thus described by

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\xi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2, \tag{2.6}$$

[4]Due to the production subsidy, the flexible-price equilibrium is constrained efficient, thus the monetary rule targets a welfare-relevant output gap. In Appendix C, we provide extensive robustness analysis on the monetary rule.

where $\delta \in (0, 1]$ and $\xi > 0$ stand for the depreciation rate and capital adjustment costs, respectively. The existing capital stock can be rented to firms at price r_t or retained within the household for home production purposes. Let $K_{m,t}$ be the capital stock available to firms and $K_{n,t}$ the capital stock available for non-market activity. Hence,

$$K_t = K_{m,t} + K_{n,t}. \tag{2.7}$$

The time endowment, which we normalize to 1, can be allocated to market work in exchange for a real wage W_t or to housework, so that

$$1 = h_{m,t} + h_{n,t} + l_t. \tag{2.8}$$

$h_{m,t}$ and $h_{n,t}$ represent hours worked on the market and at home, respectively, while l_t is the residual time that can be enjoyed as leisure. We assume that households are price-takers in all markets and that financial markets are complete. Hence, the optimal allocation of expenditure across varieties i implies the flow budget constraint

$$E_t \left\{Q_{t,t+1} B_{t+1}\right\} + P_t(C_{m,t} + I_t) \leq B_t + W_t P_t h_{m,t} + r_t P_t K_{m,t} + T_t. \tag{2.9}$$

B_{t+1} is a portfolio of state-contingent assets, $Q_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs[5] and T_t are all lump-sum taxes and transfers, including firms' profits. The household has the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(C_t)^b (l_t)^{1-b}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{2.10}$$

where $b \in (0, 1)$ and $\sigma \geq 1$. C_t is an index that combines aggregate market and home goods, $C_{n,t}$:

$$C_t = \left[\alpha_1 (C_{m,t})^{b_1} + (1-\alpha_1)(C_{n,t})^{b_1}\right]^{\frac{1}{b_1}}; \ \alpha_1 \in [0, 1] \text{ and } b_1 < 1; \tag{2.11}$$

$$C_{n,t} = (K_{n,t})^{\alpha_2} (h_{n,t})^{1-\alpha_2}; \ \alpha_2 \in [0, 1]. \tag{2.12}$$

Home goods cannot be traded, but rather have to be produced within the household by combining capital and labor. According to our preference specification, aggregate market and home goods can be substituted at a constant elasticity[6] $(1-b_1)^{-1}$.

[5]The stochastic discount factor in period t is the price of a bond that delivers one unit of currency if a given state of the world realizes in period $t+1$, divided by the conditional probability that the state of the world occurs given the information available in t. The nominal interest rate, R_t, relates to the discount factor according to $(1 + R_t) = \{E_t Q_{t,t+1}\}^{-1}$ by a standard no-arbitrage argument.

[6]Recall the following limiting cases: when b_1 approaches one, $C_{m,t}$ and $C_{n,t}$ are perfect substitutes. They are instead perfect complements if b_1 tends to minus infinity. $b_1 = 0$ nests the Cobb-Douglas specification. We restrict to a Cobb-Douglas home production technology. Appendix B shows that our findings extend to the case of a constant elasticity of substitution (CES) production function.

Given initial values of the capital stock K_0 and assets B_0, and all prices and policies, households maximize their lifetime utility by choosing state-contingent sequences of market and home consumption, capital and hours worked, as well as the total stock of capital and bonds to carry over to the next period. The solution to the households' problem needs to satisfy three intratemporal conditions:

$$\begin{aligned}
\frac{\alpha_1}{1-\alpha_1}\left[\frac{C_{m,t}}{C_{n,t}}\right]^{b_1-1} &= \frac{1-\alpha_2}{W_t}\left(\frac{C_{n,t}}{h_{n,t}}\right), && (2.13)\\
\frac{\alpha_1}{1-\alpha_1}\left[\frac{C_{m,t}}{C_{n,t}}\right]^{b_1-1} &= \frac{\alpha_2}{r_t}\left(\frac{C_{n,t}}{K_{n,t}}\right), && (2.14)\\
W_t(1-h_{n,t}-h_{m,t}) &= \frac{1-b}{b\alpha_1}C_{m,t}^{1-b_1}C_t^{b_1}. && (2.15)
\end{aligned}$$

Equation (2.13) drives the optimal allocation of time between the home and the market sector. It establishes that the marginal rate of substitution between home and market consumption has to equalize the corresponding relative price, which is the ratio between the return to housework, i.e., the marginal productivity of labor in the non-market sector, and the return to market work, i.e., the real wage. Similarly, equation (2.14) requires that the marginal rate of substitution between the two consumption goods is equal to the ratio of returns to capital in the two sector, marginal productivity of capital at home and the rental rate of market capital. Taken together, the two conditions imply that returns to labor, relative to capital, are equalized across sectors. In fact, the household can freely reallocate both time and capital between market and non-market activity. Equation (2.15) is the standard intratemporal optimality condition solving for the leisure-consumption trade-off. Finally, two conventional Euler equations are required for the allocation to be optimal intertemporally, one for the capital stock and one for financial assets:

$$\begin{gathered}
\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[1+\frac{\xi}{K_t}\left(\frac{K_{t+1}}{K_t}-1\right)\right]^{-1}\right.\\
\left.\left[1-\delta+r_{t+1}+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^2}\right)\right]\right\}=1,
\end{gathered} \qquad (2.16)$$

$$\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+R_t)\Pi_{t+1}^{-1}\right\}=1, \qquad (2.17)$$

where λ denotes the marginal utility of market consumption and reads as

$$\lambda_t = b\alpha_1(1-h_{n,t}-h_{m,t})^{(1-b)(1-\sigma)}C_{m,t}^{b_1-1}(C_t)^{b(1-\sigma)-b_1}. \qquad (2.18)$$

2.3 Firms

In the economy, there are infinitely many monopolistically competitive firms, $i \in [0,1]$. Each firm buys market capital and hours worked on perfectly competitive markets in order to produce a variety i of the market good, according to the following production function:

$$Y_t(i) = (K_{m,t}(i))^{\alpha_3} (h_{m,t}(i))^{1-\alpha_3}, \quad \alpha_3 \in [0,1]. \tag{2.19}$$

Cost minimization yields

$$\alpha_3 RMC_t \left(\frac{Y_t(i)}{K_{m,t}(i)} \right) = r_t, \tag{2.20}$$

$$(1-\alpha_3) RMC_t \left(\frac{Y_t(i)}{h_{m,t}(i)} \right) = W_t. \tag{2.21}$$

The real marginal cost, RMC_t, is constant across firms because of constant returns to scale in production and perfect competition on factor markets. We follow Calvo (1983) and we assume that in any given period each firm resets its price $P_t(i)$ with a constant probability $(1-\theta)$. At a given price $P_t(i)$, production has to satisfy demand:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} [C_{m,t} + I_t + G_t]. \tag{2.22}$$

We assume that production is subsidized by the government, which pays a fraction τ of the cost per unit of production. Maximization of profits

$$E_t \left\{ \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[P_t(i) Y_{t+j}(i) - P_{t+j}(1-\tau) RMC_{t+j} Y_{t+j}(i) \right] \right\} \tag{2.23}$$

subject to constraint (2.22) yields the following first-order condition for any firm i that is allowed to re-optimize in period t:

$$E_t \left\{ \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{t+j}(i) \left[\frac{P_t^*}{P_t} - \frac{\varepsilon(1-\tau)}{\varepsilon - 1} RMC_{t+j} \Pi_{t,t+j} \right] \right\} = 0. \tag{2.24}$$

P_t^* is the optimal price, $Q_{t,t+j}$ denotes the stochastic discount factor in period t for nominal profits j periods ahead and it is such that

$$Q_{t,t+j} = \beta^j E_t \left\{ \frac{\lambda_{t+j}}{\lambda_t} \Pi_{t,t+j}^{-1} \right\}, \tag{2.25}$$

while $\Pi_{t,t+j} \equiv (P_{t+j}/P_t)$. Calvo pricing implies the following conventional relation between inflation and the relative price charged by re-optimizing firms:

$$\frac{P_t^*}{P_t} = \left(\frac{1 - \theta \Pi_t^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}}. \tag{2.26}$$

The necessary condition for profit maximization (2.24) can easily be rewritten as

$$\frac{P_t^*}{P_t} = \frac{x_{1,t}}{x_{2,t}}, \tag{2.27}$$

where the auxiliary variables $x_{1,t}$ and $x_{2,t}$ are recursively defined by

$$x_{1,t} = [C_{m,t} + I_t + G_t] \left(\frac{\varepsilon(1-\tau)}{\varepsilon - 1} \right) RMC_t + \beta\theta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{\varepsilon} x_{1,t+1} \right\}, \tag{2.28}$$

$$x_{2,t} = [C_{m,t} + I_t + G_t] + \beta\theta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{\varepsilon-1} x_{2,t+1} \right\}. \tag{2.29}$$

2.4 Aggregation and Market Clearing

After defining aggregate production

$$Y_t = \left[\int_0^1 (Y_t(i))^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.30}$$

the clearing of the goods market implies

$$Y_t = C_{m,t} + I_t + G_t. \tag{2.31}$$

Define the market capital-labor ratio, $k_t \equiv (K_{m,t}(i)) \, / \, (h_{m,t}(i))$. By equations (2.20) and (2.21), the ratio is constant across firms and satisfies

$$k_t = \frac{\alpha_3 W_t}{(1-\alpha_3) r_t}. \tag{2.32}$$

By the clearing of the labor market,

$$h_{m,t} = \int_0^1 h_{m,t}(i) \, di. \tag{2.33}$$

Integrating equation (2.19) over all firms i yields

$$Y_t = \Delta_t^{-1} k_t^{\alpha_3} h_{m,t}, \tag{2.34}$$

where Δ_t denotes relative price dispersion

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} \, di, \tag{2.35}$$

and evolves according to

$$\Delta_t = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} + \theta \Pi_t^{\varepsilon} \Delta_{t-1}. \tag{2.36}$$

It is well known that $log\,(\Delta_t)$ is a second-order term and can thus be neglected when the model is approximated to the first order around the non-stochastic steady state. By the clearing of the capital rental market,

$$K_{m,t} = \int_0^1 K_{m,t}(i) \, di, \tag{2.37}$$

which implies

$$K_{m,t} = k_t h_{m,t}. \tag{2.38}$$

Finally, by using (2.38) into (2.34), one can obtain the aggregate production function

$$Y_t = \Delta_t^{-1} \left(K_{m,t}\right)^{\alpha_3} \left(h_{m,t}\right)^{1-\alpha_3}, \tag{2.39}$$

as well as the aggregate counterparts of equations (2.20) and (2.21):

$$\alpha_3 RMC_t \left(\frac{\Delta_t Y_t}{K_{m,t}}\right) = r_t, \tag{2.40}$$

$$(1-\alpha_3) RMC_t \left(\frac{\Delta_t Y_t}{h_{m,t}}\right) = W_t. \tag{2.41}$$

3 Parametrization of the Model

We resort to data in order to choose the values of structural parameters that capture the importance of the home sector, relative to the market economy. In particular, we calibrate these parameters in order to match the value of endogenous variables at the non-stochastic steady state with their observable counterparts. We discipline the remaining parameters by using independent microeconomic evidence as well as information coming from previous studies.[7] After a brief description of the data, this section illustrates the details of our calibration strategy. Table 1 summarizes parameter values and the corresponding source and/or calibration targets.

3.1 Data

We collect time series of capital, investment, market consumption, government expenditure and the consumer price index (price index for personal consumption expenditure) from the U.S. Bureau of Economic Analysis. All the series refer to the time period 1950:Q1–2007:Q2, which excludes the financial crisis. Data are available at a quarterly frequency, with the exception of capital, which is annual. All series are seasonally adjusted. The series have been downloaded in current dollars and divided by the consumer price index. The series of market consumption includes non-durable goods and services, after subtracting the value of services from housing and utilities that in turn are considered as part of the non-market sector.[8] Consistently, we assign fixed non-residential assets to market capital, while we consider residential assets and the stock of durable goods as part of the home capital. We obtain total investment by adding purchases of durable goods to the fixed investment component, both residential and non-residential, but we leave out inventories, as in Smets and Wouters (2007).

[7] In the exercises inspecting the transmission mechanism of fiscal shocks, we keep most of these parameters fixed. However, we extensively check the robustness of our findings in Section 4.4.

[8] This is conventional in the home production literature. See, for instance, McGrattan, Rogerson and Wright (1997).

For government expenditure, we only include purchases of goods, while we omit purchases of non-military durable goods and structures. We finally derive a measure of GDP consistent with the model by summing up market consumption, investment and government expenditure. We obtain average capital-output ratios equal to 1.29 and 1.69 in the market and in the home sectors, respectively. Government expenditure as a share of GDP amounts to 0.18.

We measure time use by relying on the information contained in the American Time Use Survey (ATUS), as summarized by Aguiar, Hurst and Karabarbounis (2013). The ATUS provides nationally representative estimates of how Americans spend their time, supplying data on a wide range of non-market activities, from childcare to volunteering, for a cross-section of roughly $100,000$ individuals over the period 2003–2010. Respondents are randomly selected from a subset of households that have completed their eighth and final month of interviews for the Current Population Survey (CPS). As a fraction of the weekly endowment, time allocated to market work is 0.33, time for housework amounts to 0.19 and the rest is devoted to leisure, which excludes sleeping, eating and personal-care time.[9]

3.2 Baseline Calibration

We choose parameters β, ε, θ, ξ and σ by referring to independent microeconomic evidence and/or previous studies. We set the discount factor β to 0.99, which implies an annual interest rate of roughly 4 percent per year. The elasticity of substitution between market varieties, $\varepsilon = 11$, matches a 10 percent steady-state markup, while $\theta = 0.75$ implies a conventional price duration of four quarters. A production subsidy, $\tau = 1/\varepsilon$, offsets the steady-state distortion due to monopolistic competition. As far as capital adjustment costs are concerned, estimates display great variability, ranging from $\xi = 3$ to $\xi = 110$.[10] We restrict to a value in the middle range, $\xi = 50$. Parameter σ is determined so as to match an elasticity of intertemporal substitution equal to 0.5, a reasonable value for models matching growth and/or fluctuations facts.[11] Hence, we restrict to this case for our main parametrization, while we check for robustness in the next section.

When inspecting the transmission mechanism of fiscal shocks, for the sake of clarity we restrict to a simple monetary rule and we assume $\rho_m = \Phi_y = \Phi_{dy} = 0$ and $\Phi_\pi = 1.5$. However, it is well known that the monetary response significantly affects the impact of spending shocks on macroeconomic variables.[12]

[9]As reported by Aguiar, Hurst and Karabarbounis (2013) in Table B1 of their online Appendix, the average respondent devotes 31.62 hours to market work and 18.12 hours to home production per week. Our figures obtain after subtracting from the weekly time endowment sleeping, personal care and eating, for a total of 72.92 hours. Instead, if those activities are included, market work and home production time result in 0.18 and 0.11, respectively. Both ways of accounting time are used in the home production literature. We choose the former in our baseline calibration, but our results are robust to the latter definition.

[10]For a survey, see Neiss and Pappa (2002).

[11]See Hall (2009b) and Guvenen (2006) for an overview.

[12]For an illustration of the empirical importance of monetary accommodation after fiscal shocks,

Hence, in Section 4.4, we conduct an extensive robustness analysis by considering alternative parameter values. We also report the case of alternative classes of rules in Appendix C.

Parameters α_1, α_2, α_3, G, δ and b deserve particular attention. In fact, they drive the size of the home sector, relative to the market economy. We recover their values by using the model equilibrium conditions, evaluated at the non-stochastic steady state, to target the average value of the following variables: the ratio of investment to total capital stock; the capital-output ratio and hours worked, both in the market and the home sector; and the share of government expenditure in GDP. All variables without time subscript denote a steady state. The steady-state version of the capital accumulation equation, (2.6), determines the depreciation rate, δ, by using data on capital and investment. The Euler equation on capital, (2.16), thus implies that the steady-state rental rate is

$$r = \frac{1-\beta(1-\delta)}{\beta}. \tag{3.1}$$

Equations (2.26)-(2.29), together with the monetary rule (2.4), imply a unitary real marginal cost at the steady state, while $\Pi = P^*/P = \Delta = 1$ and $(1+R) = \beta^{-1}$. Given the real interest rate and the target for the capital-output ratio in the market sector, α_3 is easily retrieved by equation (2.40). Therefore, given the target on h_m, one can easily solve for Y and K_m via the production function (2.39) at the steady state:

$$\left(\frac{h_m}{Y}\right)^{\alpha_3-1} = \left(\frac{K_m}{Y}\right)^{\alpha_3}. \tag{3.2}$$

It follows from definitions (2.28) and (2.29) that $x_1 = x_2 = (1-\beta\theta)Y$, while equation (2.41) determines the real wage:

$$W = (1-\alpha_3)\left(\frac{Y}{h_m}\right). \tag{3.3}$$

Conditions (2.13) and (2.14) imply

$$\frac{h_n}{Y} = \left[\frac{(1-\alpha_2)r}{\alpha_2 W}\right]\frac{K_n}{Y} \tag{3.4}$$

and α_2 must be chosen such that the target on hours worked and the observed capital-output ratio in the home sector are consistent. Knowing α_2, K_n is also pinned down and C_n can be found by using (2.12). Define g as the share of government expenditure in GDP. The resource constraint (2.31) together with (2.13) yields

$$\frac{\alpha_1}{1-\alpha_1}\left[\frac{(1-g)Y-I}{C_{n,t}}\right]^{b_1-1} = \frac{1-\alpha_2}{W_t(1-\tau_h)}\left[\frac{h_{n,t}}{C_{n,t}}\right], \tag{3.5}$$

see for instance, Canova and Pappa (2011) and Bouakez and Eyquem (2012).

where all endogenous variables have been determined. As a consequence, α_1 must be chosen such that (3.5) holds. Finally, $G = gY$, while the labor supply equation (2.15) recovers the value of b consistent with all our targets.

Even though parameters have been calibrated jointly, heuristically α_1 and α_2 match hours worked and the capital-output ratio in the home sector; b and α_3 match hours worked and the capital-output ratio in the market sector; δ matches the investment-capital ratio; and G implements a 0.18 share of government purchases in GDP. The corresponding parameter values are reported in Table 1 and are consistent with the home production literature. See, for instance, Aruoba, Davis and Wright (2012).

We are finally left with the elasticity of substitution between market and home goods, the parameter we are primarily interested in. In our baseline parametrization, we fix the elasticity to 4, which implies $b_1 = 0.75$. However, in most of our exercises, we leave the parameter free to vary, so as to assess its importance for the transmission of G shocks. We finally take a stand on the magnitude of b_1 in Section 4.3, where we discuss the relevance of our findings for the literature on fiscal multipliers.

4 Housework and Fiscal Multipliers

The purpose of this section is threefold. We first document that the size of the home sector and the substitutability between market and home goods positively affect the size of fiscal multipliers. Then, we investigate the transmission mechanism of fiscal shocks by mapping values of the elasticity of substitution between market and home goods into the degree of complementarity between market goods and hours worked on the market. After summarizing the implications of our findings for the literature, we conclude by performing extensive robustness analysis following the methodology proposed by Canova and Paustian (2011).

4.1 Impulse Responses

We consider an exogenous increase in government expenditure, normalized to one percentage point of steady-state GDP, and analyze its impact on market consumption, hours worked on the market, real wages, GDP and investment. We express the responses of hours worked and the real wage in terms of percentage deviations from their steady state. We report GDP, market consumption and investment in percentage points of GDP, so that their responses on impact can be read as fiscal multipliers, and directly compared to the corresponding VAR evidence. We maintain this normalization in the rest of the paper. All parameters are as in Table 1. For the monetary rule, we assume $\rho_m = \Phi_y = \Phi_{dy} = 0$ and $\Phi_\pi = 1.5$.

Figure 1 makes clear the contribution of home production by comparing our model, labeled as "GHP,"with a counterfactual model, labeled as "Baseline,"where hours worked and capital in the home sector are set to zero. It is evident that the "GHP" model implies larger multipliers and, for $b_1 = 0.75$, it predicts a positive response of market consumption. In both models, a fiscal expansion conventionally generates a negative wealth effect: the shock reduces the present discounted value of disposable income. Therefore, labor supply shifts outward because it is optimal to work more, for any given wage. Since consumption is a normal good, the wealth effect drives consumption down, *ceteris paribus*. On the other hand, in both models, price stickiness boosts real wages, hours worked and market consumption. The fiscal expansion stimulates aggregate demand, reduces price markups and consequently raises the real wage, making it relatively more attractive to work in the market sector. The wealth and the aggregate demand effects reinforce each other in increasing employment, but they push the real wage and market consumption in opposite directions. The final outcome is ultimately a quantitative question. In a model without home production, hours worked on the market do not increase enough to prevent market consumption from falling. The outcome is reversed when the household has the possibility of reallocating time from housework to market activity.

Figure 2 shows that fiscal multipliers increase with the incentive to substitute home and market goods, as long as prices are sticky. In fact, if we reduce the elasticity of substitution or price stickiness, the output multiplier is dampened and the response of market consumption turns to negative, as in the "Baseline" version. When prices become more flexible, the outward shift of the labor supply curve becomes more important, relative to the fall in markups, and the real wage does not increase much or it even falls. In the latter case, the household falls back on home consumption and spends on the market just as much time as needed to optimally smooth the shock. Without an outward shift of labor demand, market consumption is doomed to fall.[13]

4.2 Inspecting the Mechanism

We now turn our attention to why housework and the substitutability between home and market goods amplify fiscal multipliers. We start by recalling the equivalence result by Benhabib, Rogerson and Wright (1990, 1991). Housework does not add outcomes that would be impossible without it: for any given preferences specified in the housework model, there exists a reduced-form utility function that generates the same market equilibrium in a model without home sector. Accordingly, inspecting our mechanism amounts to characterizing the reduced-from preferences and relating their features to the transmission of fiscal shocks.[14] The literature has pointed out that the major preference-related drivers of fiscal multipliers are the elasticity of labor supply, the degree of complementarity between market goods and hours worked on the market, and the

[13] Quoting Hall (2009a), "For the purpose of understanding fiscal policy, the issue is the markup, not price stickiness itself." Price stickiness might then be replaced with any alternative mechanism that generates countercyclical markups, conditional on the shock.

[14] Such reduced-form preferences do not necessarily have a closed form, as in our case. However, one can characterize their main properties as we do below.

strength of the wealth effect.[15] Therefore, we map the size of the home sector and values of the elasticity of substitution between market and home goods into the following Frisch elasticities:[16]

$$\eta_{hm,W} \equiv \frac{\partial h_{m,t}}{\partial W_t}\frac{W_t}{h_{m,t}}\bigg|_{\lambda_t}, \eta_{Cm,W} \equiv \frac{\partial C_{m,t}}{\partial W_t}\frac{W_t}{C_{m,t}}\bigg|_{\lambda_t}, \eta_{l,W} \equiv \frac{\partial l_t}{\partial W_t}\frac{W_t}{l_t}\bigg|_{\lambda_t}, \tag{4.1}$$
$$\eta_{hm,\lambda} \equiv \frac{\partial h_{m,t}}{\partial \lambda_t}\frac{\lambda_t}{h_{m,t}}\bigg|_{W_t}, \eta_{Cm,\lambda} \equiv -\frac{\partial C_{m,t}}{\partial \lambda_t}\frac{\lambda_t}{C_{m,t}}\bigg|_{W_t}, \eta_{l,\lambda_t} \equiv \frac{\partial l_t}{\partial \lambda_t}\frac{\lambda_t}{l_t}\bigg|_{W_t},$$

and we evaluate them at the steady state. $\eta_{Cm,W}$ is the only measure that might require some additional explanation. In the absence of complementarity between consumption and hours worked on the market, $\eta_{Cm,W}$ must be zero: an increase of the real wage positively affects labor supply, but it leaves market consumption unaffected after controlling for higher labor income. In contrast, if consumption and hours worked in the market sector are complements, they both increase following a rise in the real wage, even if wealth is kept constant. Hence, $\eta_{Cm,W}$ is positive and its magnitude measures the degree of complementarity.

Figure 3 compares the elasticities across the "GHP" and the "Baseline" versions of the model for different values of $(1 - b_1)^{-1}$. Both the Frisch elasticity of labor supply and the degree of complementarity, $\eta_{hm,W}$ and $\eta_{Cm,W}$, increase in the elasticity of substitution between home and market goods. Moreover, in the "GHP," they are larger than in the "Baseline" case. Intuitively, if home and market goods are good substitutes, the household is more willing to reallocate hours and consumption to the market sector when the return to market work increases relative to the return to housework. The substitution margin between consumption and leisure and the strength of the wealth effect are not affected by housework, since $\eta_{l,W}$, $\eta_{l,\lambda}$ and $\eta_{hm,\lambda}$ do not vary with b_1, nor with the size of the home sector.[17]

As the labor supply elasticity and the degree of complementarity increase, fiscal multipliers are amplified, because the household has a greater incentive to substitute hours worked in the home sector with hours worked in the market sector, conditional on a fiscal shock.[18] Equivalently, the strength of the wealth

[15]See Hall (2009a), Bilbiie (2009, 2011) and Monacelli and Perotti (2008, 2010) for an extensive discussion.

[16]Following Frisch (1959), we rewrite decision rules as functions of relative prices and the marginal utility of market consumption, λ. Equations (4.1) emphasize that we fix either wealth or the real wage, depending on whether we are interested in isolating income or substitution effects. Notice that we define $\eta_{Cm,\lambda}$ as the opposite of the elasticity of market consumption to λ and thus it can be read as the elasticity of intertemporal substitution. Appendix D provides details about the Frisch system.

[17]Consumption and leisure are normal goods; hence, the restrictions derived by Bilbiie (2009) are satisfied in our model.

[18]An elastic labor supply makes the response of hours worked on the market larger, contributing to larger output multipliers, while its effect on the response of the real wage and market consumption can go either way, depending on parameter values. Following an outward shift of labor demand, a more elastic labor supply dampens the rise of the real wage. However, it also dampens its fall when labor

effect, relative to the aggregate demand effect, is diminished because of the higher elasticity of labor supply and consumption-hours complementarity in the market sector.

Figure 4 repeats the previous exercise for $\sigma = 1$, showing that when utility is separable in consumption and leisure, the transmission channel, though weakened, is still active.

4.3 Implications of Our Findings for the Literature

The equivalence result, far from rendering home production irrelevant, allows us to organize data in a useful way. If home production were excluded, reduced-form preferences would have to be different in order to recover the observationally equivalent model. As an implication, to the extent that microeconomic evidence on preferences over home goods is available, home production models offer valuable guidance in specifying restrictions on the functional form and parameters of the utility function in models that abstract from housework. By the same token, this class of models can be used to discriminate among alternative theories that advocate particular preferences to rationalize macroeconomic data. We apply this logic to the literature on fiscal multipliers by taking a stand on b_1 and on the Frisch elasticities for which some evidence is available.

To have a broad idea of an empirically relevant range for the elasticity of substitution, one can refer to a variety of micro- and macroeconomic studies. The preferred calibration chosen by Benhabib, Rogerson and Wright (1991) in their seminal contribution about home production is 5. McGrattan, Rogerson and Wright (1997) use macroeconomic data to estimate a version of the model by Benhabib, Rogerson and Wright (1991) via maximum likelihood and find a value slightly below 2. In the same vein, Chang and Schorfheide (2003) use Bayesian techniques and estimate an elasticity of about 2.3. Karabarbounis (2014) shows that a value of 4 accounts for cyclical fluctuations of the labor wedge. Aguiar, Hurst and Karabarbounis (2013) use data from the American Time Use Survey (ATUS). After establishing that home production absorbs about 30 percent of foregone market work hours at business cycle frequencies, they show that the Benhabib, Rogerson and Wright (1991) model is consistent with the ATUS evidence under a 2.5 elasticity. One might consider 2.5 as a particularly relevant case, since b_1 has been chosen to match microeconomic evidence.

Our analysis delivers some key messages that are relevant to the literature on fiscal multipliers. First, if one regards the $[2, 4]$ interval as an empirically relevant range for the substitutability between market and home goods, a model of housework delivers fiscal multipliers that agree with the VAR evidence. For the middle-range value of substitutability, the consumption multiplier is mildly positive and amounts to 0.13 percent, while the output multiplier is greater than 1. The implied Frisch elasticity of labor supply, $\eta_{hm,W}$, is fairly high and about

supply shifts outwards. Since both demand and supply shift, the net effect on market consumption is a quantitative issue.

1.8, but it is consistent with the value advocated by Hall (2009b), accounting for both the intensive and the extensive margins of employment.

Second, evidence on time use supports the plausibility of substantial complementarity between consumption and hours worked in the market sector, as proposed by Bilbiie (2011). For $(1 - b_1)^{-1} = 3$, the implied degree of complementarity is indeed about 1.2 percent. However, we also differ from previous contributions, since we rationalize complementarity by invoking substitutability of market consumption along both the leisure and the home-consumption margins. Explicitly modeling housework has the advantage of generating high complementarity and a weak wealth effect, relative to the substitution induced by higher wages, without imposing an implausibly low elasticity of intertemporal substitution. For example, if we were to calibrate the "Baseline" model in order to obtain a consumption multiplier of 0.13 percent, we would need to assume $\sigma = 5$.

More broadly, our analysis suggests that if one abstracts from housework, KPR preferences might be too restrictive. Even though we assume KPR preferences in our "structural" model, the corresponding reduced-form utility function must impose restrictions on Frisch elasticities that are weaker than the ones embedded in a KPR preference specification defined on market consumption and leisure. In fact, in the same model, we can match multipliers with a calibration that is fairly in line with microeconomic evidence, while a KPR preference specification that abstracts from housework cannot. Since the reduced-form utility function cannot be derived in closed form, a model of housework and KPR preferences might be an attractive option.

Also, our exercise is less supportive of theories relying on preferences that rule out sizeable wealth effects, such as Monacelli and Perotti (2008, 2010), Schmitt-Grohé and Uribe (2012) and Nakamura and Steinsson (2014). If housework is explicitly modeled, or equivalently, if one is willing to assume more general preferences than the ones introduced by Greenwood, Hercowitz and Huffman (1988), empirically plausible degrees of complementarity can be achieved without assuming away wealth effects. We believe that this is a reassuring implication of our analysis in the light of microeconomic studies that emphasize the relevance of non-negligible wealth effects.[19] In this respect, our findings are consistent and support the results by Eusepi and Preston (2009) and Furlanetto and Seneca (2014).

Last but not least, strong complementarity can be useful to complement alternative mechanisms that contribute to rationalize estimated multipliers, but that might require a questionable parametrization if used in isolation. Canova and Paustian (2011) argue that models relying on hand-to-mouth consumers, such as Galí, López-Salido and Vallés (2007), need a fraction of non-Ricardian households that is implausibly high. The inclusion of housework could be helpful.

[19] See Imbens, Rubin and Sacerdote (2001).

4.4 Robustness

The parametrization of the model may hide some forces that under- or overstate the quantitative importance of substitutability between market and home goods. In addition to price stickiness and b_1, several parameters are naturally expected to be relevant. As already emphasized by Basu and Kimball (2002), risk aversion, σ, affects both the strength of the wealth effect and complementarity between consumption and leisure. A large cost of adjustment of the capital stock, ξ, discourages households from smoothing the fiscal shock by reducing savings and investment. The monetary reaction to the shock also plays an important role, as pointed out above. Finally, the persistence of government expenditure shocks, ρ_g, makes the wealth effect stronger, thus magnifying the incentive to reduce market consumption on impact.

As in Canova and Paustian (2011), we perform the following robustness exercise. We consider 50,000 draws of parameters from uniform distributions defined over an empirically relevant range. In particular, we consider the following parameters with their respective bounds: $\theta \in [0.2, 0.9]$, $\sigma \in [1, 4]$, $\xi \in [3, 110]$, $\rho_m \in [0, 0.9]$, $\Phi_\pi \in [1.05, 2.5]$, $\Phi_y \in [0.05, 0.25]$, $\Phi_{dy} \in [0.15, 0.30]$ and $\rho_g \in [0, 0.95]$. For convenience, we collect and report these values in Table 2. The 50,000 draws generate a distribution of the impulse response function of market consumption to government expenditure shocks. Figure 5 reports the percentage of positive responses on impact (left panel) and the median responses on impact (right panel), as functions of the elasticity of substitution between home and market goods. We display the results for a given value of the price-stickiness parameter and for the case where θ is also randomly drawn from the uniform distribution. The median response of market consumption confirms the relation we find in Figure 2. It also shows that, for a plausible degree of complementarity and a sensible calibration of the other structural parameters, our model likely predicts a mildly positive response of market consumption.

5 Conclusion

We build an otherwise-standard New Keynesian model that encompasses a home production sector. Following the seminal intuition by Benhabib, Rogerson and Wright (1991), we use the housework model, together with evidence on time use and substitutability between home and market goods, to assess the importance of consumption-hours complementarity for fiscal multipliers. We find that the home-market consumption margin is relevant to generate complementarity, which is large enough to yield fiscal multipliers in line with the macroeconomic evidence. We also find that explicitly modeling the home sector is more appealing than restricting to the consumption-leisure margin with KPR preferences. Moreover, a model of housework, unlike GHH preferences, does not require implausibly low wealth effects.

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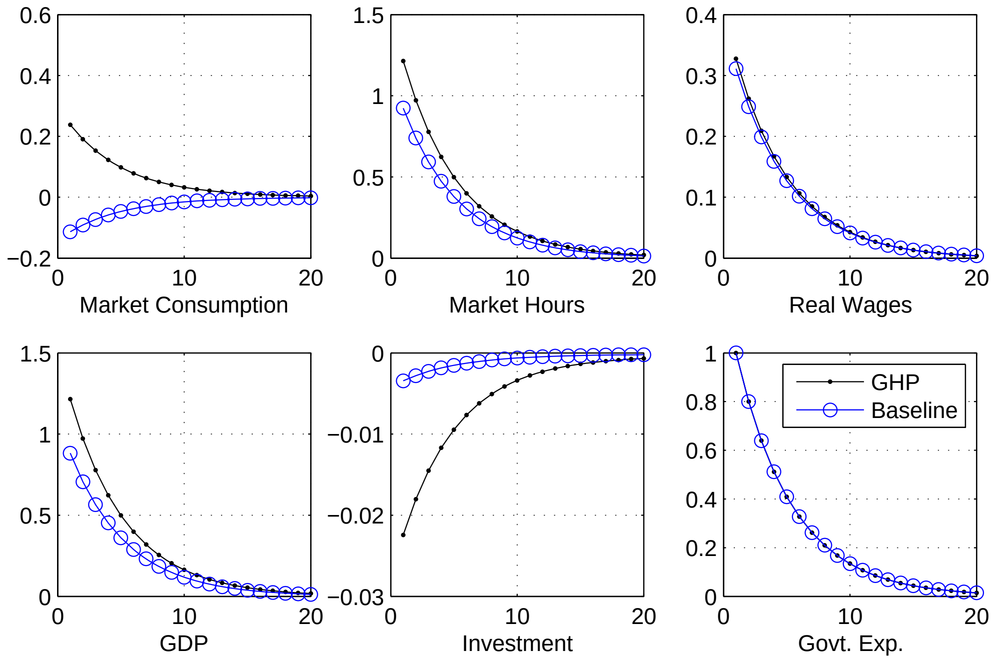

Figure 1: Impulse responses for the model calibrated as in Table 1, labeled as GHP, and impulse responses of the model without home sector, labeled as Baseline.

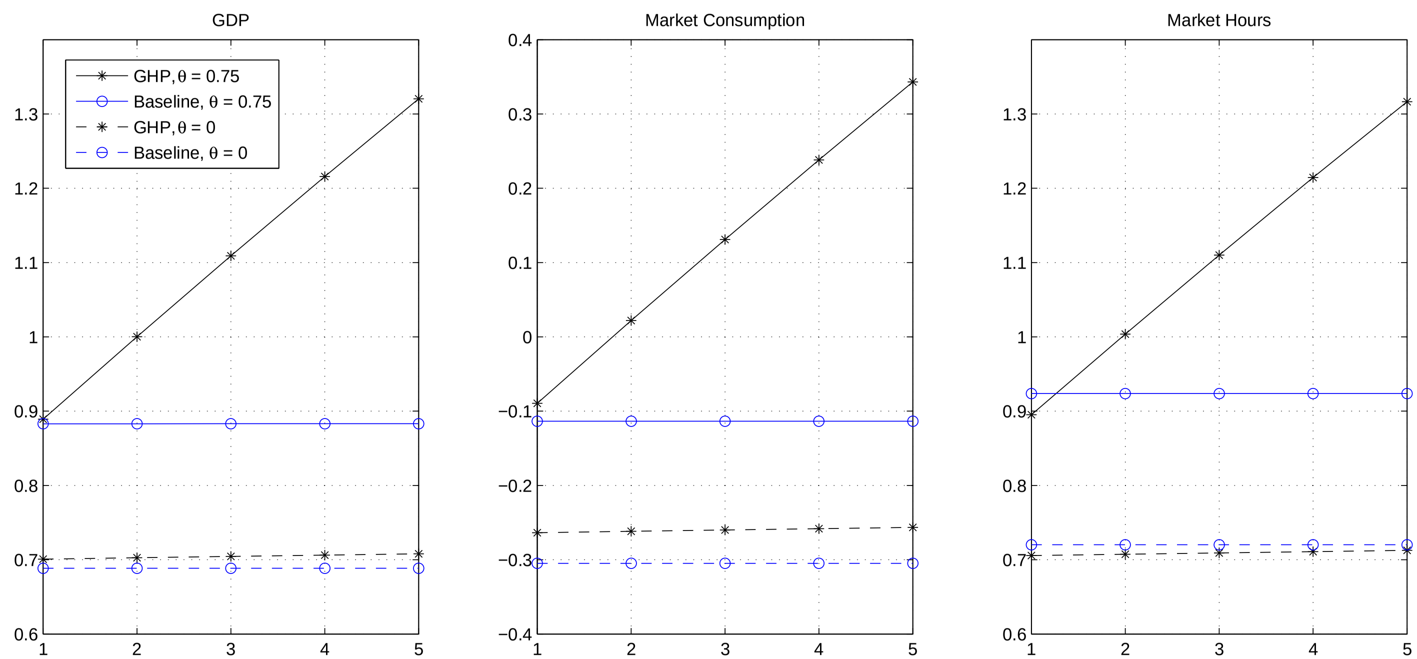

Figure 2: Impact impulse responses of GDP, market consumption and market hours to a G shock, for different values of the elasticity of substitution between home and market goods, $(1 - b_1)^{-1}$, and for different values of price stickiness, θ, for the model calibrated as in Table 1, labeled as GHP. The model without home sector is labeled as Baseline.

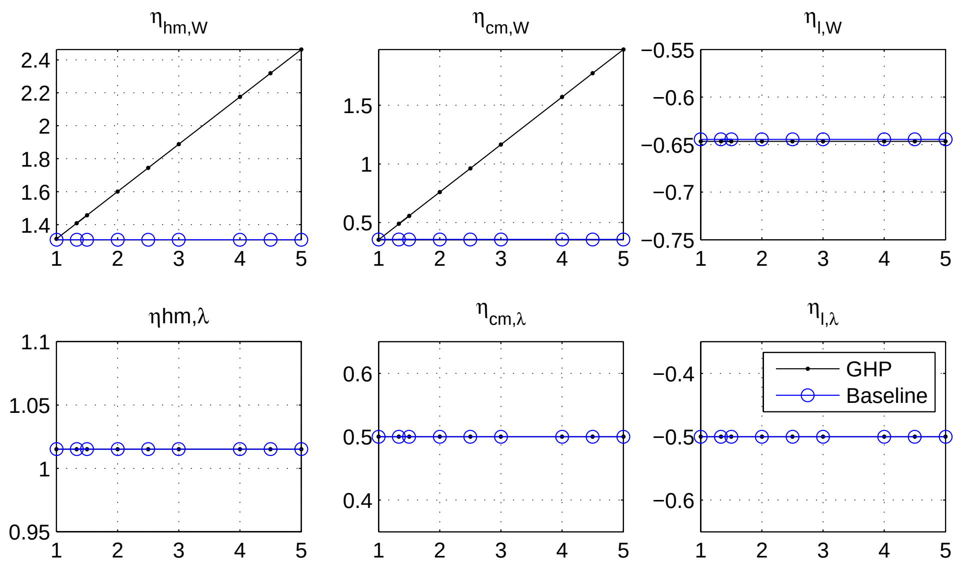

Figure 3: Frisch elasticities across the GHP and the Baseline versions of the model, for different values of $(1-b_1)^{-1}$. All remaining parameters are calibrated as in Table 1.

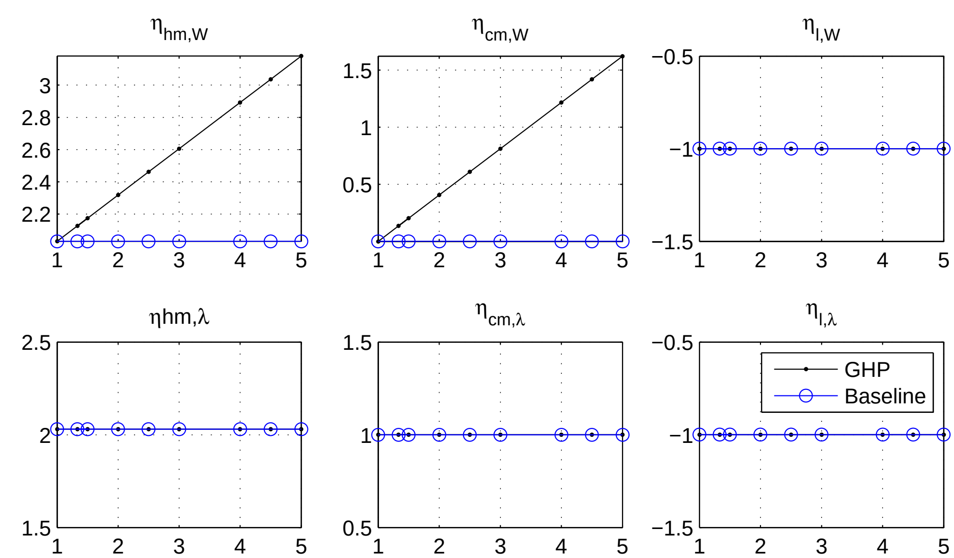

Figure 4: Frisch elasticities across the GHP and the Baseline versions of the model, for different values of $(1-b_1)^{-1}$, and $\sigma = 1$. All remaining parameters are calibrated as in Table 1.

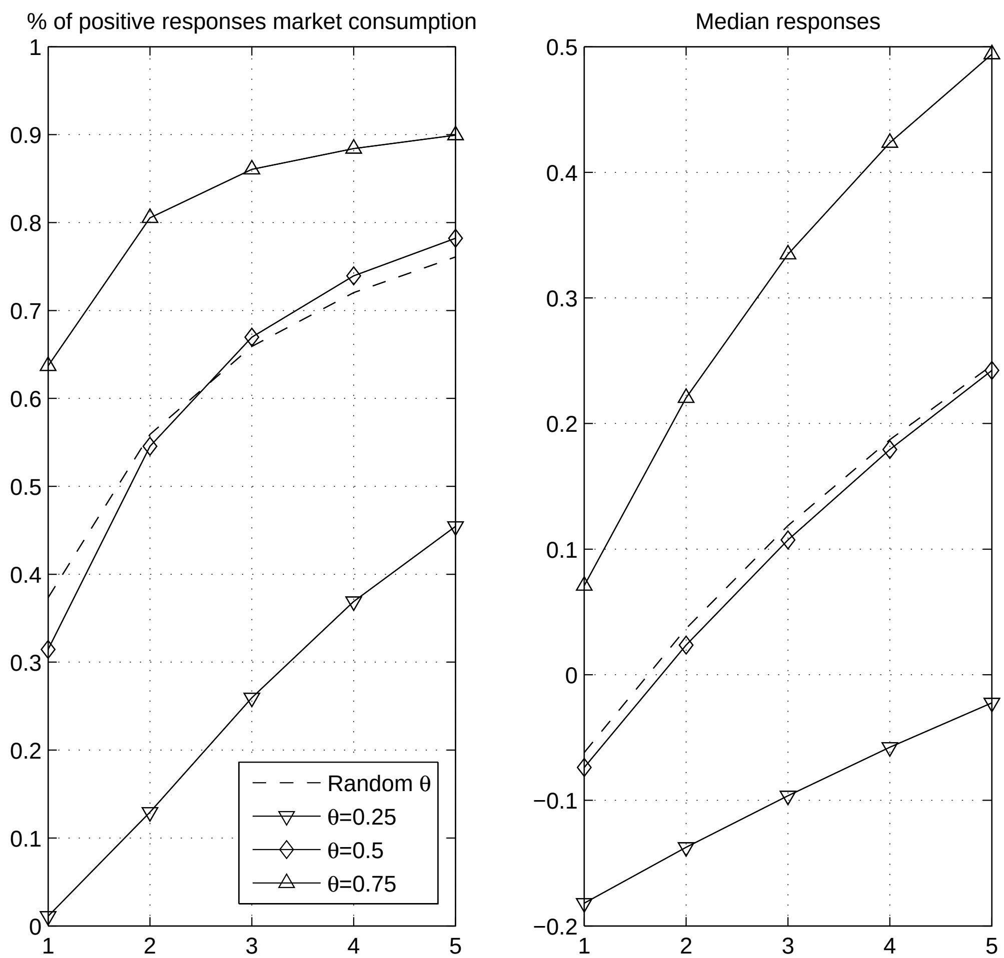

Figure 5: Percentage of positive consumption multipliers (left-hand panel) and median responses (right-hand panel) to a G shock for $50,000$ draws from uniform distributions of the following parameters, with their respective bounds, as summarized in Table 2: $\theta \in [0.2, 0.9]$, $\sigma \in [1, 4]$, $\xi \in [3, 110]$, $\rho_m \in [0, 0.9]$, $\Phi_\pi \in [1.05, 2.5]$, $\Phi_y \in [0.05, 0.25]$, $\Phi_{dy} \in [0.15, 0.30]$, $\rho_g \in [0, 0.95]$. All the other parameters are chosen as in Table 1.

Mnemonic	**Value**	**Target/Source**
β	0.99	4% average real return
ϵ	11	10% price markup
θ	3/4	price duration
ξ	50	Neiss and Pappa (2002)
σ	2	intertemporal elasticity of substitution 0.5
ρ_g	0.8	Monacelli and Perotti (2008, 2010)
α_1	0.5199	$K_m/Y = 1.29$
α_2	0.0950	$h_m = 0.33$
b	0.4938	$K_n/Y = 1.69$
α_3	0.0441	$h_n = 0.19$
δ	0.0241	$I/K = 0.0241$
G	0.0601	$G/Y = 0.18$
b_1	0.75	4% elasticity of substitution between C_m and C_n

Table 1: Baseline calibration

Parameter	Description	Support
θ	price stickiness	$[0.2, 0.9]$
σ	risk aversion	$[1, 4]$
ξ	capital adjustment cost	$[3, 110]$
ρ_g	AR(1) parameter government spending	$[0, 0.95]$
ρ_m	interest rate smoother	$[0, 0.9]$
Φ_π	policy response to inflation	$[1.05, 2.5]$
Φ_y	policy response to output gap	$[0.05, 0.25]$
Φ_{dy}	policy response to growth in output gap	$[0.15, 0.30]$
Φ_Y	policy response to output (Appendix D)	$[0, 0.1]$

Table 2: Support for the structural parameters used in the simulations in Section 4.4 and in Appendix C

A Equilibrium Definition

The equilibrium of the model is a set of state-contingent plans for variables C_t, $C_{m,t}$, $C_{n,t}$, $K_{m,t}$, $K_{n,t}$, K_t, $h_{m,t}$, $h_{n,t}$, I_t, λ_t, Y_t, Π_t, Δ_t, $\frac{P_t^*}{P_t}$, $x_{1,t}$, $x_{2,t}$, RMC_t, R_t, W_t and r_t that satisfy the following system of equations

$$C_t = \left[\alpha_1 (C_{m,t})^{b_1} + (1-\alpha_1)(C_{n,t})^{b_1}\right]^{\frac{1}{b_1}} \tag{A.1}$$

$$C_{n,t} = (K_{n,t})^{\alpha_2} (h_{n,t})^{1-\alpha_2} \tag{A.2}$$

$$K_t = K_{m,t} + K_{n,t} \tag{A.3}$$

$$I_t = K_{t+1} - (1-\delta)K_t + \frac{\xi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 \tag{A.4}$$

$$\frac{\alpha_1}{1-\alpha_1}\left[\frac{C_{m,t}}{C_{n,t}}\right]^{b_1-1} = \frac{1-\alpha_2}{W_t(1-\tau_h)}\left(\frac{C_{n,t}}{h_{n,t}}\right) \tag{A.5}$$

$$\frac{\alpha_1}{1-\alpha_1}\left[\frac{C_{m,t}}{C_{n,t}}\right]^{b_1-1} = \frac{\alpha_2}{(1-\tau_k)r_t + \delta\tau_k}\left(\frac{C_{n,t}}{K_{n,t}}\right) \tag{A.6}$$

$$W_t(1-\tau_h)(1-h_{n,t}-h_{m,t}) = \frac{1-b}{b\alpha_1} C_{m,t}^{1-b_1} C_t^{b_1} \tag{A.7}$$

$$\begin{gathered}\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[1+\frac{\xi}{K_t}\left(\frac{K_{t+1}}{K_t}-1\right)\right]^{-1}\right.\\ \left.\left[1-\delta+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^2}\right)+(1-\tau_k)r_{t+1}+\delta\tau_k\right]\right\} = 1\end{gathered} \tag{A.8}$$

$$\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+R_t)\Pi_{t+1}^{-1}\right\} = 1 \tag{A.9}$$

$$\lambda_t = b\alpha_1(1-h_{n,t}-h_{m,t})^{(1-b)(1-\sigma)} C_{m,t}^{b_1-1}(C_t)^{b(1-\sigma)-b_1} \tag{A.10}$$

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} \tag{A.11}$$

$$\frac{P_t^*}{P_t} = \frac{x_{1,t}}{x_{2,t}} \tag{A.12}$$

$$x_{1,t} = Y_t\left(\frac{\varepsilon}{\varepsilon-1}\right)RMC_t + \beta\theta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\Pi_{t+1}^{\varepsilon}x_{1,t+1}\right\} \tag{A.13}$$

$$x_{2,t} = Y_t + \beta\theta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{\varepsilon-1} x_{2,t+1} \right\} \tag{A.14}$$

$$Y_t = C_{m,t} + I_t + G_t \tag{A.15}$$

$$Y_t = \Delta_t^{-1} \left(K_{m,t}\right)^{\alpha_3} \left(h_{m,t}\right)^{1-\alpha_3} \tag{A.16}$$

$$\alpha_3 RMC_t \left(\frac{\Delta_t Y_t}{K_{m,t}} \right) = r_t \tag{A.17}$$

$$(1-\alpha_3) RMC_t \left(\frac{\Delta_t Y_t}{h_{m,t}} \right) = W_t \tag{A.18}$$

$$\Delta_t = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} + \theta \Pi_t^{\varepsilon} \Delta_{t-1} \tag{A.19}$$

$$(1+R_t) = \beta^{-1} \Pi_t^{\Phi_\pi} \tag{A.20}$$

for all t, for given tax rates and government expenditure. To close the equilibrium definition, we furthermore need a specification for monetary policy and a law of motion for government expenditure.

B Robustness: Distortionary Taxation and CES Production Functions

In this section, we show that our findings continue to hold in the case of distortionary taxation on capital and labor, and in the more general case of constant elasticity of substitution (CES) production functions both in the home and the market sector

$$C_{n,t} = \left[\alpha_2 \left(K_{n,t}\right)^{b_2} + \left(1-\alpha_2\right)\left(h_{n,t}\right)^{b_2}\right]^{\frac{1}{b_2}} \tag{B.1}$$

$$Y_t = \left[\alpha_3 \left(K_{m,t}\right)^{b_3} + \left(1-\alpha_3\right)\left(h_{m,t}\right)^{b_3}\right]^{\frac{1}{b_3}}, \tag{B.2}$$

where (B.1) and (B.2) replace equations (2.12) and (2.19), respectively. Assuming the presence of distortionary taxes on capital and labor, the household's budget constraint, (2.9), becomes

$$\begin{aligned} & E_t\left\{Q_{t,t+1}B_{t+1}\right\} + P_t(C_{m,t}+I_t) \\ & \qquad \leq B_t + (1-\tau_h)W_tP_th_{m,t} + (1-\tau_k)r_tP_tK_{m,t} + \delta\tau_kP_tK_{m,t} + T_t. \end{aligned} \tag{B.3}$$

Accordingly, the household's intratemporal conditions, (2.13) - (2.15), and the Euler equation for the optimal intertemporal allocation of the capital stock, (2.16), are replaced by

$$\frac{\alpha_1}{1-\alpha_1}\left[\frac{C_{m,t}}{C_{n,t}}\right]^{b_1-1} = \frac{1-\alpha_2}{W_t(1-\tau_h)}\left[\frac{C_{n,t}}{h_{n,t}}\right]^{1-b_2} \tag{B.4}$$

$$\frac{\alpha_1}{1-\alpha_1}\left[\frac{C_{m,t}}{C_{n,t}}\right]^{b_1-1} = \frac{\alpha_2}{(1-\tau_k)r_t+\delta\tau_k}\left[\frac{C_{n,t}}{K_{n,t}}\right]^{1-b_2} \tag{B.5}$$

$$W_t(1-\tau_h)(1-h_{n,t}-h_{m,t}) = \frac{1-b}{b\alpha_1}C_{m,t}^{1-b_1}C_t^{b_1} \tag{B.6}$$

$$\begin{aligned} & \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[1+\frac{\xi}{K_t}\left(\frac{K_{t+1}}{K_t}-1\right)\right]^{-1}\right. \\ & \left.\left[1-\delta+\xi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left(\frac{K_{t+2}}{K_{t+1}^2}\right)+(1-\tau_k)r_{t+1}+\delta\tau_k\right]\right\} = 1 \end{aligned} \tag{B.7}$$

Finally, the firms' optimality conditions, (2.20) and (2.21), become

$$\alpha_3 RMC_t\left(\frac{K_{m,t}(i)}{Y_t(i)}\right)^{b_3-1} = r_t \tag{B.8}$$

$$(1-\alpha_3)RMC_t\left(\frac{h_{m,t}(i)}{Y_t(i)}\right)^{b_3-1}\left[exp\{s_{m,t}\}\right]^{b_3} = W_t. \tag{B.9}$$

All remaining equilibrium conditions remain unaffected. We set tax rates according to the base case in McGrattan, Rogerson and Wright (1997), i.e., $\tau_k = 0.55$

and $\tau_h = 0.24$, and set $b_2 = 0.269$ and $b_3 = 0.054$ according to the estimates in McGrattan, Rogerson and Wright (1997). Figure B.1 shows that the relative performance of our model, labeled as "GHP,"compared to a counterfactual model, labeled as "Baseline,"where hours worked and capital in the home sector are set to zero, is unaffected by the presence of distortionary taxes and by the assumption of CES production functions in both sectors.

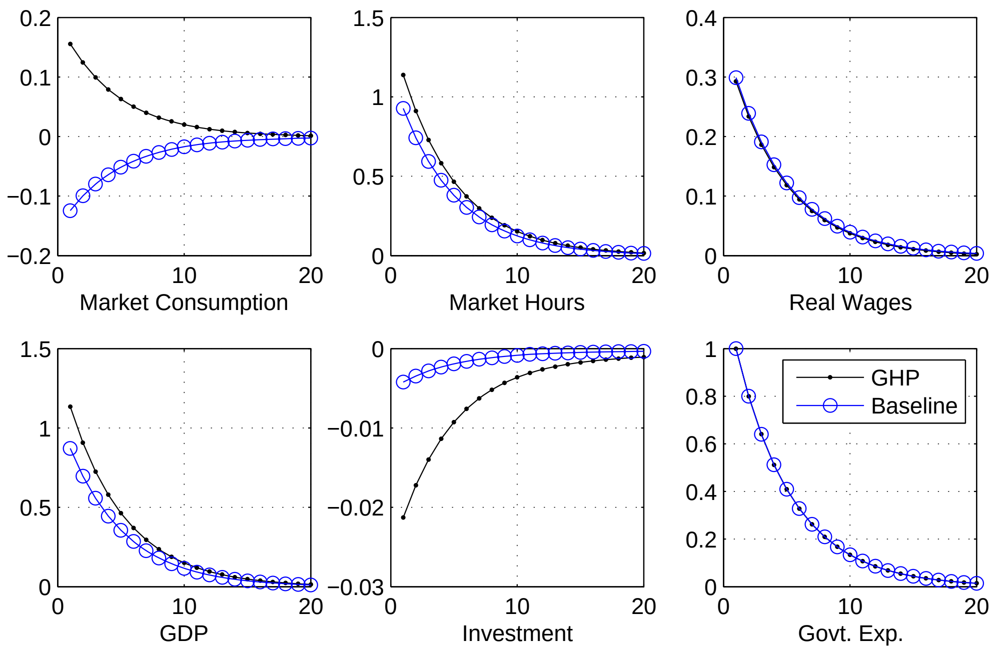

Figure B.1: Impulse responses with distortionary taxation, $\tau_k = 0.55$ and $\tau_h = 0.24$, and CES production functions both in the market ($b_2 = 0.269$) and the home sector ($b_3 = 0.054$). All the other parameters are calibrated as in Table 1.

C Robustness: Monetary Policy Rules

In this section, we assess the robustness of our findings to two additional monetary policy rules for which we repeat the exercise presented in Section 4.4, following Canova and Paustian (2011). In particular, we consider the following monetary policy rules.

- **Taylor Rule with Output (in deviation from steady state) and Interest Rate Smoother (Rule 1):**

$$(1 + R_t) = (1 + R_{t-1})^{\rho_m} \left(\beta^{-1} \Pi_t^{\Phi_\pi} \left(\frac{Y_t}{Y} \right)^{\Phi_Y} \right)^{1-\rho_m} \tag{C.1}$$

  Among others, this rule has been considered by Del Negro and Schorfheide (2004), Rabanal and Rubio-Ramírez (2005), Del Negro, Schorfheide, Smets and Wouters (2007), and Canova and Paustian (2011).

- **Simple Taylor Rule with Interest Rate Smoother (Rule 2):**

$$(1 + R_t) = (1 + R_{t-1})^{\rho_m} \left(\beta^{-1} \Pi_t^{\Phi_\pi} \right)^{1-\rho_m} \tag{C.2}$$

We take $50,000$ draws from uniform distributions of the following parameters, with their respective bounds: $\theta \in [0.2, 0.9]$, $\sigma \in [1, 4]$, $\xi \in [3, 110]$, $\rho_m \in [0, 0.9]$, $\Phi_\pi \in [1.05, 2.5]$, $\rho_g \in [0, 0.95]$ and $\Phi_Y \in [0, 0.1]$ for Rule 1. Supports for the structural parameters used in the simulations are summarized in Table 2. All the other parameters are chosen as in Table 1. As it becomes clear from Figure C.1, the two monetary policy rules do not differ significantly from our main specification, neither in terms of the percentage of positive consumption multipliers, nor in terms of the median responses of market consumption to government expenditure shocks. However, notice that we choose to be conservative and report the rule delivering the lowest multipliers in the main text.

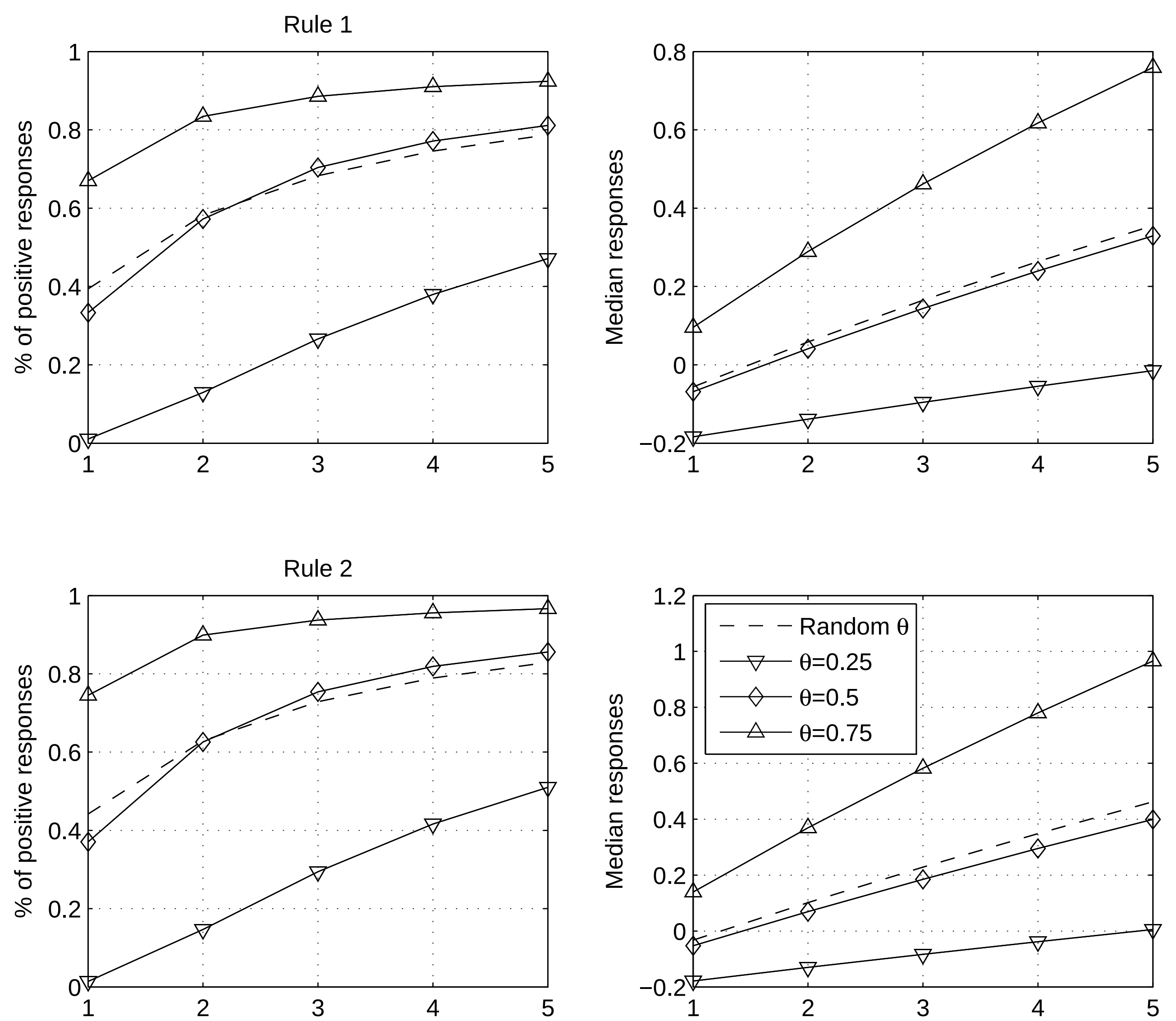

Figure C.1: Robustness analysis on the monetary policy rule. Percentage of positive consumption multipliers (left-hand panel) and median responses (right-hand panel) to a G shock for $50,000$ draws from uniform distributions of the following parameters, with their respective bounds, as summarized in Table 2: $\theta \in [0.2, 0.9]$, $\sigma \in [1, 4]$, $\xi \in [3, 110]$, $\rho_m \in [0, 0.9]$, $\Phi_\pi \in [1.05, 2.5]$, $\rho_g \in [0, 0.95]$ and $\Phi_Y \in [0, 0.1]$ for Rule 1. All the other parameters are chosen as in Table 1.

D Frisch System

Following Frisch (1959), we define the Frisch system of our model. Given the utility function, (2.10), and the budget constraint, (2.9), the choice variables are $y_t = \{C_t, C_{m,t}, C_{n,t}, h_{m,t}, h_{n,t}, k_{n,t}\}$, whereas $x_t = \{\lambda_t, W_t, r_t\}$ are taken as given. Six equations define the Frisch system:

$$\begin{aligned}
b\alpha_1(1-h_{n,t}-h_{m,t})^{(1-b)(1-\sigma)}C_{m,t}^{b_1-1}C_t^{b(1-\sigma)-b_1} - \lambda_t &= f1 \quad \text{(D.1)}\\
(1-b)C_t^{b(1-\sigma)}(1-h_{n,t}-h_{m,t})^{(1-b)(1-\sigma)-1} - \lambda_t W_t &= f2 \quad \text{(D.2)}\\
(1-h_{n,t}-h_{m,t})b(1-\alpha_1)(1-\alpha_2)\frac{c_{n,t}^{b_1}}{h_{n,t}} - (1-b)C_t^{b_1} &= f3 \quad \text{(D.3)}\\
(1-h_{n,t}-h_{m,t})^{(1-b)(1-\sigma)}b(1-\alpha_1)\alpha_2 C_t^{b(1-\sigma)-b_1}h_{n,t}^{(1-\alpha_2)} - &\\
(r_t+\delta)\lambda_t C_{n,t}^{(1-b_1)}k_{n,t}^{(1-\alpha_2)} &= f4 \quad \text{(D.4)}\\
\left[\alpha_1(C_{m,t})^{b_1} + (1-\alpha_1)(C_{n,t})^{b_1}\right]^{\frac{1}{b_1}} - C_t &= f5 \quad \text{(D.5)}\\
(K_{n,t})^{\alpha_2}(h_{n,t})^{1-\alpha_2} - C_{n,t} &= f6 \quad \text{(D.6)}
\end{aligned}$$

Define $f = [f1; f2; f3; f4; f5; f6]$ and the matrix of unknown derivatives we are interested in

$$Z_{y,x} = \begin{pmatrix}
\frac{\partial C_t}{\partial \lambda_t} & \frac{\partial C_t}{\partial W_t} & \frac{\partial C_t}{\partial r_t}\\
\frac{\partial C_{m,t}}{\partial \lambda_t} & \frac{\partial C_{m,t}}{\partial W_t} & \frac{\partial C_{m,t}}{\partial r_t}\\
\frac{\partial C_{n,t}}{\partial \lambda_t} & \frac{\partial C_{n,t}}{\partial W_t} & \frac{\partial C_{n,t}}{\partial r_t}\\
\frac{\partial h_{m,t}}{\partial \lambda_t} & \frac{\partial h_{m,t}}{\partial W_t} & \frac{\partial h_{m,t}}{\partial r_t}\\
\frac{\partial h_{n,t}}{\partial \lambda_t} & \frac{\partial h_{n,t}}{\partial W_t} & \frac{\partial h_{n,t}}{\partial r_t}\\
\frac{\partial k_{n,t}}{\partial \lambda_t} & \frac{\partial k_{n,t}}{\partial W_t} & \frac{\partial k_{n,t}}{\partial r_t}
\end{pmatrix} \quad \text{(D.7)}$$

We then solve the following system for matrix $Z_{y,x}$

$$\underset{6*6}{J_y}\ \underset{6*3}{Z_{y,x}} + \underset{6*3}{J_x} = 0, \quad \text{(D.8)}$$

where J_y is the Jacobian matrix of function f with respect to the control variables, J_x is the Jacobian matrix of function f with respect to the state variables.