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Information, Amplification and Financial Crisis

by Toni Ahnert and Ali Kakhbod

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Abstract

We propose a parsimonious model of information choice in a global coordination game of regime change that is used to analyze debt crises, bank runs or currency attacks. A change in the publicly available information alters the uncertainty about the behavior of other investors. Greater strategic uncertainty makes private information more valuable, so more investors acquire information. This change in the proportion of informed investors amplifies the impact of the initial change in public information on the probability of a crisis. Our amplification result explains how a small deterioration in public information can cause a financial crisis.

JEL classification: D83, G01
Bank classification: Financial institutions; Financial stability

Résumé

Les auteurs proposent un modèle parcimonieux du choix de l’information dans le cadre d’un jeu global de coordination autour d’un changement de régime – jeu utilisé pour analyser les crises d’endettement, les retraits massifs de dépôts bancaires ou les attaques contre la monnaie. Toute modification de l’information rendue publique fait varier l’incertitude associée au comportement des autres investisseurs. Plus l’incertitude stratégique grandit, plus l’information privée est prisée, et donc plus les investisseurs acquièrent de l’information. Cette évolution de la proportion d’investisseurs avertis amplifie les effets que produit la modification de l’information publique sur la probabilité d’une crise. Grâce à cette amplification, les auteurs parviennent à expliquer comment une légère aggravation dans les données à disposition peut engendrer une crise financière.

Classification JEL : D83, G01
Classification de la Banque : Institutions financières; Stabilité financière

Non-technical summary

Financial stability is a key concern for central bankers. This paper focuses on financial crises, especially the coordination aspect that is at the heart of currency crises, bank runs and debt crises. A crisis occurs if sufficiently many depositors withdraw from a bank, currency speculators attack a peg or creditors do not roll over debt. In this context, we analyze the incentives of investors to acquire information about an unobserved level of the monetary authority's foreign reserves, the bank's profitability or the debtor's solvency.

Our main result is that information acquisition by investors amplifies the probability of a financial crisis. Here we illustrate our mechanism with the example of a debt crisis. Each investor wishes to roll over debt if other investors roll over as well. Suppose the debtor was initially considered as solvent, so each investor expects that most other investors roll over debt. After adverse news about the debtor's solvency, each investor is more uncertain about whether other investors will roll over.

Given this heightened uncertainty about the behavior of other investors, more investors acquire information about the debtor's solvency. Acquiring private information helps an individual investor align its rollover decision with that of other investors. However, informed investors tend to roll over less often than uninformed investors. First, informed investors tend to refuse to roll over for a larger range of private information than uninformed investors. Second, informed investors expect most other investors not to roll over. Consequently, a debt crisis becomes more likely as more investors become informed.

1 Introduction

Financial crises are sometimes difficult to explain without resorting to large shocks or multiple equilibria. The financial meltdown of 2007–08 is considered the worst financial crisis since the Great Depression of the 1930s. However, the recent financial crisis was not caused by a large shock (Financial Crisis Inquiry Commission Report (2011)).[1] How are small shocks amplified, resulting in a financial crisis?

Financial crises have been explained by both weak economic conditions and elevated uncertainty among investors.[2] The global games literature synthesizes both views, since weak fundamentals cause the self-fulfilling beliefs about a financial crisis.[3] In particular, global coordination games of regime change are used to analyze debt crises, bank runs and currency crises.[4] A financial crisis occurs if a sufficiently large proportion of creditors do not roll over debt, depositors withdraw from a bank or currency speculators attack a peg.

These investors base their decision on both public and private information about an unobserved fundamental that corresponds to the debtor's solvency, the bank's profitability or the monetary authority's foreign reserves. Unique predictions arise if investors are endowed with sufficiently precise private information. However, much economic activity is devoted to generating, collecting and processing information (Veldkamp (2011)), where investors seek to improve the quality of their private information ex ante at a cost, for example by hiring analysts or purchasing data (Hellwig and Veldkamp (2009)).

[1] See also Gorton and Ordonez (2014), and Brunnermeier and Sannikov (2014).

[2] A self-fulfilling financial crisis can be caused by a panic among bank depositors (Bryant (1980), Diamond and Dybvig (1983)) or currency speculators (Obstfeld (1996)). By contrast, fundamental-based financial crises are analyzed by Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998) for bank runs, and by Krugman (1979) for currency crises. See also Goldstein (2012).

[3] This literature was pioneered by Carlsson and van Damme (1993). See also Morris and Shin (2003).

[4] See, for example, Morris and Shin (1998) and Corsetti et al. (2004) for currency crises, Goldstein and Pauzner (2005) and Rochet and Vives (2004) for bank runs, Morris and Shin (2004) for debt crises, and Corsetti et al. (2006) for sovereign debt crises. For other applications, see Dasgupta (2007) for foreign direct investment and Edmond (2013) for political regime change. See Angeletos et al. (2007) for dynamic aspects of crises.

We study how the acquisition of information by investors affects the probability of a financial crisis. We propose a parsimonious model of information choice in a standard global game of regime change. Investors choose whether to improve their private information at a cost that is heterogeneous across investors. Our set-up generates strategic complementarity in information choices but retains a unique equilibrium. We analyze how changing public information about the fundamental affects the incentives to acquire private information.

Our main result is amplification. We show that the private information choices of investors amplify the effect of any change in public information on the probability of a financial crisis. Our amplification result explains how a small deterioration in publicly available information can cause a financial crisis.

The key dynamic of our model is how a change in the public information about the fundamental affects the uncertainty about the behavior of other investors. Greater strategic uncertainty makes private information more valuable, so more investors acquire information. This change in the proportion of informed investors always amplifies the impact of the initial change in public information on the probability of a financial crisis. That is, the information choices of investors imply a further reduction in the probability of a crisis after improving public news, and a further increase in this probability after deteriorating public news.

Consider the example of a simultaneous debt rollover game, where an investor wishes to roll over if other investors also roll over. When the available public information suggests that the debtor is solvent, each investor expects most other investors to roll over. However, if adverse news about the debtor's solvency becomes available subsequently, each investor becomes more uncertain about whether other investors will roll over. Given the heightened strategic uncertainty, acquiring information helps to align an investor's rollover decision with that of other investors.[5] More investors acquire information about the debtor.[6]

[5] Specifically, an informed investor forecasts the information received by other investors more precisely.

[6] The case of adverse news about the debtor's solvency when it was already considered to be little cred-

How does the proportion of informed investors change the probability of a financial crisis? Our result on the extensive margin complements Metz (2002), who studies the intensive margin (the precision of private information). More-informed investors make a debt crisis more likely if the public information suggests that the debtor is solvent. Uninformed investors roll over debt unless their private information is unfavorable. Conversely, informed investors refuse to roll over debt for a larger range of private information. First, informed investors rely more on their relatively more-precise private information. Second, they expect a larger proportion of other informed investors not to roll over. Hence, informed investors tend to roll over less often than uninformed investors, making a debt crisis more likely.[7]

The amplification result holds independently of the initial level of public information or its change. We illustrate this point in the debt rollover game. First, good news about the solvency of an initially creditworthy debtor further reduces strategic uncertainty and thus the incentive to acquire information. Fewer investors acquire information and a debt crisis becomes even less likely. Second, bad news about the solvency of an initially little-creditworthy debtor also reduces strategic uncertainty (a debt crisis is expected). Fewer investors acquire information, and this further increases the probability of a debt crisis.

Literature Our amplification result implies that a small deterioration in public information can cause a financial crisis due to changes in the information acquisition of investors. A related implication also arises in Dang et al. (2012) in the context of adverse selection in secondary debt markets (Gorton and Pennacchi (1990)). The authors show that symmetric ignorance in liquidity provision is welfare maximizing. Debt is the optimal form of private money, since it provides the least incentive to acquire information. Rather than adverse selection, our focus is on strategic complementarity.

itworthy initially is symmetric. The behavior of other investors becomes less uncertain. Lower strategic uncertainty makes private information less valuable, so fewer investors acquire information.

[7] We have another symmetry result here. If the public information about the debtors suggests a low solvency, a larger proportion of informed investors reduces the probability of a debt crisis.

Endogenous information in coordination games can lead to multiple equilibria. In Angeletos and Werning (2006), aggregating dispersed private information into a publicly observed market price, similar to Grossman and Stiglitz (1980), results in multiple equilibria despite a global game refinement. Hellwig et al. (2006) formulate a market-based model of currency attacks with multiple equilibria. Angeletos et al. (2006) examine how the endogenous public information from a policy intervention generates multiple equilibria. By contrast, our parsimonious model of private information choice retains a unique equilibrium.[8]

Hellwig and Veldkamp (2009) first studied the optimal information choices in coordination games. They show that these choices inherit the strategic motive of the underlying beauty contest game (complementarity or substitutability). This inheritance result extends to our game of regime change with parsimonious information choice. Hellwig and Veldkamp (2009) also show that a binary information choice and a homogeneous information cost generates multiple equilibria. Uniqueness arises in our set-up since the information cost is sufficiently heterogeneous across investors. Szkup and Trevino (2013) also analyze the information choice in a game of regime change. They study a continuous information choice subject to a convex cost function that is homogeneous across investors. In contrast to their paper, strategic complementarity in information choices always arises in our parsimonious set-up. In contrast to both papers, we explain how the information choices of investors amplify the impact of changes in public information on the probability of regime change.

In recent work on information acquisition in coordination games, Myatt and Wallace (2012) and Colombo et al. (2014) analyze costly information acquisition in the beauty contest framework.[9] In Myatt and Wallace (2012), agents have access to a variety of information sources, whereby the precision of each signal depends on both its accuracy and the attention

[8] Zwart (2007) studies the signalling effect of public intervention and obtains a unique equilibrium.

[9] Morris and Shin (2002) show that more transparency can be detrimental to welfare within the beauty contest framework, initiating a debate on the social value of public information. See Angeletos and Pavan (2007) for a substantial generalization with an exogenous information structure.

devoted to the signal. Uniqueness obtains if signals are not fully private. Colombo et al. (2014) show that the social value of public information is higher when there is private information choice, since the provision of public information crowds out private information acquisition.[10] By contrast, we explain how the information choices of investors amplify the impact of changes in public information on the probability of regime change.

This paper is organized as follows. In section 2 we provide a parsimonious model of private information choice in a global coordination game of regime change. We solve for the unique equilibrium in section 3. We explain the amplification result in section 4. Analyzing an extension in Appendix A, we show the robustness of our main result. Appendix B contains numerical examples. Derivations and proofs are provided in Appendices C and D.

2 Model

A unit continuum of agents indexed by $i \in [0,1]$ play a coordination game in which a status quo is either maintained, $R = 0$, or a regime change occurs, $R = 1$. A fundamental $\theta \in \mathbb{R}$ parameterizes the strength of the status quo. Agents simultaneously decide whether to attack the status quo, $a_i = 1$, or not, $a_i = 0$. Regime change occurs if and only if the aggregate attack size $A \equiv \int_0^1 a_i \, di$ exceeds the fundamental, $R = \mathbf{1}\{A \geq \theta\}$.

Attacking the status quo yields a benefit $b \in (0,1)$ if regime change occurs, and a loss $\ell \in (0,1)$ otherwise. The constant payoff from not attacking is normalized to zero, $u(a_i = 0) \equiv 0$. If $\mathbf{1}$ is the indicator function, agent i's (relative) payoff from attacking is

$$u(a_i = 1, A, \theta) = b \, \mathbf{1}\{A \geq \theta\} - \ell \, \mathbf{1}\{A < \theta\}. \tag{1}$$

[10]For the effect of information acquisition on bank runs, see also Nikitin and Smith (2008) for the costly verification of bank solvency in a Diamond and Dybvig (1983) set-up, and He and Manela (2012) for information acquisition and withdrawal-redeposit decisions in a continuous-time model with rumors. Yang (2013) studies flexible information acquisition in coordination games.

Greater conservativeness, measured by $\kappa \equiv \frac{\ell}{b+\ell} \in (0,1)$, reduces an individual agent's incentive to attack. The attacking decisions are strategic complements: the individual incentive to attack increases in the mass of attacking agents, resulting in a strong coordination motive.[11]

Our preferred interpretation of a regime change is a financial crisis, such as a bank run (Diamond and Dybvig (1983); Goldstein and Pauzner (2005); Rochet and Vives (2004)); a currency crisis (Krugman (1979); Obstfeld (1986); Morris and Shin (1998); Corsetti et al. (2004)); and a sovereign debt crisis (Corsetti et al. (2006); Zwart (2007)). Hence, the fundamental measures a bank's investment profitability; a central bank's ability or willingness to maintain a currency peg; and a sovereign's taxation power, respectively. In turn, agents correspond to bank creditors who withdraw funds from the bank; speculators who attack a currency; and investors who do not roll over government debt, respectively.

Following the global games literature pioneered by Carlsson and van Damme (1993), there is incomplete information about the fundamental θ. Agents share a common prior that is normally distributed with finite mean $\mu \in (-\infty, \infty)$ and finite precision $\alpha \in (0, \infty)$:[12]

$$\theta \sim \mathcal{N}\left(\mu, \alpha^{-1}\right). \tag{2}$$

Agents receive noisy private information about the fundamental (Morris and Shin (2003)):

$$x_i \equiv \theta + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}\left(0, \gamma^{-1}\right), \tag{3}$$

where the idiosyncratic noise ϵ_i is identically and independently distributed across agents and independent of the fundamental. Its distribution is normal with zero mean and an endogenous precision $\gamma > 0$, described below. The information structure is common knowledge.

[11] This coordination motive is starkest if information about the fundamental is complete. Two symmetric equilibria in pure strategies exist for $\theta \in (0,1)$: the status quo, $A^* = 0 = R$, and regime change, $A^* = 1 = R$.

[12] If agents initially had an improper uniform prior, this common prior would be induced by a public signal $\mu \equiv \theta + \eta$, where the noise term $\eta \sim \mathcal{N}\left(0, \alpha^{-1}\right)$ is independent of the fundamental θ.

This coordination stage is preceded by an information stage (see timeline in Table 1). Agents simultaneously make a costly binary information choice $n_i = z \in \{I, U\}$.[13] The private signal received by informed agents at the coordination stage is more precise:

$$\gamma_U < \gamma_I. \tag{4}$$

We focus on the vanishing noise of informed agents, $\gamma_I \to \infty$. This specification provides a natural and parsimonious benchmark that maintains uniqueness at the coordination stage.

An information cost captures the resources required to acquire and process information. Differences in the skill of generating and processing information is captured by heterogeneity in the information cost c_i, which is uniformly distributed over a unit interval:[14]

$$c_i \sim \mathcal{U}\,[0, 1]. \tag{5}$$

Information stage	**Coordination stage**
1. Common prior μ about fundamental	1. Private signal x_i about fundamental • Signal more precise if informed
2. Simultaneous information choice • binary action $n_i \in \{I, U\}$ • heterogeneous information cost c_i	2. Simultaneous attacking decision • binary action $a_i \in \{0, 1\}$
	3. Status quo or regime change realized
	4. Payoffs received

Table 1: Timeline of the model.

[13] See Szkup and Trevino (2013) for a model with continuous private information choice and a homogeneous and convex information cost. While strategic complementarity in information choices may not be attained in such a set-up, strategic complementarity in information choices is present in our parsimonious model.

[14] The amplification result is robust to other information cost specifications. In Appendix A, we consider a homogeneous information cost and show that amplification occurs in all stable equilibria.

3 Equilibrium

We analyze a pure-strategy perfect Bayesian equilibrium in threshold strategies. It comprises (i) information choices $n_i^* \in \{I, U\}$ for each agent; (ii) the aggregate proportion of informed agents $n^* \in [0,1]$; (iii) attack decisions $a_i^* \in \{0,1\}$ for each agent; and (iv) the aggregate attack size $A^* \in [0,1]$. Individual choices are consistent with aggregate proportions:

$$A^* = \int_0^1 a_i^* di, \qquad\qquad n^* = \int_0^1 \mathbf{1}\{n_i^* = I\} di. \tag{6}$$

Agents optimally attack the regime at the coordination stage and make optimal information choices at the information stage. The former depends on the individual information choice, the aggregate proportion of informed agents and the received private signal, and the latter depends on the aggregate proportion of informed agents and the individual information cost:

$$\begin{aligned} a_i^*(n_i^*, n^*; x_i) &= \arg\max_{a_i \in \{0,1\}} a_i \left[b \Pr(A^* \geq \theta | n_i^*, n^*; x_i) - \ell \Pr(A^* < \theta | n_i^*, n^*; x_i) \right], \ \forall i, \\ n_i^*(n^*; c_i) &= \arg\max_{n_i \in \{I,U\}} \mathbf{1}\{n_i = I\}[EU^I(n^*) - c_i] + \mathbf{1}\{n_i = U\} EU^U(n^*), \ \forall i, \end{aligned} \tag{7}$$

where EU^I (EU^U) is the expected utility of an informed (uninformed) agent. The benefit of acquiring information is the improvement in expected utility:

$$D(n^*) \equiv EU^I(n^*) - EU^U(n^*). \tag{8}$$

Two parameter constraints ensure equilibrium uniqueness. First, private information about the fundamental must be sufficiently precise. This standard condition in the global games literature places a lower bound on the precision of the signal of *uninformed* agents.[15]

[15] However, this lower bound is more restrictive than the usual condition in a stand-alone global coordination game (e.g., Morris and Shin (2003)), in order to ensure uniqueness of equilibrium not only at the coordination stage but in the overall game. See also Ahnert (2014).

Second, the constraint stated in Condition 1 and illustrated in Figure 1, which holds almost surely, ensures a link between the information and coordination stages. More precisely, a change in the proportion of informed agents affects the probability of a crisis (see Lemma 1 in Appendix C).

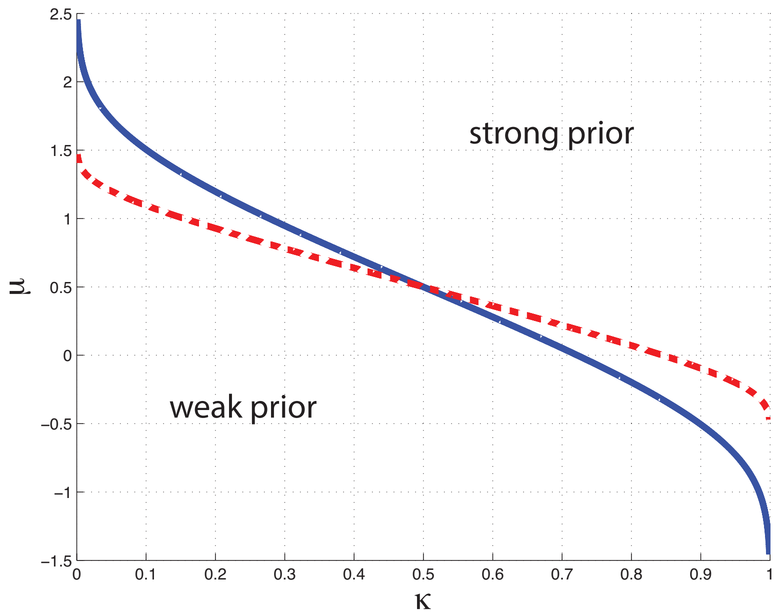

Figure 1: The values of prior mean μ and conservativeness κ that *violate* Condition 1. Parameter values: $\alpha = 0.25$, $\gamma_U = 1$ (solid) and $\alpha = 1.5$, $\gamma_U = 10$ (dashed). The properties of $\hat{\mu}$ are: $\hat{\mu}(\frac{1}{2}) = \frac{1}{2}$, $\frac{\partial \hat{\mu}}{\partial \kappa} < 0$, and $\hat{\mu} \to \infty$ if $\kappa \to 0$, $\hat{\mu} \to -\infty$ if $\kappa \to 1$. Proximity to $\mu = \hat{\mu}$ corresponds to greater strategic uncertainty and thus a higher value of private information.

Condition 1. *Let Φ^{-1} denote the inverted cumulative distribution function of a standard normal random variable. The mean of the common prior about the fundamental satisfies:*

$$\mu \neq \hat{\mu}(\kappa) \to 1 - \kappa - \frac{\sqrt{\alpha + \gamma_U} - \sqrt{\gamma_U}}{\alpha} \Phi^{-1}(\kappa). \tag{9}$$

Definition 1. *The common prior is strong if $\mu > \hat{\mu}(\kappa)$. Likewise, it is weak if $\mu < \hat{\mu}(\kappa)$.*

Proposition 1. ***Existence of a unique equilibrium.*** *If private information is sufficiently precise, $\underline{\gamma}_U \equiv \left(\frac{\alpha}{\sqrt{2\pi}-2}\right)^2 < \gamma_U$, and Condition 1 holds, $\mu \neq \hat{\mu}(\kappa)$, then there exists a unique pure-strategy perfect Bayesian equilibrium in threshold strategies characterized by:*

(i) a unique threshold of the information cost $\overline{c}$ that is implicitly defined by $\overline{c} = D(\overline{c})$. At the information stage, agents become informed if and only if their individual information cost is below this threshold, $n_i^* = I \Leftrightarrow c_i < \overline{c}$, and the proportion of informed agents is $n^* = \overline{c}$;

(ii) unique attack thresholds for informed agents, $\overline{x}_I(\overline{c})$, and uninformed agents, $\overline{x}_U(\overline{c})$, and a unique aggregate threshold $\overline{\theta}(\overline{c})$. At the coordination stage, agents attack the status quo if they receive sufficiently low private information, $x_i < \overline{x}_z(\overline{c})$ for $n_i^* = z \in \{I, U\}$, and a regime change occurs if and only if the realized fundamental is sufficiently low, $\theta < \overline{\theta}(\overline{c})$.

Proof. *See Appendix C.*

Our uniqueness result rests on the heterogeneity of information costs. For a homogeneous information cost, Hellwig and Veldkamp (2009) show that multiple equilibria arise in case of a binary information choice. Although there is strategic complementarity in information choices (Lemma 2 in Appendix C), inherited from the strategic complementarity in actions as in Hellwig and Veldkamp (2009), the amount of ex-ante heterogeneity suffices for uniqueness. The range of information costs nests the range of the benefit of becoming informed, resulting in lower and upper dominance regions at the information stage. Specifically, there exists a lower dominance region $c_j \in [0, D(0))$ in which acquiring information is a dominant action for agents j. Likewise, there exists an upper dominance region $c_j \in (D(1), 1]$ in which not acquiring information is a dominant action for agents j.

Our object of interest is the equilibrium probability of a regime change interpreted as a financial crisis such as a currency crisis, a (sovereign) debt crisis or a bank run:

$$\Pr\{\theta \leq \overline{\theta}\} = \Phi\left(\sqrt{\alpha}[\overline{\theta}(\overline{c}) - \mu]\right). \tag{10}$$

4 Amplification

This section reports our main result. We demonstrate how the information choice of agents amplifies the impact of changing public information on the probability of a crisis. We show that this amplification result is independent of both the strength of the common prior and the direction of its change.

The amplification result is best illustrated by comparing the case of endogenous and exogenous information. Let $\tilde{\theta}$ denote the aggregate threshold if information is exogenous. Then, a standard result is that a lower common prior μ leads to a larger aggregate threshold and thus to a larger probability of a crisis (e.g., Metz (2002)):

$$\left.\frac{d\tilde{\theta}}{d\mu}\right|_{\overline{n},\tilde{\theta}} = \frac{A_\mu}{1 - A_\theta} < 0, \tag{11}$$

where A_μ and A_θ are the partial derivatives of the aggregate attack size with respect to the prior mean and the fundamental, respectively, that we derive in Appendix C. All partial derivatives are evaluated at the aggregate threshold $\tilde{\theta}$ and an (exogenous) proportion of informed agents $\overline{n}$. To ensure comparability, this proportion is set to the equilibrium proportion of informed agents in case of endogenous information, $\overline{n} = n^*$.

A novel effect arises if information is endogenous. Changes in the prior mean now also affect the incentives to acquire information (captured by D_μ) and the impact of a change in the proportion of informed agents on the aggregate threshold (captured by A_n). Therefore, the total effect of a change in the prior mean on the probability of a crisis is given by

$$\left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} = \frac{A_\mu + A_n D_\mu}{1 - A_\theta - A_n D_\theta} < 0, \tag{12}$$

where A_n is the partial derivative of the aggregate attack size with respect to the proportion of informed agents, while D_μ and D_θ are the partial derivatives of the benefit of becoming informed with respect to the prior mean and the fundamental. Equation (12) is derived in Appendix D by totally differentiating the equilibrium conditions at both stages. Our main result contrasts the cases of exogenous and endogenous information.

Proposition 2. ***Amplification.*** *If private information is sufficiently precise, $\gamma_U > \underline{\gamma}_U$, and Condition 1 holds, $\mu \neq \hat{\mu}(\kappa)$, then the information choices of investors amplify the impact of changes in public information on the probability of a financial crisis:*

$$- \left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} > - \left.\frac{d\tilde{\theta}}{d\mu}\right|_{\overline{n}=n^*,\overline{\theta}}. \tag{13}$$

Proof. *See Appendix D.*

The conditions sufficient for uniqueness in Proposition 1 also generate amplification. A change in the proportion of informed agents affects the aggregate threshold, $A_n \neq 0$. This is implied by Condition 1, which holds almost surely (Lemma 1 in Appendix C). Moreover, the lower bound on the precision of private information ensures a positive denominator of equation (12), since this bound implies that $\frac{\partial D}{\partial n} < 1$ (see Lemma 2 in Appendix C).

Intuition To illustrate the intuition of the amplification result, consider a deterioration in the prior mean that could be caused by an adverse public signal about the fundamental. Analyzing a deterioration is purely expositional, and the opposite results holds for an improvement. Furthermore, our amplification result holds for both strong and weak priors.

First, consider a strong prior, $\mu > \hat{\mu}(\kappa)$. A reduction in the prior mean, $d\mu < 0$, brings it closer to the interim value $\hat{\mu}(\kappa)$ defined in Condition 1. This makes the behavior of other agents more uncertain. As strategic uncertainty increases, private information becomes more valuable because information supports coordination. Since the benefit of becoming informed increases, so does the equilibrium proportion of informed agents. In turn, a larger proportion of informed agents raises the aggregate threshold, since informed agents attack more aggressively than uninformed agents if the prior is strong. Overall, the indirect effect via the change in the proportion of informed agents amplifies the direct effect of a reduction in the prior mean on the probability of a crisis. In sum, the increase in the probability of a financial crisis after a reduction in the prior mean is greater when information is endogenous.

Second, consider a weak prior. A reduction in the prior mean brings it further away from the interim value $\hat{\mu}(\kappa)$. Thus, strategic uncertainty is smaller, which decreases the value of private information, so fewer agents acquire information in equilibrium. However, the effect of the proportion of informed agents on the aggregate threshold is reversed for a weak prior. Fewer informed agents *raise* the aggregate threshold, since informed agents attack less aggressively than uninformed agents if the prior is weak. Therefore, as with the strong prior, the increase in the probability of a financial crisis after a reduction in the prior mean is greater when information is endogenous. Overall, the indirect effect via the change in the proportion of informed agents amplifies the direct effect from a reduction in the prior.

These cases illustrate that the mechanism behind our amplification result works differently depending on the mean of the common prior. To shed more light on this mechanism, and to provide intuition for the factors in play, we analyze these in detail below. Specifically, we describe how a change in the prior mean affects the incentives to acquire information (section 4.1) and the equilibrium proportion of informed agents (section 4.2). The consequences for the aggregate threshold, and thus the probability of a crisis, follow (section 4.3).

4.1 Effect on the incentives to acquire information

Changes in the prior mean affect the value of private information and thus the incentives to acquire information. Private information supports the coordination of agents in general, but information is more valuable the more uncertain is the behavior of other agents. We show in Appendix D that Condition 1 suffices for:

$$\frac{\partial D}{\partial \mu}\left[\mu - \hat{\mu}(\kappa)\right] > 0. \tag{14}$$

Figure 2 illustrates this intuitive link between strategic uncertainty and the value of private information. First, the left panel shows the case in which the prior moves toward the interim value $\mu = \hat{\mu}(\kappa)$, whereby it is either high and deteriorating or low and improving. The closer the prior is to the interim value, the larger the sensitivity of the aggregate behavior to small changes in the fundamental. Therefore, strategic uncertainty increases and private information becomes more valuable, since it facilitates the coordination of agents. In turn, agents have greater incentives to acquire information.

Second, consider the right panel, which shows the case in which the prior moves away from the interim value $\hat{\mu}(\kappa)$. That is, the prior mean is either high and improving or low and deteriorating. The further away the prior mean is, the more certain is the outcome of the coordination game and thus the behavior of other agents. Smaller strategic uncertainty implies a lower value of private information, which is less required for the coordination of agents for more extreme prior means. Hence, the incentive to acquire information decreases.

4.2 Effect on the proportion of informed agents

Changes in the incentive to acquire information directly map into the equilibrium proportion of informed agents, which equals the threshold investment cost, $n^* = \overline{c}$ that solves the fixed-point problem $\overline{c} = D(\overline{c})$. Figure 3 shows how a change in the benefit of becoming informed affects the threshold information cost and thus the equilibrium proportion of informed agents.

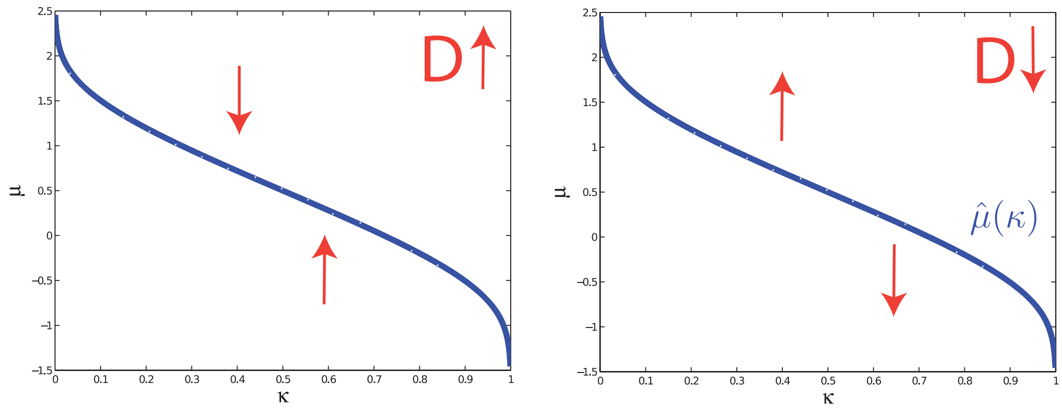

Figure 2: Strategic uncertainty and the value of private information. The left panel shows changes of the prior mean associated with higher strategic uncertainty and, consequently, a higher value of private information (higher D). In turn, the right panel shows changes associated with lower strategic uncertainty and, consequently, a lower value of private information (lower D).

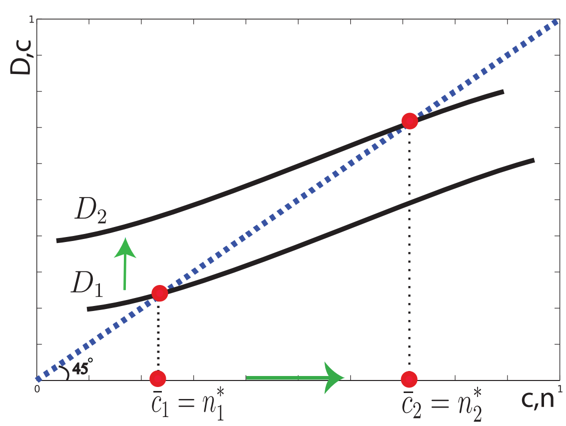

Figure 3: The benefit of becoming informed and the equilibrium proportion of informed agents. A larger benefit of becoming informed translates into a strictly larger proportion of informed agents, because there is a unique fixed point of the threshold information cost.

As an ancillary result, we obtain strategic complementarity in information choices.[16] This result was first shown by Hellwig and Veldkamp (2009) in a beauty contest framework. We extend this result to a game of regime change with parsimonious information choice (binary choice and heterogeneous information cost). In contrast, Szkup and Trevino (2013) study a model with continuous information choice and convex but homogeneous information cost. Strategic complementarity in information choices may not occur in their set-up, however.

To gather intuition for the strategic complementarity in information choices, consider the two benefits of acquiring information. First, informed agents forecast the fundamental more precisely, but this benefit is independent of the proportion of informed agents. Second, informed agents forecast the information received by other agents more precisely. In particular, an informed agent is relatively better at forecasting the information of other informed agents than of uninformed agents. Since information supports coordination, the benefit of forecasting more precisely the information of informed agents increases with the proportion of informed agents. Since the second benefit of becoming informed increases with the proportion of informed agents, there is strategic complementarity in information choices.[17]

4.3 Effect on the probability of a crisis

The probability of a crisis is determined by the aggregate threshold. Figure 4 shows the effect of the proportion of informed agents on this threshold. The proportion of informed

[16] In Lemma 2 in Appendix C, we show that the benefit of becoming informed increases with the proportion of informed agents, $\frac{\partial D}{\partial n^*} > 0$. Therefore, the strategic complementarity in attacking decisions at the coordination stage translates into strategic complementarity at the information stage.

[17] Grossman and Stiglitz (1980) show that information choices are strategic substitutes in an economy with uncertainty about the value of an asset and subsequent trade. Uninformed traders only learn from the market price, which is more informative if the proportion of informed traders is larger. Then, the private incentive to become informed is low, so no competitive equilibrium exists as the private information cost vanishes. In contrast, there is always an equilibrium in our model. Existence in our paper arises from the fact that the information choices of agents are strategic complements. The closest equivalent to Grossman and Stiglitz (1980) is the extension of a homogeneous information cost that we study in Appendix B.

agents is the extensive margin of information, while the precision of private information is the intensive margin of information. The latter is analyzed in a stand-alone coordination game by Metz (2002), who finds that the effect of more precise private information on the aggregate threshold depends on the strength of the prior. More specifically, the aggregate threshold increases (decreases) in the precision of private information if the prior is weak (strong). We show in Appendix C that these insights generalize to the extensive margin. Condition 1, which holds almost surely, is again sufficient for the following strict inequality:

$$\frac{\partial \bar{\theta}}{\partial n^*}\left[\mu - \mu(\kappa)\right] > 0. \tag{15}$$

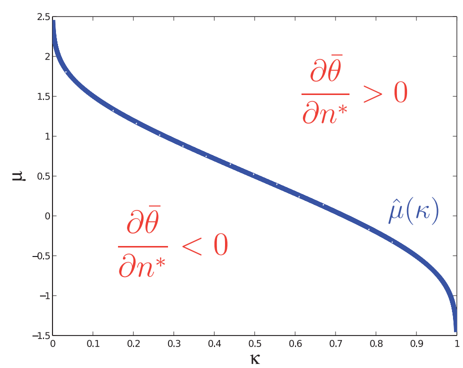

Figure 4: The proportion of informed agents and the probability of a crisis. For a strong prior, $\mu > \hat{\mu}(\kappa)$, a larger proportion of informed agents leads to a higher probability of a crisis. Conversely, for a weak prior, a smaller proportion of informed agents leads to a higher probability of a crisis.

We illustrate the forces at work for a strong prior. An uninformed agent only attacks upon receiving a very low signal, while an informed agent also attacks for moderately low signals. This is for two reasons. First, an informed agent relies more on its private information, since it is more precise than the private information of an uninformed agent. Since the low private signal implies a low fundamental, an informed agent attacks the regime. Second, an informed agent expects other informed agents to join the attack because they must have received a low signal as well. In sum, both fundamental and strategic reasons induce an informed agent to attack the status quo for a larger range of private information, relative to an uninformed agent. Hence, a larger proportion of informed agents increases the probability of a crisis if the prior is strong.

5 Conclusion

How can a small shock have large consequences? In this paper we analyze the information choices of investors and their consequences for the probability of a financial crisis. We study a global coordination game of regime change, in which a crisis occurs if a sufficient number of investors attack a currency peg, withdraw from a bank or do not roll over debt. Investors receive noisy public and private information about the central bank's foreign reserves, the health of a bank's balance sheet, or the solvency of a debtor. We propose a parsimonious model of information choice. Investors choose ex ante whether to improve the quality of their private information at a cost that is heterogeneous across investors. This set-up features strategic complementarity in information choices but retains a unique equilibrium.

We establish an amplification result. The information choices of investors amplify the impact of changes in public information on the probability of a financial crisis. This result holds independent of the level of public information and the direction of its change. An implication of our main result is that a small adverse shock can have large detrimental consequences, resulting in a financial crisis. Our amplification result highlights the importance of information acquisition, especially during times of crisis.

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A Homogeneous information cost

To evaluate the robustness of our main result, we study the extension of a homogeneous information cost, $c_i = c$. As in Hellwig and Veldkamp (2009), the ex-ante homogeneity in the information cost combined with a discrete information choice produces multiple equilibria. We show that the amplification effect prevails in both stable equilibria.

Several papers show multiplicity in global coordination games. Angeletos and Werning (2006) endogenize the precision of public information in the set-up of Morris and Shin (2003), using the market price to aggregate private information as in Grossman and Stiglitz (1980). They show that better private information improves public information (at a rate fast enough) to result in multiple equilibria. Similarly, Hellwig et al. (2006) endogenize the interest rate in a currency crisis model and obtain multiple equilibria. Angeletos et al. (2007) consider dynamic global games of a regime change and show how the arrival of information over time can lead to multiplicity of equilibria. Furthermore, Angeletos et al. (2006) show that endogenous information generated by policy interventions can lead to multiplicity.

Proposition 3. ***Multiplicity and amplification in stable equilibria.*** *Consider instead a homogeneous information cost, $c_i = c$. Suppose that the private information of the uninformed agent is sufficiently precise, $\gamma_U > \underline{\gamma}_U$, and that Condition 1 holds, $\mu \neq \hat{\mu}(\kappa)$. Then, the number of equilibria in the overall game depends on the information cost:*

- *If the information cost is low, $c < D(0)$, then there exists a unique equilibrium in which all agents acquire information, $n^* = 1$. This equilibrium is symmetric and stable.*
- *If the information cost is high, $c > D(1)$, then there exists a unique equilibrium in which no agent acquires information, $n^* = 0$. This equilibrium is symmetric and stable.*
- *However, if the information cost takes an interim value, $D(0) \leq c \leq D(1)$, then there exist three equilibria. The two equilibria described above prevail and there is also*

an asymmetric and unstable equilibrium. In the asymmetric equilibrium, agents are indifferent between becoming informed and remaining uninformed, and the aggregate proportion of informed agents is determined by the indifference of the marginal agent to become informed: $n^* = D^{-1}(c)$.

The optimal behavior of agents at the coordination stage is again uniquely pinned down for a given proportion of informed agents, and is characterized by the three thresholds $\overline{x}_I(n^*)$, $\overline{x}_U(n^*)$, *and* $\overline{\theta}(n^*)$. *Amplification occurs weakly in both stable equilibria.*

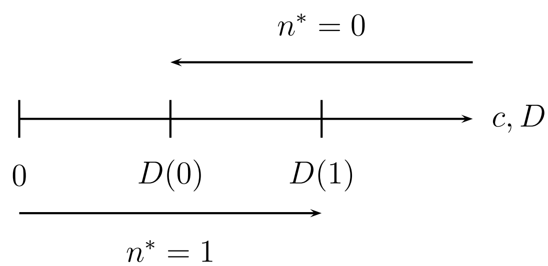


Figure 5: Multiple equilibria with information choice and homogeneous information cost.

Figure 5 illustrates the link between the information cost and the number of equilibria. Given the strict monotonicity of the benefit of becoming informed in the proportion of informed agents (shown in Appendix C.2), the proportion of informed agents is uniquely determined in the asymmetric equilibrium. While the amplification effect does not occur for the asymmetric equilibrium, we exclude this equilibrium based on its instability. That is, we consider the two stable equilibria in which the information choices of agents are symmetric.

As described in section 4.1, the benefit of becoming informed changes with the mean of the fundamental, $\frac{\partial D}{\partial \mu}(\mu - \hat{\mu}(\kappa)) < 0$, if Condition 1 holds. These changes affect the equilibrium proportion of informed agents, as shown in Figures 6 and 7.

Consider first a strong prior. Adverse public news about the fundamental, a lower μ, increases strategic uncertainty, since the prior is now closer to the interim value $\hat{\mu}$. Therefore,

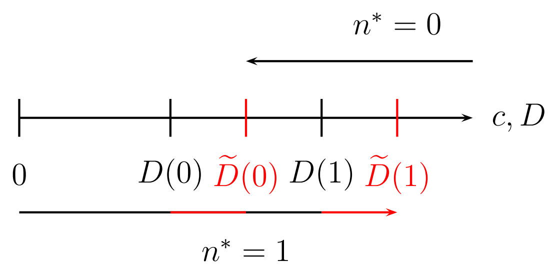


Figure 6: Equilibria with a homogeneous information cost and strong prior. The figure shows the case before and after (in tildes) a reduction in the prior.

the benefit of becoming informed increases. Thus, there exist a range of information costs for which information acquisition, $n^* = 1$, becomes the unique equilibrium, where it was previously one of several equilibria. Similarly, there is a range of information costs for which information acquisition, $n^* = 1$, becomes one of several equilibria, where it was not previously an equilibrium. Hence, the equilibrium proportion of informed agents increases weakly for these information cost ranges. If more agents become informed, such as a switch from $n^* = 0$ to $n^* = 1$, the probability of a crisis increases. If Condition 1 holds and the equilibrium proportion of informed agents changes, the probability of a crisis increases strictly, since a larger proportion of informed agents leads to a higher probability of crisis for a high prior.

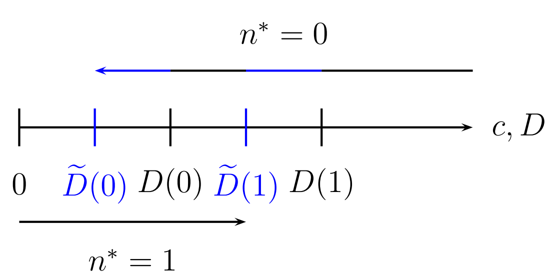


Figure 7: Equilibria with a homogeneous information cost and weak prior. The figure shows the case before and after (in tildes) a reduction in the prior.

Second, consider a weak prior. Adverse public news about the fundamental reduces strategic uncertainty, since the prior is now further away from the interim value $\hat{\mu}$. Therefore, the benefit of becoming informed decreases. Hence, there is a range of information costs for which no information acquisition, $n^* = 0$, becomes the unique equilibrium, where it was previously one of several equilibria. Similarly, there is a range of information costs for which no information acquisition, $n^* = 0$, becomes one of several equilibria, where it was not previously an equilibrium. The equilibrium proportion of informed agents decreases weakly for these information cost ranges. If fewer agents become informed, such as a switch from $n^* = 1$ to $n^* = 0$, the probability of a crisis increases. If Condition 1 holds and the equilibrium proportion of informed agents changes, the probability of a crisis increases strictly, since a larger proportion of informed agents leads to a smaller probability of crisis for a weak prior.

B Numerical examples

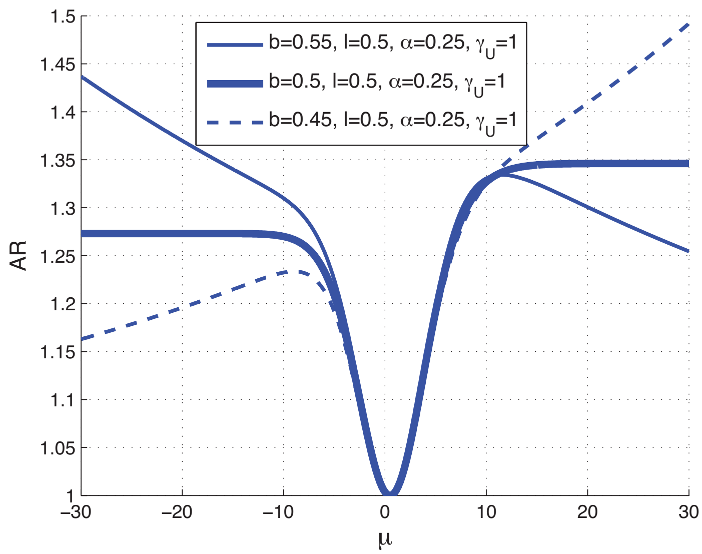
To determine the quantitative impact of the amplification effect, we conduct two simple numerical exercises. First, we calculate the *amplification ratio* AR that measures the strength of the amplification effect:

$$AR \equiv \frac{A_\mu + A_n D_\mu}{1 - A_\theta - A_n D_\theta} \Big/ \frac{A_\mu}{1 - A_\theta} \geq 1. \tag{16}$$

Second, we calculate the contribution of information acquisition to the total effect on the aggregate threshold. This measure is the percentage contribution of the information stage as a proportion of the total effect of a change in the prior:

$$Contribution \equiv \frac{A_n D_\mu}{A_\mu + A_n D_\mu} \in (0,\ 1). \tag{17}$$

Figure 8 shows the amplification ratio and the contribution of the information stage for different prior means.



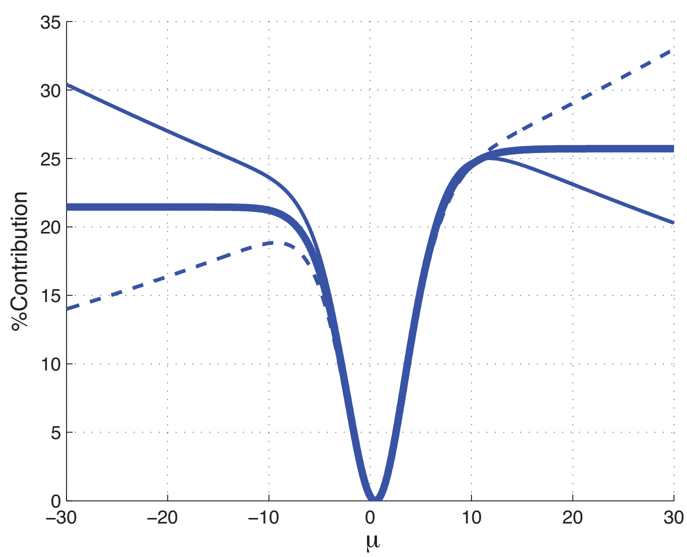


Figure 8: The amplification ratio (top) and the contribution of the information stage (bottom). The top figure shows, for various priors, the factor with which the aggregate threshold is amplified after a marginal change in the prior. The bottom figure shows, for various priors, the contribution of a change in the proportion of informed agents to the total effect on the aggregate threshold after a marginal change in the prior.

C Derivation of equilibrium and uniqueness proof

It is useful to construct the equilibrium sequentially. We start by deriving the optimal behavior at the coordination stage, for any given set of information choices $\{n_i^*\}_{i\in[0,1]}$ that summarizes the behavior at the information stage. Next, we analyze the optimal behavior at the information stage.

C.1 Stage 2: Coordination stage

For a given proportion of informed agents n^*, the coordination stage is a standard global coordination game. It is a game of imperfect information, once an agent's signal x_i is established as the type. A strategy s_i is a mapping from the signal into the binary action space: $s_i : \mathbb{R} \rightarrow \{0, 1\}$ for a given individual information choice n_i^* and the aggregate proportion of informed agents n^*. An agent's expected utility from attacking conditional on the private information x_i, the information choice n_i^*, and the aggregate proportion of informed agents n^* is $E[u(a_i = 1)|n_i^*, n^*; x_i] = -l + (b + l) \Pr[A(s_{-i}) \geq \theta | n_i^*, n^*; x_i]$. Optimality for agent i at the coordination stage requires that strategy s_i maximizes the conditional expected utility, taking all other agents' strategies s_{-i}^* as given. Since each agent is atomistic, the aggregate attack size is unaffected by the individual attack decision. We focus on symmetric monotone equilibria at the coordination stage throughout.[18]

The equilibrium at the coordination stage for a given proportion of informed agents is fully characterized by an **attacking threshold** for the informed and uninformed agent, $\overline{x}_I(n^*)$ and $\overline{x}_U(n^*)$, and an **aggregate threshold**, $\overline{\theta}(n^*)$. Agent i optimally attacks the status

[18]This is without loss of generality, since Frankel et al. (2003) and Morris and Shin (2003) show that this is the unique Bayesian equilibrium with an argument based on iterated deletion of strictly dominated strategies.

quo if and only if the private signal is below an information-specific attacking threshold:

$$a_i^* = 1 \Leftrightarrow x_i \leq \overline{x}(n_i^* = z, n^*) \equiv \overline{x}_z(n^*). \tag{18}$$

Regime change occurs if and only if the realized fundamental is below an aggregate threshold:

$$R^* = 1 \Leftrightarrow \theta \leq \overline{\theta}(n^*). \tag{19}$$

These thresholds are determined by a critical mass condition at the aggregate level and indifference conditions at the individual level.

An agent i uses the private signal to form a posterior about the fundamental:

$$\theta | n_i^* = z; x_i \sim \mathcal{N}\left(\frac{\alpha\mu + \gamma_z x_i}{\alpha + \gamma_z}, \frac{1}{\alpha + \gamma_z}\right), \tag{20}$$

where normality is preserved, the posterior mean is a weighted average of the prior mean and the private signal, and the posterior precision is the sum of the precisions of the prior and of the signal (DeGroot (1970)). Because of the high informativeness of the private signal of informed agents, they base their prior completely on it: $\theta | n_i^* = I; x_i \rightarrow x_i$. Therefore, an agent with information choice $n_i^* = z$ assigns the following probability to a regime change:

$$\Pr\{\theta \leq \overline{\theta} | n_i^* = z, n^*; x_i\} = \Phi\left(\sqrt{\alpha + \gamma_z}\left[\overline{\theta}(n^*) - \frac{\alpha\mu + \gamma_z x_i}{\alpha + \gamma_z}\right]\right), \tag{21}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Therefore, an agent with information choice $n_i^* = z$ that receives the threshold signal $x_i = \overline{x}_z(n^*)$ is indifferent between attacking and not attacking the status quo. This **indifference condition** states that the probability of regime change evaluated at the attacking threshold must equal the conservativeness ratio, $\Pr\{\theta \leq \overline{\theta} | n_i^* = z, n^*; x_i = \overline{x}_z\} = \kappa$, for both informed and uninformed agents. This yields the attacking thresholds:

$$\overline{x}_z(n^*) = \overline{\theta}(n^*) + \frac{\alpha}{\gamma_z}[\overline{\theta}(n^*) - \mu] - \frac{\sqrt{\alpha+\gamma_z}}{\gamma_z}\Phi^{-1}(\kappa). \tag{22}$$

Again, this simplifies for informed agents who use the attacking threshold $\overline{x}_I(n^*) \to \overline{\theta}(n^*)$.

Third, since all agents play the threshold strategy with thresholds $\overline{x}_I(n^*)$ if informed, and $\overline{x}_U(n^*)$ if uninformed, the aggregate attack size for any fundamental θ is

$$\begin{aligned} A(n^*;\theta,\overline{x}_I(n^*),\overline{x}_U(n^*)) &= \int_0^1 \mathbf{1}\{x_i \le \overline{x}_z(n^*)|n_i^* = z,\theta\}di \\ &= n^*\Phi\left(\sqrt{\gamma_I}\left[\overline{x}_I(n^*) - \theta\right]\right) + (1-n^*)\underbrace{\Phi\left(\sqrt{\gamma_U}\left[\overline{x}_U(n^*) - \theta\right]\right)}_{\equiv\,\Phi_U}. \end{aligned}$$

The **critical mass condition** states that the aggregate attack size is just sufficient for regime change when the fundamental equals the aggregate threshold:

$$\overline{\theta}(n^*) = A\left(n^*;\overline{\theta}(n^*),\overline{x}_I(n^*),\overline{x}_U(n^*)\right). \tag{23}$$

Combining indifference conditions with the critical mass condition, the aggregate threshold at the coordination stage for a given proportion of informed agents, $\overline{\theta} = \overline{\theta}(n^*)$, is defined by

$$\begin{aligned} \overline{\theta}(n^*) &= n^*\Phi\left(\frac{\alpha}{\sqrt{\gamma_I}}\left[\overline{\theta}(n^*) - \mu\right] - \sqrt{1+\frac{\alpha}{\gamma_I}}\Phi^{-1}(\kappa)\right)\cdots \qquad (24)\\ &\quad \cdots + (1-n^*)\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}(n^*) - \mu\right] - \sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right) \\ &\to n^*(1-\kappa) + (1-n^*)\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}(n^*) - \mu\right] - \sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right). \qquad (25) \end{aligned}$$

A unique solution to equation (25), for any given n^*, is ensured by a sufficiently precise private signal of the uninformed agent, $\gamma_U > \underline{\gamma}'_U \equiv \frac{\alpha^2}{2\pi}$ (Morris and Shin (2003)). Under this condition, the slope of the left-hand side of equation (25) exceeds the slope of the right-hand side for any proportion of informed agents, $1 > A_\theta \equiv \frac{\partial A(n^*,\overline{\theta}(n^*))}{\partial \theta}$ $\forall\, n^* \in [0,1]$. Thus, there exists at most one solution. Since $A \in [0,1]$ and $A_\theta > 0\ \forall n^*$, there exists a unique fixed point $\overline{\theta}(n^*)$ and it is in the interval $[0,1]$. Once the unique aggregate threshold is determined, the attack thresholds $\overline{x}_z(n^*)$ are backed out from the indifference conditions. Our object of interest is the probability of a crisis, which increases strictly in the aggregate threshold:

$$\Pr\{\theta \leq \overline{\theta}(n^*)\} = \Phi\left(\sqrt{\alpha}[\overline{\theta}(n^*) - \mu]\right). \tag{26}$$

For a given proportion of informed agents, the outcome of the coordination stage is characterized by fundamental and strategic forces. That is, a crisis is less likely if the average fundamental is stronger or the relative benefit of attacking is lower. First, a *fundamental effect* states that the aggregate threshold decreases in the mean of the common prior, $\frac{\partial \overline{\theta}(n^*)}{\partial \mu} = -\frac{A_\theta(\overline{\theta})}{1-A_\theta(\overline{\theta})} < 0$. A higher mean requires a higher aggregate attack size to abandon the regime, which induces an individual agent not to attack the status quo. Second, a *strategic effect* states that the aggregate threshold decreases in the conservativeness ratio ($\frac{\partial \overline{\theta}(n^*)}{\partial \kappa} < 0$). If the relative benefit of attacking is low, few other agents are expected to attack. This leads to a low expected aggregate attack size, so an individual agent tends not to attack the status quo. In summary, the strategic effect suggests that the probability of a crisis is small when the conservativeness ratio is high.

Lemma 1 summarizes the responsiveness of the aggregate threshold at the coordination stage to changes in the proportion of informed agents. In particular, it states conditions under which the probability of a crisis increases or decreases in a larger proportion of informed agents.

Lemma 1. *Let the private information of uninformed agents be sufficiently precise,* $\gamma_U > \underline{\gamma}'_U$, *so* $\overline{\theta}(n^*)$ *is unique for any* $n^* \in [0,1]$. *If Condition 1 holds, then the aggregate threshold at* *the coordination stage responds to changes in the proportion of informed agents:*

$$\frac{\partial \overline{\theta}}{\partial n^*} \neq 0. \tag{27}$$

Furthermore, the aggregate threshold increases (decreases) in the proportion of informed agents if the mean of the common prior is above (below) $\hat{\mu}(\kappa)$ defined by Condition 1:

$$\frac{\partial \overline{\theta}}{\partial n^*}\left(\mu - \hat{\mu}(\kappa)\right) > 0. \tag{28}$$

Proof. *The proof is in three steps. First, differentiating the aggregate threshold with respect to the proportion of informed agents yields:*

$$\frac{\partial \overline{\theta}}{\partial n^*} = \frac{\Phi\left(\frac{\alpha}{\sqrt{\gamma_I}}[\overline{\theta}-\mu] - \sqrt{1+\frac{\alpha}{\gamma_I}}\Phi^{-1}(\kappa)\right) - \Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\overline{\theta}-\mu] - \sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)}{1-A_\theta(\overline{\theta})} \tag{29}$$

$$\rightarrow \frac{1-\kappa-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\overline{\theta}-\mu] - \sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)}{1-A_\theta(\overline{\theta})}. \tag{30}$$

Second, note that $0 < A_\theta(\overline{\theta}) < 1$, where the second inequality follows from the sufficient condition for uniqueness at the coordination stage, $\gamma_U > \underline{\gamma}'_U$. Third, Condition 1 ensures that the numerator of the partial derivative is non-zero, which can be seen by contradiction.

Suppose that the numerator is zero. Since $0 = \Phi^{-1}(\kappa) + \Phi^{-1}(1-\kappa)$, this yields $\hat{\theta} \equiv \mu + \frac{\sqrt{\alpha+\gamma_U}-\sqrt{\gamma_U}}{\alpha}\Phi^{-1}(\kappa)$. Inserting $\hat{\theta}$ in the defining equation of the aggregate threshold, equation (25), yields $\mu = \hat{\mu}(\kappa)$. In summary, the partial derivative $\frac{\partial \overline{\theta}}{\partial n^*}$ is non-zero if Condition 1 holds and the private information of uninformed agents is sufficiently precise.

Using the same argument, the numerator of the partial derivative is positive if and only if the mean of the common prior is high relative to the conservativeness ratio: $\mu > \hat{\mu}(\kappa)$. Likewise, the numerator is negative if $\mu < \hat{\mu}(\kappa)$.

C.2 Stage 1: Information acquisition stage

Equipped with the results from the coordination stage, we next evaluate the incentive to acquire information. This is achieved by comparing the expected utility of an informed agent EU^I with that of an uninformed agent EU^U. Each expression of the expected utility has two terms. An agent receives the payoff b when attacking, $x_i < \overline{x}_z$, and the status quo is abandoned, $\theta < \overline{\theta}$. Likewise, an agent receives the payoff $-\ell$ when attacking and the status quo is maintained. This yields

$$\begin{aligned} EU^I &= b\int_{-\infty}^{\overline{\theta}} \int_{-\infty}^{\overline{x}_I} f^I(x|\theta) dx\, dG(\theta) - \ell \int_{\overline{\theta}}^{\infty} \int_{-\infty}^{\overline{x}_I} f^I(x|\theta) dx\, dG(\theta), \\ EU^U &= b\int_{-\infty}^{\overline{\theta}} \int_{-\infty}^{\overline{x}_U} f^U(x|\theta) dx\, dG(\theta) - \ell \int_{\overline{\theta}}^{\infty} \int_{-\infty}^{\overline{x}_U} f^U(x|\theta) dx\, dG(\theta), \end{aligned}$$

where $G(\theta)$ is the cumulative distribution function of the fundamental that is distributed as $\mathcal{N}(\mu, \alpha^{-1})$, while $f^z(x)$ is the probability distribution function of private signals conditional on a realized fundamental θ and on the information choice z, which is distributed as $\mathcal{N}(\theta, \gamma_z^{-1})$. The dependence of the three thresholds on the proportion of informed agents is suppressed for brevity.

Since informed agents receive a private signal that is perfectly informative in the limit, they never make errors in attacking the regime. That is, they always attack if the regime is abandoned and they never attack when the status quo is maintained. Thus, $EU^I(n^*) = b\int_{-\infty}^{\overline{\theta}} \Phi\left((\overline{x}_I - \theta)\sqrt{\gamma_I}\right) dG(\theta) - \ell \int_{\overline{\theta}}^{\infty} \Phi\left((\overline{x}_I - \theta)\sqrt{\gamma_I}\right) dG(\theta) \to bG(\overline{\theta}(n^*))$. By contrast, uninformed agents make mistakes of two types. First, an uninformed agent sometimes attacks the status quo when there is no regime change (type-I error, proportional to ℓ). Second, an uninformed agent sometimes does not attack the status quo when there is a regime change (type-II error, proportional to b). Thus $EU^U \to bG(\overline{\theta}) - \ell \int_{\overline{\theta}}^{\infty} \int_{-\infty}^{\overline{x}_U} f^U(x|\theta) dx\, dG(\theta) - b\int_{-\infty}^{\overline{\theta}} \int_{\overline{x}_U}^{\infty} f^U(x|\theta) dx\, dG(\theta)$.

The benefit of acquiring information is measured in terms of the expected utility difference $D(n^*) \equiv EU^I(n^*) - EU^U(n^*)$. This sums the errors of an uninformed agent, so the benefit of becoming informed is

$$D = \ell \int_{\overline{\theta}}^{\infty} \int_{-\infty}^{\overline{x}_U} f^U(x|\theta) dx \, dG(\theta) + b \int_{-\infty}^{\overline{\theta}} \int_{\overline{x}_U}^{\infty} f^U(x|\theta) dx \, dG(\theta). \tag{31}$$

Lemma 2. *If private information is sufficiently precise, $\gamma_U > \underline{\gamma}'_U$, then there is strategic complementarity in information choices:*

$$\frac{\partial D}{\partial n^*} \geq 0, \tag{32}$$

with strict inequality if Condition 1 holds.

Furthermore, a more-restrictive lower bound on the precision of private information, $\gamma_U > \left(\frac{\alpha}{\sqrt{2\pi}-2}\right)^2 = \underline{\gamma}_U$, ensures that

$$\frac{\partial D}{\partial n^*} < 1. \tag{33}$$

Proof. *First, $\gamma_U > \underline{\gamma}'_U$, so $\overline{\theta}(n^*)$ is unique for any $n^* \in [0,1]$. Total differentiation yields $\frac{\partial D}{\partial n^*} = \frac{\partial D}{\partial \overline{\theta}} \frac{\partial \overline{\theta}}{\partial n^*} + \frac{\partial D}{\partial \overline{x}_U} \frac{\partial \overline{x}_U}{\partial \overline{\theta}} \frac{\partial \overline{\theta}}{\partial n^*}$. As we prove below, $\frac{\partial D}{\partial \overline{x}_U} = 0$, so we obtain by using the Leibniz rule:*

$$\frac{\partial D}{\partial n^*} = (b + l) \left[1 - A_\theta(\overline{\theta})\right] g(\overline{\theta}) \left(\frac{\partial \overline{\theta}}{\partial n^*}\right)^2 \geq 0. \tag{34}$$

The partial derivative comprises four terms. The first and second terms are strictly positive because $b > 0$, $l > 0$, $A_\theta > 0$ and $A_\theta(\overline{\theta}) < 1$, as implied by the sufficient condition for uniqueness of $\overline{\theta}(n^*)$, $\gamma_U > \underline{\gamma}'_U$. The third term is the probability distribution function of the standard normal random variable and is always positive, $g > 0$. Finally, the fourth term is a square and thus non-negative. If the fourth term is strictly positive, as implied by Condition 1 because of Lemma 1, then the partial derivative is strictly positive.

Why is $\frac{\partial D}{\partial \overline{x}_U} = 0$? This indirect effect of the proportion of informed agents is always *zero by an envelope theorem argument. That is, the threshold $\overline{x}_U$ is chosen by a first-order condition that requires that the marginal cost of attacking the status quo when it is maintained balances with the marginal benefit of attacking when the status quo is abandoned:*

$$\frac{\partial D}{\partial \overline{x}_U} = -b \int_{-\infty}^{\overline{\theta}} f^U(\overline{x}_U|\theta) g(\theta) d\theta + \ell \int_{\overline{\theta}}^{\infty} f^U(\overline{x}_U|\theta) g(\theta) d\theta = 0. \tag{35}$$

Furthermore, using Lemma 1, the slope of the benefit of becoming informed as more agents acquire information can be written as

$$\frac{\partial D}{\partial n^*} = \frac{(b+\ell) g(\overline{\theta}) [1-\kappa-\Phi(\cdot)]^2}{1-A_\theta(\overline{\theta})}, \tag{36}$$

where $g \leq \frac{1}{\sqrt{2\pi}}$ and $b+\ell < 2$. In addition, $[1-\kappa-\Phi(\cdot)]^2 \leq \max\{\kappa^2, (1-\kappa)^2\} < 1$. Next, $A_\theta(\overline{\theta}) = (1-n^*)\phi(\cdot)\frac{\alpha}{\sqrt{\gamma_U}} \leq \frac{\alpha}{\sqrt{2\pi}\sqrt{\gamma_U}}$ since $n^* \geq 0$. Therefore, $\frac{\partial D}{\partial n^*} < 1$ is ensured by $\gamma_U > \underline{\gamma}_U \equiv \left(\frac{\alpha}{\sqrt{2\pi}-2}\right)^2 > \underline{\gamma}'_U$.

Since Condition 1 excludes a parameter space of measure zero, the benefit of becoming informed almost surely increases strictly in the proportion of informed agents. In order to understand the strategic complementarity in information acquisition, consider the two effects of becoming informed. First, informed agents forecast the fundamental θ more precisely, but this effect is independent of the proportion of informed agents. Second, informed agents forecast the behavior of other agents more precisely. In particular, an informed agent is better at forecasting other informed agents than other uninformed agents. Therefore, the second benefit of becoming informed increases in the proportion of informed agents, establishing the strategic complementarity in information acquisition. In short, information supports coordination.

Lemma 3 describes the boundaries of the benefit of becoming informed:

Lemma 3. *The benefit of becoming informed is strictly positive but smaller than one,*

$D(n^*) \in (0,1)$.

Proof. *First, we show $D > 0$ for all n^*. Since $\gamma_U < \infty$ and $\overline{\theta}(n^*) \in [0,1]$ for all n^*, there is positive probability mass on the type-I and type-II errors of an uninformed agent. Therefore, $D > 0$. Second, we show $D < 1$ for all n^*. Since $\int_{-\infty}^{\infty} f^U(x|\theta)dx = 1$ for all θ, $D < \ell + (b-\ell)G(\overline{\theta})$. Since $G(\overline{\theta}) \in [0,1]$, we have that $D \leq \max\{b, l\} < 1$. This completes the proof.*

Consider an agent's optimal information acquisition choice. Given the binary action $n_i \in \{I, U\}$ at stage 1, an agent optimally acquires information if and only if the individual information cost is at most the benefit of becoming informed:

$$n_i^* = I \Leftrightarrow c_i \leq D(n^*). \tag{37}$$

Since each agent is atomistic and has no effect on the aggregate proportion of informed agents, the benefit of becoming informed depends on the proportion of informed agents only. The perfect Bayesian equilibrium is constructed by combining these individual optimality conditions, Lemmas 2 and 3, and the consistency between individually optimal information acquisition choices and the aggregate proportion of informed agents.

The optimal information acquisition is characterized by a threshold strategy, where $\overline{c}$ denotes the cut-off value of the information cost below which an agent acquires information. Consequently, an agent acquires information if and only if $c_i < \overline{c}$, so the proportion of informed agents is $n^* = \overline{c}$. While this result follows from our parsimonious specification of the information cost, it generalizes for other continuous distributions, so n^* is an increasing function in $\overline{c}$. Next, the marginal agent is indifferent between the information choices, so the threshold information cost is any solution of

$$\overline{c} = D(\overline{c}). \tag{38}$$

Uniqueness requires that there is only one fixed point of $D(\overline{c})$. First, the left-hand side of equation (38) is continuous, within $[0, 1]$, and has a unit slope. Second, the right-hand side is continuous and strictly positive by Lemma 3, $D > 0$. Because $n^* = \overline{c}$, it has a slope that is strictly within $(0, 1)$ under the parameter conditions by Lemma 2. Therefore, if a solution exists, it is unique. Next, $D < 1$ by Lemma 3 ensures the existence of a unique interior solution $\overline{c}$.

D Proof of amplification effect

Here we derive the total effect of a change in the mean of the prior, μ, on the aggregate threshold $\overline{\theta}$ to prove the amplification result of Proposition 2. Specifically, we need to show that

$$-\frac{A_\mu + A_n D_\mu}{1 - A_\theta - A_n D_\theta} > -\frac{A_\mu}{1 - A_\theta}. \tag{39}$$

The equilibrium is given by a set of two equations, (25) and (38). Rewriting yields

$$\begin{aligned} 0 &= -\overline{\theta} + A(n, \overline{\theta}, \mu), \\ 0 &= -n + D(\overline{\theta}, \mu, \overline{x}_z), \end{aligned} \tag{40}$$

where the benefit of becoming informed D depends on the proportion of informed investors only indirectly. As for notation, if $w = m(x, y)$, then m_x denotes the partial derivative $\frac{\partial m}{\partial x}$. Therefore, total differentiation yields

$$\begin{aligned} 0 &= -\frac{d\overline{\theta}}{d\mu} + A_n \frac{dn}{d\mu} + A_{\overline{\theta}} \frac{d\overline{\theta}}{d\mu} + A_\mu, \\ 0 &= -\frac{dn}{d\mu} + D_{\overline{\theta}} \frac{d\overline{\theta}}{d\mu} + D_\mu, \end{aligned} \tag{41}$$

where we used $D_{\bar{x}_z} = 0$ by the envelope theorem, as shown below. Rewriting yields equation (12).

To evaluate this total derivative, observe that $A_\mu = -A_\theta = -(1-n^*)\phi_U(\cdot)\frac{\alpha}{\sqrt{\gamma_U}} < 0$ and $A_n = 1 - \kappa - \Phi_U = [1 - A_\theta]\frac{\partial \overline{\theta}}{\partial n^*}$, where all of these partial derivatives are evaluated at the equilibrium quantities $n^*, \overline{\theta}$.

Next, we determine the partial derivatives associated with the benefit of becoming informed. A change in the (mean of the) prior μ affects the benefit of becoming informed via: (i) the aggregate threshold $\overline{\theta}$, (ii) the attacking thresholds $\overline{x}_z$, and (iii) the distribution of fundamentals $g(\theta)$. We investigate each of these effects below.

Aggregate threshold Using the Leibniz rule, we have that $\frac{\partial D}{\partial \overline{\theta}} = g(\overline{\theta})[b-(b+l)F^U(\overline{x}_U|\overline{\theta})]$. Inserting $\overline{x}_U$ from equation (22), we see that $F^U(\overline{x}_U|\overline{\theta})] = \Phi_U$ as defined earlier. Using $\frac{\partial \overline{\theta}}{\partial n^*}$, which is stated in the proof of Lemma 1, the partial effect via the aggregate threshold is

$$D_\theta = (b+l)\left[(1-A_\theta)\frac{\partial \overline{\theta}}{\partial n^*}\right] g(\overline{\theta}), \tag{42}$$

where all expressions are evaluated at the equilibrium levels and $0 < A_\theta < 1$, $\forall\, n^* \in [0,1]$ if $\gamma_U > \underline{\gamma}_U$, as shown in Appendix C.1.

Attacking thresholds As shown in Appendix C.2, specifically in equation (35), the effect on the attacking threshold of an uninformed agent is zero by an envelope theorem argument. This result generalizes to an informed agent as well. Therefore,

$$\frac{\partial D}{\partial \overline{x}_z} = 0,\ \ z \in \{I, U\}. \tag{43}$$

Distribution of fundamentals In the proof, we make use of the following lemma.

Lemma 4. Bromiley (2013). *The product of two normal probability density functions is a scaled normal probability density function. That is, if*

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_f} \exp\left(-\frac{(x-\mu_f)^2}{2\sigma_f^2}\right) \quad \text{and} \quad g(x) = \frac{1}{\sqrt{2\pi}\sigma_g} \exp\left(-\frac{(x-\mu_g)^2}{2\sigma_g^2}\right),$$

then

$$f(x)g(x) = \frac{S}{\sqrt{2\pi}\sigma_{fg}} \exp\left(-\frac{(x-\mu_{fg})^2}{2\sigma_{fg}^2}\right),$$

where

$$\sigma_{fg} = \sqrt{\frac{\sigma_f^2\sigma_g^2}{\sigma_f^2+\sigma_g^2}} \quad \text{and} \quad \mu_{fg} = \frac{\mu_f\sigma_g^2+\mu_g\sigma_f^2}{\sigma_f^2+\sigma_g^2},$$

and

$$S = \frac{1}{\sqrt{2\pi(\sigma_f^2+\sigma_g^2)}} \exp\left(-\frac{(\mu_f-\mu_g)^2}{2(\sigma_f^2+\sigma_g^2)}\right).$$

The partial effect via a change in the distribution is obtained as follows. First, we rewrite the benefit of becoming informed D:

$$D = bG(\overline{\theta}) + \ell\int_{\overline{\theta}}^{\infty} F^U(x|\theta)\, g(\theta)d\theta - b\int_{-\infty}^{\overline{\theta}} F^U(x|\theta)\, g(\theta)d\theta. \tag{44}$$

Next, partially differentiating this equation, using $\frac{\partial g(\theta)}{\partial \mu} = g(\theta)\alpha(\theta-\mu)$, we have

$$D_\mu = -bg(\overline{\theta}) + \ell\int_{\overline{\theta}}^{\infty} F^U(\overline{x}_U|\theta)\alpha(\theta-\mu)g(\theta)d\theta - b\int_{-\infty}^{\overline{\theta}} F^U(\overline{x}_U|\theta)\alpha(\theta-\mu)g(\theta)d\theta, \tag{45}$$

where the first term is due to the fact that $\frac{\partial g(\theta)}{\partial \theta} = (-1)g(\theta)\alpha(\theta-\mu)$.

Thus, we require $\int F^U(\overline{x}_U|\theta)\alpha(\theta-\mu)g(\theta)d\theta$, which is found by using integration by parts. We set $u(\theta) \equiv F^U(\overline{x}_U|\theta)$ and $v'(\theta) \equiv \alpha(\theta-\mu)g(\theta)$, so $u'(\theta) = -f^U(\overline{x}_U|\theta) = -\phi(\sqrt{\gamma_U}[\overline{x}_U-\theta])$ and $v(\theta) = -g(\theta) = -\phi(\sqrt{\alpha}[\theta-\mu])$. When partially integrating, we use the result that $f^U(\overline{x}_U|\theta)g(\theta) = S\,h(\theta)$, where $S \equiv \sqrt{\frac{\alpha\gamma_U}{\alpha+\gamma_U}}\phi\left(\sqrt{\frac{\alpha\gamma_U}{\alpha+\gamma_U}}[\mu-\overline{x}_U]\right)$ is a positive constant and $h(\theta)$ is the probability density function of a normal random variable:

$$h : \theta \sim \mathcal{N}\left(\frac{\alpha\mu+\gamma_U\overline{x}_U}{\alpha+\gamma_U}, \frac{1}{\alpha+\gamma_U}\right). \tag{46}$$

Therefore,

$$\begin{aligned}\ell\int_{\overline{\theta}}^{\infty} F^U(\overline{x}_U|\theta)\alpha(\theta-\mu)g(\theta)d\theta &= \ell g(\overline{\theta})\Phi((\overline{x}_U-\overline{\theta})\sqrt{\gamma_U}) + \ell\int_{\overline{\theta}}^{\infty} g(\theta)f^U(\overline{x}_U|\theta)d\theta \\ &= \ell g(\overline{\theta})\Phi((\overline{x}_U-\overline{\theta})\sqrt{\gamma_U}) + \ell S\ \Pr_h\{\theta > \overline{\theta}\}, \end{aligned} \tag{47}$$

where $\Pr_h$ denotes the probability induced by the probability density function $h(\cdot)$. Similarly,

$$-b\int_{-\infty}^{\overline{\theta}} F^U(\overline{x}_U|\theta)\alpha(\theta-\mu)g(\theta)d\theta = bg(\overline{\theta})\Phi((\overline{x}_U-\overline{\theta})\sqrt{\gamma_U}) - bS\ \Pr_h\{\theta < \overline{\theta}\}. \tag{48}$$

Note that since $\kappa = \frac{\ell}{b+\ell}$ and $\sqrt{\alpha+\gamma_U}\left[\theta - \frac{\alpha\mu+\gamma_U\overline{x}_U}{\alpha+\gamma_U}\right] = \Phi^{-1}(\kappa)$, thus $\ell S\ \mathrm{Prob}_h\{\theta > \overline{\theta}\} - bS\ \mathrm{Prob}_h\{\theta < \overline{\theta}\} = 0$.

Finally, plugging (47) and (48) into (45) yields

$$D_\mu = -(b+l)g(\overline{\theta})[1-A_\theta(\overline{\theta})]\frac{\partial\overline{\theta}}{\partial n^*}.$$

Putting these steps together, the overall partial derivatives associated with the benefit of becoming informed becomes $\frac{dD}{d\mu} = D_\mu + D_\theta\frac{d\theta}{d\mu} = D_\mu\left(1-\frac{d\theta}{d\mu}\right)$. Further, recall that $\mathrm{sign}\left\{\frac{d\theta}{d\mu}\right\} < 0$ (see (12)). This is because $\mathrm{sign}\{A_\mu\} = -\mathrm{sign}\{A_\theta\} = \mathrm{sign}\{A_n D_\mu\} < 0$. In addition,

$$\begin{aligned}
1 - A_\theta - A_n D_\theta &= 1 - A_\theta - (1 - A_\theta)\frac{\partial \bar{\theta}}{\partial n^*} D_\theta \\
&= (1 - A_\theta)\left(1 - D_\theta \frac{\partial \bar{\theta}}{\partial n^*}\right) \\
&= (1 - A_\theta)\left(1 - (b + \ell)\left(\frac{\partial \bar{\theta}}{\partial n^*}\right)^2 [1 - A_\theta] g(\bar{\theta})\right) \\
&= (1 - A_\theta)\left(1 - \frac{\partial D}{\partial n^*}\right) \\
&> 0,
\end{aligned}$$

where the last inequality follows by (33).

As a result, since $\frac{d\theta}{d\mu} < 0$, thus

$$\text{sign}\left\{\frac{dD}{d\mu}\right\} = \text{sign}\left\{D_\mu\right\}. \tag{49}$$

We can next turn to the amplification result. Rewriting equation (13), since $A_\theta = -A_\mu$ and $D_\theta = -D_\mu$, amplification obtains if $A_n D_\mu < 0$. This inequality is always true if Condition 1 holds, $\mu \neq \hat{\mu}(\kappa)$.

Finally, using (27), (32) and (49), we obtain

$$\text{sign}\left\{A_n D_\mu\right\} = \text{sign}\left\{\frac{\partial \bar{\theta}}{\partial n^*}\right\} \text{sign}\left\{D_n\right\} \text{sign}\left\{\frac{dD}{d\mu}\right\}, \tag{50}$$

under the parameter Condition 1, which completes the proof.