BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2014-23

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www.bank-banque-canada.ca

Bank of Canada Working Paper 2014-23

June 2014

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ISSN 1701-9397

Acknowledgements

This paper has been prepared by the author under the Lamfalussy Fellowship Program sponsored by the ECB. I wish to thank Kartik Anand, Christoph Bertsch, Margaret Bray, Jonathan Chiu, Co-Pierre Georg, Stefan Gissler, Philipp Hartmann, Peter Hoffmann, Benjamin Nelson, Dimitri Vayanos, and seminar participants at Copenhagen IRMC 2013, LSE, Nottingham CFCM 2013, and UPF for useful feedback.

Abstract

I study rollover risk in the wholesale funding market when intermediaries can hold liquidity ex ante and are subject to fire sales ex post. Precautionary liquidity restores multiple equilibria in a global rollover game. An intermediate liquidity level supports both the usual run equilibrium and an efficient equilibrium. I provide a uniqueness refinement to characterize the privately optimal liquidity choice. Because of fire sales, liquidity holdings are strategic substitutes. Intermediaries free ride on the liquidity of other intermediaries, causing excessive liquidation. A macroprudential authority internalizes the *systemic* nature of liquidity and restores constrained efficiency by imposing a macroprudential liquidity buffer.

JEL classification: G01, G11, G28
Bank classification: Financial institutions; Financial system regulation and policies

Résumé

Nous étudions le risque de refinancement sur le marché du financement de gros lorsque les intermédiaires ont la possibilité de détenir ex ante de la liquidité et d'être confrontés ex post à des liquidations. La liquidité conservée à titre préventif restaure des équilibres multiples dans un jeu mondial de refinancement. Le niveau intermédiaire de liquidité permet à la fois l'équilibre habituel en cas de retraits massifs et un équilibre efficace. Nous fournissons des conditions à la singularité pour caractériser le choix de la quantité optimale de liquidité privée. Les liquidations font des réserves de liquidité des actifs de remplacement stratégiques. Étant donné que les intermédiaires profitent des réserves des autres intermédiaires, on aboutit à des liquidations excessives. Une autorité macroprudentielle internalise la nature *systémique* de la liquidité et rétablit une efficacité contrainte en imposant une réserve macroprudentielle de liquidité.

Classification JEL : G01, G11, G28
Classification de la Banque : Institutions financières; Réglementation et politiques relatives au système financier

Non-technical summary

The stability of the financial system is a key concern to central bankers. A particular aspect that has received much attention recently is the wholesale funding of banks. Before the financial crisis, banks funded themselves increasingly with short-term debt. However, such funding dried up as economic news deteriorated. This dark side of wholesale funding in part inspired the Basel III regulation. Specifically, a Liquidity Coverage Ratio (LCR) and a Net Stable Funding Ratio (NSFR) require banks to maintain a minimum level of liquidity.

In this paper, I study rollover risk in wholesale funding markets. Investors may roll over funding to their intermediaries after receiving some noisy information about their solvency. My set-up has two noteworthy features. First, financial intermediaries are allowed to hold precautionary liquidity in order to prepare for the possible future drying up of funding. That is, intermediaries choose how much liquidity to hold initially, while the remainder is invested in illiquid projects with high expected returns. Second, a fire sale occurs when intermediaries liquidate their projects jointly.

This paper derives two results. The first is technical and states that the methods previously used to ensure a unique equilibrium require refinement. Uniqueness matters, since it puts subsequent policy implications on a strong theoretical footing. The second result is the inefficiency of private liquidity holdings. Since intermediaries free ride on the liquidity holdings of other intermediaries, there is excessive liquidation. By contrast, a macroprudential authority understands the *systemic* nature of liquidity, whereby one intermediary's holdings mitigate the risk of fire sales for other intermediaries. Imposing a macroprudential liquidity buffer restores efficiency.

Finally, I link this welfare result to the regulatory debate of Basel III. I argue why this set-up provides a theoretical foundation for the Liquidity Coverage Ratio. It follows a brief discussion about requiring intermediaries to hold additional liquidity versus creating a systemic liquidity fund that all intermediaries contribute to. While the proposed set-up also provides some support for the Net Stable Funding Ratio, I describe how the current set-up could be extended to address the trade-offs associated with the NSFR more directly.

1 Introduction

Wholesale funding markets have received a great deal of attention since the financial crisis of 2007–09. Financial institutions funded themselves with very short-term debt provided by uninsured investors. This funding dried up after adverse economic news, such as disappointing data on the U.S. housing market. Huang and Ratnovski (2011) point to the dark side of wholesale funding when investors are relatively uninformed. During the financial crisis, money market mutual funds also experienced large outflows from institutional investors (Schmidt et al. (2013)), and even secured short-term borrowing was a highly unstable source of funds (Martin et al. (2013)). These events in part inspired the recent proposals for the regulation of financial intermediaries, which include balance-sheet rules such as maintaining a Liquidity Coverage Ratio (LCR) and a Net Stable Funding Ratio (NSFR) (Basel Committee on Banking Supervision (2010a,b)).

To address these issues, I study rollover risk in the wholesale funding market, where uninsured investors can withdraw funding from intermediaries at an interim date.[1] Intermediaries choose their portfolio at the initial date, either holding safe and low-return assets (liquidity) or making long-term investment. While promising a higher expected return, investment is costly to liquidate at the interim date because of a lower and diminishing marginal product in alternative use (Shleifer and Vishny (1992), Kiyotaki and Moore (1997)). This cost of liquidation is exacerbated by fire sales that occur when many intermediaries liquidate jointly (Allen and Gale (1994), Gromb and Vayanos (2002)). The stochastic return on investment is determined by aggregate economic conditions such as business cycle movements, which determine default rates.[2]

Each investor receives a noisy private signal about the return on investment at the interim date, based on the global games approach in Morris and Shin (2003). Using the private information, each investor decides whether to roll over funding to the intermediary. As a result, a low (high) realization of the investment return implies that many (few) investors receive unfavorable private information. Hence, a small (large) proportion of investors rolls over their funding to the intermediary, which results in a large (small) amount of investment that the intermediary liqui-

[1] I use "refusing to roll over" and "withdrawing" interchangeably.

[2] In the case of financial investment, changing aggregate conditions can represent a shock to an asset class in which all intermediaries are invested, such as the effect of a drop in U.S. house prices on asset-backed securities.

dates. However, holding precautionary liquidity allows the intermediary to drive a wedge between the withdrawal and liquidation volumes.

The first result is that the presence of liquidity generates multiple equilibria in the global rollover game (Proposition 1). The usual condition of fairly precise private information (e.g., Morris and Shin (2000)) is not sufficient for uniqueness. On the one hand, an efficient equilibrium exists if an intermediary holds an intermediate or high amount of liquidity (Lemma 1). On the other hand, the typical run equilibrium also exists if an intermediary holds a low or intermediate amount of liquidity (Lemma 2). Hence, an intermediate choice of liquidity supports multiple equilibria in the rollover game between investors, even if the private information received is fairly precise. This result, which hinges on the presence of precautionary liquidity holdings, contributes to a recent literature on multiplicity in global coordination games. For example, Angeletos et al. (2006) examine how the endogenous public information from a policy intervention generates multiple equilibria, and Hellwig and Veldkamp (2009) analyze when the acquisition of information prior to coordination leads to multiple equilibria.[3]

To obtain intuition for the multiplicity result, consider the strategic incentives of investors to roll over funding (see section 2 and Remark 1). Despite the risk neutrality of all agents, the incomplete-information game with liquidity holdings yields intriguing strategic interactions, which differ across the cases with and without liquidation. First, intermediate liquidity holdings can support the efficient equilibrium in which no liquidation is forecast by the threshold investor, resulting in a low equilibrium threshold. Exactly at this point, the strategic incentives to roll over funding change from complementarity to substitutability. Second, intermediate liquidity holdings can also support the run equilibrium in which positive liquidation is forecast. Then, there is global strategic complementarity in rollover decisions that leads to coordination failure between investors, resulting in a high equilibrium threshold (e.g., Morris and Shin (2000)).[4]

[3] Angeletos and Werning (2006) show that the aggregation of dispersed private information into a publicly observed market price, similar to Grossman and Stiglitz (1980), re-establishes multiplicity in a global game. See also Hellwig et al. (2006) for a market-based model of currency attacks with multiple equilibria.

[4] Rochet and Vives (2004) analyze the impact of balance-sheet variables, including liquidity, on the run threshold in the context of delegated management, which gives rise to a unique Bayesian equilibrium. By contrast, the rollover game I study may yield multiple equilibria. However, once the uniqueness refinement is applied, I replicate the effect of liquidity on the run threshold stated in their second proposition. Another difference is the analysis of two intermediaries that allows me to study the private and social incentives to hold liquidity from a systemic perspective.

To analyze the private incentives to hold liquidity, I provide a uniqueness refinement (Corollary 2). Consider first the benchmark of a single intermediary. The choice of liquidity balances the opportunity cost of a higher expected return on investment with the saved cost of liquidation, which reduces the equilibrium threshold in the run equilibrium. The intermediary optimally holds scarce liquidity and implements the run equilibrium in the rollover game if the expected return on investment takes an intermediate value (Proposition 2). Additionally, the intermediary holds no liquidity at all if the expected return takes a sufficiently large value, where this lower bound on the expected return intuitively increases in the liquidation cost.

The portfolio choice of the single intermediary is constrained efficient (Corollary 3). A planner who takes the optimal rollover behavior of investors as given holds the same level of liquidity. As such, there is no role for a *microprudential* regulation of liquidity. Despite the constrained efficiency, the incomplete information about the return on investment may prevent the decentralized economy from achieving first-best.[5]

Consider now the main case of multiple intermediaries. The run equilibrium is again implemented if the expected return is sufficiently high, so the private choices of liquidity are strategic substitutes (Proposition 3). Therefore, each intermediary free rides on the liquidity holdings of other intermediaries. If one intermediary holds more liquidity, the liquidation cost of another intermediary is reduced as the effect of fire sales is less pronounced. Since holding liquidity is costly due to the forgone return on investment, the other intermediary optimally reduces its liquidity holding. As a result, excessive liquidation occurs that renders the private choice of liquidity as constrained inefficient from an ex-ante perspective.

This yields a role for a *macroprudential* regulation of liquidity (Proposition 4). A constrained planner internalizes the systemic nature of liquidity and is therefore interpreted as a macroprudential authority. Specifically, it takes into account that more liquidity held by one intermediary reduces the liquidation cost of other intermediaries in the case of ex-post fire sales. Therefore, the social choice of liquidity exceeds the private choice, so imposing a macroprudential liquidity buffer restores

[5]That is, if the expected return on investment is low, the intermediary holds abundant liquidity. This implements the efficient equilibrium in the rollover game and attains first-best. In contrast, if the expected return is high, the intermediary holds scarce liquidity and implements the run equilibrium. First-best is not attained since coordination failure between investors leads to inefficient liquidation ex post.

constrained efficiency. As a consequence, inefficient liquidation occurs for the smallest possible range consistent with incomplete information.

The most-related papers are the global rollover games of Morris and Shin (2000) and Eisenbach (2013). Building on the seminal works of Carlsson and van Damme (1993) and Diamond and Dybvig (1983), Morris and Shin (2000) solve for the unique equilibrium in a bank run game, using global games techniques.[6] By contrast, I analyze the ex-ante portfolio choice of intermediaries and show how precautionary liquidity can restore multiple equilibria under the sufficient condition proposed by Morris and Shin (2000).[7] Next, I extend the analysis to multiple intermediaries and fire sales to explore macroprudential regulation of liquidity. Eisenbach (2013) also analyzes an ex-post coordination game in which investors roll over short-term debt. He studies the ex-ante optimal maturity choice of funding to discipline a bank manager tempted by moral hazard and derives a two-sided inefficiency. In contrast, I study the optimal portfolio choice of intermediaries on the asset side.

Other consequences of fire sales have already been analyzed. Wagner (2011) studies the diversification-diversity trade-off in ex-ante portfolio choices. Since joint liquidation is costly ex post, investors have an incentive to hold diverse portfolios ex ante.[8] In contrast, I examine the consequences of fire sales for the ex-ante liquidity holdings of intermediaries subject to rollover risk. Uhlig (2010) studies endogenous liquidation costs in a model of a two-tiered banking sector, where a system-wide externality also generates strategic complementarities in withdrawal decisions between investors. Instead of a positive analysis of the previous financial crisis and a discussion of ex-post policy interventions, I take an ex-ante perspective by focusing on macroprudential regulation of liquidity. Studying ex-ante regulation has the advantage of precluding the issue of moral hazard arising from an ex-post policy intervention, as stressed by Farhi and Tirole (2012).

[6] Multiple equilibria occur in Diamond and Dybvig (1983) because of self-fulfilling beliefs. In such a set-up, Goldstein and Pauzner (2005) obtain a unique equilibrium by extending the global games techniques to the case of one-sided strategic complementarities. See section 5 for a comparison of the strategic incentives of investors to their set-up.

[7] Vives (2013) and Morris and Shin (2010) also analyze the role of liquidity in rollover coordination games. However, they do not study the ex-ante portfolio choice of intermediaries and abstract from cases that induce the efficient equilibrium in the rollover game.

[8] Wagner (2009) analyzes the effect of ex-post fire sales on the ex-ante diversification choice of banks. He finds ambiguous welfare implications of optimal diversification choices, since banks may be 'too correlated' (as in the standard case) or 'too diversified' under laissez-faire.

2 Model

I present a simple model of financial intermediation that extends the single-intermediary model of Morris and Shin (2000) in two ways. First, there are many intermediaries whose ex-post liquidation decisions impose fire sale externalities on each other. Second, intermediaries are allowed to hold precautionary liquidity. This set-up is suitable to revisit the issue of equilibrium uniqueness in a global rollover game and to analyze the ex-ante liquidity choices and their welfare properties.

Agents and preferences The economy extends over three dates, $t \in \{0, 1, 2\}$, and there is a single good for consumption and investment. A finite number of intermediaries $N \in \{1, 2\}$ raise funds from a continuum of risk-neutral uninsured investors $i \in [0, N]$.[9] Investors consume at the final date and receive a payoff $\pi_i = c_2$. They are akin to wholesale short-term debt holders who played a pivotal role in the recent financial crisis.

Investment technology and funding Intermediaries simultaneously choose their portfolios at the initial date. Intermediaries can hold a liquid asset y_n, such as central bank reserves and government bonds, which yield a unit safe return at the subsequent date. They can also invest by originating loans to the real economy at the initial date, which constitutes a constant returns-to-scale technology, that yield a risky payoff r at the final date. The portfolio choice of intermediaries is publicly observed at the interim date.

Investors are endowed with one unit of the good at the initial date. Apart from claims on the intermediary, investors can hold liquidity but direct investment is infeasible because of inferior skills in monitoring or loan collection. The intermediary is funded purely with debt.[10] Because of free entry, an intermediary maximizes the expected utility of its investors (see also Gale (2010)). Combined with the intermediary's access to investment, the participation constraint of investors is satisfied and each intermediary attracts one unit of funding at the initial date.[11]

[9]This specification ensures a constant average amount of funding per intermediary.

[10]Absent an agency conflict between the intermediary and its debt holders, there is no role for microprudential regulation of intermediaries (Corollary 3). Thus, equity is not required as an incentive device.

[11]Instead of providing liquidity insurance for risk-averse investors (Diamond and Dybvig (1983)), financial intermediation occurs in this paper because of an intermediary's superior monitoring or loan collection skills.

Information The following information structure is common knowledge. Investors share a common prior about the profitability of risky investment:

$$r \sim \mathcal{N}\left(\bar{r}, \alpha^{-1}\right), \tag{1}$$

where investment produces an expected return superior to liquidity, $\bar{r} > 1$, and $\alpha \in (0, \infty)$ measures the precision (inverse variance) of public information.[12] Following the seminal contribution of Carlsson and van Damme (1993), the return on the risky investment is realized at the interim date but not publicly observed. Instead, each investor receives a private signal at the interim date:

$$x_i = r + \epsilon_i, \;\; \epsilon_i \sim \mathcal{N}\left(0, \gamma^{-1}\right), \tag{2}$$

where the idiosyncratic noise ϵ_i is identically and independently distributed as well as independent of the investment return, and $\gamma \in (0, \infty)$ measures the precision of private information.

Liquidity-return trade-off Loans are costly to liquidate at the interim date. As in Morris and Shin (2000), the liquidation of an amount $l_n \in [0, 1]$ by intermediary n reduces the final-date return by $\chi_n(l_n, l_{-n})$, where l_{-n} is the amount of liquidation by the other intermediary. This specification allows for many intermediaries compared to the case of a single intermediary in Morris and Shin (2000). Costs occur only in the case of liquidation, so $\chi_n(0, a) = 0 \;\forall a \geq 0$. To ensure that liquidity is not dominated, I assume $\bar{r} < 1 + \chi(1, 1) < \infty$ throughout.

The liquidation cost is determined endogenously from a downward-sloping demand for liquidated investment, as in Eisenbach (2013). For example, assets are relocated to another, less-productive sector (Shleifer and Vishny (1992), Kiyotaki and Moore (1997)).[13] Since the marginal product of liquidated assets in alternative use is strictly positive, continuous, and diminishing, the cost function is symmetric, weakly convex, and increases in an intermediary's liquidation volume

[12] This prior may be induced by a public signal: $\bar{r} = r + \eta$, where the noise $\eta \sim \mathcal{N}\left(0, \alpha^{-1}\right)$ is independent of the return. Furthermore, the aggregate noise is independent of each of the idiosyncratic noise terms ϵ_i.

[13] In Shleifer and Vishny (1992), liquidation values are depressed after an industry-specific shock, since distress sales take place to unlevered industry outsiders who value industry-specific assets less.

conditional on a positive amount of liquidation, $l_n > 0$:

$$\frac{\partial \chi_n}{\partial l_n} > 0 \text{ if } l_n > 0. \tag{3}$$

Fire sales occur when intermediaries jointly liquidate investment:

$$\frac{\partial \chi_n}{\partial l_{-n}} > 0 \text{ if } l_n > 0. \tag{4}$$

Limited participation in asset markets can lead to cash-in-the-market pricing and therefore underpricing of assets (Allen and Gale (1994)). In the interpretation of financial investment, financial arbitrageurs cannot pick up assets in fire sales, since they are constrained by losses and outflows themselves (Gromb and Vayanos (2002)). Where loans are secured on real estate, for example, foreclosures generate negative spillovers for the owners of nearby property. Campbell et al. (2011) quantify this effect for housing in Massachusetts, finding that forced sales following bank foreclosures take place at discounts of up to 27 per cent.

A simple specification of the liquidation cost function that satisfies all constraints is

$$\chi_n(l_n, l_{-n}) = \chi \cdot [l_n + l_{-n}]\, 1\{l_n > 0\}, \tag{5}$$

for some constant $\chi > 0$. I use the general specification in the first part of the paper. When analyzing welfare, I restrict attention to the simple specification, which produces succinct results.

Upon receiving the private information x_i, investors may withdraw their funds at par at the interim date.[14] A proportion of investors $w_n \in [0,1]$ withdraws and is served with liquidity. In the case of high withdrawals, $w_n > y_n$, intermediaries also liquidate some investment $l_n \equiv w_n - y_n \in [0, 1 - y_n]$. In this case, more liquidity reduces the liquidation cost by driving a wedge between withdrawals and liquidation. Hence, an intermediary may hold precautionary liquidity ex ante to avoid costly liquidation in states with high withdrawals ex post.

[14] Similar to Dasgupta (2004), Goldstein (2005), and Shapiro and Skeie (2013), this ensures the viability of intermediaries at the interim date, since the promised payment does not exceed the liquidation value.

2.1 Payoffs and strategic incentives to roll over

What are the strategic incentives to roll over funding to an intermediary? To address this, consider for now the benchmark case of common knowledge about the investment return. The payoffs, and thus the strategic incentives to rolling over, depend critically on the liquidation volumes of each intermediary. I consider the cases of no liquidation and some liquidation in turn.

First, if many investors withdraw from intermediary n, all liquidity is exhausted and some liquidation occurs. Then the payoff to an investor who rolls over to this intermediary simplifies to

$$c_{2n} = r - \chi_n(l_n, l_{-n}). \tag{6}$$

Second, if withdrawals are small relative to the intermediary's liquidity holdings, then no liquidation occurs and the payoff to an investor who rolls over is

$$c_{2n} = \frac{y_n - w_n + (1 - y_n)r}{1 - w_n}, \tag{7}$$

where investors that roll over share the proceeds from some excess liquidity and investment.

An investor's incentive to roll over is affected by more withdrawals from the same intermediary. There is *strategic complementarity* in rollover decisions if liquidation occurs, $l_n > 0$. The more investors roll over, the smaller the liquidation cost, the higher the relative payoff from rolling over. However, if no liquidation occurs, the strategic incentives depend on the realized investment return (if the intermediary makes some investment, $y_n < 1$). If the investment return is low, $r < 1$, then there is again strategic complementarity in rolling over decisions.

By contrast, if the investment return is high, $r > 1$, and no liquidation occurs, then the payoff exhibits *strategic substitutability* in rolling over decisions. Then, the more investors withdraw from intermediary n, the larger is a given investor's incentive to roll over. Strategic substitutability in rolling over decisions is a 'last-man-standing effect' (see also Perotti and Suarez (2011)). In the absence of liquidation, withdrawing investors receive less than their equal share of an intermediary's total resources since the investment return is high. Thus, investors who roll over receive more than an equal share and consequently have a greater incentive to roll over.

Zero liquidation and the last-man-standing effect can only occur with precautionary liquidity holdings. This holds in the complete-information case with a positive amount of withdrawals. Furthermore, in the case of incomplete information, there are always some investors who receive a bad signal and withdraw at the interim date. Therefore, precautionary liquidity holdings, which are absent in Morris and Shin (2000), are necessary to prevent some liquidation from taking place.

How is an investor's incentive to roll over affected by more withdrawals from another intermediary? First, there is no effect if these withdrawals are small, so the liquidity of the other intermediary $-n$ suffices to serve withdrawals. There is also no effect if the withdrawals from intermediary n are small because no liquidation occurs at this intermediary. Second, if withdrawals from both intermediaries are large and joint liquidation (fire sales) occurs, there is strategic complementarity in rollover decisions between investors across intermediaries. The relative payoff from rolling over funding increases in the proportion of investors who roll over to another intermediary.

The strategic incentives of investors to roll over at the interim date are rich and nest, for example, the set-up of Goldstein (2005) that features the interaction of strategic complementarities. In a model of twin crises, the incentive of investors to withdraw from a bank as well as the incentive of speculators to attack a currency increase in the proportions of attacking speculators and withdrawing investors. Such a 'triple-decker' of strategic complementarity in rollover decisions also arises here if both intermediaries liquidate some investment at the interim date. Then, each investor's incentive to withdraw from the intermediary increases in the proportion of investors who withdraw from either intermediary. However, other forms of strategic interaction are also possible. In particular, strategic substitutability in rollover decisions to the same intermediary can arise if no liquidation occurs.

Remark 1. *The ex-ante portfolio choices of intermediaries determine the ex-post liquidation volume and therefore shape the strategic incentives of investors to roll over funding.*

The strategic interaction between investors at the interim date is driven by the portfolio choice of intermediaries at the initial date. This link arises because an intermediary's liquidity holdings affect the liquidation amount for some withdrawal volumes. The more liquidity held by the intermediary, the larger the wedge between withdrawals and liquidation for any given adverse realization of the investment return. However, liquidity has no marginal benefit for low amounts of withdrawals, since no liquidation takes place. In the incomplete-information model studied here, the amount of withdrawals is tied to the realized investment return. Hence, a low realized return leads to adverse signals for many investors and thus many withdrawals. Liquidation occurs and liquidity has a positive marginal benefit. Likewise, a high realized return leads to high signals for many investors and thus few withdrawals. There is no liquidation and no marginal benefit of liquidity.

Solving for the equilibrium Working backwards, I start by analyzing Bayesian equilibria in the incomplete-information rollover game at the interim date, which is a proper subgame. An investor's strategy is a plan of action for each private signal x_i. For any portfolio choice (y_1, y_2), a profile of strategies is a Bayesian equilibrium in the subgame if the actions of each investor's strategy maximize the expected utility conditional on the private information x_i, taking as given the strategies followed by all other investors. I focus on threshold strategies, whereby an investor rolls over if and only if the private information is sufficiently good relative to an intermediary-specific threshold that depends on the portfolio choices of intermediaries:

$$x_i \geq x_n^*(y_n, y_{-n}). \tag{8}$$

Turning to the initial date, the liquidity holdings (y_1^*, y_2^*) constitute a Nash equilibrium in the complete-information portfolio choice game if each intermediary's liquidity choice y_n^* maximizes its objective function subject to the withdrawal threshold $x_n^*(y_n, y_{-n})$, taking as given the level of liquidity held by the other intermediary y_{-n}^*. The following timeline summarizes the model.

Initial date ($t = 0$)	**Interim date** ($t = 1$)	**Final date** ($t = 2$)
1. Endowed investors fund intermediaries.	1. Private information x_i about the return on investment.	1. Investment matures and return is publicly observed.
2. Intermediaries simultaneously choose investment, $1 - y_n$.	2. Investors simultaneously decide whether to roll over.	2. Remaining investors withdraw.
	3. Intermediaries may liquidate some investment, l_n.	3. Consumption.

Table 1: Timeline of the model.

3 Equilibrium

Analyzing the case of a single intermediary first ($N = 1$), I show that introducing liquidity can restore multiple equilibria despite the standard global game refinement of slightly noisy but precise private information. In order to analyze the optimal ex-ante portfolio choice, I provide a stronger condition sufficient for overall uniqueness. The privately optimal liquidity choice is characterized and shown to be constrained efficient, so there is no role for *microprudential* regulation of liquidity. This contrasts with the case of multiple intermediaries studied in section 4, where the private choices of intermediaries are constrained inefficient, so there is a role for *macroprudential* regulation.

3.1 Rollover subgame

Consider the equilibrium withdrawal behavior of investors at the interim date. Each investor uses the private information x_i to form a posterior about the return on investment, $R_i \equiv \mathrm{E}[r|x_i]$, and the proportion of withdrawing investors, $W_i \equiv w|x_i$, both of which are derived in Appendix A.1. By definition, a 'threshold investor' is indifferent between withdrawing and rolling over upon receiving the threshold signal $x_i = x^*$, which is defined by

$$\mathrm{E}[\pi_i|x_i = x^*] \equiv 1, \tag{9}$$

where the left-hand side is the expected payoff from rolling over conditional on receiving the threshold signal x^* and the right-hand side is the payoff from withdrawing. Because of the one-to-one mapping between the posterior mean R_i and the private signal x_i (see Appendix A.1), equation (9) defines an equilibrium threshold R^*, which is more convenient to use than the threshold signal x^*.

Since liquidation affects the payoff from rolling over, the cases of whether or not the threshold investor forecasts liquidation to occur are considered in turn below.

Zero liquidation forecast by threshold investor If the threshold investor forecasts zero liquidation, $W^* \leq y$, then the indifference condition yields

$$(1-y)(R^* - 1) = 0. \tag{10}$$

If there is maturity transformation, $y < 1$, then the equilibrium threshold is $R^* \equiv 1$.[15] For the consistency of this equilibrium threshold with the supposed zero liquidation forecast by the threshold investor, liquidity must be abundant. This lower bound on liquidity is derived by using the posterior about the proportion of investors who withdraw from the intermediary derived in Appendix A.1:

$$y \geq \underline{y} \equiv \Phi\left(\sqrt{\delta}[1-\bar{r}]\right) < \frac{1}{2}, \tag{11}$$

where $\delta \equiv \frac{\alpha^2(\alpha+\gamma)}{\gamma(\alpha+2\gamma)}$ collects precision parameters. Lemma 1 summarizes the case of zero liquidation forecast by the threshold investor.

Lemma 1. ***Efficient equilibrium.*** *In the single-intermediary case, $N = 1$, with abundant liquidity, $y \in [\underline{y}, 1)$, there exists a threshold equilibrium in which the threshold investor forecasts zero liquidation, $W^* \leq y$. For any finite precision of private information, $\gamma \in (0, \infty)$, this equilibrium prescribes an investor to roll over if and only if $R_i \geq R^* = 1$, generating efficient runs on the intermediary ($R^{FB} \equiv 1$).*

The strategic incentives of investors to roll over determine the ex-post efficiency of equilibrium. In the case of abundant liquidity, no liquidation is forecast by the threshold investor. As described in detail in section 2, strategic complementarity in rolling over arises for an investment return below unity, while strategic substitutability arises for a return above unity. Therefore, the unique equilibrium threshold must be unity. The privately optimal rollover rule, $R^* = 1$, coincides with the first-best rollover rule, $R^{FB} \equiv 1$, so no coordination failure occurs among investors. Whenever the realized return on investment is below the unit return on liquidity, it is efficient not to roll over to the intermediary to terminate investment.

[15] If there is no maturity transformation, $y = 1$ or 'narrow banking,' then the rollover decision of investors is irrelevant, since the asset value of the intermediary is unity irrespective of the return on investment. I set $R^* \to -\infty$ in this case without loss of generality.

Threshold investor forecasts liquidation If the threshold investor forecasts some liquidation, $W^* > y$, then the indifference condition yields

$$R^{**} = 1 + \chi\left(\Phi\left(\sqrt{\delta}[R^{**} - \bar{r}]\right) - y\right), \tag{12}$$

which defines R^{**} implicitly. As in Morris and Shin (2003), uniqueness of the equilibrium threshold R^{**} requires a sufficiently precise private signal $\gamma > \underline{\gamma} < \infty$.[16] For the consistency of this equilibrium threshold with the supposed positive liquidation forecast by the threshold investor, liquidity must be scarce:

$$y < \overline{y} \equiv \Phi\left(\sqrt{\delta}[R^{**} - \bar{r}]\right). \tag{13}$$

Consequently, the ranking of equilibrium thresholds is $R^{**} > 1 = R^*$, which also determines the ranking of bounds on liquidity, $\overline{y} > \underline{y}$. Lemma 2 summarizes the case of positive liquidation forecast by the threshold investor.

Lemma 2. ***Run equilibrium.*** *In the single-intermediary case, $N = 1$, with scarce liquidity, $y \in [0, \overline{y})$, there exists a threshold equilibrium in which the threshold investor forecasts some liquidation, $W^* > y$. If private information is sufficiently precise, $\gamma \in \left(\underline{\gamma}, \infty\right)$, then this equilibrium prescribes an investor to roll over if and only if $R_i \geq R^{**} > 1$, as defined by equation (12). There are inefficient runs on the intermediary, $R^{**} > R^{FB}$.*

The strategic incentives of investors to roll over again determine the efficiency of equilibrium. In the case of scarce liquidity, however, liquidation is forecast by the threshold investor. As described in detail in section 2, strategic complementarity in rollover decisions of investors arises globally. Hence, there is coordination failure between investors that pushes the equilibrium threshold above the efficient level, $R^{**} > 1$. Fearing that other investors refrain from rolling over funding to the intermediary, and thereby cause costly liquidation, each investor has an incentive not to roll over to the intermediary for a larger range of investment returns.

[16] More precisely, since the right-hand side of equation (12) is bounded, the uniqueness of the threshold is guaranteed if the slope of the left-hand side exceeds the slope of the right-hand side: $1 > \sqrt{\delta}\chi'(l)\phi\left(\sqrt{\delta}[R^* - \bar{r}]\right) \equiv D^*$, where $\phi(\cdot)$ is the probability distribution function of the standard normal distribution. Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$ and $\chi'(l) > 0$, the finite precision of private information must be sufficiently high. If the liquidation cost function takes the simple linear form, the lower bound on the private precision $\underline{\gamma} \in (0, \infty)$ is defined as the largest γ that solves $\delta(\gamma) = \frac{2\pi}{\chi^2}$.

Optimal rollover decisions Lemmas 1 and 2 allow one to characterize the optimal behavior of investors in the rollover subgame at the interim date. Proposition 1 follows directly and describes how the number of threshold equilibria depends on the intermediary's liquidity choice at the initial date.

Proposition 1. ***Multiple Bayesian equilibria in rollover subgame.*** *Consider the rollover subgame at the interim date with a single intermediary and a sufficiently high but finite precision of private information, $\gamma \in \left(\underline{\gamma}, \infty\right)$. The number of Bayesian equilibria in threshold strategies is determined by the intermediary's liquidity choice at the initial date:*

- *If the intermediary holds abundant liquidity, $y \geq \overline{y}$, then the efficient equilibrium is the unique Bayesian equilibrium in the rollover subgame.*
- *If the intermediary holds scarce liquidity, $y < \underline{y}$, then the run equilibrium is the unique Bayesian equilibrium in the rollover subgame.*
- *However, if the intermediary holds an intermediate amount of liquidity, $y \in [\underline{y}, \overline{y})$, both the efficient equilibrium and the run equilibrium exist.*

The standard condition in global games does not guarantee uniqueness. That is, high but finite precision of private information is insufficient for uniqueness in the current rollover subgame once intermediaries are allowed to hold liquidity ex ante. Apart from the usual run equilibrium when the intermediary holds scarce liquidity, there exists also an efficient equilibrium. For such an equilibrium to be present, the threshold investor must forecast zero liquidation to occur, which requires the intermediary to hold abundant liquidity. This explains the existence of different equilibria in the subgame, depending on the liquidity choice of an intermediary (see also Rochet and Vives (2004)).

Why can intermediate liquidity holdings support multiple equilibria? To obtain intuition for the multiplicity result, recall the strategic incentives to roll over funding described in section 2. An important insight is that the strategic incentives of investors to roll over differ across the cases with and without liquidation (see also Remark 1 and the preceding discussion of strategic incentives). First, intermediate liquidity holdings can support an equilibrium with no liquidation forecast by the threshold investor. The strategic incentives described in detail below Lemma 1 ensure a low equilibrium threshold. Exactly at this point, the strategic incentives to roll over funding change from complementarity to substitutability. Second, intermediate liquidity holdings can also support an equilibrium with positive liquidation forecast. There are global strategic complementarities in rollover decisions of investors that lead to coordination failure and a high equilibrium threshold.

Proposition 1 nests the unique Bayesian equilibrium in Morris and Shin (2000), which corresponds to the run equilibrium in the rollover subgame, as a special case. In the absence of liquidity, $y = 0 < \underline{y}$, there is always positive liquidation forecast by the threshold investor. There are always some investors who receive adverse signals and do not roll over, even if the realized return on investment is high. Thus, the efficient equilibrium cannot be obtained, resulting in the run equilibrium as the unique equilibrium in the subgame.

Actual withdrawals A consequence of Proposition 1 is that some intermediate realizations of the investment return are consistent with both equilibria. To develop this point, I determine the amount of actual liquidation in each equilibrium, which depends on the realized return on investment.

No liquidation occurs if the liquidity holdings suffice to serve actual withdrawals, $w|r \leq y$, which requires a sufficiently high realized return on investment for a given ex-ante liquidity choice. The private signal conditional on the return is distributed as $x_i|r \sim \mathcal{N}\left(r, \gamma^{-1}\right)$, so the proportion of investors who withdraw is $\Phi(\sqrt{\gamma}[\bar{x} - r])$ for any signal threshold $\bar{x}$. Hence, the lower bound on the realized return on investment is, for any signal threshold $\bar{x}$:

$$r \geq \bar{x} + \frac{\Phi^{-1}(y)}{\sqrt{\gamma}}, \tag{14}$$

where $\Phi^{-1}(\cdot)$ is the inverse of the cumulative probability function of the standard normal.

First, consider the efficient equilibrium. From the posterior distribution of the return on investment (Appendix A.1), we can express the signal threshold as $x^* \equiv 1 - \frac{\alpha}{\gamma}(\bar{r} - 1) < 1$. Therefore, zero actual liquidation occurs if the realized investment return is sufficiently high:

$$r \geq r_L \equiv 1 - \frac{\alpha}{\gamma}(\bar{r} - 1) - \frac{\Phi^{-1}(y)}{\sqrt{\gamma}}.$$

Intuitively, more liquidity allows the intermediary to serve more withdrawals without liquidating investment. More withdrawals occur for lower realized investment returns, so the lower bound on the investment return decreases in the liquidity holding, $\frac{\partial r_L}{\partial y} < 0$.

Second, consider the run equilibrium. The lower bound on the investment return is determined analogously and now depends on the implicitly defined equilibrium threshold R^{**}. Zero actual liquidation occurs if the realized return on investment is sufficiently high:

$$r \geq r_H \equiv R^{**} - \frac{\alpha}{\gamma}(\bar{r} - R^{**}) - \frac{\Phi^{-1}(y)}{\sqrt{\gamma}}.$$

For the same reasons, the lower bound on the investment return decreases in the liquidity holding, $\frac{\partial r_H}{\partial y} < 0$. Since investors roll over less frequently in the run equilibrium than in the efficient equilibrium, $R^{**} > 1$, a higher realized return on investment is required to ensure zero actual liquidation in the run equilibrium, $r_H > r_L$.

Corollary 1 expresses the multiplicity result of Proposition 1 in terms of actual liquidation at the interim date. In sum, the liquidity holding of intermediaries determines whether the threshold investor forecasts liquidation to occur, while the realized return on investment determines, for a given level of liquidity, whether liquidation actually occurs.

Corollary 1. *If the intermediary holds an intermediate level of liquidity, $y \in [\underline{y}, \overline{y})$, then any realized return on investment $r \in [r_L, r_H)$ is consistent with actual liquidation (in the run equilibrium) and no liquidation (in the efficient equilibrium).*

3.2 Uniqueness refinement

For an analysis of the private and social incentives to hold liquidity at the initial date to be meaningful, the equilibrium in the rollover subgame at the interim date must be unique. In this section, I provide a simple condition sufficient for uniqueness in the rollover subgame.

Uniqueness requires the range of intermediate liquidity holdings $[\underline{y}, \overline{y})$ to vanish. These bounds are defined in equations and (11) and (13). Since the threshold in the run equilibrium is always higher, $R^{**} > 1$, $\delta \to 0$ is required to ensure that $\underline{y} - \overline{y} \to 0$. This means that the precision of public information relative to private information must vanish, $\frac{\alpha}{\gamma} \to 0$, which is ensured by vanishing private noise, $\gamma \to \infty$. As a consequence, the bounds on the liquidity holding that define scarcity and abundance converge, while the lower bounds on the realized return on investment required for no actual liquidation converge to the equilibrium thresholds:

$$\begin{aligned} \underline{y} \to \frac{1}{2} & \leftarrow & \overline{y} \\ r_L \to R^* & = & 1 \\ r_H \to R^{**} & \to & 1 + \chi\left(\frac{1}{2} - y\right) > 1. \end{aligned} \tag{15}$$

These results are summarized in Corollary 2 and shown in Figure 1.

Corollary 2. ***Uniqueness refinement for the rollover subgame.*** *Consider the rollover subgame at the interim date with a single intermediary. If fundamental uncertainty vanishes, $\gamma \to \infty$, then there exists a unique Bayesian equilibrium in threshold strategies:*

- *If the intermediary holds abundant liquidity, $y \geq \frac{1}{2}$, then the efficient equilibrium occurs. Actual liquidation occurs if and only if the realized return on investment is low, $r \leq r_L$.*
- *If the intermediary holds scarce liquidity, $y < \frac{1}{2}$, then the run equilibrium occurs. Actual liquidation occurs if and only if the realized return on investment is low, $r \leq r_H$.*

The refinement establishes a unique link between an intermediary's liquidity choice and the withdrawal threshold of investors in the rollover subgame. Rochet and Vives (2004) obtain uniqueness away from the limit in the case of delegated management. While I replicate their result under the uniqueness refinement, I also analyze the optimal liquidity choice, as reported below.

3.3 Liquidity choice

Using the uniqueness refinement, I analyze the intermediary's optimal liquidity choice at the initial date and derive consequences for individual fragility. See section 4 for multiple intermediaries and systemic fragility.

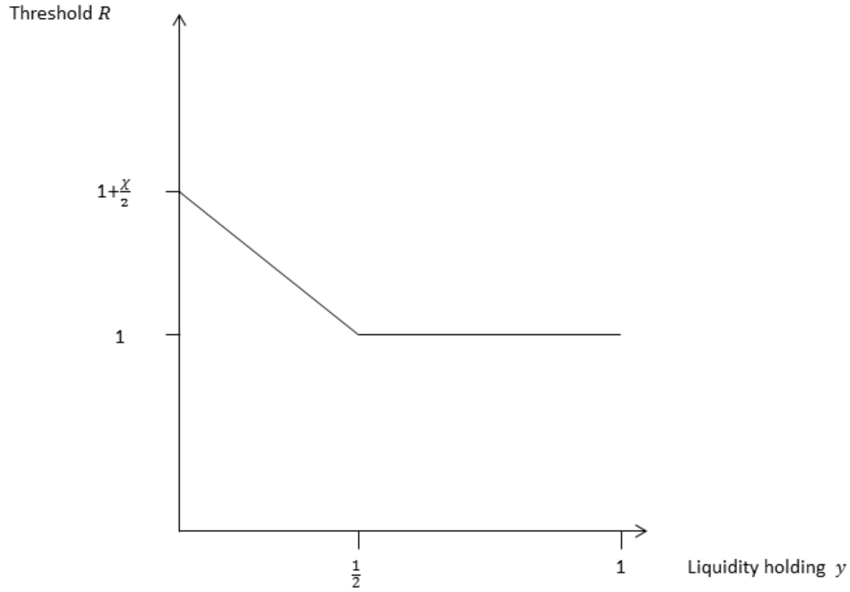

Figure 1: The rollover threshold and an intermediary's liquidity for vanishing private noise.

The liquidity choice has both marginal and discrete impact. The marginal impact is present only in the run equilibrium, where more liquidity reduces the equilibrium threshold R^{**}. The discrete impact is the selection of the equilibrium in the rollover subgame as summarized by Corollary 2. Therefore, the optimal liquidity levels in cases of abundant and scarce liquidity are derived in turn and the objective function is compared globally next. As derived in Appendix A.2, the objective function of the intermediary is the expected utility of an investor:

$$EU(y) = y + (1-y)\left[F(R) + (1-F(R))\mathrm{E}[r|r > R]\right], \tag{16}$$

where R is any threshold, and $f(r) = \phi(\sqrt{\alpha}[r - \bar{r}])$ is the probability distribution function of the investment return with the associated cumulative distribution function $F(r)$.

The expected utility comprises the unit payoff from liquidity and the proceeds from investment. Investment is liquidated if its realized return falls short of a threshold R, which occurs with probability $F(R)$. Otherwise, investment is continued, where the expected investment return conditional on continuation is $\mathrm{E}[r|r > R] = \bar{r} + \frac{f(R)}{\sqrt{\alpha}(1-F(R))}$.

What are the effects of changes in liquidity holdings or the equilibrium threshold on the expected utility? As derived in Appendix A.2, the marginal cost of liquidity from an ex-ante perspective is the forgone net return on investment, which is always positive:

$$\frac{\partial EU}{\partial y} = -(1-F(R))\left(\mathrm{E}[r|r \geq R] - 1\right) < 0. \tag{17}$$

The marginal cost of liquidity is exacerbated by liquidating investment that shields investors from low returns. First, the expected return on investment conditional on liquidation exceeds the unit return of liquidity, where no liquidation occurs with probability $1 - F(R)$. Second, the liquidation value of investment equals the unit return of liquidity, so the case of liquidation, which occurs with probability $F(R)$, drops out. Next, a lower equilibrium threshold increases expected utility, since it results in a smaller area of inefficient runs on the intermediary $[1, R]$:

$$\frac{\partial EU}{\partial R} = -(1-y)(R-1)\sqrt{\alpha}f(R) \leq 0. \tag{18}$$

Abundant liquidity, $y \geq \frac{1}{2}$, ensures the efficient equilibrium in the rollover subgame. The equilibrium threshold $R^* = 1$ is independent of liquidity, so there is no marginal benefit of liquidity in this subgame. The marginal cost of liquidity is positive, so the optimal amount of liquidity to implement the efficient equilibrium is the lower bound $y^* = \frac{1}{2}$, which yields the expected utility:

$$EU(y^*) = 1 + \frac{1}{2}\left[(\bar{r} - 1)(1 - F[1]) + \frac{f[1]}{\sqrt{\alpha}}\right] > 1 = EU(y = 1), \tag{19}$$

so the intermediary prefers to implement the efficient equilibrium over narrow banking, $y = 1$.

Second, consider the case of scarce liquidity $y < \frac{1}{2}$, which implies the run equilibrium R^{**}. In contrast to the efficient equilibrium, the marginal benefit of liquidity is strictly positive:

$$\frac{\partial R^{**}}{\partial y} = -\chi'\left(\frac{1}{2} - y\right) < 0. \tag{20}$$

Holding more liquidity, within the feasible bounds of $y < \frac{1}{2}$, allows the intermediary to serve a larger proportion of withdrawing investors without liquidating investment. As liquidation is costly, this reduces the amount of coordination failure between investors. This lowers the equilibrium threshold and therefore increases the expected utility indirectly:

$$\frac{\partial EU}{\partial R^{**}}\frac{\partial R^{**}}{\partial y} > 0.$$

The intermediary's optimal amount of liquidity to implement the run equilibrium solves

$$y^{**} \equiv \arg\max EU(y) \text{ s.t. } R^{**} = R^{**}(y).$$

If an interior solution $y^{**} \in (0, \frac{1}{2})$ exists, then the first-order condition balances the benefits of liquidity in terms of reducing the equilibrium threshold with the cost of liquidity in terms of forgone investment return:

$$\overbrace{-\frac{\partial EU}{\partial y}}^{MC} = \overbrace{\frac{\partial EU}{\partial R^{**}}\frac{\partial R^{**}}{\partial y}}^{MB}. \tag{21}$$

Then, the associated level of expected utility is

$$EU^{**} = 1 + (1 - y^{**}) \left[(\bar{r} - 1)(1 - F(R^{**})) + \frac{f(R^{**})}{\sqrt{\alpha}} \right]. \tag{22}$$

We can now characterize the optimal liquidity choice of the intermediary. Having determined the optimal liquidity choices that implement the run equilibrium and the efficient equilibrium, respectively, the intermediary compares the expected utility in either case. Therefore, the intermediary holds scarce liquidity to implement the run equilibrium in the rollover subgame if and only if $EU^{**} \geq EU^{*}$. Let $y^{global} \equiv \arg\max\{EU^{*}, EU^{**}\}$.

The liquidity choice trades off the forgone expected investment return with reducing the equilibrium threshold by avoiding costly liquidation. The intermediary optimally implements the run equilibrium if the liquidation cost, and therefore the benefit of liquidity, is low relative to the opportunity cost of liquidity, as summarized by Lemma 3.

Lemma 3. *Consider the case of a single intermediary and vanishing fundamental uncertainty. If the expected return on investment is high relative to the cost of liquidation, $\bar{r} \geq \bar{r}_0 \equiv 1 + \chi(0.5)$, then the intermediary always implements the run equilibrium in the rollover subgame, $y^{global} = y^{**}$.*

Proof. See Appendix A.3. ■

Furthermore, the intermediary optimally holds no liquidity, $y^{**} = 0$, if the opportunity cost of liquidity is particularly large relative to the cost of liquidation. Proposition 2 summarizes the optimal portfolio choice of the intermediary.

Proposition 2. ***Optimal portfolio choice.*** *Consider the case of a single intermediary and vanishing fundamental uncertainty. If the expected return on investment is high relative to the cost of liquidation, $\bar{r} \geq 1 + \chi(0.5)$, then the intermediary implements the run equilibrium in the rollover subgame. Furthermore, the optimal liquidity level takes an interior value $y^{global} = y^{**} \in (0, \frac{1}{2})$ that is uniquely defined by equation (21) if and only if the expected return on investment satisfies $\bar{r}_0 < \bar{r} < \bar{r}_H$, where $\bar{r}_0$ is defined in Lemma 3 and $\bar{r}_H$ in the proof. However, the intermediary holds no liquidity, $y^{global} = 0$, if $\bar{r} \geq \bar{r}_H$.*

Proof. See Appendix A.4, which also proves the existence and uniqueness of $\bar{r}_H$. ■

Appendix A.4 also contains an analysis of the linear liquidation cost function. For example, I show that the larger the unit cost of liquidation, the larger the expected return on investment $\bar{r}_H$ that is required for the intermediary to hold zero liquidity, $\frac{d\bar{r}_H}{d\chi} > 0$.

I conclude this section by briefly analyzing whether there is a role for government intervention. To address this question, I adopt the notion of constrained efficiency. While a direct choice of the equilibrium threshold would achieve the first-best rollover rule, such an intervention is infeasible. By contrast, the social planner is only allowed to choose the liquidity holdings and takes the optimal rollover decisions of investors at the interim date as given.

Corollary 3. ***Constrained efficiency.*** *The liquidity choice of the intermediary is constrained efficient. Given the incomplete information of investors about the return on investment, a constrained social planner does not choose a different liquidity holding. As such, there is no role for a microprudential regulation of liquidity.*

4 Macroprudential liquidity regulation

While Corollary 3 concluded that there is no role for microprudential liquidity regulation, I analyze multiple intermediaries ($N = 2$) in this section and show that the liquidity choices of intermediaries are constrained inefficient. This implies a role for *macroprudential* liquidity regulation.

Since part of the analysis overlaps with the previous section, I focus on the case where the existence of multiple intermediaries matters for the private and social incentives to hold liquidity. That is, I focus on the run equilibrium in the rollover subgame.[17] This is the relevant equilibrium for a sufficiently large expected return on investment, as shown below.

This section has three parts. I start by establishing the existence of a unique equilibrium in the rollover subgame. Next, I derive conditions sufficient for the existence of a unique private choice of liquidity at the initial date. This equilibrium is characterized by partial free riding on the liquidity

[17] For example, the efficient equilibrium generalizes directly, where the equilibrium threshold $R_1^* = 1 = R_2^*$ arises for vanishing fundamental uncertainty and abundant liquidity, $y_1^* = \frac{1}{2} = y_2^*$.

holdings of other intermediaries and leads to a constrained inefficiently large range of liquidation. As a result, I analyze the problem of a constrained planner who internalizes the beneficial effect of an intermediary's liquidity holding on other intermediaries. Since it internalizes the *systemic* nature of liquidity, I prefer to interpret the constrained planner as a macroprudential authority. More liquidity held by one intermediary reduces the equilibrium threshold of another intermediary by mitigating the coordination failure between investors across intermediaries. Therefore, the macroprudential authority holds more liquidity than private intermediaries, so inefficient liquidation and runs on intermediaries occur less frequently than under laissez-faire.

4.1 Rollover subgame

Suppose that the threshold investor at intermediary n forecasts positive liquidation, $W_{n,n}^{***} > y_n$. Furthermore, suppose that the threshold investor at intermediary n also forecasts positive liquidation by the other intermediary, $W_{n,-n}^{***} > y_{-n}$, which is verified below.[18] The threshold investor's forecast of the proportion of investors who withdraw from another intermediary is derived in Appendix A.1. For any finite precision of private information, the indifference condition of the threshold investor at intermediary n yields the following best-response correspondence:

$$R_n^{***}(R_{-n}^{***}; y_n, y_{-n}) = 1 + \chi\left(W_{n,n}^{***} - y_n, W_{n,-n}^{***} - y_{-n}\right). \tag{23}$$

While the analysis goes through for a general liquidation cost function, the linear specification $\chi_n(l_n + l_{-n}) \equiv \chi \cdot [l_n + l_{-n}]$ produces succinct results and is used in the remainder of the paper. For example, the unique and symmetric equilibrium threshold in the run equilibrium is identical across intermediaries, as described by Lemma 4.

Lemma 4. *Consider the case of multiple intermediaries and vanishing fundamental uncertainty. If the liquidation cost function takes the linear form of equation (5), then there exists a unique run equilibrium characterized by the equilibrium threshold:*

$$R^{***} \equiv 1 + \chi(1 - y_1 - y_2) = R_1^{***} = R_2^{***} > 1, \tag{24}$$

[18] In contrast, if the threshold investor forecasts $W_{n,-n}^{***} \leq y_{-n}$, then there is no effect of the presence of the other intermediary on the equilibrium threshold.

for scarce liquidity choices $y_1, y_2 \in [0, \frac{1}{2})$. The threshold investor at each intermediary forecasts liquidation by both intermediaries.

Proof. See Appendix A.5. ■

The equilibrium threshold in the run equilibrium is larger in the case of multiple intermediaries than in the case of a single intermediary, $R^{***} > R^{**} > R^{FB} = 1$. As stated in Remark 1 and the preceding description, two forms of strategic complementarity in rollover decisions are present in the incomplete-information set-up. First, as in the case of a single intermediary, there is strategic complementarity in rollover decisions between investors of the same intermediary. This leads to coordination failure among investors of each intermediary and pushes the equilibrium threshold above the first-best level, $R^{**} > 1$.

However, there is also strategic complementarity in rollover decisions between investors *across* intermediaries. This leads to coordination failure among investors of different intermediaries and pushes the equilibrium threshold in the run equilibrium further up, $R^{***} > R^{**}$. An investor fears that other investors refuse to roll over to one intermediary, thereby increasing the extent of fire sales, since the marginal product of liquidated investment in alternative use decreases. As a consequence, an investor has a higher incentive not to roll over to the other intermediary. Comparable to Goldstein (2005), this is a 'triple-decker' model of strategic complementarity in rollover decisions of investors.

I complete the description of the run equilibrium in the rollover subgame with two observations. First, a more restrictive lower bound on the expected return on investment, $\bar{r} > \bar{r}_1 \equiv 1 + \chi(0.5, 0.5) = 1 + \chi > \bar{r}_0$ ensures that $R^{***} < \bar{r}$, which is useful for constructing subsequent proofs. Second, the marginal benefit of liquidity is strictly positive in the run equilibrium:

$$\frac{\partial R^{***}}{\partial y_1} = -\chi = \frac{\partial R^{***}}{\partial y_2} < 0. \tag{25}$$

In the multiple-intermediary case, liquidity has a *systemic* nature. More liquidity held by one intermediary reduces the equilibrium threshold of another intermediary R^{***}_{-n} by mitigating the coordination failure between investors across intermediaries. This is in addition to the result of the single-intermediary case, where more liquidity held by an intermediary also reduces the coordination failure among investors of the same intermediary and thereby lowers the equilibrium threshold R_n^{***}.

4.2 Private choice of liquidity

Turning to the initial date, I solve for the equilibrium in the portfolio choice game. Each intermediary chooses its precautionary liquidity holding by taking the equilibrium in rollover subgame $R^{***}(y_n, y_{-n})$ into account and another intermediary's liquidity holding y_{-n} as given:

$$y_n^{***}(y_{-n}) \equiv \arg\max_{y_n} EU_n(y_n, y_{-n}) \text{ s.t. } R^{***} = R^{***}(y_n, y_{-n}). \tag{26}$$

If an interior solution exists, $y^{***} > 0$, then the best-response correspondence $y_n^{***}(y_{-n})$ is defined by the first-order condition:

$$\underbrace{-\frac{\partial EU_n}{\partial y_n}}_{MC} = \underbrace{\frac{\partial EU_n}{\partial R_n^{***}} \frac{\partial R_n^{***}(y_n, y_{-n})}{\partial y_n}}_{MB}, \tag{27}$$

where the private choice of liquidity again balances the private marginal benefit of liquidity (avoiding costly liquidation and thus reducing the equilibrium threshold) with the private marginal cost (forgone investment return).

In Appendix A.6, I derive conditions on the expected return on investment, $\bar{r}_L \leq \bar{r} \leq \bar{r}_H$, that are sufficient for the existence of a best-response *function* $y_n^{***}(y_{-n})$. A sketch of the argument follows. Since the objective function EU_n is globally concave in the level of liquidity y_n when evaluated at the equilibrium threshold $R^{***}(y_n, y_{-n})$, there exists at most one solution. Next, the upper and lower bounds on the expected return on investment ensure the existence of a solution, where the upper bound is identical to $\bar{r}_H$, which guaranteed an interior solution of y^{**} in the single-intermediary case.

As derived in Appendix A.7, the private choices of liquidity are strategic substitutes. If another intermediary holds more liquidity, the liquidation cost of a given intermediary is reduced. The fire sale effect is less pronounced since the marginal product of liquidated investment in alternative use is higher. Since holding liquidity is costly (forgone investment return), the given intermediary optimally reduces its liquidity holding. That is, intermediaries free ride on the liquidity held by other intermediaries.

However, another intermediary's liquidity holding is only partially useful. While other liquidity holdings help to reduce the cost from liquidation by mitigating the fire sale effect, only the liquidity held by an intermediary can be used to serve withdrawing investors and to prevent costly liquidation. For this reason, liquidity holdings of intermediaries are only partial substitutes, despite the full symmetry in their effect on the equilibrium threshold in the run equilibrium:

$$\frac{dy_n^{***}}{dy_{-n}} \in (-1, 0). \tag{28}$$

There exists a unique crossing of best-response functions because the slope of the best-response function lies strictly within the unit circle and the best-response function is symmetric. Therefore, the private choice of liquidity is identical across intermediaries, $y_n^{***} \equiv y^{***}$. Since the previous bounds on the expected return on investment are sufficient for $y^{***} \in [0, \frac{1}{2})$, the private choice of liquidity is implicitly given by $\frac{dEU_n}{dy_n}(y^{***}, y^{***}) = 0$.

To evaluate the consequences of free riding on other liquidity holdings, I compare the range of ex-post inefficient liquidation in the multiple-intermediary case to the single-intermediary case. Ex-post inefficient liquidation, which is a consequence of incomplete information, always occurs in the run equilibrium, $R^{**} > R^{FB}$. In this statement, 'inefficient' refers to a comparison with the first-best liquidation rule. It is critical to emphasize that the welfare criterion adopted in this paper is not first-best but constrained efficiency from an ex-ante perspective. Therefore, I compare the extent of ex-post inefficient liquidation across various choices of ex-ante liquidity holdings. Then, ex-ante *constrained* inefficiency occurs whenever 'excessive' liquidation occurs in the run equilibrium, which is defined as $R > R^{SP}$. In a nutshell, inefficient refers to (the absence of) first-best, while excessive refers to (the absence of) second-best.

Proposition 3 summarizes the private choice of liquidity and its welfare properties. It can be shown that the free riding on liquidity holdings leads to an excessive ex-post liquidation in the run equilibrium, $R^{***} > R^{**}$. Since the range of inefficient liquidation $[1, R]$ is larger in the multiple-intermediary case than in the single-intermediary case, the private portfolio choice is constrained inefficient from an ex-ante perspective.

Proposition 3. ***Free riding and constrained inefficiency.*** *Consider the case of multiple intermediaries and vanishing fundamental uncertainty. If the expected return on investment takes an intermediate value, $\bar{r} \in [\bar{r}_L, \bar{r}_H]$, then there exists a unique and symmetric equilibrium. This equilibrium is characterized by a private choice of liquidity at the initial date, $y_1^{***} = y_2^{***} \equiv y^{***} \in \left[0, \frac{1}{2}\right)$, that implements the run equilibrium in the rollover subgame between investors at the interim date. Each investor rolls over funding to the intermediary if and only if $R_i \geq R^{***}$. The quantities are implicitly defined by*

$$R^{***} = 1 + \chi(1 - 2y^{***}) > 1 \tag{29}$$

$$\chi(1 - y^{***})(R^{***} - 1) = \frac{1}{\alpha} + (\bar{r} - 1)\frac{1 - F(R^{***})}{\sqrt{\alpha} f(R^{***})}. \tag{30}$$

This equilibrium is characterized by partial free riding on the liquidity of other intermediaries. As a result, the coordination failure among investors across intermediaries leads to excessive liquidation ex post and renders the private choice of liquidity as constrained inefficient:

$$R^{***} > R^{**}. \tag{31}$$

The constrained inefficiency from an ex-ante perspective is related to the strategic complementarity in rollover decisions between investors across intermediaries. Fearing that investors refuse to roll over to another intermediary, thereby exacerbating the fire sale effect of joint liquidation, investors have a greater incentive not to roll over funding to their intermediary. However, an intermediary does not consider the beneficial impact of its private choice of liquidity on other intermediaries, which would reduce the coordination failure between investors across intermediaries. Instead, an intermediary free rides on the liquidity held by other intermediaries in the system and holds insufficient liquidity itself. Therefore, excessive liquidation occurs and the private liquidity holdings are constrained inefficient.

4.3 Social choice of liquidity

A constrained planner chooses the liquidity holdings of both intermediaries at the initial date, taking the optimal rollover decision of investors at the interim date into account. In contrast to an intermediary, the planner internalizes the benefit of one intermediary's liquidity holding on another intermediary. Since the constrained planner captures these systemic effects of liquidity, my preferred interpretation is that of a macroprudential authority.

The social choice of liquidity that implements the run equilibrium, $y_k^{SP} \in [0, \frac{1}{2})$, maximizes the social welfare function SWF:

$$(y_1^{SP}, y_2^{SP}) \equiv \arg\max_{y_1, y_2} SWF \equiv EU_1 + EU_2 \text{ s.t. } R_k^{SP} = R^{SP}(y_1, y_2). \tag{32}$$

If an interior solution exists, it is characterized by the first-order condition:

$$\overbrace{-\frac{\partial EU_n}{\partial y_n}}^{SMC} = \overbrace{\frac{\partial EU_n}{\partial R_n^{SP}} \frac{\partial R_n^{SP}}{\partial y_n} + \frac{\partial EU_{-n}}{\partial R_{-n}^{SP}} \frac{\partial R_{-n}^{SP}}{\partial y_n}}^{SMB}. \tag{33}$$

The social choice of liquidity balances the social marginal cost of liquidity (forgone investment return) with the social marginal benefits of liquidity (lower equilibrium thresholds by avoiding costly liquidation). The private and social marginal costs of liquidity coincide because the opportunity cost of holding liquidity is unchanged. By contrast, the social marginal benefit of liquidity exceeds the private marginal benefit. Apart from the beneficial effect of liquidity on the investors of one intermediary ($\partial R_n^{SP}/\partial y_n < 0$), which is identical to the private marginal benefit of liquidity, the planner also considers the beneficial effect on the other intermediary's investors ($\partial R_{-n}^{SP}/\partial y_n < 0$). More liquidity allows an intermediary to serve more withdrawing investors and therefore mitigates the effect of fire sales. Therefore, the marginal product of liquidated investment in alternative use is higher, which reduces the coordination failure between investors across intermediaries.

The optimization problem is fully symmetric. From a social perspective, there is full substitutability between liquidity held at one intermediary and that held at another in order to reduce the equilibrium threshold $R_n^{SP} = R^{SP} = 1 + \chi[1 - y_1^{SP} - y_2^{SP}]$. Furthermore, both first-order conditions yield the same condition, equation (33). Therefore, only the total amount of liquidity is determined $y^{SP}_{total} \equiv y^{SP}_1 + y^{SP}_2$. I adopt the convention that $y^{SP}_n \equiv \frac{y^{SP}_{total}}{2}$. As shown in Appendix A.8, a unique social liquidity choice exists if the expected return on investment again takes an intermediate value, $\bar{r} \in [\bar{r}_L, \bar{r}^{SP}_H]$.

The macroprudential authority holds zero liquidity for a smaller range of parameters than a private intermediary. That is, the upper bound for the social choice of liquidity is less restrictive than the upper bound for the private choice of liquidity, $\bar{r}^{SP}_H > \bar{r}_H$. Intuitively, the macroprudential authority is more reluctant to hold zero liquidity since it considers the systemic benefits of liquidity, not only the individual benefits. If the expected return on investment follows $\bar{r} \in [\bar{r}_H, \bar{r}^{SP}_H]$, then the run equilibrium is implemented. While the intermediary optimally holds zero liquidity, $y^{***} = 0$, the macroprudential authority holds some liquidity, $y^{SP} > 0$.

Proposition 4 summarizes the social choice of liquidity and compares it to the private choice of liquidity. The more restrictive bounds on the expected return on investment are used to allow comparison.

Proposition 4. ***Social liquidity choice and constrained efficiency.*** *Consider the case of multiple intermediaries and vanishing fundamental uncertainty. If the expected return on investment takes an intermediate value, $\bar{r} \in [\bar{r}_L, \bar{r}_H]$, then there exists a unique equilibrium. This equilibrium is characterized by a social choice of liquidity $y^{SP}_{total} \in [0, 1)$ that optimally implements the run equilibrium in the rollover subgame. Each investor rolls over to the intermediary if and only if $R_i \geq R^{SP} = 1 + \chi(1 - y^{SP}_{total}) > 1$, where the social choice of liquidity is the unique solution of*

$$\chi^2(2 - y^{SP}_{total})(1 - y^{SP}_{total}) = \frac{1}{\alpha} + \frac{\bar{r} - 1}{\sqrt{\alpha}} \frac{1 - F[1 + \chi(1 - y^{SP}_{total})]}{f[1 + \chi(1 - y^{SP}_{total})]}. \tag{34}$$

A macroprudential authority holds more liquidity than private intermediaries. It also generates a smaller range of ex-post inefficient liquidation:

$$y^{SP}_{total} > 2y^{***} \tag{35}$$

$$1 < R^{SP} < R^{**} < R^{***}. \tag{36}$$

Proof. See Appendix A.8. ■

A macroprudential authority takes the *systemic* nature of liquidity into account. As a result, the social choice of liquidity exceeds the private choice, because the former internalizes the social marginal costs of liquidation due to fire sales (that is, the social marginal benefit of liquidity). Therefore, the macroprudential authority mitigates the coordination failure between investors across intermediaries and reduces the range of ex-post inefficient liquidation. The difference between the social and private choices of liquidity, $y^{SP} - y^{***}$, is interpreted as a *macroprudential liquidity buffer*.

5 Discussion

The purpose of this section is threefold. I start by discussing some assumptions of the model and their impact on the results. Next, I argue that this microfounded set-up is suitable for an analysis of macroprudential regulation more broadly and highlight avenues for further work. Finally, I relate my findings to the regulatory debate of Basel III.

5.1 Model assumptions and extensions

First, I analyze the welfare implications in the case of two intermediaries. This can be generalized to any finite number of intermediaries without losing either the strategic substitutability in private choices of liquidity or their welfare properties. While the strategic aspect of liquidity choices vanishes in the limiting case of a continuum of intermediaries, the welfare result prevails.

Next, I consider an exogenous debt contract whereby an investor receives a unit payment upon not rolling over. This approach is similar to Dasgupta (2004) and Shapiro and Skeie (2013). It ensures the viability of intermediaries at the interim date, since the promised payment does not exceed the liquidation value. I do not attempt to endogenize the debt contract in the current paper, but this could be achieved by assuming idiosyncratic liquidity risk that creates demand for liquidity at the interim date as in Diamond and Dybvig (1983). However, I do endogenize the portfolio choice of intermediaries, which improves upon Morris and Shin (2000), who explicitly abstract from both the optimal contract design and the ex-ante portfolio choice of intermediaries.

The strategic interaction between investors arises from costly liquidation of investment due to diminishing marginal product in alternative use, which contrasts with Goldstein and Pauzner (2005), where it arises since the promised interim-date payment exceeds the liquidation value of investment. The current set-up allows an analysis not only of the ex-ante portfolio choice of intermediaries subject to rollover risk, but also of the systemic nature of liquidity in the case of multiple intermediaries and fire sales.

Third, investors deposit with a single intermediary for simplicity. However, this result can allow for a diversification of intermediaries with whom investors deposit. For concreteness, consider the case in which a proportion ω of investors deposit with both intermediaries. Then, the difference between the objective functions of the intermediary and the macroprudential authority shrinks but the welfare result prevails. While the objective functions coincide in the extreme case of $\omega = 1$, this scenario is unlikely, because some investors are unsophisticated or face high costs of diversifying their deposits.

5.2 Relation to regulatory debate

I develop a model of financial intermediaries that optimally choose their portfolio composition before facing rollover risk in their funding. Because assets are jointly liquidated in a fire sale, private choices of liquidity are strategic substitutes, so each intermediary free rides on the liquidity of other intermediaries. This set-up results in constraint inefficient liquidity choices of intermediaries and creates a role for macroprudential regulation. A constrained planner, preferably interpreted as a macroprudential authority, internalizes the systemic nature of liquidity and requires intermediaries to hold more liquidity than under laissez-faire. Furthermore, the macroprudential authority holds positive liquidity for a larger range of parameters than a private intermediary, which is particularly welfare-improving whenever fire sales are severe, such as during a financial crisis.

There are different options for a macroprudential authority to require intermediaries to hold larger amounts of liquidity. Two of these are currently discussed and implemented within the endorsed Basel III framework. The first option is to require intermediaries to hold a short-term liquidity buffer against unforeseen liquidity outflows. Translated into the model set-up, this Liquidity

Coverage Ratio (LCR) rule requires intermediaries to hold more liquidity in the initial period, that is *before* shocks are realized (and investors receive their private signal about the solvency of the intermediary).[19] In the words of the Basel Committee on Banking Supervision (2013): "It does this by ensuring that banks have an adequate stock of unencumbered high-quality liquid assets (HQLA) that can be converted easily and immediately in private markets into cash to meet their liquidity needs for a 30 calendar day liquidity stress scenario." The additional liquidity holdings required from intermediaries by the macroprudential authority (Proposition 4) are therefore best interpreted and implemented in the form of an LCR.

This leads to the question of the optimal implementation of the LCR. One option is to request each intermediary to hold additional liquidity to offset the effect of liquidity free riding. Another option is to create a systemic liquidity fund that all intermediaries contribute to. While another intermediary's liquidity can be used to dampen the effects of a fire sale, investors who withdraw can only be served with the liquidity held by an intermediary. Therefore, another intermediary's liquidity is only a partial substitute. Hence, this suggests that the preferable way to implement a macroprudential liquidity buffer is in the form of increased liquidity requirements for individual intermediaries.

In contrast to the LCR, the Net Stable Funding Ratio (NSFR) rule proposed by Basel III restricts the availability of demandable debt instruments to investors. Thus, it directly addresses the ratio of stable to unstable funding of the intermediary. The NSFR is computed as the ratio of the available amount of stable funding to the required amount of stable funding and should always exceed 100%. Unsecured wholesale funding, as discussed in this paper, is attributed a weight of 50% in the available amount of stable funding. As a result, the NSFR is likely to impose a strong restriction on the optimal funding structure of intermediaries. However, the current model is more suitable for analyzing the LCR to which it links directly. By including long-term debt and equity in the analysis (see also the discussion above), an extension of this model could inform the policy debate on the NSFR more directly.

Finally, a key feature of the financial crisis of 2007-09 was the specific role of information. After the insolvency of Lehman Brothers and the subsequent insolvency of the Reserve Primary Fund, the

[19] To match the LCR, the duration between the initial and final date is required to be one month.

fear of further contagion triggered substantial liquidity outflows from many money market mutual funds (see Schmidt et al. (2013)) that, in turn, drew liquidity from the banking system. The global games approach of this paper appears to be a very suitable modelling choice: coordination between investors of one intermediary, as well as between investors across intermediaries, is at the heart of the analysis reported in this paper (recall Remark 1 and the preceding description of the strategic incentives of investors to roll over funding).

6 Conclusion

This paper studied rollover risk in the wholesale funding market when intermediaries can hold liquidity ex ante and are subject to fire sales ex post. I showed that the presence of liquidity, which drives a wedge between the amount of withdrawals and the liquidation volume, restores multiple equilibria – even if a global game refinement is used. Apart from the usual equilibrium with coordination failure and a high equilibrium threshold, a constrained efficient equilibrium with a low equilibrium threshold exists for sufficiently high levels of liquidity. Liquidity holdings serve withdrawing investors and therefore support an equilibrium in which the threshold investor forecasts no liquidation to take place. I provided a simple uniqueness refinement to characterize the privately optimal liquidity choice.

Furthermore, I explored fire sales in a simple set-up with two intermediaries, whereby one intermediary's liquidation volume increases the liquidation costs of another intermediary. The positive implication of fire sales is that the liquidity holdings are partial substitutes. Consequently, intermediaries free ride on each other's liquidity holdings, causing excessive liquidation of productive investment. The normative implication of fire sales is that intermediaries hold insufficient liquidity relative to a constrained planner, leading to a larger range of inefficient liquidation. Such a planner is best interpreted as a macroprudential authority, since it internalizes the systemic nature of liquidity.

This framework provides a natural laboratory for studying macroprudential policies in a microfounded setting more generally. There are other elements relevant to the conduct of macroprudential regulation omitted in this framework, such as capital requirements, portfolio diversification, or penalties on early withdrawals. These are all exciting avenues for subsequent research.

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A Appendix

A.1 Posterior distributions

Return on investment The posterior distribution of the return on investment is also normal. The posterior precision is the sum of the prior precisions and the precision of the private information. The mean is a weighted average of the prior and the private signal with the respective precisions as weights:

$$r|x_i \sim \mathcal{N}\left(\frac{\alpha\bar{r} + \gamma x_i}{\alpha + \gamma}, \frac{1}{\alpha + \gamma}\right). \tag{37}$$

The ratio of the precision of the prior (public signal) relative to the private signal, $\frac{\alpha}{\gamma}$, determines the extent to which the posterior mean depends on the private signal. The more precise the private signal is relative to the prior, the more the posterior is determined by the private signal. As private noise vanishes, $\gamma \to \infty$, the posterior mean converges to the private signal.

Proportion of investors who withdraw from the same intermediary Using a law of large numbers, the posterior proportion of investors who withdraw from intermediary n is

$$W_{in} = \Phi\left(\sqrt{\delta}\left[R_n^* - \bar{r}\right] + \sqrt{\frac{\gamma(\alpha+\gamma)}{\alpha+2\gamma}}\left[R_n^* - R_i\right]\right) \tag{38}$$

$$\delta \equiv \frac{\alpha^2(\alpha+\gamma)}{\gamma(\alpha+2\gamma)}, \tag{39}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal and δ summarizes precision parameters. Therefore, the threshold investor has the following posterior:

$$W_n^* \equiv W_i|_{x_i=x^*} = \Phi(z_{1n}) \tag{40}$$

$$z_{1n} \equiv \sqrt{\delta}\left[R_n^* - \bar{r}\right]. \tag{41}$$

If private noise vanishes, $\gamma \to \infty$, then $\delta \to 0$ and $W_n^* \to \frac{1}{2}$.

Proportion of investors who withdraw from another intermediary In the case of multiple banks, an investor needs to forecast the proportion of investors who withdraw from another intermediary

$$W_{i,n,-n} = \Phi\left(\sqrt{\delta}\left[R^*_{-n} - \bar{r}\right] + \sqrt{\frac{\gamma(\alpha+\gamma)}{\alpha+2\gamma}}\left[R^*_{-n} - R_{in}\right]\right) \tag{42}$$

$$(W_{n,-n})^* \equiv W_{i,n,-n}|_{x_i=x^*_n} = \Phi(z_{2n}) \tag{43}$$

$$z_{2n} \equiv \sqrt{\delta}\left[R^*_{-n} - \bar{r}\right] + \sqrt{\delta}\frac{\gamma}{\alpha}\left[R^*_{-n} - R^*_n\right]. \tag{44}$$

A.2 Derivation of expected utility

Consider the case of a single intermediary, $n = 1$. As private noise vanishes, $\gamma \to \infty$, the realized proportion of investors who withdraw from the intermediary at the interim date is

$$w^*(r) = \Phi\left(\frac{\alpha}{\sqrt{\gamma}}[R - \bar{r}] + \sqrt{\gamma}[R - r]\right) \to \begin{cases} 0 & & r > R \\ \frac{1}{2} & \text{if} & r = R, \\ 1 & & r < R \end{cases} \tag{45}$$

for any equilibrium threshold R. Therefore, there is no (full) liquidation of investment if the return on investment is above (below) the threshold R. Investors receive the continuation payoff c_2 or a unit liquidation payoff. These payoffs follow from the timing of liquidation costs, which occur at the final date as in Morris and Shin (2000). This timing of liquidation costs is implied by a lender-of-last-resort policy, for instance. Adding up, the expected utility is

$$EU(y) = \int_{-\infty}^{R} \phi(\sqrt{\alpha}[r - \bar{r}])dr + \int_{R}^{\infty} [y + (1-y)r]\phi(\sqrt{\alpha}[r - \bar{r}])dr \tag{46}$$

$$= y + (1-y)\left[F(R) + (1 - F(R))\bar{r} + \frac{f(R)}{\sqrt{\alpha}}\right]. \tag{47}$$

The partial derivatives of the expected utility are

$$\begin{aligned}
\frac{\partial EU}{\partial y} &= -[1-F(R)](\bar{r}-1)-\frac{f(R)}{\sqrt{\alpha}}<0 && (48)\\
\frac{\partial EU}{\partial R} &= -(1-y)(R-1)\sqrt{\alpha}f(R)\leq 0 && (49)\\
\frac{\partial EU}{\partial y\partial R} &= (R-1)\sqrt{\alpha}f(R)\geq 0 && (50)\\
\frac{\partial^2 EU}{\partial R^2} &= -(1-y)\sqrt{\alpha}f(R)[1+\alpha(R-1)(\bar{r}-R)]\leq 0 && (51)\\
\frac{\partial^2 EU}{\partial y^2} &= 0. && (52)
\end{aligned}$$

A.3 Proof of Lemma 3

The optimal liquidity choice to implement the efficient equilibrium is $y^* = \frac{1}{2}$, which yields

$$EU(y^*) = 1+\frac{1}{2}\left[(\bar{r}-1)(1-F[1])+\frac{f[1]}{\sqrt{\alpha}}\right]. \tag{53}$$

If $y = 0$, which may not be optimal, then inefficient equilibrium is implemented. The expected utility in this case is

$$EU(y=0) = 1+(\bar{r}-1)(1-F[1+\chi(0.5)])+\frac{f[1+\chi(0.5)]}{\sqrt{\alpha}}. \tag{54}$$

Thus, we have that $EU(y=0) > EU(y^*)$ if two conditions hold. First, $\Phi(\sqrt{\alpha}[\bar{r}-1-\chi(0.5)]) > 0.5\Phi(\sqrt{\alpha}[\bar{r}-1])$. Since the right-hand side of this equation is smaller than one half, $\bar{r}\geq 1+\chi(0.5)$ suffices to ensure this constraint. Second, $\Phi(\sqrt{\alpha}[\bar{r}-1-\chi(0.5)]) > 0.5\Phi(\sqrt{\alpha}[\bar{r}-1])$. Since $\phi(\cdot)$ is larger the closer the argument is to zero, the bound on the expected return is again sufficient for this constraint.

A.4 Proof of Proposition 2

Rewriting equation (22) yields $LHS \equiv (1-y)\chi(\frac{1}{2}-y)\chi'(\frac{1}{2}-y) = \frac{1}{\alpha}+\frac{(\bar{r}-1)\Phi(\sqrt{\alpha}[\bar{r}-R^{**}])}{\sqrt{\alpha}\phi(\sqrt{\alpha}[\bar{r}-R^{**}])} \equiv RHS$, where $R^{**} = 1+\chi(0.5-y)$.

Below I show that the left-hand side (LHS) is smaller than the right-hand side (RHS) at the upper bound and that the LHS (RHS) decreases (increases) in the liquidity level y if the lower bound on the expected return on investment is maintained. First, observe that $LHS \to 0 < \frac{1}{\alpha} + \frac{(\bar{r}-1)\Phi(\sqrt{\alpha}[\bar{r}-1])}{\sqrt{\alpha}\phi(\sqrt{\alpha}[\bar{r}-1])} \leftarrow RHS$ as $y \to 0.5$. Second, $LHS'(y) = -\chi\chi' - (1-y)[(\chi')^2 + \chi\chi''] < 0$. Third, introduce $\Lambda(R) \equiv \frac{1-\Phi(\sqrt{\alpha}[R-\bar{r}])}{\phi(\sqrt{\alpha}[R-\bar{r}])} > 0$, so $\Lambda'(R) = -\sqrt{\alpha} - \alpha(\bar{r} - R)\Lambda(R)$. Thus, $RHS'(y) = -\frac{\bar{r}-1}{\sqrt{\alpha}}\chi'(0.5-y)\Lambda'(R^{**})$. Therefore, $RHS'(y) > 0$ if $\bar{r} > R^{**}$, which is again ensured by the condition $\bar{r} \geq 1 + \chi(0.5)$ that we already used in the proof of Lemma 3.

As a consequence, the corner solution $y^{**} = 0$ is the optimal liquidity choice of the intermediary if $LHS(0) \leq RHS(0)$, which imposes the constraint on the liquidation cost function:

$$\chi(0.5)\chi'(0.5) \leq \frac{1}{\alpha} + \frac{\Phi(\sqrt{\alpha}[\bar{r} - 1 - \chi(0.5)])}{\sqrt{\alpha}\phi(\sqrt{\alpha}[\bar{r} - 1 - \chi(0.5)])}. \tag{55}$$

Define $\bar{r}_H$ as the value of $\bar{r}$ for which condition (55) holds with equality.

Since the left-hand side is independent of the expected return on investment, the proof of the existence and uniqueness of $\bar{r}_H$ can be completed for the linear liquidation cost function without loss of generality. If the liquidation cost function takes the simple linear form, this constraint simplifies to

$$\frac{\chi^2}{2} \leq \frac{1}{\alpha} + \frac{(\bar{r}-1)\Phi\left(\sqrt{\alpha}[\bar{r} - 1 - \frac{\chi}{2}]\right)}{\sqrt{\alpha}\phi\left(\sqrt{\alpha}[\bar{r} - 1 - \frac{\chi}{2}]\right)}. \tag{56}$$

Then, $\bar{r}_H$ exists and is unique. First, as $\bar{r} \to \infty$, the left-hand side of equation (62) is positive and finite and the right-hand side goes to infinity. Next, as the expected return converges to its minimum $\bar{r} \to 1 + \frac{\chi}{2}$, $LHS = \frac{\chi^2}{2}$ and $RHS \to \frac{1}{\alpha} + \frac{\sqrt{2\pi}\chi}{4\sqrt{\alpha}}$. A necessary condition is $LHS(\bar{r} = 1 + \frac{\chi}{2}) > RHS(1 + \frac{\chi}{2})$, which yields

$$\chi\sqrt{\alpha} > \sqrt{\frac{\pi}{8}} + \sqrt{\frac{\pi}{8} + 2}, \tag{57}$$

where the right-hand side reflects the specifics of the probability distribution function. Condition (57) is satisfied if the precision of the public information is sufficiently high, $\alpha > \alpha_L \equiv$

$\left(\frac{\sqrt{\frac{\pi}{8}}+\sqrt{\frac{\pi}{8}+2}}{\chi}\right)^2$. Third, the right-hand side increases in the expected investment return:

$$\frac{\partial RHS}{\partial \bar{r}} = (\bar{r}-1)\sqrt{\alpha} + \left(1+\alpha[\bar{r}-1-\frac{\chi}{2}]\right)\frac{\Phi(z)}{\phi(z)} > 0, \tag{58}$$

where $z \equiv \sqrt{\alpha}[\bar{r}-1-\frac{\chi}{2}]$ is shorthand. Taking all of the three steps together, there exists a unique $\bar{r}_H$ as claimed.

As for the dependence of $\bar{r}_H$ on χ, total differentiation yields

$$\frac{d\bar{r}_H}{d\chi} = \frac{\chi - \frac{\partial RHS}{\partial \chi}}{\frac{\partial RHS}{\partial \bar{r}}}. \tag{59}$$

To evaluate this, $\frac{\partial RHS}{\partial \chi}$ is required:

$$\frac{\partial RHS}{\partial \chi} = -\frac{\bar{r}_H - 1}{\sqrt{\alpha}}\frac{1}{2}\left(1+\alpha\frac{\Phi(z)}{\phi(z)}[\bar{r}_H - 1 - \frac{\chi}{2}]\right) < 0. \tag{60}$$

Therefore, $\frac{d\bar{r}_H}{d\chi} > 0$ as claimed.

A.5 Proof of Lemma 4

This proof partially builds on Goldstein (2005), but proceeds differently.[20]

First, observe that the cumulative distribution function of the standard normal is bounded within $[0,1]$ as R_{-n} diverges. Second, I claim that the equilibrium thresholds converge to $R_1^{***} = R_2^{***} = R^{***} \equiv 1+\chi[1-y_1-y_2]$ as the fundamental uncertainty vanishes. This is proven by contradiction.

I start by supposing that $R_1^{***} > R_2^{***}$. Then, $W_{1,2}^{***} \to 0$ and $W_{2,1}^{***} \to 1$ as $\gamma \to \infty$. Insert these results into equation (23); the implied expressions for the thresholds can never satisfy the supposed inequality $R_1^{***} > R_2^{***}$. By extension, the argument applies for $R_1^{***} < R_2^{***}$ as well. Therefore, $R_1^{***} = R_2^{***}$ as claimed. Hence, the expression for R^{***} obtains.

[20] In that paper, the uniqueness proof is in two steps. Using my notation here, a unique solution R_n^{***} must be obtained for any R_{-n} first, so $R_n^{***}(R_{-n})$ is a best-response function for all n. Second, there must be a unique intersection of best-response functions.

Third, liquidity must again be scarce to support positive liquidation as forecast by the threshold investor, $y_n < \frac{1}{2}$ for $n = 1, 2$. Furthermore, the equilibrium threshold confirms the supposition of $W^{***}_{n,-n} \leq y_{-n}$ if the other bank's liquidity holding is also scarce, $y_{-n} < \frac{1+\lambda}{2}$.

Digression: another liquidation cost specification The equilibrium threshold places an equal weight on the liquidity levels of each intermediary, as implied by the equal weight of the liquidation volumes in the liquidation cost function. This feature can be relaxed by either a larger weight on the own liquidation volume or by specifying a convex liquidation cost function. Both specifications imply a larger weight of an intermediary's equilibrium threshold on its level of liquidity. For example, take a liquidation cost function that is proportional to both its individual liquidation volume and the aggregate liquidation volume: $\chi(l_n, l_{-n}) = \chi l_n(l_n + l_{-n})$. This yields $R^{***}_n \to 1 + \chi(1 - y_A - y_B)(\frac{1}{2} - y_n)$.

A.6 Best-response function of private liquidity choice

This section derives conditions sufficient for $y^{***}_n(y_{-n})$ to be unique, thereby constituting a best-response function. This proof mirrors and extends the single-intermediary case in Appendix A.4.

Recall that $\Lambda(R^{***}_n) = \frac{1-\Phi\left(\sqrt{\alpha}[R^{***}_n - \bar{r}]\right)}{\phi\left(\sqrt{\alpha}[R^{***}_n - \bar{r}]\right)} > 0$, where $R^{***}_n = 1 + \chi(1 - y^{***}_n - y_{-n})$. Thus, $\Lambda'(R^{***}_n) = -\sqrt{\alpha} - \alpha(\bar{r} - R^{***}_n)\Lambda(R^{***}_n) < 0$. Using $\Lambda(R^{***}_n)$, the first-order condition that determines the best-response function $y^{***}_n(y_{-n})$ becomes

$$\chi^2(1 - y^{***}_n)(1 - y^{***}_n - y_{-n}) = \frac{1}{\alpha} + (\bar{r} - 1)\Lambda(R^{***}_n), \tag{61}$$

for any $y_{-n} \in [0, 1]$.

Four checks are required to establish a best-response function. First, the left-hand side of equation (61) is decreasing in the liquidity level y_n over the relevant range $y_n \in [0, \frac{1}{2})$ for all y_{-n}, while the right-hand side is increasing in it:

$$\begin{aligned}\frac{\partial LHS}{\partial y_n} &= \chi^2[2y_n + y_{-n} - 2-] < -\chi^2(1 - y_{-n}) \leq 0 \\ \frac{\partial RHS}{\partial y_n} &= -\chi(\bar{r} - 1)\Lambda'(R_n^{***}) > 0.\end{aligned}$$

Second, the left-hand side exceeds the right-hand side when evaluated at the lower bound $y_n = 0$ for any liquidity level y_{-n}. This inequality is hardest to satisfy for $y_{-n} \to \frac{1}{2}$. Therefore, $LHS(0, \frac{1}{2}) \geq RHS(0, \frac{1}{2})$ is sufficient and yields an upper bound on the expected investment return $\bar{r}_H$; this bound is identical to the one that ensures an interior solution y^{**} in the one-bank case, $\bar{r} \leq \bar{r}_H$.

Third, the right-hand side exceeds the left-hand side when evaluated at the upper bound $y_n \to \frac{1}{2}$ for any liquidity level y_{-n}. This inequality is hardest to satisfy for $y_{-n} = 0$. Therefore, $LHS(\frac{1}{2}, 0) \leq RHS(\frac{1}{2}, 0)$ is sufficient and yields a lower bound on the expected investment return $\bar{r}_2$ implicitly defined by

$$\frac{\chi^2}{4} \equiv \frac{1}{\alpha} + \frac{(\bar{r}_2 - 1)\Phi\left(\sqrt{\alpha}[\bar{r}_2 - 1 - \chi\frac{1}{4}]\right)}{\sqrt{\alpha}\phi\left(\sqrt{\alpha}[\bar{r}_2 - 1 - \chi\frac{1}{2}]\right)}. \tag{62}$$

Define the lower bound on the expected return as $\bar{r}_L \equiv \max\{\bar{r}_1, \bar{r}_2\}$. Therefore, a solution $y_n^{***}(y_{-n})$ to the first-order condition in equation (22) exists if $\bar{r} \in [\bar{r}_L, \bar{r}_H]$.

Fourth, the global concavity of the objective function in y_n when evaluated at R_n^{***} ensures that this solution is unique:

$$\frac{d^2 EU_n}{dy_n^2} = \frac{\partial R_n^{***}}{\partial y_n}\left[2\frac{\partial^2 EU_n}{\partial y_n \partial R_n^{***}} + \frac{\partial^2 EU_n}{\partial (R_n^{***})^2}\frac{\partial R_n^{***}}{\partial y_n}\right] + \frac{\partial EU_n}{\partial R_n^{***}}\frac{\partial^2 R_n^{***}}{\partial y_n^2} < 0,$$

where $\frac{\partial^2 R_n^{***}}{\partial y_n^2} = 0$ and the sign follows directly from the previously established signs on the partial derivatives of the expected utility EU_n for the equilibrium threshold $R_n^{***} \in (1, \bar{r})$, where the upper bound is implied by $\bar{r} > \bar{r}_1$.

In sum, $\bar{r}_L \leq \bar{r} \leq \bar{r}_H$ ensures that a best-response function $y_n^{***}(y_{-n})$ exists.

A.7 Strategic substitutability in liquidity holdings

Strategic substitutability of bank liquidity holdings is established by totally differentiating equation (22):

$$\frac{dy_n^*}{dy_{-n}} = -\frac{\chi^2[1 - y_n^{***}] + \chi(\bar{r} - 1)[-\Lambda(R_n^{***})]}{\chi^2[2 - 2y_n^{***} - y_{-n}] + \chi(\bar{r} - 1)[-\Lambda(R_n^{***})]} \in (-1, 0), \tag{63}$$

where the lower bound is ensured by $y_k < \frac{1}{2},\ k = 1, 2$.

A.8 Constrained planner's problem

Unique constraint efficient liquidity level This section shows that there exists a unique constrained efficient liquidity level. First, we require that the left-hand side of equation (34) exceeds the right-hand side at the lower bound $y_{total}^{SP} = 0$, which implicitly defines an upper bound on the expected investment return $\bar{r} < \bar{r}_H^{SP}$:

$$2\chi^2 \equiv \frac{1}{\alpha} + \frac{\bar{r}_H^{SP} - 1}{\sqrt{\alpha}} \Lambda\left(\sqrt{\alpha}[\bar{r}_H^{SP} - (1 + \chi)]\right). \tag{64}$$

Second, the right-hand side of equation (34) exceeds the left-hand side at the upper bound $y_{total} \to 1$, which is never binding. Third, the left-hand side of equation (34) strictly decreases in the liquidity level over the required range $[0, 1)$, while the right-hand side of equation (34) strictly increases in it. Finally, verify that it is indeed a local maximum of the social welfare function:

$$\frac{d^2 SWF(y = y_{total}^{SP})}{d(y^{SP})^2} = -\sqrt{\alpha}\chi f(R^{SP})\left(\sqrt{\alpha}(\bar{r} - 1)\Lambda(R^{SP}) + \chi(3 - y_{total}^{SP}) + (\bar{r} - 1)\right) < 0.$$

Taking these points together, there exists a unique level of total liquidity y_{total}^{SP} that maximizes social welfare and is implicitly given by $\frac{dSWF}{dy_n}(y_{total}^{SP}) = 0$.

Rankings of liquidity levels and equilibrium thresholds Solving for y^{***} and y^{SP} and substituting in defining equations, we get:

$$(R^{***}-1)\left(\frac{R^{***}-1+\chi}{2}\right) = \frac{1}{\alpha}+\frac{\bar{r}-1}{\sqrt{\alpha}}\Lambda(R^{***}) \tag{65}$$

$$(R^{SP}-1)\left(R^{SP}-1+\chi\right) = \frac{1}{\alpha}+\frac{\bar{r}-1}{\sqrt{\alpha}}\Lambda(R^{SP}). \tag{66}$$

The right-hand side is identical, while the left-hand side is larger for the social planner allocation than for the private allocation, yielding $R^{SP} < R^{**} < R^{***}$. Therefore, the privately optimal liquidity levels are in sum smaller than the social planner's allocation, $y^{SP}_{total} < 2y^{***}$.