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by

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Abstract

We incorporate a participation decision in a standard New Keynesian model with matching frictions and show that treating the labor force as constant leads to incorrect evaluation of alternative policies. We also show that the presence of a participation margin mitigates the Shimer critique.

*JEL classification: E24, E32, E52*
*Bank classification: Labour markets; Business fluctuations and cycles; Transmission of monetary policy*


# Résumé

Les auteurs incorporent la décision de participer ou non au marché du travail à un modèle néokeynésien standard avec frictions d'appariement. Ils montrent que le fait de considérer comme constant le taux de participation à la population active entraîne une évaluation incorrecte des différentes politiques envisagées et que la présence d'une marge de participation atténue les problèmes soulevés par Shimer.

*Classification JEL : E24, E32, E52*
*Classification de la Banque : Marchés du travail; Cycles et fluctuations économiques; Transmission de la politique monétaire*

## 1. Introduction

Recent empirical evidence has shown that movements in the labor force explain between one-fourth (Barnichon and Figura (2010)) and one-third (Elsby et al. (2013)) of the cyclical variation in the unemployment rate. This fact is in stark contrast with most recent dynamic stochastic general-equilibrium models with nominal rigidities and matching frictions. This paper builds an otherwise-standard New Keynesian model with matching frictions featuring a labor market participation decision, and compares it to an alternative model assuming an exogenously constant labor force.

We contribute to the literature by showing that even though the labor force is the least volatile among labor market variables, neglecting it might be very misleading. We first find that an exogenous participation model understates the volatility of labor market tightness, as compared to a model with an endogenous labor force. Hence, including a participation margin is helpful to address the Shimer critique. Then, we show that treating the labor force as constant, and thus invariant to policy, leads to incorrect evaluation of a change in the monetary policy rule and, more broadly, of a change in any policy shaping the response of aggregate demand to shocks. For a given participation rate, expanding aggregate demand affects labor market tightness and the unemployment rate by boosting vacancy posting and labor demand. However, it also affects labor market variables through the participation decision via two channels. On the one hand, a change in tightness makes job search more or less attractive, inducing the household to vary the size of the labor force. As a result, the policy change transmits to labor market variables by acting also on labor supply. On the other hand, by making search more or less attractive, policy affects workers' outside option, which feeds back on vacancy posting and thus on the initial policy effect on labor demand. As a consequence, policies cannot be evaluated abstracting from the endogenous response of participation.

We model entry to the labor market as the outcome of an optimal time-allocation problem among market work, housework and search activity, and use micro evidence to calibrate the household's opportunity cost of search in terms of forgone home production.[1]

In both the exogenous and the endogenous participation models, the marginal rate of substitution, and thus workers' outside option, is procyclical conditional on market productivity shocks. As pointed out by Chodorow-Reich and Karabarbounis (2013), this fact implies that a high relative average value of non-work to work activity does not necessarily address the Shimer critique, contrary to the proposal by Hagedorn and Manovskii (2008). However, in the endogenous participation model the household chooses participation so as to tie the marginal rate of substitution between market and home goods to labor market tightness, which indeed captures the opportunity cost of

[1] Changes in time devoted to home-related activity upon entering the labor force roughly match time spent in job search, as we document in Table 1.

home producing. At equilibrium, the marginal rate of substitution does not increase as much as under exogenous participation, because *ceteris paribus* the household is relatively more willing to substitute home production with market activity. As a result, firms find it cheaper to post vacancies and the unemployment rate is about five times more volatile than under constant participation.

Before conducting our policy analysis, we make sure that the model delivers correct predictions about the labor force once the volatility of other labor market variables is in line with the data. We allow for a combination of technology and preference shocks to ensure that both the endogenous and the exogenous models match key second moments of labor market variables.[2] Then, we validate our mechanism by showing that our model also accounts for observed fluctuations of the participation rate in the U.S. economy, a moment that we have *not* targeted.

Finally, we show that even if both models successfully match key second moments of labor market variables, the endogenous and the exogenous participation models deliver remarkably different implications for the effects of monetary policy on macroeconomic variables. For instance, the exogenous participation model overpredicts the surge in the unemployment rate volatility due to inflation stabilization, since it neglects the fact that most of the policy change is absorbed by the participation margin. Moreover, we also show that the welfare ranking of alternative monetary policy rules varies across models. Under our calibration, we find that a monetary policy rule assigning some weight to the unemployment rate is welfare improving compared to a rule that neither fully stabilizes inflation nor targets the unemployment rate. The opposite holds if participation is assumed to be constant.

Two papers in particular are related to ours. Galí (2010) considers a model of adjustment costs to employment and an endogenous labor force. Differently from us, he does not tackle the role of the participation margin for monetary policy and he assumes a large opportunity cost of search, as compared to the micro evidence, downplaying the importance of labor force fluctuations. Christiano et al. (2012) consider a New Keynesian model with endogenous search intensity. However, they abstract from matching frictions neglecting the mechanism we highlight here, which is entirely driven by the general-equilibrium interaction between participation decisions and vacancy posting.

The paper is organized as follows. In section 2 we describe our model economy, section 3 explains the calibration strategy, section 4 investigates the incentives driving the participation decision, section 5 performs the policy analysis and section 6 concludes.

[2]The contributions by Diamond (1982) and Mortensen and Pissarides (1999) spurred a rich literature investigating whether matching friction models are successful in replicating business cycle evidence. Various solutions to the Shimer critique have been proposed, such as staggered wage bargaining (Gertler and Trigari (2009)), fixed matching costs (Pissarides (2009)), and demand shocks (Sveen and Weinke (2008); Balleer (2012)).

## 2. The model

The economy is populated by a representative large household whose utility depends on market- and home-produced goods, firms producing a homogeneous intermediate good under perfect competition, and final retailers selling a differentiated market good under monopolistic competition. There are search frictions à la Diamond (1982) and Mortensen and Pissarides (1999) in the labor market, and the household's members can be employed, unemployed or non-participant. The employed engage in market production; the unemployed home-produce in the residual time unoccupied by job search; the non-participant is devoted solely to housework. In the intermediate-good sector, firms need to be matched with a household member in order to produce and, when searching, they are subject to a vacancy posting cost. Jobs can be exogenously discontinued at any time. In the final-goods sector prices are sticky, as in Calvo (1983).[3]

In this section we describe the primitives of the model and we refer to the appendix for most of the derivations.

### *2.1. Households*

The representative household consists of a mass 1 continuum of family members. The mass of employed, unemployed and non-participant members is denoted by $E_t$, $U_t$ and $L_t$, respectively. The pool of labor market participants is given by $N_t = 1 - L_t$, which can be interpreted as the participation rate.

In a generic period, say $t-1$, after all decisions have been taken and executed, a fraction $\rho$ of the employed are separated from their job. The unemployed, the non-participants and separated workers $U_{t-1} + L_{t-1} + \rho E_{t-1} = 1 - (1-\rho)E_{t-1}$ form the non-employment pool, out of which some members become searchers in the following period, $S_t$, and the remaining ones enter non-participation, $L_t$. Hence, we can write

$$S_t + L_t = 1 - (1-\rho)E_{t-1}; \qquad S_t = N_t - (1-\rho)E_{t-1}, \tag{1}$$

where $S_t \geq 0$, and $N_t \geq (1-\rho)E_{t-1}$. Since we do not model endogenous separation, the employed flowing out of the labor force cannot be more than the workers who separated in the previous period, while flows from unemployment to out of the labor force can be as large as $U_{t-1}$.[4] Denote the job-finding rate by $f_t$ and assume instantaneous hiring.[5]

[3]We use the two-sector set-up to keep matching frictions separated from price rigidity, as in Walsh (2005) and Trigari (2006).

[4]We assume that if the household wishes to reduce participation, there are always enough unemployed workers to choose from. This is the case if steady-state unemployment is large enough relative to shocks. We check ex post that the assumption holds at equilibrium.

[5]Period $t$ searchers can be matched and start producing in period $t$. This is a standard assumption in sticky-price models with no capital and exogenous separation, which keeps the model simple and seems to be reasonable if a period is interpreted as a quarter.

Therefore, $U_t \equiv (1-f_t)S_t$ and employment evolves as follows:

$$\begin{aligned} E_t &= (1-\rho)E_{t-1} + f_t S_t \\ &= (1-\rho)(1-f_t)E_{t-1} + f_t N_t \\ &= (1-\rho)E_{t-1} + \frac{f_t}{1-f_t}U_t, \end{aligned} \tag{2}$$

where the second equality follows from (1) and the third from the definition of an unemployed worker. Hence, if the household is allowed to choose $N_t$ conditional on the finding rate and the stock of non-employed $E_{t-1}$, she can also decide indirectly on $E_t$ and $U_t$ by suitably assigning members to search and non-participation.

The employed earn a nominal wage $W_t$ and the unemployed are entitled to real unemployment benefits $b$. Consumption risks across members are pooled and all decisions are collectively taken by the household.[6] She buys market goods $C_t(i)$ at price $P_t(i)$, $i \in [0,1]$, zero-coupon nominal bonds $D_t$ at a price $R_t^{-1}$, and pays lump-sum taxes $T_t$. Hence, the budget constraint is

$$\int_0^1 P_t(i)C_t(i)\ di + R_t^{-1}D_t \leq D_{t-1} + W_t E_t + P_t b U_t + T_t. \tag{3}$$

We assume that the household values consumption of both market- and home-produced goods according to[7]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ Z_t \log(C_t) + \phi \frac{h_t^{1+\nu}}{1+\nu} \right]; \quad \nu \leq 0 \tag{4}$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$, $\varepsilon \geq 1$, $P_t$ is the price of the market consumption bundle minimizing total nominal expenditure and $h_t$ represents home-produced goods.

We assume that home goods enter the utility function separably from market consumption as leisure does in most of the business cycle literature.[8] $\phi$ is a scaling parameter that can be chosen to match a target value for the steady-state participation rate, and $-\nu$ is the inverse intertemporal elasticity of substitution of home consumption. We allow for a preference shock, $Z_t$, where $\log(Z_t) = \rho_z \log(Z_{t-1}) + \xi_t^z$, and $\xi_t^z$ is an i.i.d. shock with zero mean and variance $\sigma_z^2$.

Home goods are the product of housework time. Since the responsiveness of time use to changes in market time is stable across labor market statuses, we focus on

[6] See Andolfatto (1996) and Merz (1995).

[7] As suggested by the microeconomic evidence, the market-housework margin is three to four times more elastic than the conventional one between consumption and leisure. Hence, for simplicity we abstract from the latter. See Aguiar et al. (2013) for further discussion in this respect.

[8] Separability has been motivated by early contributions, such as by Eichenbaum et al. (1988) and Campbell and Mankiw (1990).

the extensive margin.[9] After normalizing to one the housework time forgone by the employed, as compared to the non-participant, the search cost $\Gamma \in (0,1)$ is the fraction of housework time forgone by the unemployed relative to the employed. We assume a production function that has decreasing returns to scale:

$$h_t = [A_t^h(1 - E_t - \Gamma U_t)]^{1-\alpha_h}, \tag{5}$$

where $\alpha_h \in [0,1)$, $\log(A_t^h) = \rho_{ah}\log(A_{t-1}^h) + \xi_t^{ah}$, and $\xi_t^{ah}$ is an i.i.d. shock with zero mean and variance $\sigma_{ah}^2$.[10]

The household decides on $C_t(i)$, $h_t$, $D_t$, $E_t$, $U_t$, $S_t$, $L_t$ and $N_t$ for any $t$ and $i$ taking as given all prices, the finding rate, $E_{-1}$, and $D_{-1}$ in order to maximize (4) subject to (1), (2), (3), (5) as well as $L_t = 1 - E_t - U_t$ and $N_t = E_t + U_t$.

*2.2. Intermediate-good producers and final retailers*

As in the standard Mortensen and Pissarides (1999) framework, any intermediate-good firm $j \in [0,1]$ that is not matched with a worker at time $t$ might decide to post a vacancy and pay a cost of $\kappa$ units of the final good $C$. If the vacancy is filled, the firm immediately starts producing $X_t(j) = A_t$, where $\log(A_t) = \rho_a \log(A_{t-1}) + \xi_t^a$ and $\xi_t^a$ is an i.i.d. shock with zero mean, variance $\sigma_a^2$. The firm keeps producing until the job is exogenously discontinued. $P_t^x$ denotes the price of the intermediate good, and $V_t^J$ represents the value of a filled vacancy expressed in terms of the final consumption bundle.[11]

We model sticky-price producers à la Calvo (1983) following the standard textbook version, as in Galí (2008) or Woodford (2003). Any firm $i \in [0,1]$ produces a differentiated good using technology $Y_t(i) = X_t(i)^{1-\alpha}$ in a regime of monopolistic competition. In any period, with probability $1-\delta$ each firm has the chance to reset the price $P_t(i)$ so as to maximize profits. With probability $\delta$ the firm sticks to the price charged in the previous period.

*2.3. Employment and wages*

Searchers and job vacancies $V_t$ are matched according to a standard constant-returns-to-scale technology

$$M_t = \omega V_t^{1-\gamma} S_t^{\gamma}. \tag{6}$$

[9] For instance, see Aguiar et al. (2013).

[10] We also allow for positive correlation between innovations to home and market technology, denoted by $\rho_\xi$. Some types of technology might indeed be relevant for both the home and the market sector – for example, in the case of Internet-related innovations – and the quantity or the quality of home capital goods might covary with market productivity, as suggested by positive co-movements of home appliances consumption with market production (see the appendix for details). Since we abstract from capital accumulation for the sake of simplicity, those co-movements may be captured in reduced form by positive correlation of technology across sectors.

[11] See the appendix for the derivation.

Define labor market tightness as $\theta_t \equiv V_t/S_t$. Hence, participation and vacancy posting decisions jointly determine the finding rate $f_t = \omega\theta_t^{1-\gamma}$, the filling rate $q_t = \omega\theta_t^{-\gamma}$ and employment through (2).

After computing the surplus of employing one additional member, $V_t^w$, we assume that the wage is determined according to Nash bargaining so that $\eta V_t^w = (1-\eta)V_t^J$, where $\eta$ is the firm's bargaining power.[12]

## 3. Steady state and calibration

In this section, we explain the choice of parameter values other than the shocks, which we discuss in the following section. We restrict to a zero-inflation non-stochastic steady state.

Some parameters are relatively uncontroversial in the literature and are set to values consistent with microeconomic evidence and/or previous contributions. This is the case for $\rho$, $\beta$, $\alpha$, $\varepsilon$ and $\delta$, all displayed in Table 2.

A second group of parameters is calibrated so as to match steady-state values of labor market variables to their observed counterpart. This is the case for $\omega$ and $\phi$. Targeting the employment rate, the participation rate and the job-filling rate delivers the values reported in Table 2 and a steady-state value for the job-finding rate equal to 0.6572.[13]

Concerning the home sector, some parameters are hardly identifiable and we fix them in order to maintain symmetry with respect to the market sector and/or comparability with previous studies. For instance, we assume that $\alpha_h = \alpha = 1/3$. Our interpretation of $h$ as home production rather than leisure disconnects $\nu$ from the Frisch elasticity of labor supply. However, one can compute its steady-state value. Given $\alpha_h = 1/3$, $\nu = -5$ implies an elasticity of about 0.2, a value commonly used in the literature and in line with the microeconomic evidence.[14]

A subset of parameters is crucial for labor market dynamics and deserves more in-depth discussion. This is the case for $b$, $\eta$, $\gamma$, $\kappa$ and $\Gamma$: we calibrate these parameters by resorting to independent microeconomic evidence. First, from Petrongolo and Pissarides (2001) and Mortensen and Nagypal (2007), we know that $\eta$ has to lie on the interval $(0.3, 0.5)$. We choose the midpoint $\eta = 0.4$. The Hosios condition requires $\gamma = 0.6$.[15] We are left with $\kappa$, $\Gamma$ and $b$. Define the replacement rate as $RR \equiv bP/W$.

[12] The case of stochastic bargaining power does not affect our main results and we leave it to the appendix.

[13] We use U.S. quarterly data from Federal Reserve Economic Data (FRED II) on employment and the labor force over the period 1964Q1-2006Q3.

[14] In the appendix, we show derivations and values of labor supply elasticity for alternative parameterizations.

[15] In the appendix, we characterize Pareto efficiency and we show that the conventional Hosios condition applies to our model.

At the steady state, the following relations must hold:

$$\kappa = \frac{\varepsilon - 1}{\varepsilon}(1-\alpha)E^{-\alpha}\frac{q\bar{v}}{1+\bar{v}[1-\beta(1-\rho)]}; \quad \bar{v} \equiv \frac{\kappa}{q}\left(\frac{W}{P}\right)^{-1} \tag{7}$$

$$RR = \Gamma\left\{1 - \frac{1-\eta}{\eta}(1-f)\left[1-\beta(1-\rho)\right]\bar{v}\right\} - \frac{1-\eta}{\eta}f\bar{v} \tag{8}$$

$$b = \frac{\varepsilon - 1}{\varepsilon}(1-\alpha)E^{-\alpha}\frac{RR}{1+\bar{v}[1-\beta(1-\rho)]}, \tag{9}$$

where variables without a time subscript denote the steady state. We pin down $\kappa$ by targeting vacancy costs per filled job as a fraction of the real wage, $\bar{v}$, which has been estimated at about 0.045.[16]

If $\bar{v}$ is targeted, setting the search cost implies a value for the unemployment benefit and the replacement rate. Some information on $RR$ and $\Gamma$ is available. Petrongolo and Pissarides (2001) and Mortensen and Nagypal (2007) argue that replacement rates vary over the range $(0.2, 0.4)$. We use the American Time Use Survey (ATUS) to recover some information on $\Gamma$ by exploiting its home-production interpretation. The ATUS provides nationally representative estimates of how Americans spend their time supplying data on a wide range of non-market activities, from child care to volunteering.[17] Table 1 shows time devoted to home production, measured in minutes per day, depending on the employment status. $\Gamma = 0.44$ implies $RR = 0.4$ and is the only value that squares the evidence from Petrongolo and Pissarides (2001) and Mortensen and Nagypal (2007) with home-production data, through equation (8). Hence, we choose $b$ accordingly.

The implied relative average value of non-work to work activity is large and about 0.95. However, it depends only on the targets for the job-filling and the employment rates, and it is thus independent of all other parameters, including $\Gamma$, which, as we show below, drives the response of labor market variables to shocks.

## 4. Participation dynamics and labor market variables

The objective of this section is threefold: we illustrate how shocks propagate to labor market variables; we compare our model to an alternative one that assumes exogenous participation; and we evaluate our model against the data.

[16] We are following Hagedorn and Manovskii (2008) and Galí (2010).

[17] ATUS individuals are randomly selected from a subset of households that have completed their eighth and final month of interviews for the Current Population Survey (CPS). In the sample, we can observe minutes per day devoted to paid activities and home production for a cross-section of approximately 98,000 individuals over the period 2003-2009. In Table 1, we consider the whole sample and a shorter one excluding the years of the recent crisis.

*4.1. Endogenous participation*

Log-linearizing the model around the deterministic steady state eases intuition. Wage bargaining and the participation choice imply

$$\hat{\theta}_t = (1 - \Phi_b)\widehat{MRS}_t, \tag{10}$$

where the following definition applies:

$$\widehat{MRS}_t \equiv \left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\hat{h}_t + \widehat{C}_t + \widehat{A}_t^h - \widehat{Z}_t. \tag{11}$$

$\Phi_b$ is a steady-state efficiency wedge and variables with a hat denote log-deviations from the steady state.[18] Equation (10) states that the household chooses participation to make the marginal rate of substitution between market consumption and home production proportional to labor market tightness. Assume that tightness increases: the finding rate rises and so does the opportunity cost of home producing. At an optimum, the marginal rate of substitution also has to increase and, given market consumption, the household reduces home production by adjusting participation upward: according to a conventional substitution effect, participation increases in labor market tightness. On the other hand, for a given tightness, the desired home-production level increases in market consumption according to a conventional income effect. Hence, as long as market consumption increases with tightness at equilibrium, the participation rate can increase or decrease, depending on which of the two effects prevails.

Price stickiness introduces a conventional Phillips curve:

$$\widehat{\pi}_t = \beta E_t\{\widehat{\pi}_{t+1}\} - \lambda\widehat{\mu}_t, \tag{12}$$

where $\widehat{\pi}_t \equiv logP_t - logP_{t-1}$, $\widehat{r}_t \equiv \widehat{R}_t - E_t\{\widehat{\pi}_{t+1}\}$, $\lambda \equiv \frac{(1-\delta)(1-\beta\delta)}{\delta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$, and $\widehat{\mu}_t$ denotes the average price markup, which evolves according to

$$-\widehat{\mu}_t = \widehat{\chi}_t - \left[(1-\alpha)\widehat{A}_t - \alpha\widehat{E}_t\right], \tag{13}$$

where $\widehat{\chi}_t$ is the labor cost faced by intermediate-goods producers, including the option value of being in a match, and it equalizes the cost that final-goods producers have to pay for an additional unit of the intermediate good. The average price markup increases when the labor cost falls below the marginal factor product. Because of matching frictions, the labor cost can be written as[19]

$$\begin{aligned}\widehat{\chi}_t \equiv \frac{\varepsilon E^{\alpha}}{(1-\alpha)(\varepsilon-1)}\Bigg\{\frac{\gamma\kappa\theta}{(1-\gamma)f}\Big[\widehat{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widehat{\theta}_{t+1}\} \\ +\beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widehat{r}_t\Big] + (1-\Gamma)MRS\widehat{MRS}_t\Bigg\}.\end{aligned} \tag{14}$$

[18]In the appendix we show that at the Pareto efficient equilibrium, $\Phi_b = 0$.

[19]This is similar to Ravenna and Walsh (2011).

The value of being in a match, and thus the real wage, rises when the labor market is tight. Accordingly, labor cost increases and decreases in current and future tightness, respectively. Similarly, workers' outside option co-moves with the marginal rate of substitution and exerts upward pressure on the labor cost. When $\Gamma$ tends to 1, the contribution of an additional unemployed worker in the home sector is nil, and fluctuations of $\widehat{MRS}_t$ do not affect the outside option, the real wage, or the incentive to post vacancies. We close the model by assuming that the central bank follows a simple Taylor rule:

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t - \phi_u \widehat{u}_t, \tag{15}$$

where $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ is aggregate output and $u_t \equiv U_t/N_t$ is the unemployment rate. We assume that $\phi_\pi = 1.5$ and $\phi_Y = \phi_u = 0$, and we analyze impulse-response functions to technology and preference shocks.[20]

As shown in Figure 1, after a market technology shock, tightness increases and participation drops, making the labor market much tighter. The unemployment rate falls and employment is boosted. Because of sticky prices, output and tightness cannot increase as much as under flexible prices and the cost of labor falls below its marginal product, with the result that inflation falls. Conditional on market technology shocks, the income effect prevails and participation is countercyclical. A preference shock exogenously reduces $\widehat{MRS}_t$. The outside option, and thus the bargaining position of workers, worsens and tightness increases because of vacancy posting. The household's response is driven by a substitution effect and participation adjusts upward. At general equilibrium, the unemployment rate falls as the surge in the labor force does not outbalance the larger finding rate. After a shock to home technology, it is feasible to home produce more without reducing the labor force. The income effect drives the household's decision and the surge in participation is so large that both employment and the unemployment rate increase on impact. Finally, note that when home and market technology are positively correlated, participation and employment increase after a productivity shock, while the unemployment rate falls.[21]

*4.2. Endogenous versus exogenous participation*

To emphasize the role of the participation decision, we compare our model to an alternative one, where the household values and produces home goods according to (4) and (5), respectively, but participation is constrained by $N_t = N \in (0, 1)$. Any part of the model other than the household's decision problem is not affected by the additional constraint. Hence, equilibrium conditions are the same across models up to equation

[20] We consider more general rules when performing our policy analysis.

[21] Similarly, Campbell and Ludvigson (2001) assume a positive correlation between market and home technology shocks in a real business cycle model with home production so as to match the procyclicality of hours worked.

(10), which fails under exogenous participation. We calibrate the exogenous participation model as described above and we obtain the same parameter values reported in Table 2. Hence, differences across models are entirely attributable to the propagation channel created by the participation margin. Figure 2 plots impulse responses of the participation rate, the marginal rate of substitution and labor market tightness to a market technology shock for different values of the search cost in both models.

Three facts are important. First, labor market volatility increases in $\Gamma$ in the exogenous participation model. After a market technology shock, the marginal rate of substitution is procyclical. Hence, the outside option is also procyclical and it is so to a smaller extent the larger is $\Gamma$. This fact discourages vacancy posting. Chodorow-Reich and Karabarbounis (2013) argue that the way out of the Shimer puzzle proposed by Hagedorn and Manovskii (2008) is not viable if an outside option as procyclical as in the data is incorporated in a standard model with matching frictions. For empirically plausible values of $\Gamma$, our finding is in line with Chodorow-Reich and Karabarbounis (2013). Second, the relation between labor market volatility and the search cost is inverted in the case of endogenous participation. When $\Gamma$ is small, moving members in and out of the labor force is less costly; the volatility of participation increases and it transmits to labor market tightness. The marginal rate of substitution and the outside option move consistently. Hence, the presence of a participation margin helps mitigate the Shimer critique. Indeed, conditional on market technology shocks, the volatility of the unemployment rate relative to output is 5.40, against only 0.12 under exogenous participation.[22] Finally, and related to the last fact, when the search cost is large, the volatility of participation falls dramatically and the two models move closer in terms of labor market volatility.

### *4.3. Second moments*

The conditional evidence on the response of the participation rate to technology shocks is controversial. Galí (2010) finds a negative response by using conventional long-run restrictions to identify shocks to productivity. Christiano et al. (2012) find the opposite under the same identification strategy but using a different specification of the vector autoregression.[23] Hence, we opt to assess the model by examining the unconditional evidence.

We set the serial correlation of all shocks to 0.9 and then we calibrate their standard deviation as well as $\rho_\xi$ to minimize the average distance of simulated unconditional moments from their empirical counterparts. We target the standard deviation of output, the standard deviation of employment, and that of the unemployment rate relative to

[22] See Table 4, column R2.

[23] Christiano et al. (2012) rationalize their finding with a variable search effort model that delivers procyclical search intensity as an equilibrium outcome. However, Mukoyama et al. (2013) examine evidence from ATUS and CPS and argue that, in the data, search effort is countercyclical.

output and the correlation of the unemployment rate with output.[24] We determine parameters simultaneously by performing a grid search. Then, we assess our model by examining the standard deviation of the participation rate relative to output and the correlation of the participation rate with output, two moments that we have *not* targeted. We repeat the calibration exercise for the exogenous participation model. Table 3 reports the empirical and the simulated moments for both models. By construction, they both account well for fluctuations in targeted labor market variables. In addition, the participation dynamics implied by our model is well in line with the data.

## 5. Participation and monetary policy

In this section, we show that the endogenous and the exogenous participation models deliver remarkably different implications for the effects of monetary policy on labor market variables, and different rankings of alternative monetary policy rules.

We start by considering a simplified version of the model that assumes efficiency at the steady state.[25] Following a linear-quadratic approach, it is straightforward to show after some algebra that, in both models, price stability implements Pareto efficiency and is thus optimal.[26]

The calibrated version of the model features steady-state distortions. For this case, we resort to a conventional second-order perturbation method and evaluate the performance of alternative simple rules in the class defined by (15) against strict inflation targeting, i.e. $\pi_t = 0$ at all times. Table 4 shows the policy rules we consider and the results conditional on each shock.[27] As compared to a strict target on inflation, both rules $R2$ and $R3$ always rank lower. Thus, strict inflation targeting provides a good benchmark against which to evaluate the simple rules.

We can draw several conclusions. First, a rule that neither fully stabilizes inflation nor targets the unemployment rate, $R2$, is suboptimal in both models, and switching to strict inflation targeting is always welfare improving. However, the predicted change in volatilities associated with the policy switch differs across models. Conditional on technology shocks, in both models a strict target on inflation magnifies the volatility of labor market variables, since it eliminates inefficient fluctuations in price markups and

[24]We apply a Hodrick-Prescott filter with a conventional smoothing parameter of 1,600 to extract the business-cycle component from seasonally adjusted data.

[25]We show in the appendix that this is the case when the unemployment benefit is zero and final-goods production is appropriately subsidized to undo the monopolistic distortion.

[26]We also show that the introduction of bargaining shocks changes our result only marginally, since strict inflation targeting delivers negligible welfare losses. This finding extends the results of Ravenna and Walsh (2011) to an endogenous participation model.

[27]For the sake of brevity, and to emphasize that our results are robust to the calibration of shocks, we report only conditional moments. In the appendix we report unconditional moments and the associated welfare losses. Also, we do not show results referring to a classical Taylor rule, assuming that $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, and $\phi_u = 0$, because it is always dominated by the others.

boosts the response of aggregate demand and, through vacancy posting, of labor market tightness. However, the participation rate, which reacts to tightness, also becomes more responsive under a strict inflation target. Hence, the exogenous participation model overstates the change, since it neglects the surge in volatility of the countercyclical movement in participation that counterbalances the rise in vacancy posting. A similar argument applies to home technology and preference shocks. If participation is exogenous, the policy switch affects the volatility of output, but not the volatility of labor market variables relative to output.[28] In contrast, if the participation rate is free to move, the volatility of the unemployment rate falls after the policy change. Overall, under our calibration of the shocks, the endogenous participation model predicts that after switching to strict inflation targeting the volatility of the unemployment rate will fall, while the exogenous participation model predicts the opposite.

Second, whether a rule assigning some weight to the unemployment rate, $R3$, ranks higher or lower than $R2$ largely depends on the joint behavior of inflation, unemployment and participation. Under market technology shocks, $R3$ performs better than $R2$ only if participation is constant. After a market technology shock, inflation falls and price markups increase. In the exogenous model, targeting the unemployment rate destabilizes inflation and markups but it helps stabilize employment, and overall $R3$ ranks higher. However, if participation is free to reallocate labor from the market to the home sector, employment is stabilized automatically by the more-volatile participation rate, the concern for price stability prevails and $R2$ ranks higher. The rank inverts under home technology shocks which, acting as positive labor supply shocks, tend to reduce the labor cost and increase price markups.[29] Conditional on those shocks, the unemployment rate is procyclical under endogenous participation and countercyclical under exogenous participation, making $R3$ more accommodative in the former case but tighter in the latter. Hence, only in the former case does some weight on unemployment help sustain aggregate demand and thus contain price markup volatility. Finally, under preference shocks, the unemployment rate is countercyclical in both models, while inflation increases after the shock. Now $R3$ is always preferred to $R2$ because, given the behavior of the unemployment rate, it allows inflation and price markup volatility to be contained. Overall, under our calibration of the shocks, the endogenous participation model predicts that $R3$ ranks higher than $R2$, while the opposite holds if participation is constant.

## 6. Conclusions

We incorporate a labor market participation decision in an otherwise-standard New Keynesian model with matching frictions. We show that monetary policy, by affecting

[28] Looking at the production functions, it is clear that the volatility of employment relative to output has to be equal to $1/(1-\alpha) = 1.5$, while for the unemployment rate the coefficient of proportionality is $E/(U(1-\alpha)) = 23.97$.

[29] This fact is made evident by the negative response of inflation shown in Figure 1.

vacancy posting, also affects the incentive to participate in the labor market which, in turn, feeds back into the vacancy posting decisions by changing the workers' outside option (via changes in the marginal rate of substitution between home and market goods). As a consequence, the effects and desirability of alternative policies cannot be evaluated abstracting from the participation margin. Indeed, the ranking of simple monetary policy rules against strict inflation targeting can be reversed across models. Finally, we contribute to the literature by showing that the presence of a participation margin boosts the volatility of both labor market tightness and the unemployment rate, thereby mitigating the Shimer critique.

Table 1: Time allocated to home production and search activity (minutes per day). Data are from the American Time Use Survey (ATUS) and were collected over the period 2003-2009.

| Status | 2003-2009 | | 2003-2006 | |
|---|---|---|---|---|
| | Home Prod. | Search | Home Prod. | Search |
| Employed | 119 | 0.52 | 118 | 0.49 |
| Unemployed | 154 | 23 | 155 | 20 |
| Not in labor force | 178 | 0.37 | 182 | 0.28 |
| Search cost $\Gamma$ | 0.41 | | 0.44 | |

Table 2: Calibration of all parameters other than policy and shock parameters. We use U.S. quarterly data from Federal Reserve Economic Data (FRED II) on employment and the labor force over the period 1964Q1-2006Q3 to compute the steady state of the employment and participation rates.

| Mnemonic | Value | Target/Source |
|---|---|---|
| $\beta$ | 0.99 | 4% average real return |
| $\epsilon$ | 6 | 20% price markup |
| $\delta$ | 2/3 | Price duration |
| $\rho$ | 0.12 | Shimer (2005) |
| $\alpha$ | 1/3 | Galí (2010) |
| $\alpha_h$ | 1/3 | Symmetry with market sector |
| $\nu$ | -5 | 0.2 Frisch elasticity |
| $\omega$ | 0.66 | 94% employment rate |
| | | 2/3 job-filling rate |
| $\phi$ | 0.04 | 64% participation rate |
| $\eta$ | 0.4 | Petrongolo and Pissarides (2001)<br>Mortensen and Nagypal (2007) |
| $\gamma$ | 0.6 | Hosios condition |
| $\kappa$ | 0.0196 | |
| $\Gamma$ | 0.44 | $\bar{v} = 0.045$ - Galí (2010)<br>ATUS |
| $b$ | 0.2617 | |

Table 3: Selected unconditional moments. We compare standard deviations (relative to output) of selected variables in the endogenous and exogenous participation models, and in the data. Volatilities are expressed in percentage standard deviations. The parameters of the shocks have been chosen so as to give the best possible fit for the first four moments. Data moments are computed using U.S. quarterly data from Federal Reserve Economic Data (FRED II), to which we applied a standard Hodrick-Prescott filter.

| Unconditional Moments | Data | Endogenous | Exogenous |
|---|---|---|---|
| Output volatility | 1.53 | 1.43 | 1.56 |
| Unemployment rate volatility | 7.36 | 7.36 | 7.55 |
| Employment volatility | 0.63 | 0.67 | 0.47 |
| Correlation of unempl. rate with output | -0.85 | -0.76 | -1 |
| Participation rate volatility | 0.20 | 0.24 | - |
| Correlation of participation with output | 0.42 | 0.56 | - |
| Calibrated Parameters | | | |
| $\sigma_a$ | | 0.0070 | 0.0070 |
| $\sigma_{ah}$ | | 0.0037 | 0.0074 |
| $\sigma_z$ | | 0.0147 | 0 |
| $\rho_\xi$ | | 0.9474 | 1 |

Table 4: Conditional moments and welfare losses. We compare standard deviations (relative to output) of selected variables in the endogenous and exogenous participation models under three rules. Volatilities are expressed in percentage standard deviations. We also report welfare losses (in terms of steady-state consumption) of deviating from strict inflation targeting (R1) and adopting one of the two alternative rules. R2: $\phi_\pi = 1.5$ and $\phi_y = \phi_u = 0$. R3: $\phi_\pi = 1.5$, $\phi_y = 0$ and $\phi_u = 0.5/4$.

| | Endogenous | | | Exogenous | | |
|---|---|---|---|---|---|---|
| | R1 | R2 | R3 | R1 | R2 | R3 |
| Market Technology Shocks (i.e., $\sigma_a = 0.01$, $\sigma_{ah} = \sigma_z = 0$) | | | | | | |
| Output (%) | 1.77 | 1.61 | 1.42 | 1.67 | 1.54 | 1.53 |
| Unemployment rate | 8.34 | 5.40 | 1.11 | 2.07 | 0.12 | 0.03 |
| Employment | 0.20 | 0.07 | 0.12 | 0.13 | 0.01 | 0.002 |
| Participation rate | 0.32 | 0.27 | 0.19 | | | |
| Welfare Losses | | 0.0155 | 0.0205 | | 0.0319 | 0.0093 |
| Home Technology Shocks (i.e., $\sigma_{ah} = 0.01$, $\sigma_a = \sigma_z = 0$) | | | | | | |
| Output (%) | 0.53 | 0.50 | 0.60 | 0.70 | 0.66 | 0.19 |
| Unemployment rate | 7.28 | 9.63 | 1.49 | 23.97 | 23.97 | 23.97 |
| Employment volatility | 1.50 | 1.50 | 1.50 | 1.5 | 1.5 | 1.5 |
| Participation rate | 1.90 | 2.10 | 1.59 | | | |
| Welfare Losses | | 0.0068 | 0.0013 | | 0.0033 | 0.1293 |
| Preference Shocks (i.e., $\sigma_z = 0.01$, $\sigma_a = \sigma_{ah} = 0$) | | | | | | |
| Output (%) | 0.20 | 0.41 | 0.27 | 0.26 | 0.44 | 0.12 |
| Unemployment rate | 7.88 | 15.91 | 4.38 | 23.97 | 23.97 | 23.97 |
| Employment | 1.50 | 1.50 | 1.50 | 1.5 | 1.5 | 1.5 |
| Participation rate | 1.94 | 0.52 | 1.23 | | | |
| Welfare Losses | | 0.7250 | 0.0100 | | 0.5743 | 0.1261 |

Figure 1: Impulse responses to a one percent market technology (“Mkt”), preference (“Pref”), and home technology (“Home”) shock. “Mix”is the response of simultaneous shocks to market and home technology of 1 and 0.5 percent, respectively.

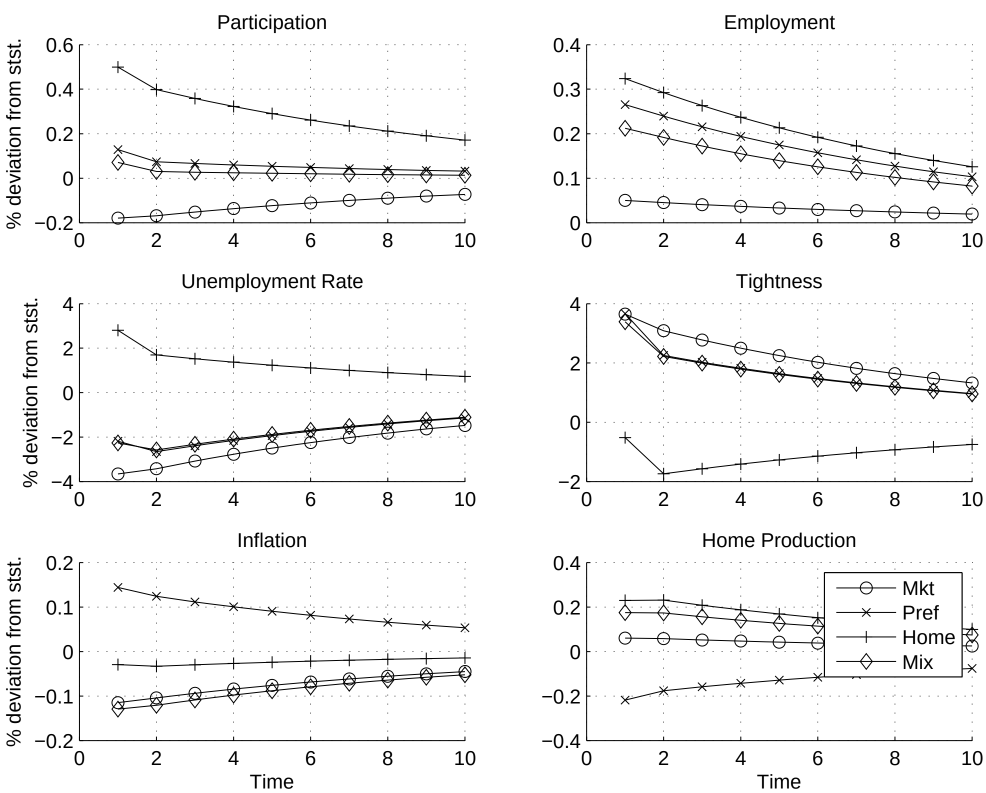
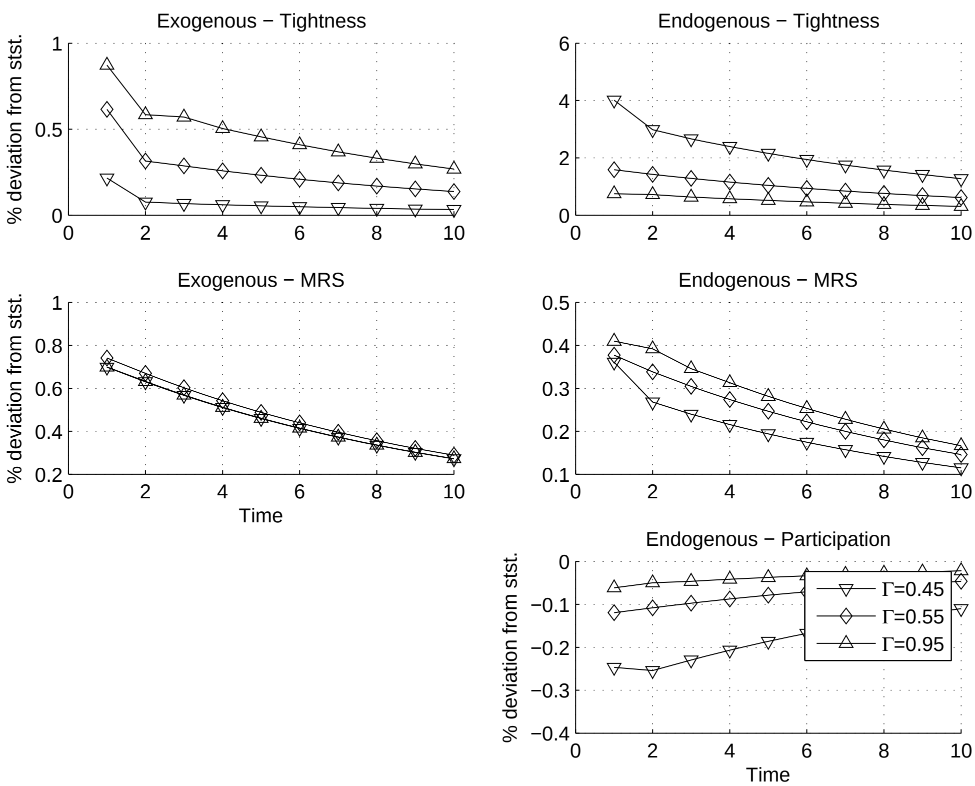


Figure 2: Impulse responses of labor market tightness, the marginal rate of substitution, and the participation rate to a one percent market technology shock in the exogenous (left panel) and endogenous (right panel) participation model for different values of Γ.



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## Appendix A. Endogenous participation model

### *Appendix A.1. Households*

Solving the household's optimization problem we obtain a conventional Euler equation

$$\beta R_t E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\} = 1, \tag{A.1}$$

and the following participation condition:

$$\begin{aligned} \left[ \frac{1-f_t}{f_t} \right] (\Gamma MRS_t - b) = \frac{W_t}{P_t} - MRS_t + \\ (1-\rho) E_t \left\{ Q_{t,t+1} \frac{(1-f_{t+1})}{f_{t+1}} (\Gamma MRS_{t+1} - b) \right\}, \end{aligned} \tag{A.2}$$

where $Q_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t}$, and $MRS_t \equiv \frac{\phi h_t^{\nu} C_t}{Z_t} A_t^h (1-\alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}}$. Note that, absent matching frictions, $f_t = 1$ $\forall t$ and equation (A.2) simplifies to a standard labor supply where the real wage equals the marginal rate of substitution between market consumption and home production.

### *Appendix A.2. Intermediate-good producers*

For firms in the intermediate sector, the value of a filled vacancy, $V_t^J$, expressed in terms of the final consumption bundle $P_t$, is given by

$$V_t^J = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1-\rho) E_t \left\{ Q_{t,t+1} V_{t+1}^J \right\}. \tag{A.3}$$

The free-entry condition ensures that

$$\frac{\kappa}{q_t} = V_t^J. \tag{A.4}$$

Substituting (A.4) into (A.3) gives the job-creation condition

$$\frac{\kappa}{q_t} = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1-\rho) E_t \left\{ Q_{t,t+1} \frac{\kappa}{q_{t+1}} \right\}. \tag{A.5}$$

### *Appendix A.3. Final retailers*

Final retailers face a downward-sloping demand function

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\varepsilon} [C_t + \kappa V_t]. \tag{A.6}$$

When price rigidity à la Calvo (1983) is assumed, optimal pricing for a firm allowed to reoptimize in $t$ requires

$$\sum_{T=0}^{\infty} \delta^T E_t \left\{ Q_{t,t+T} \frac{Y_{t+T}(i)}{P_{t+T}} \left[ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} \tau_p MC_{t+T}(i) \right] \right\} = 0, \tag{A.7}$$

where $1-\delta$ represents the probability of reoptimizing and $\tau_p < 1$ is a production subsidy. Log-linearizing (A.7) around the zero-inflation symmetric steady state, we obtain the New Keynesian Phillips Curve (NKPC)

$$\widehat{\pi}_t = \beta E_t\{\widehat{\pi}_{t+1}\} + \lambda \widehat{mc}_t, \tag{A.8}$$

where $\lambda \equiv \frac{(1-\delta)(1-\beta\delta)}{\delta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$ and lowercase variables with a hat represent log-deviations from steady state.

*Appendix A.4. Wage determination*

Define the value function associated with the household's optimization problem as

$$\mathcal{V}_t = \max_{E_t, D_t} \{\mathcal{U}_t + \beta E_t \{\mathcal{V}_{t+1}\}\}\,; \quad \mathcal{U}_t \equiv Z_t \log(C_t) + \phi \frac{h_t^{1+\nu}}{1+\nu}, \tag{A.9}$$

where constraints

$$S_t = N_t - (1-\rho)E_{t-1} \tag{A.10}$$

$$\begin{aligned} E_t &= (1-\rho)E_{t-1} + f_t S_t \\ &= (1-\rho)(1-f_t)E_{t-1} + f_t N_t \\ &= (1-\rho)E_{t-1} + \frac{f_t}{1-f_t}U_t \end{aligned} \tag{A.11}$$

$$\int_0^1 P_t(i)C_t(i)\, di + R_t^{-1}D_t \leq D_{t-1} + W_t E_t + P_t b U_t + T_t \tag{A.12}$$

$$h_t = [A_t^h(1 - E_t - \Gamma U_t)]^{1-\alpha_h} \tag{A.13}$$

$$L_t = 1 - E_t - U_t \tag{A.14}$$

$$N_t = E_t + U_t \tag{A.15}$$

have been taken into account. The envelope condition and the first-order condition with respect to employment are

$$\frac{\partial \mathcal{V}_{t+1}}{\partial E_t} = (1-\rho)\frac{Z_{t+1}}{C_{t+1}}\frac{1-f_{t+1}}{f_{t+1}}\left(\Gamma MRS_{t+1} - b\right), \tag{A.16}$$

$$\begin{aligned} \frac{\partial \mathcal{V}_t}{\partial E_t} &= \frac{\partial \mathcal{U}_t}{\partial E_t} + \beta \frac{\partial \mathcal{V}_{t+1}}{\partial E_t} \\ &= \frac{Z_t}{C_t}\left[\frac{W_t}{P_t} + \frac{(b - \Gamma MRS_t)(1-f_t)}{f_t} - MRS_t\right] + \beta \frac{\partial \mathcal{V}_{t+1}}{\partial E_t}, \end{aligned} \tag{A.17}$$

respectively.[30] The household bargains the wage on behalf of her members as soon as one of them meets a firm. Such an event happens conditional on incurring the search

[30] By substituting (A.16) into (A.17), one gets the participation condition (A.2).

cost $\Gamma$. Equivalently, if the household and the firm do not reach an agreement and deviate from equilibrium, the member enters the unemployment rather than the employment pool after the participation rate has been chosen. Therefore, we compute the surplus of employing one additional member, $V_t^w$, by keeping constant the participation rate at $t$:

$$\begin{aligned} V_t^w \equiv \left.\frac{\partial \mathcal{V}_t}{\partial E_t}\right|_{N_t=\bar{N}} &= \left.\frac{\partial \mathcal{U}_t}{\partial E_t}\right|_{N_t=\bar{N}} + \beta \frac{\partial \mathcal{V}_{t+1}}{\partial E_t} \\ &= \frac{W_t}{P_t} - b - \frac{\phi h_t^{\nu}(1-\Gamma)C_t}{Z_t} A_t^h (1-\alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} + \\ & E_t\left\{Q_{t,t+1}(1-\rho)(1-f_{t+1})V_{t+1}^w\right\}, \end{aligned} \tag{A.18}$$

where (A.18) assumes a constant participation rate at time $t$ and the second equality makes use of (A.16). The surplus from the match is split according to Nash bargaining:

$$\eta_t V_t^w = (1-\eta_t) V_t^J. \tag{A.19}$$

$\eta_t$ denotes the bargaining power of firms, is exogenous and evolves according to

$$\log(\eta_t) = (1-\rho_\eta)\log(\eta) + \rho_\eta \log(\eta_{t-1}) + \xi_t^\eta, \tag{A.20}$$

where $\eta \in (0,1)$ and $\xi_t^\eta$ is an i.i.d. shock with zero mean and variance $\sigma_\eta^2$. The equations in the paper refer to the simplified version with no shocks to the bargaining power, i.e. $\eta_t = \eta \ \forall t$. Using the definitions of $V_t^J$ and $V_t^w$ in (A.19), together with the free-entry (A.4) and the job-creation condition (A.5), it is possible to derive the wage equation:

$$\begin{aligned} \frac{W_t}{P_t} &= (1-\eta_t)\frac{P_t^x}{P_t}A_t + \eta_t\left[b + (1-\Gamma)MRS_t\right] + \\ (1-\rho)&E_t\left\{\beta \frac{C_t}{C_{t+1}}\frac{Z_{t+1}}{Z_t}\kappa\theta_{t+1}(1-\eta_{t+1})\right\}. \end{aligned} \tag{A.21}$$

*Appendix A.5. Market clearing conditions*

The aggregate production of the intermediate sector is given by

$$X_t = \int_0^1 X_t(j)dj = A_t E_t. \tag{A.22}$$

Integrating the demand for good $i$ (A.6) yields the conventional aggregate resource constraint

$$Y_t = C_t + \kappa V_t, \tag{A.23}$$

after defining aggregate output as

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.24}$$

Combining the demand for final goods (A.6) with their production function and integrating delivers the aggregate production function

$$Y_t = X_t^{1-\alpha} \Delta_t^{\alpha-1}, \tag{A.25}$$

where the following definition applies:

$$\Delta_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} di, \tag{A.26}$$

and $\Delta_t$, bounded by 1 from below, is a measure of price dispersion.

## Appendix B. The planner's solution

In this section we first define the social planner's problem for both the endogenous and the exogenous participation models. Then, after reducing the dimension of the problems, we show that the necessary conditions for efficiency can be nested into a single system of equations that applies to both models. This fact eases the derivation of the welfare function. Finally, we compute a first-order approximation of efficiency conditions around the non-stochastic steady state.

*Appendix B.1. Defining efficiency in the exogenous participation model*

The planner chooses $\{C_t, h_t, E_t, U_t, N_t, V_t, S_t, f_t\}_{t=0}^{\infty}$ in order to maximize

$$E_0 \sum_{t=0}^{\infty} \left[ Z_t \log(C_t) + \phi \frac{h_t^{1+\nu}}{1+\nu} \right], \tag{B.1}$$

subject to

$$E_t = (1-\rho) E_{t-1} + f_t S_t \tag{B.2}$$

$$f_t = \omega \left( \frac{V_t}{S_t} \right)^{1-\gamma} \tag{B.3}$$

$$S_t = E_t + U_t - (1-\rho) E_{t-1} \tag{B.4}$$

$$E_t + U_t = N_t \tag{B.5}$$

$$(A_t E_t)^{1-\alpha} = C_t + \kappa V_t \tag{B.6}$$

$$h_t = [A_t^h (1 - E_t - \Gamma U_t)]^{1-\alpha_h}, \tag{B.7}$$

and to the additional constraint that $N_t = N$, where $N < 1$ is exogenously given.

*Appendix B.2. Defining efficiency in the endogenous participation model*

The planner chooses $\{C_t, h_t, E_t, U_t, N_t, V_t, S_t, f_t\}_{t=0}^{\infty}$ in order to maximize (B.1) subject to (B.2)-(B.7). We restrict parameter $\phi$ and the stochastic processes for exogenous shocks so that there exists an interior solution to the endogenous participation problem, i.e., $N_t^* < 1$ for all $t$.

*Appendix B.3. Characterizing efficiency*

Before solving the social planner's problem, it is useful to reduce the dimension of the system. Combine equations (B.2)-(B.7) to obtain

$$C_t = (A_t E_t)^{1-\alpha} - \frac{\kappa}{\omega}\theta_t^{\gamma}\left[E_t - (1-\rho)E_{t-1}\right], \tag{B.8}$$

$$h_t^{\frac{1}{1-\alpha_h}} = A_t^h\left(1 - E_t - \Gamma\frac{1-\omega\theta_t^{1-\gamma}}{\omega\theta_t^{1-\gamma}}E_t + \Gamma(1-\rho)\frac{1-\omega\theta_t^{1-\gamma}}{\omega\theta_t^{1-\gamma}}E_{t-1}\right), \tag{B.9}$$

and

$$N_t = \frac{1}{\omega\theta_t^{1-\gamma}}E_t - \frac{(1-\rho)(1-\omega\theta_t^{1-\gamma})}{\omega\theta_t^{1-\gamma}}E_{t-1}. \tag{B.10}$$

We are now ready to restate the definition of efficiency in a more compact form. An allocation $\{C_t^*, h_t^*, \theta_t^*, E_t^*, N_t^*\}_{t=0}^{\infty}$ is efficient in the exogenous participation model if it maximizes (B.1) subject to (B.8), (B.9), (B.10) and $N_t = N$, where $N < 1$ is exogenously given. An allocation $\{C_t^*, h_t^*, \theta_t^*, E_t^*, N_t^*\}_{t=0}^{\infty}$ is efficient in the endogenous participation model if it maximizes (B.1) subject to (B.8), (B.9), (B.10) and $N_t < 1$. The problem can be solved in two steps: first choose $\{C_t^*, h_t^*, \theta_t^*, E_t^*\}_{t=0}^{\infty}$ so as to maximize (B.1) subject to (B.8) and (B.9); then, pick the participation rate that, given $E_t^*$, $E_{t-1}^*$, and $\theta_t^*$, satisfies (B.10). By definition, (B.10) is redundant in the first-stage problem and, if included, the associated Lagrangian multiplier must be equal to zero. In contrast, after substituting $N_t = N$, the constraint is binding in the exogenous participation problem. Now use (B.8) and (B.9) to substitute for $C_t^*$ and $h_t^*$ in (B.1) and let $\lambda_t$ be the Lagrange multiplier associated with (B.10). Then, the first-order conditions (FOCs) associated with the exogenous participation model can be written as

- FOC w.r.t. $\theta_t^*$:

$$\kappa\theta_t^*\frac{\gamma}{1-\gamma} = \Gamma MRS_t^* - \lambda_t\frac{C_t^*}{Z_t}. \tag{B.11}$$

- FOC w.r.t. $E_t^*$:

$$\frac{\kappa}{\omega}(\theta_t^*)^\gamma + MRS_t^*\Gamma\frac{1-f_t^*}{f_t^*} = (1-\alpha)A_t^{1-\alpha}(E_t^*)^{-\alpha} - MRS_t^*$$
$$+\beta(1-\rho)E_t\left\{\frac{C_t^*}{C_{t+1}^*}\frac{Z_{t+1}}{Z_t}\left[\frac{\kappa}{\omega}(\theta_{t+1}^*)^\gamma + \Gamma\frac{1-f_{t+1}^*}{f_{t+1}^*}MRS_{t+1}^*\right]\right\}$$
$$+\lambda_t\frac{C_t^*}{Z_t}\left(1+\frac{1-f_t^*}{f_t^*}\right) - \beta(1-\rho)E_t\left\{\lambda_{t+1}\frac{C_t^*}{Z_t}\frac{1-f_{t+1}^*}{f_{t+1}^*}\right\}. \tag{B.12}$$

Equations (B.11) and (B.12) apply to both models, but only under endogenous participation $\lambda_t = 0$, which follows from optimizing with respect to $N_t$.[31]

*Appendix B.4. Efficient steady state*

At the steady state, we assume that $A = A^h = Z = 1$. At the efficient steady state, equation (B.11) becomes

$$\kappa\theta\frac{\gamma}{1-\gamma} = \Gamma MRS - \lambda C, \tag{B.13}$$

while the dynamic efficiency condition (B.12) can be rewritten as follows:

$$(1-\beta(1-\rho))\left[\frac{\kappa\theta^\gamma}{\omega} + MRS\Gamma\frac{1-f}{f}\right] = (1-\alpha)E^{-\alpha} - MRS+$$
$$\frac{\lambda C}{f}\left[1-\beta(1-\rho)(1-f)\right]. \tag{B.14}$$

For the model with endogenous participation we also have

$$\lambda = 0. \tag{B.15}$$

*Appendix B.5. First-order approximation of the efficient solution*

In the derivations of the welfare function we will need a first-order approximation of (B.11) and (B.12) around the efficient steady state. Log-linearizing (B.11) we obtain

$$\kappa\frac{\gamma}{1-\gamma}\theta\hat{\theta}_t^* = \Gamma MRS\widehat{MRS}_t^* - \lambda C\left(\hat{\lambda}_t + \hat{C}_t^* - \hat{Z}_t\right). \tag{B.16}$$

Log-linearizing (B.12) we obtain

$$MRS\frac{1-f}{f}\Gamma\widehat{MRS}_t^* = (1-\alpha)^2E^{-\alpha}\hat{A}_t - (1-\alpha)\alpha E^{-\alpha}\hat{E}_t^* - MRS\widehat{MRS}_t^*$$
$$+(1-\rho)\beta\left[\frac{1-f}{f}\Gamma MRS + \frac{\kappa\theta^\gamma}{\omega}\right]E_t\left\{\hat{Q}_{t,t+1}^*\right\} + \beta(1-\rho)\Gamma MRS\frac{1-f}{f}E_t\left\{\widehat{MRS}_{t+1}^*\right\}$$
$$+\frac{C\lambda}{f}\hat{\lambda}_t - \beta(1-\rho)C\lambda\frac{1-f}{f}E_t\{\hat{\lambda}_{t+1}\} + \lambda C\left[\frac{1}{f} - \beta(1-\rho)\frac{1-f}{f}\right]\left(\hat{C}_t^* - \hat{Z}_t\right). \tag{B.17}$$

[31]Obviously, if the endogenous participation problem has a corner solution, the value of the Lagrange multiplier associated with (B.10) is non-zero and its solution coincides with that of the exogenous participation model for $N = 1$. Here, we restrict to cases where an interior solution exists.

We can use (B.16) to substitute out $\hat{\lambda}_t$ from (B.17)

$$
\begin{aligned}
MRS\frac{1-f}{f}\Gamma\widehat{MRS}_t^* &= (1-\alpha)^2E^{-\alpha}\hat{A}_t-(1-\alpha)\alpha E^{-\alpha}\hat{E}_t^*-MRS\widehat{MRS}_t^* \\
&+(1-\rho)\beta\left[\frac{1-f}{f}\Gamma MRS+\frac{\kappa\theta^\gamma}{\omega}-\frac{1-f}{f}\lambda C\right]E_t\left\{\hat{Q}_{t,t+1}^*\right\} \\
&+\Gamma MRS\frac{1}{f}\widehat{MRS}_t^*-\frac{\kappa\theta}{f}\frac{\gamma}{1-\gamma}\hat{\theta}_t^*+\beta(1-\rho)\frac{1-f}{f}\kappa\theta\frac{\gamma}{1-\gamma}E_t\left\{\hat{\theta}_{t+1}^*\right\}.
\end{aligned}
\tag{B.18}
$$

Using the steady-state relation (B.13), equation (B.18) can be further simplified to

$$
\begin{aligned}
(1-\Gamma)\,MRS\widehat{MRS}_t^* &= (1-\alpha)^2E^{-\alpha}\hat{A}_t-(1-\alpha)\alpha E^{-\alpha}\hat{E}_t^* \\
&+(1-\rho)\beta\frac{\kappa}{\omega\theta^{-\gamma}}\frac{1-\gamma f}{1-\gamma}E_t\left\{\hat{Q}_{t,t+1}^*\right\} \\
&-\frac{\kappa}{\omega\theta^{-\gamma}}\frac{\gamma}{1-\gamma}\hat{\theta}_t^*+\beta(1-\rho)\frac{\kappa}{\omega\theta^{-\gamma}}(1-f)\frac{\gamma}{1-\gamma}E_t\left\{\hat{\theta}_{t+1}^*\right\}.
\end{aligned}
\tag{B.19}
$$

## Appendix C. Hosios condition

In this section we show that, for both versions of the model, the conventional Hosios condition holds.

### *Appendix C.1. Exogenous participation*

When participation is exogenous, (B.11) can be used to substitute for $\lambda_t$ in (B.12). The efficient allocation is then characterized by the following dynamic condition:

$$
\begin{aligned}
(1-\Gamma)MRS_t^*+\frac{\kappa}{q_t^*(1-\gamma)} &= (1-\alpha)A_t^{1-\alpha}(E_t^*)^{-\alpha} \\
&+(1-\rho)\beta E_t\left\{\frac{C_t^*Z_{t+1}}{C_{t+1}^*Z_t}\frac{\kappa}{q_{t+1}^*}\frac{1-\gamma f_{t+1}^*}{1-\gamma}\right\}.
\end{aligned}
\tag{C.1}
$$

Note that equation (C.1) coincides with the job-creation condition if prices are flexible, the monopolistic distortion has been eliminated ($(\varepsilon\tau_p)/(\varepsilon-1)=1$), there is no unemployment benefit ($b=0$), and the Hosios condition is verified ($\eta_t=1-\gamma$). Hence, absent distortions unrelated to Nash bargaining, the Hosios condition sustains the efficient allocation.

### *Appendix C.2. Endogenous participation*

When participation is endogenous, after substituting $\lambda_t=0$ in (B.11) and (B.12), we obtain a static and a dynamic condition for efficiency:

$$
\kappa\theta_t^*\frac{\gamma}{1-\gamma}=\Gamma MRS_t^*,
\tag{C.2}
$$

$$
\begin{aligned}
&\frac{\kappa}{\omega}(\theta_t^*)^\gamma + MRS_t^*\Gamma\frac{1-f_t^*}{f_t^*} = (1-\alpha)A_t^{1-\alpha}(E_t^*)^{-\alpha} - MRS_t^* \\
&+ \beta(1-\rho)E_t\left\{\frac{C_t^*}{C_{t+1}^*}\frac{Z_{t+1}}{Z_t}\left[\frac{\kappa}{\omega}(\theta_{t+1}^*)^\gamma + \Gamma\frac{1-f_{t+1}^*}{f_{t+1}^*}MRS_{t+1}^*\right]\right\}.
\end{aligned} \tag{C.3}
$$

First note that if $b = 0$, one can recover equation (C.3) by combining the participation condition (A.2) and the job-creation condition (A.5) after imposing $\eta_t = \eta$. The dynamic efficiency condition is therefore always satisfied by the market allocation. Moreover, equations (A.4), (A.5), (A.18), and (A.19) imply that

$$
\begin{aligned}
&\frac{\kappa}{\omega}\theta_t^\gamma + \frac{1-\eta_t}{\eta_t}\frac{\kappa}{\omega}\theta_t^\gamma - \Gamma MRS_t = (1-\alpha)A_t^{1-\alpha}E_t^{-\alpha} - MRS_t \\
&\quad + \beta(1-\rho)E_t\left\{\frac{C_t}{C_{t+1}}\frac{Z_{t+1}}{Z_t}\left[\frac{\kappa}{\omega}\theta_{t+1}^\gamma + (1-f_{t+1})\frac{1-\eta_{t+1}}{\eta_{t+1}}\frac{\kappa}{\omega}\theta_{t+1}^\gamma\right]\right\},
\end{aligned} \tag{C.4}
$$

and by combining the efficiency conditions (C.2) and (C.3) we obtain

$$
\begin{aligned}
&\frac{\kappa}{\omega}(\theta_t^*)^\gamma + \frac{\gamma}{1-\gamma}\kappa\frac{\theta_t^*}{f_t^*} - \Gamma MRS_t^* = (1-\alpha)A_t^{1-\alpha}(E_t^*)^{-\alpha} - MRS_t^* \\
&\quad + \beta(1-\rho)E_t\left\{\frac{C_t^*}{C_{t+1}^*}\frac{Z_{t+1}}{Z_t}\left[\frac{\kappa}{\omega}(\theta_{t+1}^*)^\gamma + \frac{1-f_{t+1}^*}{f_{t+1}^*}\frac{\gamma}{1-\gamma}\kappa\theta_{t+1}^*\right]\right\}.
\end{aligned} \tag{C.5}
$$

Equations (C.4) and (C.5) are equivalent if $\eta_t = 1-\gamma$.

## Appendix D. Frisch elasticity of labor supply

Recall that the production function in the home sector can be rewritten as

$$
h_t = A_t^h\left(1 - E_t\left(1+\Gamma\frac{1-f_t}{f_t}\right) + \Gamma(1-\rho)\frac{1-f_t}{f_t}E_{t-1}\right)^{1-\alpha_h}. \tag{D.1}
$$

Let $w_t$ be the real wage. By using the participation condition (A.2) and the definition of the marginal rate of substitution, one can write

$$
\begin{aligned}
&\frac{\partial E_t}{\partial w_t}\frac{w_t}{E_t} = \frac{\partial E_t}{\partial MRS_t}\frac{MRS_t}{E_t}\frac{\partial MRS_t}{\partial w_t}\frac{w_t}{MRS_t} = \\
&\left(\frac{\partial MRS_t}{\partial h_t}\frac{h_t}{MRS_t}\frac{\partial h_t}{\partial E_t}\frac{E_t}{h_t}\right)^{-1}\frac{\partial MRS_t}{\partial w_t}\frac{w_t}{MRS_t} = \\
&-\frac{w_t h_t^{1-\alpha_h}}{(1-\alpha_h)E_t(\nu - \frac{\alpha_h}{1-\alpha_h})}\left[MRS_t\left(1+\Gamma\frac{1-f_t}{f_t}\right)^2\right]^{-1}.
\end{aligned} \tag{D.2}
$$

We compute the elasticity at the steady state for different values of $\nu$ and find the following numbers: 0.1650 if $\nu = -6$; 0.1950, if $\nu = -5$; 0.2383, if $\nu = -4$; 0.3064, if $\nu = -3$; 0.4290, if $\nu = -2$; 0.7150, if $\nu = -1$. Our baseline calibration implies a reasonable value. Given $\nu = -5$, we also let $\alpha_h$ vary and we find the following values: 0.1297, if $\alpha_h = 1/6$; 0.1093, if $\alpha_h = 1/10$; 0.0837, if $\alpha_h = 0$, i.e., under the assumption of constant returns.

## Appendix E. A purely quadratic welfare criterion

We approximate to second order the utility function. Then, we use the feasibility constraints faced by the policy-maker, also approximated to second order, in order to substitute out the linear terms in consumption and home production. The resulting second-order expression contains both linear and quadratic terms. For the sake of clarity, we analyze separately the linear terms, the squares and the cross-products. We show that all linear terms can be substituted with linear combinations of second-order terms so that the resulting welfare criterion is purely quadratic.

*Appendix E.1. Taylor expansion of the utility function*

A second-order approximation of the instantaneous utility function yields

$$U_t \simeq U + \hat{C}_t + \phi h^{1+\nu}\hat{h}_t + \frac{1}{2}\phi(1+\nu)h^{1+\nu}\hat{h}_t^2 + \hat{C}_t\hat{Z}_t + t.i.p., \tag{E.1}$$

where $t.i.p.$ stands for terms that are independent of policy.

*Appendix E.2. Approximation of the feasibility constraints*

Take the first feasibility constraint

$$C_t = \Delta_t^{\alpha-1}\left(A_t E_t\right)^{1-\alpha} - \frac{\kappa}{\omega}\theta_t^{\gamma}E_t + (1-\rho)\frac{\kappa}{\omega}\theta_t^{\gamma}E_{t-1}, \tag{E.2}$$

and approximate it to second order:

$$\begin{aligned}
&C\left[\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right] = -(1-\alpha)E^{1-\alpha}\hat{\Delta}_t + \\
&\left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^{\gamma}}{\omega}\right]E\hat{E}_t - \rho\gamma\frac{\kappa\theta^{\gamma}}{\omega}E\hat{\theta}_t + (1-\rho)\frac{\kappa\theta^{\gamma}}{\omega}E\hat{E}_{t-1} + \\
&\frac{1}{2}\left[(1-\alpha)^2E^{-\alpha} - \frac{\kappa\theta^{\gamma}}{\omega}\right]E\hat{E}_t^2 - \frac{1}{2}\gamma^2\rho\frac{\kappa\theta^{\gamma}}{\omega}E\hat{\theta}_t^2 + \frac{1}{2}(1-\rho)\frac{\kappa\theta^{\gamma}}{\omega}E\hat{E}_{t-1}^2 + \\
&-\gamma\frac{\kappa\theta^{\gamma}}{\omega}E\hat{\theta}_t\left(\hat{E}_t - (1-\rho)\hat{E}_{t-1}\right) + (1-\alpha)^2E^{1-\alpha}\hat{A}_t\hat{E}_t + t.i.p.
\end{aligned} \tag{E.3}$$

A second-order Taylor expansion of the second feasibility constraint, (B.9), reads as

$$
\begin{aligned}
&\hat{h}_t + \frac{1}{2(1-\alpha_h)}\hat{h}_t^2 = (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\left\{-\left(1+\Gamma\frac{1-f}{f}\right)E\hat{E}_t + \right.\\
&+(1-\rho)\Gamma\frac{1-f}{f}E\hat{E}_{t-1} + \rho\Gamma E\frac{1-\gamma}{\omega}\theta^{\gamma-1}\hat{\theta}_t - \frac{1}{2}\left(1+\Gamma\frac{1-f}{f}\right)E\hat{E}_t^2 + \\
&\frac{1}{2}(1-\rho)\Gamma\frac{1-f}{f}E\hat{E}_{t-1}^2 - \frac{1}{2}\rho\Gamma\frac{(1-\gamma)^2}{\omega}\theta^{\gamma-1}E\hat{\theta}_t^2 + \\
&\Gamma\frac{1-\gamma}{\omega}\theta^{\gamma-1}E\hat{\theta}_t\left(\hat{E}_t - (1-\rho)\hat{E}_{t-1}\right) + \\
&\left. -\left(1+\Gamma\frac{1-f}{f}\right)E\hat{E}_t\hat{A}_t^h + (1-\rho)\Gamma\frac{1-f}{f}E\hat{E}_{t-1}\hat{A}_t^h + \rho\Gamma E\frac{1-\gamma}{\omega}\theta^{\gamma-1}\hat{\theta}_t\hat{A}_t^h\right\}\\
&+t.i.p.
\end{aligned}
\tag{E.4}
$$

Finally, take equation (B.10) and substitute $N_t = N$. A second-order Taylor expansion yields

$$
\begin{aligned}
&\frac{E}{f}\hat{E}_t - (1-\rho)\frac{E(1-f)}{f}\hat{E}_{t-1} - \rho(1-\gamma)\frac{E}{f}\hat{\theta}_t\\
&+\frac{1}{2}\frac{E}{f}\hat{E}_t^2 - \frac{1}{2}(1-\rho)\frac{E(1-f)}{f}\hat{E}_{t-1}^2 + \frac{1}{2}(1-\gamma)^2\frac{\rho E}{f}\hat{\theta}_t^2\\
&-(1-\gamma)\frac{E}{f}\hat{\theta}_t\hat{E}_t + (1-\rho)(1-\gamma)\frac{E}{f}\hat{E}_{t-1}\hat{\theta}_t = 0.
\end{aligned}
\tag{E.5}
$$

Thus, it follows that

$$
\begin{aligned}
&-\lambda(1-\rho)\frac{(1-f)E}{f}\hat{E}_{-1} - \lambda\sum_{t=0}^{\infty}\beta^t\left[-\frac{E}{f}(1-\beta(1-\rho)(1-f))\hat{E}_t + (1-\gamma)\rho\frac{E}{f}\hat{\theta}_t\right]\\
&=\lambda\frac{1}{2}(1-\rho)\frac{E(1-f)}{f}\hat{E}_{-1}^2 + \\
&\lambda\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2}\frac{E}{f}\hat{E}_t^2 + \frac{1}{2}\beta(1-\rho)\frac{E(1-f)}{f}\hat{E}_t^2 - \frac{1}{2}(1-\gamma)^2\frac{\rho E}{f}\hat{\theta}_t^2\right.\\
&\left.+(1-\gamma)\frac{E}{f}\hat{\theta}_t\hat{E}_t - (1-\rho)(1-\gamma)\frac{E}{f}\hat{E}_{t-1}\hat{\theta}_t\right],
\end{aligned}
\tag{E.6}
$$

being $\lambda$ the Lagrangian multiplier associated with the planner's problem, evaluated at the steady state. Equation (E.6) is satisfied both in the exogenous and the endogenous participation model. It holds indeed for any $\lambda$ in the former and in the latter $\lambda = 0$.

*Appendix E.3. Linear terms*

We now use equations (E.3) and (E.4) into (E.1) to substitute for $\hat{C}_t$ and $\hat{h}_t$. After the substitution, one can collect all linear terms appearing in the generic period $t$

$$\begin{aligned}
&\left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^{\gamma}}{\omega} - \phi(1-\alpha_h)Ch^{\nu-\frac{\alpha_h}{1-\alpha_h}}\left(1+\Gamma\frac{1-f}{f}\right)\right]\frac{E}{C}\hat{E}_t + \\
&\left\{\Gamma\phi(1-\alpha_h)Ch^{\nu-\frac{\alpha_h}{1-\alpha_h}} - \frac{\gamma}{1-\gamma}\kappa\theta\right\}\frac{(1-\gamma)\theta^{\gamma-1}\rho E}{\omega C}\hat{\theta}_t + \\
&\left\{(1-\rho)\frac{\kappa\theta^{\gamma}}{\omega} + \Gamma\phi(1-\alpha_h)Ch^{\nu-\frac{\alpha_h}{1-\alpha_h}}(1-\rho)\frac{1-f}{f}\right\}\frac{E}{C}\hat{E}_{t-1},
\end{aligned} \tag{E.7}$$

so that all the first-order terms in the approximated discounted lifetime utility read as

$$\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t\left\{\delta_1\hat{E}_t + \delta_2\hat{E}_{t-1} + \delta_3\hat{\theta}_t\right\} = \\
&\delta_2\hat{E}_{-1} + \sum_{t=0}^{\infty}\beta^t\left\{(\delta_1+\beta\delta_2)\hat{E}_t + \delta_3\hat{\theta}_t\right\},
\end{aligned} \tag{E.8}$$

where

$$\begin{aligned}
\delta_1+\beta\delta_2 &= \Big\{(1-\alpha)E^{-\alpha} - \phi(1-\alpha_h)Ch^{\nu-\frac{\alpha_h}{1-\alpha_h}} - \\
&(1-\beta(1-\rho))\left(\frac{\kappa\theta^{\gamma}}{\omega} + \phi(1-\alpha_h)Ch^{\nu-\frac{\alpha_h}{1-\alpha_h}}\Gamma\frac{1-f}{f}\right)\Big\}\frac{E}{C} \\
\delta_3 &= \left\{\Gamma\phi(1-\alpha_h)Ch^{\nu-\frac{\alpha_h}{1-\alpha_h}} - \frac{\gamma}{1-\gamma}\kappa\theta\right\}\frac{(1-\gamma)\theta^{\gamma-1}\rho E}{\omega C},
\end{aligned} \tag{E.9}$$

and the term $\delta_2\hat{E}_{-1}$ is given at time $t=0$ and thus is independent of policy. It follows from steady-state efficiency that

$$\begin{aligned}
\delta_1+\beta\delta_2 &= -[1-\beta(1-\rho)(1-f)]\frac{\lambda E}{f} \\
\delta_3 &= \lambda\frac{(1-\gamma)\rho E}{f}.
\end{aligned} \tag{E.10}$$

Neglecting terms that are independent of policy, the right-hand side of (E.8) coincides with the left-hand side of equation (E.6), so that

$$\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t\left\{\delta_1\hat{E}_t + \delta_2\hat{E}_{t-1} + \delta_3\hat{\theta}_t\right\} = \\
&\frac{\lambda E}{2f}\sum_{t=0}^{\infty}\beta^t\Big[(1-\beta(1-\rho)(1-f))\,\hat{E}_t^2 + (1-\gamma)^2\rho\hat{\theta}_t^2 \\
&-2(1-\gamma)\left(\hat{E}_t - (1-\rho)\hat{E}_{t-1}\right)\hat{\theta}_t\Big] + t.i.p.,
\end{aligned} \tag{E.11}$$

which we can add to the other quadratic terms in the next section.

*Appendix E.4. Quadratic terms for endogenous variables*

Leaving the cross-products with the shocks for the next section, we collect the purely quadratic terms

$$\begin{aligned}
&\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\delta_E^1\hat{E}_t^2+\delta_\theta^1\hat{\theta}_t^2+\delta_c\hat{C}_t^2+\delta_h\hat{h}_t^2+\delta_p\hat{\Delta}_t+2\delta_{\theta,E}\left(\hat{E}_t-(1-\rho)\hat{E}_{t-1}\right)\hat{\theta}_t\right]\\
&+\frac{\lambda E}{2f}\sum_{t=0}^{\infty}\beta^t\left[(1-\beta(1-\rho)(1-f))\,\hat{E}_t^2+(1-\gamma)^2\rho\hat{\theta}_t^2\right.\\
&\left.-2(1-\gamma)\left(\hat{E}_t-(1-\rho)\hat{E}_{t-1}\right)\hat{\theta}_t\right],
\end{aligned} \tag{E.12}$$

where the following relations hold:

$$\begin{aligned}
\delta_E^1 &\equiv \left[-\alpha(1-\alpha)E^{-\alpha}+(1-\alpha)E^{-\alpha}-\right.\\
&\left.MRS-(1-\beta(1-\rho))\left(\frac{\kappa\theta^\gamma}{\omega}+MRS\Gamma\frac{1-f}{f}\right)\right]\frac{E}{C}\\
&=-\alpha(1-\alpha)\frac{E^{1-\alpha}}{C}-[1-\beta(1-\rho)(1-f)]\frac{\lambda E}{f}\\
\delta_\theta^1 &\equiv -\frac{E\rho}{fC}\left[\gamma^2\kappa\theta+(1-\gamma)^2\Gamma MRS\right]\\
\delta_c &\equiv -1\\
\delta_p &\equiv -2\frac{1-\alpha}{C}E^{1-\alpha}\\
\delta_h &\equiv \phi h^{1+\nu}\left(\nu-\frac{\alpha_h}{1-\alpha_h}\right)\\
\delta_{\theta,E} &\equiv -\frac{E(1-\gamma)}{fC}\left[\frac{\gamma\kappa\theta}{1-\gamma}-\Gamma MRS\right].
\end{aligned} \tag{E.13}$$

Finally, notice that because of steady-state efficiency

$$\begin{aligned}
\delta_\theta^1+(1-\gamma)^2\rho\frac{\lambda E}{f} &= -\frac{E\rho}{Cf}\left[\gamma^2\kappa\theta+(1-\gamma)^2\left(\Gamma MRS-\lambda C\right)\right]\\
&=-\frac{E\rho\gamma\kappa\theta^\gamma}{\omega C},
\end{aligned} \tag{E.14}$$

and

$$\delta_{\theta,E}=\frac{\lambda E}{f}(1-\gamma). \tag{E.15}$$

Therefore, the squares appearing in the welfare function are

$$\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\delta_E\hat{E}_t^2+\delta_\theta\hat{\theta}_t^2+\delta_c\hat{C}_t^2+\delta_h\hat{h}_t^2+\delta_p\hat{\Delta}_t\right\}, \tag{E.16}$$

where

$$\begin{aligned}\delta_E &\equiv -\alpha(1-\alpha)\frac{E}{C}E^{-\alpha},\\ \delta_\theta &\equiv -\frac{E}{C}\rho\kappa\gamma\frac{\theta^\gamma}{\omega}.\end{aligned} \tag{E.17}$$

*Appendix E.5. Cross-products with the shocks*

We are left with second-order terms where the shocks are multiplied by endogenous variables. They are

$$\sum_{t=0}^{\infty}\beta^t\left\{\delta_{EA}\hat{E}_t\hat{A}_t+\delta_{1,EA^h}\hat{E}_t\hat{A}_t^h+\delta_{2,EA^h}\hat{E}_{t-1}\hat{A}_t^h+\delta_{\theta A^h}\hat{\theta}_t\hat{A}_t^h+\hat{C}_t\hat{Z}_t\right\}, \tag{E.18}$$

where

$$\begin{aligned}\delta_{EA} &\equiv (1-\alpha)^2\frac{E}{C}E^{-\alpha}\\ \delta_{1,EA^h} &\equiv -MRS\frac{E}{C}\left(1+\Gamma\frac{1-f}{f}\right)\\ \delta_{2,EA^h} &\equiv MRS\frac{E}{C}\Gamma\frac{1-f}{f}(1-\rho)\\ \delta_{\theta A^h} &\equiv MRS\frac{E}{C}\rho\Gamma\frac{1-\gamma}{\omega}\theta^{\gamma-1}.\end{aligned} \tag{E.19}$$

*Appendix E.6. Second-order approximation of utility*

Collecting all quadratic terms, we have

$$\begin{aligned}&W \simeq \frac{U}{1-\beta}+\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\delta_E\hat{E}_t^2+\delta_\theta\hat{\theta}_t^2+\delta_c\hat{C}_t^2+\delta_h\hat{h}_t^2+\delta_p\hat{\Delta}_t\right\}+\\ &E_0\sum_{t=0}^{\infty}\beta^t\left\{\delta_{EA}\hat{E}_t\hat{A}_t+(\delta_{1,EA^h}\hat{A}_t^h+\beta\delta_{2,EA^h}\hat{A}_{t+1}^h)\hat{E}_t+\delta_{\theta A^h}\hat{\theta}_t\hat{A}_t^h+\hat{C}_t\hat{Z}_t\right\}\\ &+t.i.p.\end{aligned} \tag{E.20}$$

*Appendix E.7. Welfare function*

The second-order approximation of the utility function can be rewritten as a sum of squared deviations of endogenous variables from their Pareto efficient level. For a generic variable $X_t$, let us define $\widetilde{X}_t \equiv \log(X_t)-\log(X_t^*)$. Such a variable represents the gap between the market solution and the efficient one. It is also useful to define the deviation of the efficient allocation from the steady state as $\hat{X}_t^* \equiv \log(X_t^*)-\log(X)$.

Using these definitions, (E.20) becomes

$$
\begin{aligned}
W \simeq \frac{U}{1-\beta} &+ \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\delta_E\widetilde{E}_t^2+\delta_\theta\widetilde{\theta}_t^2+\delta_c\widetilde{C}_t^2+\delta_h\widetilde{h}_t^2+\delta_p\widetilde{\Delta}_t\right\} \\
&+\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\widetilde{E}_t\left[\delta_E\hat{E}_t^*+\delta_{EA}\hat{A}_t+\delta_{1,EA^h}\hat{A}_t^h+\beta\delta_{2,EA^h}\hat{A}_{t+1}^h\right]\right. \\
&\left.+\delta_\theta\widetilde{\theta}_t\hat{\theta}_t^*+\delta_{\theta A^h}\widetilde{\theta}_t\hat{A}_t^h+\widetilde{C}_t\left[\hat{Z}_t-\hat{C}_t^*\right]+\delta_h\widetilde{h}_t\hat{h}_t^*\right\}+t.i.p.
\end{aligned}
\tag{E.21}
$$

Define

$$
\begin{aligned}
AA_0 \equiv E_0\sum_{t=0}^{\infty}\beta^t&\left\{\widetilde{E}_t\left[\delta_E\hat{E}_t^*+\delta_{EA}\hat{A}_t+\delta_{1,EA^h}\hat{A}_t^h+\beta\delta_{2,EA^h}\hat{A}_{t+1}^h\right]\right. \\
&\left.+\widetilde{\theta}_t\left[\delta_\theta\hat{\theta}_t^*+\delta_{\theta A^h}\hat{A}_t^h\right]+\widetilde{C}_t\left[\hat{Z}_t-\hat{C}_t^*\right]+\delta_h\widetilde{h}_t\hat{h}_t^*\right\}.
\end{aligned}
\tag{E.22}
$$

One can show that around an efficient steady state the sum of all cross-products, $AA_0$, is equal to zero and only squared deviations are left in (E.21). First, recall that both the market and the Pareto efficient equilibrium have to satisfy resource constraints. Hence, we can write

$$
\widetilde{C}_t=\left[(1-\alpha)E^{-\alpha}-\frac{\kappa\theta^\gamma}{\omega}\right]\frac{E}{C}\widetilde{E}_t-\rho\gamma\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\widetilde{\theta}_t+(1-\rho)\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\widetilde{E}_{t-1}+t.i.p., \tag{E.23}
$$

and

$$
\begin{aligned}
\widetilde{h}_t = (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}&\left\{-\left(1+\Gamma\frac{1-f}{f}\right)E\widetilde{E}_t+(1-\rho)\Gamma\frac{1-f}{f}E\widetilde{E}_{t-1}+\right. \\
&\left.\rho\Gamma E\frac{1-\gamma}{\omega}\theta^{\gamma-1}\widetilde{\theta}_t\right\}+t.i.p.
\end{aligned}
\tag{E.24}
$$

Equations (E.23)-(E.24) imply that

$$
\begin{aligned}
E_0\sum_{t=0}^{\infty}\beta^t\widetilde{C}_t\hat{C}_t^* = E_0\sum_{t=0}^{\infty}\beta^t&\left\{\left[(1-\alpha)E^{-\alpha}-\frac{\kappa\theta^\gamma}{\omega}\right]\frac{E}{C}\widetilde{E}_t\hat{C}_t^*-\rho\gamma\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\widetilde{\theta}_t\hat{C}_t^*\right. \\
&\left.+\beta(1-\rho)\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\widetilde{E}_t\hat{C}_{t+1}^*\right\}+t.i.p.
\end{aligned}
\tag{E.25}
$$

$$
\begin{aligned}
E_0\sum_{t=0}^{\infty}\beta^t\delta_h\widetilde{h}_t\hat{h}_t^* = E_0\sum_{t=0}^{\infty}\beta^t&\left\{MRS\frac{E}{C}\left(\nu-\frac{\alpha_h}{1-\alpha_h}\right)\left[-\left(1+\Gamma\frac{1-f}{f}\right)\widetilde{E}_t\hat{h}_t^*\right.\right. \\
&\left.\left.+\beta(1-\rho)\Gamma\frac{1-f}{f}\widetilde{E}_t\hat{h}_{t+1}^*+\rho\Gamma\frac{1-\gamma}{\omega}\theta^{\gamma-1}\widetilde{\theta}_t\hat{h}_t^*\right]\right\}+t.i.p.
\end{aligned}
\tag{E.26}
$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{C}_t \hat{Z}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ (1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega} \right] \frac{E}{C} \widetilde{E}_t \hat{Z}_t - \rho\gamma \frac{\kappa\theta^\gamma}{\omega} \frac{E}{C} \widetilde{\theta}_t \hat{Z}_t \right. \\ \left. + \beta(1-\rho) \frac{\kappa\theta^\gamma}{\omega} \frac{E}{C} \widetilde{E}_t \hat{Z}_{t+1} \right\} + t.i.p. \tag{E.27}$$

Therefore, we can use equations (E.25)-(E.27) to substitute for $\widetilde{C}_t$ and $\widetilde{h}_t$ in (E.22)

$$AA_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \widetilde{E}_t \frac{E}{C} \left[ -\alpha(1-\alpha)E^{-\alpha} \hat{E}_t^* + (1-\alpha)^2 E^{-\alpha} \hat{A}_t \right. \right. \\ -MRS \left( 1 + \Gamma \frac{1-f}{f} \right) \hat{A}_t^h + \beta(1-\rho) MRS \Gamma \frac{1-f}{f} \hat{A}_{t+1}^h \\ - \left( (1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega} \right) (\hat{C}_t^* - \hat{Z}_t) - \beta(1-\rho) \frac{\kappa\theta^\gamma}{\omega} (\hat{C}_{t+1}^* - \hat{Z}_{t+1}) \\ \left. -MRS \left( \nu - \frac{\alpha_h}{1-\alpha_h} \right) \left( 1 + \Gamma \frac{1-f}{f} \right) \hat{h}_t^* + \beta(1-\rho) \Gamma MRS \left( \nu - \frac{\alpha_h}{1-\alpha_h} \right) \frac{1-f}{f} \hat{h}_{t+1}^* \right] \\ \left. + \widetilde{\theta}_t \frac{E}{C} \rho \left[ \frac{\gamma\kappa\theta^\gamma}{\omega} \left( -\hat{\theta}_t^* + \hat{C}_t^* - \hat{Z}_t \right) + MRS \Gamma \frac{1-\gamma}{\omega} \theta^{\gamma-1} \left( \left( \nu - \frac{\alpha_h}{1-\alpha_h} \right) \hat{h}_t^* + \hat{A}_t^h \right) \right] \right\} \\ + t.i.p. \tag{E.28}$$

Using the static efficiency condition (both in its steady-state version (B.13) and the linear version (B.16)), we can rewrite the term in $\widetilde{\theta}_t$ in (E.28) as follows:

$$\widetilde{\theta}_t \frac{E}{C} \rho \left[ \frac{\gamma\kappa\theta^\gamma}{\omega} \left( -\hat{\theta}_t^* + \hat{C}_t^* - \hat{Z}_t \right) + MRS \Gamma \frac{1-\gamma}{\omega} \theta^{\gamma-1} \left( \left( \nu - \frac{\alpha_h}{1-\alpha_h} \right) \hat{h}_t^* + \hat{A}_t^h \right) \right] = \\ \widetilde{\theta}_t \frac{E}{C} \rho \frac{1-\gamma}{f} \lambda C \hat{\lambda}_t. \tag{E.29}$$

If participation is endogenous, $\lambda = 0$ and the term in $\tilde{\theta}_t$ drops out. If participation is exogenous, from (E.5) we have that, up to first order,

$$\widetilde{\theta}_t = \frac{1}{\rho(1-\gamma)} \widetilde{E}_t - \frac{(1-\rho)(1-f)}{\rho(1-\gamma)} \widetilde{E}_{t-1}, \tag{E.30}$$

which implies that we can substitute for $\widetilde{\theta}_t$ in equation (E.28) and rewrite it as follows:

$$
\begin{aligned}
AA_0 = E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{E}_t \frac{E}{C} \Bigg\{ & -\alpha(1-\alpha)E^{-\alpha}\hat{E}_t^* + (1-\alpha)^2 E^{-\alpha}\hat{A}_t - MRS\left(1+\Gamma\frac{1-f}{f}\right)\hat{A}_t^h \\
& +\beta(1-\rho)MRS\Gamma\frac{1-f}{f}\hat{A}_{t+1}^h - \left((1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega}\right)(\hat{C}_t^* - \hat{Z}_t) \\
& -\beta(1-\rho)\frac{\kappa\theta^\gamma}{\omega}(\hat{C}_{t+1}^* - \hat{Z}_{t+1}) - MRS\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\left(1+\Gamma\frac{1-f}{f}\right)\hat{h}_t^* \\
& +\beta(1-\rho)\Gamma MRS\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\frac{1-f}{f}\hat{h}_{t+1}^* + \frac{\lambda C}{f}\left[\hat{\lambda}_t - \beta(1-\rho)(1-f)\hat{\lambda}_{t+1}\right]\Bigg\} \\
& + t.i.p.
\end{aligned}
\tag{E.31}
$$

This equation holds under both exogenous and endogenous participation. We can use (B.17) to substitute for $\hat{E}_t^*$ in (E.31)

$$
\begin{aligned}
AA_0 = E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{E}_t \frac{E}{C} \Bigg\{ & MRS\left(1+\Gamma\frac{1-f}{f}\right)\left[\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\hat{h}_t^* + \hat{C}_t^* + \hat{A}_t^h - \hat{Z}_t\right] \\
& -(1-\rho)\beta\left(\frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f}\Gamma MRS\right)\left(\hat{C}_t^* - \hat{C}_{t+1}^* + \hat{Z}_{t+1}^* - \hat{Z}_t^*\right) \\
& -\beta(1-\rho)\Gamma MRS\frac{1-f}{f}\left[\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\hat{h}_t^* + \hat{C}_t^* + \hat{A}_t^h - \hat{Z}_t\right] \\
& -\frac{\lambda C}{f}(1-\beta(1-\rho)(1-f))(\hat{C}_t^* - \hat{Z}_t) - MRS\left(1+\Gamma\frac{1-f}{f}\right)\hat{A}_t^h \\
& +\beta(1-\rho)MRS\Gamma\frac{1-f}{f}\hat{A}_{t+1}^h - \left((1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega}\right)(\hat{C}_t^* - \hat{Z}_t) \\
& -\beta(1-\rho)\frac{\kappa\theta^\gamma}{\omega}(\hat{C}_{t+1}^* - \hat{Z}_{t+1}) - MRS\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\left(1+\Gamma\frac{1-f}{f}\right)\hat{h}_t^* \\
& +\beta(1-\rho)\Gamma MRS\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\frac{1-f}{f}\hat{h}_{t+1}^*\Bigg\} + t.i.p.
\end{aligned}
\tag{E.32}
$$

Finally, using the steady state of the dynamic efficiency condition (B.12) into (E.32), and collecting terms in $\hat{C}_t^*$, $\hat{Z}_t^*$, $\hat{C}_{t+1}^*$ and $\hat{Z}_{t+1}^*$, it is easy to see that $AA_0 = 0$:

$$\begin{aligned}
AA_0 = E_0 \sum_{t=0}^{\infty} \beta^t \widetilde{E}_t \frac{E}{C} \Big\{ \hat{C}_t^* \Big[ & MRS \left(1 + \Gamma \frac{1-f}{f}\right) - \beta(1-\rho) \left( \frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f} \Gamma MRS \right) \\
& + (1-\alpha) E^{-\alpha} - MRS - \frac{\kappa\theta^\gamma}{\omega} - MRS \Gamma \frac{1-f}{f} + \beta(1-\rho) \left( \frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f} \Gamma MRS \right) \\
& - (1-\alpha) E^{-\alpha} + \frac{\kappa\theta^\gamma}{\omega} \Big] + \hat{Z}_t^* \Big[ -MRS \left(1 + \Gamma \frac{1-f}{f}\right) + \beta(1-\rho) \left( \frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f} \Gamma MRS \right) \\
& + [1 - \beta(1-\rho)] \left( \frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f} \Gamma MRS \right) - (1-\alpha) E^{-\alpha} + MRS + (1-\alpha) E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega} \Big] \\
& + \hat{C}_{t+1}^* \Big[ \beta(1-\rho) \left( \frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f} \Gamma MRS \right) - \beta(1-\rho) \Gamma MRS \frac{1-f}{f} + \beta(1-\rho) \frac{\kappa\theta^\gamma}{\omega} \Big] \\
& + \hat{Z}_{t+1}^* \Big[ -\beta(1-\rho) \left( \frac{\kappa\theta^\gamma}{\omega} + \frac{1-f}{f} \Gamma MRS \right) + \beta(1-\rho) \Gamma MRS \frac{1-f}{f} - \beta(1-\rho) \frac{\kappa\theta^\gamma}{\omega} \Big] \Big\} \\
& + t.i.p. = 0.
\end{aligned} \tag{E.33}$$

Finally, recall that up to second order

$$\Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\alpha}{1-\alpha}} di \simeq 1 + \frac{1}{2} \frac{\varepsilon}{1-\alpha} \frac{1-\alpha+\alpha\varepsilon}{1-\alpha} Var_i \log(P_t(i)).$$

Then,

$$\widetilde{\Delta_t} \simeq \frac{1}{2} \frac{\varepsilon}{1-\alpha} \frac{1-\alpha+\alpha\varepsilon}{1-\alpha} Var_i \log(P_t(i)). \tag{E.34}$$

From Woodford (2003) we know that

$$\sum_{t=0}^{\infty} \beta^t Var_i \log(P_t(i)) = \sum_{t=0}^{\infty} \beta^t \frac{\delta}{(1-\delta)(1-\beta\delta)} \hat{\pi}_t^2. \tag{E.35}$$

So finally,

$$\sum_{t=0}^{\infty} \beta^t \delta_p \widetilde{\Delta}_t = \sum_{t=0}^{\infty} \beta^t \delta_\pi \hat{\pi}_t^2 = \sum_{t=0}^{\infty} \beta^t \delta_\pi \tilde{\pi}_t^2, \tag{E.36}$$

where

$$\delta_\pi \equiv -\frac{Y}{C} \frac{\varepsilon}{\lambda}. \tag{E.37}$$

Therefore, a second-order approximation to the utility function simply yields

$$W \simeq \frac{U}{1-\beta} + \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \delta_E \widetilde{E}_t^2 + \delta_\theta \widetilde{\theta}_t^2 + \delta_c \widetilde{C}_t^2 + \delta_h \widetilde{h}_t^2 + \delta_\pi \tilde{\pi}_t^2 \right\} + t.i.p. \tag{E.38}$$

The result can be applied both to the exogenous and the endogenous participation models. Given our calibration, we have $\delta_E = -0.2222$, $\delta_\theta = -0.0017$, $\delta_c = -1$, $\delta_h = -1.2034$ and $\delta_\pi = -126.6473$. Following the linear-quadratic approach, we can compute the optimal monetary policy by maximizing (E.38) subject to a set of linear constraints defining the market equilibrium. In the next section we derive those constraints for both versions of the model.

## Appendix F. Linearized system of equations

In this section we derive a log-linear representation of both models as a system of six equations in seven variables: $\widetilde{\pi}_t$, $\widetilde{C}_t$, $\widetilde{r}_t$, $\widetilde{\theta}_t$, $\widetilde{E}_t$, $\widetilde{N}_t$ and $\widetilde{h}_t$. To close the system, an equation defining monetary policy is needed.

### *Appendix F.1. Equation 1: Labor market tightness*

When participation is endogenous, the law of motion of employment is given by

$$E_t = (1-\rho)(1-f_t)E_{t-1} + f_t N_t. \tag{F.1}$$

A first-order approximation around the steady state yields

$$E\widehat{E}_t = (1-\rho)(1-f)E\widehat{E}_{t-1} - (1-\rho)fE\widehat{f}_t + fN\widehat{f}_t + fN\widehat{N}_t. \tag{F.2}$$

Using the steady-state relation

$$\frac{E}{N} = \frac{f}{1-(1-\rho)(1-f)} \iff f\frac{N}{E} - (1-\rho)f = \rho, \tag{F.3}$$

and

$$\widehat{f}_t = (1-\gamma)\widehat{\theta}_t, \tag{F.4}$$

the law of motion can be written as

$$\widehat{\theta}_t = \frac{1}{\rho(1-\gamma)}\widehat{E}_t - \frac{\rho_e}{\rho(1-\gamma)}\widehat{E}_{t-1} - \frac{1-\rho_e}{\rho(1-\gamma)}\widehat{N}_t, \tag{F.5}$$

where

$$\rho_e \equiv (1-\rho)(1-f). \tag{F.6}$$

If participation is exogenous, $\widehat{N}_t = 0$ and (F.5) simplifies to

$$\widehat{\theta}_t = \frac{1}{\rho(1-\gamma)}\widehat{E}_t - \frac{\rho_e}{\rho(1-\gamma)}\widehat{E}_{t-1}. \tag{F.7}$$

In deviation from the efficient equilibrium we have

$$\widetilde{\theta}_t = \frac{1}{\rho(1-\gamma)}\widetilde{E}_t - \frac{\rho_e}{\rho(1-\gamma)}\widetilde{E}_{t-1} - \frac{1-\rho_e}{\rho(1-\gamma)}\widetilde{N}_t, \tag{F.8}$$

if participation is endogenous, while

$$\widetilde{\theta}_t = \frac{1}{\rho(1-\gamma)}\widetilde{E}_t - \frac{\rho_e}{\rho(1-\gamma)}\widetilde{E}_{t-1}, \tag{F.9}$$

if participation is exogenous.

*Appendix F.2. Equation 2: Euler equation*

A log-linear version of the Euler equation reads for both models as

$$\widehat{C}_t = E_t\{\widehat{C}_{t+1}\} - \widehat{r}_t + (1-\rho_z)\widehat{Z}_t, \tag{F.10}$$

where $\widehat{r}_t = \widehat{R}_t - E_t\{\widehat{\pi}_{t+1}\} = -E_t\{\widehat{Q}_{t,t+1}\}$ and, in deviation from the efficient equilibrium, it becomes

$$\widetilde{C}_t = E_t\{\widetilde{C}_{t+1}\} - \widetilde{r}_t. \tag{F.11}$$

*Appendix F.3. Equation 3: Home production*

Using the law of motion of employment, we can rewrite the home-production function as follows:

$$\begin{aligned} h_t &= \left[A_t^h(1-E_t-\Gamma U_t)\right]^{1-\alpha_h} \\ &= \left[A_t^h\left(1-E_t-\Gamma\left(E_t\frac{1-\omega\theta_t^{1-\gamma}}{\omega\theta_t^{1-\gamma}}-(1-\rho)\frac{1-\omega\theta_t^{1-\gamma}}{\omega\theta_t^{1-\gamma}}E_{t-1}\right)\right)\right]^{1-\alpha_h}, \end{aligned} \tag{F.12}$$

and log-linearize it

$$\begin{aligned} \widehat{h}_t = (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\Bigg[&-\left(1+\Gamma\frac{1-f}{f}\right)E\widehat{E}_t + (1-\rho)\Gamma\frac{1-f}{f}E\widehat{E}_{t-1} + \rho\Gamma E\frac{1-\gamma}{f}\widehat{\theta}_t \\ &+\left(1-E\left(1+\Gamma\frac{1-f}{f}\rho\right)\right)\widehat{A}_t^h\Bigg]. \end{aligned} \tag{F.13}$$

This equation applies to both models. In deviation from the efficient equilibrium we have

$$\widetilde{h}_t = (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\left[-\left(1+\Gamma\frac{1-f}{f}\right)E\widetilde{E}_t + (1-\rho)\Gamma\frac{1-f}{f}E\widetilde{E}_{t-1} + \rho\Gamma E\frac{1-\gamma}{f}\widetilde{\theta}_t\right]. \tag{F.14}$$

*Appendix F.4. Equation 4: Participation condition*

Let

$$\Omega_t \equiv \frac{1-f_t}{f_t}\left[MRS_t - b\right]. \tag{F.15}$$

Hence,

$$\widehat{mrs}_t = \widehat{C}_t + \left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widehat{h}_t + \widehat{A}_t^h - \widehat{Z}_t, \tag{F.16}$$

$$\widehat{\Omega}_t = -\frac{1}{1-f}\widehat{f}_t + \frac{\Gamma MRS}{\Gamma MRS - b}\widehat{mrs}_t. \tag{F.17}$$

Log-linearizing (A.2) and (A.21), we get

$$
\begin{aligned}
\widehat{\Omega}_t = {} & \frac{W}{P}\frac{f}{1-f}(\Gamma MRS - b)^{-1}\widehat{w}_t - \frac{f}{1-f}\frac{MRS}{\Gamma MRS - b}\widehat{mrs}_t - \\
& \beta(1-\rho)\widehat{r}_t + \beta(1-\rho)E_t\left\{\widehat{\Omega}_{t+1}\right\},
\end{aligned}
\tag{F.18}
$$

$$
\begin{aligned}
\frac{W}{P}\hat{w}_t = (1-\Gamma)\mathcal{M}\widehat{mrs}_t - \frac{1-\eta}{\eta}\frac{\kappa}{q}\hat{q}_t - \frac{1}{\eta}\frac{\kappa}{q}\hat{\eta}_t \\
+\beta(1-\rho)\frac{1-\eta}{\eta}\frac{\kappa}{q}E_t\left\{(1-f)\hat{r}_t + f\hat{f}_{t+1} + \frac{1-f}{1-\eta}\hat{\eta}_{t+1} + (1-f)\hat{q}_{t+1}\right\}.
\end{aligned}
\tag{F.19}
$$

After substituting (F.17) and (F.19) into (F.18), we obtain

$$
\begin{aligned}
& \frac{\Gamma MRS}{(\Gamma MRS - b)(1-f)}\widehat{mrs}_t = \beta(1-\rho)\left(\frac{(1-\eta)\kappa}{\eta q}\frac{1-f}{\Omega} - 1\right)\hat{r}_t \\
& + \beta(1-\rho)\frac{\Gamma(1-f)}{f}\frac{MRS}{\Omega}E_t\left\{\widehat{mrs}_{t+1}\right\} \\
& + \frac{1}{1-f}\left[\hat{f}_t + \beta\frac{1-\rho}{\Omega}\frac{\kappa}{q}\frac{1-\eta}{\eta}(1-f)fE_t\left\{\hat{f}_{t+1}\right\} - \beta(1-\rho)E_t\left\{\hat{f}_{t+1}\right\}\right] \\
& - \frac{1}{\Omega}\frac{(1-\eta)\kappa}{\eta q}\left[\hat{q}_t - \beta(1-\rho)(1-f)E_t\left\{\hat{q}_{t+1}\right\}\right] \\
& - \frac{1}{\Omega}\frac{\kappa}{\eta q}\left[\hat{\eta}_t - \beta(1-\rho)(1-f)E_t\left\{\hat{\eta}_{t+1}\right\}\right].
\end{aligned}
$$

After using the relation between the finding rate, filling rate and tightness, and imposing the Hosios condition, we can rewrite

$$
\begin{aligned}
& \frac{\Gamma MRS}{(\Gamma MRS - b)(1-f)}\widehat{mrs}_t = \beta(1-\rho)\left(\frac{\gamma\kappa}{(1-\gamma)q}\frac{1-f}{\Omega} - 1\right)\hat{r}_t \\
& + \beta(1-\rho)\frac{\Gamma(1-f)}{f}\frac{MRS}{\Omega}E_t\left\{\widehat{mrs}_{t+1}\right\} + \left[\frac{\gamma^2\kappa}{(1-\gamma)q\Omega} + \frac{1-\gamma}{1-f}\right]\hat{\theta}_t \\
& - \left[\frac{\gamma^2\kappa(1-f)}{(1-\gamma)q\Omega} + \frac{1-\gamma}{1-f} - \gamma\frac{f}{\Omega}\frac{\kappa}{q}\right]\beta(1-\rho)E_t\{\hat{\theta}_{t+1}\} \\
& - \frac{1}{\Omega}\frac{\kappa}{(1-\gamma)q}\left[\hat{\eta}_t - \beta(1-\rho)(1-f)E_t\left\{\hat{\eta}_{t+1}\right\}\right].
\end{aligned}
\tag{F.20}
$$

At the steady state

$$
\Omega = \frac{1-f}{f}\left[\Gamma MRS - b\right],
\tag{F.21}
$$

$$
\Omega(1-\beta(1-\rho)) = \frac{W}{P} - MRS,
\tag{F.22}
$$

$$\frac{W}{P} = b + (1-\Gamma)MRS + \frac{1-\eta}{\eta}\frac{\kappa\theta^{\gamma}}{\omega}(1-\beta(1-\rho)(1-f)). \tag{F.23}$$

Combine (F.21), (F.22), and (F.23). Since the Hosios condition holds, the market equilibrium satisfies

$$\frac{\gamma}{1-\gamma} = \frac{\frac{\Omega}{1-f}}{\frac{\kappa\theta^{\gamma}}{\omega}}, \tag{F.24}$$

which can be rewritten as

$$\frac{\gamma}{1-\gamma}\kappa\theta = \Gamma MRS - b. \tag{F.25}$$

Define an efficiency wedge $\Phi_b$, which is equal to zero if $b = 0$:

$$(1-\Phi_b) \equiv \frac{\Gamma MRS}{\Gamma MRS - b}. \tag{F.26}$$

Hence,

$$\Gamma MRS = \frac{\gamma}{1-\gamma}\kappa\theta(1-\Phi_b). \tag{F.27}$$

The coefficients in (F.20) simplify to

$$\frac{\Gamma MRS}{(\Gamma MRS - b)(1-f)} = \frac{1-\Phi_b}{1-f} \tag{F.28}$$

$$\frac{\gamma\kappa}{(1-\gamma)q}\frac{1-f}{\Omega} - 1 = \frac{\gamma\kappa\theta}{1-\gamma}\frac{1}{\Gamma MRS} - 1 = 0 \tag{F.29}$$

$$\frac{\Gamma(1-f)}{f}\frac{MRS}{\Omega} = 1-\Phi_b \tag{F.30}$$

$$\left[\frac{\gamma^2\kappa}{(1-\gamma)q\Omega} + \frac{1-\gamma}{1-f}\right] = \frac{1}{1-f} \tag{F.31}$$

$$\left[\frac{\gamma^2\kappa(1-f)}{(1-\gamma)q\Omega} + \frac{1-\gamma}{1-f} - \gamma\frac{f}{\Omega}\frac{\kappa}{q}\right] = 1 \tag{F.32}$$

$$\frac{1}{\Omega}\frac{\kappa}{\eta q} = \frac{1}{\gamma(1-f)}. \tag{F.33}$$

Therefore, (F.20) becomes

$$\begin{aligned}(1-\Phi_b)\widehat{mrs}_t - \widehat{\theta}_t =& \beta(1-\rho)(1-f)E_t\left\{(1-\Phi_b)\widehat{mrs}_{t+1} - \widehat{\theta}_{t+1}\right\} \\ & - \gamma^{-1}\left[1-\beta(1-\rho)(1-f)\rho_\eta\right]\widehat{\eta}_t.\end{aligned} \tag{F.34}$$

Iterating forward equation (F.34), one gets

$$
\begin{aligned}
(1-\Phi_b)\widehat{mrs}_t - \widehat{\theta}_t &= -\gamma^{-1}\left[1-\beta(1-\rho)(1-f)\rho_\eta\right]\sum_{j=0}^{\infty}\left(\beta(1-\rho)(1-f)\right)^j E_t\{\widehat{\eta}_{t+j}\} \\
&= -\gamma^{-1}\widehat{\eta}_t,
\end{aligned}
\tag{F.35}
$$

so that

$$
(1-\Phi_b)\left[\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widehat{h}_t + \widehat{C}_t + \widehat{A}^h_t - \widehat{Z}\right] = \widehat{\theta}_t - \gamma^{-1}\widehat{\eta}_t. \tag{F.36}
$$

At an efficient equilibrium, it must be that

$$
\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widehat{h}^*_t + \widehat{C}^*_t + \widehat{A}^h_t - \widehat{Z} = \widehat{\theta}^*_t. \tag{F.37}
$$

Moreover, $\Phi_b = 0$ implies that we can write

$$
\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widetilde{h}_t + \widetilde{C}_t = \widetilde{\theta}_t - \gamma^{-1}\widehat{\eta}_t. \tag{F.38}
$$

This equation is specific to the model with endogenous participation. It is straightforward to see that, in the absence of bargaining shocks, (F.37) coincides with the first-order approximation of the static efficiency condition (B.16) in the endogenous participation model.

*Appendix F.5. Equation 5: NKPC*

We define the average real marginal cost as follows:

$$
MC_t \equiv \frac{P^x_t X^\alpha_t}{P_t(1-\alpha)} \tag{F.39}
$$

and the average markup as its inverse:

$$
\mu_t \equiv MC_t^{-1}, \tag{F.40}
$$

so that

$$
\widehat{\mu}_t = -\widehat{p}^x_t - \alpha\widehat{x}_t, \tag{F.41}
$$

where $\widehat{p}^x_t$ is the log-deviation of the relative price of the intermediate good. One can write

$$
\frac{P^x_t}{P_t}A_t = A_t MC_t(1-\alpha)x_t^{-\alpha} = \frac{(1-\alpha)A_t}{\mu_t X^\alpha_t}, \tag{F.42}
$$

and rearrange the job-creation condition

$$
\frac{(1-\alpha)A_t}{\mu_t X^\alpha_t} = \frac{W_t}{P_t} + \frac{\kappa}{q_t} - (1-\rho)E_t\left\{Q_{t,t+1}\frac{\kappa}{q_{t+1}}\right\}. \tag{F.43}
$$

The wage equation can be written as

$$\frac{W_t}{P_t} = b + \frac{\phi h_t^{\nu}(1-\Gamma)C_t}{Z_t}(1-\alpha_h)A_t^h h_t^{-\frac{\alpha_h}{1-\alpha_h}} + \\ + \frac{1-\eta_t}{\eta_t}\frac{\kappa}{q_t} - E_t\left\{Q_{t,t+1}(1-\rho)(1-f_{t+1})\frac{1-\eta_{t+1}}{\eta_{t+1}}\frac{\kappa}{q_{t+1}}\right\}. \tag{F.44}$$

We can now use (F.43) into (F.44) to eliminate the real wage and obtain

$$\frac{1}{\mu_t} = \frac{X_t^{\alpha}}{(1-\alpha)A_t}\chi_t, \tag{F.45}$$

where $\chi_t$ denotes the cost of hiring a worker, and form a match. It is related to the markup in the final-good sector, and it is defined as

$$\chi_t = b + \frac{\phi h_t^{\nu}(1-\Gamma)C_t}{Z_t}(1-\alpha_h)A_t^h h_t^{-\frac{\alpha_h}{1-\alpha_h}} \\ + \frac{\kappa}{\eta_t q_t} - (1-\rho)E_t\left\{Q_{t,t+1}\frac{\kappa}{q_{t+1}}\frac{1-f_{t+1}(1-\eta_{t+1})}{\eta_{t+1}}\right\}. \tag{F.46}$$

The log-linear form of equation (F.45) reads as

$$-\widehat{\mu}_t = \widehat{\chi}_t - \left[(1-\alpha)\widehat{A}_t - \alpha\widehat{E}_t\right], \tag{F.47}$$

where the relation $X_t = A_t E_t$ has been used to substitute for $X_t$. Log-linearizing the expression for $\chi_t$, we obtain

$$\chi\widehat{\chi}_t = A_1\gamma\widehat{\theta}_t - \beta(1-\rho)A_1(\gamma-\theta q(1-\eta))E_t\{\widehat{\theta}_{t+1}\} + A_1\beta(1-\rho)[1-\theta q(1-\eta)]\widehat{r}_t \\ - A_2\widehat{\eta}_t + A_3\left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widehat{h}_t + A_3(\widehat{C}_t + \widehat{A}_t^h - \widehat{Z}_t), \tag{F.48}$$

where the following definitions apply:

$$A_1 = \frac{\kappa}{\eta q}, \tag{F.49}$$

$$A_2 = A_1[1+(1-\rho)\beta(1-\theta q)\rho_\eta], \tag{F.50}$$

$$A_3 = \frac{\phi h^{\nu}(1-\Gamma)C}{Z}A^h(1-\alpha_h)h^{-\frac{\alpha_h}{1-\alpha_h}}, \tag{F.51}$$

and where we used the fact that, up to first order, $\hat{q}_t = -\gamma\hat{\theta}_t$ and $\hat{f}_t = (1-\gamma)\hat{\theta}_t$.

*Appendix F.5.1. General version*

We can rewrite the coefficients of the markup equation imposing the Hosios (but without using steady-state relations)

$$\gamma A_1 = \gamma \frac{\kappa}{\eta q} = \frac{\gamma \kappa \theta}{f(1-\gamma)}, \tag{F.52}$$

$$A_1(\gamma - \theta q(1-\eta)) = A_1 \gamma (1-f) = \frac{\gamma \kappa \theta (1-f)}{f(1-\gamma)}, \tag{F.53}$$

$$A_1(1 - \theta q(1-\eta)) = \frac{\kappa \theta (1-\gamma f)}{f(1-\gamma)}, \tag{F.54}$$

$$A_2 = \frac{\kappa \theta}{f(1-\gamma)}[1 + (1-\rho)\beta(1-f)\rho_\eta], \tag{F.55}$$

$$A_3 = (1-\Gamma)MRS. \tag{F.56}$$

Therefore, for both the endogenous and the exogenous model we can write

$$\begin{aligned} \chi \widehat{\chi}_t = & \frac{\gamma \kappa \theta}{(1-\gamma)f} \left\{ \widehat{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widehat{\theta}_{t+1}\} + \beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widehat{r}_t \right. \\ & \left. - \frac{[1+(1-\rho)\beta(1-f)\rho_\eta]}{\gamma}\widehat{\eta}_t \right\} + (1-\Gamma)MRS \left[ \left( \nu - \frac{\alpha_h}{1-\alpha_h} \right) \widehat{h}_t + \right. \\ & \left. \widehat{C}_t + \widehat{A}_t^h - \widehat{Z}_t \right]. \end{aligned} \tag{F.57}$$

We can now use (F.47) and (F.57) to write the NKPC:

$$\begin{aligned} \widehat{\pi}_t &= \beta E_t\{\widehat{\pi}_{t+1}\} + \lambda \widehat{mc}_t \\ &= \beta E_t\{\widehat{\pi}_{t+1}\} - \lambda \hat{\mu}_t \\ &= \beta E_t\{\widehat{\pi}_{t+1}\} + \frac{\lambda}{\chi}\chi\widehat{\chi}_t - \lambda((1-\alpha)\widehat{A}_t - \alpha\widehat{E}_t) \\ &= \beta E_t\{\widehat{\pi}_{t+1}\} + \frac{\lambda \mu E^\alpha}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f} \left\{ \widehat{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widehat{\theta}_{t+1}\} + \beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widehat{r}_t \right. \\ & \left. - \frac{[1+(1-\rho)\beta(1-f)\rho_\eta]}{\gamma}\widehat{\eta}_t \right\} + \frac{\lambda\mu E^\alpha}{1-\alpha}(1-\Gamma)MRS\widehat{MRS}_t - \lambda((1-\alpha)\widehat{A}_t - \alpha\widehat{E}_t). \end{aligned} \tag{F.58}$$

When the steady state is efficient, we can write the NKPC in terms of efficiency gaps. At an efficient equilibrium, (F.58) becomes

$$\begin{aligned} 0 = & \frac{\lambda E^\alpha}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f} \left\{ \widehat{\theta}_t^* - \beta(1-\rho)(1-f)E_t\{\widehat{\theta}_{t+1}^*\} + \beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widehat{r}_t^* \right\} \\ & + \frac{\lambda E^\alpha}{1-\alpha}(1-\Gamma)MRS\widehat{MRS}_t^* - \lambda((1-\alpha)\widehat{A}_t - \alpha\widehat{E}_t^*), \end{aligned} \tag{F.59}$$

which coincides with the dynamic efficiency condition (B.19). Then, if we evaluate (F.58) at the efficient steady state and we subtract (F.59), NKPC becomes

$$\widetilde{\pi}_t = \beta E_t\{\widetilde{\pi}_{t+1}\} + \frac{\lambda E^{\alpha}}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f}\left\{\widetilde{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widetilde{\theta}_{t+1}\} + \beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widetilde{r}_t \right.$$
$$\left. -\frac{[1+(1-\rho)\beta(1-f)\rho_\eta]}{\gamma}\widehat{\eta}_t\right\} + \frac{\lambda E^{\alpha}}{1-\alpha}(1-\Gamma)MRS\widetilde{MRS}_t + \lambda\alpha\widetilde{E}_t, \tag{F.60}$$

where

$$\widetilde{MRS}_t = \left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widetilde{h}_t + \widetilde{C}_t. \tag{F.61}$$

*Appendix F.5.2. Endogenous participation*

For the model with endogenous participation, (F.60) can be further simplified using the participation condition (F.38) and the steady-state efficiency conditions (B.13) and (B.15):

$$\widetilde{\pi}_t = \beta E_t\{\widetilde{\pi}_{t+1}\} + \frac{\lambda E^{\alpha} MRS}{1-\alpha}\left\{\left[1+\Gamma\frac{1-f}{f}\right]\widetilde{\theta}_t - \beta(1-\rho)\Gamma\frac{(1-f)}{f}E_t\{\widetilde{\theta}_{t+1}\}\right.$$
$$\left. +\beta(1-\rho)\Gamma\frac{(1-\gamma f)}{\gamma f}\widetilde{r}_t - \left[\frac{\Gamma}{\gamma f}\left(1+(1-\rho)\beta(1-f)\rho_\eta\right) + \frac{1-\Gamma}{\gamma}\right]\widehat{\eta}_t\right\} + \lambda\alpha\widetilde{E}_t. \tag{F.62}$$

*Appendix F.6. Equation 6: Resource constraint*

For both models, the resource constraint is given by

$$C_t = \Delta_t^{\alpha-1}\left(A_t E_t\right)^{1-\alpha} - \frac{\kappa}{\omega}\theta_t^{\gamma} E_t + (1-\rho)\frac{\kappa}{\omega}\theta_t^{\gamma} E_{t-1}. \tag{F.63}$$

Log-linearizing it, we have

$$\widehat{C}_t = \left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^{\gamma}}{\omega}\right]\frac{E}{C}\widehat{E}_t - \rho\gamma\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\widehat{\theta}_t + (1-\rho)\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\widehat{E}_{t-1} +$$
$$(1-\alpha)\frac{E}{C}E^{-\alpha}\widehat{A}_t, \tag{F.64}$$

and

$$\widetilde{C}_t = \left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^{\gamma}}{\omega}\right]\frac{E}{C}\widetilde{E}_t - \rho\gamma\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\widetilde{\theta}_t + (1-\rho)\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\widetilde{E}_{t-1}. \tag{F.65}$$

## Appendix G. Optimal monetary policy: the case of the efficient steady state

### *Appendix G.1. Optimal policy without bargaining shocks*

Assume that the steady state is efficient and there are no bargaining shocks. If $\widehat{\pi}_t = 0$ for all $t$, then NKPC (F.58) becomes

$$\begin{aligned} 0 = &\frac{\gamma\kappa\theta}{(1-\gamma)f}\left\{\widehat{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widehat{\theta}_{t+1}\} + \beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widehat{r}_t\right\} \\ &+ (1-\Gamma)MRS\widehat{MRS}_t - (1-\alpha)^2E^{-\alpha}\widehat{A}_t + \alpha(1-\alpha)E^{-\alpha}\widehat{E}_t, \end{aligned} \tag{G.1}$$

which coincides with the dynamic efficiency condition (B.19). Moreover, as we pointed out above, the participation condition implies the static efficiency condition. Therefore, a zero-inflation policy implements the efficient allocation and it is thus optimal in both models.

### *Appendix G.2. Optimal policy with bargaining shocks*

If $\hat{\eta}_t$ fluctuates, a cost-push shock arises: full inflation stabilization induces inefficient fluctuations in the cost of hiring new workers, $\hat{\chi}_t$, which in turn generates inefficient movements in labor market tightness. In fact, the dynamic efficiency condition is not satisfied. Rather, it has a residual that coincides with the cost-push term. Therefore, the central bank will have to trade off inflation against tightness fluctuations. In what follows, we compute the optimal monetary policy for this more general case. We do so for both versions of the model so as to emphasize the role played by the endogenous participation choice in the monetary policy design.

#### *Appendix G.2.1. Endogenous participation*

The monetary authority chooses $\{\widetilde{\pi}_t, \widetilde{C}_t, \widetilde{r}_t, \widetilde{\theta}_t, \widetilde{E}_t, \widetilde{N}_t, \widetilde{h}_t, \widetilde{MRS}_t\}_{t=0}^{\infty}$ so as to maximize

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\delta_E\widetilde{E}_t^2 + \delta_\theta\widetilde{\theta}_t^2 + \delta_c\widetilde{C}_t^2 + \delta_h\widetilde{h}_t^2 + \delta_\pi\tilde{\pi}_t^2\right\} \tag{G.2}$$

subject to

- $\lambda_{1,t}$

$$\widetilde{\theta}_t = \frac{1}{\rho(1-\gamma)}\widetilde{E}_t - \frac{\rho_e}{\rho(1-\gamma)}\widetilde{E}_{t-1} - \frac{1-\rho_e}{\rho(1-\gamma)}\widetilde{N}_t \tag{G.3}$$

- $\lambda_{2,t}$

$$\widetilde{C}_t = E_t\{\widetilde{C}_{t+1}\} - \widetilde{r}_t \tag{G.4}$$

- $\lambda_{3,t}$

$$\widetilde{\pi}_t = \beta E_t\{\widetilde{\pi}_{t+1}\} + \frac{\lambda E^{\alpha}}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f}\left\{\widetilde{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widetilde{\theta}_{t+1}\} + \beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widetilde{r}_t - \frac{[1+(1-\rho)\beta(1-f)\rho_{\eta}]}{\gamma}\widehat{\eta}_t\right\} + \frac{\lambda E^{\alpha}}{1-\alpha}(1-\Gamma)MRS\widetilde{MRS}_t + \lambda\alpha\widetilde{E}_t \tag{G.5}$$

- $\lambda_{4,t}$

$$\widetilde{h}_t = (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\left[-\left(1+\Gamma\frac{1-f}{f}\right)E\widetilde{E}_t + (1-\rho)\Gamma\frac{1-f}{f}E\widetilde{E}_{t-1} + \rho\Gamma E\frac{1-\gamma}{f}\widetilde{\theta}_t\right] \tag{G.6}$$

- $\lambda_{5,t}$

$$\widetilde{C}_t = \left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^{\gamma}}{\omega}\right]\frac{E}{C}\widetilde{E}_t - \rho\gamma\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\widetilde{\theta}_t + (1-\rho)\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\widetilde{E}_{t-1} \tag{G.7}$$

- $\lambda_{6,t}$

$$\widetilde{MRS}_t = \left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widetilde{h}_t + \widetilde{C}_t \tag{G.8}$$

- $\lambda_{7,t}$

$$\widetilde{MRS}_t = \widetilde{\theta}_t - \gamma^{-1}\widehat{\eta}_t \tag{G.9}$$

FOC:

- $\widetilde{\pi}_t$:

$$\delta_{\pi}\widetilde{\pi}_t + \lambda_{3,t-1} - \lambda_{3,t} = 0 \tag{G.10}$$

- $\widetilde{C}_t$:

$$\delta_C\widetilde{C}_t + \frac{\lambda_{2,t-1}}{\beta} - \lambda_{2,t} - \lambda_{5,t} + \lambda_{6,t} = 0 \tag{G.11}$$

- $\widetilde{r}_t$:

$$-\lambda_{2,t} + \beta(1-\rho)\frac{\lambda E^{\alpha}}{1-\alpha}\frac{\kappa\theta(1-\gamma f)}{(1-\gamma)f}\lambda_{3,t} = 0 \tag{G.12}$$

- $\widetilde{\theta}_t$:

$$\delta_{\theta}\widetilde{\theta}_t - \lambda_{1,t} + \frac{\lambda E^{\alpha}}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f}[\lambda_{3,t} - (1-\rho)(1-f)\lambda_{3,t-1}] + + (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\rho\Gamma E\frac{1-\gamma}{f}\lambda_{4,t} - \rho\gamma\frac{\kappa\theta^{\gamma}}{\omega}\frac{E}{C}\lambda_{5,t} + \lambda_{7,t} = 0 \tag{G.13}$$

- $\widetilde{E}_t$:

$$\delta_E \widetilde{E}_t + \frac{1}{\rho(1-\gamma)}\lambda_{1,t} - \frac{\beta\rho_e}{\rho(1-\gamma)}E_t\{\lambda_{1,t+1}\} + \lambda\alpha\lambda_{3,t} -$$
$$(1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\left(1+\Gamma\frac{1-f}{f}\right)E\lambda_{4,t}+$$
$$+\beta(1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}(1-\rho)\Gamma\frac{1-f}{f}EE_t\{\lambda_{4,t+1}\} + \left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega}\right]\frac{E}{C}\lambda_{5,t}+$$
$$+\beta(1-\rho)\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}E_t\{\lambda_{5,t+1}\} = 0 \tag{G.14}$$

- $\widetilde{h}_t$:

$$\delta_h \widetilde{h}_t - \lambda_{4,t} + \left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\lambda_{6,t} = 0 \tag{G.15}$$

- $\widetilde{MRS}_t$:

$$\frac{\lambda E^\alpha}{1-\alpha}(1-\Gamma)MRS\lambda_{3,t} - \lambda_{6,t} - \lambda_{7,t} = 0 \tag{G.16}$$

- $\widetilde{N}_t$:

$$\lambda_{1,t} = 0 \tag{G.17}$$

*Appendix G.2.2. Exogenous participation*

The monetary authority chooses $\{\widetilde{\pi}_t, \widetilde{C}_t, \widetilde{r}_t, \widetilde{\theta}_t, \widetilde{E}_t, \widetilde{h}_t, \widetilde{MRS}_t\}_{t=0}^\infty$ so as to maximize

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\delta_E\widetilde{E}_t^2 + \delta_\theta\widetilde{\theta}_t^2 + \delta_c\widetilde{C}_t^2 + \delta_h\widetilde{h}_t^2 + \delta_\pi\tilde{\pi}_t^2\right\} \tag{G.18}$$

subject to

- $\lambda_{1,t}$

$$\widetilde{\theta}_t = \frac{1}{\rho(1-\gamma)}\widetilde{E}_t - \frac{\rho_e}{\rho(1-\gamma)}\widetilde{E}_{t-1} \tag{G.19}$$

- $\lambda_{2,t}$

$$\widetilde{C}_t = E_t\{\widetilde{C}_{t+1}\} - \widetilde{r}_t \tag{G.20}$$

- $\lambda_{3,t}$

$$\widetilde{\pi}_t = \beta E_t\{\widetilde{\pi}_{t+1}\} + \frac{\lambda E^\alpha}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f}\Big\{\widetilde{\theta}_t - \beta(1-\rho)(1-f)E_t\{\widetilde{\theta}_{t+1}\}+$$
$$\beta(1-\rho)\frac{(1-\gamma f)}{\gamma}\widetilde{r}_t - \frac{[1+(1-\rho)\beta(1-f)\rho_\eta]}{\gamma}\widehat{\eta}_t\Big\} + \tag{G.21}$$
$$\frac{\lambda E^\alpha}{1-\alpha}(1-\Gamma)MRS\widetilde{MRS}_t + \lambda\alpha\widetilde{E}_t$$

- $\lambda_{4,t}$

$$\widetilde{h}_t = (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\left[-\left(1+\Gamma\frac{1-f}{f}\right)E\widetilde{E}_t + (1-\rho)\Gamma\frac{1-f}{f}E\widetilde{E}_{t-1} + \rho\Gamma E\frac{1-\gamma}{f}\widetilde{\theta}_t\right] \tag{G.22}$$

- $\lambda_{5,t}$

$$\widetilde{C}_t = \left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega}\right]\frac{E}{C}\widetilde{E}_t - \rho\gamma\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\widetilde{\theta}_t + (1-\rho)\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\widetilde{E}_{t-1} \tag{G.23}$$

- $\lambda_{6,t}$

$$\widetilde{MRS}_t = \left(\nu - \frac{\alpha_h}{1-\alpha_h}\right)\widetilde{h}_t + \widetilde{C}_t \tag{G.24}$$

The FOCs of the model with exogenous participation coincide with (G.10)-(G.16) after imposing $\lambda_{7,t} = 0$:

- $\widetilde{\pi}_t$:

$$\delta_\pi\widetilde{\pi}_t + \lambda_{3,t-1} - \lambda_{3,t} = 0 \tag{G.25}$$

- $\widetilde{C}_t$:

$$\delta_C\widetilde{C}_t + \frac{\lambda_{2,t-1}}{\beta} - \lambda_{2,t} - \lambda_{5,t} + \lambda_{6,t} = 0 \tag{G.26}$$

- $\widetilde{r}_t$:

$$-\lambda_{2,t} + \beta(1-\rho)\frac{\lambda E^\alpha}{1-\alpha}\frac{\kappa\theta(1-\gamma f)}{(1-\gamma)f}\lambda_{3,t} = 0 \tag{G.27}$$

- $\widetilde{\theta}_t$:

$$\delta_\theta\widetilde{\theta}_t - \lambda_{1,t} + \frac{\lambda E^\alpha}{1-\alpha}\frac{\gamma\kappa\theta}{(1-\gamma)f}\left[\lambda_{3,t} - (1-\rho)(1-f)\lambda_{3,t-1}\right] + (1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\rho\Gamma E\frac{1-\gamma}{f}\lambda_{4,t} - \rho\gamma\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}\lambda_{5,t} = 0 \tag{G.28}$$

- $\widetilde{E}_t$:

$$\begin{aligned}&\delta_E\widetilde{E}_t + \frac{1}{\rho(1-\gamma)}\lambda_{1,t} - \frac{\beta\rho_e}{\rho(1-\gamma)}E_t\{\lambda_{1,t+1}\} + \lambda\alpha\lambda_{3,t} - \\ &(1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}\left(1+\Gamma\frac{1-f}{f}\right)E\lambda_{4,t} + \\ &+\beta(1-\alpha_h)h^{-\frac{1}{1-\alpha_h}}(1-\rho)\Gamma\frac{1-f}{f}EE_t\{\lambda_{4,t+1}\} + \left[(1-\alpha)E^{-\alpha} - \frac{\kappa\theta^\gamma}{\omega}\right]\frac{E}{C}\lambda_{5,t} + \\ &+\beta(1-\rho)\frac{\kappa\theta^\gamma}{\omega}\frac{E}{C}E_t\{\lambda_{5,t+1}\} = 0\end{aligned} \tag{G.29}$$

- $\widetilde{h}_t$:

$$\delta_h \widetilde{h}_t - \lambda_{4,t} + \left( \nu - \frac{\alpha_h}{1-\alpha_h} \right) \lambda_{6,t} = 0 \tag{G.30}$$

- $\widetilde{MRS}_t$:

$$\frac{\lambda E^\alpha}{1-\alpha}(1-\Gamma) MRS \lambda_{3,t} - \lambda_{6,t} = 0 \tag{G.31}$$

*Appendix G.2.3. The near-optimality of strict inflation targeting*

Let the superscript $A$ indicate the optimal policy and the superscript $B$ indicate a policy of strict inflation targeting. Define $\Lambda$ as the fraction of consumption that the household would have to give up to be as well off under regime $A$ as under regime $B$. Denote by $W^A$ and $W^B$ the utility associated with the respective policies evaluated according to (E.38). One can show that

$$\Lambda = 1 - e^{(1-\beta)(W^B - W^A)}, \tag{G.32}$$

where $\Lambda$ measures the cost of switching from policy $A$ to policy $B$. We compute the cost of a strict target on inflation relative to the optimal policy and we find that, in both models, it is never larger than $10^{-3}$ percentage points of steady-state consumption. We conclude that strict inflation targeting is nearly optimal under bargaining shocks.

## Appendix H. Optimal monetary policy: the case of a distorted steady state

When calibrating the model to U.S. data, the unemployment benefit is chosen such that $b/w = 0.4$ and the final-good sector charges a steady-state markup of 20%, which is not corrected for by an appropriate production subsidy. Therefore, steady-state efficiency conditions are violated. We can no longer use (E.38) to approximate welfare. For this more general case, we follow a conventional second-order perturbation method. For the sake of clarity, we list below all equations defining the competitive equilibrium in both models.

*Appendix H.1. Non-linear system - Endogenous model*

- Law of motion of employment

$$E_t = (1-\rho)(1-f_t)E_{t-1} + f_t N_t \tag{H.1}$$

- Job-filling rate

$$q_t = \omega \theta_t^{-\gamma} \tag{H.2}$$

- Job-finding rate

$$f_t = \theta_t q_t \tag{H.3}$$

- Home-production technology

$$h_t = \left[A_t^h(1 - E_t - \Gamma U_t)\right]^{1-\alpha_h} \tag{H.4}$$

- Definition of participation

$$N_t = E_t + U_t \tag{H.5}$$

- Euler equation

$$\beta R_t E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{1}{\pi_{t+1}} \right\} = 1 \tag{H.6}$$

- Participation condition

$$\begin{aligned} \frac{1-f_t}{f_t}\left(\Gamma MRS_t - b\right) = \frac{W_t}{P_t} - MRS_t + \\ \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{(1-\rho)(1-f_{t+1})}{f_{t+1}} \left(\Gamma MRS_{t+1} - b\right) \right\} \end{aligned} \tag{H.7}$$

- Definition of marginal rate of substitution

$$MRS_t = \phi(1-\alpha_h) \frac{A_t^h C_t h_t^{\nu - \frac{\alpha_h}{1-\alpha_h}}}{Z_t} \tag{H.8}$$

- Job-creation condition

$$\frac{\kappa}{q_t} = \frac{P_t^X}{P_t} A_t - \frac{W_t}{P_t} + (1-\rho) E_t \left\{ \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{\kappa}{q_{t+1}} \right\} \tag{H.9}$$

- Real wage

$$\begin{aligned} \frac{W_t}{P_t} = (1-\eta_t) \frac{P_t^x}{P_t} A_t + \eta_t \left[ b + (1-\Gamma) MRS_t \right] + \\ (1-\eta_t)(1-\rho) E_t \left\{ \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \kappa \theta_{t+1} \right\} \end{aligned} \tag{H.10}$$

- Market clearing

$$Y_t = C_t + \kappa V_t \tag{H.11}$$

- Final-good sector production function

$$Y_t = (A_t E_t)^{1-\alpha} \Delta_t^{\alpha-1} \tag{H.12}$$

- Evolution of price dispersion

$$\Delta_t = (1-\delta) \left( \frac{\bar{P}_t}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} + \delta \pi_t^{\frac{\varepsilon}{1-\alpha}} \Delta_{t-1} \tag{H.13}$$

- Price index

$$\frac{\bar{P}_t}{P_t} = \left( \frac{1 - \delta \pi_t^{\varepsilon-1}}{1-\delta} \right)^{\frac{1}{1-\varepsilon}} \tag{H.14}$$

- Optimal pricing equation

$$\frac{\bar{P}_t}{P_t} = \frac{K_t}{F_t} \tag{H.15}$$

- Recursive formulation for $K_t$

$$K_t = \frac{Y_t}{C_t} \mu RMC_t + \beta \delta E_t \left\{ \pi_{t+1}^{\varepsilon} \frac{Z_{t+1}}{Z_t} K_{t+1} \right\} \tag{H.16}$$

- Recursive formulation for $F_t$

$$F_t = \frac{Y_t}{C_t} + \beta \delta E_t \left\{ \pi_{t+1}^{\varepsilon-1} \frac{Z_{t+1}}{Z_t} F_{t+1} \right\} \tag{H.17}$$

- Definition of real marginal cost

$$RMC_t = \frac{1}{1-\alpha} \frac{P_t^x}{P_t} \left( \frac{\bar{P}_t}{P_t} \right)^{-\varepsilon \frac{\alpha}{1-\alpha}} (C_t + \kappa V_t) \tag{H.18}$$

- Monetary policy rule

$$\log(R_t) = -\log(\beta) + \phi_\pi \log(\pi_t) + \phi_y \log\left( \frac{Y_t}{Y} \right) - \phi_u \left( \log\left( \frac{U_t}{N_t} \right) - \log\left( \frac{U}{N} \right) \right) \tag{H.19}$$

- Definition of market tightness

$$\theta_t = \frac{V_t}{S_t} \tag{H.20}$$

- Definition of searching workers

$$S_t = N_t - (1-\rho) E_{t-1} \tag{H.21}$$

*Appendix H.2. Non-linear system - Exogenous model*

- Law of motion of employment

$$E_t = (1-\rho)(1-f_t) E_{t-1} + f_t N \tag{H.22}$$

- Job-filling rate

$$q_t = \omega \theta_t^{-\gamma} \tag{H.23}$$

- Job-finding rate

$$f_t = \theta_t q_t \tag{H.24}$$

- Home-production technology

$$h_t = \left[A_t^h (1 - E_t - \Gamma U_t)\right]^{1-\alpha_h} \tag{H.25}$$

- Definition of participation

$$N = E_t + U_t \tag{H.26}$$

- Euler equation

$$\beta R_t E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{1}{\pi_{t+1}} \right\} = 1 \tag{H.27}$$

- Definition of marginal rate of substitution

$$MRS_t = \phi(1-\alpha_h) \frac{A_t^h C_t h_t^{\nu - \frac{\alpha_h}{1-\alpha_h}}}{Z_t} \tag{H.28}$$

- Job-creation condition

$$\frac{\kappa}{q_t} = \frac{P_t^X}{P_t} A_t - \frac{W_t}{P_t} + (1-\rho) E_t \left\{ \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{\kappa}{q_{t+1}} \right\} \tag{H.29}$$

- Real wage

$$\begin{aligned} \frac{W_t}{P_t} = (1-\eta_t) \frac{P_t^x}{P_t} A_t + \eta_t \left[b + (1-\Gamma) MRS_t\right] + \\ (1-\eta_t)(1-\rho) E_t \left\{ \beta \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \kappa \theta_{t+1} \right\} \end{aligned} \tag{H.30}$$

- Market clearing

$$Y_t = C_t + \kappa V_t \tag{H.31}$$

- Final-good sector production function

$$Y_t = (A_t E_t)^{1-\alpha} \Delta_t^{\alpha - 1} \tag{H.32}$$

- Evolution of price dispersion

$$\Delta_t = (1-\delta) \left( \frac{\bar{P}_t}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} + \delta \pi_t^{\frac{\varepsilon}{1-\alpha}} \Delta_{t-1} \tag{H.33}$$

- Price index

$$\frac{\bar{P}_t}{P_t} = \left(\frac{1-\delta\pi_t^{\varepsilon-1}}{1-\delta}\right)^{\frac{1}{1-\varepsilon}} \tag{H.34}$$

- Optimal pricing equation

$$\frac{\bar{P}_t}{P_t} = \frac{K_t}{F_t} \tag{H.35}$$

- Recursive formulation for $K_t$

$$K_t = \frac{Y_t}{C_t}\mu RMC_t + \beta\delta E_t\left\{\pi_{t+1}^{\varepsilon}\frac{Z_{t+1}}{Z_t}K_{t+1}\right\} \tag{H.36}$$

- Recursive formulation for $F_t$

$$F_t = \frac{Y_t}{C_t} + \beta\delta E_t\left\{\pi_{t+1}^{\varepsilon-1}\frac{Z_{t+1}}{Z_t}F_{t+1}\right\} \tag{H.37}$$

- Definition of real marginal cost

$$RMC_t = \frac{1}{1-\alpha}\frac{P_t^x}{P_t}\left(\frac{\bar{P}_t}{P_t}\right)^{-\varepsilon\frac{\alpha}{1-\alpha}}(C_t+\kappa V_t) \tag{H.38}$$

- Monetary policy rule

$$\log(R_t) = -\log(\beta) + \phi_\pi\log(\pi_t) + \phi_y\log\left(\frac{Y_t}{Y}\right) - \phi_u\left(\log\left(U_t\right) - \log(U)\right) \tag{H.39}$$

- Definition of market tightness

$$\theta_t = \frac{V_t}{S_t} \tag{H.40}$$

- Definition of searching workers

$$S_t = N - (1-\rho)E_{t-1} \tag{H.41}$$

*Appendix H.3. Welfare losses*

Let the superscript $A$ indicate a policy of strict inflation targeting, while the superscript $B$ indicates an alternative policy. Let $\Lambda$ be the fraction of consumption that the household would have to give up to be as well off under regime $A$ as under regime $B$, i.e., let $\Lambda$ be such that

$$E_0\sum_{t=0}^{\infty}\beta^t\left[Z_t\log((1-\Lambda)C_t^A) + \phi\frac{(h_t^A)^{1+\nu}}{1+\nu}\right] = E_0\sum_{t=0}^{\infty}\beta^t\left[Z_t\log(C_t^B) + \phi\frac{(h_t^B)^{1+\nu}}{1+\nu}\right]. \tag{H.42}$$

We compute the expected welfare conditional on being at the non-stochastic steady state (which is the same across all monetary policy regimes) at time 0. Therefore, $\Lambda$ measures the welfare cost in terms of steady-state consumption of switching from regime $A$ to regime $B$. Let

$$W_0^i \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[ Z_t \log(C_t^i) + \phi \frac{(h_t^i)^{1+\nu}}{1+\nu} \right], \tag{H.43}$$

for $i \in A, B$. Then,

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_t \log(1-\Lambda) + W_0^A = W_0^B. \tag{H.44}$$

If there are no preference shocks, $Z_t = 0 \; \forall t$, and

$$\Lambda = 1 - e^{(1-\beta)(W_0^B - W_0^A)}. \tag{H.45}$$

In the more general case instead,

$$\Lambda = 1 - e^{(1-\beta\rho_Z)(W_0^B - W_0^A)}. \tag{H.46}$$

Table I.1 reports the values of $\Lambda$. The cost of implementing a simple rule ($R2$ or $R3$) instead of adopting strict inflation targeting ($R1$) ranges from a minimum of 0.12 to a maximum of 1.73.

## Appendix I. Consumption of home appliances

In the model, we allow for a positive correlation between innovations to home and market technology, denoted by $\rho_\xi$. Some types of technology might indeed be relevant for both the home and the market sector, for example in the case of Internet-related innovations. Also, since in the model we abstract from capital accumulation for the sake of simplicity, we see this as a reduced way of capturing the fact that the quantity or the quality of home capital goods might covary with market productivity. In support of this hypothesis, we use the data set from Boivin et al. (2009) to obtain the "Quantities of major household appliances from personal consumption expenditure (Q1FNR10)," compute the correlation between this variable and the real GDP over the period 1976Q1 - 2004Q4, and find it to be 0.81.

Table I.1: Unconditional moments and welfare losses. We compare standard deviations (relative to output) of selected variables in the endogenous and exogenous participation models under three rules. Volatilities are expressed in percentage standard deviations. We also report welfare losses (in terms of steady-state consumption) of deviating from strict inflation targeting (R1) and adopting one of the two alternative rules. R2: $\phi_\pi = 1.5$ and $\phi_y = \phi_u = 0$. R3: $\phi_\pi = 1.5$, $\phi_y = 0$ and $\phi_u = 0.5/4$. Stochastic processes are calibrated as in section 4.3.

| | Endogenous | | | Exogenous | | |
|---|---|---|---|---|---|---|
| | R1 | R2 | R3 | R1 | R2 | R3 |
| Output (%) | 1.46 | 1.43 | 1.26 | 1.69 | 1.56 | 1.21 |
| Unemployment rate | 6.55 | 7.36 | 1.50 | 8.77 | 7.55 | 2.77 |
| Employment | 0.48 | 0.67 | 0.49 | 0.55 | 0.47 | 0.17 |
| Participation rate | 0.40 | 0.24 | 0.41 | 0 | 0 | 0 |
| Welfare losses | | 1.7335 | 0.1228 | | 0.3018 | 1.2101 |