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Abstract

We present CoMargin, a new methodology to estimate collateral requirements for central counterparties (CCPs) in derivatives markets. CoMargin depends on both the tail risk of a given market participant and its interdependence with other participants. Our approach internalizes market interdependencies and enhances the stability of CCPs, thereby reducing the systemic risk concerns associated with them. CoMargin can be estimated using a model-free and scenario-based methodology, validated using formal statistical tests, and generalized to any number of market participants. We assess and illustrate our methodology using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). Our data set, the first one of its kind to be used in an academic study, includes daily observations of the actual trading positions of all CDCC members from 2003 to 2011. We show theoretically and empirically that CoMargin outperforms existing margining systems by stabilizing the probability and minimizing the shortfall of simultaneous margin-exceeding losses. The relative performance of our methodology increases when trading similarities across clearing members or co-movements among underlying assets increase, as was the case during the recent financial crisis.

JEL classification: G13
Bank classification: Financial stability; Financial markets; Financial institutions; Econometric and statistical methods

Résumé

Les auteurs présentent la CoMargin, une nouvelle méthodologie permettant d'estimer les exigences en matière de dépôts de marge que devraient imposer les contreparties centrales sur le marché de produits dérivés. La méthode CoMargin se base sur le risque extrême auquel sont exposés les participants au marché, ainsi que sur les interdépendances qui existent entre leurs pertes. Cette approche améliore la stabilité des contreparties centrales, atténuant ainsi les appréhensions liées au risque systémique suscitées par ces contreparties. La CoMargin peut être estimée à l'aide de scénarios sans recourir à des modèles, et ce, pour n'importe quel nombre d'acteurs de marché. Les résultats de l'estimation sont validés au moyen de tests statistiques formels. Les auteurs utilisent des données exclusives de la Corporation canadienne de compensation de produits dérivés (CDCC) afin d'évaluer et d'illustrer leur méthodologie. Cette base de données, la première du genre à être employée dans une étude universitaire, comprend les positions quotidiennes de négociation réelles de tous les membres de la CDCC de 2003 à 2011. Les auteurs montrent, de manière théorique et empirique, que la CoMargin domine les systèmes actuels d'établissement de marges en stabilisant la probabilité de pertes et en minimisant le déficit attribuable à des pertes simultanées supérieures aux dépôts de marge. La méthodologie s'avère particulièrement efficace lorsque la similitude entre les positions de négociation des membres compensateurs s'accroît ou en présence de forte corrélation entre les actifs sous-jacents, comme ce fut le cas pendant la récente crise financière.

Classification JEL : G13
Classification de la Banque : Stabilité financière; Marchés financiers; Institutions financières; Méthodes économétriques et statistiques

Introduction

How much collateral should a market participant post against its derivatives positions? In this paper, we argue that margin requirements should increase with both the variability and the interdependence of the profits and losses (P&Ls) of market participants. We show that commonly used margining systems, such as the Standard Portfolio Analysis of Risk (SPAN) or the value-at-risk (VaR) approach, often fail to properly allocate collateral requirements because they disregard interdependencies across the portfolios of different market members. Our objective is to address this issue by proposing a new margining system, called CoMargin, which explicitly internalizes P&L interdependence. Our methodology is a model-free, scenario-based approach that can be generalized to any number of market participants and, unlike the SPAN system, formally backtested by using an extension of existing statistical techniques.

We focus on clearing houses in derivatives markets because these institutions concentrate a significant amount of credit risk in the financial system (Pirrong, 2009). However, our margining and backtesting methodology is general enough that it can be applied to any context where counterparty risk needs to be managed. Examples include, but are not limited to, collateral and capital requirements for repo transactions, over-the-counter (OTC) securities dealers, banks, insurance companies and newly proposed swap execution facilities (SEFs).

In a derivatives exchange, the clearing house is in charge of confirming, matching and settling all trades. Clearing houses operate with a small number of firms, called clearing members (CMs), who are allowed to submit their own trades for clearing through their firm accounts (i.e., conduct proprietary trading), as well as those of their customers or other non-clearing members through their client accounts.

The process of novation allows the clearing house to become the only counterparty to every contract. For this reason, clearing houses are sometimes known as central counterparties (CCPs). Throughout this process, the clearing house remains market-risk neutral because, by construction, the number of long positions is equal to the number of short positions outstanding for all contracts. However, the clearing house accumulates a significant amount of credit risk, which is primarily managed through the use of margining systems.[1]

A clearing house margining system requires members to post cash or financial assets as collateral in a margin account. These funds are used to protect the clearing house against the losses and potential default of its members over a period of time, usually one day. However, CMs sometimes experience losses that exceed their posted collateral, which leaves them with a negative balance in their margin accounts. We refer to these losses as *margin-exceeding losses* or just *exceedances* for short. CMs that experience exceedances may delay payment or in some cases default on their obligations, thus creating a shortfall in the market. The CCP has to cover this shortfall with its own funds and compensate the clearing members that profited from their trading

[1] Other common credit-risk-management tools used by CCPs include requiring members to hold minimum capital levels, contribute to a default fund, enter into private insurance arrangements and segregate between client and firm margin accounts (see Jones and Pérignon, 2013).

positions. Usually, financing the shortfall of a single CM over a limited time does not impose a hefty financial burden on the clearing house. However, when two or more large CMs experience simultaneous margin-exceeding losses, the consequences tend to be more severe. If these CMs delay their payments temporarily, the resulting shortfall tends to be short-lived, but it can significantly affect market liquidity, particularly during volatile periods. On the other hand, if these CMs default, the shortfall tends to be long-lived or even permanent, which can erode the resources of the clearing house to the point of financial distress or even failure.

While clearing house failures are rare events, the cases of Paris in 1973, Kuala Lumpur in 1983 and Hong Kong in 1987 demonstrate that these extreme scenarios are not only possible, but also very economically significant (Knott and Mills, 2002).[2] In addition, the systemic importance of CCPs has increased in recent years due to their consolidation through economic integration and mergers and acquisitions, and due to the strong pressure from governments and market participants to facilitate or mandate the central clearing of OTC derivatives (Acharya et al., 2009; U.S. Congress' OTC Derivatives Market Act of 2009; U.S. Department of Treasury, 2009; Duffie, Li, and Lubke, 2010; Duffie and Zhu, 2011).[3] Therefore, it is increasingly necessary to devise appropriate risk-management systems that enhance the stability and resiliency of clearing facilities.

Current margining systems employed by derivatives exchanges set collateral requirements based on a coverage level or a target probability of an exceedance event for an individual contract or a portfolio of contracts (Figlewski 1984; Booth et al. 1997; Cotter 2001).[4] However, by focusing only on individual contracts or member portfolios, these systems ignore the fact that sometimes CMs face homogeneous risk exposures that make their losses highly interdependent. This situation exposes the clearing house to simultaneous exceedance events that could undermine its stability.

The level of P&L dependence across clearing members increases with trade crowdedness and underlying asset co-movement (Cruz Lopez, 2013). Trade crowdedness refers to the similarity of trading positions across CMs. When positions across portfolios are very similar, they tend to have equivalent exposures and returns, regardless of how underlying assets behave. Underlying asset co-movement refers to asset returns moving in unison. When underlying assets experience high levels of co-movement, CMs tend to face similar risk

[2] Default of CMs is of course much more frequent. Recent examples in the Chicago Mercantile Exchange (CME) include Refco in 2005, Lehman in 2008, and MF Global in 2011 (see Jones and Pérignon, 2013).
[3] The CME, Intercontinental Exchange (ICE), EUREX, Euronext Liffe, and LCH.Clearnet have each recently created clearing facilities for credit default swaps.
[4] For example, Kupiec (1994 and 1995) shows the empirical performance of the SPAN system for selected portfolios of S&P 500 futures and futures-options contracts and finds that, over the period from 1988 to 1992, the historical margin coverage exceeds 99% for most portfolios included in the sample.

exposures regardless of the composition of their portfolios, because securities in all portfolios tend to move in the same direction.[5]

Both dimensions of P&L dependence are related. However, trade crowdedness is directly influenced by the individual trading behaviour of clearing members, while asset co-movement is determined by aggregate market behaviour. Similar trading positions, or crowded trades, tend to arise among large clearing members, because they share a common (and superior) information set. This informational advantage leads them to pursue similar directional trades, arbitrage opportunities and hedging strategies.[6] On the other hand, underlying assets tend to move in the same direction during economic slowdowns or during periods of high volatility, both of which are rarely the result of the actions undertaken by an individual market participant.[7]

In this paper we depart from the traditional view of setting margin requirements based on individual member positions, and propose a methodology that accounts for their interdependence. We estimate the margin requirement of each CM conditional on one or several other members being in financial distress. A CM is said to be in financial distress if its losses exceed its P&L VaR. By adopting this approach, we obtain individual margin requirements that increase with P&L dependence, stabilize the probability of exceedance events given financial distress and reduce the risk that the clearing house exhausts its funds due to large or sudden shortfalls.

Our method builds on the CoVaR concept introduced by Adrian and Brunnermeier (2011) to identify systemically important financial institutions. CoVaR is defined as the VaR of the financial system (i.e., the banking sector) conditional on a given institution being in financial distress (i.e., exceeding its VaR). The core of their analysis, denominated ΔCoVaR, measures the marginal contribution of a particular institution to the overall risk in the system. ΔCoVaR is calculated as the difference between the VaR of the financial system conditional on a given institution being in distress and the VaR of the financial system in the median state of the institution. Similarly, by inverting the conditioning relationship, one can assess the exposure of a given institution to the state of the financial system.

There are, however, some key differences between the CoMargin and CoVaR methodologies that are worth noticing. First, the objective of CoMargin is not to measure systemic risk. Instead, it is used to estimate margin requirements that account for the interdependence of market participants. Thus, we are not concerned with the state (i.e., VaR) of the financial system, but with the coverage that the CCP derives from collecting

[5] The importance of asset co-movement has been identified in previous studies. For example, in an early attempt to analyze the default risk of a clearing house, Gemmill (1994) highlights the dramatic diversification benefit from combining contracts on uncorrelated or weakly correlated assets.
[6] Much of the proprietary trading activity in derivatives exchanges consists of arbitraging futures and OTC or cash markets (e.g. cash-futures arbitrage of the S&P 500 index, eurodollar-interest rate swap arbitrage, etc.).
[7] Extreme dependence and contagion across assets is discussed in Longin and Solnik (2001); Bae, Karolyi and Stulz (2003); Longstaff (2004); Poon, Rockinger and Tawn (2004); Boyson, Stahel and Stulz (2010); and Harris and Stahel (2011), among others.

collateral. Second, CoMargin does not define financial distress in terms of the VaR of bank stock returns, but in terms of the VaR of the potential (one-day-ahead) P&Ls of clearing members. Furthermore, unlike CoVaR, which can be estimated by conditioning on the financial distress of all members in the banking system, CoMargin can only be estimated by conditioning on a subset of market participants, because by construction the aggregate P&L across all CMs in a derivatives exchange is zero. Finally, the estimation of CoMargin is semi-parametric and much simpler than that of CoVaR, which requires a quantile regression approach.

The CoMargin estimation process starts by taking the trading positions of all clearing members at the end of the trading day as given. A series of one-day-ahead scenarios based on projected changes in the price and volatility of the underlying assets is used to assess changes in the value of the portfolio of each member. For each scenario, we mark-to-model the portfolio of each CM and obtain its hypothetical one-day-ahead P&L. We use these hypothetical P&L calculations to compute margin requirements that target the probability of margin exceedances conditional on the financial distress of other members.

Our results show that the CoMargin system enhances the stability of the CCP by maintaining the probability of margin exceedances conditional on the financial distress of other members constant and by reducing the occurrence of simultaneous margin-exceeding losses. In addition, our method increases financial resiliency because it actively adjusts the allocation of collateral as a function of market conditions that influence P&L dependence. As a result, the average magnitude of the shortfall given simultaneous exceedances is minimized relative to other collateral systems. Both of these conditions greatly reduce the systemic risk concerns associated with CCPs.

The remainder of the paper is organized as follows. In section 2, we describe how margin requirements are currently estimated under the SPAN and VaR margining systems. In section 3 we define a list of properties needed to achieve a sound margining system. We describe the theoretical foundations of the CoMargin system in section 4 and examine its empirical effectiveness in section 5. Finally, section 6 concludes.

2. Standard Margining Systems

2.1. Derivatives market

Consider a derivatives exchange with N clearing members and D derivatives securities (futures, options, credit default swaps, etc.) written on U underlying assets. Let $w_{i,t}$ be the number of contracts in the derivatives portfolio of clearing member i, for $i = 1, \dots, N$, at the end of day t:

$$w_{i,t} = \begin{bmatrix} w_{1,i,t} \\ \vdots \\ w_{D,i,t} \end{bmatrix}. \tag{1}$$

Margins are collected every day from each clearing member to guarantee the performance of their obligations and to guard the clearing house against default. Let $B_{i,t}$ be the performance bond or margin collected by the clearing house from clearing member i at the end of day t. This performance bond is a function of the outstanding trading positions of that member, $w_{i,t}$.

The variation margin, $V_{i,t}$, represents the aggregate portfolio P&L of clearing member i on day t. In this paper, we are interested in cases when trading losses exceed margin requirements; that is, when $V_{i,t} \leq -B_{i,t-1}$. In these cases, we say that firm i has experienced a margin-exceeding loss or an exceedance. Identifying firms in this state is important because they have an incentive to default on their positions or to delay payment on their obligations, which generates a shortfall in the market that needs to be covered by the CCP. Given the limited funds available to the CCP, simultaneous exceedance events can threaten its stability and survival.

2.2. SPAN margin

The CME introduced the Standard Portfolio Analysis of Risk margining methodology in 1988. It has since become the most widely used margining system in derivatives exchanges around the world. Every day following the market close, clearing houses such as the Canadian Derivatives Clearing Corporation (CDCC), the Chicago Mercantile Exchange (CME), Eurex, LCH.Clearnet, Nymex and the Options Clearing Corporation (OCC), among others, use the SPAN system to estimate the margin requirements of their members.

SPAN is a scenario-based methodology that is used to assess potential changes in the value of the derivatives held by each clearing member. However, SPAN does not take a portfolio-wide approach. Instead, it divides each portfolio into contract families, defined as groups of contracts that share the same underlying asset, and estimates a charge for each family independently. Thus, for a portfolio with $d \in D$ derivatives written on $u \in U$ underlying assets, the SPAN system computes u contract family charges.

To compute a contract family charge in a portfolio, the SPAN system simulates one-day-ahead changes in the value of each contract by using sixteen scenarios that vary the price and the volatility of the underlying asset, as well as the time to expiration of the contract (see Table 1). The range of the potential price changes of the underlying asset usually covers 99% of its daily price movements over a historical calibration window. A similar approach is adopted for the volatility. The extreme price changes are used to assess potential changes in deep out-of-the-money options. The scenario analysis yields a risk array for each contract that contains sixteen one-day-ahead potential value changes (i.e., each maturity and each strike price has its own array).[8] The scenario with the worst potential loss for the entire contract family is identified and that loss becomes the first part of the contract family charge.

[8] The projected price changes of non-linear contracts, such as options, are obtained by using numerical valuation methods or option-pricing models.

The second part of the contract family charge consists of a discretionary adjustment that is needed because contracts with different expiration months are assumed to be equivalent in the scenario analysis. In other words, long and short positions written on the same underlying asset but with different expiration months offset each other. Therefore, risk managers are required to add an *intra-commodity spread charge* to the worst-case scenario loss to account for time-spread trading. The resulting value is the contract family charge.

The collateral requirement for an entire portfolio is computed by aggregating the charges across all of its contract families. However, once again, risk managers are required to use discretionary aggregation rules to account for commodity-spread trading (i.e., simultaneous long and short positions in contracts with the same expiration months but written on different though correlated underlying assets). These adjustments are known as *inter-commodity spread charges.*

It is important to note that both intra- and inter-commodity spread charges involve the discretion of risk managers. Thus, these adjustments are rarely consistent across commodities, market conditions or clearing houses. This situation, coupled with the fact that the SPAN system targets underlying price and volatility ranges, instead of the probability of portfolio-wide margin-exceeding losses, makes the exceedance coverage of the SPAN system inconsistent across time and markets.

2.3. VaR margin

VaR is defined as a lower quantile of a P&L distribution. It is the standard measure used to assess the aggregate risk exposure of banks (Berkowitz and O'Brien, 2002; Berkowitz, Christoffersen and Pelletier, 2011), as well as their regulatory capital requirements (Jorion, 2007). VaR can also be used to set margins on a derivatives exchange. In this case, the margin requirement corresponds to a given quantile of a clearing member's one-day-ahead P&L distribution.

Definition 1: The VaR margin of firm i, B_i, corresponds to the $\alpha\%$ quantile of its P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t}\right) = \alpha. \tag{2}$$

Like the SPAN system, the VaR margin method is applied on a firm-by-firm basis using a scenario analysis. However, the scenarios are applied to the entire portfolio (Cruz Lopez, Harris and Pérignon, 2011). More specifically, we consider S scenarios derived from simulated one-day-ahead changes in the value of the price and the volatility of the underlying assets and use them to evaluate each clearing member's entire portfolio. The hypothetical P&L or variation margin of each CM is computed by *marking-to-model* its positions in each scenario. Thus, for each CM and date t, we obtain a simulated sample of $V_{i,t+1}$ denoted $\{v_{i,t+1}^s\}_{s=1}^S$ that can be used to estimate the VaR margin requirement as follows:

$$\hat{B}_{i,t} = percentile\left(\{v_{i,t+1}^s\}_{s=1}^S, 100\alpha\right). \quad (3)$$

Compared to market risk VaR (Jorion, 2007), the estimation of the VaR margin is much simpler. When estimating market risk VaR, there is only one observation available for each asset on date t. Therefore, the quantile of a return distribution of a given asset at time t cannot be estimated without making some strong distributional assumptions. For example, the historical simulation approach broadly used by financial institutions for market risk VaR estimations assumes that the asset returns are independently and identically distributed over time. Under these assumptions, the unconditional VaR is stationary and can be estimated from the historical path of past returns. The estimation of more refined conditional measures also requires some specific assumptions regarding quantile dynamics. For instance, the CAViaR approach proposed by Engle and Manganelli (2004) assumes an autoregressive process for the quantiles.

In the context of the VaR margin, however, the situation is quite different and much simpler because we have S simulated observations of the P&L distribution of each clearing member at time t. This is an ideal situation from an econometric standpoint because the quantile of the P&L distribution can be directly implied without making any assumptions regarding its behavior over time. Thus, $\hat{B}_{i,t}$, which represents the empirical quantile based on the S simulated observations (equation (3)), is a consistent estimate of the P&L VaR when S tends to infinity.

3. Characteristics of a Sound Margining System

Remarkably, there is very little guidance in the literature regarding the properties that a sound collateral system should satisfy. Nevertheless, this is a fundamental issue that needs to be addressed in order to assess the relative merits of different margining methodologies. In this section, we attend to this issue by proposing five main properties that any well designed margining system must satisfy. These properties and the rationale behind them are explained below.

i. Margins must increase with P&L variability

Let $\sigma_{i,t}$ be a measure of the variability of the P&L of clearing member i at time t:

$$\text{If } \sigma_{i,t}^1 \geq \sigma_{i,t}^2, \text{ then } B_{i,t}(w_{i,t}, \sigma_{i,t}^1) \geq B_{i,t}(w_{i,t}, \sigma_{i,t}^2). \quad (4)$$

As Table 2 shows, this basic property is at the heart of existing margining methods. Intuitively, it means that, since riskier trading portfolios (as measured by their variability) tend to have larger potential losses, more collateral must be collected to guarantee their performance. Or, in simple words, riskier clearing members should post higher margins. The SPAN and VaR methods comply with this property because both the worst-case loss of the SPAN system and the quantile that determines the VaR margin tend to increase with the variability of P&Ls.[9]

ii. Margins must increase with P&L dependence

Let $\delta_{i,t}$ be a measure of dependence between the losses of market participant i and those of other market participants at time t. P&L dependence can originate from similarities in trading positions, correlated asset prices, or both:

$$\text{If } \delta_{i,t}^{1} \geq \delta_{i,t}^{2}, \text{ then } B_{i,t}\left(w_{i,t}, \delta_{i,t}^{1}\right) \geq B_{i,t}\left(w_{i,t}, \delta_{i,t}^{2}\right). \quad (5)$$

The intuition behind this property is that a sound margining system should prevent (or minimize) the occurrence of simultaneous margin-exceeding losses across market participants. As shown in section 2, both the SPAN and VaR margin methods set margins on a firm-by-firm basis and hence completely disregard P&L dependence across clearing members.

iii. Margins should not be excessively procyclical

When margins are procyclical, market downturns and excess volatility can lead to higher initial margins and more frequent margin calls. This situation can adversely affect funding conditions and market liquidity, and can force traders to close out their positions simultaneously, thus intensifying market declines. Brunnermeier and Pedersen (2009) explain and model this sequence of reinforcing events, which they refer to as a "margin spiral." Current margin requirements are prone to trigger these spirals because they are only a function of expected price and volatility changes. In addition, discretionary parameters, such as the intra- and inter-commodity charges used in the SPAN system, are usually

[9] Artzner et al. (1999) define a coherent risk measure using four axioms: *monotonicity* (if the returns of portfolio 1 (P1) are always lower than those of portfolio 2 (P2), then P1 is riskier than P2), *translation invariance* (adding $K in cash to P1 reduces its risk by the same amount), *homogeneity* (increasing the size of P1 by a factor *S* increases its risk by the same factor), and *subadditivity* (risk measures need to account for diversification). Conceptually, with non-subadditive margins, it may be optimal for participants to break down their trading portfolio into smaller subportfolios in order to reduce their total margin requirements. However, in practice, clearing houses prevent financial institutions from having multiple clearing members. Furthermore, netting rules allow clearing members to post considerably less margin than what they would be required to post if they had dislocated portfolios.

modified after significant or persistent market shocks, thus causing more variability in the margining process, which can be destabilizing for the market.

iv. Margins must be robust to outliers

Erratic margin swings due to outliers should be prevented as much as possible, since they may lead to severe operational problems, such as sudden margin calls. Since SPAN margins are based on the maximum simulated loss and not a quantile, they are much more sensitive to outliers than the VaR margins.

v. Margins must be testable ex post

The only way to systematically measure the effectiveness of a margining system is by backtesting it. Backtesting aims to identify misspecified models that lead to either excessive or insufficient coverage for the CCP relative to a target. Therefore, if a margining system cannot be backtested using formal statistical methods, we cannot identify its potential shortcomings and fine-tune it to meet its objectives.

The VaR margins can be easily backtested because they are defined by the quantile of the P&L distribution. The intuition behind the backtesting procedure is that the actual trading losses of a given clearing member should only exceed its VaR margin α% of the time. Well-known VaR validation tests can be found in Jorion (2007), and a more refined approach in Hurlin and Pérignon (2012).

On the other hand, backtesting SPAN margin requirements is extremely challenging, because they are based on the minimum of a simulated P&L distribution, which is very hard to identify. Validation tests, in this case, cannot be performed without making strong distributional assumptions. More important, however, is the fact that the SPAN system cannot be backtested in terms of a coverage probability of margin-exceeding losses, since it targets the ranges of the underlying asset prices and volatilities instead of the coverage of portfolio-wide P&Ls. Nevertheless, since the objective of collecting margins is to guarantee the overall performance of clearing member obligations, the probability of margin exceedances is the relevant measure of the effectiveness of a margining system.

In summary, the SPAN system complies only with the first key property, whereas the VaR margin system complies with properties one, four and five. Table 2 summarizes these findings. It is interesting to notice, however, that existing margining techniques are unable to account for P&L dependence across market participants and to produce margin requirements that are not highly procyclical.

4. CoMargin

4.1. Concept

The VaR and SPAN collateral systems focus only on firm-specific risk; that is, the unconditional probability of an individual clearing member experiencing a margin-exceeding loss. By adopting either method, the clearing house guards itself from unique or independent exceedances, but it leaves itself exposed to simultaneous exceedance events. These events, however, tend to be much more economically significant, because they place a more substantial burden on the resources of the clearing house.

Consider the VaR margin of firms i and j. The probability of simultaneous exceedances is given by

$$\begin{aligned}&\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\ &= \Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) \times \Pr\left(V_{j,t+1} \leq -B_{j,t}\right).\end{aligned} \tag{6}$$

Equation (6) shows that simultaneous exceedance events tend to happen more frequently not only when firm-specific risk increases (i.e., when $\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)$ increases), but also when P&L dependence increases (i.e., when $\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right)$ increases). In the first case, firms are more likely to experience losses that exceed their collateral levels in all states of the world. In the second case, firms are more likely to experience these losses at the same time as other firms, either because they hold similar positions (i.e., trade crowdedness is high) or because underlying assets have a tendency to move together (i.e., underlying asset co-movement is high). However, VaR and SPAN margins completely disregard P&L dependence and its potential effect on the stability of the CCP. In the case of the VaR system, risk managers only target unconditional exceedance probabilities by setting a coverage level, $1-\alpha$, for each clearing member individually. In the case of the SPAN system, risk managers do not have direct control over the unconditional exceedance probabilities, so the clearing house is potentially left even more vulnerable to simultaneous exceedance events.

Next, consider a fully orthogonal market; that is, a market that has firms with orthogonal trading positions and orthogonal underlying asset returns. In this case, firms have orthogonal risk exposures and their exceedance probabilities are independent. Under the VaR system, this means that

$$\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha, \tag{7}$$

and

$$\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2. \tag{8}$$

Equation (8) shows that given a common coverage probability, a fully orthogonal market minimizes the probability of simultaneous exceedance events across clearing members. In addition, a fully orthogonal market provides the best possible level of market stability, regardless of the collateral system being adopted by the clearing house, because once the risk manager selects α, the probabilities of simultaneous events are also fixed (i.e., $\alpha^2$for two events, $\alpha^3$for three events and so on). Therefore, a fully orthogonal market can be seen as a conceptual construct that provides a common benchmark for all margining systems.

With this in mind and in the spirit of the CoVaR measure of Adrian and Brunnermeier (2011), we propose a new collateral system, called CoMargin, which enhances financial stability by taking into account the P&L dependence of clearing members. Our starting point is the framework used to estimate VaR margin requirements, which was described in the previous section. Once we establish the S scenarios for each underlying asset, we jointly evaluate the portfolios of firms i and j and compute their associated hypothetical P&Ls or variation margins, $V_{i,t+1}$ and $V_{j,t+1}$, respectively, such that for each date t, we obtain a panel of simulated P&Ls, denoted $\left\{v_{i,t+1}^{s}, v_{j,t+1}^{s}\right\}_{s=1}^{S}$.

The CoMargin of firm i, denoted $B_t^{i|j}$, conditional on the realization of an event affecting firm j, is

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | \mathbf{C}\left(V_{j,t+1}\right)\right) = \alpha. \quad (9)$$

The conditioning event that we consider is the financial distress of firm j, which we define as a loss in its portfolio in excess of its α% VaR, or equivalently, a loss in excess of its VaR margin; i.e., $\mathbf{C}\left(V_{j,t+1}\right) = \left\{V_{j,t+1} \leq -B_{j,t}\right\}$.

Definition 2: The CoMargin of firm i conditional on the financial distress of firm j, $B^{i|j}$, corresponds to the α% conditional quantile of their joint P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha. \quad (10)$$

Through the Bayes theorem, we know that

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]}{\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)}, \quad (11)$$

where the numerator represents the joint probability of i exceeding its CoMargin requirement and j experiencing financial distress. From definitions 1 and 2, we can see that the CoMargin of firm i is defined as the margin level $B_t^{i|j}$ such that

$$\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2. \tag{12}$$

Notice that the CoMargin system starts by defining the financial distress level of a CM as its VaR margin. This part accounts for firm-specific risk. P&L dependence is then incorporated by directly targeting the conditional exceedance probability of firm i, such that it behaves as if the market was fully orthogonal when firm j is in financial distress. Thus, when the market is indeed fully orthogonal, the CoMargin and VaR margin requirements are equivalent and produce the same results. When the market is not fully orthogonal, any differences between the collateral requirements of these two systems can be attributed to P&L dependence. More specifically, $B_t^{i|j}$ can be interpreted as the margin level that guarantees with probability α that firm i remains solvent at an optimal level when firm j experiences financial distress. The optimal level of solvency corresponds to that seen in a fully orthogonal market, where, given the financial distress of firm j, firm i always has enough funds in its margin account to cover its potential losses $1-\alpha\%$ of the time. Thus, by providing coverage levels similar to those prevalent in an orthogonal market, the CoMargin system greatly enhances financial stability.

4.2. Illustration

4.2.1. Properties

We consider a simple case with two firms that have normally distributed P&Ls. For simplicity, we consider an unconditional distribution, with respect to past information, and consequently neglect the time index t. Let

$$(V_1, V_2)' \sim N(0, \Sigma)$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

In this setting, the CoMargins of both members, denoted ($B^{1|2}$, $B^{2|1}$), are defined by

$$\Pr\left(V_i \leq B^{i|j} | V_j \leq -B_j\right) = \alpha, \tag{13}$$

for $i = 1, 2$ and where $B_i = -\sigma_i \Phi^{-1}(\alpha)$ denotes the unconditional VaR of firm i and $\Phi(\cdot)$ the cumulative distribution function of the standard normal distribution. The conditional distribution of V_i given $V_j < c$, $\forall c \in \mathbb{R}$ is a skewed distribution (Horrace, 2005) and is denoted by $g(\cdot)$. The CoMargin for the firm i is the solution to

$$\int_{-\infty}^{-B^{i|j}} g(u;\sigma_i,\sigma_j,\rho)du = \alpha, \tag{14}$$

$$g(u;\sigma_i,\sigma_j,\rho) = \frac{1}{\alpha\sigma_i} \times \phi(\frac{u}{\sigma_i}) \times \Phi(\frac{-B_j/\sigma_j-\rho u/\sigma_i}{\sqrt{1-\rho^2}}), \tag{15}$$

where $\phi(\cdot)$ denotes the probability density function (pdf) of the standard normal distribution (Arnold et al., 1993). Using the expression of CoMargin in equation (14), we can illustrate some of its properties:

i. The CoMargin of firm i increases with the variability of its P&L:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} > 0. \tag{16}$$

See Appendix A1 for the proof.

ii. When there is no P&L dependence between firms i and j, CoMargin and the VaR margin converge. In this example, P&L dependence is fully characterized by the correlation coefficient, ρ; thus,

$$B^{i|j} = B_i \text{ when } \rho = 0. \tag{17}$$

Notice, however, that this result is not specific to the normal distribution case. When there is no dependence (linear or otherwise) between the P&L of the two firms, CoMargin converges to the VaR margin. See Appendix A2 for the proof.

iii. The CoMargin of firm i increases with the dependence between its P&L and that of other firms. In this example, the only other member is firm j, so

$$\frac{\partial B^{i|j}}{\partial \rho} > 0. \tag{18}$$

See Appendix A3 for the proof.

iv. When firms i and j have perfect P&L dependence, their CoMargin converges to α^2% VaR margin, $B_i(\alpha^2)$,

$$\lim_{\rho \to 1} B^{i|j} = B_i(\alpha^2). \tag{19}$$

This property shows that CoMargin is not explosive when P&L dependence is high. See Appendix A4 for the proof.

v. The CoMargin of firm i does not depend on the variability of the P&L of firm j:

$$\frac{\partial B^{i|j}}{\partial \sigma_j} = 0. \tag{20}$$

See Appendix A5 for the proof.

4.2.2. Theoretical performance

In order to illustrate the performance of the CoMargin system, we next consider the case of four CMs, where two of them, members 1 and 2, have correlated P&Ls, such that

$$V \sim N(0, \Sigma),$$

where

$$V = (V_1, V_2, V_3, V_4)' \text{ and } \Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

We allow the correlation between the P&Ls of firms 1 and 2, ρ, to increase from 0 to 0.8. As explained earlier, the rising correlation between the P&Ls of these firms can reflect an increase in the similarity of their trading positions or an increase in the co-movement of the underlying assets. The left panel of Table 3 shows the margin requirements for each CM under both the VaR and CoMargin systems for the levels of correlation being considered. To estimate the CoMargin of a given CM, we define the conditioning event as at least one of the other three firms being in financial distress.

The table shows that the VaR margin remains constant for all firms regardless of their correlation level because this method does not take into account P&L dependence. Consistent with the explanation in the previous section, CoMargin and the VaR margin are equal for all firms when $\rho = 0$; that is, when there is no P&L dependence. However, notice that CoMargin is greater than the VaR margin when $\rho > 0$; that is, when there is P&L dependence. In addition, CoMargin increases as ρ increases; that is, as P&L dependence increases.

It is important to notice that CoMargin could be more effective than the VaR margin either because it provides a better allocation of collateral or simply because it collects additional funds. Thus, we address this issue by reporting what we call a budget-neutral VaR (BNVaR) margin. This margining system is designed to neutralize the second effect by collecting as much aggregate collateral as the CoMargin system, but it does so evenly across all clearing members.[10] Thus, the BNVaR margin of firm i at time t, $B^0_{i,t}$, is defined as

$$B^0_{i,t} = B_{i,t} + \frac{\sum_{i=1}^{N} B_t^{i|j} - \sum_{i=1}^{N} B_{i,t}}{N}. \quad (21)$$

Panels A, B and C of Figure 1 show the theoretical performance of the VaR, BNVaR and CoMargin systems at the clearing member level. The horizontal line in each of these charts highlights the values prevalent when $\rho = 0$; that is, when the market is orthogonal. Panel A plots the margin levels reported in Table 3. Panel B shows the probability of a given CM exceeding its margin conditional on at least another CM being in financial distress. When $\rho = 0$, all three margining systems provide the same level of coverage. However, as ρ increases, the VaR and BNVaR margins provide less coverage when at least one clearing member is in financial distress. On the other hand, CoMargin keeps the coverage level constant. Panel C shows the probability of a CM exceeding its margin conditional on at least another CM having an exceedance. In this case, CoMargin keeps the conditional probabilities of the uncorrelated CMs stable and, unlike VaR and BNVaR, it reduces the conditional probabilities of the correlated CMs; that is, those that are more likely to experience simultaneous exceedances.

Table 4 reports the theoretical performance of the different margining systems at the CCP level. The table reports the unconditional probability of having a minimum number of exceedances and the probability of having additional exceedances given that one has occurred. In addition, it reports the expected shortfalls associated with these events. Panels C, D and E of Figure 1 extend these results to up to four exceedance events. Our findings show once again that when $\rho = 0$, all three margining systems provide the same coverage to the CCP, but as ρ increases, CoMargin provides the best overall coverage.

The unconditional probabilities in Table 4 and Panel D of Figure 1 suggest that the BNVaR margin provides the best unconditional coverage as correlation increases. Nevertheless, this result is expected. In our example, all four firms are identical in all respects except for their correlation level. Since BNVaR collects more aggregate funds than the VaR margin and it does so evenly across CMs, it is equivalent to a VaR margin with a higher coverage level (i.e., lower α). This higher coverage level embedded in BNVaR reduces the unconditional probability of individual margin exceedances. However, as Panels D and E of Figure 1 show, this does not improve the unconditional and conditional probabilities of experiencing additional (i.e. simultaneous)

[10] An alternative budget-neutral margin scheme would be to redistribute the additional collateral collected from firms 1 and 2 to firms 3 and 4; that is, to collect the additional collateral from the firms that have uncorrelated P&Ls. A previous version of this paper conducted that experiment and the relative effectiveness of CoMargin is even higher than that reported here. The results are available upon request from the authors.

exceedance events, particularly as P&L dependence increases. Simply put, collecting more VaR margin indiscriminately across CMs does not optimize the coverage to the CCP.

Finally, Panel F in Figure 1 shows the shortfall that the CCP is expected to cover given a minimum number of margin exceedances. Notice that both CoMargin and BNVaR margin provide similar results that outperform VaR for $\rho > 0$. CoMargin, however, has a slightly lower shortfall when simultaneous exceedances occur. In addition, recall from Panels D and E that the probability of simultaneous exceedances is lower under the CoMargin system. Therefore, the ex-ante expected shortfall for simultaneous exceedance events under the CoMargin system is less than that under the BNVaR margin system.

The right panels of Tables 3 and 4 and Figure 2 repeat the previous exercise but for P&Ls that are jointly Student t distributed with degrees of freedom υ, $V \sim t_\upsilon(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case we set $\rho = 0.4$ and let the degrees of freedom decrease from 30 to 5.[11] Thus, the resulting P&L distributions have progressively fatter tails.

As explained in Cruz Lopez, Harris and Pérignon (2011), changing the distributional assumption of the previous exercise from a normal to a Student t multivariate distribution allows us to create a situation where all CMs have some level of tail dependence in their P&Ls. This is consistent with empirical evidence.

The fact that a Student t multivariate distribution allows for tail dependence becomes apparent when one considers that in the bivariate case the (upper and lower) tail dependence coefficient of firms i and j, denoted $\tau_{i,j}$, is defined as

$$\tau_{i,j} = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{1+\rho}}\right), \tag{22}$$

provided that $\rho > -1$ (Schmidt and Stadtmuller, 2006).

The results of this exercise are consistent with those presented for the Gaussian assumption, but they highlight an important finding: CoMargin is able to capture P&L dependence structures that go beyond correlation. Recall that the P&L dependence structure is fully characterized by correlation only if P&Ls are normally distributed. However, asset prices and P&Ls, particularly those of non-linear portfolios, rarely follow normal distributions. Thus, at least in theory, CoMargin is more robust than other methods for a wide range of P&L distributions.

[11] We conducted a similar experiment using $\rho = 0$ which leads to the same conclusions. The results are available upon request from the authors.

4.3. Scenario generation

One common feature of all margin methods is that they are scenario based. As a consequence, generating meaningful scenarios is a crucial stage when setting margin requirements. The scenario-generating process used for CoMargin incorporates different dimensions of P&L dependence to simulate potential changes in the price and volatility of the underlying assets.

Since the scenarios used for computing the VaR margin are the starting point for the estimation of CoMargin, let us start by explaining first the VaR margin scenario-generating process. Unlike the SPAN margining system, the VaR margin uses a portfolio-wide approach. This allows us to take into account the asset co-movement within the portfolio of each clearing member without the need for ad hoc adjustments (i.e., inter- and intra-commodity spreads). In order to assess the potential P&L of the entire portfolio of each CM, we jointly simulate one-day-ahead changes in the underlying asset prices from a semi-parametric copula.

A copula is a function that links marginal probability distribution functions, say $F_1(r_1)$, $F_2(r_2)$,..., $F_U(r_U)$, to form a multivariate probability distribution function, $F(r_1, r_2, \dots, r_U)$, where r_x is the standardized return of underlying asset x and U is the number of assets underlying the derivatives cleared by the CCP. According to Sklar's theorem (Sklar, 1959), if the marginal distributions are continuous, there exists a unique copula function C, such that

$$F(r_1, r_2, \dots, r_U) = C(F_1(r_1), F_2(r_2), \dots, F_U(r_U)). \qquad (23)$$

As stated in Cruz Lopez, Harris and Pérignon (2011), using copulas to model the multivariate structure of underlying asset returns is useful in this context. First, marginal distributions do not need to be similar to each other to be linked together with a copula structure. Second, the choice of the copula or multivariate structure is not constrained by the choice of the marginal distributions. Third, copulas can be used with U marginal distributions to cover all of the underlying assets cleared by the CCP (see Oh and Patton, 2012). Finally, the use of copulas allows us to model the tails of the marginal distributions and the tail dependence across underlying assets separately, which is particularly important in our case, since the likelihood of an extreme underlying asset return might increase either because of fatter tails in the marginal distributions or because of fatter tails in the multivariate distribution function.

We use Student t copulas in our modelling because, unlike their Gaussian counterparts, they resemble more closely some of the stylized features of asset returns, such as fat tails in the marginal distributions and multivariate tail dependence (Cruz Lopez, 2008). Let R be a symmetric, positive definite matrix with $diag(R) = I_U$, where I_U is the identity matrix of dimension U. Let $t_{R,\upsilon}$ be the standardized multivariate Student t distribution with correlation matrix R and υ degrees of freedom. Then, the multivariate Student t copula, $T_{R,\upsilon}$ is defined as

$$T_{R,\upsilon}(F_1(r_1), F_2(r_2), \dots, F_U(r_U)) = t_{R,\upsilon}(r_1, r_2, \dots, r_U). \quad (24)$$

A Student t copula corresponds to the dependence structure implied by a multivariate Student t distribution. It is fully characterized by the variance-covariance matrix of the standardized returns and the degrees of freedom, υ. The degrees of freedom define the probability mass assigned to simultaneous extreme returns (both positive and negative); the lower the degrees of freedom, the higher the probability of experiencing simultaneous extreme returns relative to the Gaussian copula. However, as $\upsilon \to \infty$ the Student t copula converges to its Gaussian counterpart. In addition, notice that the Student t copula allows us to readily obtain an estimate of the coefficient of tail dependence across pairs of underlying asset returns, as shown in equation (22).

We implement a two-stage semi-parametric approach to estimate a U-dimensional copula for the underlying asset returns. The first stage consists of estimating the empirical marginal distributions of the returns of each underlying asset. The second stage consists of estimating the t-copula parameters, R and υ, through maximum likelihood. This approach is commonly known as the canonical maximum likelihood estimation (CMLE) method (Genest, Ghoudi and Rivest 1995). Once the copula parameters are estimated, we use the implied multivariate structure to simulate potential changes in the price of the underlying assets.

If the CCP is clearing instruments that depend on volatility, such as options contracts, the exercise can be repeated for different variations of R; that is, for different variance-covariance structures. For simplicity, in this paper we set these variations according to a predefined range relative to current values. The predefined range is obtained from 95% confidence intervals used to forecast R through the dynamic conditional correlation method proposed by Engle (2002).

We use a fixed-length estimation window that is rolled daily to simulate new scenarios every day. Thus, once the S potential changes in the price of the underlying assets have been simulated, we mark-to-model all of the derivatives in the portfolio of each CM to obtain the simulated sample path $\{v_{i,t+1}^{s}\}_{s=1}^{S}$ that is required to estimate the VaR margin as described in section 2.3.

As described in section 4.1, we use the same scenarios for estimating CoMargin; however, in this case we mark-to-model the portfolios of all clearing members simultaneously for each scenario to obtain $\{v_{1,t+1}^{s}, v_{2,t+1}^{s}, \dots, v_{n,t+1}^{s}\}_{s=1}^{S}$. This allows us to capture P&L dependence across clearing members.

4.4. Estimation

In the case of two clearing members, given the simulated path $\{v_{i,t+1}^s, v_{j,t+1}^s\}_{s=1}^S$, conditional on $B_t^{i|j}$, a simple estimate of the joint probability $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]$, denoted $P_t^{i,j}$, is given by

$$\hat{P}_t^{i,j} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1}^s \leq -B_{j,t}\right), \tag{25}$$

where $v_{i,t+1}^s$ and $v_{j,t+1}^s$ correspond to the s^{th} simulated P&L of firms i and j, respectively. Given this result, we can now estimate $B_t^{i|j}$. For each time t and for each firm i, we look for the value $B_t^{i|j}$, such that the distance $\hat{P}_t^{i,j} - \alpha^2$ is minimized:

$$\hat{B}_t^{i|j} = \arg\min_{\left\{B_t^{i|j}\right\}} \left(\hat{P}_t^{i,j} - \alpha^2\right)^2. \tag{26}$$

Thus, for each firm i, we end up with a time series of CoMargin requirements $\left\{\hat{B}_t^{i|j}\right\}_{t=1}^T$ for which confidence bounds can be bootstrapped.

4.5. Backtesting

Just like with the VaR margin, CoMargin allows us to test the null hypothesis of an individual member exceeding its margin requirement. More importantly, however, is the fact that we can also test the probability of exceedances conditional on the financial distress of other firms, as defined by the CoMargin of firm i, $B_t^{i|j}$. The null hypothesis in this case becomes

$$H_0\text{: } \Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha. \tag{27}$$

Since the null implies that $E\left[\mathbf{I}\left(V_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha$, then a simple likelihood-ratio (*LR*) test can also be used (Christoffersen, 2009). To assess the conditional probability exceedances, we use the historical paths of the P&Ls for both members i and j; i.e., $\{v_{i,t+1}\}_{t=1}^T$ and $\{v_{j,t+1}\}_{t=1}^T$. The corresponding LR test statistic, denoted $LR_{i|j}$, takes the same form as LR_i:

$$LR_{i|j} = -2\ln\left[(1-\alpha)^{T-N_{i|j}}\alpha^{N_{i|j}}\right] + 2\ln\left[\left(1 - \frac{N_{i|j}}{T}\right)^{T-N_{i|j}} \frac{N_{i|j}}{T}^{N_{i|j}}\right], \tag{28}$$

except that in this case $N_{i|j}$ denotes the total number of joint past violations observed for both members i and j; that is, $N_{i|j} = \sum_{t=1}^{T} \mathbf{I}\left(v_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1} \leq -B_{j,t}\right)$.

4.6. Extension to n conditioning firms

Consider next that the conditioning event depends on two firms denoted j and k. In this case, the CoMargin of firm i, denoted by $B_t^{i|j,k}$, is defined as follows:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j,k} | \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right) = \alpha, \tag{29}$$

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]} = \alpha. \tag{30}$$

The conditioning event that we consider is either firm j or firm k, or both, being in financial distress; i.e., $\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right) = V_{j,t+1} \leq -B_{j,t} \ or \ V_{k,t+1} \leq -B_{k,t}$. In this case, the probability of the conditioning event is equal to 2α only if the financial distress events of firms j and k are mutually exclusive. In the general case, we have

$$\begin{aligned}
\Pr\left[\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right] &= \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right) \text{ or } \left(V_{k,t+1} \leq -B_{k,t}\right)\right] \\
&= \Pr\left(V_{j,t+1} \leq -B_{j,t}\right) + \Pr\left(V_{k,t+1} \leq -B_{k,t}\right) \\
&\quad -\Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right] \\
&= 2\alpha - \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right].
\end{aligned} \tag{31}$$

Hence, CoMargin $B_t^{i|j,k}$ satisfies the following condition:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]}{2\alpha - \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right]} = \alpha. \tag{32}$$

Given this result, we proceed to estimate CoMargin $B_t^{i|j,k}$. First, notice that the probability $\Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right]$, denoted $P_t^{j,k}$, does not depend on the CoMargin level $B_t^{i|j,k}$; thus, it can simply be estimated by

$$\hat{P}_t^{j,k} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{j,t+1}^{s} \leq -B_{j,t}\right) \times \mathbf{I}\left(v_{k,t+1}^{s} \leq -B_{k,t}\right). \tag{33}$$

Second, conditional on $B_t^{i|j,k}$, the joint probability in the numerator of equation (32), denoted $P_t^{i,j,k}$, becomes

$$\begin{aligned} P_t^{i,j,k} &= \Pr\left[\left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right] \\ &= \Pr\left[\left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \left[\left(V_{j,t+1} \le -B_{j,t}\right) \text{ or } \left(V_{k,t+1} \le -B_{k,t}\right)\right]\right] \\ &= \Pr\left[\left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \left(V_{j,t+1} \le -B_{j,t}\right)\right. \\ &\quad \left.\text{or } \left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \left(V_{k,t+1} \le -B_{k,t}\right)\right] \\ &= \Pr\left[\left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \left(V_{j,t+1} \le -B_{j,t}\right)\right] \\ &\quad + \Pr\left[\left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \left(V_{k,t+1} \le -B_{k,t}\right)\right] \\ &\quad - \Pr\left[\left(V_{i,t+1} \le -B_t^{i|j,k}\right) \cap \left(V_{j,t+1} \le -B_{j,t}\right) \cap \left(V_{k,t+1} \le -B_{k,t}\right)\right]. \end{aligned} \tag{34}$$

Thus, a simple estimator of this probability is given by

$$\begin{aligned} \hat{P}_t^{i,j,k} &= \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \le -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{j,t+1}^s \le -B_{j,t}\right) \\ &+ \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \le -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{k,t+1}^s \le -B_{k,t}\right) \\ &- \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \le -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{j,t+1}^s \le -B_{j,t}\right) \times \mathbf{I}\left(v_{k,t+1}^s \le -B_{k,t}\right), \end{aligned} \tag{35}$$

and the CoMargin $B_t^{i|j,k}$ can be estimated by

$$\hat{B}_t^{i|j,k} = \arg\min_{\left\{B_t^{i|j,k}\right\}} \left(\frac{\hat{P}_t^{i,j,k}}{2\alpha - \hat{P}_t^{j,k}} - \alpha\right)^2. \tag{36}$$

Following a similar argument, CoMargin can be generalized to n conditioning firms, with $n < N - 1$. In this case, the conditioning event is that at least one of the n clearing members is in financial distress (see the appendix for details).

5. Empirical Analysis

5.1. Data

In this section we compare the empirical performance of the SPAN, VaR and CoMargin systems by using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). The CDCC is the clearing house of the TMX Montreal Exchange, the largest derivatives exchange in Canada. The data set includes the daily open interest (i.e., the daily trading positions at market close) on the three-month Canadian Bankers' Acceptance Futures (BAX), the ten-year Government of Canada Bond Futures (CGB) and the S&P/TSX 60 Index Standard Futures (SXF) for the 48 clearing members active in the CDCC between 2 January 2003 and 31 March 2011. To the best of our knowledge, no other study has ever used actual clearing member positions to analyze the performance of competing margining systems. Nevertheless, due to the proprietary nature of the data, we are only able to report aggregate results. Table 5 presents a short description of the data.

In a derivatives exchange, on any given day, there are many delivery dates available on each underlying asset. Over the sample period there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Thus, the sample includes a total of 113 futures contracts. Table 6 summarizes the specifications of these contracts. The contracts in our sample do not constitute the full set of derivatives cleared by the CDCC. However, they represent a significant portion of its clearing activity. The documentation provided by CDCC states that the BAX, CGB and SXF are among the most actively traded derivatives in Canada. Furthermore, BAX and CGB are the most actively traded cleared interest rate contracts in the country (Campbell and Chung 2003; TMX Montreal Exchange 2013a, b and c).

Table 7 shows the summary statistics for the contracts in the sample. Panel A shows the aggregate statistics for all 113 contracts and Panels B, C and D report the summary statistics by underlying asset. On a typical day, there were approximately 20 active contracts; 12 of them were BAX, 4 of them were CGB and the remaining 4 were SXF. On average, contracts remained active for 363 trading days. However, there is a significant dispersion across underlying assets. BAX contracts remained active for 551 days, whereas CGB and SXF contracts remained active for 239 and 237 days, respectively. BAX contracts were also the most actively traded, with an average daily gross open interest of 275,000. The corresponding value for CGB and SXF contracts was less than half of that for BAX, at 131,000 and 111,000, respectively.

CDCC members have access to three accounts to submit trades for clearing: a firm, a client and an omnibus account. The firm account is used by clearing members to submit their own trades (i.e., conduct proprietary trading). The client account is used to submit trades on behalf of clearing members’ clients. The omnibus account is used for all other clearing activities and is the least active account across all clearing members.

Our analysis includes 21 firm, 23 client and 16 omnibus accounts that were active on at least one day of the sample period. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Due to disclosure restrictions, we are unable to report the owners of each active account by type. However, Table 8 provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period. Notice that this list includes more clearing members than those currently affiliated with the CDCC, because some of them entered and exited the market during this period.

Since our objective in this section is to present an assessment of our methodology by using actual trading positions, instead of by assuming them, we report results only for the firm or proprietary-trading accounts. We consider this appropriate because these accounts best represent the actual trading decisions of the clearing members. Nevertheless, our results are consistent across all three accounts.

The daily settlement prices for the underlying assets and the futures contracts in the sample were obtained from Bloomberg and are plotted in Figure 3. Panel A shows the time series of underlying asset prices, Panel B shows the underlying asset returns and Panel C shows the settlement futures prices for all delivery dates. Lines in different colours represent different delivery dates. It is evident from Panel B that the volatility of the underlying assets increased dramatically after the onset of the financial crisis in mid-2007. In addition, Panel C shows an increase in the spread of futures prices during the same period, particularly for BAX contracts.

Figure 4 plots the daily stacked P&L values implied from the positions in active firm accounts. For each date t, $n^a \in N$ observations are plotted, which correspond to the P&L of the n^a clearing members that had an active account on that day. Notice how the volatility of the P&Ls increased dramatically at the beginning of the financial crisis. This is consistent with the trends described for the underlying and futures prices in Figure 3. Therefore, we consider two subperiods in our analysis. The first is the pre-crisis period, from 2 January 2003 to 31 July 2007, and the second is the crisis period, from 1 August 2007 to 31 March 2011.

Table 9 reports the summary statistics for the firm accounts in the sample. Panel A shows the values for the full sample period, and Panels B and C show the values for the pre-crisis and crisis periods, respectively. On a typical day, there were approximately 12 clearing members with active firm accounts. This number remained relatively stable during the pre-crisis and crisis periods. The average account was active for 56% of the days in the sample (1,146 out of 2,066 days). The corresponding proportion is 75% (858 out of 1,148 days) for the pre-crisis period and 56% (516 out of 918 days) for the crisis period. The relatively shorter activity during the crisis period was partially influenced by the fact that some clearing members exited the market. The P&L numbers reported in the table focus exclusively on active accounts. The typical active account reported an implied daily loss of $60,000 on the futures contracts listed in the sample. During the pre-crisis period, these accounts reported daily losses of $164,000. However, during the crisis period, the average account reported a daily profit of $65,000. These profits were mostly derived from short positions in BAX contracts. Over the entire sample period, the typical account reported an implied loss of $38,000. The corresponding numbers are a loss of $119,000 and a profit of $39,000 for the pre-crisis and crisis periods, respectively.[12]

5.2. Empirical performance

Using the daily open interest in each firm account, we compute the initial margin that should be collected from each clearing member under the SPAN, VaR and CoMargin systems. The underlying price range for the SPAN approach is set at 99% and $\alpha = 2\%$ for the VaR and CoMargin systems. We use a rolling estimation window of 500 trading days in all cases. As mentioned in section 2.2, by construction, the SPAN system is estimated using the sixteen scenarios in Table 1. The extreme scenarios (scenarios 15 and 16) are ignored, since we are dealing only with futures (i.e., linear) contracts. For the VaR and CoMargin systems, we consider $S = 100{,}000$ scenarios that are obtained using the methodology described in section 4.3. Consistent with our earlier discussion and theoretical illustration, we set the financial distress threshold for CoMargin at the VaR margin level of the conditioning firms. The conditioning firms are the two clearing members with the highest one-day-ahead expected shortfall (ES) given financial distress.

For consistency across time periods, we ignore the ad hoc inter- and intra-commodity spreads used in the SPAN system and impose a minimum margin of $10,000 on all active accounts under all systems. This amount allows us to avoid cases when clearing members are not required to post any collateral because they have matched long and short positions. These cases are likely to result in small exceedances, since P&Ls in different contracts do not always offset each other. Thus, imposing a minimum margin amount prevents an upward bias in the number of SPAN exceedances. This amount, however, does not influence the rest of our results, since it represents a constant that accounts for less than 0.2% of the average individual daily margin required under all systems.[13]

Table 10 reports the summary statistics for the daily margin collected over the full sample period under different margining methods. It also reports budget-neutral margins for the SPAN (BNSPAN) and VaR (BNVaR) systems using the same approach as that described in equation (21) of section 4.2.2. The average aggregate daily margin collected across all clearing members is $112 million, $101 million and $161 million for the SPAN, VaR and CoMargin systems, respectively. The typical clearing member posted $9.76 million, $8.83 million and $14.14 million of SPAN, VaR and CoMargin collateral when it entered the market. However, on a typical day, clearing members posted $5.85 million, $5.34 million and $8.50 million, respectively. The discrepancy between

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[12] It is important to notice that the P&L values reported in this paper are those implied by the positions held by the clearing members in their firm accounts on the contracts included in the sample. The actual accounts of these clearing members, however, included positions in other contracts cleared by the CDCC that are not included in our sample. In addition, our P&L values do not include trading revenues from other sources, such as non-cleared OTC transactions.

[13] We computed our results under different minimum collateral amounts ranging from $0 to $100,000. The results are consistent in all cases. For a minimum collateral of $0, however, the SPAN system yields a high number of small exceedances.

the cross-sectional and time-series averages derives from the fact that the number of active clearing members changed daily due to entry and exit.

Tables 11 and 12 report the summary statistics for the daily margin collected over the pre-crisis and crisis periods, respectively. As would be expected given the increased volatility during the financial crisis, both aggregate and individual collateral levels are higher during the crisis period. However, the ranking of margin collections is consistent throughout the full sample period and the two subperiods being considered. The VaR margin consistently collects the least collateral and CoMargin consistently collects the most. Similarly, the VaR margin consistently shows the least dispersion and CoMargin consistently shows the most dispersion of the collected margin as measured by standard deviation. This occurs because CoMargin takes into account the variation of more factors than the other margining methods (i.e., the factors causing P&L dependence).

Panel A of Figure 5 shows the daily stacked initial margin requirements under the SPAN, VaR and CoMargin systems. The stacking process is the same as that used in the previous section for the P&L values of Figure 4. Notice that all three approaches produce margin requirements that are highly correlated. Table 13 shows the average cross-sectional correlation for the full sample period and the two subperiods, and displays the standard deviations in brackets. The high correlation and low dispersion between the SPAN and VaR systems, coupled with the average collection values shown in Tables 10, 11, and 12, indicate that, at the individual CM level, SPAN margins behave much like VaR margins, but at a higher coverage (i.e., lower α) probability. However, notice that CoMargin is the least correlated of the three systems and shows the widest dispersion. This dispersion is more pronounced during the crisis period, when P&L dependence is higher. As mentioned in the previous section, this can be explained by the fact that CoMargin converges to the VaR margin as P&L dependence decreases, and diverges as P&L dependence increases (see section 4.2.1).

Panel B of Figure 5 plots the daily P&L of each active CM against its initial margin requirement. The 45-degree and -45-degree lines are indicated in red. Observations falling below the -45-degree line denote margin-exceeding losses. Notice that, of the three margining systems, CoMargin shows the least number of margin exceedances. In addition, unlike the other systems, CoMargin tends to concentrate exceedances in low-initial-margin, low P&L points. These points represent clearing members with the smallest or least-active portfolios; that is, those that are the least likely to pose a systemic threat to the CCP.

Panel C of Figure 5 shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to the posted initial margin ($V_{i,t}/B_{i,t-1}$). Once again, the stacking process is the same as that used in Figure 4. Observations with a relative variation margin below -1, the level depicted with a red line, represent margin exceedances. Notice that the CoMargin system exhibits the lowest number of simultaneous (i.e., clustered) margin exceedances.

Table 14 summarizes the performance of different margining systems over the full sample period. Tables 15 and 16 show the corresponding values for the pre-crisis and crisis periods, respectively. The left panel of the tables measures unconditional performance in terms of the probability of experiencing at least one exceedance, the average number of exceedances and the expected shortfall if at least one exceedance occurs. The right panel of the tables reports the same measures but conditional on at least one member exceeding its margin.

Consistent with the theoretical results reported in section 4, our empirical results show that the CoMargin system outperforms the SPAN and VaR systems in all dimensions, whether these are estimated unconditionally or conditionally. At the CCP level, notice how CoMargin consistently has the lowest probability of exceedances and the lowest average number of exceedances across the three systems. A lowest number of simultaneous exceedances also allows CoMargin to have the lowest expected shortfall.

In addition, the relative performance of CoMargin increases when we condition on at least one exceedance event. This finding shows that CoMargin does a better job of protecting the CCP from simultaneous exceedances, even after a clearing member has surpassed its margin. Furthermore, Tables 15 and 16 show that the relative performance of CoMargin also improves during the crisis period, when P&L dependence is more persistent. This indicates that CoMargin tends to provide more protection to the CCP when it is needed most; that is, when simultaneous exceedance events are more likely to occur.

The cross-sectional panels of Tables 14, 15 and 16 show the probability that a typical CM surpasses its margin on any given day, and the expected shortfall associated with this event. Once again, the conditional probabilities and expected shortfalls are lower for CoMargin than for the SPAN and VaR margins. This implies that, under the SPAN and VaR systems, the typical CM is more likely to experience a margin-exceeding loss when another CM has exceeded its margin than under the CoMargin system. In addition, notice how both the conditional and unconditional probabilities increase for the SPAN and VaR systems during the crisis period relative to the pre-crisis period. These results imply that, when P&L dependence increases, SPAN and VaR margin requirements are more likely to be exceeded by the typical CM than CoMargin requirements.

The time-series panels in Tables 14, 15 and 16 show the average values over the sample period for each CM. By definition, the unconditional probability of exceedances corresponds to one minus the coverage level. For the VaR margin, this probability corresponds to $\alpha = 2\%$, which was used for its estimation. Similarly, CoMargin shows a constant unconditional exceedance probability of 1% during the full sample period and both subperiods. However, notice that for the SPAN system this probability increases (i.e., its unconditional coverage decreases) during the crisis period. This situation arises because the SPAN system targets price and volatility ranges instead of portfolio-wide P&L quantiles, as is the case in the VaR and CoMargin systems.

Tables 14, 15 and 16 also show the performance of the budget-neutral SPAN and VaR systems (BNSPAN and BNVaR, respectively). As explained in section 4, these artificial constructs allow us to test whether CoMargin performs better than its counterparts due to its allocation of collateral or due to the fact that it collects more funds. At first glance, the results show that, at the CCP and CM level, budget-neutral margins tend to perform better than CoMargin in terms of unconditional exceedance probabilities. However, as explained in the previous section, this is consistent with our theoretical results. By construction, budget-neutral methods tend to collect more aggregate collateral than the SPAN and VaR margining systems and spread the additional requirements evenly across clearing members. This allocation of collateral increases the coverage level of the SPAN and VaR systems across all CMs and reduces their unconditional exceedance probabilities.

For the pre-crisis period, Table 15 shows that BNSPAN slightly outperforms CoMargin in terms of conditional exceedance probabilities. However, there is an explanation for this finding that is also consistent with our theoretical results. During periods of low P&L dependence, such as the pre-crisis period, unconditional exceedance probabilities tend to be more important than conditional probabilities in determining the likelihood of simultaneous distress events (see equation (6)). By collecting more collateral across all CMs, BNSPAN further reduces the unconditional exceedance probabilities of SPAN margins, to the point that it reduces the likelihood (but not the expected shortfall) of simultaneous distress events. This effect is further reinforced by the fact that, during low P&L dependence periods, the CoMargin of many CMs converges to the VaR margin (see equation (17)), which Table 15 also shows has a higher unconditional exceedance probability than SPAN.

Nevertheless, despite the fact that budget-neutral measures sometimes have lower exceedance probabilities, in all cases, the results show that CoMargin yields the lowest expected shortfalls. This finding is once again consistent with our theoretical results. Relative to the CoMargin system, budget-neutral methods effectively transfer margin requirements from firms with high P&L dependence to those with low P&L dependence, thus leaving the CCP exposed to simultaneous exceedance events. These simultaneous events account for the higher (conditional and unconditional) expected shortfalls under the budget-neutral systems. Since expected shortfalls ultimately determine the impact on the funds available to the CCP, our findings show that CoMargin allocations enhance the resilience of clearing houses by minimizing the likelihood and economic impact of adverse events.

Figure 6 extends our empirical findings by conditioning on up to three margin-exceedance events. The results on the charts confirm that CoMargin tends to perform better than the SPAN and VaR systems even after accounting for the additional amount of collateral required (i.e., after using budget-neutral measures).

6. Conclusion

In this paper, we present a new methodology, called CoMargin, to estimate margin requirements in derivatives CCPs. Our approach is innovative because it explicitly takes into account both the individual risk and the interdependence of the P&Ls of market participants. As a result, CoMargin produces collateral allocations that enhance the stability and resilience of the CCPs, which in turn reduce their systemic risk.

We show theoretically and empirically that CoMargin outperforms the widely popular SPAN and VaR margining approaches. Our method performs particularly well relative to these alternatives when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, CoMargin provides more protection to the CCP when it needs it most.

At a technical level, we show how credit risk can be assessed using a scenario-based approach that takes into account the co-movement of underlying assets and similarities across portfolios. We also contribute to the literature by developing a backtesting methodology that relies on formal statistical tests and that can be generalized to any number of market participants. Formal backtesting techniques are particularly important in the context of structural market changes, such as those that occurred during the financial crisis as a result of policy changes (e.g., the current G-20 mandate to centrally clear OTC derivatives). The use of systems that cannot be backtested, such as SPAN, imposes a challenge to both risk managers and regulators, who need to assess the effectiveness and consistency of their risk-management policies.

Finally, at a more general level, we show the importance of accounting for simultaneous extreme events, or interdependencies, when managing credit risk. Our approach can be seen as a stepping stone that can be generalized and used in different situations, such as estimating collateral requirements for repo and other OTC transactions, assessing capital requirements for banks and insurance companies, or monitoring the accumulation of credit risk across market participants for regulatory purposes.

Appendix A: Proofs for CoMargin Properties (Section 4.2.1)

[Proof A1]: Let $H(B^{i|j}, \sigma_i)$ be a function such that

$$H(B^{i|j}, \sigma_i) = \int_{-\infty}^{-B^{i|j}} g(u, \sigma_i) du - \alpha = 0. \tag{A1}$$

Note that we simplified the notation of the pdf $g(u; \sigma_i)$ compared to equation (15). Then, the CoMargin can be defined as an implicit function $B^{i|j} = h(\sigma_i)$. By the implicit functions theorem, we have

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j}, \sigma_i)}{H_B(B^{i|j}, \sigma_i)}. \tag{A2}$$

The derivative $H_B(B^{i|j}, \sigma_i)$ can be expressed as follows:

$$H_B(B^{i|j}, \sigma_i) = -g(-B^{i|j}; \sigma_i) < 0, \tag{A3}$$

and is negative since $g(u; \sigma_i)$ is a pdf. Thus, the sign of $\frac{\partial B^{i|j}}{\partial \sigma_i}$ is given by the sign of $H_{\sigma_i}(B^{i|j}, \sigma_i)$:

$$H_{\sigma_i}(B^{i|j}, \sigma_i) = \frac{\partial}{\partial \sigma_i}\left(\int_{-\infty}^{-B^{i|j}} g(u; \sigma_i)\, du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u;\sigma_i)}{\partial \sigma_i} du. \tag{A4}$$

For simplicity, let us consider the case where $\rho = 0$:

$$\frac{\partial g(u; \sigma_i)}{\partial \sigma_i} = \frac{\partial}{\partial \sigma_i}\left(\frac{1}{\sigma_i}\phi\left(\frac{u}{\sigma_i}\right)\right) = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right) - \frac{u}{\sigma_i^3}\phi'\left(\frac{u}{\sigma_i}\right). \tag{A5}$$

Since $\phi'(x) = -x\,\phi(x)$, we have

$$\frac{\partial g(u; \sigma_i)}{\partial \sigma_i} = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right)\left(1 - \left(\frac{u}{\sigma_i}\right)^2\right). \tag{A6}$$

For any value of u such that $u < -\sigma_i$, we have $\partial g(u; \rho)/\partial \sigma_i > 0$. This condition is satisfied when $u \in [-\infty, -B^{i|j}]$, since $-B^{i|j} = \sigma_i \Phi^{-1}(\alpha) = -\sigma_i \Phi^{-1}(1-\alpha)$ and $\Phi^{-1}(1-\alpha) > 1$ for most of the considered coverage rates (e.g., 1%, 5%). Consequently, the integral AX8 is also positive and $H_{\sigma_i}(B^{i|j}, \sigma_i) > 0$. Then we conclude that

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j}, \sigma_i)}{H_B(B^{i|j}, \sigma_i)} > 0. \tag{A7}$$

A similar result can be obtained when we relax the assumption.

[Proof A2]: If $\rho = 0$, the last term in equation (15) becomes $\Phi(-B_j/\sigma_j) = \Phi(\Phi^{-1}(\alpha)) = \alpha$, since $B_i = -\sigma_i \Phi^{-1}(\alpha)$. Consequently, the CoMargin of firm i is the solution of the following integral:

$$\int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) du = \alpha. \tag{A8}$$

By properties of the normal distribution, we have $B^{i|j} = -\sigma_i \Phi^{-1}(\alpha) = B_i$.

[Proof A3]: Let $F(B^{i|j}, \rho)$ be a function such that

$$F(B^{i|j}, \rho) = \int_{-\infty}^{-B^{i|j}} g(u; \rho)\, \mathrm{du} - \alpha = 0. \quad \text{(A9)}$$

Note that we simplified the notation of the pdf $g(u; \rho)$ compared to equation (15). Then, the CoMargin can be defined as an implicit function $B^{i|j} = f(\rho)$. By the implicit functions theorem, we have

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j}, \rho)}{F_B(B^{i|j}, \rho)}, \quad \text{(A10)}$$

where $F_\rho(.)$ and $F_B(.)$ denote, respectively, the first derivative of the F function with respect to ρ and B. For any function $H(x)$ defined as

$$H(x) = \int_{-\infty}^{-b(x)} h(t)\, dt, \quad \text{(A11)}$$

we have $H'(x) = h(b(x)) \times \partial b(x)/\partial x$. Consequently, the derivative $F_B(B^{i|j}, \rho)$ can be expressed as follows:

$$F_B(B^{i|j}, \rho) = -g(-B^{i|j}; \rho) < 0, \quad \text{(A12)}$$

and it is negative, since $g(u; \rho)$ is a pdf. Thus, the sign of $\partial B^{i|j}/\partial \rho$ is given by the sign of $F_\rho(B^{i|j}, \rho)$:

$$F_\rho(B^{i|j}, \rho) = \frac{\partial}{\partial \rho}\left(\int_{-\infty}^{-B^{i|j}} g(u; \rho)\, du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u; \rho)}{\partial \rho}\, du. \quad \text{(A13)}$$

Given the expression of the pdf $g(u; \rho)$, we have

$$\begin{aligned} \frac{\partial g(u; \rho)}{\partial \rho} &= \overbrace{-\frac{1}{\alpha \sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \phi\left(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right)}^{A} \\ &\quad \times \left(\frac{-u/\sigma_i \sqrt{1-\rho^2} - (B_j/\sigma_j + \rho u/\sigma_i)\rho(1-\rho^2)^{-1/2}}{1-\rho^2}\right) \\ &= A \times \left(\frac{1}{1-\rho^2}\right)^{3/2} \times \left(\frac{u}{\sigma_i} + \frac{\rho B_j}{\sigma_j}\right). \end{aligned} \quad \text{(A14)}$$

This function is positive for any value of u such that $u \leq \rho B_i = -\rho \sigma_i \Phi^{-1}(\alpha)$ with $-\rho \sigma_i \Phi^{-1}(\alpha) > 0$. Since $B^{i|j} \geq 0$ by definition, this condition is satisfied for the interval $]-\infty, -B^{i|j}]$ and $F_\rho(B^{i|j}, \rho) > 0$. Then we conclude that

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j}, \rho)}{F_B(B^{i|j}, \rho)} > 0\,. \quad \text{(A15)}$$

[Proof A4]: For $\rho = 1$, the pdf $g(u; \sigma_i, \sigma_j, \rho)$ in equation (15) is degenerated. However, when ρ tends to one, we have

$$\lim_{\rho\to 1} \Phi\left(\frac{-\mathrm{B_i}/\sigma_j - \rho u}{\sqrt{1-\rho^2}}\right) = 1, \tag{A16}$$

as long as $u < \frac{-\mathrm{B_i}}{\sigma_j} = \Phi^{-1}(\alpha)$. If we assume that the standardized CoMargin for i is larger than the standardized VaR margin for i, i.e., $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$, then we have

$$\lim_{\rho\to 1} g(u) = \frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right), \tag{A17}$$

and consequently the CoMargin corresponds to the VaR margin defined for a coverage rate α^2, since

$$\lim_{\rho\to 1} \int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi\left(\frac{x}{\sigma_i}\right) dx = \alpha^2, \tag{A18}$$

$$\lim_{\rho\to 1} B^{i|j} = -\,\sigma_i \Phi^{-1}(\alpha^2). \tag{A19}$$

We can check that condition $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$ is satisfied since $\Phi^{-1}(\alpha^2) \leq \Phi^{-1}(\alpha)$.

[Proof A5]: Since $B_j = -\sigma_i \Phi^{-1}(\alpha)$, the pdf $g(.)$ in equation (15) can be rewritten as

$$g(u; \sigma_i, \sigma_j, \rho) = \frac{1}{\alpha\sigma_i} \times \phi\left(\frac{u}{\sigma_i}\right) \times \;\Phi\left[\frac{\Phi^{-1}(\alpha) - \frac{\rho u}{\sigma_i}}{\sqrt{1-\rho^2}}\right], \tag{A20}$$

since $g(.)$ does not depend on σ_j, $\partial B^{i|j}/\partial\sigma_j = 0$.

Appendix B: CoMargin with *n* Firms

With n conditioning firms, $n < N - 1$, the conditioning event of the CoMargin is that at least one of the n clearing members is in financial distress. Thus, the definition of CoMargin becomes

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right]} = \alpha, \tag{B1}$$

where the probability to observe the conditioning event is

$$\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right] = \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \, or \ldots or \, \left(V_{n,t+1} \leq -B_{n,t}\right)\right]. \tag{B2}$$

Using Poincaré's formula for the probability of the union of events, we can see that

$$\begin{aligned}
\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right] = & \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\
& - \underbrace{\sum_{1 \leq j_1 < j_2 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right)\right]}_{\boldsymbol{2\ events}} \\
& + \underbrace{\sum_{1 \leq j_1 < j_2 < j_3 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right) \cap \left(V_{j_3,t+1} \leq -B_{j_3,t}\right)\right]}_{\boldsymbol{3\ events}} \\
& \ldots \underbrace{+(-1)^{\boldsymbol{n-1}} \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \cap \ldots \cap \left(V_{n,t+1} \leq -B_{n,t}\right)\right]}_{\boldsymbol{n\ events}}.
\end{aligned} \tag{B3}$$

Thus, the probability of the conditioning event can be rewritten as follows:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right] = n\alpha - P_t^n, \tag{B4}$$

where P_t^n denotes the sum of the probabilities of all common events (for two events, three events, etc.). An estimator of this value, $\hat{P}_t^n$, can be obtained from the simulated path $\left\{V_{1,t+1}^s, \ldots, V_{n,t+1}^s\right\}_{s=1}^{S}$. When the financial distress events of the conditioning firms are mutually exclusive, however, the probability of the conditioning events simplifies to $n\alpha$. Therefore, an estimator of the CoMargin of firm i conditional on n clearing members, $B_t^{i|n}$, is the solution of the program:

$$\hat{B}_t^{i|n} = \arg\min_{\left\{B_t^{i|n}\right\}} \left(\frac{\hat{P}_t^{i,n}}{n\alpha - \hat{P}_t^n} - \alpha\right)^2, \tag{B5}$$

where $\hat{P}_t^{i,n}$ denotes the estimator of $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right]$, which is obtained by generalizing equation (34) conditioning on $B_t^{i|n}$.

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Table 1: Scenarios used in the SPAN system

Scenario	Underlying Asset Price Change	Volatility Change	Time to Expiration
1	0	+ volatility range	$-1/252$
2	0	- volatility range	$-1/252$
3	$+1/3$ x price range	+ volatility range	$-1/252$
4	$+1/3$ x price range	- volatility range	$-1/252$
5	$-1/3$ x price range	+ volatility range	$-1/252$
6	$-1/3$ x price range	- volatility range	$-1/252$
7	$+2/3$ x price range	+ volatility range	$-1/252$
8	$+2/3$ x price range	- volatility range	$-1/252$
9	$-2/3$ x price range	+ volatility range	$-1/252$
10	$-2/3$ x price range	- volatility range	$-1/252$
11	$+3/3$ x price range	+ volatility range	$-1/252$
12	$+3/3$ x price range	- volatility range	$-1/252$
13	$-3/3$ x price range	+ volatility range	$-1/252$
14	$-3/3$ x price range	- volatility range	$-1/252$
15	Positive extreme change	0	$-1/252$
16	Negative extreme change	0	$-1/252$

Note: The table shows the sixteen scenarios used to determine the contract family charge in the SPAN system. Price and volatility ranges usually cover 99% of the data points over a rolling historical estimation window. Positive and negative extreme changes are designed to assess the effect of deep out-of-the-money options.

Table 2: Desirable properties of margin requirements

Properties	SPAN Margin	VaR Margin	CoMargin
Reflects P&L variability	**Yes**	**Yes**	**Yes**
Reflects P&L dependence across participants	No	No	**Yes**
Exhibits low procyclicality	No	No	**Moderate**
Is robust to outliers	No	**Yes**	**Yes**
Can be backtested	No	**Yes**	**Yes**

Table 3: Theoretical margin collected under VaR and CoMargin systems

	Jointly Normally Distributed P&Ls				**Jointly Student t Distributed P&Ls**			
	CM1	**CM2**	**CM3**	**CM4**	**CM1**	**CM2**	**CM3**	**CM4**
	ρ = 0				**ν = 30**			
VaR	1.645	1.645	1.645	1.645	1.697	1.697	1.697	1.697
CoMargin	1.645	1.645	1.645	1.645	2.136	2.136	1.791	1.791
BNVaR	1.645	1.645	1.645	1.645	1.964	1.964	1.964	1.964
	ρ = 0.4				**ν = 10**			
VaR	1.645	1.645	1.645	1.645	1.812	1.812	1.812	1.812
CoMargin	1.981	1.981	1.645	1.645	2.505	2.505	2.138	2.138
BNVaR	1.813	1.813	1.813	1.813	2.322	2.322	2.322	2.322
	ρ = 0.8				**ν = 5**			
VaR	1.645	1.645	1.645	1.645	2.015	2.015	2.015	2.015
CoMargin	2.374	2.374	1.645	1.645	3.248	3.249	2.840	2.840
BNVaR	2.009	2.009	2.009	2.009	3.044	3.044	3.044	3.044

Note: This table presents the VaR (equation (2)), CoMargin (equation (10)), and Budget-neutral VaR (BNVaR, equation (21)) margin requirements, assuming four clearing members whose P&Ls are jointly normally or Student t distributed. The left panel presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0,\Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, and reports the results for different levels of the correlation parameter, ρ, that range from 0 to 0.8. The right panel shows the case when P&Ls are Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0,\Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case we set ρ=0.4 and let the degrees of freedom decrease from 30 to 5.

Table 4: Theoretical performance of VaR and CoMargin systems

	Jointly Normally Distributed P&Ls				Jointly Student t Distributed P&Ls			
	Unconditional		Conditional on One Exceedance		Unconditional		Conditional on One Exceedance	
	Prob. of at least one Exceedance	Expected Shortfall	Prob. of Additional Exceedances	Expected Shortfall	Prob. of at least one Exceedance	Expected Shortfall	Prob. of at least one Exceedance	Expected Shortfall
	ρ = 0				**ν = 30**			
VaR	0.185	0.084	0.076	0.451	0.177	0.094	0.123	0.531
CoMargin	0.185	0.084	0.076	0.451	0.115	0.056	0.074	0.485
BNVaR	0.185	0.084	0.076	0.451	0.108	0.052	0.088	0.485
	ρ = 0.4				**ν = 10**			
VaR	0.179	0.084	0.109	0.466	0.171	0.119	0.151	0.695
CoMargin	0.138	0.060	0.069	0.433	0.081	0.053	0.090	0.650
BNVaR	0.129	0.055	0.083	0.430	0.077	0.051	0.104	0.658
	ρ = 0.8				**ν = 5**			
VaR	0.165	0.084	0.193	0.505	0.164	0.175	0.191	1.068
CoMargin	0.110	0.048	0.062	0.432	0.051	0.060	0.129	1.171
BNVaR	0.077	0.033	0.144	0.428	0.049	0.059	0.141	1.193

Note: This table presents the theoretical performance of the VaR (equation (2)), CoMargin (equation (10)), and Budget-neutral VaR (BNVaR, equation (21)) systems, assuming four clearing members whose P&Ls are jointly normally or Student t distributed. The left panel presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, and reports the results for different levels of the correlation parameter, ρ, that range from 0 to 0.8. The right panel show the case when P&Ls are Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case we set ρ=0.4 and let the degrees of freedom decrease from 30 to 5.

Table 5: Description of the data used in the empirical analysis

Item	Number	Comments
Clearing members	48	There is entry and exit in the sample, so the number of clearing members varies over time.
Trading Days	2066	The sample period is from 2 January 2003 to 31 March 2011.
Underlying Assets	3	The three underlying assets are: 1. Yield on the three-month Canadian bankers' acceptance. 2. Yield on the ten-year Government of Canada Bond Futures 3. Level of the S&P/TSX 60 Index
Three-Month Canadian Bankers' Acceptance Futures Contracts (BAX)	45	Delivery dates range from January 2003 to December 2013.
Ten-Year Government of Canada Bond Futures Contracts (CGB)	34	Delivery dates range from March 2003 to June 2011.
S&P/TSX 60 Index Standard Futures Contracts (SXF)	34	Delivery dates range from March 2003 to June 2011.
Total futures contracts	113	These represent all the futures contracts (i.e., all delivery dates) written on the three underlying assets during the sample period.
Active firm accounts	21	We report results only for this type of account.
Active client accounts	23	
Active omnibus accounts	16	

Note: The table presents an overview of the data set used in the empirical analysis, which was obtained from the Canadian Derivatives Clearing Corporation. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset.

Table 6: Specifications of the contracts included in the empirical analysis

	S&P/TSX 60 Index Standard Futures (SXF)	Three-Month Canadian Bankers' Acceptance Futures (BAX)	Ten-Year Government of Canada Bond Futures (CGB)
Underlying Interest	The S&P/TSX 60 Index C$200 times the S&P/TSX 60 index futures value	C$1,000,000 nominal value of Canadian bankers' acceptance with a three-month maturity.	C$100,000 nominal value of Government of Canada Bond with 6% notional coupon.
Expiration Months	March, June, September and December.	March, June, September and December plus two nearest non-quarterly months (serials).	March, June, September and December.
Price Quotation	Quoted in index points, expressed to two decimals.	Index: 100 minus the annualized yield of a three-month Canadian bankers' acceptance.	Par is on the basis of 100 points where 1 point equals C$1,000.
Price Fluctuation	0.10 index points for outright positions. 0.01 index points for calendar spreads	0.005 = C$12.50 per contract for the nearest three listed contract months, including serials. 0.01 = C$25.00 per contract for all other contract months.	0.01 = C$10
Price Limits	A trading halt will be invoked in conjunction with the triggering of "circuit breaker" in the underlying stocks.	None	None
Settlement	Cash settlement	Cash settlement	Physical delivery of eligible Government of Canada Bonds.
Trading Hours (EST)	Early session*: 6:00 a.m. to 9:15 a.m. Regular session: 9:30 a.m. to 4:15 p.m. * A trading range of -5% to +5% (based on previous day's settlement price) has been established only for this session.	Early session: 6:00 a.m. to 7:45 a.m. Regular session: 8:00 a.m. to 3:00 p.m. Extended session*: 3:09 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.	Early session: 6:00 a.m. to 8:05 a.m. Regular session: 8:20 a.m. to 3:00 p.m. Extended session*: 3:06 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.

Source: TMX Montreal Exchange (http://www.m-x.ca).

Table 7: Summary statistics of the contracts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
		Panel A: All Contracts			
Active contracts per day	19.81	20.00	0.9279	13.00	21.00
Trading days per contract	362.25	253.00	221.72	6.00	756.00
		Panel B: BAX Contracts			
Active contracts per day	11.99	12.00	0.14	8.00	13.00
Trading days per contract	550.58	699.00	249.25	6.00	756.00
Open interest long	137.81	131.32	49.21	48.35	310.97
Open interest short	-137.81	-131.32	49.21	-310.97	-48.35
Open interest gross	275.61	262.65	98.42	96.71	621.94
		Panel C: CGB Contracts			
Active contracts per day	3.93	4.00	0.49	1.00	5.00
Trading days per contract	238.91	253.00	42.73	55.00	255.00
Open interest long	65.26	60.81	27.36	16.15	176.97
Open interest short	-65.26	-60.81	27.36	-176.97	-16.15
Open interest gross	130.52	121.62	54.72	32.30	353.94
		Panel D: SXF Contracts			
Active contracts per day	3.89	4.00	0.43	1.00	4.00
Trading days per contract	236.32	250.00	42.58	52.00	255.00
Open interest long	55.28	55.09	14.01	24.40	98.49
Open interest short	-55.28	-55.09	14.01	-98.49	-24.40
Open interest gross	110.56	110.17	28.02	48.79	196.99

Note: The table shows the summary statistics of the 113 futures contracts included in the empirical analysis. These contracts are divided according to their underlying assets as follows: Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF). Over the sample period (2 January 2003 to 31 March 2011), there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Open interest values are reported in thousands.

Table 8: Clearing members included in the empirical analysis

Number	Name	Number	Name
1	Newedge Canada Inc.	25	Morgan Stanley Canada LTD.
2	RBC Dominion Securities Inc.	26	CFG Financial Group Inc.
3	Union Securities LTD.	27	MF Global Canada Co.
4	T.D. Securities Inc.	28	Haywood Securities Inc.
5	BMO Nesbitt Burns LTD.	29	Goldman Sachs Canada Inc.
6	Macquarie Private Wealth Inc.	30	Timber Hill Canada Co.
7	UBS Securities Canada Inc.	31	Credit Suisse Securities
8	Desjardins Securities Inc.	32	CIBC World Markets Inc.
9	Macquarie Capital Markets Inc.	33	NBCN Clearing Services Inc.
10	Name not reported	34	HSBC Securities (Canada) Inc.
11	Merrill Lynch Canada Inc.	35	Mackie Research Capital Corporation
12	Odlum Brown LTD.	36	Benson-Quinn GMS Inc.
13	Penson Financial Services Inc.	37	Scotia Capital Inc.
14	Dundee securities corporation	38	E*trade Canada Securities Corporation
15	Daex Commodities Inc.	39	Raymond Kames LTD.
16	Canaccord Capital Corporation	40	Lévesque Beaubien Geoffrion Inc.
17	Friedberg Mercantile Group LTD.	41	TD Waterhouse Canada Inc.
18	W.D. Latimer Co. LTD.	42	Citigroup Global Markets Canada Inc.
19	Canadian Imperial Bank of Commerce (CIBC)	43	National Bank of Canada
20	Jones, Gable & Co. LTD.	44	J.P. Morgan Securities Canada Inc.
21	Name not reported	45	Merrill Lynch Canada Inc.
22	Timber Hill Canada Company	46	Name not reported
23	Laurentian Bank Securities Inc.	47	Fidelity Clearing Canada ULC
24	Deutsche Bank Securities LTD.	48	Maple Securities Canada LTD.

Note: The table provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period (2 January 2003 to 31 March 2011). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Notice that this list includes more clearing members than those currently affiliated with the Canadian Derivatives Clearing Corporation (CDCC) because some of them entered and exited the market during the sample period.

Table 9: Summary statistics of the firm accounts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
		Panel A: Full Sample period			
Active accounts per day	11.64	12.00	1.09	8.00	15.00
Active days for an account	1145.19	1420.00	911.72	3.00	2066.00
Daily P&L across CMs	-60.92	-97.80	2659.44	-15014.20	17502.52
P&L over time	-37.83	0.43	160.57	-455.52	237.50
		Panel B: Pre-Crisis period			
Active accounts per day	11.96	12.00	0.95	9.00	15.00
Active days for an account	858.00	1148.00	431.12	3.00	1148.00
Daily P&L across CMs	-163.50	-156.15	2027.37	-6813.50	10381.36
P&L over time	-119.12	-0.78	225.46	-671.62	39.80
		Panel C: Crisis period			
Active accounts per day	11.24	11.00	1.13	8.00	15.00
Active days for an account	516.05	684.00	418.29	3.00	918.00
Daily P&L across CMs	65.18	-57.43	3280.36	-15014.20	17502.52
P&L over time	39.83	-0.60	135.28	-110.76	484.42

Note: The table presents the summary statistics of the 23 active firm accounts used in the empirical analysis. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from 2 January 2003 to 31 March 2011 and there are $N = 48$ clearing members in the sample. The pre-crisis period is from 2 January 2003 to 31 July 2007 and the crisis period is from 1 August 2007 to 31 March 2011. P&L values are reported in thousands of dollars and are estimated only for active firm accounts.

Table 10: Daily margin requirements under the SPAN, VaR and CoMargin systems over the full sample period

	Mean	Median	St.Dev.	Min	Max
			Aggregate Market (CCP level)		
SPAN	112.04	105.71	38.49	49.83	328.77
VaR	101.40	95.80	36.03	42.90	301.97
CoMargin	161.31	156.20	64.93	56.89	475.27
BNSPAN	161.31	156.20	64.93	56.88	475.27
BNVaR	161.31	156.20	64.93	56.88	475.27
			Cross-Sectional (CM level)		
SPAN	9.76	9.16	3.54	3.83	29.86
VaR	8.83	8.26	3.28	3.30	27.45
CoMargin	14.14	13.41	6.08	4.38	43.01
BNSPAN	14.14	13.41	6.08	4.38	43.01
BNVaR	14.14	13.41	6.08	4.38	43.01
			Time Series (CM level)		
SPAN	5.85	1.48	7.63	0.01	22.65
VaR	5.34	1.44	7.03	0.01	20.38
CoMargin	8.50	1.87	11.69	0.01	35.00
BNSPAN	10.16	5.77	7.67	3.06	27.02
BNVaR	10.55	6.95	7.06	3.81	25.69

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems for the 23 active firm accounts during the full sample period, from 2 January 2003 to 31 March 2011. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 11: Daily margin requirements under the SPAN, VaR and CoMargin systems during the pre-crisis period

	Mean	Median	St.Dev.	Min	Max
			Aggregate Market (CCP level)		
SPAN	100.27	100.80	24.19	49.83	154.91
VaR	91.20	92.31	21.57	42.90	140.43
CoMargin	140.14	139.08	45.63	56.89	262.98
BNSPAN	140.14	139.07	45.63	56.88	262.97
BNVaR	140.14	139.07	45.63	56.88	262.97
			Cross-Sectional (CM level)		
SPAN	8.49	8.32	2.38	3.83	14.98
VaR	7.72	7.47	2.15	3.30	13.79
CoMargin	11.94	11.62	4.45	4.38	25.98
BNSPAN	11.94	11.62	4.45	4.38	25.98
BNVaR	11.94	11.62	4.45	4.38	25.98
			Time Series (CM level)		
SPAN	6.30	1.76	7.87	0.01	22.05
VaR	5.73	1.56	7.27	0.01	19.62
CoMargin	8.80	2.02	11.40	0.01	32.46
BNSPAN	9.73	5.58	7.94	2.14	25.50
BNVaR	9.86	6.01	7.40	2.67	23.83

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems for the 23 active firm accounts during the pre-crisis period, from 2 January 2003 to 31 July 2007. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 12: Daily margin requirements under the SPAN, VaR and CoMargin systems during the crisis period

	Mean	Median	St.Dev.	Min	Max
			Aggregate Market (CCP level)		
SPAN	126.76	115.88	47.05	62.72	328.77
VaR	114.17	102.57	45.25	50.76	301.97
CoMargin	187.78	174.05	75.01	72.88	475.27
BNSPAN	187.78	174.05	75.01	72.88	475.27
BNVaR	187.78	174.05	75.01	72.88	475.27
			Cross-Sectional (CM level)		
SPAN	11.36	10.85	4.07	4.88	29.86
VaR	10.21	9.51	3.87	4.46	27.45
CoMargin	16.89	16.21	6.69	5.64	43.01
BNSPAN	16.89	16.21	6.69	5.64	43.01
BNVaR	16.89	16.21	6.69	5.64	43.01
			Time Series (CM level)		
SPAN	7.21	2.24	10.44	0.01	37.10
VaR	6.58	2.09	9.67	0.01	34.05
CoMargin	10.95	3.11	16.72	0.01	60.15
BNSPAN	12.29	7.61	10.86	3.30	42.64
BNVaR	12.72	8.55	10.10	4.20	40.73

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems for the 23 active firm accounts during the crisis period, from 1 August 2007 to 31 March 2011. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 13: Average cross-sectional correlation between the SPAN, VaR and CoMargin systems

Variable	SPAN	VaR	CoMargin
Panel A: Full sample period			
SPAN	1.00 (0.00)		
VaR	0.99 (0.01)	1.00 (0.00)	
CoMargin	0.90 (0.28)	0.90 (0.29)	1.00 (0.00)
Panel B: Pre-Crisis period			
SPAN	1.00 (0.00)		
VaR	0.98 (0.03)	1.00 (0.00)	
CoMargin	0.94 (0.05)	0.94 (0.05)	1.00 (0.00)
Panel C: Crisis period			
SPAN	1.00 (0.00)		
VaR	0.99 (0.01)	1.00 (0.00)	
CoMargin	0.90 (0.29)	0.90 (0.29)	1.00 (0.00)

Note: The table shows the average cross-sectional correlation between the SPAN, VaR and CoMargin requirements of the 23 active firm accounts in the sample. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from 2 January 2003 to 31 March 2011. The pre-crisis period is from 2 January 2003 to 31 July 2007 and the crisis period is from 1 August 2007 to 31 March 2011. Standard deviations are reported in brackets.

Table 14: Performance of the SPAN, VaR and CoMargin systems over the full sample period

	Unconditional			Conditional on at least one exceedance		
	Prob. of Exceedances	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**	**Prob. of Exceedances**	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**
			Aggregate Market (CCP level)			
SPAN	0.09	0.15	0.35	0.36	1.63	3.78
VaR	0.14	0.25	0.44	0.42	1.80	3.20
CoMargin	0.07	0.10	0.13	0.28	1.44	1.85
BNSPAN	0.02	0.02	0.16	0.38	1.47	10.15
BNVaR	0.02	0.03	0.16	0.36	1.47	9.45
			Cross-Sectional (CM level)			
SPAN	0.01	-	0.03	0.14	-	0.34
VaR	0.02	-	0.04	0.15	-	0.29
CoMargin	0.01	-	0.01	0.12	-	0.16
BNSPAN	0.00	-	0.01	0.12	-	0.90
BNVaR	0.00	-	0.01	0.12	-	0.84
			Time Series (CM level)			
SPAN	0.01	-	0.02	0.15	-	0.25
VaR	0.02	-	0.02	0.15	-	0.21
CoMargin	0.01	-	0.01	0.12	-	0.11
BNSPAN	0.00	-	0.01	0.11	-	0.74
BNVaR	0.00	-	0.01	0.11	-	0.69

Note: The table compares the empirical performance of the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems computed for the 23 active firm accounts over the sample period, from 2 January 2003 to 31 March 2011. The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

Table 15: Performance of the SPAN, VaR and CoMargin systems during the pre-crisis period

	Unconditional			**Conditional on at least one exceedance**		
	Prob. of Exceedances	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**	**Prob. of Exceedances**	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**
			Aggregate Market (CCP level)			
SPAN	0.09	0.12	0.08	0.31	1.35	0.90
VaR	0.12	0.19	0.12	0.40	1.60	1.03
CoMargin	0.07	0.09	0.04	0.25	1.27	0.60
BNSPAN	0.01	0.01	0.03	0.23	1.23	2.22
BNVaR	0.01	0.01	0.03	0.31	1.31	2.36
			Cross-Sectional (CM level)			
SPAN	0.01	-	0.01	0.11	-	0.08
VaR	0.02	-	0.01	0.13	-	0.09
CoMargin	0.01	-	0.00	0.10	-	0.05
BNSPAN	0.00	-	0.00	0.10	-	0.17
BNVaR	0.00	-	0.00	0.10	-	0.18
			Time Series (CM level)			
SPAN	0.01	-	0.00	0.12	-	0.06
VaR	0.02	-	0.01	0.14	-	0.07
CoMargin	0.01	-	0.00	0.11	-	0.04
BNSPAN	0.00	-	0.00	0.09	-	0.17
BNVaR	0.00	-	0.00	0.10	-	0.18

Note: The table compares the empirical performance of the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems computed for the 23 active firm accounts over the pre-crisis period, from 2 January 2003 to 31 July 2007. The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

Table 16: Performance of the SPAN, VaR and CoMargin systems during the crisis period

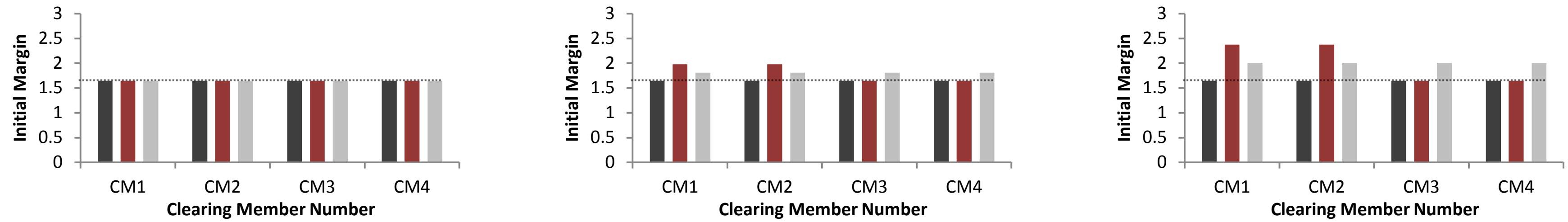
	Unconditional			**Conditional on at least one exceedance**		
	Prob. of Exceedances	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**	**Prob. of Exceedances**	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**
			Aggregate Market (CCP level)			
SPAN	0.10	0.19	0.69	0.42	1.93	6.91
VaR	0.16	0.31	0.83	0.45	1.99	5.32
CoMargin	0.07	0.12	0.24	0.32	1.63	3.33
BNSPAN	0.02	0.03	0.32	0.47	1.63	15.58
BNVaR	0.03	0.04	0.34	0.39	1.57	13.46
			Cross-Sectional (CM level)			
SPAN	0.02	-	0.06	0.17	-	0.63
VaR	0.03	-	0.08	0.18	-	0.48
CoMargin	0.01	-	0.02	0.14	-	0.30
BNSPAN	0.00	-	0.03	0.14	-	1.40
BNVaR	0.00	-	0.03	0.14	-	1.21
			Time Series (CM level)			
SPAN	0.02	-	0.04	0.18	-	0.51
VaR	0.02	-	0.05	0.16	-	0.36
CoMargin	0.01	-	0.01	0.13	-	0.24
BNSPAN	0.00	-	0.02	0.12	-	1.15
BNVaR	0.00	-	0.02	0.12	-	0.99

Note: The table compares the empirical performance of the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems computed for the 23 active firm accounts over the crisis period, from 1 August 2007 to 31 March 2011. The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

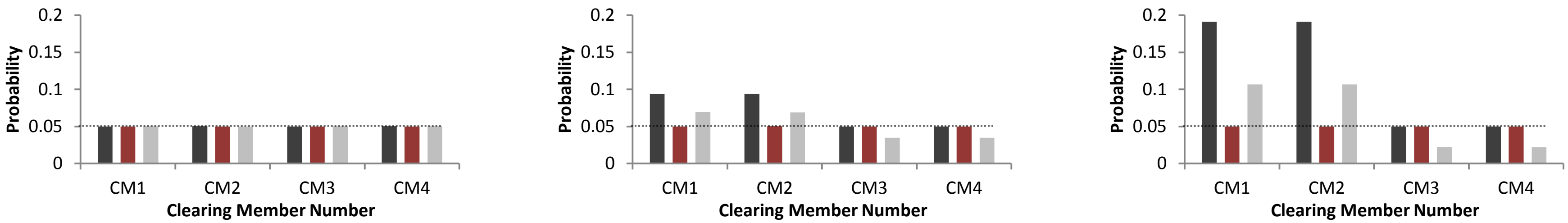
Figure 1: Theoretical performance of VaR and CoMargin systems assuming jointly normally distributed P&Ls

ρ = 0	ρ = 0.4	ρ = 0.8

Panel A: Initial margin collected from each clearing member (dollars)



Panel B: Probability of a given CM exceeding its margin conditional on at least another CM being in financial distress



Panel C: Probability of a CM exceeding its margin conditional on at least another CM having a margin exceedance

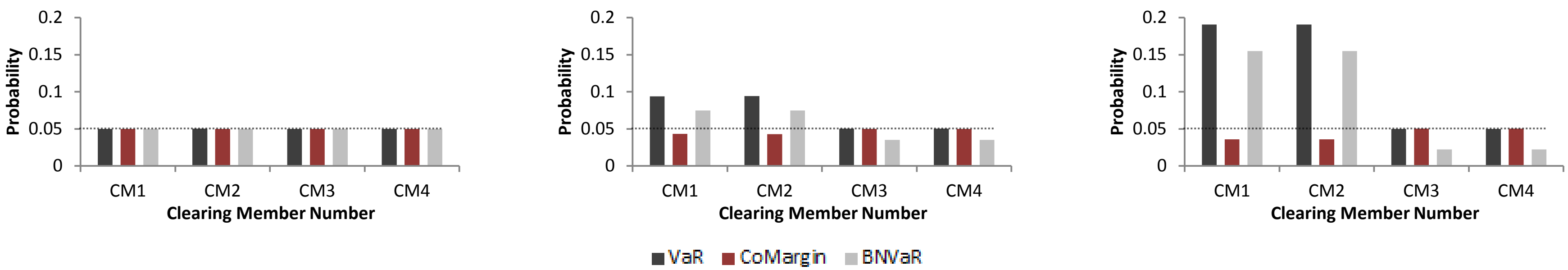


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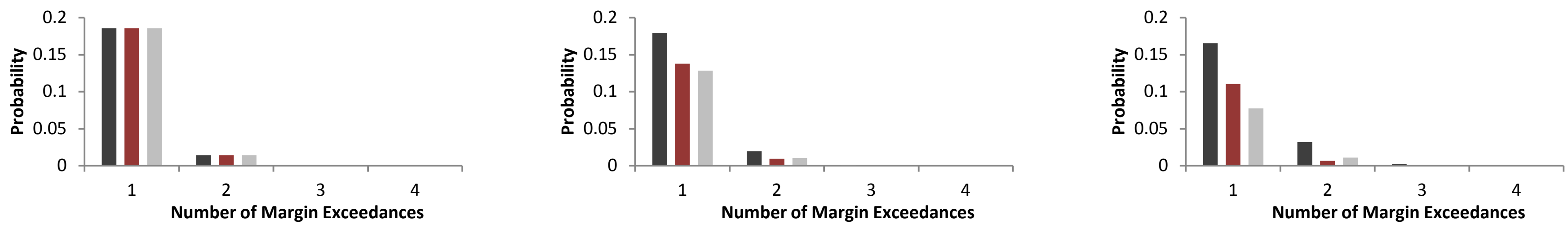
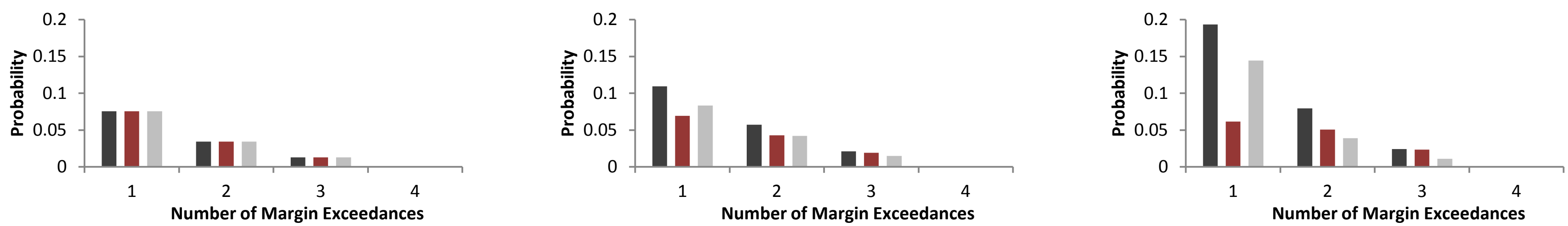
Figure 1 (Continued): Theoretical performance of VaR and CoMargin systems assuming jointly normally distributed P&Ls

ρ = 0	ρ = 0.4	ρ = 0.8

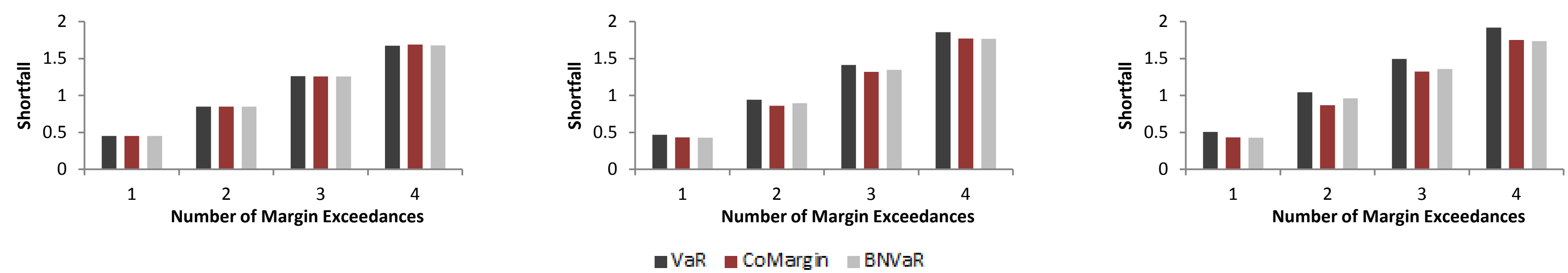
Panel D: Probability of a minimum number of margin exceedances



Panel E: Probability of additional margin exceedances given that a number of exceedances have occurred



Panel F: Conditional expected shortfall for the CCP given a minimum number of margin exceedances

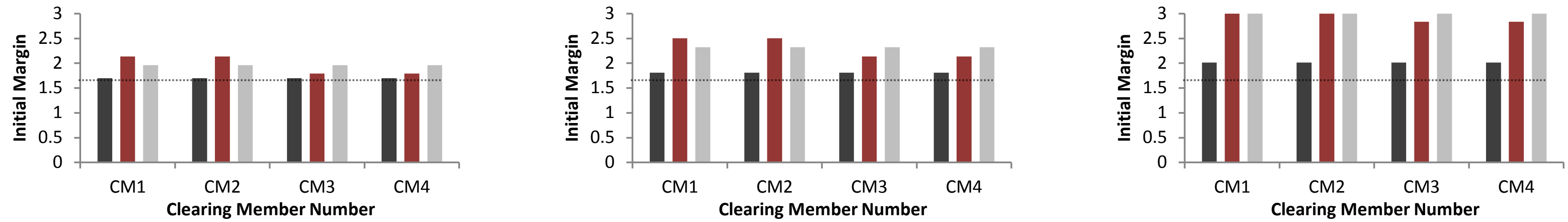


Note: This figure presents the theoretical performance of VaR margin (equation (2)), CoMargin (equation (10)) and BNVaR margin (equation (21)). We consider four firms with jointly normally distributed P&Ls, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. We report our results for different levels of the correlation parameter, ρ, that range from 0 to 0.8.

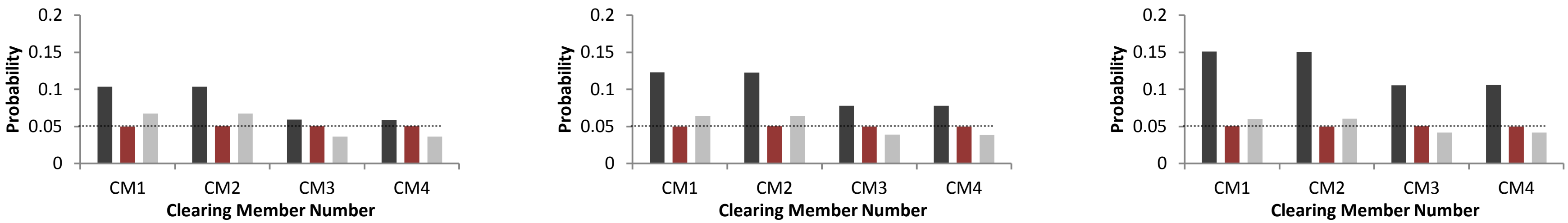
Figure 2: Theoretical performance of VaR and CoMargin systems assuming jointly Student t distributed P&Ls

v = 30	v = 10	v = 5

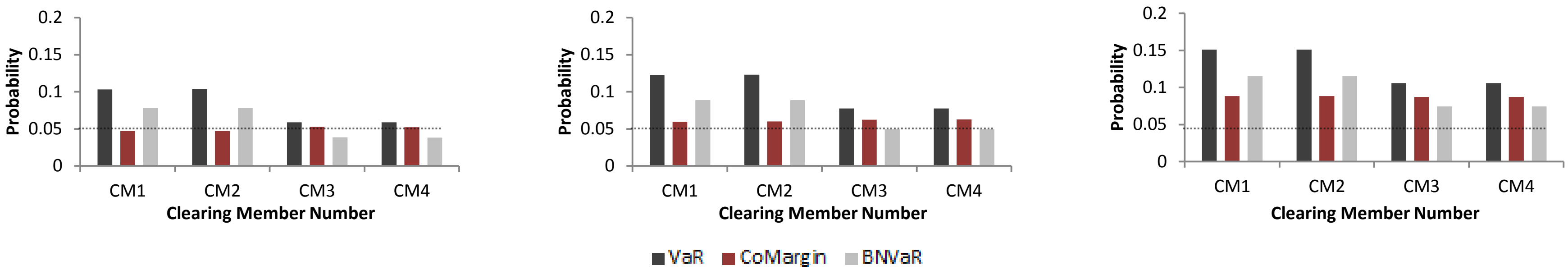
Panel A: Initial margin collected from each clearing member (dollars)



Panel B: Probability of a given CM exceeding its margin conditional on at least another CM being in financial distress



Panel C: Probability of a CM exceeding its margin conditional on at least another CM having a margin exceedance

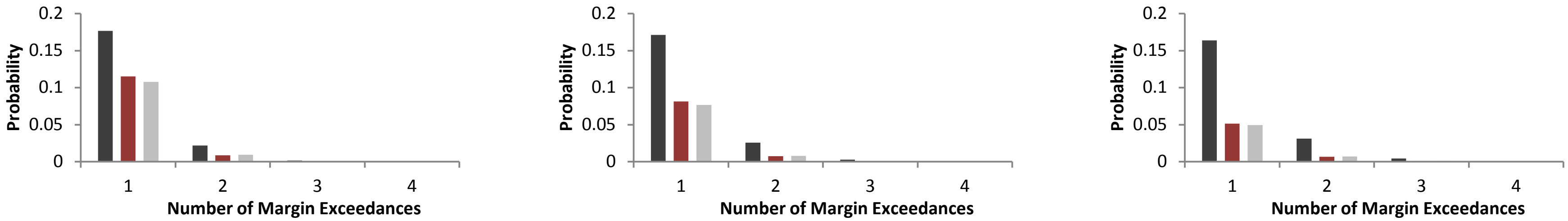


continued . . .

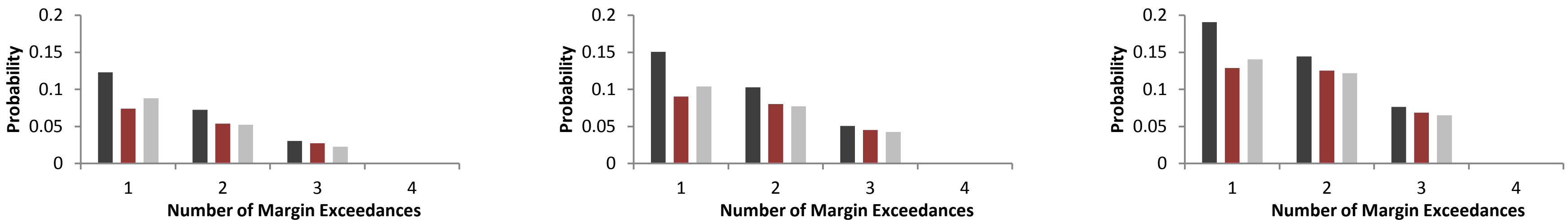
Figure 2 (Continued): Theoretical performance of VaR and CoMargin systems assuming jointly Student t distributed P&Ls

v = 30	v = 10	v = 5

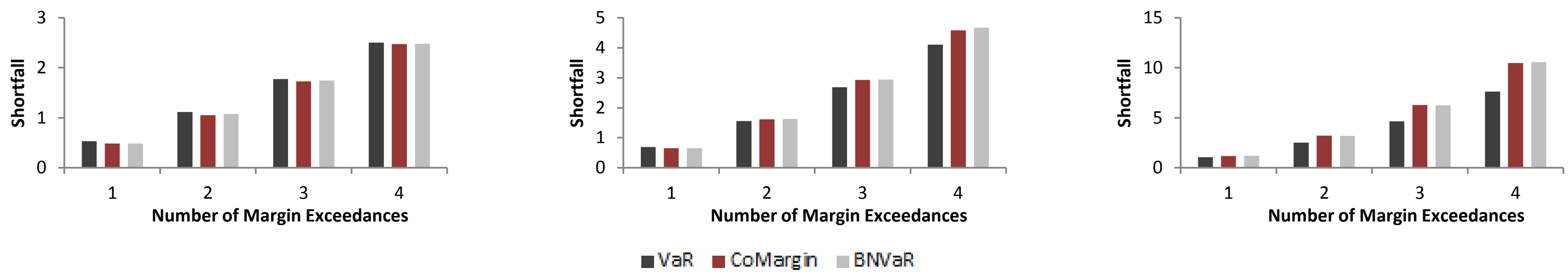
Panel D: Probability of a minimum number of margin exceedances



Panel E: Probability of additional margin exceedances given that a number of exceedances have occurred



Panel F: Conditional expected shortfall for the CCP given a minimum number of margin exceedances

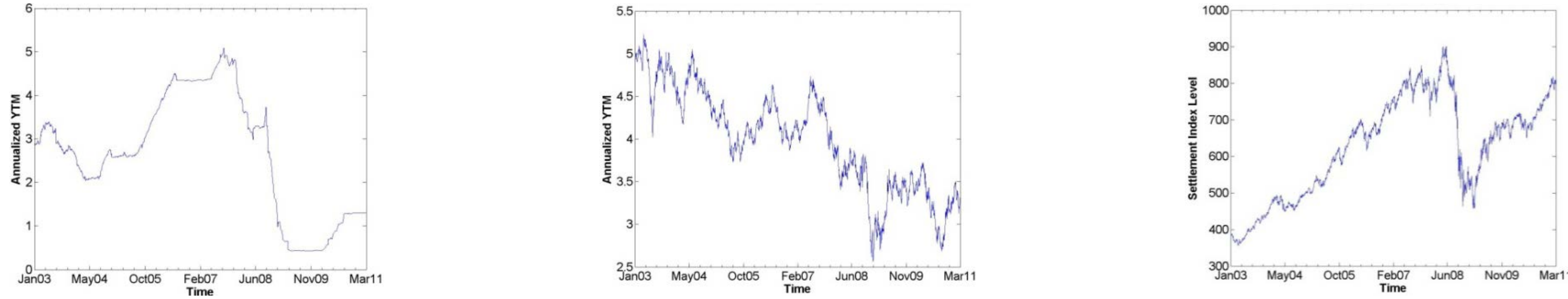


Note: This figure presents the theoretical performance of VaR margin (equation (2)), CoMargin (equation (10)) and BNVaR margin (equation (21)). We consider four firms with joint Student t distributed P&Ls with degrees of freedom υ, such that $V \sim t_{\upsilon}(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$, $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and $\rho = 0.4$. We report our results for different levels of the degrees of freedom parameter that range from 30 to 5.

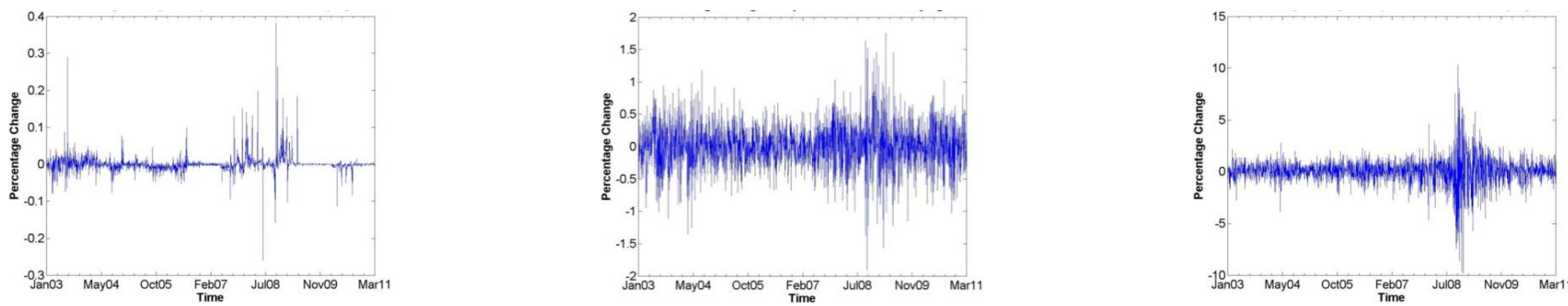
Figure 3: Underlying assets and futures contracts used in the empirical analysis

Three-month Canadian Bankers' Acceptance (BAX) | **Ten-year Government of Canada Bond (CGB)** | **S&P/TSX 60 Index Standard (SXF)**

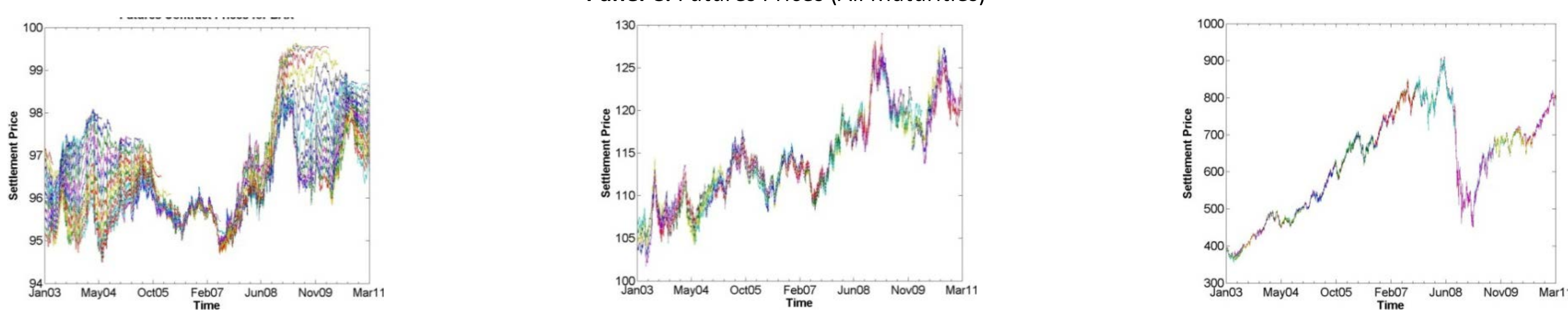
Panel A: Underlying Asset Prices



Panel B: Underlying Asset Returns

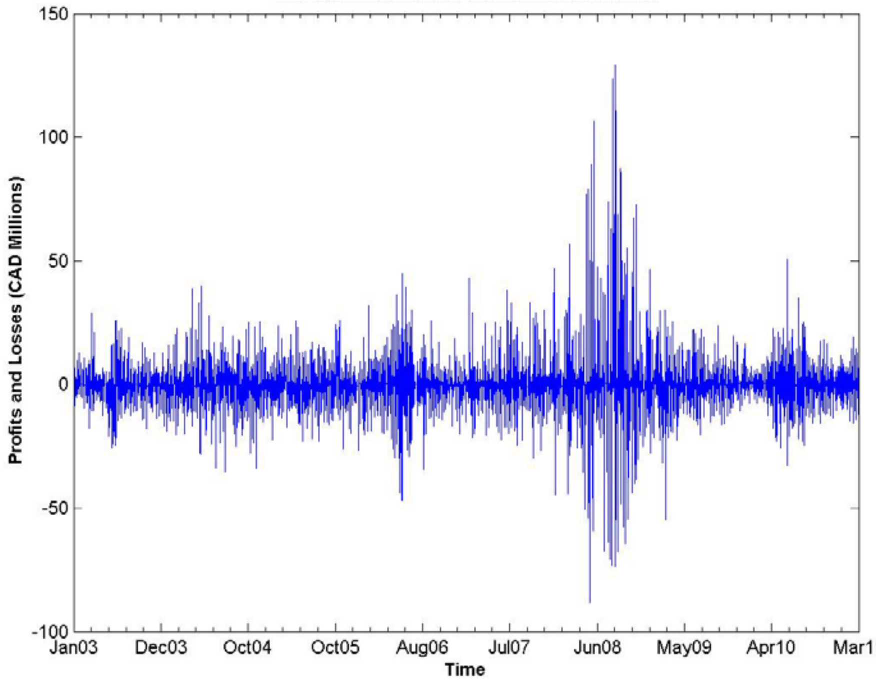


Panel C: Futures Prices (All Maturities)



Note: Panel A presents the daily annualized settlement yield for the Three-month Canadian Bankers' Acceptance, the annualized yield on the Ten-year Government of Canada Bond and the settlement level of the S&P/TSX 60 Index. Panel B shows the daily returns (i.e., percentage changes) of the variables presented in Panel A. Panel C presents the daily settlement futures prices for the futures contracts written on the Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF), traded in the Montreal Exchange. Lines in different colours represent different delivery dates. The sample period is from 2 January 2003 to 31 March 2011. **Source:** Bloomberg.

Figure 4: P&L for active firm accounts

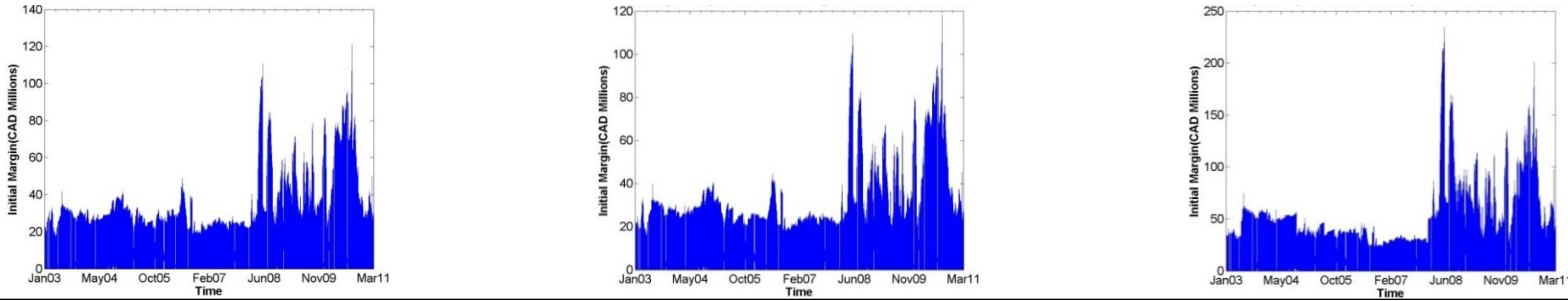


Note: The figure shows the daily stacked P&L implied from the positions of the 23 active firm accounts included in the sample; that is, accounts with an open interest (i.e., long or short position) in at least one underlying asset at the end of the trading day. For each date t, $n^a \in N$ observations are plotted, which correspond to the P&L of the n^a clearing members with an active account. The sample period is from 2 January 2003 to 31 March 2011 and there are $N = 48$ clearing members in the sample.

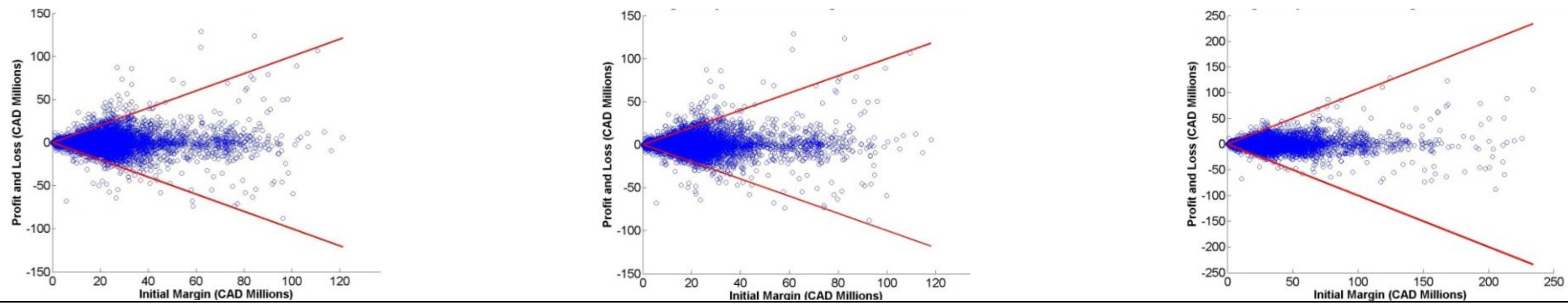
Figure 5: SPAN, VaR and CoMargin collateral requirements over the full sample period

SPAN	VaR	CoMargin

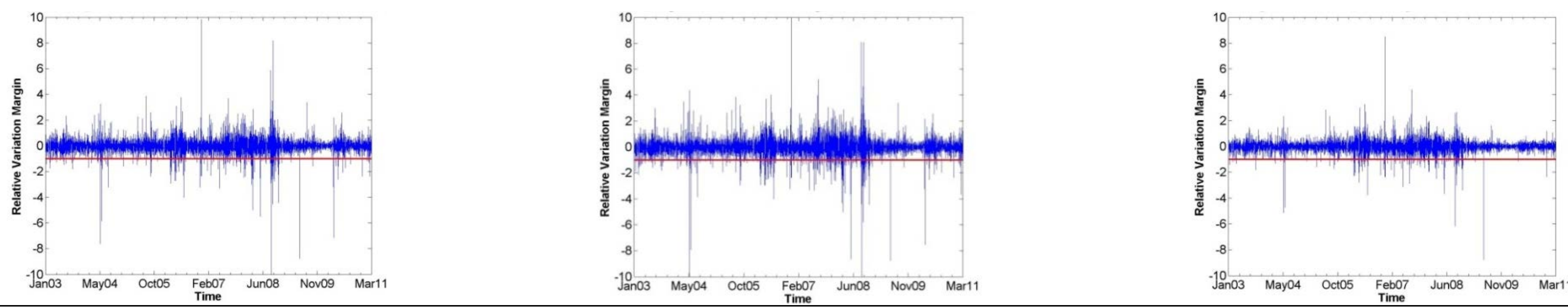
Panel A: Initial Margin



Panel B: Daily P&L and Initial Margins

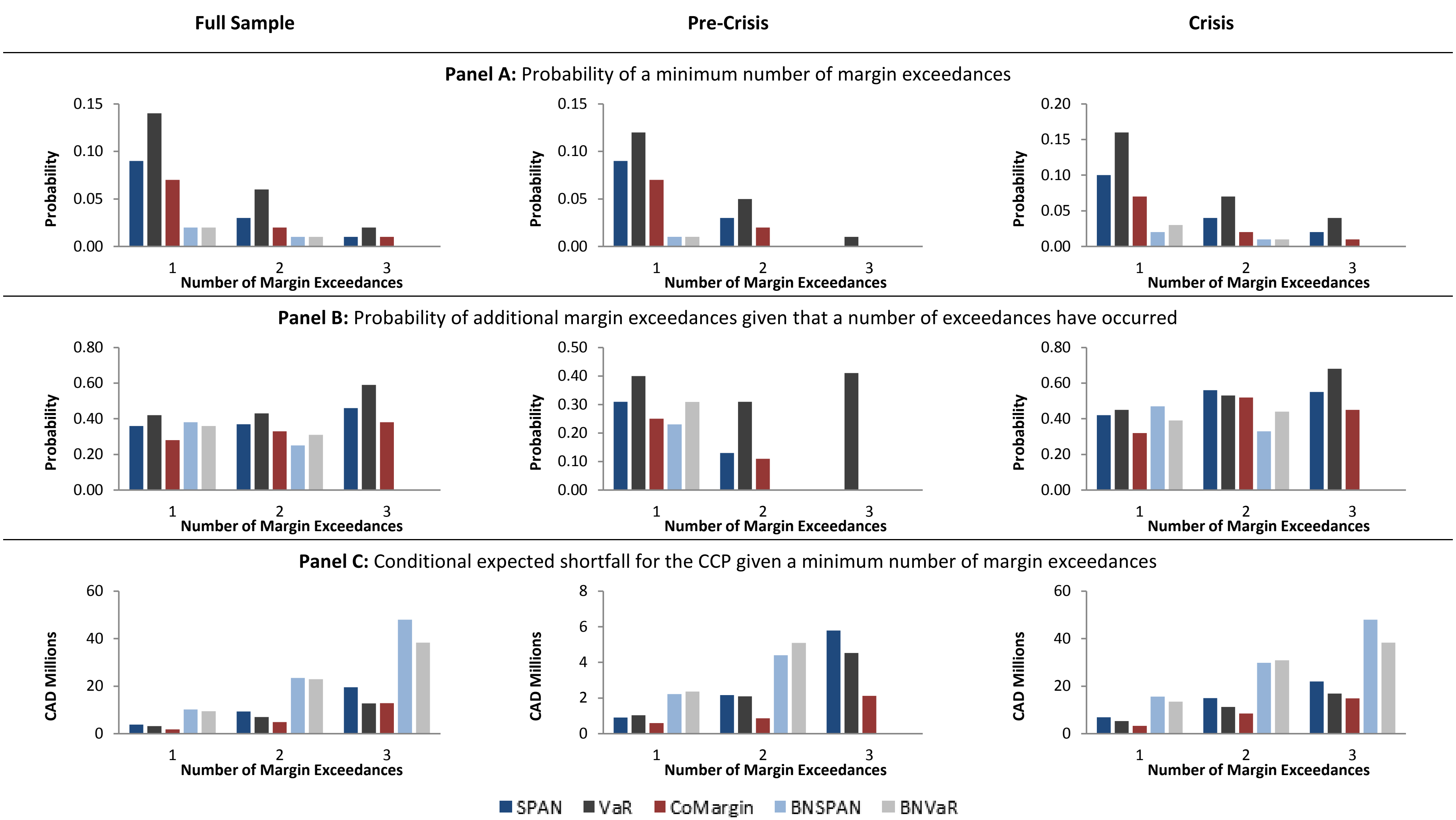


Panel C: Relative Variation Margin



Note: The figure shows the implied margin requirements from the positions in the 23 active firm accounts of the $N = 48$ clearing members in the sample under the SPAN, VaR and CoMargin systems. Panel A shows the daily stacked initial margin requirements. Panel B plots the daily implied P&L against its initial margin requirement. Panel C shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin. The stacking method used in panels A and C is as follows: for each date t, $n^a \in N$ observations are plotted, which correspond to the observations of the n^a clearing members with an active account. The sample period is from 2 January 2003 to 31 March 2011.

Figure 6: Empirical performance of the SPAN, VaR and CoMargin systems



Note: The figure compares the empirical performance of the SPAN, VaR (equation (2)) and CoMargin (equation (10)) systems computed for the 23 active firm accounts in the sample. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation (21). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from 2 January 2003 to 31 March 2011. The pre-crisis period is from 2 January 2003 to 31 July 2007, and the crisis period is from 1 August 2007 to 31 March 2011.