Perceived Inflation Persistence

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Canadian Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
jain@bankofcanada.ca
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Abstract

The Survey of Professional Forecasters (SPF) has had vast influence on research related to better understanding expectation formation and the behaviour of macroeconomic agents. Inflation expectations, in particular, have received a great deal of attention, since they play a crucial role in determining real interest rates, the expectations-augmented Phillips curve and monetary policy. One feature of the SPF that has surprisingly not been explored is the natural way in which it can be used to extract useful measures of inflation persistence. This paper presents a new measure of U.S. inflation persistence from the point of view of a professional forecaster, referred to as perceived inflation persistence. It is built via the implied autocorrelation function that follows from the estimates obtained using a forecaster-specific state-space model. Findings indicate that perceived inflation persistence has changed over time and that forecasters are more likely to view unexpected shocks to inflation as transitory, particularly since the mid-1990s. When compared to the autocorrelation function for actual inflation, forecasters react less to shocks than the actual inflation data would suggest, since they may engage in forecast smoothing.

JEL classification: E31, E37
Bank classification: Inflation and prices; Econometric and statistical methods

Résumé

L’enquête auprès des prévisionnistes professionnels a eu une influence considérable sur les recherches visant à acquérir une meilleure compréhension de la formation des anticipations et du comportement des agents macroéconomiques. Plus particulièrement, les attentes en matière d’inflation ont fait l’objet de nombreuses études, étant donné le rôle majeur qu’elles jouent dans la détermination des taux d’intérêt réels, de la courbe de Phillips dotée d’anticipations et de la politique monétaire. Étonnamment, un aspect qui n’a pas été exploré est la manière par laquelle cette enquête peut être utilisée pour extraire des mesures utiles de la persistance de l’inflation. L’auteure présente une nouvelle mesure de la persistance de l’inflation aux États-Unis telle qu’elle est perçue par les prévisionnistes professionnels. Cette mesure, qu’elle appelle persistance perçue de l’inflation, s’appuie sur la fonction d’autocorrélation implicite qui découle d’estimations obtenues à l’aide d’un modèle espace-état tenant compte de la spécificité de chaque prévisionniste. Les résultats indiquent que la persistance perçue de l’inflation a changé au fil du temps et que les prévisionnistes sont plus susceptibles de considérer les chocs d’inflation imprévus comme temporaires, particulièrement depuis le milieu des années 1990. La comparaison avec la fonction d’autocorrélation révèle que les prévisionnistes réagissent aux chocs dans une moindre mesure que ce que laissent entrevoir les données sur l’inflation réalisée, car ils lisseraient leurs prévisions.

Classification JEL : E31, E37
Classification de la Banque : Inflation et prix; Méthodes économétriques et statistiques
1 Introduction

Inflation persistence is a heavily debated property of inflation that can be defined as the duration of shocks hitting inflation. At present, there is no consensus regarding the degree or stability of inflation persistence: the empirical evidence differs across various measures of inflation and time spans, as well as across different measures of persistence. An important consequence of high inflation persistence is that it can undermine people’s confidence in the central bank’s policies. Consequently, it may cause inflation expectations to differ from the central bank’s objectives as inflation expectations become sensitive to shocks that hit the economy. This may influence the level of actual inflation, despite the central bank’s policies targeted at controlling it. For example, consider a substantial increase in the price of gasoline. If the public viewed the central bank’s mandate to maintain price stability as credible, then they would view the resulting increase in inflation as temporary and their expectations of future inflation would be in line with the central bank’s objectives. Conversely, if the public did not view the central bank as credible in maintaining price stability, persistently high inflation could result, since the public would expect higher inflation in the future, and so make price and wage decisions accordingly. In this case, the central bank may be required to take a more aggressive response following an inflationary shock than would otherwise be necessary, solely to achieve credibility with the public. Thus, persistence in expected inflation may raise the output costs associated with stabilizing actual inflation, and without a concrete consensus regarding inflation persistence, it is difficult to determine the extent of these costs.

This paper takes a different approach to tackle this challenge by introducing a novel measure, called perceived inflation persistence, to extract persistence estimates from inflation-expectations data. Perceived inflation persistence measures the persistence of inflation from the point of view of a forecaster. Obtaining this measure is not a straightforward task, since people are not asked to quantify or report their views on inflation persistence. Moreover, agents are heterogeneous in the way in which they form inflation expectations, making it
difficult to deduce their implied persistence. One reason for this heterogeneity is that agents may have different loss functions. Alternatively, agents may have heterogeneous information sets, so the differences in their expectations reflect this different information.

The methodology developed in this paper to identify perceived inflation persistence can be applied to many related issues, such as understanding forecasters’ views on the persistence of shocks to different macroeconomic variables. In addition, recognizing that forecasters are heterogeneous, they are not assumed to have identical information sets: differences in their perceptions are allowed for, making it possible to generate a distribution of implied estimates of persistence that can be tested for stability across forecasters.

The method used to identify perceptions of inflation persistence in this paper does not presume a particular model, but, in the context of a state-space model, provides a useful means of comparison between the measure introduced in this paper and other measures of inflation persistence. In a state-space model, inflation comprises a persistent, forecaster-specific state variable that evolves according to an AR(1) process, plus a transitory noise component. State persistence is measured as the autocorrelation in the state variable and is shown to be closely related to inflation predictability, which has been equated with inflation persistence in recent literature. Furthermore, once an estimate of state persistence is obtained for each forecaster, it can be used to compute forecasters’ implied autocorrelation functions, giving an estimate of perceived inflation persistence that may be directly compared to actual inflation persistence. The results suggest that, likely due to forecast smoothing by forecasters, their forecasts react less to shocks than actual inflation data would suggest.

This paper is organized as follows. Section 2 reviews the literature related to the study of inflation persistence and the use of data obtained from surveys of professional forecasters. Section 3 discusses the relationship between forecast revisions and persistence, and the state-space model. Section 4 describes the data used and section 5 reports the estimation results. Section 6 concludes.
2 Research Context

2.1 U.S. inflation persistence

Much research has been dedicated to determining the degree of inflation persistence along with its stability over time. Cogley and Sargent (2001) use a vector autoregression (VAR) with random coefficients to measure parameter drift in U.S. inflation-unemployment-interest rate dynamics. They find that, since the early 1980s, the degree of persistence in CPI inflation has been drifting downwards. This result coincides with the findings of Taylor (2000), who uses the GDP deflator, and Brainard and Perry (2000), who use the CPI.

Similar to Cogley and Sargent (2001), Pivetta and Reis (2007) use Bayesian methods to estimate inflation dynamics using the GDP deflator, but allow for the possibility that inflation may have a unit root. They compute alternative statistical measures of inflation persistence and, as a result, find that inflation persistence in the United States has been unchanged over the past three decades and that the data cannot reject the possibility of a unit root. Sims (2001) and Stock (2001) also maintain that inflation persistence has been unchanged over this time period using the CPI and GDP deflator, respectively.

Levin and Piger (2004) apply both classical and Bayesian econometric methods to characterize the dynamic behaviour of inflation from 1984 to 2003. They consider four different measures of inflation: GDP price inflation, CPI inflation, core CPI inflation and personal consumption expenditure (PCE) price inflation. Contrary to the findings by Pivetta and Reis (2007), they find that once the model allows for structural breaks in the intercept (particularly in 1991 for the United States), all measures of inflation exhibit little persistence over the sample period.

A possible source of disagreement stems from the fact that the literature focuses on diverse features of inflation. In more recent work, Cogley, Primiceri and Sargent (2010) study inflation-gap persistence in the United States using two measures of inflation: the log-differences of the GDP and PCE chain-weighted price indices, measuring persistence in terms
of short- to medium-term predictability. They argue that the inflation gap, defined as the difference between inflation and the Federal Reserve’s long-run inflation target, has become less persistent since the Volcker disinflation, due to both monetary policy and non-policy factors.

In the inflation-forecasting literature, Stock and Watson (2007) examine the extent to which changes in the inflation process have made U.S. GDP inflation more difficult to forecast. Using an unobserved component trend-cycle model with stochastic volatility, their findings suggest that there have been major changes to the univariate inflation process that has made inflation less persistent and volatile since 1984. Post-1984 inflation forecasting models have smaller root-mean-squared errors (RMSE) than prior to 1984, suggesting that inflation has become easier to forecast. At the same time, post-1984, it has become difficult to improve on forecasts made beyond simple univariate models, making the job of an inflation forecaster more difficult.

For the most part, there is agreement that inflation persistence was high from the 1960s until the disinflationary period in the early 1980s. The controversy occurs in the subsequent period from the early 1980s onwards; some researchers suggest that inflation persistence has declined, while others contend that it has been unchanged. For example, Pivetta and Reis (2007) find that postwar U.S. inflation persistence is always close to 1, with no evidence of a decrease in recent years. They use multiple measures of inflation persistence to obtain this result, including the largest autoregressive root and the sum of the autoregressive coefficients, when inflation follows an AR(3) process. Conversely, Levin and Piger (2004) find that, since 1984, estimates of U.S. inflation persistence are less than 0.7 and the null hypothesis of a unit root can be rejected at the 95% confidence level. Levin and Piger measure the degree of inflation persistence in terms of the sum of the autoregressive coefficients when inflation follows an AR(4) process.

In a related context, the literature on inflation persistence for an inflation-targeting country such as Canada indicates that the persistence of both core and headline CPI inflation
has shown signs of significant decline since the 1980s. In fact, Mendes and Murchison (2009–2010) find that the adoption of an explicit inflation target has likely played a key role in this decline. Benati (2008) documents a similar decline in inflation persistence for Canada as well as other countries that have adopted inflation-targeting regimes.

2.2 Forecast survey data

Many researchers have found that data obtained from surveys of professional forecasters outperform simple time-series benchmarks in forecasting inflation: see Grant and Thomas (1999), Thomas (1999), and Mehra (2002). Faust and Wright (2012) compare several methods of forecasting inflation and find that private sector survey forecasts are quite difficult to beat. In addition, Ang, Bekaert and Wei (2007) examine the forecasting power of four alternative methods of forecasting U.S. inflation out-of-sample: autoregressive integrated moving-average (ARIMA) models, forecasts from a Phillips curve, term-structure models and survey-based measures. They find that the Livingston Survey and the Survey of Professional Forecasters (SPF) outperform the other forecasting methods. Capistran and Timmermann (2009) use data from the SPF to explain the dispersion in inflation expectations across forecasters. Their paper is relevant because it highlights aspects of forecaster heterogeneity, suggesting that the differences in beliefs are correlated with both the level and volatility of inflation based on asymmetries in costs of over- and underpredicting inflation. Similarly, Patton and Timmermann (2010) find that differences in agents’ forecasts of inflation are greater at longer forecast horizons and that these differences persist over time.

Models in which agents are inattentive to new information have been proposed to explain heterogeneity in forecasts. Mankiw, Reis and Wolfers (2003), for example, document disagreement in inflation expectations among both consumers and professional economists. They suggest that this disagreement can provide insight into macroeconomic dynamics. Their paper accounts for this heterogeneity by using a sticky-information model which suggests that agents update their expectations only periodically, and therefore some agents may
form expectations based on outdated information. They find that the sticky-information model is able to explain some (but not all) features of the data. Other forms of agent inattention include noisy information models proposed by Woodford (2002), Sims (2003), and Maćkowiak and Wiederholt (2009), in which agents continuously update their information but do not have perfect access to it. Andrade and Le Bihan (2010) estimate a model in which forecasters have both sticky information and noisy information. Their work is one of the few to highlight and exploit the panel dimension of a survey of professional forecasters, in particular, the European Central Bank’s SPF.

This paper uses data from the U.S. SPF to measure perceived inflation persistence. In light of the debate surrounding actual U.S. inflation persistence, I explore the panel dimension of the SPF, enabling a comparison of estimates across forecasters.

3 Forecast Revisions and Persistence

3.1 Notation and test equation

The notation used to represent a forecast is $E_{jt}\pi(t + h)$, which is the inflation forecast made by forecaster $j$, conditional on his or her time $t$ information set, $h$-steps ahead. Forecasts are typically made at the beginning of time $t + 1$, so the most recent period of information the forecaster would have available would be time $t$. For example, $E_{jt}\pi(t + 1)$ and $E_{jt}\pi(t + 2)$ would be the one-step-ahead and two-step-ahead inflation forecasts that forecaster $j$ makes based on time $t$ information, respectively.

An $h$-step-ahead forecast revision is defined as

$$r_{\pi}(j, t, h) = E_{jt}\pi(t + h) - E_{jt-1}\pi(t + h), \quad h = 1, ..., H \text{ and } J = 1, ..., J; \quad (1)$$

where the revision is made with time $t$ information. Notice that $E_{jt}\pi(t + h)$ and $E_{jt-1}\pi(t + h)$ are both forecasts of the same thing, $\pi(t + h)$. However, revisions can be made to this forecast
from one period to the next as forecaster j’s information set changes.

In this paper, the central idea is that if forecasters believe that an inflationary shock would have only temporary effects, they would not make substantial revisions to multiple future inflation forecasts, and their perception of shock persistence would be low. On the other hand, if forecasters believe that the central bank may not be able to maintain low and stable inflation following a shock, they would make revisions to their inflation forecasts for multiple future periods, and thus perceive the shock as being more persistent. Therefore, the coefficient linking revisions made to inflation forecasts between multiple horizons can be used to back out forecasters’ views on inflationary shock persistence. Specifically, we will see that, using the chain rule of forecasting, we can determine the relationship between revisions at horizons $h$ and $h + 1$ as follows:

$$r_\pi(j, t, h + 1) = a(j)r_\pi(j, t, h) \quad h = 1, \ldots, H. \quad (2)$$

Thus $a(j)$ measures the state persistence for forecaster $j$, which we will see later is very closely linked to forecaster $j$’s perceptions of inflation persistence.

### 3.2 State-space model

The state-space model described in this section is one example of a model that can help identify perceptions of inflation persistence; there may be other suitable models that are useful in this regard. The advantage here is that the model will provide a means to identify perceptions with minimal assumptions on the models used by forecasters. This measure is comparable to another measure of persistence used in the literature, namely, predictability, which is discussed in more detail later in this section and in Appendix C.

To represent forecasts, I attribute to each forecaster $j$ a state-space time-series model
for inflation:

\[ \pi(t) = x(j,t) + \varepsilon(j,t) \]  \hspace{1cm} (3)

\[ x(j,t) = a(j)x(j,t-1) + \eta(j,t), \]  \hspace{1cm} (4)

where \( \varepsilon(j,t) \) and \( \eta(j,t) \) are serially uncorrelated error terms with \( \varepsilon(j,t) \sim (0, \sigma_{\varepsilon(j)}^2) \) and \( \eta(j,t) \sim (0, \sigma_{\eta(j)}^2) \).

One interpretation of this model is that forecaster \( j \) decomposes inflation into persistent and transitory components, where \( \varepsilon(j,t) \) and \( \eta(j,t) \) are transitory and persistent shocks to inflation, respectively. As pointed out by Gourinchas and Tornell (2004), these shocks may capture agents’ uncertainty regarding the conduct of monetary policy since the central bank’s inflation target and information set are imperfectly known to forecasters.

The state variable \( x(j,t) \) can be interpreted as a scalar index of the variables that forecaster \( j \) uses when calculating inflation expectations. The heterogeneity in this model arises through \( x(j,t) \), which varies across forecasters and is unobserved (except to forecaster \( j \)). I do not place any assumptions on the identity of \( x(j,t) \); I require only that it evolve according to (4). First, \( x(j,t) \) may, for example, be \( \pi(t) \) itself. In this case, the state-space model would become an AR(1) representation for inflation, allowing forecasters to have their own views of inflation persistence. This framework includes several interesting special cases and is explored further in Jain (2010). However, my more general state-space representation allows for heterogeneity in both information sets and in state persistence.

Second, \( x(j,t) \) may be core inflation, the persistent part of inflation. Core inflation is often used to forecast the CPI inflation rate (i.e., headline inflation). Since core inflation excludes the most volatile components of headline inflation, it is a smoother series that can better capture the trend in headline inflation. Hence, one can think of actual inflation as being core inflation plus noise, as in (3).

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1 The set-up here is a forecaster-specific, stationary version of the unobserved component model in Stock and Watson (2007), without a drift. It is useful to note that the inclusion of a forecaster-specific constant term in the state variable in (4) does not alter the calculation of state persistence. In both cases, the constants cancel out during the calculation.
Third, if \( a(j) = 1 \) for all \( j \), and \( x(j, t) \) is not directly observable by forecasters, then this state-space model would be the unobserved components model of Stock and Watson (2007). They model inflation as the sum of a stochastic trend (which follows a random walk) and a serially uncorrelated disturbance. Their paper shows how to forecast inflation out-of-sample by filtering the unobservable stochastic trend. However, if all forecasters had the same unobserved components model, then all forecasters would have the same inflation forecasts, but, in general, inflation forecasts are not the same across forecasters. The state-space model presented here allows for forecaster heterogeneity in two ways: through \( a(j) \) and through \( x(j, t) \). In the case that \( a(j) = 1 \) for all forecasters, a different \( x(j, t) \) observed by the forecaster would lead to a different forecast.

Inflation is modelled in a similar fashion as in Stock and Watson (2007) – having a persistent component and a transitory component – but forecasters’ estimates of state persistence will be captured by \( a(j) \) in this model, which can then be used to measure forecasters’ perceptions of inflation persistence. The question then becomes: how is it possible to obtain \( a(j) \) without making any additional assumptions on the unobservable state variable \( x(j, t) \)? I start by writing out the one-step-ahead forecast revision made to inflation using the state-space model and the notation described earlier in this section:

\[
\begin{align*}
    r_x(j, t, 1) &= E_{jt} \pi(t + 1) - E_{jt-1} \pi(t + 1) \\
    &= E_{jt} x(j, t + 1) - E_{jt-1} x(j, t + 1) \\
    &= a(j) x(j, t) - a(j) E_{jt-1} x(j, t) \\
    &= a(j) \eta(j, t). \quad (5)
\end{align*}
\]

It is not possible to estimate (5), since \( \eta(j, t) \) is unknown, given that \( x(j, t) \) itself is unob-
servable. However, consider the two-step-ahead forecast revision made to inflation:

\[
\begin{align*}
    r_x(j, t, 2) &= E_{jt}\pi(t + 2) - E_{jt-1}\pi(t + 2) \\
    &= E_{jt}x(j, t + 2) - E_{jt-1}x(j, t + 2) \\
    &= a(j)E_{jt}x(j, t + 1) - a(j)E_{jt-1}x(j, t + 1) \\
    &= a(j)r_x(j, t, 1),
\end{align*}
\]

where (6) follows using the result found in (5). Hence, using only inflation forecasts, we can identify forecaster \(j\)'s state persistence without any additional assumptions on \(x(j, t)\). This result holds for any \(h\)-step-ahead forecast, as depicted in (2).

We can also consider the case where the unobserved state variable, \(x(j, t)\), follows a higher-order AR\((p)\) process. Appendix A derives the corresponding relationship between revisions in the case that \(x(j, t)\) follows an AR\((2)\) or AR\((3)\) process. I then test the coefficients on the additional lags of \(x(j, t)\), and find that, for over 75% of forecasters in the sample, the coefficients on these additional lags are not significantly different from zero. Therefore, throughout the paper I continue with the case in which \(x(j, t)\) follows an AR\((1)\) process.

Forecast revisions are an intuitive but indirect way to identify \(a(j)\).\(^2\) It is possible to estimate \(a(j)\) using a more direct approach, but the estimation in this case becomes more difficult if a constant term is present in the transition equation of the state-space model, while the forecast revision method remains unchanged. Appendix B further explores a direct

\(^2\)The link between forecast revisions and forecasters’ perceptions of shocks has been used in recent work. Krane (2011), for example, uses professional forecasters’ GDP forecast revisions from the Blue Chip economic consensus to identify forecasters’ views of shocks to GDP. Krane discusses the merits of using forecast revisions, and in particular, the correlations in forecast revisions to understand forecasters’ views in response to shocks, as compared to the conventional use of forecast errors. The analysis presented here, however, differs from Krane’s (2011) in several ways. In terms of the data, Krane focuses on consensus forecasts, which does not allow for heterogeneity across forecasters to be captured. In this paper, a key focus is on understanding the heterogeneity of perceptions across forecasters, as well as how these perceptions have changed over time. In terms of the statistical model, Krane proposes a specific parameterization of forecast revisions to identify perceptions of permanent and transitory shocks using method-of-moments estimation. As shown above, a parameterization will not be necessary in this paper, since perceptions of persistence become identified more naturally following the forecaster-specific state-space representation for inflation. I then use SUR estimation to allow for potential efficiency gains due to contemporaneous correlation across equations.
method of estimating $a(j)$ in this case.

In this paper, state persistence is measured in a unique way. It would be an informative exercise to link $a(j)$ to existing measures of persistence in the literature. Cogley, Primiceri and Sargent (2010) measure persistence in terms of predictability using an $R^2$ statistic for $h$-step-ahead forecasts. Essentially, predictability and persistence are related concepts, since past shocks may cause forecastable movements while future shocks contribute to forecast errors. Thus, the influence of past shocks can be measured by the proportion of predictable variance. Appendix C(I) derives an analogous measure of perceived inflation predictability and shows that state persistence and predictability are related, as follows:

$$R^2(j, h) = 1 - \frac{\sigma^2_\eta(j)(1-a(j)^2h) + \sigma^2_\varepsilon(j)(1-a(j)^2)}{\sigma^2_\eta(j) + \sigma^2_\varepsilon(j)(1-a(j)^2)}. \tag{7}$$

Notice that $R^2(j, h)$ will converge to zero as the forecast horizon increases. For a non-persistent state variable (i.e., an $a(j)$ close to 0), predictability is also very low, since the state variable contains mostly noise and therefore forecasts for shorter horizons are just as accurate as forecasts for longer horizons. Conversely, for a highly persistent $x(j, t)$ (i.e., an $a(j)$ close to 1), predictability is also close to 1 (provided $\sigma^2_\varepsilon(j)$ is not high), and steadily declines as the forecast horizon increases.

Appendix C(I) also shows that $\frac{\partial R^2(j, h)}{\partial a(j)} > 0$, and Figure C3 in Appendix C(II) shows that this relationship also holds empirically. This result reinforces the importance of studying $a(j)$, since it affects predictability. It is also possible to show that for $a(j) \in (0, 1)$, $\frac{\partial R^2(j, h)}{\partial \sigma^2_\eta(j)} > 0$ and $\frac{\partial R^2(j, h)}{\partial \sigma^2_\varepsilon(j)} < 0$. The former result is consistent with the findings of Stock and Watson (2007). They show that if $a(j) = 1$ and $x(j, t)$ is viewed as an unobserved component, then the moving-average component of the IMA(1,1) model of inflation depends on the ratio of the permanent innovation variance ($\sigma^2_\eta(j)$) to the transitory innovation variance ($\sigma^2_\varepsilon(j)$). Over time, they find that the permanent innovation variance fell relative to the transitory

\footnote{A related measure of predictability is proposed in Diebold and Kilian (2001). They use actual data for several U.S. macroeconomic time series (not including inflation), and mention that it would be of interest for future work to examine survey-based predictability estimates.}
innovation variance, so inflation persistence declined. The $R^2$ predictability measure in this state-space model also includes this effect: if $\sigma^2_{\eta(j)}$ falls, $R^2(j,h)$ falls.

4 Expectations Data

The expectations data come from the *Survey of Professional Forecasters* (SPF), the oldest quarterly survey of macroeconomic forecasts in the United States. The survey began in 1968 and was conducted by the American Statistical Association and the National Bureau of Economic Research. The Federal Reserve Bank of Philadelphia assumed responsibility for the survey in June 1990. A survey participant’s affiliation is kept confidential, but the individual responses are coded with an identification number for each forecaster that is used to track their forecasts. The inflation forecasts used in this paper are quarterly forecasts for the CPI inflation rate, seasonally adjusted, at annual rates, in percentage points. Quarterly forecasts are annualized quarter-over-quarter percent changes. The particular quarterly forecasts series used are one-quarter-ahead through five-quarter-ahead forecasts of CPI inflation (“CPI2” to “CPI6”). The data run from 1984Q1 to 2010Q1. The sample contains observations from forecasters who have submitted enough forecasts to generate at least 10 revisions. This yields a panel of 80 forecasters.

The advantage of using CPI inflation is that the monthly U.S. CPI releases by the Bureau of Labor Statistics are not subject to revisions. On the other hand, GDP deflator data are usually subject to systematic revisions, which could result in forecasts that differ systematically from announced values solely due to data revisions. To avoid this issue, all the results reported in this paper use CPI inflation.

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429 forecasters are financial service providers, 31 are non-financial service providers, 7 switched between the financial and non-financial industry over the sample period, the industry of 6 forecasters is unknown, and the industry of 7 forecasters is not reported. The SPF began the industry classification in 1990Q2, hence the forecasters whose industry is not reported are usually participants early in the sample, before the classification was started.
5 Estimation Results

5.1 Seemingly unrelated regression (SUR) estimation

For each forecaster $j$, $a(j)$ can be interpreted as the slope of equation (2). I proceed by estimating the following equations jointly using SUR estimation:

\begin{align*}
    r_\pi(j, t, 2) &= v(j, 2) + a(j)r_\pi(j, t, 1) + \nu(j, t, 2) \\
    r_\pi(j, t, 3) &= v(j, 3) + a(j)r_\pi(j, t, 2) + \nu(j, t, 3) \\
    r_\pi(j, t, 4) &= v(j, 4) + a(j)r_\pi(j, t, 3) + \nu(j, t, 4),
\end{align*}

where $v$ is the intercept and $\nu$ is the error term.\(^5\) The set-up follows, since the SPF provides forecasts up to five quarters ahead, allowing state persistence to be estimated using three equations.\(^6,7\) Estimating these equations jointly using a SUR estimator enables me to impose cross-equation restrictions.\(^8\)

Ideally, estimation would be performed with a panel of fixed composition, with most forecasters participating and relatively few missing observations. But since many observations are missing, it is not possible to find overlapping time periods for a large number of forecasters. In addition, calculating a forecast revision requires at least two consecutive forecasts. I proceed to the estimation keeping in mind this limitation of the data set.

Figure 1 shows the estimates of state persistence for each of the 80 forecasters, along with the 95% confidence interval given by the height of the bar associated with the point

\(^5\)Forecasters’ loss functions are not explicitly modelled in this paper, since they are generally unknown to the researcher. However, in order for the estimation of $a(j)$ to be performed in this manner, we require the underlying assumption of similar, symmetric loss functions that have a certainty equivalence feature.

\(^6\)Estimation is limited to three equations, since forecast revisions require an additional horizon (e.g., when $h = 4$, the forecast revision is $r_\pi(j, t, 4) = E_{j(t+4)}\pi(t+4) - E_{j(t)}\pi(t+4)$).

\(^7\)Allowing for a forecaster- and horizon-specific intercept allows us to incorporate the case in which shocks are learned but do not affect forecasts until some future date.

\(^8\)It remains a challenge to estimate a system of equations that is ideal for all 80 forecasters in the sample. I find that over half of the sample of forecasters do not reject the cross-equation restrictions at the 1% significance level. Since this suggests that the majority of forecasters do in fact use a similar model for these horizons, I continue estimation as described above.
estimate. Notice that, first, very few forecasters have point estimates near 1, which would indicate a unit root in state persistence. Second, of the 80 forecasters, 13 have estimates that are not statistically different from zero at the 5% significance level; the majority of these forecasters can be seen in Figure 1 as having slightly negative estimates of \( a(j) \) or estimates of \( a(j) \) very close to zero. Third, there is substantial variation in the precision of the estimates across forecasters; some of this is due to the inclusion of forecasters with smaller sample sizes. The box-whiskers plot in Figure 2 shows that \( \hat{a}(j) \) is primarily clustered around relatively low levels of state persistence, with the third quartile of estimates of \( a(j) \) below 0.5. A cross-plot of accuracy (RMSE) and \( \hat{a}(j) \) is shown in Figure 3. Each point on the cross-plot represents a forecaster in the sample, and there does not appear to be any clear indication that forecasters with state persistence near the consensus make more accurate forecasts than those who perceive \( x(j,t) \) to be more highly persistent.

**Figure 1: Estimates of \( a(j) \) and 95% Confidence Interval**

Note: The solid dots indicate the SUR estimates of \( a(j) \) from (8)-(10) for each of the 80 forecasters. The height of the bar provides the corresponding 95% confidence interval.
Figure 2: Estimates of State Persistence

Note: The box portion of the box-whiskers plot depicts the 25th percentile of estimates on the lower hinge and the 75th percentile of estimates on the upper hinge. The line inside the box is the median estimate. The bottom and top whiskers portray the lower and upper adjacent values, respectively. The dots represent outside values.

Figure 3: Accuracy and State Persistence

Note: The RMSE here is the average RMSE for each forecaster based on their one-quarter-ahead inflation forecasts.
5.2 Changes in state persistence over time

5.2.1 Split sample estimation

It is possible that state persistence has changed over time. Along with the debate surrounding the degree of actual inflation persistence, its stability over time remains controversial. Studies by Levin and Piger (2004) and Stock and Watson (2007) have investigated structural breaks in the inflation process.

Ideally, a long span of survey participation would allow us to test for breaks in state persistence with an unknown breakpoint and for individual forecasters, but, given the gaps in the data set, that is not viable in our case. One way to account for possible changes in state persistence over time is to perform split sample estimation, splitting the sample roughly in half. The midway point in our sample would be 1996Q4. However, a group of new forecasters entered in 1995Q2 and, to avoid splitting their observations after only six quarters, I pick a split date of 1995Q2. Splitting the sample in this way yields 40 forecasters in the first portion of the sample (pre-1995Q2) and 53 forecasters in the second (post-1995Q2). Figure 4a shows a smoothed histogram of state persistence point estimates across forecasters, where the dashed kernel density represents the estimates in the pre-1995Q2 sample and the solid kernel density represents the estimates in the post-1995Q2 sample. The estimation results are consistent with those in the literature, which suggests that there has been a decline in inflation persistence over time. Prior to the mid-1990s, there appeared to be two types of forecasters: those with high state persistence and those with lower state persistence. That is, forecasters likely responded to recent macroeconomic news differently and had different views on the credibility of the central bank. By the mid-1990s, the group of ‘sceptical’ forecasters had decreased. Of course, there is a different composition of forecasters in both portions of the sample, but the more recent consensus is clearly one of low state persistence. Given papers that have documented disagreement in inflation expectations across forecasters (e.g., Capistran and Timmermann (2009); Mankiw, Reis and Wolfers (2003)), this finding
can be interpreted as less disagreement across forecasters since the mid-1990s.\footnote{Another advantage of choosing this split date is that it coincides with Fuhrer (2009), who extends Pivetta and Reis’s (2007) rolling sample estimates of the first-order autocorrelation coefficient to 2008Q4; Fuhrer finds that the estimates experience a drop in the mid-1990s.}

**Figure 4: Split-Sample Estimates of State Persistence: Split Date 1995Q2**

- **a) All Forecasters**
- **b) Forecasters in Both Samples**

Note: The kernel densities show the distribution of state persistence estimates across forecasters over each portion of the sample using the Epanechnikov kernel.

Engelberg, Manski and Williams (2011) warn against drawing conclusions from a forecaster panel of varying composition. Only 13 forecasters from the full sample of 80 forecasters submitted enough surveys to have estimates available in both portions of the sample. Figure 4b shows the smoothed histogram of estimates for this subsample; the estimates for them are similar to those in Figure 4a. Only one forecaster has state persistence above 0.45 in the post-1995Q2 sample estimates, versus seven forecasters who had state persistence above 0.45 in the pre-1995Q2 sample. More formally, a test of the null hypothesis that the estimate was the same before and after 1995Q2 was rejected for 6 of the 13 forecasters at the 10% significance level.

Hence, the findings are not driven by a different composition of forecasters in the earlier and more recent portions of the sample; state persistence does appear to have fallen.
findings are consistent with many studies that have reported that actual inflation persistence has decreased over time (for examples, see Cogley and Sargent (2001); Taylor (2000); Stock and Watson (2007)), though a more in-depth comparison between perceived inflation persistence and actual inflation persistence is provided in section 5.3.

### 5.2.2 Rolling regression estimation

We have seen evidence that state persistence has changed from the first portion of the sample to the second. It is possible that it could be time-varying even within the two sample portions. To examine this possibility further, I perform rolling regressions over a window of 12 periods to estimate the average $\hat{a}$ across the full sample of forecasters. Figure 5 shows the estimation results, along with the 95% confidence intervals.

**Figure 5: Rolling Regression Estimates**

![Figure 5: Rolling Regression Estimates](image)

Note: Rolling regression estimates of $\hat{a}$ are indicated by the solid line. The dashed lines depict the 95% confidence interval of the point estimates.

The rolling estimates convey similar findings as in split-sample estimation: there is a general downward trend in $\hat{a}$ starting in the mid-1990s. In the mid-2000s, there is a brief period when $\hat{a}$ increases slightly, which occurs when the rolling sample period incorporates 2001Q3, likely associated with forecasters expecting shocks to have a longer duration following the
September 2001 terrorist attacks. The estimates then fall to very low levels for a brief period, increasing slightly toward the end of the sample period. Overall, the estimates of $\hat{a}$ are lower after the mid-1990s than in the first portion of the sample.

5.3 A comparison with actual inflation data

The measure of state persistence discussed thus far is obtained indirectly using forecast revisions. To form a concrete means of comparison with actual inflation data, we can obtain a measure of perceived inflation persistence by calculating the implied autocorrelation function (ACF) using the state persistence estimates and comparing it with the ACF of actual inflation. Fuhrer (2009) advocates use of the ACF to measure persistence, since it summarizes much of the information in the time series. It is defined as the vector of correlations of current period inflation with each of its own lags from $i = 0, 1, 2, \ldots, I$: $[\rho_1, \ldots, \rho_I]$. Appendix D uses the Yule-Walker equations to calculate the following implied autocorrelations for each forecaster $j$ in the state-space model:

$$\rho_1(j) = \frac{\gamma_1}{\gamma_0} = \frac{\sigma_{\eta(j)}^2 a(j)^3}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 [1 - a(j)^2]}$$  \hspace{1cm} (11)

$$\rho_2(j) = \frac{\gamma_2}{\gamma_0} = \frac{\sigma_{\eta(j)}^2 a(j)^4}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 [1 - a(j)^2]}$$  \hspace{1cm} (12)

$$\vdots$$

$$\rho_i(j) = \frac{\gamma_i}{\gamma_0} = \frac{\sigma_{\eta(j)}^2 a(j)^{2+i}}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 [1 - a(j)^2]}$$  \hspace{1cm} (13)

where $\rho_i(j)$ denotes the implied autocorrelation for forecaster $j$ at lag $i$. In general, we can show that

$$\rho_i(j) = a(j)\rho_{i-1}(j) \quad \forall \ i \geq 2.$$  \hspace{1cm} (14)

At any lag, these autocorrelations depend on $a(j)$ and both shock variances $\sigma_{\eta(j)}^2$ and $\sigma_{\varepsilon(j)}^2$. If we can obtain sample estimates of $\sigma_{\varepsilon(j)}^2$ and $\sigma_{\eta(j)}^2$, we can identify our measure of perceived
inflation persistence. Taking expectations from the state-space model yields

\[ E_{jt}(t + 1) = E_{jt}x(j, t + 1) = a(j)x(j, t), \]

and using the sample estimates of \( \hat{a}(j) \), we can back out an estimate of the state variable:

\[ \hat{x}(j, t) = \frac{E_{jt}(t + 1)}{\hat{a}(j)}. \]  \hspace{1cm} (15)

Then, from the measurement equation in the state-space model, we have

\[ \hat{\epsilon}(j, t) = \pi(t) - \hat{x}(j, t) = \pi(t) - \frac{E_{jt}(t + 1)}{\hat{a}(j)}, \]  \hspace{1cm} (16)

and therefore,

\[ \hat{\sigma}^2_{\epsilon(j)} = \frac{1}{n - 1} \sum_{t=1}^{n} \hat{\epsilon}(j, t)^2, \quad j = 1, \ldots, J. \]  \hspace{1cm} (17)

Similarly, rearranging the transition equation in terms of sample estimates gives

\[ \hat{\eta}(j, t) = \hat{x}(j, t) - \hat{a}(j)\hat{x}(j, t - 1) = \frac{E_{jt}(t + 1)}{\hat{a}(j)} - E_{jt-1}\pi(t). \]  \hspace{1cm} (18)

Thus, computing \( \hat{\eta}(j, t) \) requires that forecasts be submitted in at least two consecutive periods. We can then calculate the corresponding variance estimate:

\[ \hat{\sigma}^2_{\eta(j)} = \frac{1}{n - 1} \sum_{t=1}^{n} \hat{\eta}(j, t)^2, \quad j = 1, \ldots, J. \]  \hspace{1cm} (19)

Figure 6 shows the perceived inflation persistence estimates. Each connected line represents a different forecaster in the sample, while the successive dots represent lag \( i \). For the majority of forecasters, the implied autocorrelations start with \( \rho_1(j) < 0.2. \)\(^{11, 12} \) This is

\(^{10}\)Please see Appendix C(II) for further details related to the sample estimates of the shock variances.

\(^{11}\)State persistence is typically larger than the perceived inflation persistence, primarily as a result of a large \( \sigma^2_{\epsilon(j)} \) for the vast majority of forecasters.

\(^{12}\)It could be argued that the use of the short-term, current period, forecast revision, \( r(j, t, 1) \), in estimation...
smaller than the first lag in the actual inflation ACF, shown in Figure 7. The ACF for actual inflation suggests the strong presence of first-order autocorrelation.

**Figure 6: Implied ACF for Inflation**

Note: Each line represents the implied ACF of a different forecaster \( j \), where \( j = 1, \ldots, 80 \).

**Figure 7: ACF for Actual Inflation**

Note: Data for actual inflation come from Federal Reserve Economic Data (FRED). To be consistent with the SPF data, monthly CPI data (seasonally adjusted) running from 1984 to 2010 are averaged to quarterly data and then converted to an annualized quarterly rate. The box represents the 95% confidence bands.

Previously, we found that there has likely been a change in state persistence over the sample period. It is therefore likely that the implied ACFs have also experienced some change could be the cause of a lower \( \rho_1(j) \). To check this, I redo estimation of \( a(j) \), omitting \( r(j, t, 1) \) (so the estimation involves (9) – (10) only and sample variances are recalculated). In this case, estimates of \( \rho_1(j) \) are slightly higher, but the conclusion of Figure 6 remains unaltered: the large majority of forecasters still have implied autocorrelations that start with \( \rho_1(j) < 0.2 \).
over time in a similar direction, since the ratio of the variances does not appear to show substantial movement over the sample period (see Appendix C). Consider the ACF of actual inflation before and after the 1995Q2 split date used earlier, shown in both panels of Figure 8. There appears to be a clear fall in the autocorrelations at all four lags. In fact, before 1995Q2, the portmanteau $Q$ test indicates that we can reject the null hypothesis that the actual inflation series is white noise at standard significance levels; however, after 1995Q2, this null cannot be rejected.

**Figure 8: ACF of Actual Inflation**

a) Pre-1995Q2  
b) Post-1995Q2

![ACF of Actual Inflation](image)

Note: Refer to the notes in Figure 7 regarding data. Panel a comprises data running from 1984Q1 to 1995Q1. Panel b uses data from 1995Q2 to 2010Q1. The box represents 95% confidence bands.

Perceived inflation persistence before and after 1995Q2 is shown in both panels of Figure 9 and also suggests a decrease in the implied correlations, since an even larger majority of forecasters’ $\rho_1(j)$ is below 0.2, post-1995Q2. The fall in the autocorrelation, however, is not nearly as large as that experienced by the actual inflation data. A closer look indicates that the fall in actual inflation autocorrelations was primarily due to the 2008 financial crisis, in which CPI inflation drastically dropped to negative levels, reaching -9.2% in 2008Q4 and -2.2% in 2009Q1. Removing both these quarterly observations yields the ACF for inflation in

---

13There are 40 forecasters in the pre-1995Q2 portion of the sample, where the explosive ACF of one forecaster has been removed to allow for easier comparison between both panels. There are 53 forecasters in the post-1995Q2 portion of the sample.
the post-1995Q2 portion of the data (Figure 10), which indicates the presence of third-order autocorrelation in actual inflation.

Figure 9: Perceived Inflation Persistence: Split Sample

a) Pre-1995Q2

b) Post-1995Q2

Note: Panel a shows the ACFs of the 40 forecasters in the pre-1995Q2 portion of the sample. Panel b shows the ACFs of the 53 forecasters in the post-1995Q2 portion of the sample.

Figure 10: ACF of Actual Inflation

Post-1995Q2 (2008Q4 and 2009Q1 omitted)

Note: The data source for Figure 10 is the same as for Figure 8b, except that the 2008Q4 and 2009Q1 data points have been removed. The box represents 95% confidence bands.

It is interesting to note that perceived inflation persistence estimates in the post-1995Q2 portion of the sample (see Figure 9b) also include the financial crisis period, though they
do not cause the autocorrelations to reach levels as negative as in the actual inflation data (in fact, the lowest autocorrelation reported in the sample is -0.0079). Figure 11 shows only those forecasters who participated in the SPF during 2008Q4 and/or 2009Q1. Panel a shows that the perceived inflation persistence estimates for these 19 forecasters before 2008Q4 and 2009Q1 have been excluded. Panel b excludes these dates; notice that two forecasters experience a drop in perceived inflation persistence as a result of the exclusion (contrary to what is experienced in the actual inflation data). For the majority of the remaining forecasters, there does not appear to be much change in perceived inflation persistence estimates. In other words, even after observing the financial crisis shock, forecasters did not revise their forecasts enough at future horizons to cause their ACF to drop as the autocorrelations in the actual inflation data (see Figure 8b). This finding may support the notion of increased central bank credibility, since the shock was not expected to have a long duration, even amidst a financial crisis.

**Figure 11: Perceived Inflation Persistence: Post-1995Q2**

<table>
<thead>
<tr>
<th>Panel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>2008Q4 and 2009Q1 included</td>
</tr>
<tr>
<td>b)</td>
<td>2008Q4 and 2009Q1 omitted</td>
</tr>
</tbody>
</table>

Note: This figure shows the 19 forecasters who participated in the survey during the 2008 financial crisis period. Panel a shows these forecasters’ ACFs for the full post-1995Q2 period, and panel b calculates forecasters’ ACFs with the 2008Q4 and 2009Q1 forecasts removed from the sample.

To avoid any conclusions based on a changing composition of forecasters, we can examine the autocorrelations of only forecasters who were present in both portions of the sample. This
is shown in panels a and b of Figure 12 and includes 24 forecasters.\footnote{There are more forecasters in Figure 12 than in Figure 4b since I do not place the restriction of a minimum of 10 forecast revisions. Instead, Figure 12 allows for forecasters with a minimum of five forecast revisions in both portions of the sample, and this does not alter any previous conclusions.}

**Figure 12: Implied ACF of Inflation (Forecasters in Both Samples)**

a) Pre-1995Q2  

b) Post-1995Q2

Note: This figure shows the ACFs of the 24 forecasters who participated in both the pre-1995Q2 (panel a) and post-1995Q2 (panel b) portions of the sample.

Again, we can see that these autocorrelations have fallen over time, and unlike the actual inflation data, this decline is not largely driven by the financial crisis period. Of the 24 forecasters, nine are affected by this time period, and since most of these forecasters did not make large downward revisions to their forecasts at multiple future horizons, removing the 2008Q4 and 2009Q1 observations does not substantially alter the results; perceived inflation persistence simply did not react as much as actual inflation persistence.

### 5.4 A test of full-information rational expectations

The findings in the previous section raise an important question: why do forecasters react less to shocks than the actual inflation data? Increased central bank credibility could lead to this, but it is possible that there are also other factors at play. Coibion and Gorodnichenko (2012) (henceforth CG) propose a unique approach to test the full-information rational
expectations (FIRE) hypothesis. This approach can identify whether a rejection of FIRE arises due to informational rigidities. Alternatively, a rejection could occur as a result of a deviation from rational expectations. I employ tests along the lines of CG to gain insight into the underlying reason why forecasters’ perceptions of inflation persistence differ from actual inflation persistence. It is worth noting that although much of this paper has focused on obtaining forecaster-specific results, the tests proposed by CG apply only when averaging across agents, and are therefore not expected to hold at the individual level. Hence, the results reported here will be based on the average of the sample of 80 forecasters studied in this paper.

CG show that models of informational rigidities imply the following relationship between \textit{ex-post} mean forecast errors and \textit{ex-ante} mean forecast revisions:

\begin{equation}
\chi(t+h) - E_t\chi(t+h) = c + \beta[E_t\chi(t+h) - E_{t-1}\chi(t+h)] + error(t),
\end{equation}

where \(\chi\) is a given macroeconomic variable, and \(E_\chi\) is the mean forecast across agents. CG find that when \(\beta > 0\), informational rigidities are present. Models of informational rigidities also predict a constant of zero.

I test the following relationship between average \(h\)-quarter-ahead inflation forecast errors across forecasters and the corresponding average forecast revisions for \(h = 1, 2, 3\):

\begin{equation}
\pi(t+h) - E_t\pi(t+h) = c(h) + \beta(h)[E_t\pi(t+h) - E_{t-1}\pi(t+h)] + error(t). \tag{20}
\end{equation}

The results are reported in Table 1 for each forecast horizon, with the inclusion of two lagged quarterly changes in real oil prices (as in CG). In panel a, at horizon 1, the coefficient on forecast revisions is positive, though statistically significant only at the 15% level. When \(h = 2\), \(\beta_2\) is statistically significant, but in the opposite direction than what would be implied by informational rigidities. Furthermore, in both cases, the first lag of log changes in real oil prices is statistically significant at the 5% level, which should not be the case in the
informational rigidities framework (i.e., once forecast revisions are controlled for, no other variable should have predictive power for forecast errors). The coefficient when $h = 3$ is the correct sign, but not statistically significant. Therefore, it is possible that informational rigidities are present, but they are likely not the main source of differences between forecasters’ perceptions and the actual inflation data.

CG also provide an alternative interpretation that could result in a deviation from FIRE: forecasters could be performing forecast smoothing for reputational considerations. To test this interpretation, CG add a lagged forecast revision on the right-hand side of (20) that could have predictive power. Under forecast smoothing, the sign of the contemporaneous forecast revision coefficient should now be negative if forecasters take into account how their choice of forecast will affect the subsequent period’s adjustment cost. Panel b of Table 1 accounts for a lag of forecast revisions. When $h = 1$, this lag contains some predictive power; it is significant at the same 15% level as the contemporaneous forecast revision. When $h = 2$, the lagged forecast revision does not contain predictive power, but the coefficient on the contemporaneous forecast revisions is negative and significant, which is suggestive of forecast smoothing behaviour. When $h = 3$, neither coefficient is significant.

### Table 1: Tests of FIRE

<table>
<thead>
<tr>
<th>Test</th>
<th>Dependent Variable: Forecast Error</th>
<th>a) Informational Rigidities</th>
<th>b) Forecast Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizon:</td>
<td>$h = 1$</td>
<td>$h = 2$</td>
<td>$h = 3$</td>
</tr>
<tr>
<td>$E_t \pi(t + h) - E_{t-1} \pi(t + h)$</td>
<td>0.31*</td>
<td>-0.79**</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.44)</td>
<td>(0.87)</td>
</tr>
<tr>
<td>$E_{t-1} \pi(t + h) - E_{t-2} \pi(t + h)$</td>
<td>-0.37*</td>
<td>0.26</td>
<td>-0.44</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.63)</td>
<td>(0.54)</td>
</tr>
<tr>
<td>$c(h)$</td>
<td>0.02</td>
<td>-0.14</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.17)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>Observations</td>
<td>104</td>
<td>104</td>
<td>104</td>
</tr>
<tr>
<td>F-stat</td>
<td>15.22</td>
<td>5.06</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Notes: This table reports estimates of equation (20) augmented with two lags of log changes in oil prices (not shown). Panel b also includes a lag of forecast revisions. Newey-West standard errors are shown in parentheses. ***, **, * denote significance at the 5%, 10% and 15% levels.
Overall, these results suggest that forecast smoothing is somewhat more likely to be the source of deviation from FIRE forecasts. Professional forecasters may be required to give a compelling and well-supported explanation whenever a previous forecast is revised, causing them to eliminate or dampen the effects of certain shocks in their forecasts, and therefore lowering estimates of perceived inflation persistence relative to actual inflation persistence. This differs from the results found in CG, in which the source of deviation from FIRE is due to informational rigidities. This could be due to several factors: first, the mean measures used here represent only the 80 forecasters in the SPF that are investigated in this paper, not the full sample available in the SPF that is used by CG. Second, the estimates here are reported separately for each forecast horizon, whereas CG pool estimates across forecast horizons.\footnote{Pooling horizons would not be consistent with the model presented in this paper, since forecast revisions in the one-step-ahead case would be a fraction of the two-step-ahead forecast revisions, which, in turn, is a fraction of the three-step-ahead forecast revision. Hence, we would not expect $\beta(h)$ to be the same across forecast horizons.} Third, CG’s results are based on GDP/GNP deflator inflation forecasts dating back to 1968Q4, whereas, here, CPI inflation forecasts dating back to 1984Q1 are used.

6 Conclusions

This paper proposes a new methodology to extract measures of persistence from expectations data. Empirically, this paper shows that it is possible to estimate a measure of perceived inflation persistence while allowing for heterogeneity across forecasters and minimal assumptions on the models they use. The measure of perceived inflation persistence introduced in this paper provides important insights into expectations data, and, when used in conjunction with persistence measures of actual data, can be useful in the arsenal of both researchers and policy-makers.

Perceived inflation persistence has declined over time, with greater consensus around low levels of persistence when split sample estimation is performed. These findings are consistent with those of Stock and Watson (2007), who suggest that unexpected changes
to inflation are more likely to be viewed as transitory than they had been before the early 1990s.

From the perspective of a central bank, perceptions of inflation persistence have implications for monetary policy. The relatively lower persistence estimates obtained through inflation-expectations data appear to suggest that the Federal Reserve has gained credibility in recent years, giving them greater flexibility, and lowering the costs associated with stabilizing inflation. There may, however, be other factors driving inflation expectations, causing their underlying properties to differ from those of actual inflation, such as forecasters’ incentives to smooth their announced forecasts, leading them to differ from true values. Future work will aim to examine the testable implications of this perspective.

A natural next step for future research would also entail developing a better understanding of the perceptions of other agents in the economy, such as businesses and the public. Since the methodology proposed in this paper is applicable in different contexts, given a rich enough data set, it can be used to obtain measures of perceived inflation persistence that are comparable across agents. Subsequently, a detailed analysis of the implications of these results for monetary policy would be a key avenue to explore.
References


Appendix A

The state-space model presented in the paper derives the revision regressions in the case that \( x(j, t) \) follows an AR(1) process. Here, I will discuss how the revision regressions change when I allow \( x(j, t) \) to follow an AR(2) or AR(3) process.

I. AR(2) Case

Suppose that the state-space time series model for inflation were as follows:

\[
\begin{align*}
p(t) &= x(j, t) + \varepsilon(j, t) \quad (21) \\
x(j, t) &= a_1(j)x(j, t-1) + a_2(j)x(j, t-2) + \eta(j, t), \quad (22)
\end{align*}
\]

where state persistence would be measured by \( a_1(j) + a_2(j) \). The one-step-ahead forecast revision can then be calculated as follows:

\[
r_{\pi}(j, t, 1) = E_{jt}\pi(t + 1) - E_{jt-1}\pi(t + 1)
= E_{jt}[x(j, t + 1) + \varepsilon(j, t + 1)] - E_{jt-1}[x(j, t + 1) + \varepsilon(j, t + 1)]
= E_{jt}[a_1(j)x(j, t) + a_2(j)x(j, t - 1) + \eta(j, t + 1)]
- E_{jt-1}[a_1(j)x(j, t) + a_2(j)x(j, t - 1) + \eta(j, t + 1)]
= a_1(j)[x(j, t) - E_{jt-1}x(j, t)] + a_2(j)[x(j, t - 1) - x(j, t - 1)]
= a_1(j)\eta(j, t), \quad (23)
\]

which is the same result as in the AR(1) case. The two-step-ahead revision, however, changes:

\[
r_{\pi}(j, t, 2) = E_{jt}\pi(t + 2) - E_{jt-1}\pi(t + 2)
= E_{jt}[x(j, t + 2) + \varepsilon(j, t + 2)] - E_{jt-1}[x(j, t + 2) + \varepsilon(j, t + 2)]
= E_{jt}[a_1(j)x(j, t + 1) + a_2(j)x(j, t) + \eta(j, t + 2)]
- E_{jt-1}[a_1(j)x(j, t + 1) + a_2(j)x(j, t) + \eta(j, t + 2)]
= a_1(j)[E_{jt}x(j, t + 1) - E_{jt-1}x(j, t + 1)] + a_2(j)[x(j, t) - E_{jt-1}x(j, t)]
= a_1(j)r_{\pi}(j, t, 1) + a_2(j)\eta(j, t), \quad (24)
\]

where we cannot estimate \( a_2(j) \) directly, since \( \eta(j, t) \) is unknown. The three-step- and four-step-ahead forecast revisions are useful in this regard:

\[
r_{\pi}(j, t, 3) = E_{jt}\pi(t + 3) - E_{jt-1}\pi(t + 3)
= E_{jt}[x(j, t + 3) + \varepsilon(j, t + 3)] - E_{jt-1}[x(j, t + 3) + \varepsilon(j, t + 3)]
= E_{jt}[a_1(j)x(j, t + 2) + a_2(j)x(j, t + 1) + \eta(j, t + 3)]
- E_{jt-1}[a_1(j)x(j, t + 2) + a_2(j)x(j, t + 1) + \eta(j, t + 3)]
= a_1(j)[E_{jt}x(j, t + 2) - E_{jt-1}x(j, t + 2)] + a_2(j)[E_{jt}x(j, t + 1) - E_{jt-1}x(j, t + 1)]
= a_1(j)r_{\pi}(j, t, 2) + a_2(j)r_{\pi}(j, t, 1), \quad (25)
\]
\[ r_\pi(j, t, 4) = E_{jt}\pi(t + 4) - E_{jt-1}\pi(t + 4) \]
\[ = E_{jt}[x(j, t + 4) + \varepsilon(j, t + 4)] - E_{jt-1}[x(j, t + 4) + \varepsilon(j, t + 4)] \]
\[ = E_{jt}[a_1(j)x(j, t + 3) + a_2(j)x(j, t + 2) + \eta(j, t + 4)] \]
\[ - E_{jt-1}[a_1(j)x(j, t + 3) + a_2(j)x(j, t + 2) + \eta(j, t + 4)] \]
\[ = a_1(j)[E_{jt}x(j, t + 3) - E_{jt-1}x(j, t + 3)] + a_2(j)[E_{jt}x(j, t + 2) - E_{jt-1}x(j, t + 2)] \]
\[ = a_1(j)r_\pi(j, t, 3) + a_2(j)r_\pi(j, t, 2). \tag{26} \]

It would be possible to obtain \( \hat{a}_2(j) \) by estimating (25) or (26). Estimation results from (25) show that, from our sample of 80 forecasters, 62 forecasters do not reject the null hypothesis that \( a_2(j) = 0 \) at the 5% significance level, versus only 18 forecasters that do not reject the null hypothesis that \( a_1(j) = 0 \) at the same significance level.

II. AR(3) Case
Suppose next that the state-space time-series model for inflation were as follows:
\[ \pi(t) = x(j, t) + \varepsilon(j, t) \tag{27} \]
\[ x(j, t) = a_1(j)x(j, t - 1) + a_2(j)x(j, t - 2) + a_3(j)x(j, t - 3) + \eta(j, t). \tag{28} \]

Using a similar procedure as in the AR(2) case, the three-step-ahead and four-step-ahead revision regressions in this case become:
\[ r_\pi(j, t, 3) = E_{jt}\pi(t + 3) - E_{jt-1}\pi(t + 3) \]
\[ = a_1(j)r_\pi(j, t, 2) + a_2(j)r_\pi(j, t, 1) + a_3(j)\eta(j, t) \]
\[ r_\pi(j, t, 4) = E_{jt}\pi(t + 4) - E_{jt-1}\pi(t + 4) \]
\[ = a_1(j)r_\pi(j, t, 3) + a_2(j)r_\pi(j, t, 2) + a_3(j)r_\pi(j, t, 1), \tag{29} \]

where (29) would allow us to obtain and test \( \hat{a}_2(j) \) and \( \hat{a}_3(j) \). I find that 70 forecasters do not reject the null hypothesis that \( a_3(j) = 0 \), and 65 forecasters do not reject the null hypothesis that \( a_2(j) = 0 \), both at the 5% significance level.
Appendix B
Consider a version of the state-space model that includes a constant \( \mu(j) \) for the state equation:

\[
\begin{align*}
\pi(t) &= x(j, t) + \varepsilon(j, t) \\
x(j, t) &= \mu(j) + a(j)x(j, t - 1) + \eta(j, t).
\end{align*}
\]

One can see from the calculations of revisions in section 3 that the estimation results would not change in this case, since the constants would cancel out during the calculations.

However, when a more direct approach is used to identify \( a(j) \), such as by regressing the \( h \)-step-ahead inflation forecasts on the model-implied predictions, the estimation becomes more complicated. Start by calculating the model-implied predictions:

\[
E_{jt}\pi(t + 1) = E_{jt}x(j, t + 1) + E_{jt}\varepsilon(j, t + 1)
= E_{jt}[\mu(j) + a(j)x(j, t) + \eta(j, t + 1)]
= \mu(j) + a(j)x(j, t)
\]

\[
E_{jt}\pi(t + 2) = E_{jt}x(j, t + 2) + E_{jt}\varepsilon(j, t + 2)
= E_{jt}[\mu(j) + a(j)E_{jt}x(j, t + 1) + \eta(j, t + 2)]
= \mu(j) + a(j)E_{jt}x(j, t + 1)
= \mu(j) + a(j)[1 + a(j)]\mu(j) + a(j)x(j, t) + \eta(j, t + 1)]
= (1 + a(j))\mu(j) + a(j)^2x(j, t)
\]

\[
E_{jt}\pi(t + 3) = E_{jt}x(j, t + 3) + E_{jt}\varepsilon(j, t + 3)
= E_{jt}[\mu(j) + a(j)E_{jt}x(j, t + 2) + \eta(j, t + 3)]
= \mu(j) + a(j)E_{jt}x(j, t + 2)
= \mu(j) + a(j)[1 + a(j)]\mu(j) + a(j)^2x(j, t) + \eta(j, t + 2)]
= (1 + a(j) + a(j)^2)\mu(j) + a(j)^3x(j, t),
\]

and similarly:

\[
E_{jt}\pi(t + 4) = (1 + a(j) + a(j)^2 + a(j)^3)\mu(j) + a(j)^4x(j, t)
E_{jt}\pi(t + 5) = (1 + a(j) + a(j)^2 + a(j)^3 + a(j)^4)\mu(j) + a(j)^5x(j, t),
\]

where \( h = 5 \) to reflect the five horizons for which forecasts are available from the SPF. The SUR system then becomes a non-linear system:

\[
\begin{align*}
E_{jt}\pi(t + 1) &= \mu(j) + a(j)x(j, t) + v(j, t, 1) \\
E_{jt}\pi(t + 2) &= [1 + a(j)]\mu(j) + a(j)^2x(j, t) + v(j, t, 2) \\
E_{jt}\pi(t + 3) &= [1 + a(j)] + a(j)^2]\mu(j) + a(j)^3x(j, t) + v(j, t, 3) \\
E_{jt}\pi(t + 4) &= [1 + a(j) + a(j)^2] + a(j)^3]\mu(j) + a(j)^4x(j, t) + v(j, t, 4) \\
E_{jt}\pi(t + 5) &= [1 + a(j) + a(j)^2 + a(j)^3] + a(j)^4]\mu(j) + a(j)^5x(j, t) + v(j, t, 5).
\end{align*}
\]

35
We would like to pin down a value for \( x(j, t) \) so that it is possible to estimate \( \mu(j) \) and \( a(j) \) in this system. This is not as straightforward as in the case when a constant is not included. One method could be as follows: consider the difference between the two-step-ahead and one-step-ahead forecasts made in the same period, denoted \( d(j, t, 1) \), and the difference between the three-step-ahead and two-step-ahead forecasts made in the same period, denoted \( d(j, t, 2) \):\(^{16}\)

\[
d(j, t, 1) = E_{jt}\pi(t + 2) - E_{jt}\pi(t + 1) = [1 + a(j)]\mu(j) + a(j)^{2}x(j, t) - [\mu(j) + a(j)x(j, t)] = a(j)\mu(j) - a(j)(1 - a(j))x(j, t)
\]

\[
d(j, t, 2) = E_{jt}\pi(t + 3) - E_{jt}\pi(t + 2) = [1 + a(j) + a(j)^{2}]\mu(j) + a(j)^{3}x(j, t) - ([1 + a(j)]\mu(j) + a(j)^{2}x(j, t)) = a(j)^{2}\mu(j) - a(j)^{3}(1 - a(j))x(j, t) = a(j)[a(j)\mu(j) - a(j)(1 - a(j))x(j, t)],
\]

and, then, notice that the ratio gives the following result:

\[
\frac{d(j, t, 2)}{d(j, t, 1)} = a(j). \tag{35}
\]

Next, the remaining model-implied predictions can be used to isolate \( x(j, t) \) and solve for \( \mu(j) \):

\[
x(j, t) = \frac{E_{jt}\pi(t + 4) - (1 + a(j) + a(j)^{2} + a(j)^{3})\mu(j)}{a(j)^{4}}
\]

\[
x(j, t) = \frac{E_{jt}\pi(t + 5) - (1 + a(j) + a(j)^{2} + a(j)^{3} + a(j)^{4})\mu(j)}{a(j)^{5}}.
\]

Setting these two equations equal to one another and cross-multiplying gives

\[
a(j)^{5}[E_{jt}\pi(t + 4) - (1 + a(j) + a(j)^{2} + a(j)^{3})\mu(j)] = a(j)^{4}[E_{jt}\pi(t + 5) - (1 + a(j) + a(j)^{2} + a(j)^{3} + a(j)^{4})\mu(j)],
\]

and, simplifying, gives

\[
\mu(j) = E_{jt}\pi(t + 5) - a(j)E_{jt}\pi(t + 4). \tag{36}
\]

Rearranging the transition equation and substituting the expressions for \( \mu(j) \) and \( a(j) \), it is possible to obtain an expression for \( x(j, t) \) in terms of the observables:

\[
x(j, t) = \frac{E_{jt}\pi(t + 1) - \mu(j)}{a(j)} = \frac{E_{jt}\pi(t + 1) - [E_{jt}\pi(t + 5) - \frac{d(j, t, 2)}{d(j, t, 1)}E_{jt}\pi(t + 4)]}{\frac{d(j, t, 2)}{d(j, t, 1)}}. \tag{37}
\]

\(^{16}\)Note that \( d(j, t, 1) \) and \( d(j, t, 2) \) are not forecast revisions.
In theory, it is possible to pin down a value for \( x(j, t) \); however, using the data to perform this calculation appears to be less feasible. There are many instances where \( x(j, t) \) is not defined: whenever forecasters do not expect \( E_{jt} \pi(t + 1) \) to differ from \( E_{jt} \pi(t + 2) \), or \( E_{jt} \pi(t + 2) \) to differ from \( E_{jt} \pi(t + 3) \) (i.e., \( d(j, t, 1) \) and/or \( d(j, t, 2) \) are zero), that would make this direct approach difficult to use, in practice.

We can also consider the case in which there is a forecaster-specific bias in the measurement equation of the state-space model:

\[
\begin{align*}
\pi(t) &= \mu(j) + x(j, t) + \varepsilon(j, t) \\
x(j, t) &= a(j)x(j, t - 1) + \eta(j, t).
\end{align*}
\]

(38) (39)

Similar calculations show that using revisions, including this bias term, would not affect the estimation results, and the relationship between state persistence and predictability could still be established in the same way.
Appendix C

I. State Persistence and Perceived Predictability

Cogley, Primiceri and Sargent (2010) calculate predictability as 1 minus the ratio of the conditional forecast error variance to the unconditional forecast error variance. The one-step-ahead and two-step-ahead conditional variances in our state-space model are given by

\[
E_{jt}(\pi(t + 1) - E_{jt}\pi(t + 1))^2 = E_{jt}(\eta(j, t + 1) + \varepsilon(j, t + 1))^2
= \sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2
\]

and, in general,

\[
E_{jt}(\pi(t + 1) - E_{jt}\pi(t + 2))^2 = E_{jt}(a(j)\eta(j, t + 1) + \eta(j, t + 2) + \varepsilon(j, t + 2))^2
= \sigma_{\eta(j)}^2(1 + a(j)^2) + \sigma_{\varepsilon(j)}^2.
\]

where \(\text{cov}(\eta(j, t), \varepsilon(j, k)) = 0\) for all \(t, k\). The unconditional variance from our state-space model is given by

\[
\frac{\sigma_{\eta(j)}^2}{1 - a(j)^2} + \sigma_{\varepsilon(j)}^2.
\]

Hence, the one-step- and two-step-ahead predictability measures can be calculated as

\[
R^2(j, 1) = 1 - \frac{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2}{\frac{\sigma_{\eta(j)}^2}{1 - a(j)^2} + \sigma_{\varepsilon(j)}^2}
= 1 - \frac{\sigma_{\eta(j)}^2(1 - a(j)^2) + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)}
\]  

\[
R^2(j, 2) = 1 - \frac{\sigma_{\eta(j)}^2(1 + a(j)^2) + \sigma_{\varepsilon(j)}^2}{\frac{\sigma_{\eta(j)}^2}{1 - a(j)^2} + \sigma_{\varepsilon(j)}^2}
= 1 - \frac{\sigma_{\eta(j)}^2(1 - a(j)^4) + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)},
\]

and, in general,

\[
R^2(j, h) = 1 - \frac{\sigma_{\eta(j)}^2(1 - a(j)^{2h}) + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)}.
\]

Some algebra shows that:

\[
\frac{\partial R^2(j, h)}{\partial a(j)} = \frac{2h\sigma_{a(j)}^4a(j)^{2h-1} + 2h\sigma_{a(j)}^2(1 - a(j)^2)a(j)^{2h-1} + 2\sigma_{a(j)}^2a(j)^{2h+1}}{[\sigma_{a(j)}^2 + \sigma_{\varepsilon(j)}^2(1 - a(j)^2)]^2}.
\]

Since \(h, \sigma_{a(j)}^2\) and \(\sigma_{\varepsilon(j)}^2\) are all positive, it follows that, for \(a(j) \in (0, 1)\), \(\frac{\partial R^2(j, h)}{\partial a(j)} > 0\).17

---

17It is possible to weaken this condition to \(\frac{\partial R^2(j, h)}{\partial a(j)} \geq 0\) to incorporate more values of \(a(j)\), in particular, \(a(j) \in [0, \sqrt{1 + \frac{\sigma_{\varepsilon(j)}^2}{\sigma_{a(j)}^2}}]\). However, this interval would no longer guarantee that \(R^2(j, h) \to 0\) as \(h \to \infty\). Furthermore, if \(a(j) = 0\), it would not be possible to obtain estimates for the other components of \(R^2(j, h)\). Therefore, I will restrict attention to \(a(j) \in (0, 1)\).
II. Measuring Perceived Predictability

The variance of the persistent innovation relative to the variance of the transitory innovation is shown in Figure C1. For most forecasters, the relative variance is fairly close to one. This ratio is interesting to consider, since Stock and Watson (2007) report that the ratio had declined between the first and second portion of their data: between 1970–1983 and 1984–2004, respectively. Specifically, the variance of the persistent innovation had declined in their sample, while the variance of the transitory innovation had remained stable throughout the entire sample period.

![Figure C1: Sample Estimates of Relative Variance](image1)

Using the 1995Q2 split date presented earlier, the relative variance can be calculated for forecasters in both portions of the sample, and it can be determined whether changes have occurred between 1984Q1–1995Q1 and 1995Q2–2010Q1 using forecast data. Of these 24 forecasters, the solid red line in Figure C2 depicts the relative variance between 1984Q1–1995Q1, while the dotted blue line indicates the increase in the relative variance due to the second portion of the sample; 1995Q2–2010Q1. There does not appear to be any

![Figure C2: Split-Sample Estimate of Relative Variance](image2)
conclusive change during the sample period: 13 of the 24 forecasters have a higher relative variance in the second portion of the sample, while 8 forecasters have a higher relative variance in the first portion of the sample. The three remaining forecasters have a relative variance that has remained roughly equal throughout the sample period.

Using the sample estimates of the variances, along with the estimates of state persistence, it is possible to obtain an estimate of perceived predictability in (7). Figure C3 shows the relationship between $\hat{a}(j)$ and $\hat{R}^2(j, h)$ for $h = 1, 2$ and $h = 5$:

**Figure C3: Estimates of State Persistence and Predictability**

The aligned symbols represent the one-step-, two-step- and five-step-ahead estimates of perceived predictability for each forecaster $j$, respectively. There are five forecasters whose estimates of $a(j)$ are slightly below zero, and, therefore, the positive relationship between perceived persistence and predictability would not apply to these forecasters. For the rest of the forecasters, predictability is an increasing function of state persistence, so focusing on $a(j)$ conveys some of the same information as focusing on $R^2(j, h)$.

Furthermore, I showed in the previous section that, holding the shock variances constant, $a(j)$ is positively related to $R^2(j, h)$. Figure C3 shows a stronger result empirically: even if shock variances are unrestricted across forecasters, state persistence and predictability are still positively related.
Appendix D
By writing the state-space model as follows:

\[ \pi(t) = a(j)x(j, t - 1) + \eta(j, t) + \varepsilon(j, t), \] (46)

the autocovariances can be derived by multiplying \( \pi(t) \) by \( \pi(t - i) \) for \( i = 0, 1, 2, \ldots, I \) and then taking expectations. I will denote \( \gamma_0^z \) as the variance of variable \( z \), and \( \gamma_i^z \) as the \( i \)-period apart covariance of variable \( z \). The Yule-Walker equations are then given by

\[
\begin{align*}
E(\pi(t)\pi(t)) &= a(j)E(x(j, t - 1)\pi(t)) + E(\eta(j, t)\pi(t)) + E(\varepsilon(j, t)\pi(t)) \\
\gamma_0^\pi &= a(j)\gamma_0^{x(j)} + \sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 \\
E(\pi(t)\pi(t - 1)) &= a(j)E(x(j, t - 1)\pi(t - 1)) + E(\eta(j, t)\pi(t - 1)) + E(\varepsilon(j, t)\pi(t - 1)) \\
\gamma_1^\pi &= a(j)\gamma_1^{x(j)} \\
E(\pi(t)\pi(t - 2)) &= a(j)E(x(j, t - 1)\pi(t - 2)) + E(\eta(j, t)\pi(t - 2)) + E(\varepsilon(j, t)\pi(t - 2)) \\
\gamma_2^\pi &= a(j)\gamma_2^{x(j)} \\
\vdots \\
\gamma_i^\pi &= a(j)\gamma_i^{x(j)}.
\end{align*}
\]

Since \( x(j, t) \) is just an AR(1) process, we can show that

\[
\begin{align*}
\gamma_0^{x(j)} &= \frac{\sigma_{\eta(j)}^2}{[1 - a(j)^2]} \\
\gamma_1^{x(j)} &= \frac{\sigma_{\eta(j)}^2 a(j)}{[1 - a(j)^2]} \\
\vdots \\
\gamma_i^{x(j)} &= \frac{\sigma_{\eta(j)}^2 a(j)^i}{[1 - a(j)^2]}.
\end{align*}
\]

Using these results, the implied autocorrelations for \( \pi(t) \) can be calculated as follows:

\[
\begin{align*}
\rho_1(j) &= \frac{\gamma_1^\pi}{\gamma_0^\pi} = \frac{\sigma_{\eta(j)}^2 a(j)^3}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 [1 - a(j)^2]} \\
\rho_2(j) &= \frac{\gamma_2^\pi}{\gamma_0^\pi} = \frac{\sigma_{\eta(j)}^2 a(j)^4}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 [1 - a(j)^2]} \\
\vdots \\
\rho_i(j) &= \frac{\gamma_i^\pi}{\gamma_0^\pi} = \frac{\sigma_{\eta(j)}^2 a(j)^{2+i}}{\sigma_{\eta(j)}^2 + \sigma_{\varepsilon(j)}^2 [1 - a(j)^2]}.
\end{align*}
\]