BANK OF CANADA
BANQUE DU CANADA

Working Paper/Document de travail
2013-37

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Bank of Canada Working Paper 2013-37

October 2013

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ISSN 1701-9397

Acknowledgements

We thank Jean-Sébastien Fontaine, Peter Christoffersen, Antonio Diez de los Rios, Jun Yang, Scott Hendry, Rhys Mendes, Antonio Mele, Gaetano Antinolfi and Edit Rroji for valuable comments. We also thank Royce Mendes and Jarred Seider for their research assistance.

Abstract

We measure uncertainty surrounding the central bank's future policy rates using implied volatility computed from interest rate option prices and realized volatility computed from intraday prices of interest rate futures. Both volatility measures show that uncertainty decreased following the most important policy actions taken by the Bank of Canada as a response to the financial crisis of 2007–08, such as the conditional commitment of 2009–10, the unscheduled cut in the target rate coordinated with other major central banks, and the introduction of term purchase and resale agreements. We also find that, on average, uncertainty decreases following the Bank of Canada's policy rate announcements. Furthermore, our measures of policy rate uncertainty improve the estimation of policy rate expectations from overnight index swap (OIS) rates by predicting the risk premium in the OIS market.

JEL classification: E4
Bank classification: Uncertainty and monetary policy; Monetary and financial indicators

Résumé

Dans cette étude, les auteurs mesurent l'incertitude entourant l'évolution future du taux directeur en s'appuyant à la fois sur la volatilité implicite, calculée à partir des prix des options sur taux d'intérêt, et sur la volatilité réalisée, établie en fonction des prix intrajournaliers des contrats à terme sur taux d'intérêt. Ces deux indicateurs de la volatilité mettent en évidence des diminutions de l'incertitude, consécutives aux mesures de politique monétaire les plus importantes que la Banque du Canada a prises pour contrer la crise financière de 2007-2008, notamment l'annonce de son engagement conditionnel pour la période 2009-2010, la réduction du taux cible décidée « hors calendrier » et mise en œuvre en concertation avec d'autres grandes banques centrales, et l'introduction d'un mécanisme de prise en pension à plus d'un jour. Les auteurs notent aussi qu'en règle générale, les annonces de la Banque concernant le taux directeur sont suivies d'une baisse de l'incertitude. En outre, ils constatent que, parce qu'elle est révélatrice de la prime de risque future sur le marché des swaps indexés sur le taux à un jour, leur mesure de l'incertitude entourant le taux directeur améliore l'estimation des attentes à l'égard du taux fondée sur les taux des swaps indexés.

Classification JEL : E4
Classification de la Banque : Incertitude et politique monétaire; Indicateurs monétaires et financiers

1 Introduction

Uncertainty around future monetary policy rates is important because interest rate uncertainty affects the real economy through investment and the hiring decisions of firms. Ingersoll and Ross (1992) show that the effect of interest rate uncertainty is critical to understanding investment at the macroeconomic level. Ferderer and Zalewski (1994) argue that interest rate uncertainty contributed to the severity of the Great Depression in the 1930s. Many studies find that higher uncertainty has a dampening effect on the economy, since uncertainty leads firms to reduce or postpone investment and hiring (Bernanke, 1983; Pindyck, 1991; Dixit, 1992). A recent study by Baker, Bloom and Davis (2013) focuses on uncertainty caused by fiscal, monetary and regulatory policy. They find that an increase in economic policy uncertainty foreshadows declines in investment, output and employment.

The financial crisis of 2007–08 has highlighted the importance of monetary policy, since central banks around the world experimented with unconventional monetary policies to tackle the crisis. In this paper, we use two measures of volatility based on the prices of interest rate futures and options to examine whether major policy actions taken by the Bank of Canada (BoC) following the financial crisis succeeded in reducing uncertainty around its future policy rates. The first measure is *realized volatility* computed from intraday prices of interest rate futures. The second measure is *implied volatility* computed from prices of options on interest rate futures.

We find that major policy actions taken by the BoC as a response to the financial crisis of 2007–08, such as the conditional commitment to keep the policy rate unchanged, the unscheduled cut in the policy rate coordinated with other major central banks, and the introduction of term purchase and resale agreements, all reduced uncertainty. We also find that, on average, uncertainty decreases following the BoC's policy rate announcements. In particular, realized volatility decreased substantially during the BoC's conditional commitment period between April 2009 and April 2010, and increased after the removal of the commitment, indicating that uncertainty was greatly reduced due to the commitment.

Our results are consistent with those found in Bauer (2012) for the United States. Bauer (2012) finds that the Quantitative Easing (QE) program announcements and forward-looking statements by the FOMC are followed by a drop in implied volatility, indicating that these policy actions succeeded in lowering uncertainty around future interest rates. Among the statements made by the Federal Reserve Board (the Fed) announcing various QEs and forward guidance during this period, the largest drop in implied volatility is found to have occurred after the announcement of its conditional commitment in August 2011.

The BoC's conditional commitment provides an excellent natural experiment, because no other major policy action was taken by the BoC or the Government of Canada during the BoC's conditional commitment period, and also because the policy was in effect for a specific time period. Unlike in the United States, where the end date of the commitment was changed a few times and the commitment has yet to be removed, the BoC removed its commitment after one year, close to the end date that was initially indicated. A similar analysis of change in realized volatility due to a specific policy action may be difficult in the United States because many policies were put into place in conjunction with each other.

For the conditional commitment, in particular, our study extends the analyses in Bauer (2012) by looking at not only changes in implied volatility on the announcement days, but also realized volatility before, after and throughout the conditional commitment period. Realized volatility allows us to observe what happened to interest rate uncertainty after the policy was put into place, whereas implied volatility only tells us how the market participants' expectation of future volatility changed following the policy announcement. Therefore, using both realized volatility and implied volatility, we are able to draw a more complete picture of the effect of the conditional commitment on uncertainty.

For instance, we observe increased uncertainty after the BoC's removal of the conditional commitment based on realized volatility. Although it is difficult to make a direct comparison of different conditional commitments, the Canadian experience of exit from its conditional commitment sheds some light into what we can expect to see following the Fed's eventual exit from its conditional commitment. If the conditional commitment by the Fed had the effect of reducing uncertainty around future policy rates substantially, similar to the experience in Canada, then we would expect to see a similar increase in uncertainty following the Fed's future exit from its conditional commitment.

The first part of the paper focuses on the use of realized volatility and implied volatility as indicators of future policy rate uncertainty. Our next set of results shows that the realized volatility and implied volatility of interest rate futures are also useful in adjusting policy rate expectations extracted from overnight index swap (OIS) rates for the risk premium in the OIS market. The risk premium in the OIS market is important because OIS rates are the most widely used gauge of policy rate expectations in Canada. The presence of a time-varying risk premium in the OIS market means that ignoring a potential risk premium embedded in OIS rates can lead to biased estimates of policy rate expectations.

Piazzesi and Swanson (2008) show that, although federal funds futures provide good forecasts of future policy rates, the extracted forecasts need to be adjusted to account for the risk premium in the federal funds futures prices. They find that business-cycle indicators such as employment growth, corporate bond spread and treasury yield spread are good predictors of the risk premium, and thus can be used to improve prediction. We propose using realized volatility and implied volatility of interest rate futures as additional predictors of the risk premium in OIS rates, based on the rationale that investors will demand a higher risk premium in the OIS market when uncertainty around future policy rates is higher. We find that both realized volatility and implied volatility are positively related to the risk premium in OIS rates, and thus can be used to adjust OIS-implied expectations for a risk premium. Resulting risk-adjusted OIS rates are closer to realized policy rates than unadjusted OIS rates, providing an improvement in the prediction of future policy rates.

We compute realized volatility daily as a sum of squared changes in high-frequency intraday prices of BAX. BAX contracts are futures on the three-month CDOR, a Canadian interbank lending rate index comparable to the LIBOR in the United States. Since the three-month CDOR is a benchmark for the short-term interbank lending rate that is closely related to the central bank's policy rate (Johnson, 2003; Fay and Gravelle, 2010), we use the realized volatility of BAX as a proxy for uncertainty around future policy rates. We compute implied volatility from the end-of-day prices of options on BAX.

The main difference between realized volatility and implied volatility is that realized volatility is an ex-post measure of uncertainty while implied volatility is a measure of an ex-ante expectation of future uncertainty. For instance, if one's objective is to assess the impact of a certain policy action on uncertainty about future policy rates, then realized volatility observed in the days following the announcement of the policy would be helpful. If, instead, one is interested in assessing, on the day of an announcement, how the announcement changed expected uncertainty, then the daily change in implied volatility would be more informative. Our empirical results show that both realized volatility and implied volatility are useful indicators of uncertainty around future policy rates.

To our knowledge, our study is the first to propose realized volatility as a measure of uncertainty around future policy rates. Implied volatility has been used in the past to gauge policy rate uncertainty (Neely, 2005; Swanson, 2006; Bauer, 2012), but our study is the first to apply it to Canadian data. Carlson, Craig and Melick (2005) and Emmons, Lakdawala and Neely (2006) go beyond implied volatility and extract the option-implied probability distribution of future policy rates using federal funds futures options. However, in Canada, data on interest rate options are not rich enough for a similar exercise, so we limit our attention to implied volatility.

The rest of the paper is organized as follows. Section 2 describes the theory, data and empirical methodologies behind the computation of volatility measures. Section 3 reports the empirical results on the impact of BoC announcements on uncertainty. Section 4 explores the relation between our uncertainty measures and the risk premium in the OIS market. Section 5 concludes.

2 Constructing Measures of Uncertainty

The main difference between implied volatility (IV) and realized volatility (RV) is the fact that IV is forward-looking whereas RV is backward-looking. In other words, RV is an ex-post measure of uncertainty while IV is a measure of ex-ante expectation of future uncertainty. As we describe below, RV is a sum of squared price changes[1] when prices are sampled at a high frequency based on intraday price data. Hence, RV of interest rate futures on a particular day captures uncertainty around the underlying interest rate at the futures expiry on that day. IV, on the other hand, captures an ex-ante expectation of RV for that futures contract on all days between today and the expiry date.

Another difference between the two volatility measures is that IV is affected by a volatility risk premium, whereas this is not the case for RV. That is, IV will be adjusted upward from expected RV during periods when investors require a high premium for bearing volatility risk, and vice versa. In practice, we can either use IV as it is with the risk premium in mind or separate the volatility risk premium from IV. We take the first approach, for two reasons. First, we cannot separate the volatility risk premium from IV without making assumptions about the level of expected RV through, for instance, a time-series model estimation based on past RV. Second, in many applications, uncertainty with a risk premium is a meaningful measure of uncertainty because it reflects the economic significance of the uncertainty prevailing at the time. For instance, the same amount of uncertainty during normal times and during crisis periods can have different implications for the economy.

2.1 Theory of realized volatility and implied volatility

Consider a futures contract that references a 3-month interest rate, r_T, at the futures expiry, T. Let f_t be the price of this futures contract at time t ($t \leq T$). Then we have

$$f_t\left(T\right) = E_t^Q\left[r_T\right],$$

where Q indicates that the expectation is taken under a risk-neutral probability measure. Now, suppose that r_t follows a stochastic process of the form

$$dr_t = \alpha_t dt + \sigma_t dW_t + \sum \Delta r_s, \tag{1}$$

where $\Delta r_t \equiv r_t - r_{t-}$ denotes a jump in r at time t. The variation of r between $[t, T]$ can be measured by its quadratic variation.

[1] For equities, the sum of squared log returns is used.

The quadratic variation of a real-valued stochastic process X_t defined on a probability space $(\Omega, \mathcal{F}, P)$ is defined as

$$[X,X]_t = \lim_{\|I\| \longrightarrow 0} \sum_{k=1}^{N} \left(X_{t_k} - X_{t_{k-1}}\right)^2,$$

where I ranges over partitions of the interval $[0,t]$ and the norm, $\|I\|$, is the length of the longest subinterval. Jacod and Shiryaev (1987, p. 55) state that the quadratic variation of the interest rate r between $[t,T]$ following the stochastic process in (1) is

$$[r,r]_T - [r,r]_t = \int_t^T \sigma_s^2 ds + \sum |\Delta r_s|^2 .$$

Barndorff-Nielsen and Shephard (2002) show that, if sampled frequently, realized variance defined as

$$RV^2_{[t,T]} \equiv \sum_{j=1}^{N} \left(r_{t_j} - r_{t_{j-1}}\right)^2,$$

for an increasing sequence of random partitions of $[t,T]$, $t = t_0 \leq t_1 \leq \cdots \leq t_N = T$, converges in probability to the quadratic variation.

The implied volatility of an option on interest rate futures is simply the ex-ante expectation of the quadratic variation under a risk-neutral measure, Q:

$$IV_t^2(T) = \frac{1}{T-t} E_t^Q \left[\int_t^T \sigma_s^2 ds + \sum |\Delta r_s|^2\right] \approx \frac{1}{T-t} E_t^Q \left[RV^2_{[t,T]}\right].$$

Since IV is an expectation taken under a risk-neutral measure, it is a biased measure of the expectation of future RV under the actual probability measure. The difference between an expectation taken under a risk-neutral probability measure and an expectation taken under the actual probability measure is called volatility risk premium. Despite the bias due to the volatility risk premium, past studies have shown that IV is a good predictor of future RV, often subsuming all the information contained in past prices (Christensen and Prabhala, 1998; Poon and Granger, 2003; Chernov, 2007; Busch, Christensen and Nielsen, 2011). See also Christoffersen, Jacobs and Chang (2013, sections 2.3-2.4) for a comprehensive survey of this literature.

2.2 Data

In Canada, the overnight rate at which major financial institutions borrow and lend one-day funds among themselves is the main tool used by the BoC to conduct monetary policy. The Canadian overnight repo rate average (CORRA) is the weighted average of overnight rates, and is the rate targeted by the central bank. In terms of derivatives on the CORRA, there are two futures contracts traded on the Montreal Exchange (ONX and OIS futures) and overnight index swap (OIS) contracts traded over the counter. Options on CORRA are yet to be introduced. The ONX and OIS futures were introduced in 2002 and 2012, respectively, but their liquidity is still quite limited. As a result, OIS contracts are the main hedging instruments for the overnight rate in Canada, making OIS rates the best gauge of expectations of future policy rates.

Another important benchmark for short-term interest rates in Canada is the Canadian dealer offered rate (CDOR), which is the average of Canadian bankers' acceptance rates for specific terms to maturity. CDOR is comparable to the LIBOR in the United States. Similar to the United States, there is a liquid market on futures on the 3-month CDOR and options on these futures, called BAX futures and OBX options, respectively.

Although ideal instruments for our study would be the futures and options on CORRA itself, we use BAX futures and OBX options instead, due to the limited liquidity of futures and the absence of options on CORRA. The use of BAX and OBX is consistent with the use of Eurodollar futures and options in Abken (1995), Amin and Ng (1997), Rigobon and Sack (2002), Neely (2005), and Bauer (2012).

We obtain daily time series of CORRA and OIS rates from Bloomberg, and the following data on BAX and OBX from the Montreal Exchange:

- BAX intraday quotes (January 2002 – May 2011)
- BAX trades (January 1997 – March 2013)
- OBX end-of-day (February 2005 – March 2013)
- OBX trades (January 2005 – June 2011).

Table 1 compares the liquidity of BAX and OBX contracts in terms of the average daily trading volume and average daily number of trades. The trading volume of BAX contracts is roughly 20 times that of OBX contracts. In number of trades, the difference is an order of magnitude larger, around 400 times larger for BAX compared to OBX. This is due to the fact that the average size of OBX trades is much larger than that of BAX trades. In terms of open interest, as of late March 2013, the size of the market for BAX was approximately Can\$600 billion compared to Can\$70 billion for OBX in notional amount. Thus, a considerable amount of money is at stake in the market for both BAX and OBX.

There was zero trading of OBX contracts in 2009. This temporary stop in trading began in October 2008 at the onset of the subprime crisis in the United States, possibly due to prohibitively high margin requirements caused by the high volatility of the underlying interest rate and risk premium at the time. The zero trading period also coincides with the BoC conditional commitment between April 2009 and April 2010. Trading in OBX resumed in mid-March 2010, about one month before the removal of the commitment was announced. This lack of trading in OBX was purely market driven, and was not a result of actions by the exchange or the regulators.

We also report in Table 2 the average daily number of OBX contracts with positive open interest. We consider only options with positive open interest, to filter out any options with stale prices that are not informative of the market's current assessment of the future. The average daily total number of options is around 20. When grouped by maturity, we observe that most of the options fall into the maturities between one month and six months.

2.3 Realized volatility of BAX

Realized volatility computed from high-frequency intraday prices is another way to measure uncertainty in the market's expectation of the future interest rate. Following Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002), Meddahi (2002), and Hansen and Lunde (2005), we compute realized volatility as

$$RV = \sqrt{\sum_{i=1}^{N-1} (S_{i+1} - S_i)^2 + \Delta S^2_{overnight}},$$

where $\Delta S_{overnight}$ is the change in the BAX price between the close of the previous day and the opening of the current day, and S_i are the intraday prices observed at a certain interval (e.g., every 5 minutes or every 5 ticks). In our implementation, we sample intraday trade prices of BAX at the interval of 5 trade ticks. Details of the methodology and implementation are provided in Appendix A.

To determine how RV captures uncertainty around future policy rates, in Figure 1 we plot intraday prices of a BAX contract on three consecutive days around 21 April 2009. The plot in the middle shows intraday trade prices of a September 2009 expiry BAX contract on 21 April 2009. The right and left plots show prices of the same BAX contract one day before and after 21 April, respectively. We choose 21 April 2009 as an example because on that day the BoC announced its conditional commitment to keep its policy rate at 0.25% until the second quarter of 2010. If credible, such an announcement should decrease uncertainty around future policy rates up to the second quarter of 2010. We can see that the intraday variation of BAX prices on the day after the announcement (right) is smaller than the variation on the day before the announcement (left), as we expected. In this example, RV, which is the sum of squared changes in intraday prices, effectively captures this change in uncertainty.

The plot in the middle shows a large jump in the BAX price at 9 a.m., the time of the BoC's policy rate announcement. The observed pattern is typical of the fixed announcement dates (FADs), the BoC's pre-scheduled policy rate announcement days, and reflects a shift in expectation following the announcements. This large jump (either up or down) in the BAX price leads to distinctly larger-than-average RVs on the FADs. We explore this issue in more detail in section 3.3.

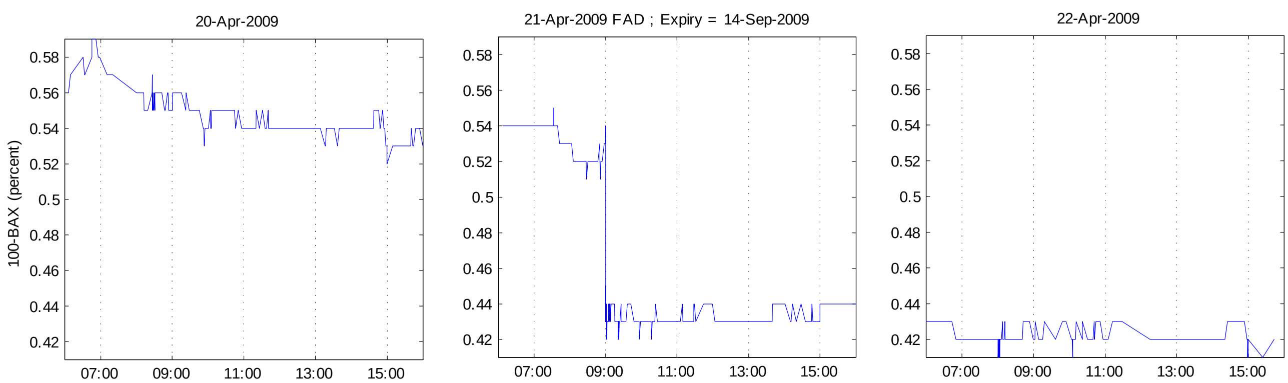

Figure 1: Intraday BAX prices

In Figure 2, we plot the average level of BAX RVs with different maturities. The bottom right panel shows that the interest rate uncertainty captured by BAX RV peaks at around the six-month maturity, and then flattens. The substantially lower level of RV in the short maturities indicates that there is less uncertainty around the future path of monetary policy at very short horizons of one to two months.

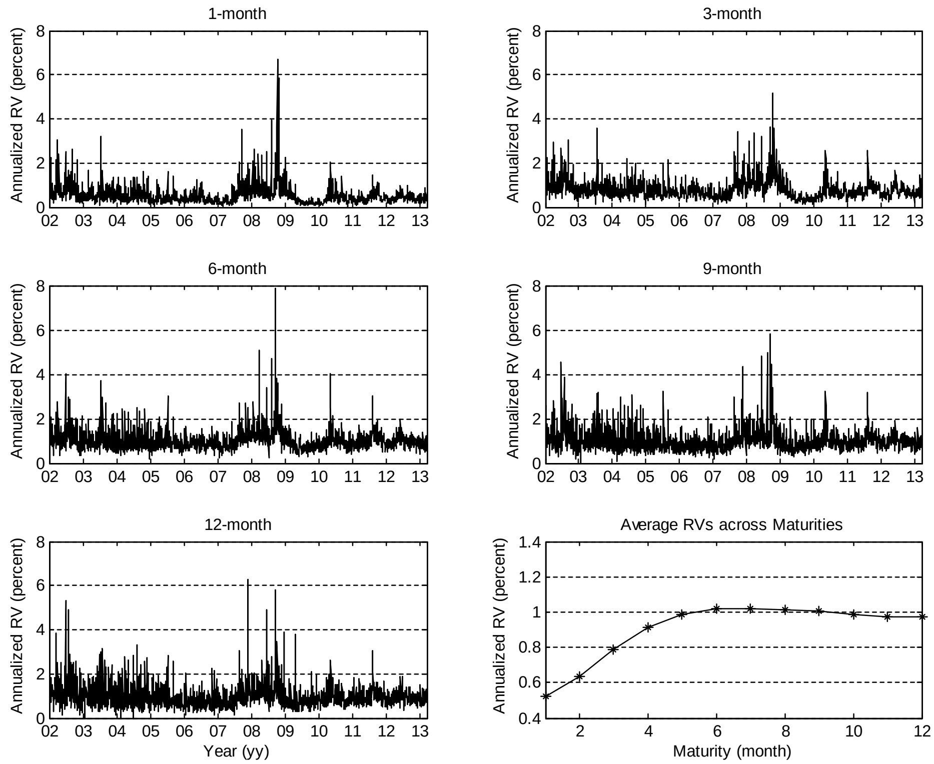

Figure 2: Realized volatility of BAX

2.4 Implied volatility of BAX

We compute the implied volatility of BAX from OBX option prices using an option valuation formula based on the Vasicek model (1977) of the short rate. Most studies that compute the implied volatility of the short-term interest rate use the Barone-Adesi and Whaley (1987) approximation of the American option pricing model. The Vasicek model provides an improved implied volatility estimate by taking into account the stochastic characteristics of the interest rate process. Amin and Ng (1997) study the ability of the implied volatility of Eurodollar futures options to forecast the future volatility of the Eurodollar futures rate. They compare implied volatilities computed based on five different volatility models: (i) Ho-Lee (1986), (ii) Cox-Ingersoll-Ross (1985), (iii) Courtadon (1982), (iv) Vasicek (1977), and (v) linear proportional (Heath-Jarrow-Morton, 1992). Amin and Ng find that the Vasicek (1977) and linear proportional volatility models perform better than the other implied volatility models. Details of the methodology and implementation are provided in Appendix B.

Ideally, we would like to compute implied volatilities for different maturities so that we can examine the term structure of implied volatility. However, due to the relatively low liquidity of OBX, we cannot compute implied volatility of BAX for different maturities in a consistent manner over time, so we compute one implied volatility for each day using all options of maturities from one to six months.

The daily time series of BAX IV is plotted in Figure 3. Note that implied volatility could not be computed between November 2008 and mid-March 2010 due to a lack of trading in OBX contracts during this period. BAX IV reached its highest level of around 8% in late 2008 during the U.S. subprime crisis, and its lowest level of around 0.25% in 2010–11 when the policy rate was kept extremely low following the crisis. Compared to the VIX index, which ranged from approximately 10% to 80% in the same period, the implied volatility of BAX is significantly smaller in magnitude. This is consistent with the fact that uncertainty in the short-term interest rate is much lower than uncertainty in the stock market.

3 Impact of Central Bank Announcements on Uncertainty

3.1 Conditional commitment between 2009 and 2010

On 21 April 2009, the BoC announced that, "Conditional on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve the inflation target." The commitment was eventually removed on 20 April 2010, and the policy rate was raised to 0.50% at the following FAD on 1 June, one month earlier than was indicated in the initial commitment.

Figure 3 plots the time series of 3-month maturity RV together with IV. We chose the 3-month maturity for RV because the average maturity of options used in the computation of IV is around three months. Since the underlying asset of both BAX futures and OBX options is the 3-month CDOR, the time horizon of uncertainty for both RV and IV in this graph reflects uncertainty around the policy rate approximately six months ahead.

As expected, the commitment led to a decline in the level of uncertainty around future policy rates six months into the future, as indicated by the low level of three-month BAX RV shown in Figure 3. The level of RV during the commitment period shows us that the ex-post uncertainty has decreased.

Although we cannot compute BAX IV throughout the conditional commitment period, we observe that the level of IV at the time of trading resumption is significantly lower than it was when the trading halted in late 2008, consistent with the evidence of decreased uncertainty exhibited by the low level of RV throughout the conditional commitment period.

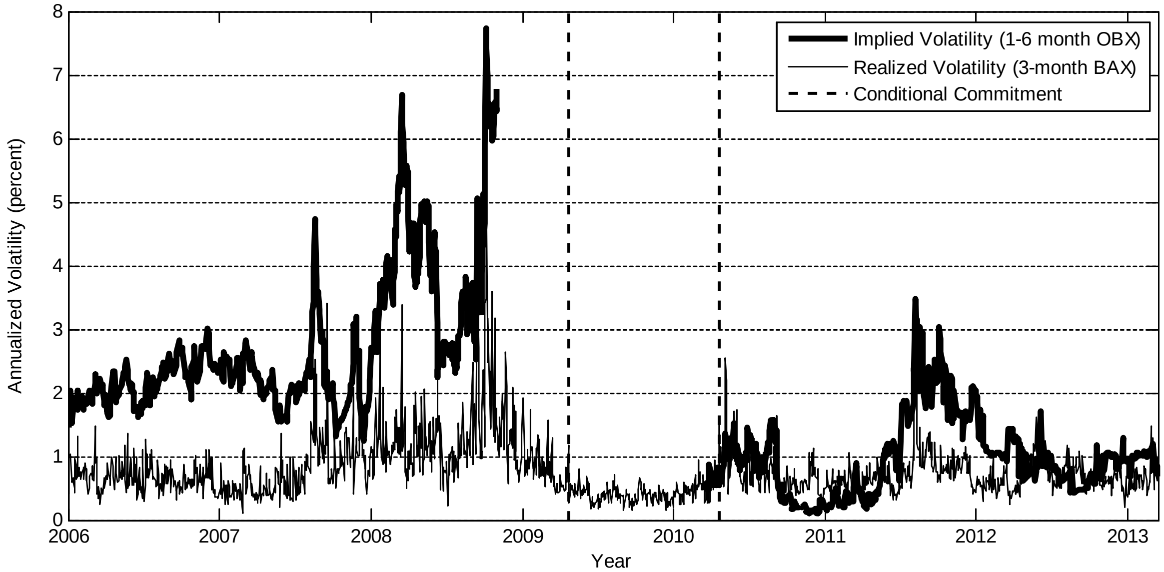

Figure 3: Implied volatility (IV) and realized volatility (RV) of BAX

The timing of trading resumption in OBX contracts also provides an interesting insight into the market's expectation on the timing of the removal of the conditional commitment. Figure 3 shows that the trading of OBX contracts (14 June 2010 expiry) resumed in mid-March of 2010, one month before the removal was announced to the public. The fact that options started trading even though they had an expiry date before the end date of the commitment suggests that the market anticipated a possible early removal of the commitment before the actual announcement was made.

The BoC's removal of the commitment resulted in a large increase in the level of both IV and RV compared to that observed during the conditional commitment period. However, both IV and RV were much lower than during the crisis between late 2007 and late 2008, and RV was comparable to its level just before the beginning of the commitment. The pattern indicates that the removal of the commitment increased uncertainty compared to the commitment period, as expected. There is no clear evidence that the commitment had any lasting effect on reducing uncertainty, since the level of RV after the removal is similar to that preceding the initial announcement of the commitment.

The Canadian experience of exiting from a conditional commitment sheds some light on what to expect following the Fed's eventual exit from its conditional commitment. In the

United States, the Fed announced its own conditional commitment in August 2011 when the FOMC noted that economic conditions “are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013.” Since the initial announcement, the end of the commitment of mid-2013 has been extended first to late 2014 (in January 2012), then again to mid-2015 (in September 2012). It is difficult to make a direct comparison of different conditional commitments, since the impact of each commitment will depend on factors such as the credibility of the commitment and other exogenous factors. For example, the end date of the commitment by the Fed has been extended twice so far, which could have affected the market’s confidence in the end date of the commitment. Nevertheless, if in fact the conditional commitment by the Fed substantially reduced uncertainty around future policy rates, similar to the experience in Canada, then we would expect to see a similar increase in uncertainty following the Fed’s future exit from its conditional commitment.

3.2 Other important policy actions following the financial crisis of 2007–08

During the financial crisis of 2007–08, the BoC put several important policy actions into effect, as did other central banks around the world. In this section, we examine the impact of these policy actions on uncertainty as reflected in the change in BAX IV on the day of the announcements.

We look at four important crisis-related policy announcements by the BoC between 2007 and 2010, and report the results in Panel A of Table 3. The policy actions considered include the introduction and modification of term purchase and resale agreements; the unscheduled cut in the policy rate in October 2008, coordinated with other major central banks; and the removal of the conditional commitment in April 2010. We find that all of these announcements led to a large decrease in IV, ranging between -12 and -51 basis points (bps). By far the largest drop in IV of 51 bps occurred on 8 October 2008 when the policy rate was cut by 75 bps, in conjunction with other central banks. This result is consistent with Bauer (2012), who finds that many of the important announcements by the Fed during the crisis also led to a larger-than-average drop in the IV of Eurodollar futures options.

We also examine the impact of recent FAD announcements when the BoC introduced a shift in bias regarding the path of monetary policy in the near future. We expect such announcements to be associated with a larger-than-average change in IV on the day of the announcement. A priori, it is unclear whether these announcements will lead to an increase or a decrease in IV. The results reported in Panel B of Table 3 show that all four FAD announcements that introduced a shift in bias between 2011 and 2012 are linked to large drops in IV, suggesting that the announcements succeeded in reducing uncertainty.

3.3 Policy rate announcements

Most central banks make their policy rate announcements only on pre-scheduled dates. In Canada, these are called fixed announcement dates, and they occur every six to eight weeks. The purpose is to reduce uncertainty about the timing of policy rate changes, a practice consistent with increased transparency in central bank communications.

After each policy rate announcement, central banks are interested in assessing the impact of their decision on the market. Typically, financial variables monitored include yields on various fixed-income securities and foreign exchange rates. These variables, however, do not tell us whether a particular decision has increased or decreased uncertainty around future policy rates. Although central banks rarely set an explicit goal to reduce uncertainty, lower uncertainty about future monetary policy is deemed desirable in most cases.

We first look at whether any pattern emerges for BAX RV and IV around policy rate decision days. The fact that the policy rate is fixed until the next policy rate announcement day, typically six to eight weeks away, means that we would expect any measure of uncertainty over the horizon that ends before the following policy rate announcement to drop sharply after each announcement. However, if the horizon covered by the uncertainty measure extends beyond the following announcement day, which is the case for our RV and IV whose horizons are around six months, uncertainty can either increase or decrease after a policy rate announcement.

In Figure 4, we plot the average levels of RV and IV between -5 and +10 business days around policy rate announcements. We observe a large spike in RV on the announcement days, followed by a slightly lower-than-average level of RV starting from day +4. In contrast, IV is lower than average on the announcement days and stays low for eight business days after the FADs.

Significantly high RVs on the FADs are due to large jumps in the expected level of the future policy rate that occur immediately following policy rate announcements, as we showed in Figure 1. Hence, a high RV reflects a high level of uncertainty that was, in a sense, resolved during the day. Unfortunately, the RV in this case cannot tell us whether uncertainty around the new expected policy rate is lower or higher than the day before. We do not observe a similar spike in IVs on the FADs because IV is computed from end-of-day option prices, which are insensitive to price variations during the day. IV computed on a FAD, then, reflects uncertainty around the new expected future policy rate after the FAD announcements, which is what we are interested in.

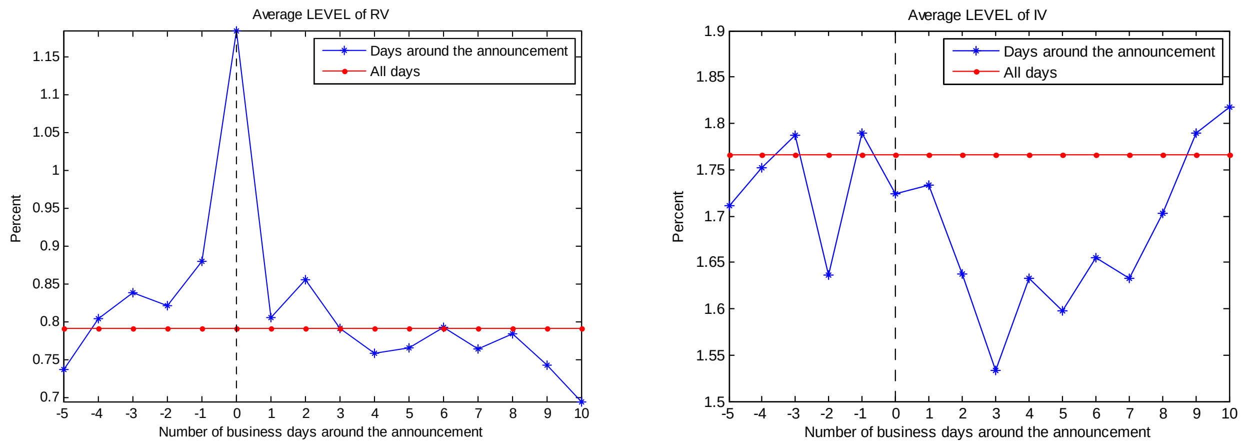

Figure 4: Average level of BAX RV and IV around policy rate announcement days

Next, we test whether the observed drop in IV after policy rate announcement days is statistically significant by running the following regression on the level and change in IV for each event day, $i \in [-5, +10]$:

$$\begin{aligned} IV_t &= \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it}, \\ \Delta IV_t &= \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it}, \end{aligned}$$

where IV_t and ΔIV_t are the level and change in IV, respectively, on date t. $\mathrm{I}_i(t)$ is an indicator function that yields 1 if date t is i business days away from a FAD, and 0 otherwise. The intercept coefficient, α_i, of the regression is then the average daily level and change in IV on all days other than the event days, i. The slope coefficient, β_i, is the deviation of daily level and change in IV on event days, i, compared to all other days. The regression results are reported in Table 4.

The level of IV is significantly lower than average starting from two business days after the FADs until seven business days after the FADs. The regression results on the change in IV show that a statistically significant drop in IV occurs on the day of the announcement (7 bps) and two business days after the announcement (5 bps). The decrease in IV two business days after the announcements can be explained by the fact that, during the sample period, the BoC released its *Monetary Policy Reports* two days after every other policy rate announcement.

The results in this section show that, on average, the BoC policy rate decisions reduced uncertainty around future policy rates in our sample period. A statistically significant reduction in uncertainty is observed on the days of policy rate announcements and releases of *Monetary Policy Reports*. The effect of the reduction in uncertainty seems to be temporary, lasting for about seven business days, on average. This gradual increase in uncertainty following the initial decrease in uncertainty after the FADs is reasonable given that the arrival of new information and new events tends to add uncertainty.

4 Forecasting the Risk Premium in the OIS Market

The previous section showed that BAX IV and RV are useful indicators of uncertainty around future policy rates. In this section, we explore whether BAX IV and RV can be used to forecast the risk premium in the OIS market. OIS rates are the most widely used gauge of policy rate expectations in Canada, and ignoring the potential risk premium embedded in OIS rates can lead to biased estimates of policy rate expectations.

Piazzesi and Swanson (2008) show that, although federal funds futures provide good forecasts of future policy rates, the extracted forecasts need to be adjusted to account for a risk premium in the federal funds futures prices. They find that business-cycle indicators such as employment growth, corporate bond spread and treasury yield spread are good predictors of the risk premium, and therefore can be used to improve prediction.

We propose using BAX IV and RV as additional predictors to those suggested by Piazzesi and Swanson (2008), to forecast the risk premium in OIS rates. Our proposal is based on the intuition that investors will demand a higher risk premium in the OIS market when uncertainty around future policy rates is higher.

We first plot the time series of prediction errors for OIS rates to determine whether such a risk premium exists. Figure 5 shows the OIS prediction error, or OIS excess return, for OIS contracts with 3-month and 9-month maturities. We find that OIS excess returns are persistent, and can be both positive and negative. OIS excess returns are also greater in magnitude for longer maturities. The fact that OIS excess returns are persistent implies that the returns are likely to be predictable.

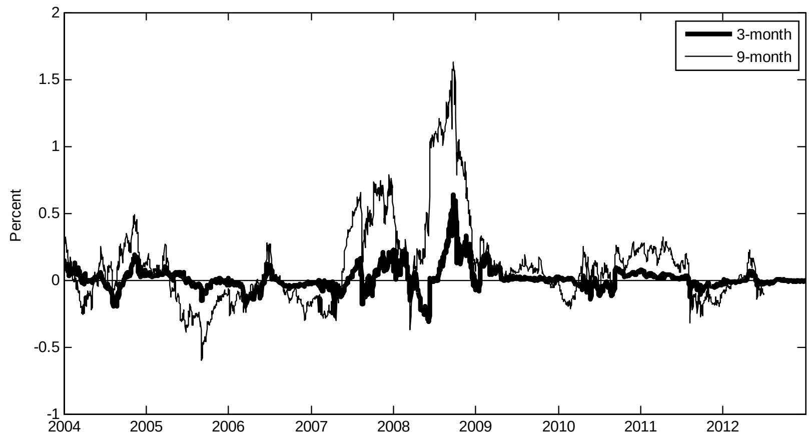

Figure 5: OIS Excess Returns

4.1 In-sample tests

We test whether BAX IV and RV can predict OIS excess returns by running univariate monthly regressions of the form

$$R_{OIS}\left(t, t+n\right) = \alpha + \beta \cdot IV_t + \varepsilon_t,$$
$$R_{OIS}\left(t, t+n\right) = \alpha + \beta \cdot RV_t + \varepsilon_t,$$

where IV_t and RV_t are the volatility measures on the last day of month t. $R_{OIS}\left(t, t+n\right)$ is the OIS excess return, defined as

$$R_{OIS}\left(t, t+n\right) \equiv OIS_{t,t+n} - CORRA_{t+1,t+h},$$

where $OIS_{t,t+n}$ is the n-month maturity OIS rate observed on the last day of month t, and $CORRA_{t+1,t+n}$ denotes the geometric mean of daily realized CORRAs between months $t+1$ and $t+n$,

$$CORRA_{t+1,t+n} = \left(\prod_{i=t+1}^{t+n}\left(1+\frac{CORRA_i}{365}\right)-1\right)\frac{365}{n}.$$

For comparison, we run the same univariate regression on the three business-cycle variables in Piazzesi and Swanson (2008): (i) the spread between BBB-rated 10-year corporate bonds and the 10-year Treasury yield, (ii) the spread between 2-year and 5-year Treasury yields, and (iii) employment growth. We also consider the index of Canadian economic policy uncertainty based on Baker, Bloom and Davis (2013), as well as the lagged OIS excess return, $R_{OIS}(t-3,t)$, and the lagged absolute OIS excess return, $|R_{OIS}(t-3,t)|$. We use the 3-month lag because we want the lag to be close enough to the current date without being too short, since the one- to two-month maturity often has relatively little uncertainty.

We report the correlations of all the predictors, excluding the lagged R_{OIS} and $|R_{OIS}|$, in Table 5. All the predictors are positively correlated except for employment growth, which is negatively correlated with all the other predictors. The negative correlation of employment growth to the other variables is consistent with the fact that high asset price volatility, high corporate bond spread and high policy uncertainty are all linked to bad states of the economy, whereas high employment growth is linked to good states of the economy.

We report the results of the univariate regressions in Table 6. We find that all slope coefficients for IV and RV are positive except for IV at the one-month maturity. This confirms our intuition that the risk premium must be positively related to uncertainty. RV performs better than IV in terms of both R^2 and the t-statistic. The regressions with RV also exhibit higher coefficients. Moreover, the coefficients on RV are highly significant at all maturities longer than one month, whereas the coefficients on IV are significant only at the nine-month maturity.

Next, we test whether BAX IV and RV have additional predictive power when the business-cycle indicators, policy uncertainty measure, lagged R_{OIS} and lagged $|R_{OIS}|$ are also used as predictors. The results of the multivariate regression, including all predictors as regressors, are reported in Table 7.

We find that adding IV and RV as regressors improves the adjusted R^2 of the regression for all maturities. The improvement in adjusted R^2 ranges from 0.01 for the one-month maturity to 0.12 for the nine-month maturity. We find that, in general, the magnitude of the improvement increases with the forecasting horizon. We also observe that the significance of IV improves in the multivariate regression compared to the univariate regression, whereas the opposite is true for RV.

4.2 Out-of-sample tests

The regression results described in the previous section are in-sample tests of the forecasting ability of IV and RV. We next report the results of out-of-sample tests. In Figure 6, we plot the root-mean-squared-error (RMSE) of four different predictions of realized CORRAs. We compare the performance of (i) unadjusted OIS, (ii) constant risk-adjusted OIS, (iii) risk-adjusted OIS using predictors excluding IV and RV, and (iv) risk-adjusted OIS including all the predictors.

The out-of-the sample predictions are conducted daily starting from January 2007, which is one year after our first data point in IV. We adjust the OIS-implied CORRAs on the first day of 2007 by using the coefficients obtained from the multivariate regression with all the predictors based on daily time series of the predictors in 2006. For the adjustment on the second day of 2007, we use the coefficients estimated from running the regression on data in the expanded window from January 2006 up to the first day of 2007. The procedure is repeated until the end of our sample.

We find that improvement in RMSE is negligible in short maturities of one to three months, but is substantial at longer maturities such as six or nine months. If we use all the predictors (black line), the improvement over the unadjusted OIS prediction (blue line) is around 10 bps for the six-month maturity and 25 bps for the nine-month maturity.

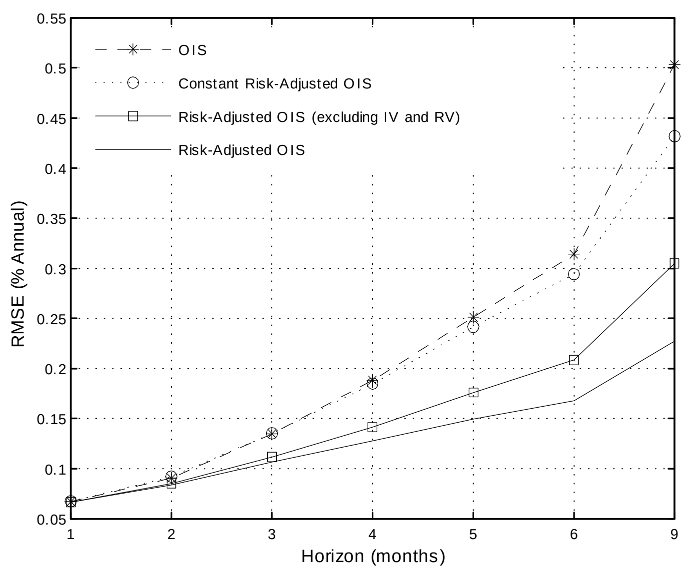

Figure 6: Out-of-sample RMSE of OIS excess-return prediction

Figure 7 compares out-of-sample forecasts of CORRAs over a nine-month horizon using three different models: (i) constant adjusted OIS, (ii) risk-adjusted OIS using all predictors except IV and RV, and (iii) risk-adjusted OIS using all predictors. The different models performed similarly in the later part of the sample in which the policy rate stayed at 1% most of the time, so we plot only the earlier period between January 2007 and October 2008. Prediction improves substantially when we adjust OIS rates using economic variables as predictors. Adding IV and RV to the regression improves forecasts slightly, especially in 2008.

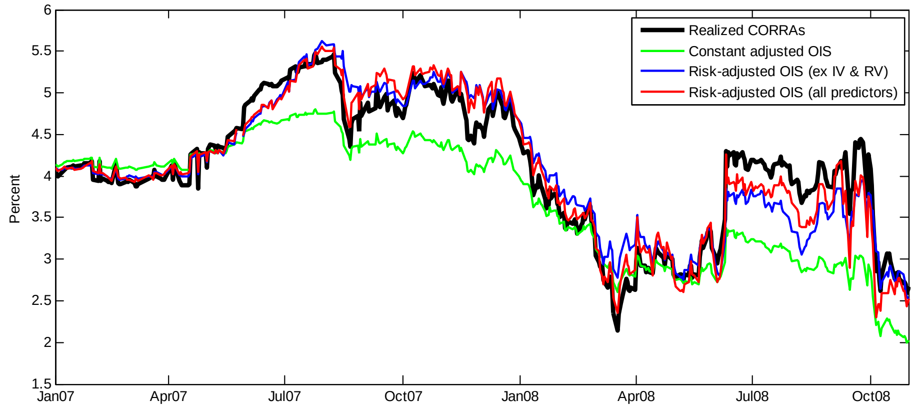

Figure 7: Risk-adjusted OIS rates

The results in this section show that BAX IV and RV are closely related to the risk premium in the OIS market, making these measures useful for adjusting OIS-implied expectations of future policy rates. The resulting risk-adjusted OIS rates are closer to realized CORRAs than unadjusted OIS rates, providing an improvement in prediction.

5 Conclusion

We show that implied volatility computed from interest rate futures options and realized volatility computed from intraday prices of interest rate futures are useful indicators of uncertainty around future central bank policy interest rates. Based on implied volatility and realized volatility computed from BAX futures and OBX options in Canada, we show that, on average, the policy rate announcements by the Bank of Canada reduced uncertainty around future policy rates between January 2006 and March 2013. We also find that some of the most important policy actions taken by the Bank of Canada as a response to the financial crisis of 2007–08, including the conditional commitment of 2009–10, the unscheduled cut in the policy rate in October 2008 coordinated with other major central banks, and the introduction of term purchase and resale agreements, all reduced uncertainty.

We also explore the relation between our two measures of uncertainty and the risk premium in the OIS market. We find that the implied volatility and realized volatility of BAX are positively related to the risk premium in OIS rates, and thus can be used to adjust OIS-implied expectations of future policy rates for a risk premium. The resulting risk-adjusted OIS rates are closer to realized overnight rates than unadjusted OIS rates, providing improvement in the prediction of future policy rates.

The volatility risk premium of BAX contains potentially useful information about risk and/or investors' attitudes toward risk for short-term interest rates in Canada. For equities, Bollerslev, Tauchen and Zhou (2009) find that the volatility risk premium of the S&P 500 index is a good predictor of future stock market returns. Mueller, Vedolin and Yen (2011) also find that bond variance risk premia predict excess returns on Treasuries, stocks, corporate bonds and mortgage-backed securities. We leave a detailed investigation of the volatility risk premium of BAX for future research.

Appendix A. Computing Realized Volatility of BAX

Following Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002), Meddahi (2002), and Hansen and Lunde (2005), we compute the realized volatility of BAX as

$$RV = \sqrt{\sum_{i=1}^{N-1} (S_{i+1} - S_i)^2 + \Delta S^2_{overnight}},$$

where $\Delta S_{overnight}$ is the change in the BAX price between the close of the previous day and the opening of the current day, and S_i is the intraday price (either trade or quote) observed at a certain interval (e.g., every 5 minutes or every 5 ticks).

To choose an optimal sampling frequency, we plot the volatility signature plots (Fang (1996) and Andersen, Bollerslev, Diebold and Labys (2000)) under four settings:

1. Tick time sampling using trade prices
2. Tick time sampling using mid-quotes
3. Calendar time sampling using trade prices
4. Calendar time sampling using mid-quotes

A volatility signature plot shows the average level of RVs at different sampling frequencies. Typically, a volatility signature plot shows a much higher level of RVs at very short sampling intervals, which decreases and converges to a certain level as the sampling interval is lengthened. This upward bias in RV when sampled very frequently is known to be due to microstructure noise. A volatility signature plot tells us beyond which sampling frequency this bias becomes negligible

The volatility signature plots for the BAX RV under four different settings are shown in the top graphs of each panel in Figure 8. An unbiased estimate of RV will have an average value that is comparable to the standard deviation of the daily change in BAX (green). We also plot the mean of the daily absolute change in BAX (red), since the absolute value of the daily change in BAX is often used to gauge the surprise in the interest rate expectation in the absence of intraday price data. The absolute value of the daily change in BAX can be also interpreted as the RV in the asymptotic limit, since it is equivalent to RV when the sampling is done only once a day.

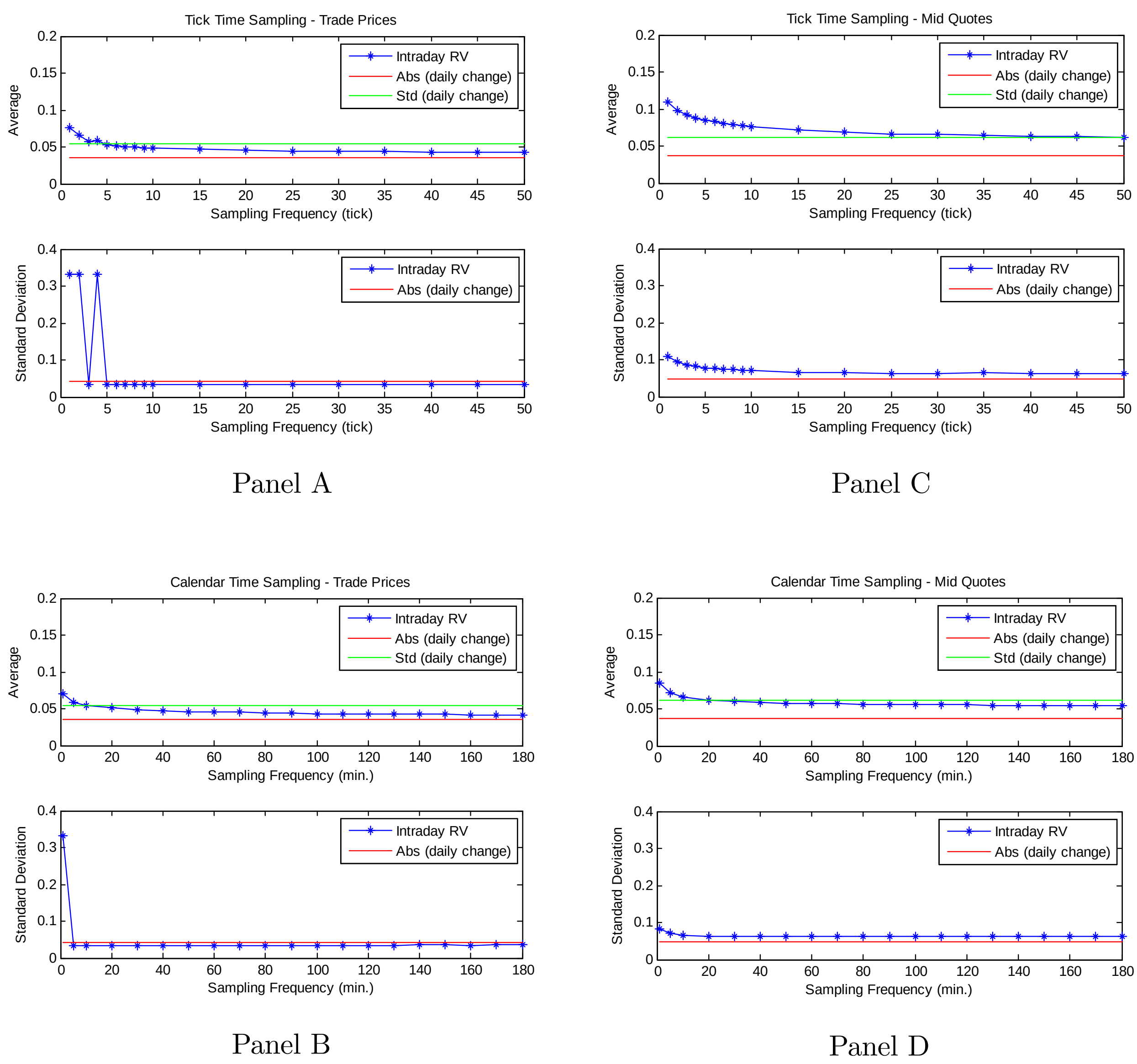

Figure 8: Volatility signature

In three of the volatility signature plots (all except tts-quote), we find that the plots flatten and coincide with the standard deviation of the daily change at around the 5-tick interval and the 20-min. interval, indicating that most of the microstructure noise is removed at these sampling frequencies. Only in the case of tts-quote is RV consistently higher than the standard deviation of the daily price change up to the sampling interval of 40-ticks.

We also examine how the volatility of the RVs changes with sampling frequency. We would expect the RV to be noisier when it is sampled less frequently. The bottom graphs of each panel show the volatility measured by the standard deviation of daily RVs at different frequencies. Again, we add the volatility of the daily absolute price change for comparison. We find that the volatility of the RVs at very high sampling frequency is higher than the volatility of the RVs sampled at low frequency. The volatility of the RVs stays more or less constant beyond certain sampling frequencies (5-tick for tts-trade, 10-tick for tts-quote, 5 min. for cts-trade, and 10 min. for cts-quote).

One puzzling observation is that the volatility of the RV is smaller than the volatility of the daily absolute change when trade prices are used (the blue line is below the red in Panels A and B), but it is larger when quotes are used (the blue line is above the red in Panels C and D). This indicates that the RV is less noisy when trade prices are used. Overall, we conclude that the RV computation using trade prices with sampling every 5-ticks yields the best results.

Appendix B. Computing Implied Volatility of BAX

We assume that the short rate, r_t, follows the risk-neutral process proposed in Vasicek (1977):

$$dr_t = a\left(b - r\right)dt + \sigma dz.$$

Yield formula

Vasicek (1977) shows that the yield to maturity h under his model is

$$R\left(t, h\right) = -\frac{1}{h}\ln A\left(h\right) + \frac{1}{h}B\left(h\right)r_t,$$

where

$$B\left(h\right) = \frac{1 - e^{-ah}}{a},$$

and

$$A\left(h\right) = \exp\left(\frac{\left(B\left(h\right) - h\right)\left(a^2 b - \frac{\sigma^2}{2}\right)}{a^2} - \frac{\sigma^2 B\left(h\right)^2}{4a}\right).$$

Futures formula

The price of a futures contract on yield $R(t+h_1,h_2)$ is

$$\begin{aligned} f(t,h_1,h_2) &= E_t\left[\exp\left(-\int_t^{t+h_1} r_s d_s\right) R(t+h_1,h_2)\right] \\ &= E_t\left[\exp\left(-\int_t^{t+h_1} r_s d_s\right)\left(-\frac{1}{h_2}\ln A(h_2)+\frac{1}{h_2}B(h_2)\,r_{t+h_1}\right)\right] \\ &= -\frac{1}{h_2}\ln A(h_2)\exp\left(-h_1 R(t,h_1)\right)+\frac{1}{h_2}B(h_2)\,E_t\left[\exp\left(-\int_t^{t+h_1} r_s d_s\right) r_{t+h_1}\right]. \end{aligned}$$

Let

$$V(h,r)\equiv E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right) r_{t+h}\right].$$

V is the solution to the Feynman-Kac partial differential equation (PDE) and the boundary condition,

$$-V_h+\frac{1}{2}\sigma^2 V_{rr}+a(b-r)V_r-rV=0,\quad V(0,r)=r. \tag{2}$$

We conjecture that

$$V(h,r)=(\alpha(h)+\beta(h)\,r)\exp(\gamma(h)\,r).$$

Then we have

$$\begin{aligned} V_h &= \left(\dot\alpha+\left(\dot\beta+\alpha\dot\gamma\right)r+\beta\dot\gamma r^2\right)\exp(\gamma r) \\ V_r &= (\alpha\gamma+\beta+\beta\gamma r)\exp(\gamma r) \\ V_{rr} &= \left(2\beta\gamma+\alpha\gamma^2+\beta\gamma^2 r\right)\exp(\gamma r) \\ \gamma(0) &= \alpha(0)=0,\ \beta(0)=1. \end{aligned}$$

Plugging these expressions into equation (2), we get

$$-\left(\dot\alpha+\left(\dot\beta+\alpha\dot\gamma\right)r+\beta\dot\gamma r^2\right)+\frac{1}{2}\sigma^2\left(2\beta\gamma+\alpha\gamma^2+\beta\gamma^2 r\right)+a(b-r)(\alpha\gamma+\beta+\beta\gamma r)-r(\alpha+\beta r)=0.$$

Hence,

$$\left\{\begin{array}{c} -\dot\alpha+\frac{1}{2}\sigma^2\left(2\beta\gamma+\alpha\gamma^2\right)+ab(\alpha\gamma+\beta)=0 \\ -\left(\dot\beta+\alpha\dot\gamma\right)+\frac{1}{2}\sigma^2\beta\gamma^2+ab\beta\gamma-a(\alpha\gamma+\beta)-\alpha=0 \\ -\beta\dot\gamma-a\beta\gamma-\beta=0 \end{array}\right. . \tag{3}$$

The last equation in the system of equations (3) implies that

$$\gamma(h)=-B(h)=-\frac{1-e^{-ah}}{a}, \tag{4}$$

and the second equation in the system of equations (3) can be rewritten as

$$\left(\frac{1}{2}\sigma^2\gamma^2 + ab\gamma - a\right)\beta = \dot{\beta}. \tag{5}$$

From equations (4) and (5), we get

$$\begin{aligned}
\beta(h) &= \exp\left(\left(ba^2 - \frac{\sigma^2}{2}\right)\frac{(B(h)-h)}{a^2} - ah - \frac{\sigma^2 B(h)^2}{4a}\right) \\
&= e^{-ah}A(h) = (1 - aB(h))A(h).
\end{aligned}$$

The first equation in the system of equations (3) implies that

$$\left(\sigma^2\gamma + ab\right)\beta + \left(\frac{1}{2}\sigma^2\gamma^2 + ab\gamma\right)\alpha = \dot{\alpha}. \tag{6}$$

Next, we look for a particular solution of type

$$\alpha = (k_1 B(h) + k_2)\beta. \tag{7}$$

The derivative of α is then

$$\begin{aligned}
\dot{\alpha} &= k_1\dot{B}(h)\beta + (k_1 B(h) + k_2)\dot{\beta} \\
&= k_1(1 - aB(h))\beta + (k_1 B(h) + k_2)\left(\frac{1}{2}\sigma^2 B(h)^2 - abB(h) - a\right)\beta \\
&= \left[k_1(1 - aB(h)) + (k_1 B(h) + k_2)\left(\frac{1}{2}\sigma^2 B(h)^2 - abB(h) - a\right)\right]\beta. \qquad (8)
\end{aligned}$$

Combining equations (6), (7) and (8), we get

$$k_1(1 - aB(h)) - a(k_1 B(h) + k_2) + \sigma^2 B(h) - ab = 0. \tag{9}$$

This equation holds if we set k_1 and k_2 to be

$$\begin{aligned}
k_1 &= \frac{\sigma^2}{2a} \\
k_2 &= \frac{\sigma^2}{2a^2} - b.
\end{aligned}$$

Hence, the general solution is

$$\alpha(h) = kA(h) + \left(\frac{\sigma^2}{2a}B(h) + \frac{\sigma^2}{2a^2} - b\right)e^{-ah}A(h).$$

The boundary condition, $\alpha(0) = 0$, implies that

$$k = b - \frac{\sigma^2}{2a^2},$$

so that

$$\begin{aligned}
\alpha(h) &= \left(\frac{\sigma^2}{2a}e^{-ah}B(h) + \left(\frac{\sigma^2}{2a^2} - b\right)e^{-ah} + b - \frac{\sigma^2}{2a^2}\right)A(h) \\
&= \left(\frac{\sigma^2}{2a}e^{-ah}B(h) + \left(b - \frac{\sigma^2}{2a^2}\right)\left(1 - e^{-ah}\right)\right)A(h) \\
&= \left(\frac{\sigma^2}{2a}e^{-ah}B(h) + a\left(b - \frac{\sigma^2}{2a^2}\right)B(h)\right)A(h) \\
&= \left(\frac{\sigma^2}{2a}\left(e^{-ah} - 1 + 1\right)B(h) + a\left(b - \frac{\sigma^2}{2a^2}\right)B(h)\right)A(h) \\
&= \left(\frac{\sigma^2}{2a}\left(-aB(h)^2 + B(h)\right) + \left(ab - \frac{\sigma^2}{2a}\right)B(h)\right)A(h) \\
&= \left(ab - \frac{\sigma^2}{2}B(h)\right)B(h)A(h).
\end{aligned}$$

Therefore, the futures price is

$$\begin{aligned}
f(t,h_1,h_2) &= \left[-\frac{1}{h_2}A(h_1)\ln A(h_2) + \frac{1}{h_2}B(h_2)\alpha(h_1) + \frac{1}{h_2}B(h_2)\beta(h_1)r_t\right]\exp(-B(h_1)r_t) \\
&\equiv (C(h_1,h_2) + D(h_1,h_2)r_t)\exp(-B(h_1)r_t).
\end{aligned}$$

Integrated variance of the futures and its expectation

We apply Ito's lemma on $f(t,h_1,h_2)$ to get

$$df = \left[f_t + a(b-r)f_r + \frac{\sigma^2}{2}f_{rr}\right]dt + \sigma f_r dz,$$

and

$$\begin{aligned}
f_r &= [D(h_1,h_2) - B(h_1)C(h_1,h_2) - B(h_1)D(h_1,h_2)r_t]\exp(-B(h_1)r_t) \\
&\equiv (E(h_1,h_2) + F(h_1,h_2)r_t)\exp(-B(h_1)r_t).
\end{aligned}$$

The integrated variance of $f(t,h_1,h_2)$ between t and $t+h$ with $h \leq h_1$ is defined as

$$\begin{aligned}
IV^f(t,t+h) &= \sigma^2\int_t^{t+h} f_{r_s}^2 d_s \\
&= \sigma^2\int_t^{t+h} (E(h_1,h_2) + F(h_1,h_2)r_s)^2\exp(-2B(h_1)r_s)d_s.
\end{aligned}$$

The expectation of the integrated variance of $f(t,h_1,h_2)$ is

$$E_t\left[IV^f(t,t+h)\right]=\sigma^2 V(h,r_t), \tag{10}$$

$$\begin{aligned} V(h,r_t) &= \int_t^{t+h} E_t\left[\left(E(h_1,h_2)+F(h_1,h_2)r_s\right)^2 \exp\left(-2B(h_1)r_s\right)\right] d_s \\ &= \int_0^h E_t\left[\left(E(h_1,h_2)+F(h_1,h_2)r_{t+s}\right)^2 \exp\left(-2B(h_1)r_{t+s}\right)\right] d_s, \end{aligned}$$

where

$$E_t\left[\left(\omega+\rho r_{t+s}\right)^2\exp\left(-ur_{t+s}\right)\right]=\omega^2E_t\left[\exp\left(-ur_{t+s}\right)\right]+2\omega\rho E_t\left[r_{t+s}\exp\left(-ur_{t+s}\right)\right]+\rho^2E_t\left[r_{t+s}^2\exp\left(-ur_{t+s}\right)\right],$$

$$E_t\left[\exp\left(-ur_{t+s}\right)\right]=\exp\left(\mu(u,s)+\phi(u,s)r_t\right),$$

and where

$$\begin{aligned} \mu(u,s) &= \frac{\sigma^2}{2a}\left(1-e^{-2as}\right)\frac{u^2}{2}-B(s)abu \\ &= \frac{\sigma^2u^2}{4}B(2s)-B(s)abu \\ \phi(u,s) &= -ue^{-as}. \end{aligned}$$

Since

$$\begin{aligned} E_t\left[r_{t+s}\exp\left(-ur_{t+s}\right)\right] &= -\left(\frac{\sigma^2}{2}B(2s)u-B(s)ab-e^{-as}r_t\right)E_t\left[\exp\left(-ur_{t+s}\right)\right], \\ E_t\left[r_{t+s}^2\exp\left(-ur_{t+s}\right)\right] &= \left(B(2s)\frac{\sigma^2}{2}\right)E_t\left[\exp\left(-ur_{t+s}\right)\right]+ \\ &\quad \left(\frac{\sigma^2}{2}B(2s)u-B(s)ab-e^{-as}r_t\right)^2E_t\left[\exp\left(-ur_{t+s}\right)\right], \end{aligned}$$

we get

$$\begin{aligned} & E_t\left[\left(\omega+\rho r_{t+s}\right)^2\exp\left(-ur_{t+s}\right)\right] \\ = & \left[\begin{array}{c} \omega^2-2\omega\rho\left(\frac{\sigma^2}{2}B(2s)u-B(s)ab-e^{-as}r_t\right)+\frac{\rho^2\sigma^2}{2}B(2s) \\ +\rho^2\left(\frac{\sigma^2}{2}B(2s)u-B(s)ab-e^{-as}r_t\right)^2 \end{array}\right]E_t\left[\exp\left(-ur_{t+s}\right)\right] \\ = & \left[\left(\rho\left(\frac{\sigma^2}{2}B(2s)u-B(s)ab-e^{-as}r_t\right)-\omega\right)^2+\frac{\rho^2\sigma^2}{2}B(2s)\right]E_t\left[\exp\left(-ur_{t+s}\right)\right]. \end{aligned}$$

Price of European options on futures

The price of a European call option on the futures, $f(t, h_1, h_2)$, with maturity h and strike $\bar{f}$, is

$$\begin{aligned}
& C(t,h,h_1,h_2) \\
= \; & E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\
= \; & E_t\left[E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\middle| r_{t+h}\right]\left(f(t+h)-\bar{f}\right)1_{[f>\bar{f}]}\right].
\end{aligned}$$

Joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$

One of the ways to characterize the joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$ is through the moment-generating function,

$$V(h,r) = E_t\left[\exp\left(-u r_{t+h} - v\int_t^{t+h} r_s d_s\right)\right].$$

V is the solution to the Feynman-Kac PDE and the boundary condition,

$$\begin{aligned}
-V_h + \frac{1}{2}\sigma^2 V_{rr} + a(b-r)V_r - vrV &= 0, \\
V(0,r) &= \exp(-ur).
\end{aligned} \tag{11}$$

We conjecture that

$$V(h,r) = \alpha(h)\exp(\gamma(h)r).$$

Then we have

$$\begin{aligned}
V_h &= (\dot{\alpha} + \alpha\dot{\gamma}r)\exp(\gamma r) \\
V_r &= \alpha\gamma\exp(\gamma r) \\
a(b-r)V_r &= (ab\alpha\gamma - a\alpha\gamma r)\exp(\gamma r) \\
V_{rr} &= \alpha\gamma^2\exp(\gamma r) \\
\gamma(0) &= -u, \alpha(0) = 1.
\end{aligned}$$

Plugging these expressions into equation (11) yields

$$\begin{aligned}
-\dot{\alpha} - \alpha\dot{\gamma}r + \frac{1}{2}\sigma^2\alpha\gamma^2 + ab\alpha\gamma - a\alpha\gamma r - v\alpha r &= 0 \\
-\dot{\alpha} + \frac{1}{2}\sigma^2\alpha\gamma^2 + ab\alpha\gamma &= 0 \\
-\dot{\gamma} - a\gamma - v &= 0.
\end{aligned}$$

Hence,

$$
\begin{aligned}
\gamma &= -e^{-ah}u - B(h)\,v \\
\alpha &= \exp\left(\begin{array}{c} ab\left(-B(h)\,u - \frac{h-B(h)}{a}v\right) \\ +\frac{\sigma^2}{2}\left(\frac{u^2}{2}B(2h) + \frac{v^2}{a^2}\left(h - 2B(h) + \frac{B(2h)}{2}\right) + \frac{2uv}{a}\left(B(h) - \frac{B(2h)}{2}\right)\right) \end{array}\right) \\
&= \exp\left(\begin{array}{c} -abB(h)\,u - b\,(h - B(h))\,v \\ +\frac{\sigma^2 B(2h)}{4}u^2 + \frac{\sigma^2}{a^2}\left(\frac{h}{2} - B(h) + \frac{B(2h)}{4}\right)v^2 + \frac{\sigma^2}{a}\left(B(h) - \frac{B(2h)}{2}\right)uv \end{array}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
& E_t\left[\exp\left(-ur_{t+h} - v\int_t^{t+h} r_s d_s\right)\right] \\
= {} & \exp\left(\begin{array}{c} -\left(abB(h) + e^{-ah}r_t\right)u - \left(b\,(h - B(h)) + B(h)\,r_t\right)v \\ +\frac{\sigma^2 B(2h)}{4}u^2 + \frac{\sigma^2}{a^2}\left(\frac{h}{2} - B(h) + \frac{B(2h)}{4}\right)v^2 + \frac{\sigma^2}{a}\left(B(h) - \frac{B(2h)}{2}\right)uv \end{array}\right).
\end{aligned}
$$

Therefore, the joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$ is

$$
\begin{pmatrix} r_{t+h} \\ \int_t^{t+h} r_s d_s \end{pmatrix} \sim N\left(\begin{pmatrix} b + e^{-ah}(r_t - b) \\ bh + B(h)(r_t - b) \end{pmatrix}, \begin{bmatrix} \frac{\sigma^2 B(2h)}{2} & \frac{\sigma^2}{a}\left(B(h) - \frac{B(2h)}{2}\right) \\ \frac{\sigma^2}{a}\left(B(h) - \frac{B(2h)}{2}\right) & \frac{\sigma^2}{a^2}\left(h - 2B(h) + \frac{B(2h)}{2}\right) \end{bmatrix}\right),
$$

and the conditional distribution of $\int_t^{t+h} r_s d_s$ given r_{t+h} is

$$
\begin{aligned}
\int_t^{t+h} r_s d_s \Big| r_{t+h} &\sim N\left(\begin{array}{c} bh + \left(B(h) - e^{-ah}\frac{(2B(h)-B(2h))}{aB(2h)}\right)(r_t - b) \\ +\frac{(2B(h)-B(2h))}{aB(2h)}(r_{t+h} - b), \left(\frac{\sigma^2}{a^2}\frac{\frac{B(2h)}{2}\left(h-2B(h)+\frac{B(2h)}{2}\right)-\left(B(h)-\frac{B(2h)}{2}\right)^2}{\frac{B(2h)}{2}}\right) \end{array}\right) \\
&\sim N\left(\begin{array}{c} bh + \left(\frac{2aB(h)^2 - 2B(h) + B(2h)}{aB(2h)}\right)(r_t - b) \\ +\frac{(2B(h)-B(2h))}{aB(2h)}(r_{t+h} - b), \frac{\sigma^2}{a^2}\frac{hB(2h)-2B(h)^2}{B(2h)} \end{array}\right) \\
&\sim N\left(\begin{array}{c} bh + \frac{2B(h)-B(2h)}{aB(2h)}(r_t - b) \\ +\frac{(2B(h)-B(2h))}{aB(2h)}(r_{t+h} - b), \frac{\sigma^2}{a^2}\frac{hB(2h)-2B(h)^2}{B(2h)} \end{array}\right) \\
&\sim N\left(bh + \frac{(2B(h) - B(2h))}{aB(2h)}(r_{t+h} - b + r_t - b), \frac{\sigma^2}{a^2}\frac{hB(2h) - 2B(h)^2}{B(2h)}\right).
\end{aligned}
$$

Going back to option price

Given the expression for the joint distribution of r_{t+h} and $\int_t^{t+h} r_s d_s$, we can compute the price of a European call option on futures as follows:

$$
\begin{aligned}
& C(t,h,h_1,h_2) \\
= \; & E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\
= \; & E_t\left[E_t\left[\exp\left(-\int_t^{t+h} r_s d_s\right)\middle| r_{t+h}\right]\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\
= \; & E_t\left[\begin{array}{c}\exp\left(-\left(bh+\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b+r_t-b\right)\right)+\frac{\sigma^2}{2a^2}\frac{hB(2h)-2B(h)^2}{B(2h)}\right) \\ \left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\end{array}\right] \\
= \; & \exp\left(-\left(bh+\frac{(2B\left(h\right)-B\left(2h\right))}{aB\left(2h\right)}\left(r_t-b\right)\right)+\frac{\sigma^2}{2a^2}\frac{hB\left(2h\right)-2B\left(h\right)^2}{B\left(2h\right)}\right)\times \\
& E_t\left[\exp\left(-\frac{(2B\left(h\right)-B\left(2h\right))}{aB\left(2h\right)}\left(r_{t+h}-b\right)\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right],
\end{aligned}
$$

where

$$
\begin{aligned}
& E_t\left[\exp\left(-\frac{(2B\left(h\right)-B\left(2h\right))}{aB\left(2h\right)}\left(r_{t+h}-b\right)\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}\right] \\
= \; & \frac{1}{\sigma\sqrt{\pi B\left(2h\right)}}\int_{-\infty}^{+\infty}\exp\left(\begin{array}{c}-\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b\right) \\ -\frac{\left(r_{t+h}-b-e^{-ah}(r_t-b)\right)^2}{\sigma^2 B(2h)}\end{array}\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}dr_{t+h} \\
= \; & \frac{1}{\sigma\sqrt{\pi B\left(2h\right)}}\int_{-\infty}^{+\infty}\exp\left(\begin{array}{c}-\frac{(2B(h)-B(2h))}{aB(2h)}\left(r_{t+h}-b\right) \\ -\frac{\left(r_{t+h}-b-e^{-ah}(r_t-b)\right)^2}{\sigma^2 B(2h)}\end{array}\right)\left(f\left(t+h\right)-\bar{f}\right)1_{[f>\bar{f}]}dr_{t+h}.
\end{aligned}
$$

Estimation strategy

We use options that meet the following criteria:

- The option has either non-zero open interest or non-zero trade on a given day.
- The option's price is greater than the minimum price. For OBX, this is 1 basis point in quote, which is equivalent to \$25.
- The option's days to maturity is between 30 and 180 days.

The Vasicek (1977) model has three unknown parameters: a, b and σ. We adopt a two-stage estimation procedure, described below:

1. In the first stage, we use the option valuation errors defined as

$$e_j = C_j^{Mkt} - C_j^{Mod},$$

and apply the Gaussian log likelihood

$$\ln L^O \propto -\frac{1}{2}\sum_{j=1}^{N}\left\{\ln\left(RMSE^2\right) + e_j^2/RMSE^2\right\}, \tag{12}$$

where

$$RMSE \equiv \sqrt{\frac{1}{N}\sum_{j=1}^{N} e_j^2}.$$

Using options prices in the past one-year period, we maximize this log-likelihood, $\ln L^O$, to compute the first-stage estimates, $\hat{a}$, $\hat{b}$ and $\hat{\sigma}$. N is the number of option contracts available.

2. In the second stage, we fix a and b to their first-stage estimates, and then apply the Gaussian log likelihood

$$\ln L_t^O \propto -\frac{1}{2}\sum_{j=1}^{N_t}\left\{\ln\left(RMSE_t^2\right) + e_j^2/RMSE_t^2\right\}, \tag{13}$$

where

$$RMSE_t \equiv \sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t} e_j^2}.$$

Each day, we maximize this log-likelihood, $\ln L_t^O$, to compute the second-stage estimate $\hat{\sigma}_t$. N_t is the number of option contracts available on day t. In this second stage, we fix the initial value of σ to its first-stage estimate $\hat{\sigma}$.

3. Finally, for a given horizon and a given day t, we use $\hat{a}$, $\hat{b}$ and $\hat{\sigma}_t$ to compute the Q-expectation of future integrated variance given in equation (10).

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Table 1. Liquidity of BAX and OBX

Year	BAX (average daily)				OBX (average daily)			
	All Dates		* FADs		All Dates		* FADs	
	Volume (x1000)	# Trades	Volume (x1000)	# Trades	Volume (x1000)	# Trades	Volume (x1000)	# Trades
2005	36	995	57	1478	2	4	3	5
2006	52	1120	100	1784	3	5	3	6
2007	48	1303	74	1605	3	5	5	7
2008	31	1478	48	1998	1	3	1	3
2009	24	1084	38	1595	0	0	0	0
2010	44	2125	89	3499	2	4	10	9
2011	74	3454	123	5173	2	6	6	11
2012	91	3969	130	4951	-	-	-	-
2013	98	3836	150	5773	-	-	-	-
all years	44	1660	71	2379	2	4	4	6

* Fixed announcement dates: Bank of Canada's pre-scheduled policy rate announcement dates.

** We do not have volume and trade data for OBX contracts in 2012 and 2013.

Table 2. Average daily number of OBX options by maturity

Year	All maturities	< 1 month	1-3 months	3-6 months	> 6 months
2006	17	2	4	7	4
2007	26	2	6	11	7
2008	21	2	7	8	3
2010	23	2	5	9	7
2011	29	2	5	10	12
2012	24	2	5	9	9
2013	23	1	2	6	15

Table 3. Important Bank of Canada policy actions between 2007 and 2012

Date	Event description	Daily change in IV (bps)
PANEL A: Crisis related policy actions, 2007–10		
2007-Dec-04	(Not a FAD*) Term Purchase and Resale Agreements (PRAs) announced for liquidity purposes (along with the Bank of England, the European Central Bank, the Federal Reserve, and the Swiss National Bank)	-12
2008-Mar-11	(Not a FAD) Term PRAs announced for liquidity purposes, coordinated with other G10 central banks	-25
2008-Oct-08	(Not a FAD) Unscheduled cut in target rate coordinated with other central banks	-51
2010-Apr-20	(FAD) Removal of conditional commitment/forward guidance	-18
PANEL B: Change in monetary policy bias, 2011–12		
2011-May-31	(FAD) Tightening bias is introduced.	-10
2011-Sep-07	(FAD) Tightening bias is removed.	-26
2012-Apr-17	(FAD) Tightening bias is reintroduced.	-10
2012-Oct-23	(FAD) Tightening bias is pushed out.	-14
	Average - all days	0
	Average - FADs	-5
	Average - non FADs	1

∗ Fixed announcement date: Bank of Canada's pre-scheduled policy rate announcement date.

Table 4. BAX IV around the Bank of Canada policy rate announcements

For each event day, $i \in [-5, +10]$, which denotes the number of business days from a fixed announcement date (FAD), we run the following regressions:

$$IV_t = \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it},$$

$$\Delta IV_t = \alpha_i + \beta_i \cdot \mathrm{I}_i(t) + \varepsilon_{it},$$

where IV_t and ΔIV_t are the daily level and the daily change in implied volatility of BAX, and $\mathrm{I}_i(t)$ is an indicator function that yields 1 if date *t* is $i \in [-5, +10]$ business days away from a FAD, and 0 otherwise. *t*-statistics (for change in IV) and Newey and West (1987) *t*-statistics with an eight-month lag (for level of IV) that are significant at the 95% confidence level are highlighted in bold red. The unit is percent (e.g., -0.07 is – 7 bps).

# business days	Level of IV				Change in IV			
from FAD	α_i	nw *t*-stat.	β_i	nw *t*-stat.	α_i	*t*-stat.	β_i	*t*-stat.
-5	1.767	**(5.431)**	-0.056	(-0.765)	-0.001	(-0.118)	0.015	(0.588)
-4	1.766	**(5.420)**	-0.014	(-0.248)	0.001	(0.115)	-0.017	(-0.717)
-3	1.765	**(5.418)**	0.023	(0.375)	-0.001	(-0.309)	0.038	(1.589)
-2	1.769	**(5.464)**	-0.132	(-1.398)	0.001	(0.147)	-0.028	(-1.032)
-1	1.765	**(5.419)**	0.025	(0.359)	0.000	(-0.032)	0.002	(0.085)
0	1.767	**(5.428)**	-0.043	(-0.561)	0.002	(0.506)	-0.068	**(-2.867)**
1	1.767	**(5.430)**	-0.033	(-0.470)	0.000	(-0.070)	0.007	(0.289)
2	1.770	**(5.436)**	-0.132	**(-2.048)**	0.002	(0.368)	-0.052	**(-2.138)**
3	1.772	**(5.468)**	-0.238	**(-3.268)**	0.001	(0.106)	-0.020	(-0.743)
4	1.770	**(5.440)**	-0.137	**(-2.535)**	0.000	(0.023)	-0.005	(-0.219)
5	1.771	**(5.449)**	-0.173	**(-3.246)**	0.001	(0.172)	-0.026	(-1.072)
6	1.769	**(5.446)**	-0.114	**(-2.321)**	-0.001	(-0.206)	0.025	(1.051)
7	1.770	**(5.453)**	-0.137	**(-2.405)**	0.000	(0.015)	-0.004	(-0.180)
8	1.767	**(5.469)**	-0.065	(-0.626)	-0.001	(-0.248)	0.037	(1.399)
9	1.765	**(5.444)**	0.024	(0.432)	0.000	(-0.002)	-0.002	(-0.084)
10	1.764	**(5.423)**	0.053	(0.812)	-0.001	(-0.106)	0.012	(0.497)

Table 5. Correlations

This table reports the correlations of monthly series of the variables between January 2006 and February 2013, excluding the period between November 2008 and March 2010, when BAX IV could not be computed. IV and RV are implied volatility and 3-month maturity realized volatility of BAX observed on the last day of the month. CBS is the spread between BBB-rated 10-year corporate bonds and the 10-year Treasury yield. TYS is the spread between 2-year and 5-year Treasury yields. EG is the employment growth. PU is the index of Canadian economic policy uncertainty published by Baker, Bloom and Davis (2013).

	IV	RV	CBS	TYS	EG	PU
IV	1.00	0.51	0.71	0.20	-0.60	0.50
RV		1.00	0.56	0.16	-0.14	0.38
CBS			1.00	0.65	-0.61	0.81
TYS				1.00	-0.32	0.82
EG					1.00	-0.54
PU						1.00

Table 6. Forecasting OIS excess returns, univariate regressions

We report the results of the following univariate regressions:

$$R_{OIS}(t, t+n) = \alpha + \beta \cdot X_t + \varepsilon_{t,}$$

where $R_{OIS}(t, t+n)$ is the prediction error of the n-month OIS contract observed on the last day of month t. X_t denotes each of the variables listed in the table below observed on the last day of month t. The maturity of RV is matched with the forecasting horizon of the regression. t-statistics that are significant at the 95% confidence level are highlighted in bold red.

	OIS Maturity	1	2	3	4	5	6	9
IV	β	0.00	0.00	0.01	0.02	0.03	0.03	0.08
	t-stat.	(-0.36)	(0.23)	(1.02)	(1.16)	(1.17)	(1.30)	**(2.12)**
	adj. R^2	-0.02	-0.02	0.00	0.01	0.01	0.01	0.07
RV	β	0.02	0.06	0.11	0.16	0.21	0.28	0.54
	t-stat.	(1.33)	**(2.26)**	**(2.94)**	**(2.82)**	**(2.80)**	**(2.95)**	**(4.69)**
	adj. R^2	0.01	0.07	0.12	0.11	0.11	0.12	0.28
Corporate bond yield spread	β	0.01	0.03	0.06	0.08	0.10	0.12	0.17
	t-stat.	(0.80)	**(2.17)**	**(3.23)**	**(3.39)**	**(3.50)**	**(3.52)**	**(3.41)**
	adj. R^2	-0.01	0.06	0.15	0.16	0.17	0.17	0.16
Treasury yield term spread	β	0.01	0.02	0.03	0.04	0.05	0.06	0.06
	t-stat.	(0.92)	(1.42)	(1.40)	(1.17)	(1.18)	(1.11)	(0.83)
	adj. R^2	0.00	0.02	0.02	0.01	0.01	0.00	-0.01
Employment growth	β	0.01	-0.03	-0.05	-0.08	-0.09	-0.11	-0.12
	t-stat.	(0.28)	(-0.58)	(-0.72)	(-0.90)	(-0.81)	(-0.78)	(-0.60)
	adj. R^2	-0.02	-0.01	-0.01	0.00	-0.01	-0.01	-0.01
Policy uncertainty	β	0.02	0.06	0.09	0.12	0.15	0.18	0.18
	t-stat.	(0.92)	**(2.29)**	**(2.67)**	**(2.63)**	**(2.62)**	**(2.43)**	(1.65)
	adj. R^2	0.00	0.07	0.10	0.10	0.10	0.08	0.03
$R_{OIS}(t-3, t)$	β	-0.01	0.00	0.01	0.02	0.02	0.03	0.05
	t-stat.	(-1.88)	(-0.48)	(1.00)	(1.30)	(1.49)	(1.55)	(1.69)
	adj. R^2	0.04	-0.01	0.00	0.01	0.02	0.03	0.03
$\|R_{OIS}(t-3, t)\|$	β	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.02
	t-stat.	(-1.41)	(-1.11)	(-0.41)	(-0.72)	(-0.79)	(-0.85)	(-0.75)
	adj. R^2	0.02	0.00	-0.02	-0.01	-0.01	-0.01	-0.01

Table 7. Forecasting OIS excess returns, multivariate regression

We report the results of the following multivariate regressions:

$$R_{OIS}(t, t+n) = \alpha + \overline{X_t} \cdot \bar{\beta} + \varepsilon_t,$$

where $R_{OIS}(t, t+n)$ is the prediction error of the n-month OIS contract observed on the last day of month t. $\overline{X_t}$ is a 1×8 vector containing the variables listed in the table below observed on the last day of month t. $\bar{\beta}$ is an 8×1 vector of slope coefficients. The maturity of RV is matched with the forecasting horizon of the regression. t-statistics that are significant at the 95% confidence level are highlighted in bold red.

	OIS Maturity	1	2	3	4	5	6	9
Constant	α	-0.04	-0.12	-0.20	-0.24	-0.29	-0.30	-0.21
	t-stat.	(-1.21)	**(-2.89)**	**(-3.41)**	**(-3.35)**	**(-3.41)**	**(-3.07)**	(-1.91)
IV	β	-0.01	-0.02	-0.04	-0.06	-0.10	-0.13	-0.19
	t-stat.	(-0.83)	(-1.53)	**(-2.00)**	**(-2.70)**	**(-3.66)**	**(-4.43)**	**(-5.32)**
RV	β	0.03	0.05	0.05	0.06	0.07	0.07	0.10
	t-stat.	(1.41)	(1.61)	(1.11)	(0.91)	(0.85)	(0.76)	(0.94)
Corporate bond yield spread	β	0.06	0.10	0.20	0.33	0.49	0.67	1.12
	t-stat.	(1.67)	**(2.15)**	**(3.14)**	**(4.49)**	**(5.73)**	**(7.20)**	**(9.79)**
Treasury yield term spread	β	-0.03	-0.08	-0.15	-0.22	-0.32	-0.41	-0.55
	t-stat.	(-1.02)	**(-2.19)**	**(-2.87)**	**(-3.63)**	**(-4.53)**	**(-5.22)**	**(-5.68)**
Employment growth	β	0.04	0.09	0.18	0.19	0.23	0.22	0.06
	t-stat.	(0.81)	(1.32)	(1.92)	(1.81)	(1.91)	(1.65)	(0.38)
Policy uncertainty	β	0.01	0.09	0.11	0.10	0.10	0.05	-0.22
	t-stat.	(0.29)	(1.58)	(1.47)	(1.10)	(1.03)	(0.48)	(-1.53)
$R_{OIS}(t-3, t)$	β	-0.01	-0.01	-0.02	-0.03	-0.04	-0.05	-0.05
	t-stat.	(-1.21)	(-1.41)	(-1.63)	**(-2.62)**	**(-2.83)**	**(-3.02)**	**(-2.40)**
$\lvert R_{OIS}(t-3, t)\rvert$	β	-0.03	-0.02	-0.01	-0.02	-0.01	-0.02	-0.03
	t-stat.	**(-2.80)**	(-1.51)	(-0.52)	(-0.73)	(-0.60)	(-0.80)	(-0.87)
adj. R^2 [all predictors]		0.18	0.31	0.40	0.55	0.65	0.72	0.81
adj. R^2 [exclude IV and RV]		0.17	0.26	0.36	0.49	0.55	0.60	0.69