The Safety of Government Debt

by Kartik Anand and Prasanna Gai
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Abstract

We examine the safety of government bonds in the presence of Knightian uncertainty amongst financial market participants. In our model, the information insensitivity of government bonds is driven by strategic complementarities across counterparties and the structure of trading relationships. We identify the frontier between safe and unsafe assets and show how the growth rate of the economy and its fiscal capacity interact with the degree of ambiguity amongst investors to determine the safe-asset equilibrium. We use our framework to illustrate a fiscal variation on the Triffin dilemma, in which the role of a country as sole provider of international liquidity is questioned as its size in the world economy – and hence its fiscal capacity – diminishes.

JEL classification: E44, D81, F02, F41, G15
Bank classification: Economic models; International financial markets; Financial stability

Résumé

Les auteurs examinent la sécurité des obligations d’État en situation d’incertitude knightienne sur les marchés financiers. Dans leur modèle, l’insensibilité des obligations d’État à l’information dépend des complémentarités stratégiques entre les contreparties et de la structure des rapports de négociation. Les auteurs établissent la frontière qui sépare les actifs sûrs des actifs risqués et montrent comment l’interaction entre, d’une part, le taux de croissance de l’économie et la capacité budgétaire de l’État et, d’autre part, le degré d’ambiguïté du comportement des investisseurs détermine l’équilibre entre l’offre et la demande d’actifs sûrs. Anand et Gai se servent de leur cadre conceptuel pour illustrer une variante du dilemme de Triffin dans laquelle le rôle d’un pays en tant que seul fournisseur de liquidité internationale est remis en cause du fait que son poids dans l’économie mondiale – et donc sa capacité budgétaire – diminue.

Classification JEL : E44, D81, F02, F41, G15
Classification de la Banque : Modèles économiques; Marchés financiers internationaux; Stabilité financière
1. Introduction

Investors typically perceive government bonds – particularly those issued by the advanced economies – as “safe” and so use them as liquid collateral in a wide range of financial transactions. But, as recent events in Europe have demonstrated, assets that are supposedly “safe” can easily lose that quality as the frontier between “safe” and “unsafe” assets shifts over time (Gourinchas and Jeanne, 2012). The downgrading of government debt in the euro area led to a sharp rise in the haircuts applied by transacting counterparties, constraining the availability of secured financing. Figure 1 shows how margin requirements were triggered once peripheral euro area government bond spreads breached a threshold of 450 basis points.¹

![Figure 1: 10-year peripheral euro area government bond spreads against German bunds (in basis points).](image)

In an important recent contribution, Dang et al. (2010) highlight the fact that safe assets are “information insensitive” – the safer the asset, the less its payoff is affected by new information, allowing investors to agree on the value of the security without collecting costly information about underlying collateral. But when doubts concerning collateral quality (or borrower solvency, in the case of government debt) emerge, it becomes profitable for potential sellers and buyers of the security to collect information on the issuer and reassess counterparty risk. In the limit, funding markets may shut down in the face of heightened adverse selection and suspicion about the motives for trade.

The health of the issuer is, however, not the sole factor driving the safety of government bonds.

¹The Bank of England (2012) estimates that, in the event of a reappraisal of U.S. Treasuries, some $850 million of additional collateral would be required for every 50 basis point increase in haircuts on U.S.-backed collateral in bilateral over-the-counter derivatives markets.
The behavior of participants in the market in which the bond is used as collateral is critical in determining whether it is traded willingly in “money-like” fashion. The proliferation of cross-border financial trades and the complex pattern of interlinked counterparty risk in modern financial markets means that while investors may be able to infer the likely behavior of their immediate counterparties, they are faced with greater (Knightian) uncertainty regarding the actions of their counterparties’ counterparties and each of these counterparties’ counterparties. The more complex the financial system, the greater is that uncertainty.

In this paper, we develop a model to clarify how the frontier between safe and unsafe government debt is determined endogenously and relates to factors such as fiscal capacity and differences in investor opinion. In so doing, we explicitly accord a role for ambiguity about the actions of other counterparties in asset trades. Multiple equilibria emerge in our set-up due to strategic complementarities across counterparties, and the safety of the bond depends on the extent to which investors’ opinions diverge from the credit rating of the asset. The smaller the divergence, the more likely the bond is information insensitive. But as investor opinion becomes increasingly distinct from the official certification, a safe bond can suddenly transform into an illiquid and informationally sensitive asset.

We demonstrate that the frontier between safe and unsafe government bonds is non-linearly related to macroeconomic fundamentals, namely the growth rate of the economy and the fiscal capacity of the government. The results are intuitive. High growth rates and sizable fiscal capacity can support information-insensitive (and hence safe) assets. But other combinations may also be feasible. When growth rates are low, safe assets can continue to be supported when fiscal capacity is large. Conversely, when growth rates are high, a weak fiscal position may be consistent with the information insensitivity of the bond. As differences in opinion on the bond’s characteristics diminish, the safe asset is supported for a wider range of fundamentals as the frontier between safe and unsafe assets shifts downward.

The possibility that a safe government bond can suddenly become unsafe has implications for the real interest rate. The advent of new information that adversely affects perceptions of safety can lead to a fall in the equilibrium interest rate in order to maintain the value of the bond. High and low interest rate solutions emerge as a consequence. Substantial differences of opinion about bond characteristics can be especially damaging in this context. The economy may well find itself stuck in a low interest rate “liquidity trap” following a credit market shock. But our results suggest that Knightian uncertainty, or ambiguity, about the decision processes of counterparties can help mitigate these effects. Greater uncertainty imparts a “safe haven” effect, as agents are more willing to trade the bond despite declining fiscal capacity and divergent opinions.

In modelling asset safety in the presence of Knightian uncertainty, we treat as discrete the
choice of whether to monitor the bond, and suppose that investors have multiplier preferences in the spirit of Hansen and Sargent (2007). Under this approach, investors take into account the possibility that their counterparties may base their investment decisions on quite different models to their own baseline model. Formally, the relative likelihood of these alternative models is given by their relative entropy, which measures the “distance” between two probability distributions. A positive parameter, \( \theta \), captures the weight ascribed by investors to the probability that their baseline model is the true model, and can be regarded as an index of ambiguity. In the Nash equilibrium of the model, the optimal choice probabilities in the decision problem take the generalized logit form.\(^2\)

As an application of our framework, we illustrate the fiscal variation of the Triffin dilemma highlighted by Obstfeld (2011) and Farhi et al. (2011) in the debate on the future role of U.S.-dollar assets as a global store of value. We extend the model to a two-region setting in which one region (the United States) provides a global safe asset. In return for providing liquidity to the world economy, that region has a negative foreign asset position. But the region’s relative size influences the reach of its fiscal policy and hence its ability to maintain the global safe asset. As the region gradually shrinks in relation to the world economy, a tipping point is reached where its fiscal capacity can no longer support information insensitivity and investor confidence collapses.

Our paper is related to several strands of literature. First, in taking a general-equilibrium perspective on the macroeconomic effects of safe-asset shortages, we build on the seminal contribution of Caballero et al. (2008), as well as recent work by Gourinchas and Jeanne (2012) and Caballero and Farhi (2013). Second, our focus on the information sensitivity of assets draws on insights developed by Gorton and Pennacchi (1990) and Dang et al. (2010). Alternative perspectives on the acquisition of information by investors are presented in Yang (2013) and Farhi and Tirole (2012), while Gorton and Ordoñez (2013) show how government bonds, by replacing private safe assets, constitute net wealth in a crisis. Finally, our analysis of fiscal capacity and the Triffin dilemma complements the literature on international monetary reform, including Gourinchas and Rey (2007), Gourinchas et al. (2010), Farhi et al. (2011) and Obstfeld (2011).

The paper proceeds as follows. In section 2, we describe the market for safe assets, showing how information sensitivity arises endogenously as the (steady-state) outcome of the interaction between financial market participants. Section 3 characterizes the frontier between safe and unsafe assets and shows how it is related to fiscal capacity and economic growth. Section 4 reports comparative static results for the equilibrium real interest rates in the model, and illustrates the

\(^2\)In this respect, our results share common ground with the literature on social dynamics (e.g., Darlauf and Young 2001; Young 2011); probabilistic choice under bounded rationality (i.e., Mattsson and Weibull 2002), and rational inattention to discrete choices (Matejka and McKay 2013).
consequences of Knightian uncertainty about the behavior of investors’ counterparties. Section 5 then applies the framework to the Triffin “dilemma.” We recast the model into a two-region world economy to consider the implications, for the world real interest rate and the net foreign asset position, of a shock that permanently reduces the share of the safe-asset issuing country in the world economy. A final section concludes.

2. The Model

Our set-up is inspired by Gourinchas and Jeanne (2012), who, in turn, extend the framework of Caballero et al. (2008) to study the market for global saving instruments. We develop our model in three steps. First, we describe the market for global safe assets. Second, we portray the decision by agents to collect information about characteristics of the bond issue. And third, we characterize the Nash equilibrium of the model under simplifying assumptions about the structure of trading relationships.

2.1. The market for safe assets

The world economy consists of \( N \) islands evolving in discrete time steps \( t = 1, 2, 3, \ldots \). There is a single financial asset – a bond – that is a perpetual claim to a fraction \( \delta \) of global GDP, namely the sum of resources on all islands, \( X_t \). Despite yielding a dividend \( \delta X_t \), there is some default risk associated with the bond, which we describe below. The world economy is assumed to grow at a constant rate, \( g \).

The population mass is constant and equal to one. In each period \( t \), a risk-neutral agent is born on every island and is economically active for the period, before dying at the start of period \( t + 1 \) to make way for the next generation. Agents born in period \( t \) receive a perishable endowment \( (1 - \delta)X_t \) at birth, but can only consume their resources at the time of death. This discrepancy between income and expenditure creates a demand for stores of value. Agents can purchase a bond from agents on islands with which there is a trading relationship. The full set of bilateral trading opportunities is represented by the adjacency matrix \( A \in \{0, 1\}^{N \times N} \), where \( A_{ij} = A_{ji} = 1 \) implies that a (symmetric) trading opportunity exists between residents of islands \( i \) and \( j \). We denote the set of possible trading partners for island \( j \) by \( \mathcal{N}_j = \{ i | A_{ij} = 1 \} \), and the number of trading partners by \( k_j = |\mathcal{N}_j| \).

An agent born on island \( j \) in period \( t \) thus purchases a bond from an agent born in period \( t - 1 \), consumes the dividend at the end of period \( t \) and sells the bond at the start of period \( t + 1 \) to a younger-generation agent residing on another island with which there are trading relations. The younger agent (residing on island \( i \)) to whom the bond is sold is randomly selected from the set of islands, \( \mathcal{N}_j \), with probability \( 1/k_j \). Any capital gain from the sale of the bond is consumed by
agent $j$ just prior to the agent’s death, at the start of period $t + 1$. We assume that agents born at $t = 0$ are each endowed with a bond.

Defaults are idiosyncratic credit events that are independent across time periods. Following Gourinchas and Jeanne (2012), we interpret default as a political “revolution” in which current bondholders are hurt to the benefit of future generations. Accordingly, the probability of default of the bond is either “high” or “low.” In the “high” default state, the authorities expropriate current bond holders for sure, while in the “low” state expropriate risk, as measured by the parameter $\alpha \ll 1$, is modest.

In the event that default occurs in period $t$, the bond is taken from agent $j$ and handed to the agent’s offspring (i.e., the agent born on the same island in period $t + 1$). As a result, agent $j$ cannot consume either the dividend from the bond or the capital gains from its potential sale. Instead, the period $t$ agents consume the resources on their islands, which diminishes output available in the next period.\(^3\) If a young agent believes that the bond will default for sure, then that agent will not purchase the bond from an old agent, and trade breaks down. We assume that, at birth, all agents hold a prior belief, $\alpha$, on the probability of default.\(^4\)

The period $t$ utility function for agent $j$ is

$$E_t \left[ u(c_{j,t}) + \beta_t u(c_{j,t+1}) | z_{j,t} \right],$$

where $c_{j,t}$ and $c_{j,t+1}$ denote per period consumption, $u(\cdot)$ is an increasing convex utility function, $\beta_t = 1/(1 + r)$ is the discount rate, $r$ is the real interest rate, and $z_{j,t} \in (0, 1)$ is a discrete decision taken by the agent to gather information about expropriation risk at the time of purchase.\(^5\) The objective of agent $j$ is to select $z_{j,t}$ to maximize utility. In section 2.2 we characterize the binary choice problem facing agent $j$ and show how it can be represented by a probability distribution over alternatives.

The market value of the bond is given by the discounted sum of future dividends. Formally, at

\[E_t[X_{t+k}] = (1 - \alpha)(1 + g)E_t[X_{t+k-1}] + \alpha g E_t[X_{t+k-1}] = E_t[X_{t+k-1}] + (1 + g - \alpha)\]

\[E_t[X_{t+k-1}] = (1 - \alpha)(1 + g)E_t[X_{t+k-2}] + \alpha g E_t[X_{t+k-2}] = \cdots = E_t[X_t](1 + g - \alpha)^k.\]

\(^3\)Accordingly, the future expected value of GDP is

\[E_t[X_{t+k}] = (1 - \alpha)(1 + g)E_t[X_{t+k-1}] + \alpha g E_t[X_{t+k-1}] = E_t[X_{t+k-1}] + (1 + g - \alpha)\]

\[E_t[X_{t+k-1}] = (1 - \alpha)(1 + g)E_t[X_{t+k-2}] + \alpha g E_t[X_{t+k-2}] = \cdots = E_t[X_t](1 + g - \alpha)^k.\]

\(^4\)This prior belief might be the result of a pronouncement on the state of sovereign political risk by a credit-rating agency outside the model.

\(^5\)To ensure dynamic efficiency, we tacitly assume the presence of an additional island $i = N + 1$, in which the agent born at $t = 0$ is endowed with a bond and lives forever. In the event of default in period $t$, this agent does not consume the dividend in period $t$, but consumes it instead at period $t + 1$. In the “high” default state, the agent will consume only in every alternate period.
time $t$, the value of the bond is

$$
V_t = \sum_{j=0}^{\infty} \frac{\delta E_t [X_{t+j}]}{(1+r)^{j+1}} = \frac{\delta X_t}{r+\alpha-g},
$$

(2)

where we assume that $g < r + \alpha < 1$ so that the value of the bond is always positive. And, at the beginning of period $t+1$, the agent attempts to sell the bond to younger-generation agents for

$$
V_{t+1} = \frac{\delta X_{t+1}}{1+r} + \frac{\delta X_{t+2}}{(1+r)^2} + \ldots = \frac{\delta X_t (1+g)}{r+\alpha-g}
$$

(3)

netting a capital gain of $(V_{t+1} - V_t)/V_t = g$.

For the bond to serve as a secure promise of future repayment in the global economy, it should be “informationally insensitive” in the sense of Dang et al. (2010) – investors should not have any incentive to challenge the initial prior belief on default, $\alpha$, and attempt to gather information about expropriation risk. But such information insensitivity depends critically on the behavior of participants in the market in which the bond is traded. In particular, the willingness of a $t+1$ agent to trade the bond will depend on the actions of their younger-generation trading partners, which, in turn, depends on the actions of their younger-generation trading partners, and so on.

Let $\bar{\pi}_{t+1} \in [0,1]$ be the fraction of period $t+1$ agents who agree that the bond’s probability of default is $\alpha$. The expected return on the bond must equal the return on the bond if there is no default, minus the expected valuation loss in the event that the credit risk materializes. If all period $t+1$ agents are in agreement, the market is liquid and there is no delay in executing transactions. In this case, $\bar{\pi}_{t+1} = 1$ and the capital gains are realized in their entirety. But if some younger-generation agents have differing assessments of default risk, $\bar{\pi}_{t+1} < 1$, and delays in executing transactions lead to real costs, $C$, for the seller. Thus,

$$
rV_t = (1 - C[1 - \bar{\pi}_{t+1}]) (V_{t+1} - V_t) + \delta X_t - \alpha V_t.
$$

(4)

Without loss of generality, we set $C = 1$ in what follows.

The aggregate financial wealth of the population, $W_t$, evolves according to

$$
\bar{\pi}_{t+1} (W_{t+1} - W_t) = -W_t + (1 - \delta)X_t + r W_t - \alpha W_t.
$$

(5)

Equation (5) states that savings decreases with the death of agents, increases with the endowment allocated to new generations, increases with the return on the bond and decreases in default risk. In the event that $\bar{\pi} = 0$, the bond is not traded across islands and the wealth of an agent born in period $t$ does not grow, since capital gains cannot be realized. In the steady state (i.e., $\bar{\pi}_{t+1} = \bar{\pi}_t$), the market value of the bond must equal the aggregate wealth, so that $W_t = X_t$. Accordingly, the
equilibrium interest rate is
\[ r \equiv r(\bar{\pi}) = \delta + \bar{\pi} \; g - \alpha, \tag{6} \]
which is decreasing in the default risk, \( \alpha \), and increasing in agents’ agreement over default risk, \( \bar{\pi} \). When default risk increases, the real interest rate must decline to keep the value of the bond constant in equilibrium. And as more investors are willing to take on the bond, it becomes easier to trade and earn a capital gain. This dictates that the real interest rate must increase to keep the value of the bond constant.

2.2. Choice over alternatives

To ascertain the stationary value of \( \bar{\pi} \), we first characterize the period \( t \) decision by agent \( j \) on whether to gather information about expropriation risk. This decision is a discrete choice. In accepting the bond, agent \( j \) must decide whether to agree with the default risk, \( \alpha \), certified by the rating agency or, instead, engage in surveillance of the collateral underlying the bond. Denote by \( z_{j,t} = 1 \) the decision by agent \( j \) to accept the credit rating and value of the bond without monitoring, and denote by \( z_{j,t} = 0 \) the decision to conduct due diligence.

Monitoring is costly. Although agents are aware that the bond is backed by a fraction \( \delta \) of global GDP, they are unsure about the underlying makeup of the collateral. Some parts of the world economy may perform above par while others perform below par, and this can change over time. The greater the size of the global economy, the more due diligence the agent must conduct on the composition of GDP to properly evaluate the bond. We therefore assume that the cost of monitoring \( M_{j,t} \) scales with the present value of tradable output, so that \( M_{j,t} = \mu_j V_t \).

Figure 2 illustrates the decision tree facing agent \( j \). If the agent decides to monitor, then \( z_{j,t} = 0 \). With probability \( \gamma \), agent \( j \) finds the bond to have been incorrectly rated and no trade takes place. The payoff to the agent in this event is
\[ -M_{j,t} = -\mu_j V_t. \]

With probability \( 1 - \gamma \), agent \( j \) agrees with the certification of the rating agency and pays \( V_t \) to buy the bond. The agent earns both a dividend and capital gains to net a payoff\(^6\)
\[ V_{t+1} - V_t + \delta X_t - M_{j,t} = V_t \left( r + \alpha - \mu_j \right) . \tag{7} \]

---

\(^6\)While reduced form, our modelling of information acquisition captures a key feature from the recent literature on Bayesian updating and search models for information diffusion and percolation (e.g., Duffie et al. 2010; Livan and Marsili 2013). Specifically, when two agents meet and exchange information, the posterior belief held by both agents is the sum of their individually held beliefs. If, however, one of the agents has an uninformed prior, then a chance meeting with that agent does not alter the views held by the other agent. In this context, \( \gamma \) can be regarded as the probability that agent \( j \) acquires information from an informed agent, while \( 1 - \gamma \) is the probability that \( j \) is matched with an uninformed agent.
If, instead, agent $j$ decides to accept the value of the bond without collecting additional information, then $z_{j,t} = 1$. The payoff to agent $j$ now depends on whether younger-generation trading partners also accept that $\alpha$ is the probability of default, or opt to conduct their own due diligence. Defining $\bar{\pi}_{j,t+1}$ to be the fraction of $j$'s younger-generation trading partners who agree with $\alpha$ being the probability of default,

$$\bar{\pi}_{j,t+1} = \frac{1}{k_j} \sum_{l \in N_j} z_{l,t+1},$$

the probability that a randomly selected younger-generation investor checks the bond and decides not to accept it is $\gamma (1 - \bar{\pi}_{j,t+1})$. In this case, the payoff to agent $j$ is

$$-V_t + \delta X_t = -V_t (1 + g - (r + \alpha)).$$

With probability $1 - \gamma (1 - \bar{\pi}_{j,t+1})$, the randomly selected younger-generation agent accepts that the bond is correctly graded. Trade occurs and the payoff to $j$ is

$$V_{t+1} - V_t + \delta X_t = V_t (r + \alpha).$$

Note that the payoff to agent $j$ from choosing $z_{j,t} = 1$ is increasing in the fraction $\bar{\pi}_{t+1,j}$. Moreover, by recursively writing out the payoffs for $j$'s future-generation trading partners, their payoffs from choosing not to monitor are also increasing in the fraction of future-generation agents who do not monitor. There are therefore strategic complementarities across counterparties.

In summary, the expected payoff in period $t$ to agent $j$ from accepting the bond without monitoring is

$$u(z_{j,t} = 1) \equiv E_t \left[ u(c_{j,t}) + \beta_t u(c_{j,t+1}) | z_{j,t} = 1 \right] = V_t \left[ r + \alpha - \gamma (1 - \bar{\pi}_{j,t+1})(1 + g) \right],$$
while the expected payoff from conducting due diligence is

\[ u(z_{j,t} = 0) = \mathbb{E}_t \left[ u(c_{j,t}) + \beta_t u(c_{j,t+1}) \right] = V_t \left[ (1 - \gamma)(r + \alpha) - \mu_j \right]. \] (12)

We next characterize the choice between these alternatives. This decision hinges on the prior beliefs that agent \( j \) holds regarding the decisions of future generations to accept the bond without verifying its characteristics. Since agent \( j \) can entertain many different priors on the choice models of these unborn agents, we suppose that the agent has multiplier preferences in the spirit of Hansen and Sargent (2007). Specifically, agent \( j \)'s baseline guess, \( q \), for the probability distribution of future agents’ behavior is an approximation. The objective for agent \( j \) is to then select a plausible choice model \( p \) that satisfies the objective function

\[
\max_p \left( \sum_{z \in \{0, 1\}} p_z \mathbb{E}_t \left[ u(c_{j,t}) + \beta_t u(c_{j,t+1}) \right| z_{j,t} = \tilde{z} \right] - \frac{R(p|q)}{\theta} \right),
\] (13)

subject to \( p_0 + p_1 = 1 \), and where

\[
R(p|q) = \sum_{z \in \{0, 1\}} p_z \log \frac{p_z}{q_z}
\] (14)

is the relative entropy between the hypothesized choice model \( p \) and the default model \( q \).

According to this formulation, agent \( j \), with default choice \( q \), considers whether there is another choice model \( p \) that better represents the agent’s preferences and knowledge, but whose plausibility diminishes proportionately to its “distance” from \( q \). The parameter \( \theta \) thus measures the relative weight given by agent \( j \) to \( q \) being the correct model. In the limit \( \theta \to 0 \), agent \( j \) treats \( q \) as being the correct model, while higher values of \( \theta \) imply greater weight on alternate models. By contrast, in the limit \( \theta \to \infty \), the relative entropy term in equation (13) drops out and agent \( j \) chooses \( p \) to maximize expected utility. Maccheroni et al. (2006) suggest that \( 1/\theta \) can be interpreted as a coefficient of ambiguity aversion, since it is concern with model misspecification that leads the agent to exercise caution in choosing the course of action.

The first-order conditions for agent \( j \)'s optimization are

\[
\mathbb{E}_t \left[ u(c_{j,t}) + \beta_t u(c_{j,t+1}) \right| z_{j,t} = \tilde{z} \right] - \frac{\log p_z + 1 - \log q_z}{\theta} = 0,
\] (15)

which, on rearranging, yields

\[
p_{\tilde{z}} = q_{\tilde{z}} \exp \left( \theta \mathbb{E}_t \left[ u(c_{j,t}) + \beta_t u(c_{j,t+1}) \right| z_{j,t} = \tilde{z} \right] - 1 \right) .
\] (16)
Enforcing the normalization constraint gives the reaction function of agent \( j \), namely

\[
p_{\tilde{z}} = \frac{q_{\tilde{z}} \exp(\theta E_t \left[ u(c_j, t) + \beta_t u(c_{j,t+1}) | z_{j,t} = \tilde{z} \right])}{\sum_{z \in \{0, 1\}} q_z \exp(\theta E_t \left[ u(c_j, t) + \beta_t u(c_{j,t+1}) | z_{j,t} = z \right])}.
\]  

(17)

Equation (17) specifies the probability \( p_{\tilde{z}} \) of agent \( j \) choosing strategy \( z_{j,t} = \tilde{z} \) and receiving expected utility from that choice, given that other agents also play the same mixed strategy.

In the face of uncertainty about the play of generations yet unborn, a natural benchmark for the default choice, \( q \), for agent \( j \) is the uniform distribution. This choice stems from Laplace’s “principle of insufficient reason” which holds that, absent a reason to expect one event to be more likely than any other, one should assume that all events are equally probable. Under this assumption, the best response for agent \( j \) is to play \( z_{j,t} = 1 \) and \( z_{j,t} = 0 \) with equal probability, and equation (17) can be expressed as

\[
p_{\tilde{z}} = \frac{\exp(\theta E_t \left[ u(c_j, t) + \beta_t u(c_{j,t+1}) | z_{j,t} = \tilde{z} \right])}{\sum_{z \in \{0, 1\}} \exp(\theta E_t \left[ u(c_j, t) + \beta_t u(c_{j,t+1}) | z_{j,t} = z \right])}.
\]  

(18)

2.3. Nash equilibrium

We next focus on the Nash equilibrium of the model and solve for the steady-state value of \( \bar{\pi} \).

In the limit \( \theta \to \infty \), there is no ambiguity concerning the actions of future generations, and agent \( j \) uses the expected-utility criterion to rank payoffs. Comparing the payoffs between conducting due diligence and not monitoring, agent \( j \) selects \( z_{j,t} = 1 \) for sure (with probability \( p_1 = 1 \)) whenever

\[
r + \alpha - \gamma \left( 1 - \bar{\pi}_{j,t+1} \right)(1 + g) > (1 - \gamma)(r + \alpha) - \mu_j,
\]  

(19)

which, upon rearranging, yields the optimal choice

\[
z^*_{j,t} = \Theta \left[ \gamma \left( \frac{1 + g}{k_j} \sum_{l \in N_j} z_{l,t+1} - \left( 1 + g - r - \alpha \right) \right) + \mu_j \right],
\]  

(20)

where the Heaviside function \( \Theta[x] = 1 \) whenever the argument \( x > 0 \), and equals zero, otherwise.

Further insight into the nature of the equilibrium can be obtained by assuming that the structure of interactions is locally tree-like; i.e., each agent has exactly \( k \) trading partners. If \( \ell \) denotes the number of trading partners who agree with the probability of expropriation risk, then agent \( j \) will also agree should the value of \( \ell \) be such that the argument of the Heaviside function in equation (20) is greater than zero; i.e., when

\[
\ell > \frac{k}{1 + g} \left[ R - \frac{\mu_j}{\gamma} \right],
\]  

(21)

where \( R = 1 + g - r - \alpha \). What matters for agent \( j \) is the absolute number of trading partners who
agree, rather than their individual identities. With \( k \) trading partners, the number of different combinations of \( \ell \) agents who agree is \( \binom{k}{\ell} \). Each of these \( \ell \) agents also has exactly \( k \) trading partners, so equation (20) provides identical conditions on the number of their trading partners who agree that “agreeing” is the best response for each of the original trading partners, and so on.

Recursively defining similar conditions for all future generations of trading partners, the probability that agent \( j \), with marginal cost of monitoring, \( \mu_j \), will agree with the probability of expropriation risk is

\[
\pi(\mu_j) = \sum_{\ell > \frac{1}{\pi_j}} \binom{k}{\ell} \bar{\pi}^{\ell} (1 - \bar{\pi})^{k-\ell},
\]

where \( \bar{\pi} \) is the unconditional probability that a randomly selected agent will agree with the probability of default and not monitor. Taking expectations over \( \mu \) in equation (22), and recalling that \( r = r(\bar{\pi}) = \delta + \bar{\pi} g - \alpha \), we obtain the following fixed-point equation for \( \bar{\pi} \) which, by the law of large numbers, is the fraction of agents who agree with the bond’s probability of default and choose not to monitor in equilibrium:

\[
\bar{\pi} = \sum_{\ell=0}^{k} \binom{k}{\ell} \bar{\pi}^{\ell} (1 - \bar{\pi})^{k-\ell} \text{Prob} \left\{ \mu > \gamma \left[ R(\bar{\pi}) - \frac{(1 + g)\ell}{k} \right] \right\}.
\]

Figure 3 plots the solutions to equation (23) for different values of \( \gamma \), where the function \( F(\bar{\pi}) \) corresponds to the right-hand side of equation (23). For small \( \gamma \) – where there is a high degree of congruence between the credit-rating agency’s appraisal and the due diligence of agents – there is a unique solution with \( \bar{\pi} = 1 \). All agents willingly trade the bond without monitoring and the asset is informationally insensitive. As \( \gamma \) increases, a second solution \( \bar{\pi} = 0 \) emerges. Here, all agents monitor and the active trading of the bond is curtailed. And as \( \gamma \) is steadily increased, the two (stable) solutions are separated by a third (unstable) solution for intermediate values of \( \bar{\pi} \). The basin of attraction for the \( \bar{\pi} = 0 \) solution grows, while that for the \( \bar{\pi} = 1 \) solution rapidly depletes.

3. The Safe-Asset Frontier

In this section, we clarify the relationship between information sensitivity, fiscal capacity and economic growth. We suppose that there are many islands, each with a large number of trading partners; i.e., \( N \) and \( k \) are large. When \( k \) is large, we can approximate the binomial distribution in equation (23) by a normal distribution that is sharply peaked around its mean.\(^7\) Defining \( s = \ell/k \),

\(^7\)Formally, according to the de Moivre-Laplace theorem, we have that

\[
\lim_{k \to \infty} \binom{k}{\ell} \bar{\pi}^{\ell} (1 - \bar{\pi})^{k-\ell} \to \frac{1}{2\pi k \bar{\pi} (1 - \bar{\pi})} e^{-\frac{(\ell - k\bar{\pi})^2}{2k(1 - \bar{\pi})}} \to \delta(\ell - k\bar{\pi}).
\]
we obtain

\[ \bar{\pi} = \int_{0}^{\infty} \delta(s - \bar{\pi}) \text{Prob}\{\mu > \gamma[R(\bar{\pi}) - (1 + g)s]\} \, ds \]

\[ = \text{Prob}\{\mu > \gamma[R(\bar{\pi}) - (1 + g)\bar{\pi}]\}. \quad (24) \]

If marginal monitoring costs, \(\mu\), are exponentially distributed, then the fixed-point equation for the fraction of agents who readily accept the bond can be written as

\[ \bar{\pi} = \min\left\{1, \exp\left(-\frac{\gamma}{\mu}\left[1 + g - \delta - (1 + 2g)\bar{\pi}\right]\right)\right\}, \quad (25) \]

where we denote the right-hand side of equation (25) by \(G(\bar{\pi})\). In what follows, the parameter \(\delta\) can be interpreted as a measure of fiscal capacity, since it amounts to a tax on (world) economic output.

Figure 4 illustrates the fixed-point solution to equation (25). Multiple \(\bar{\pi}\) solutions begin to emerge once \(G(\bar{\pi})\) becomes tangent to the 45-degree line; i.e., \(G'(\bar{\pi}) = 1\). For parameter values where \(G'(\cdot) > 1\), there is only one fixed point at \(\bar{\pi} = 1\), while there are multiple solutions at \(\bar{\pi} = 1\) and \(\bar{\pi} < 1\) once \(G'(\cdot) < 1\). We can therefore identify a frontier in terms of \(\delta\) and \(g\) at which multiplicity attains. Solving \(G'(\cdot) = 1\) together with the fixed-point condition \(\bar{\pi} = G(\cdot)\) yields

\[ \delta = H(g, \Gamma) \equiv 1 + g - \left(\frac{1 + \log\left(\frac{\Gamma(1 + 2g)}{\Gamma}\right)}{\Gamma}\right), \quad (26) \]

where \(\delta(x)\) is the Dirac-delta function.
where $\Gamma = \gamma / \bar{\mu}$. The partial derivatives of $H(g, \gamma)$ with respect to $g$ and $\Gamma$ are
\[
\frac{\partial H}{\partial g} = 1 - \frac{2}{\Gamma(1 + 2g)}, \quad \text{and} \quad \frac{\partial H}{\partial \Gamma} = \frac{\log(\Gamma(1 + 2g))}{\Gamma^2},
\]
respectively. The derivative with respect to $g$ is strictly negative for $\Gamma(1 + 2g) < 2$. Thus, if the average monitoring cost is large, the frontier between safe and unsafe assets is downward sloping in the growth rate. The derivative of $H$ with respect to $\Gamma$ is strictly positive for $\Gamma(1+2g) > 1$. Thus, as the difference in opinion over the bond increases, the frontier between safe and unsafe assets shifts upward, diminishing the region where the bond is informationally insensitive.

Figure 5 illustrates the frontier in $(g, \delta)$ space. It shows, for any given growth rate, how large fiscal capacity must be to support the global fixed asset. Intuitively, a high-growth-rate economy with sizable fiscal capacity (the upper right-hand quadrant) can support informationally insensitive (and hence safe) assets. Low growth and weak fiscal capacity have the opposite effect. But, as Figure 5 makes clear, information insensitivity can also be supported in low-growth high-
fiscal-capacity cases, as well as in high-growth low-fiscal-capacity situations.

4. Equilibrium Interest Rates

We next conduct comparative static experiments using our framework. Specifically, we explore the implications for the equilibrium world real interest rate following two types of shock: (a) an information shock, in the sense of heightened differences of opinion ($\gamma$); and (b) a fundamental shock that permanently lowers the physical collateral underlying the bond ($\delta$).

4.1. Differences in opinion

Figure 6 plots the interest rate as a function of $\gamma$ for different values of $\theta$. In the limiting case $\theta \to \infty$, there is a critical value of $\gamma$, below which $\bar{\pi} = 1$ and the world interest rate is $r = \delta + g - \alpha$, as implied by equation (6). Above this threshold, however, $\bar{\pi} = 0$ and a low interest rate solution, $r = \delta - \alpha$, emerges. Note that, immediately above the threshold, both high and low interest rate solutions coexist for a range of $\gamma$ values.

As Figure 6 suggests, far from the tipping point, an incremental change in $\gamma$ does not impact the decisions of agents to accept the value of the bond without monitoring. In particular, each agent will argue that all future-generation agents will accept the bond without monitoring, and therefore find it optimal to not monitor. This behavior persists as $\gamma$ increases beyond the tipping point, and the high interest rate solution remains. But the high $r$ equilibrium is fragile. If there is any doubt whatsoever concerning the actions of future-generation agents, all agents immediately turn to monitoring, and the interest rate collapses to the low $r$ solution. An informationally insensitive asset can, suddenly, be transformed into an illiquid and informationally sensitive one.\(^8\)

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\(^8\)By the same argument, starting from a high $\gamma$ state, where all agents choose to monitor, following an incremental
The advent of new information that changes perceptions about the safety of the bond can therefore lead to a fall in the equilibrium real interest in order to maintain the value of the asset.

Figure 6 also illustrates the consequences of ambiguity surrounding the behavior of one’s counterparties, as mentioned above. For values of $\theta < \infty$, the critical threshold values for $\gamma$ are much larger than in the case of $\theta = \infty$. As $\theta$ decreases, agents attach greater weight to their reference model, $q$, being their best guess. So they are willing to trade the bond in the face of greater divergent opinion about the bond’s underlying characteristics. Such behavior, which can be loosely considered as a “flight to safety” in favor of the safe asset, imparts an inertial bias to the world interest rate. For significantly small values of $\theta$, the decline in the interest rate is no longer abrupt. As can be seen, in the case $\theta = 100$, $r$ decreases gradually and continuously with $\gamma$.

4.2. Shocks to collateral

Figure 7 plots the equilibrium interest rate as a function of $\delta$, the underlying collateral backing the bond. From equation (6), $r$ depends both explicitly on $\delta$ and implicitly, through $\bar{\pi}$. When fundamentals are strong and the value of $\delta$ is large, equation (23) admits the unique solution, $\bar{\pi} = 1$. In this case, $r = \delta + g - \alpha$. But, as $\delta$ declines, both $\bar{\pi} = 1$ and $\bar{\pi} = 0$ emerge as stable solutions, along with an intermediate (unstable) solution. A low interest equilibrium ($r = \delta - \alpha$) is therefore possible. Figure 7(a)-(d) illustrates the critical value of $\delta$ at which multiple equilibria emerge for different values of $\theta$. Panel (a) of Figure 7 shows the relationship between $r$ and $\delta$ for $\theta = \infty$. The remaining panels (b)-(d) show how the critical value of $\delta$ is brought forward as $\theta$ is progressively lowered. Finite $\theta$ values impart an inertial bias to world interest rates as agents willingly trade the bond in spite of declining underlying collateral valuation.

Our results can also be linked to the idea of a liquidity trap. Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011) have recently highlighted the links between credit market shocks and liquidity traps. In these papers, cutbacks in the availability of consumer credit lead to a period of low or negative real interest rates. In our model, a shock to collateral or an increase in credit risk can have similar consequences. For values $\alpha > \delta$, the equilibrium real interest rate implied by the low $\bar{\pi}$ solution is negative. As Figure 3 makes clear, following a shock to the supply of safe assets, agents may become unwilling to readily trade the bond. Moreover, this behavior persists, even as fundamentals improve, since agents are confident that future-generation agents will also be unwilling to trade the bond, and the economy may find itself stuck in a liquidity trap.

decrease in $\gamma$, all agents will continue to monitor. The probability must decrease all the way to the tipping point for the high $r$ solution to emerge. If, however, a large enough fraction of agents starts to doubt whether future-generation agents will monitor, the high $r$ solution may be regained earlier.
Figure 7: The world interest rate, \( r \), as a function of \( \delta \), for different values of \( \theta \). All panels include the case \( \theta = \infty \), which is depicted by the thick black lines. Panels (b), (c) and (d) also depict the outcomes for \( \theta = 10^2 \), \( \theta = 10^3 \) and \( \theta = 10^4 \), respectively. Additional exogenous parameters are set as \( a = 0.05 \), \( g = 0.03 \) and \( k = 11 \). For the distribution of monitoring costs, we assume that 80% of islanders have \( \mu = 0.01 \), while the remaining 20% have \( \mu = 0 \).

5. The Triffin Dilemma

The analysis so far has been couched in terms of a single global store of value, backed by taxing the cumulative output of all islands in the world economy. We next consider the more realistic case where islands are grouped into two distinct regions – Red and Blue – and where, initially, only the bond of one region (Blue) is informationally insensitive. Such a situation can be likened to the pre-eminent role of the United States in the international monetary system, and we explore the consequences of a permanent shock that lowers its relative share in the world economy.\(^9\)

Suppose, therefore, that islands in the Red (Blue) region have GDP \( X_{R}^t \) (\( X_{B}^t \)), with growth and default rates \( g^R \) (\( g^B \)) and \( \alpha^R \) (\( \alpha^B \)). The structure of trading relationships, both between islands within a region and across the two regions, is assumed to be homogeneous. And denote by \( x^R \) (\( x^B = 1 - x^R \)) the share of each region’s output in the world economy.

In Appendix A, we show that the equilibrium real interest rate in this two-region economy is the weighted average of the interest rate in each region, namely

\[
r = x^R \left( \delta^B + \bar{\pi}^B g^B - \alpha^B \right) + x^R \left( \delta^R + \bar{\pi}^R g^R - \alpha^R \right),
\]

if each region issues a bond of value \( V_{i}^R \) (\( V_{i}^B \)), with take-up across agents \( \bar{\pi}^R \) (\( \bar{\pi}^B \)), respectively. If initial conditions are such that the Blue bond is informationally insensitive (\( x^B \delta^B = H(g^B, \gamma) \)), and the Red bond is informationally sensitive (\( (1 - x^B)\delta^R < H(g^R, \gamma) \)), then only Blue bonds are traded in the world economy. The total demand for Blue bonds is the sum of demands in both

\(^9\)See Eichengreen (2011) for a detailed discussion of the role of the U.S. dollar as the international reserve currency.
regions, amounting to a total Blue bond value of

$$X_t(x^B + (1 - x^B)(1 - \delta^R)), \quad (28)$$

where \(X_t = X_t^B + X_t^R\). This value, however, is greater than the Blue region’s wealth, \(X_t x^B\). So the net foreign asset of the Blue region – the difference of the wealth of islands in the Blue region and the total value of the Blue region’s assets – is strictly negative; i.e.,

$$X_t x^B - X_t(x^B + (1 - x^B)(1 - \delta^R)) < 0. \quad (29)$$

Equation (29) illustrates the advantage to the Blue region arising from its role as the sole provider of international liquidity to the world economy. As a result of the Red region’s willingness to hold liquid Blue bonds, the external constraint facing the Blue region is relaxed. This is consistent with the “exorbitant privilege” of the United States highlighted by Obstfeld and Rogoff (2005), Gourinchas and Rey (2007), Gourinchas et al. (2010), and Krishnamurthy and Vissing-Jorgensen (2012).

Consider a permanent shock to the Blue region that reduces its share in the world economy by \(\Delta\). In our model, provided that the fiscal capacity of the Blue region is sufficient to maintain the information insensitivity of the Blue bond, the effects of the shock are twofold. First, the shock reduces wealth in the Blue region. Second, on the demand side, there is an equal decrease in demand for Blue bonds within the Blue region, but a proportional increase in the enlarged Red region’s demand for the bond. Together, this implies a more negative net foreign asset position for the Blue region, namely

$$X_t x^B - X_t(x^B + (1 - x^B)(1 - \delta^R)) < X_t x^B - X_t(x^B + (1 - x^B)(1 - \delta^R)) < 0. \quad (30)$$

And the interest rate becomes

$$r = x^B \left( \delta^B + g^B - \alpha^B \right) + x^R \left( \delta^R - \alpha^R \right). \quad (31)$$

Figure 8 plots the interest rate as a function of the \(\Delta\) shock for the case where the growth rate of the Blue region is lower than that of the Red region, \(g^B < g^R\), but the fiscal capacity of the Blue region is larger, \(x^B \delta^B > (1 - x^B) \delta^R\). We set our initial conditions so that only the Blue bond is traded across all islands and the Red bond is informationally sensitive.

As Figure 8 makes clear, for sufficiently small shocks, the Blue bond remains informationally insensitive and the interest rate is decreasing in \(\Delta\). This stems from the reduction in output \(\Delta g^B\) in the Blue region. But for \(\Delta > 0.14\), a tipping point emerges where the initial fiscal capacity of
Figure 8: The world interest rate as a function of $\Delta$-shock. Additional parameters are set at $g^B = 0.05$, $g^R = 0.11$, $\delta^B = 0.05$, $\delta^R = \alpha^B = \alpha^R = 0$, and $\Gamma = 1.4$.

the Blue region is no longer sufficient to guarantee the safety of the global store of value. The Blue bond turns informationally sensitive and is no longer readily traded, $\bar{\pi}^B = 0$. As we have seen in previous sections, the sudden disappearance of the safe asset triggers a sharp fall in the real interest rate. A crystallization of confidence risk in the global safe asset can thus open the door for a Triffin-type problem, and potentially open the door to a liquidity trap.

6. Conclusion

Although simplistic, the mechanisms at work in our model shed light on both the sovereign debt crisis in Europe and debates about the future of the U.S. dollar as the pre-eminent reserve currency. Prior to the onset of the global financial crisis in 2007, spreads on debt securities issued by European states were low, reflecting the belief of investors that the bonds were liquid and readily interchangeable. But following large-scale financial sector bailouts, investors began to query whether growth rates and fiscal policies were consistent with safe-asset status. The endogenous reduction in safe-asset supply that has resulted has destabilized the euro, and highlighted that different euro area countries have very different growth prospects and very different debt burdens, with little scope for transferring fiscal resources between them.

The role of U.S. Treasuries as the key global safe asset has repeatedly been called into question in light of China’s growing importance in the world economy and the increasing internationalization of the renminbi. Our model suggests that, eventually, the relative size and fiscal capacity of the United States would be inconsistent with its role as sole provider of international liquidity. But the tipping point is unlikely to be sudden: although we characterize how safe-asset status can suddenly collapse following a collective loss of confidence, the model also suggests that ambiguity about the perceptions of other investors acts as an important brake, tempering capital flight. Investors are therefore likely to continue to favor U.S. Treasuries as the global safe asset,
particularly given the obstacles facing Europe and China as potential alternative providers of international liquidity.

Our framework can be viewed as a first step toward understanding a multipolar arrangement in which safe assets denominated in different currencies coexist. It is not clear whether the decentralization of global safe assets would be stabilizing or destabilizing, and how global safety nets—such as swap line agreements—might be designed. Extending the analysis to allow for these possibilities is an important area for future work.

Our model can also be extended to explore the conditions under which private-label safe assets arise, and can substitute for public sector safe assets. The shadow banking sectors of the advanced economies are the main providers of private-label safe assets and, in turn, of global liquidity. Problems with the shadow banks may diminish the supply of private-label safe assets, leading investors to rely more heavily on government bonds to meet liquidity needs. The interaction between banking sector risks and sovereign risks can potentially amplify the type of Triffin-dilemma mechanics that we have examined.

Finally, our analytical results point to a testable hypothesis for the determinants of safe government debt, suggesting a non-linear relationship between fiscal variables, growth and information sensitivity of government debt. Empirical analysis of the model along these lines would complement existing studies (e.g., Cecchetti et al. 2010; Fontana and Scheicher 2010), and is left for future research.
Appendix A. Interest Rates in the Two-Region Economy

The values for the bonds issued by the Red and Blue regions are

\[ r V^i_t = \bar{\pi}^i_{t+1} V^i_{t+1} - V^i_t + \delta^i X^i_t - \alpha^i V^i_t, \tag{A.1} \]

where \( i \in \{R, B\} \). The wealth of agents in region \( i \) evolves according to

\[ \bar{\pi}^i_{t+1} \left( W^i_{t+1} - W^i_t \right) = -W^i_t + (1 - \delta^i)X^i_t + r W^i_t - \alpha^i W^i_t. \tag{A.2} \]

Summing up across the two regions, we obtain

\[
\begin{align*}
    r V_t &= \left[ \bar{\pi}^B_{t+1} g^R - \alpha^B - \nu^R \left( \bar{\pi}^B_{t+1} g^R - \alpha^R \right) \right] V_t \\
    &\quad + \left[ \delta^B - x^R \left( \delta^B - \delta^R \right) \right] X_t,
\end{align*}
\tag{A.3}
\]

and

\[
\begin{align*}
    r W_t &= \left[ 1 + \bar{\pi}^B_{t+1} g^B + \alpha^B - \mu^R \left( \bar{\pi}^B_{t+1} g^B + \alpha^B - \bar{\pi}^R_{t+1} g^R - \alpha^R \right) \right] W_t \\
    &\quad + \left[ 1 - \delta^B - x^R \left( \delta^R - \delta^B \right) \right] X_t,
\end{align*}
\tag{A.4}
\]

where

\[
\begin{align*}
    V_t &= V^R_t + V^B_t, \quad X_t = X^R_t + X^B_t, \quad W_t = W^R_t + W^B_t, \tag{A.5}
\end{align*}
\]

and

\[
\begin{align*}
    \nu^R &= \frac{V^R_t}{V_t}, \quad x^R = \frac{X^R_t}{X_t}, \quad \mu^R = \frac{W^R_t}{W_t}. \tag{A.6}
\end{align*}
\]

The equilibrium interest rate is thus given by

\[ r = x^R \left( \delta^B + \bar{\pi}^B g^R - \alpha^B \right) + x^R \left( \delta^R + \bar{\pi}^R g^R - \alpha^R \right), \tag{A.7} \]

which is the weighted average of the individual region’s interest rates.
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