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ToTEM II: An Updated Version of the Bank of Canada's Quarterly Projection Model

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Abstract

This report provides a detailed technical description of an updated version of the Terms-of-Trade Economic Model (ToTEM II), which replaced ToTEM (Murchison and Rennison 2006) in June 2011 as the Bank of Canada's quarterly projection model for Canada. ToTEM has been improved along a number of dimensions, with important changes to the model structure, including: (i) multiple interest rates, (ii) sector-specific demand specifications for consumption, housing investment and inventory investment, (iii) a role for financial wealth in household consumption, and (iv) rule-of-thumb price and wage setters. These new features remove some of the restrictions on model dynamics implied by assumptions in ToTEM, making ToTEM II more general and flexible than its predecessor. Furthermore, most of ToTEM II's parameters are now formally estimated using full information estimation techniques, leading to significantly improved in-sample goodness of fit. The report discusses the model's estimation and reviews the most important changes in the model's properties. Finally, some important applications of ToTEM II in addressing recent policy questions are provided.

JEL classification: E17, E20, E30, E40, E50, F41
Bank classification: Economic models; Business fluctuations and cycles

Résumé

Ce rapport fournit une description technique détaillée de TOTEM II, la nouvelle version du modèle de projection trimestrielle que la Banque du Canada a commencé à utiliser en juin 2011, en remplacement de TOTEM (Murchison et Rennison, 2006). Le modèle a été amélioré à plusieurs égards et sa structure a fait l'objet de changements importants; à ce titre, on y a introduit 1) plusieurs taux d'intérêt; 2) des spécifications de la demande propres aux différents secteurs pour la consommation, l'investissement résidentiel et l'investissement en stocks; 3) un rôle pour la richesse financière dans la consommation des ménages; et 4) des agents qui suivent une règle rétrospective simple pour la détermination des prix et des salaires. Ces nouvelles caractéristiques confèrent à TOTEM II plus de polyvalence et de souplesse en levant certaines des contraintes imposées à la dynamique du modèle qui étaient attribuables aux hypothèses sur lesquelles se fondait TOTEM. En outre, la plupart des paramètres de TOTEM II étant maintenant estimés de façon formelle au moyen de méthodes à information complète, l'adéquation statistique du modèle pour la période d'échantillonnage s'est nettement améliorée. Les auteurs du rapport présentent l'estimation du modèle et passent en revue les principaux changements apportés aux propriétés. Enfin, le rapport illustre d'importantes applications de TOTEM II dans l'analyse de questions stratégiques récentes.

Classification JEL : E17, E20, E30, E40, E50, F41
Classification de la Banque : Modèles économiques; Cycles et fluctuations économiques

Introduction

The Terms-of-Trade Economic Model, or ToTEM, has served as the Bank's main projection and policy analysis model since December 2005 (Murchison and Rennison 2006; Fenton and Murchison 2006). An updated version of the model (ToTEM II) replaced ToTEM in June 2011. The model has been improved along a number of dimensions, with important changes to the model structure, including: (i) multiple interest rates, (ii) sector-specific demand specifications for consumption, housing investment and inventory investment,[1] (iii) a role for financial wealth in household consumption, and (iv) rule-of-thumb price and wage setters in the spirit of Galí and Gertler (1999). These new features remove some of the restrictions on model dynamics implied by assumptions in ToTEM, making ToTEM II more general and flexible than its predecessor. Moreover, most of ToTEM II's parameters are now formally estimated using full information estimation techniques, resulting in significantly improved in-sample goodness of fit.

Standard dynamic stochastic general-equilibrium (DSGE) models, including ToTEM, typically incorporate a single interest rate: the short-term risk-free interest rate. These models are good approximations of reality as long as: (i) deviations from the pure expectations theory of the term structure do not vary much over time, and (ii) there is no time variation in the spreads between risk-free rates and the rates faced by households and firms. In practice, these conditions do not appear to hold, especially in recent years.[2] For this reason, ToTEM II now includes 90-day and 5-year riskless, as well as 90-day and 5-year risky, assets for both households and firms, with time-varying risk and term premia. By including long-term rates, ToTEM II captures the effects of fluctuations of the term and risk-premium components of long-term rates on aggregate demand. In addition to improving the Bank's projection analysis, including multiple interest rates in ToTEM II also

[1]In contrast, consumption in ToTEM was defined as an aggregate of national account consumption, residential investment and inventory investment.

[2]See Dorich, Mendes and Zhang (2011).

allows Bank staff to study a broader array of policy questions than was previously possible. For instance, staff recently used the model to examine the macroeconomic implications of changes to the requirements for capital and liquidity in the banking sector.[3]

The consumption variable in ToTEM is a composite of National Income and Expenditure Accounts (NIEA) variables: personal consumption expenditures, residential investment and inventory investment. ToTEM II differentiates the latter two variables from personal consumption expenditures by allowing the paths for residential investment and inventory investment to be uniquely determined by the marginal valuation of their respective stocks. Note that the production side remains unchanged so that all relative movements in residential investment and inventory investment are demand-determined.

Both versions of the model are small open-economy models with incomplete asset markets, where consumers can borrow at an exogenous foreign interest rate. This implies that, without additional assumptions, transitory shocks will lead to permanent deviations in consumers' net foreign asset (NFA) position. In ToTEM, stationarity of net foreign assets is obtained by making the risk premium that consumers face in foreign financial markets a function of their NFA position. In ToTEM II, the stationarity stems from the link between the household discount factor and household financial wealth, which includes net foreign assets. Specifically, households become more patient when their financial wealth-to-disposable-income ratio is low, and vice versa. In addition to creating a model-consistent projection for net wealth and savings, this change creates a direct link between consumption behaviour and house prices.

Both ToTEM and ToTEM II have nominal price and wage rigidities, in the sense that not all nominal prices and wages are reoptimized every period. In the first version of the model, price and wage reoptimization on the part of firms and households is fully rational and forward looking. In ToTEM II, some firms and households behave in a forward-looking manner, while others are assumed to follow a simple rule of thumb in the spirit of Galí and Gertler (1999). The extent to which price- and wage-setting behaviour follows the rule of thumb is estimated.

Our discussion of ToTEM II will proceed as follows. In Chapter 1, we review the main elements of the theoretical framework used to develop ToTEM II. To help build intuition, the model's key linearized equations are also presented. Chapter 2 discusses the model's estimation. Chapter 3 reviews the most important changes in the model's properties. Chapter 4 presents some of the recent policy applications

[3]See Dorich and Zhang (2010).

of ToTEM II. In particular, we review the implications of short- and long-term shocks to credit spreads during the recent financial crisis, and we evaluate how different types of shocks to the supply and demand for commodities impact the Canadian economy.

Chapter 1

Model Description

In this chapter, we present the model's most important new features associated with the production of finished goods, the distribution of commodities and the household optimization problem. To provide intuition for model dynamics, linearized equations summarizing key optimality conditions for both firms and households are presented. The quantitative implications of model changes are discussed in Chapter 3.

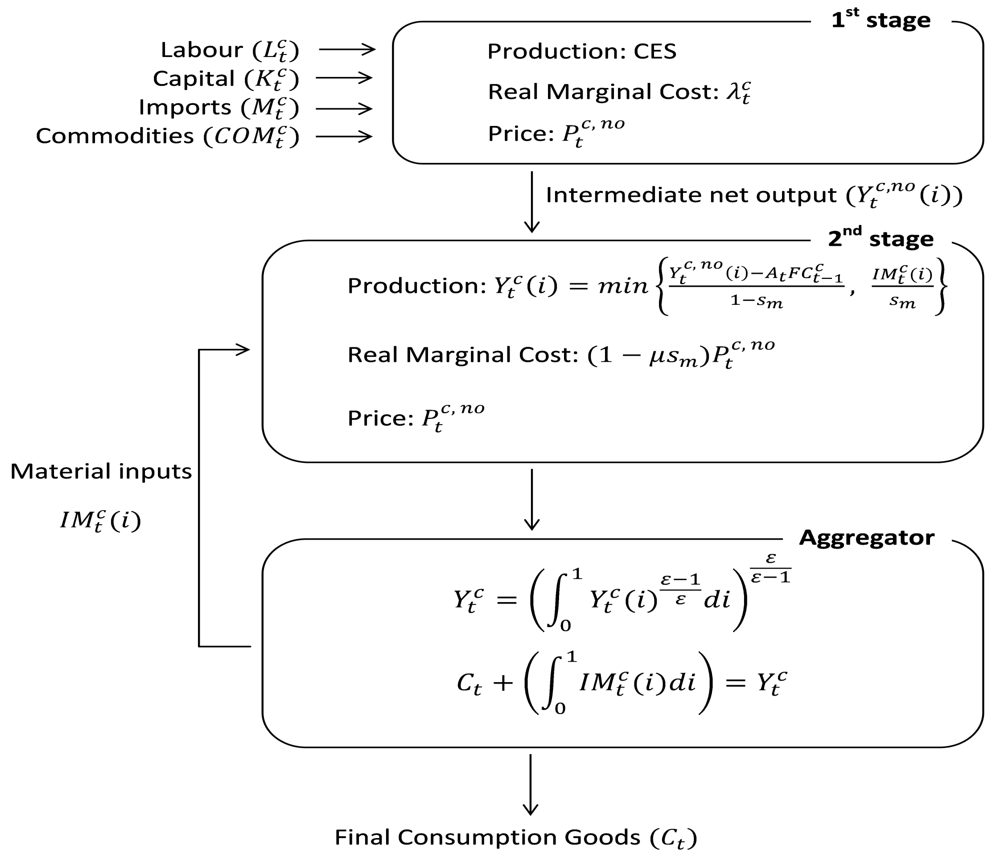
1.1 Finished-good sectors

Production of finished goods is divided across four sectors, as in ToTEM, including core consumption goods, investment goods, non-commodity export goods and government goods. Firms' behaviour is determined by profit maximization given production technology, demand for their respective output, labour supply, as well as constraints on changing the nominal price for their goods. Both ToTEM and ToTEM II feature strategic complementarities in pricing that help models in this class match some of the typical patterns of aggregate output and inflation behaviour.[4] ToTEM accomplishes this through firm-specific capital, while ToTEM II accomplishes this through multiple stages of production. In the first stage, intermediate goods are produced by identical, perfectly competitive firms using capital, labour, commodities and imports as production inputs. In the second stage, monopolistically competitive firms produce both final goods and manufactured inputs using intermediate goods and a composite of manufactured inputs. Since all sectors

[4] See Chapters 3 and 5 in Woodford (2003) for examples of strategic pricing complementarities.

are symmetric (except for commodities), we will describe only one sector.

Figure 1.1: Organization of core consumption good production sector



1.1.1 First stage: production of intermediate goods

ToTEM II uses a constant-elasticity-of-substitution (CES) production function to combine capital, labour, commodities and imports in the production of intermediate goods from the first stage. Capital and labour are supplied by households, and imports and commodities are produced in imports and commodity production sectors, respectively. The production technology is summarized in Figure 1.1.

In addition to multi-stage production, ToTEM II makes two additional changes to the production structure. First, adjustment costs take the form of a dead-weight loss of respective production inputs, as opposed to a loss of produced output used in ToTEM. Second, capital utilization is modelled as affecting depreciation instead of resource costs.

The gross output of the core consumption sector is

$$Y_t^{c,g} = \mathcal{F}(A_t E_t^c H_t^c \xi_t^{H,c}, u_t^c K_t^c \xi_t^{K,c}, COM_t^c \xi_t^{COM,c}, M_t^c \xi_t^{M,c}) \ ,$$

where $A_t, E_t^c, H_t^c, u_t^c, K_t^c, COM_t^c$ and M_t^c are the economy-wide level of labour-augmenting technology, labour effort, labour hours, the rate of capital utilization, the level of the capital stock, commodities and imports, respectively. As discussed in section 2.1 of Murchison and Rennison (2006), effective labour L_t^c is the product of observed employment H_t^c and unobserved labour effort E_t^c. The production technology (given by the function $\mathcal{F}$) is characterized by a constant elasticity of substitution.[5] $\xi_t^{j,c}$ is the factor associated with the cost of adjusting input j. The adjustment costs for hours worked are in terms of changes in hours, whereas the adjustment costs associated with capital stock, commodities and imports depend on changes in their respective shares in production:

$$\xi_t^{H,c} = 1 - \frac{\chi^H}{2}\left(\frac{H_t^c}{H_{t-1}^c} - 1\right)^2 \ , \tag{1.1}$$

$$\xi_t^{K,c} = 1 - \frac{\chi^K}{2}\left(\frac{K_{t+1}^c/Y_t^{c,g}}{K_t^c/Y_{t-1}^{c,g}} - 1\right)^2 \ , \tag{1.2}$$

$$\xi_t^{COM,c} = 1 - \frac{\chi^{COM}}{2}\left(\frac{COM_t^c/Y_t^{c,g}}{COM_{t-1}^c/Y_{t-1}^{c,g}} - 1\right)^2 \ , \tag{1.3}$$

$$\xi_t^{M,c} = 1 - \frac{\chi^M}{2}\left(\frac{M_t^c/Y_t^{c,g}}{M_{t-1}^c/Y_{t-1}^{c,g}} - 1\right)^2 \ , \tag{1.4}$$

where χ^j determines the size of the adjustment cost for changing input j. Specification of input adjustment costs in terms of production shares implies that input adjustment is costly when it is associated with changes in input composition (e.g., in response to relative-price disturbances).

[5] See Appendix A.1.1 for the specification of the production function.

The capital stock in period $t+1$ is determined by

$$K_{t+1}^{c} = (1 - d\left(u_t^c\right))K_t^c + I_t^c \ , \tag{1.5}$$

where d is the quarterly rate of capital depreciation, expressed as a function of the capital utilization in the consumption sector, u_t^c,

$$d\left(u_t^c\right) = d_0 + \overline{d}e^{\rho^c\left(u_t^c - 1\right)} \ . \tag{1.6}$$

Parameter ρ^c measures the marginal change in the rate of capital depreciation resulting from its higher or lower capacity utilization. This cost is measured in units of capital, whereas in ToTEM this cost was measured in terms of output for the sector.[6]

After we incorporate quadratic investment adjustment costs, the net output of the core consumption intermediate good, $Y_t^{c,no}$, can be defined as

$$Y_t^{c,no} = Y_t^{c,g} - \frac{\chi_I}{2}\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)^2 I_t^c \ , \tag{1.7}$$

where I_t^c is gross investment in the core consumption sector, and χ_I determines the size of investment adjustment costs.

The firm's objective in period t is then to choose $Y_t^{c,no}$, $Y_t^{c,g}$, H_t^c, K_{t+1}^c, I_t^c, u_t^c, COM_t^c and M_t^c subject to equations (1.1)–(1.7) in order to maximize

$$\mathbf{E}_t \sum_{s=t}^{\infty} \mathcal{R}_{t,s}\left(P_s^{c,no}Y_s^{c,no} - W_s H_s^c - P_s^{com}COM_s^c - P_s^I I_s^c - P_s^M M_s^c\right) \ , \tag{1.8}$$

where P_s^I is the price of investment, P_s^{com} is the price of commodities used in production, W_s is the aggregate nominal wage,[7] P_s^M is the price of the imported good, $P_s^{c,no}$ is the price of the core consumption intermediate good and $\mathbf{E}_t$ denotes the expected value conditional on information through period t. The stochastic

[6] One implication of this treatment of capacity utilization is that the utilization rate appears in the linearized net output production function in ToTEM II, whereas it did not in ToTEM.

[7] Wages are equal across sectors in ToTEM, since labour supply is assumed to be homogeneous.

discount factor, $\mathcal{R}_{t,s}$, is defined as

$$\mathcal{R}_{t,s} \equiv \prod_{v=t}^{s-1} \left(\frac{1}{1+R_v} \right) \ , \ \mathcal{R}_{t,t} \equiv 1$$

where R_t is the quarterly nominal interest rate.

In what follows, we present only the linearized aggregate first-order conditions that differ from those in the first version of ToTEM. The linearized first-order condition for hours worked is

$$\widehat{H}_t^c = \Phi \widehat{H}_{t-1}^c + (1-\Phi) E_t \widehat{H}_{t+1}^c - \frac{\Phi}{\chi^H} \left[\widehat{w}_t - \widehat{\lambda}_t^c - \widehat{\mathcal{F}}_h(\cdot, t) \right] \ , \tag{1.9}$$

where $\widehat{H}_t^c$,$\widehat{w}_t$, $\widehat{\lambda}_t^c$ and $\widehat{\mathcal{F}}_h(\cdot, t)$ are log deviations from steady state and $\Phi = \frac{1}{1+\beta}$. In this expression, w_t is the nominal wage divided by the price level in the finished core consumption good sector P_t^c, λ_t^c is the real marginal cost in the first stage of production (nominal marginal cost in the first stage of the consumption sector deflated by P_t^c), and $\mathcal{F}_h(\cdot, t)$ is the marginal product of hours worked in the absence of adjustment costs for hours worked.[8] As in ToTEM, the demand for hours is characterized by a second-order difference equation with a lead and a lag of hours, and the markup term that depends on the wage (relative to the marginal cost in the first stage of production) and the marginal product of hours in the absence of adjustment costs.

The linearized first-order conditions for commodity and imports are

$$\begin{aligned} \widehat{COM}_t^c &= \widehat{Y}_t^{c,g} + \Phi \left[\widehat{COM}_{t-1}^c - \widehat{Y}_{t-1}^{c,g} \right] + (1-\Phi) \left[E_t(\widehat{COM}_{t+1}^c - \widehat{Y}_{t+1}^{c,g}) \right] \\ &\quad - \frac{\Phi}{\chi^{COM}} \left[\widehat{p}_t^{COM} - \widehat{\lambda}_t^c - \widehat{\mathcal{F}}_{com}(\cdot, t) \right] \ , \end{aligned} \tag{1.10}$$

and

$$\begin{aligned} \widehat{M}_t^c &= \widehat{Y}_t^{c,g} + \Phi \left[\widehat{M}_{t-1}^c - \widehat{Y}_{t-1}^{c,g} \right] + (1-\Phi) \left[E_t(\widehat{M}_{t+1}^c - \widehat{Y}_{t+1}^{c,g}) \right] \\ &\quad - \frac{\Phi}{\chi^M} \left[\widehat{p}_t^M - \widehat{\lambda}_t^c - \widehat{\mathcal{F}}_m(\cdot, t) \right] \ . \end{aligned} \tag{1.11}$$

[8] Both the nominal wage and the nominal marginal cost in every sector are deflated by the price level in the finished core consumption good sector.

Given that the commodity and import adjustment costs are introduced in a way that is analogous to the one used for hours, the demands for these inputs are also characterized by a second-order difference equation. Notice also that each input price is again deflated by P_t^c and that $\mathcal{F}_{com}(\cdot, t)$ and $\mathcal{F}_m(\cdot, t)$ denote the marginal products of commodities and imports, respectively, in the absence of adjustment costs.

The linearized first-order condition for capital provides the following expression for its shadow value, $\widehat{q}_t^c$:

$$\begin{aligned}\widehat{q}_t^c &= \frac{1}{1+\overline{r}}\mathbf{E}_t\left(\begin{array}{c}\left(\overline{r}+\overline{d}\right)\left(\widehat{\lambda}_{t+1}^c+\widehat{\mathcal{F}}_K(\cdot,t+1)-\widehat{u}_{t+1}^c\right)+\left(1-\overline{d}\right)\widehat{q}_{t+1}^c-\widehat{r}_t \\ +\left(\overline{r}+\overline{d}\right)\chi_K\left(\widehat{K}_{t+2}^c-\widehat{K}_{t+1}^c-\left[\widehat{Y}_{t+1}^{c,g}-\widehat{Y}_t^{c,g}\right]\right)\end{array}\right) \\ &\quad -\chi_K\left(\overline{r}+\overline{d}\right)\left(\widehat{K}_{t+1}^c-\widehat{K}_t^c-\left[\widehat{Y}_t^{c,g}-\widehat{Y}_{t-1}^{c,g}\right]\right)\ ,\end{aligned} \tag{1.12}$$

where $\widehat{r}_t$ captures variations in the ex ante real interest rate in the consumption sector relative to the steady state, $\mathcal{F}_k(\cdot, t+1)$ denotes the marginal product of capital in the absence of adjustment costs and $\widehat{q}_t^c$ captures the net present discounted value to the firm of an additional unit of installed capital. Notice that the terms arising due to capital adjustment costs in the first-order condition for capital are different from those included in ToTEM. Moreover, changing the assumption regarding capital utilization adjustment costs does not affect the elasticity of the shadow value of capital with respect to utilization.[9]

The linearized first-order condition for investment is

$$\widehat{I}_t^c = \Phi^i \widehat{I}_{t-1}^c + \left(1-\Phi^i\right) E_t \widehat{I}_{t+1}^c + \frac{\overline{p^I}\Phi^i}{\overline{\lambda^c}\chi_I}\left(\widehat{q}_t^c - \widehat{p}_t^I\right)\ , \tag{1.13}$$

where $\Phi^i = \frac{1+\overline{r}}{2+\overline{r}}$. The steady-state real interest rate is close to zero, so $\Phi^i \approx 0.5$.

[9] To understand why, it is useful to compare both elasticities. In ToTEM, this elasticity was given by $\beta\lambda^c f'(u)$, whereas in ToTEM II it is given by $\beta d'(u)$, where $\lambda^c f'(u)$ measures the additional cost of capital utilization in gross output units and $d'(u)$ measures the same cost but in capital units. The first-order condition for capital utilization in both models equates this cost with the additional benefit of capital utilization. Then, in ToTEM and ToTEM II, this implies that $MPK = f'(u)q$ and that $\lambda^c MPK = d'(u)q$. Combining the previous optimality conditions in steady state with the formulas for the elasticities, in both cases the elasticity is given by $\beta\lambda^c MPK/q$. Finally, since $\beta\lambda^c MPK/q$ is pinned down by the first-order condition for capital in the steady state, which is the same in both models, we conclude that both elasticities are the same and equal to $\left(\overline{r}+\overline{d}\right)/(1+\overline{r})$.

The fact that capital adjustment costs now do not depend on the level of investment makes the investment equation different from the one in ToTEM in two ways. First, the weights on the first lead and first lag of investment are independent of the parameters that govern the size of investment and capital adjustment costs. Second, the level of capital does not affect the investment equation. As in ToTEM, the sensitivity of investment to changes in $\widehat{q}_t^c$ relative to $\widehat{p}_t^I$ is governed by the same four parameters $\overline{p^I}, \Phi^i, \overline{\lambda^c}$ and χ_I.

The linearized first-order condition for capital utilization is

$$\widehat{u}_t^c = \frac{1}{\rho^c}\left\{\widehat{\lambda}_t^c + \widehat{\mathcal{F}}_u(\cdot,t) - \widehat{k}_t^c - \widehat{q}_t^c\right\} \ , \tag{1.14}$$

where $\widehat{\mathcal{F}}_u(\cdot,t)$ is the marginal product of capital utilization in the absence of capital adjustment costs. As in ToTEM, the high marginal product of capital encourages higher capital utilization. By contrast, in ToTEM II it is a higher shadow value of capital instead of a higher relative price of investment that tends to reduce capital utilization. This difference is driven by the assumption that the costs of capital utilization are measured in capital units in ToTEM II, whereas they are measured in gross output units in ToTEM.[10]

1.1.2 Second stage: production of final goods and manufactured inputs

We assume that there exists a continuum of monopolistically competitive firms indexed by i. Each firm produces a good that can be used for consumption $C_t(i)$ and as a manufactured input $IM_t^C(i)$. In order to produce the final good, firms use intermediate goods $Y_t^{c,no}(i)$ and a composite of manufactured inputs $IM_t^c(i)$ according to the technology

$$y_t^c(i) = C_t(i) + IM_t^C(i) = \min\left\{\frac{Y_t^{c,no}(i) - A_t FC_t^c}{1-s_m}, \frac{IM_t^c(i)}{s_m}\right\} \ ,$$

[10]See Christiano, Eichenbaum and Evans (2005) on how different assumptions for capital utilization adjustment costs lead to different propagation of monetary policy shocks.

where

$$IM_t^c(i) = \left(\int_0^1 \left(M_t^C(i,j)\right)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} ,$$
$$C_t = \left(\int_0^1 (C_t(i))^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} ,$$

and where $M_t^C(i,j)$ denotes the quantity of the type j manufactured input demanded by firm i. This production function specification allows incorporation of a strategic complementarity between the price-setting decisions of the suppliers of different material inputs.[11] We use this input-output structure to help explain some observed degree of sluggishness of aggregate-price adjustment in response to variations in nominal expenditure in the data. $A_t FC_t^c$ represents a sector-specific fixed cost of production, which depends on two different components. A_t is the economy-wide level of labour-augmenting technology, and FC_t^c is proportional to the size of the sector. We calibrate the value of FC_t^c such that the profits in the consumption sector are zero in the steady state. The assumptions of the fixed production cost and material production inputs imply that labour productivity is more procyclical, which allows the model to better capture the behaviour of labour productivity in the Canadian data.

We assume that firms take the price of their inputs as given. Furthermore, since the manufactured input i and the consumption good i are the same, the price of the core consumption bundle is equal to that of the composite of manufactured inputs. Therefore, the total cost function for the firm i, deflated by the core consumption price level, is

$$TRC_t(i) = IM_t^c(i) + p_t^{c,no} Y_t^{c,no}(i) ,$$

where $p_t^{c,no}$ is the price of the core consumption intermediate good deflated by the core consumption price level.

Taking into account the optimality conditions associated with the cost minimization, we can rewrite the total real cost function as

$$TRC_t(i) = s_m y_t^c(i) + p_t^{c,no} \left\{(1 - s_m) y_t^c(i) + A_t FC_t^c\right\} ,$$

[11] See Woodford (2003). Similarly, Basu (1995) and Bergin and Feenstra (2000) introduce strategic complementarity by assuming a Cobb-Douglas production function in labour and material inputs.

where $p_t^{c,no}$ is equal to the real marginal cost in the first stage of production, λ_t^c.

The real marginal cost in the final stage is

$$rmc_t^c(i) = s_m + p_t^{c,no}(1 - s_m) \ .$$

Notice that the real marginal cost of firm i depends only on the price of manufactured inputs, $p_t^{c,no}$, which is the same for every firm. Therefore, real marginal costs are the same for all firms, $rmc_t^c(i) = rmc_t^c$. When log-linearized, the above expression is

$$\widehat{rmc}_t^c = (1 - \mu \ s_m)\widehat{p}_t^{c,no} \ , \tag{1.15}$$

where μ is the steady-state markup, $\widehat{rmc_t^c}$ is the log deviation of average marginal cost from its steady state and the variable $\widehat{p_t^{c,no}}$ corresponds to what the literature and the first version of ToTEM refer to as the real marginal cost of production.

1.1.3 Price setting

Following Galí and Gertler (1999), we assume that there are two different groups of price setters: rule-of-thumb (RT, whose share is given by ω) and forward-looking (FL, with a share of $1-\omega$). For each group of price setters, there are two different price-setting rules. With probability θ, both RT and FL firms index their own price to the inflation target. With probability $1-\theta$, the price-setting behaviours are different between RT firms and FL firms.

More specifically, with probability $1-\theta$, RT firms set their price $p_t^{b,c}$ according to the rule

$$p_t^{b,c} = p_{t-1}^c + \gamma\pi_{t-1}^c + (1-\gamma)\overline{\pi_t} + \Theta\widehat{\mu}_t^n \ , \tag{1.16}$$

where $p_t^c, \pi_t^c, \overline{\pi_t}$ and $\widehat{\mu}_t^n$ denote the price level in the core consumption sector, the core inflation rate, the inflation target and deviations of the markup from steady state, respectively. Notice that this rule of thumb nests the model proposed by Galí and Gertler (1999), who implicitly set $\gamma = 1$ and $\Theta = 0$. By allowing γ to be lower than 1 (i.e., less than full indexation to past inflation), we obtain a more flexible specification, which allows us to have a high proportion of rule-of-thumb agents without requiring a large weight on lagged inflation.

With probability $1-\theta$, FL firms choose the optimal price $p_t^{*,c}$ that satisfies the optimality condition

$$p_t^{*,c} - p_t^c = \beta\theta E_t \left\{ p_{t+1}^{*,c} - p_{t+1}^c \right\} + \{1 - \beta\theta\} \left[\widehat{rmc_t^c} + \widehat{\mu}_t^n \right] + \beta\theta E_t \left\{ \pi_{t+1}^c - \overline{\pi_{t+1}} \right\} \ , \tag{1.17}$$

where β is the steady-state household discount rate. The average price level for firms that do not index their own price to the inflation target ($\widetilde{p_t}$) is

$$\widetilde{p_t} - p_t^c = \omega(p_t^{b,c} - p_t^c) + (1 - \omega)(p_t^{*,c} - p_t^c) \ . \tag{1.18}$$

The relation between $\widetilde{p_t}$ and the aggregate price level is

$$\widetilde{p_t} - p_t^c = \frac{\theta}{1 - \theta} \left\{ \pi_t^c - \overline{\pi_t} \right\} \ . \tag{1.19}$$

Finally, by combining the previous five equations (from (1.15) to (1.19)), we get an aggregate supply "Phillips curve" equation:

$$\pi_t^c = (1 - \theta)\, \gamma\omega\phi^{-1}\pi_{t-1}^c + \beta\theta\phi^{-1} E_t \left\{ \pi_{t+1} \right\} + \widetilde{\lambda}(1 - \mu s_m)\widehat{p_t^{c,no}} + \varepsilon_t^p \ , \tag{1.20}$$

where ε_t^p is a linear combination of the inflation target and the deviation of the markup from its steady state. ϕ and $\widetilde{\lambda}$ are given by

$$\phi \equiv \theta + \omega\,(1 - \theta)\,(1 + \gamma\beta\theta) \ , \tag{1.21}$$
$$\widetilde{\lambda} \equiv (1 - \omega)\,(1 - \theta)\,(1 - \beta\theta)\,\phi^{-1} \ . \tag{1.22}$$

The existence of manufactured inputs implies greater strategic complementarity (i.e., a flatter Phillips curve), which is needed in the model to obtain aggregate inflation dynamics that are consistent with micro evidence on pricing behaviour.[12] In the first version of ToTEM, greater strategic complementarity was allowed by assuming firm-specific capital. The main advantage of using the assumption of manufactured inputs instead of firm-specific capital is that higher-order approximations of the model can be used to conduct welfare analysis. In contrast, in the first version of ToTEM, the analysis was limited to a first-order approximation because, under Calvo price setting, higher-order approximations of the model would require that the distribution of capital be kept track of.

[12] See, for example, Dotsey and King (2005); Martin (2004); Amirault, Kwan and Wilkinson (2004-05).

Introducing RT price setters ($\omega > 0$) decreases the effect of expected real marginal costs on inflation. To illustrate this, consider the case when $\omega > 0, \gamma = 0, E_t \widehat{\mu}^n_{t+j} = 0 \; \forall \; j > 0$. Inflation dynamics are represented by

$$\pi_t = \overline{\pi_t} + \frac{\widetilde{\lambda}\phi}{\theta + \omega\,(1-\theta)} \sum_{i=0}^{\infty} \left(\frac{\beta\theta}{\theta + \omega\,(1-\theta)} \right)^i \widehat{rmc^c_{t+i}} + \varepsilon^\mu_t \;, \tag{1.23}$$

where ε^μ_t is proportional to the deviation of the markup from its steady state.

Notice that the weights applied to real marginal costs are decreasing in ω. This leads to "overdiscounting," whereby expected marginal costs in the distant future receive a smaller weight relative to near-term conditions. In contrast, in the standard Calvo set-up with indexation used in ToTEM, the weights were given by the household's discount factor β.

1.2 Import sector

This sector is modelled as in ToTEM, except that we allow for the existence of RT price setters. Therefore, the rate of inflation for the prices of imported intermediate goods is given by an equation that is analogous to the one that describes the evolution of core inflation (equation (1.20)).

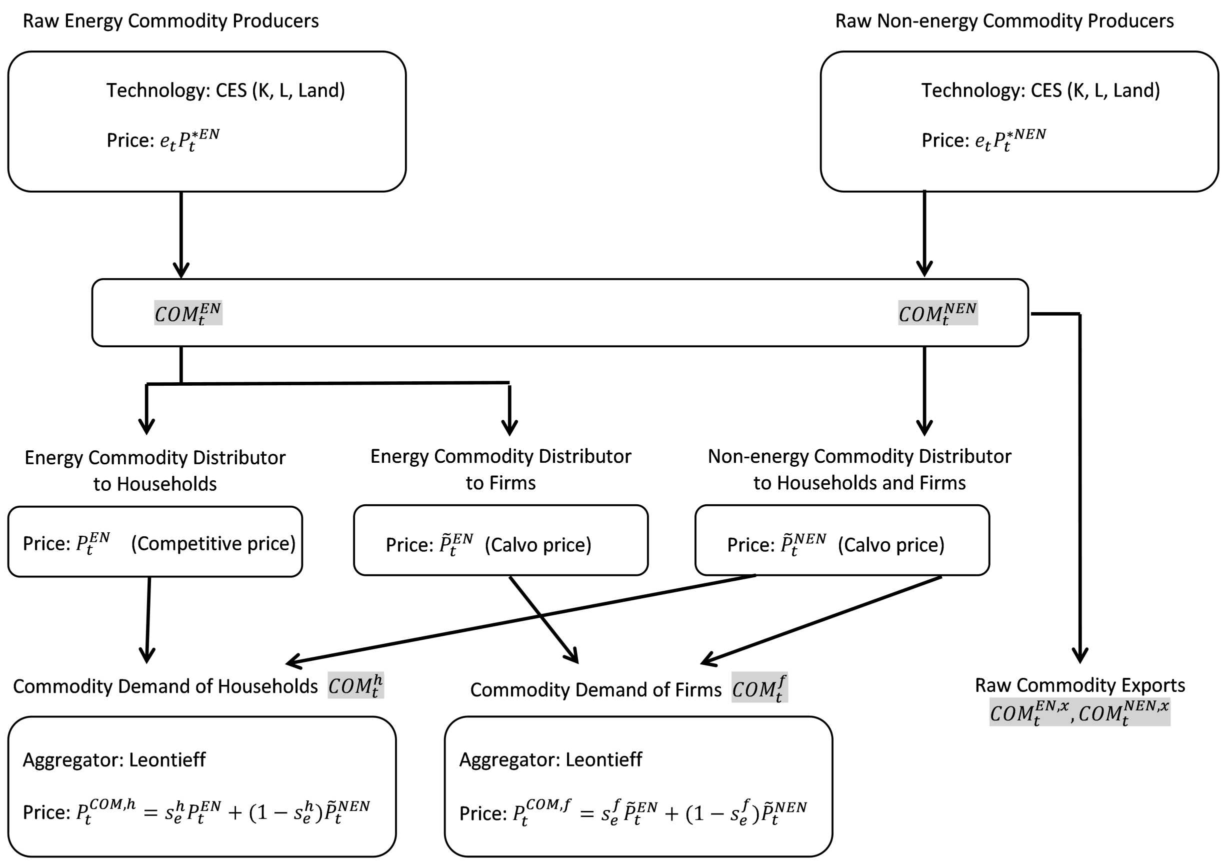
1.3 Commodity sector

The domestic commodity sector consists of competitive producers who export raw energy and non-energy commodities to the rest of the world or sell them to distributors, who convert them into final energy and non-energy commodities and then sell them to households and firms. Figure 1.2 illustrates the organization of the commodity sector in ToTEM II.

1.3.1 Commodity producers

There are two sectors of competitive firms producing raw energy and non-energy commodities. Each firm in each sector produces raw commodities by combining capital services, labour and land in a nested constant-elasticity-of-substitution production function. Just as in the finished-good production sectors, the firm faces

Figure 1.2: Organization of the commodity sector in ToTEM II



labour, capital and investment adjustment costs in addition to costs of capital utilization. The adjustment costs are specified in the same form as in other finished-good sectors. Production functions and adjustment cost functions are assumed to be identical in the two sectors. Outputs of raw energy and non-energy commodities are then exported at rest-of-world prices for raw energy and non-energy commodities, P_t^{*EN} and P_t^{*NEN}, or sold to the domestic commodity distributors at competitive prices, $e_t P_t^{*EN}$ and $e_t P_t^{*NEN}$, respectively, where e_t is the nominal Canada/rest-of-world exchange rate.

1.3.2 Commodity distributors

There are three sectors of competitive commodity distributors who buy raw energy and non-energy commodities from commodity producers and use them to produce final energy and non-energy commodities that they sell to households and firms.

Distribution of final non-energy commodities consists of two types of firms. First, there are a large number of imperfectly competitive firms who purchase raw non-energy commodities from producers at a competitive price $e_t P_t^{*NEN}$. These firms face price rigidities when setting prices, in the same way as finished-goods producers do. They produce their own differentiated variety of non-energy commodity and sell it to a large number of competitive firms, who aggregate all differentiated non-energy commodities into a final non-energy commodity, which is then sold to households and firms. The price for the final non-energy commodity is characterized by a Phillips curve equation, similar to the one for the price of final goods:

$$\pi_t^{NEN} = (1-\theta_n)\,\gamma_n \omega_n \phi_n^{-1} \pi_{t-1}^{NEN} + \beta \theta_n \phi_n^{-1} E_t \left\{ \pi_{t+1}^{NEN} \right\} + \lambda_n \widehat{mc}_t^{NEN} + \varepsilon_t^{p,NEN} \ , \quad (1.24)$$

where

$$\begin{aligned} \phi_n &\equiv \theta_n + \omega_n (1-\theta_n)(1+\gamma_n \beta \theta_n) \\ \lambda_n &\equiv (1-\omega_n)(1-\theta_n)(1-\beta\theta_n)\,\phi_n^{-1} \end{aligned}$$

and

$$\widehat{mc}_t^{NEN} = \ln e_t + \ln P_t^{*NEN} - \ln \widetilde{P}_t^{NEN} - \ln mc^{EN} \ . \quad (1.25)$$

The distribution of final energy commodities to firms is organized in a way that is similar to the distribution of final non-energy commodities. The price of the final energy commodity for firms, $\widetilde{P}_t^{EN}$, stems from a Phillips curve specification that is identical to the one for final non-energy commodities, except that the real marginal cost is $\frac{e_t P_t^{*EN}}{\widetilde{P}_t^{EN}}$.

In contrast, the distribution of final energy commodities to households is done by an industry of competitive firms, so that the price for the final energy commodity, faced by households, P_t^{EN}, is determined by the international price:

$$\ln P_t^{EN} = \ln e_t + \ln P_t^{*EN} \ . \tag{1.26}$$

Price flexibility is assumed in the distribution of the final energy commodity to households in order to capture the high degree to which exchange rate or world commodity-price movements pass through to the energy prices faced by consumers.[13]

1.3.3 Commodity aggregation

We assume that households and firms that buy final energy and non-energy commodities aggregate them into a final commodity according to a Leontieff technology with energy shares s_e^h and s_e^f, respectively.

The price of the final commodity used by firms is

$$P_t^{com,f} = s_e^f \widetilde{P}_t^{EN} + (1 - s_e^f)\widetilde{P}_t^{NEN} \ , \tag{1.27}$$

and the price of the final commodity used by households is

$$P_t^{com,h} = s_e^h P_t^{EN} + (1 - s_e^h)\widetilde{P}_t^{NEN} \ . \tag{1.28}$$

The assumption of a Leontieff aggregation of commodities is convenient, since it implies that the relative quantities of domestic final energy and non-energy commodities are constant.

1.4 Households

In ToTEM II, we assume the existence of three types of consumers: (i) unrestricted lifetime-income consumers, (ii) restricted lifetime-income consumers and

[13] See Chacra (2002) for the effects of global energy price shocks on retail energy prices in Canada.

(iii) current-income consumers.

Unrestricted households face a lifetime budget constraint and can freely borrow or save to reallocate consumption across time by trading in both short- and long-term bond markets. We assume that these households view short- and long-term securities as imperfect substitutes. They incur some disutility from holding long-term assets and therefore demand a premium to do so. We model the disutility as a transaction cost that the unrestricted households have to pay for purchasing one unit of a long-term bond. This assumption breaks the perfect arbitrage opportunity between the two assets and allows the long-term rate to deviate from the level implied by the pure expectations theory of the term structure.[14] The deviation is modelled as the term premium and its presence implies that long-term rates can vary independently of the expected path of short-term rates.

Restricted households are similar to unrestricted households, except that they can trade only in long-term bond markets and do not have to pay a transaction cost when they do so. The existence of households with restricted asset market participation ensures that consumption decisions of this subset of households are driven by long-term rates. As a result, aggregate consumption and residential investment in ToTEM II depend on both short- and long-term interest rates.[15]

Current-income consumers face a period-by-period budget constraint which equates their current consumption with their disposable income, including government transfers. These consumers represent those households in the economy that do not have access to credit or asset markets to smooth consumption. Therefore, changes in taxes and transfers can have larger effects on the consumption of such households.

The consumption variable in ToTEM represents several National Income and Expenditure Accounts (NIEA) series: personal expenditures, residential structures investment and inventory investment. By contrast, the stocks of residential structures and inventories enter the household utility function separately in ToTEM II, allowing for separate dynamic paths for personal consumption expenditure,

[14] See Andres, Lopez-Salido and Nelson (2004).

[15] The division of lifetime-income households into restricted and unrestricted allows ToTEM to capture important aspects of households' behaviour in the asset markets. Unrestricted households can be thought of as representing the portion of the private sector that saves through commercial bank deposits; restricted households can be thought of as consisting of individuals who save primarily through pension funds, or individuals who have a specific preference for long-term bonds. The latter interpretation is consistent with the preferred-habitat view, proposed by Culbertson (1957) and Modigliani and Sutch (1966). Greenwood and Vayanos (2009) discuss different market episodes supporting the preferred-habitat view.

residential structures investment and inventory investment. The housing and inventory stocks enter utility in an additively separable way, and they are subject to adjustment costs and habit formation. While households have separate demands for personal consumption expenditures, residential structures investment and inventory investment, all three goods are produced and supplied by the consumption sector, so there is no variation in their relative prices.

We assume that households supply labour differentiated into a continuum of types. The economy has a continuum of unions, each representing a type of differentiated labour. Each union sets the wage rate for its members and faces nominal wage rigidities. Inside each union, there are only unrestricted and restricted lifetime-income consumers. The current-income consumers receive the aggregate wage rate of the economy.

We begin by presenting the optimization problem for unrestricted lifetime-income consumers. The optimization problem for restricted agents is the same except for bonds; restricted households do not trade in short-term bonds and do not pay transaction costs when trading long-term bonds.

The period t utility function for the representative unrestricted lifetime-income household is

$$\begin{aligned} \mathbb{U}_t^{ul} \quad = \quad & \frac{\mu}{\mu-1}(C_t^{ul}-\xi C_{t-1}^{ul})^{\frac{\mu-1}{\mu}} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)}\int_0^1 \left(N_t^{ul}(h)\right)^{\frac{1+\eta}{\eta}} dh\right) \\ & +\boldsymbol{\zeta}_t^{hl,ul}\frac{\mu_{HL}}{\mu_{HL}-1}\left(RS_t^{ul}-\xi_{HL}RS_{t-1}^{ul}\right)^{\frac{\mu_{HL}-1}{\mu_{HL}}} \\ & +\boldsymbol{\zeta}_t^{inv,ul}\frac{\mu_{INV}}{\mu_{INV}-1}\left(INV_t^{ul}-\xi_{INV}INV_{t-1}^{ul}\right)^{\frac{\mu_{INV}-1}{\mu_{INV}}} \quad , \end{aligned} \tag{1.29}$$

where C_t^{ul} is the level of total consumption, $N_t^{ul}(h)$ is hours worked for labour type h, $\boldsymbol{\zeta}_t^{hl,ul}$ is a housing preference shock, RS_t^{ul} is the level of residential structures, $\boldsymbol{\zeta}_t^{inv,ul}$ is an inventory preference shock and INV_t^{ul} is the stock of inventories. Inventory shock $\boldsymbol{\zeta}_t^{inv,ul}$ captures the reduced-form impact on the household utility of temporary movement in the technology for producing inventories, as reflected in the storage of physical goods. The parameters that appear in the utility function are as follows: μ, μ_{HL} and μ_{INV} are intertemporal elasticities of substitution; ξ, ξ_{HL} and ξ_{INV} are habit-persistence parameters; η is the wage elasticity of labour supply.[16]

[16]Iacoviello, Schiantarelli and Schuh (2010) specify a similar CES aggregator of goods and output inventories. Low-frequency evolution in the storage and distributing technology (such

Total consumption is obtained by combining core consumption and commodity goods, and we use a fixed-factor Leontieff technology, instead of the constant-elasticity-of-substitution specification used in ToTEM. The latter assumption helps the model better match Canadian data in two respects. First, consumer demand for commodities is very inelastic and there is little substitution between core consumption and commodity consumption in response to relative-price movements. This, in turn, implies that commodity exports also respond less to commodity-price movements.

Given our assumption of Leontieff technology, the aggregate consumption price level $P^{tot}_{c,t}$ is

$$P^{tot}_{c,t} = (1+\tau_{c,t})\left\{s_c P^c_t + (1-s_c)P^{com,h}_t\right\} \ , \tag{1.30}$$

where $\tau_{c,t}$ is the indirect tax rate on consumer expenditures, and s_c is the share of core consumption goods in total consumption.

The laws of motion for the stock of residential structures and inventories are

$$RS^{ul}_{t+1} = (1-\delta_{RS})RS^{ul}_t + I^{ul}_{RS,t} - \frac{\chi_{RS}}{2} I^{ul}_{RS,t}\left(\frac{I^{ul}_{RS,t}}{I^{ul}_{RS,t-1}} - 1\right)^2 \ , \tag{1.31}$$

$$INV^{ul}_{t+1} = (1-\delta_{INV})INV^{ul}_t + I^{ul}_{INV,t} - \frac{\chi_{INV}}{2} I^{ul}_{INV,t}\left(\frac{I^{ul}_{INV,t}}{I^{ul}_{INV,t-1}} - 1\right)^2 \ , \tag{1.32}$$

where δ_{RS} and δ_{INV} are the respective depreciation rates. The sizes of the investment adjustment cost for residential and inventory investment are given by χ_{RS} and χ_{INVT}, respectively.

The budget constraint that these households face is

$$\begin{aligned} &P^{tot}_{c,t}C^{ul}_t + P^c_t I^{ul}_{RS,t} + P^c_t I_{INV,t} + \frac{B^{ul}_t}{1+R_t} + \frac{e_t B^{ul*}_t}{1+R^*_t} + \frac{(1+\phi_t)^{20} B^{20ul}_t}{(1+R^{20}_t)^{20}} \\ &= B^{ul}_{t-1} + e_t B^{ul*}_{t-1} + B^{20ul}_{t-20} + (1-\tau_{w,t})\int_0^1 N^{ul}_t(h)W_t(h)dh + TF^{ul}_t + \Pi_t \ , \end{aligned}$$

where $P^{tot}_{c,t}$ is the aggregate consumption price level, B^{ul}_t are holdings of domestic short-term bonds, R_t is the domestic short-term interest rate, e_t is the nominal

as online shopping, or just-in-time inventory management) might also be reflected in changes of structural parameters such as μ_{INV} or the volatility of the inventory shock $\boldsymbol{\zeta}^{inv,ul}_t$.

exchange rate, B_t^{ul*} are holdings of foreign short-term bonds, B_t^{20ul} denotes holdings of domestic long-term bonds, R_t^{20} is the domestic long-term interest rate, ϕ_t is the transaction cost of trading domestic long-term bonds, $\tau_{w,t}$ is the labour income tax rate, $W_t(h)$ is the nominal wage rate for labour type h, TF_t^{ul} are the level of nominal transfers received from the government and Π_t denotes the total profits of firms in the economy.[17]

Both versions of the model are small open-economy models with incomplete asset markets, where consumers can borrow at an exogenous foreign interest rate. This implies that, without additional assumptions, transitory shocks will lead to permanent deviations in consumers' net foreign asset (NFA) position.[18] In ToTEM, stationarity of net foreign assets is obtained by making the risk premium that consumers face in foreign financial markets a function of their NFA position.

In turn, in ToTEM II, stationarity of net foreign assets is achieved by assuming that households become more patient when their financial wealth-to-disposable-income ratio is low, and vice versa. Financial wealth, in addition to net foreign assets, includes housing, holdings of government debt and stock market wealth evaluated at the 'fundamental' shadow value of capital.[19]

The change in assumptions regarding NFA in ToTEM II allows for a more direct link between household wealth and consumption behaviour, as well as for a model-consistent projection for net wealth and savings. Furthermore, since house prices affect the wealth gap positively, and wealth affects consumption, there is now a direct link between house prices and consumption.

The unrestricted lifetime-income household seeks to maximize the objective function

$$\mathbf{E}_t \sum_{s=t}^{\infty} \boldsymbol{\beta}_{t,s} \mathbb{U}_s^{ul} \tag{1.33}$$

with $\boldsymbol{\beta}_{t,s} \equiv \prod_{v=t}^{s-1} \beta_v$, $\boldsymbol{\beta}_{t,t} \equiv 1$ and $\beta_v = \beta \left(fw_v\right)^{-\psi} \left(\epsilon_v^C\right)^{-1}$, subject to the budget constraint and the laws of motion for residential structures and inventories. Here

[17]Unrestricted households can also trade risk-free government bonds. Due to no arbitrage opportunities, households are indifferent between holding private and government bonds. We therefore omit government bond holdings from the budget constraint.

[18]Schmitt-Grohe and Uribe (2003) describe alternative approaches to obtain stationarity in small open-economy models. One of these approaches consists in assuming that the discount factor is endogenous. In particular, they assume that the discount factor depends on the per-capita levels of consumption and effort.

[19]A detailed derivation of household financial wealth is provided in Appendix A.5.

fw_v is the ratio of aggregate net wealth to disposable income, and ϵ_v^C is a positive consumption preference shock. This optimization problem yields the first-order conditions that characterize unrestricted lifetime-income agents' economic decisions. As mentioned earlier, these first-order conditions are the same for restricted lifetime-income agents, except for the one associated with long-term bonds.

1.4.1 Asset prices

By combining the unrestricted agents' first-order conditions for short- and long-term domestic bond holdings, we obtain the following linearized relationship between long- and short-term interest rates faced by households:

$$\widehat{R}_t^{20} = \frac{1}{20}\sum_{j=0}^{19} E_t\widehat{R}_{t+j} + \widehat{\phi}_t \ , \tag{1.34}$$

where $\widehat{\phi}_t$ is the (log deviation) transaction cost of trading long-term bonds. As we can see, this term allows for deviations of the pure expectations theory of the term structure for private bonds. To allow the model to take account of factors underlying deviations between long- and short-term rates, as well as those between risky and risk-free rates, we introduce a few additional specifications.

Let $R_{RF,t}$ and $R_{RF,t}^{20}$ be the one-period and twenty-period risk-free rates. It is assumed that the one-period risk-free rate, $R_{RF,t}$, is perfectly controlled by the monetary authority.[20] The term premium on risk-free rates, tp_t, is the difference between the twenty-period risk-free rate and the average of the expected one-period future risk-free rates,

$$\widehat{R}_{RF,t}^{20} = \frac{1}{20}\sum_{j=0}^{19} E_t\widehat{R}_{RF,t+j} + tp_t \ , \tag{1.35}$$

The short-term household interest rate is the one-period risk-free rate plus a short-term risk premium (or short-term spread), $stsp_t$,

$$\widehat{R}_t = \widehat{R}_{RF,t} + stsp_t \ , \tag{1.36}$$

and the long-term household interest rate is the long-term risk-free rate plus the

[20] Section 1.7 characterizes sequences of risk-free interest rates chosen by the monetary authority.

sum of expected future short-term spreads and the (net) long-term spread, $ltsp_t$,

$$\widehat{R}_t^{20} = \widehat{R}_{RF,t}^{20} + \frac{1}{20}\sum_{j=0}^{19} stsp_{t+j} + ltsp_t \ , \tag{1.37}$$

where both risk premia, $stsp_t$ and $ltsp_t$, are treated as exogenous in the model.[21]

The assumption of exogenous risk spreads is an important limitation of the interest rate structure in ToTEM II. We would expect risk spreads to be related to endogenous variables such as leverage ratios. Modelling such relationships would allow macroeconomic shocks and policies to affect risk spreads, and would therefore have implications for the policy prescriptions that emerge from the model. Other authors have modelled risk spreads as endogenous, but only in environments without an independent role for long-term rates.[22]

Next, by combining the unrestricted agents' first-order conditions for domestic and foreign short-term bond holdings, we obtain the conventional uncovered interest parity (UIP) condition,

$$\widehat{e}_t = E_t\widehat{e}_{t+1} + \widehat{R}_t^* - \widehat{R}_t \ . \tag{1.38}$$

To better match the business-cycle properties of the nominal and real exchange rate, we assume a hybrid UIP condition,

$$\widehat{e}_t = \varkappa\widehat{e}_{t-1} + (1-\varkappa)\left[E_t\widehat{e}_{t+1} + \widehat{R}_t^* - \widehat{R}_t + \boldsymbol{\zeta}_t^e\right] \ , \tag{1.39}$$

where $\varkappa \in [0,1]$ and $\boldsymbol{\zeta}_t^e$ is an exogenous exchange rate shock. There are two differences with respect to the UIP used in ToTEM. First, the country risk premium affects the UIP through the exogenous exchange rate shock term in ToTEM II, whereas in ToTEM the country risk premium is a function of the NFA position. Second, now the weighting factor $(1-\varkappa)$ applies not only to the expected future exchange rate but also to the interest differential. Thus, under the modified UIP condition, the current exchange rate is a weighted average of the past exchange rate and the 'textbook' UIP condition. The main implication of this modification

[21] This specification implies that the term premium on risky rates equals the sum of the term premium on risk-free rates and the net long-term risk premium: $\phi_t = ltsp_t + tp_t$.

[22] For example, Bernanke, Gertler and Gilchrist (1999) derive a model in which the risk spread that a firm must pay to borrow is a function of its leverage ratio. Similarly, Basant Roi and Mendes (2007) assume that the risk spread faced by a household depends on the household's ratio of debt to housing wealth.

is that the elasticity of the exchange rate with respect to the contemporaneous interest rate differential is smaller compared to the previous version of ToTEM.

1.4.2 Consumption and investment for lifetime consumers

By using the unrestricted and restricted agents' linearized first-order conditions for consumption and domestic bond holdings, we obtain the equation that describes the consumption of both types of lifetime-income households,

$$\begin{aligned}\widehat{C}^l_t &= \xi\widehat{C}^l_{t-1} + E_t\left[\widehat{C}^l_{t+20} - \xi\widehat{C}^l_{t+19}\right] + (1-\xi)(1-\mu)\overline{L}^{1+1/\eta}\left[\widehat{L}^l_t - E_t\widehat{L}^l_{t+20}\right] \\ &\quad -\mu(1-\xi)\left[s_u\sum_{j=0}^{19}E_t\widehat{R}_{t+j} + (1-s_u)20\widehat{R}^{20}_t\right] \\ &\quad +\mu(1-\xi)\psi\sum_{j=0}^{19}E_t\widehat{fw}_{t+j} + \mu(1-\xi)\sum_{j=0}^{19}\left[E_t\widehat{\pi}^{tot}_{c,t+j+1} + \epsilon^C_{t+j}\right], \qquad (1.40)\end{aligned}$$

where $\widehat{C}^l_t = s_u\widehat{C}^{ul}_t + (1-s_u)\widehat{C}^{rl}_t$, s_u is the percentage of lifetime-income consumers that are unrestricted and $\overline{L}$ is the steady-state level of labour.[23] Equation (1.40) differs from the one in ToTEM in two main ways. First, it allows the long-term interest rate to play a meaningful role in consumption decisions, over and above the traditional role for short-term rates. This role arises because: (i) we allow for imperfect substitutability between short-term and long-term bonds ($\widehat{\phi}_t \neq 0$), and (ii) the long-term rate is the only rate that matters for restricted lifetime-income consumers. Second, equation (1.40) allows the net financial wealth gap to influence consumption. For example, given that housing wealth is part of net financial wealth, in ToTEM II there is a link between house prices and consumption.

Notice that when there are no term-premium or long-rate risk-premium shocks (i.e., when $\widehat{\phi}_t = 0$), long- and short-term bonds are perfect substitutes, and the distinction between restricted and unrestricted households is irrelevant. In this case, equation (1.40) becomes a standard Euler equation for consumption under

[23]Aggregation of consumption is possible in this way because the model is linearized and we assume that steady-state consumption is the same for both types of agents.

external habit formation,[24]

$$\widehat{C}_t^l = \frac{\xi}{(1+\xi)}\widehat{C}_{t-1}^l + \frac{1}{(1+\xi)}\widehat{C}_{t+1}^l - \Gamma\Delta\widehat{L}_{t+1} - \mu\Lambda(\widehat{R}_t - E_t\pi_{c,t+1}^{tot}) + \mu\Lambda\psi\widehat{fw}_t + \mu\Lambda\epsilon_t^C \ , \tag{1.41}$$

where $\Lambda = (1-\xi)/(1+\xi)$ and $\Gamma = \Lambda(1-\mu)\overline{N}^{\frac{1+\eta}{\eta}}$.

Turning to the log-linearized first-order condition for the investment in residential structures, we obtain the following demand equation:

$$\widehat{I}_{RS,t}^l = \frac{1}{1+\beta}\widehat{I}_{RS,t-1}^l + \frac{\beta}{1+\beta}\widehat{I}_{RS,t+1}^l + \frac{1}{\chi_{RS}(1+\beta)}\left\{\widehat{\Psi}_t^{RSl} - \widehat{\lambda}_t^l - (\widehat{P}_t^c - \widehat{P}_{c,t}^{tot})\right\} \ , \tag{1.42}$$

where $\widehat{\Psi}_t^{RSl}$ is the shadow price of residential structures and $\widehat{\lambda}_t^l$ is the marginal utility of consumption. Log deviations of investment in residential structures are driven by a forward-looking second-order difference equation, as in the case for business investment. In the absence of adjustment costs ($\chi_{RS} = 0$), the level of investment adjusts instantaneously to ensure that the household is indifferent between investing in residential structures and consuming, which occurs when the ratio of marginal utility of investment in residential structures and marginal utility of consumption are equal to their relative prices. With adjustment costs, households optimally trade off the costs of having a suboptimal level of investment with the costs of adjusting their level of investment. A lag of investment therefore appears in the log-linearized first-order condition, helping the model to generate a hump-shaped response of residential structures investment to movements in interest rates.

By combining the first-order conditions for investment in residential structures, the stock of residential structures and domestic long-term bonds, we obtain an

[24]Habits were assumed to be internal in ToTEM. Levin et al. (2008) show how identical log-linear approximations may be consistent with different preference specifications, implying different optimal steady-state policies. For example, external habits, in contrast to internal habits, introduce an externality into consumption behaviour and thereby have important implications for optimal steady-state inflation.

alternative equation for the demand for investment in residential structures,

$$
\begin{aligned}
\widehat{I}^{l}_{RS,t} &= \frac{1}{1+\beta}\widehat{I}^{l}_{RS,t-1} + \frac{\beta}{1+\beta}\widehat{I}^{l}_{RS,t+1} - \frac{1}{\chi_{RS}(1+\beta)}(\widehat{P}^{c}_{t} - \widehat{P}^{tot}_{c,t}) \\
&\quad - \frac{1}{\chi_{RS}(1+\beta)}\left\{ s_u \sum_{j=0}^{\infty} E_t\widehat{R}_{t+j} + (1-s_u)20\sum_{j=0}^{\infty} E_t\widehat{R}^{20}_{t+20j} \right\} \\
&\quad + \frac{1}{\chi_{RS}(1+\beta)}\left\{ \psi \sum_{j=0}^{\infty}\left[1-\beta^j(1-\delta_{RS})^j\right] E_t\widehat{fw}_{t+j} + \sum_{j=0}^{\infty} E_t\widehat{\pi}^{tot}_{c,t+j+1} \right\} \\
&\quad - \frac{1-\beta(1-\delta_{RS})}{\chi_{RS}(1+\beta)}\frac{(1-\xi_{HL})^{-1}}{\mu_{HL}}\sum_{j=0}^{\infty}\beta^j(1-\delta_{RS})^j E_t\left[\widehat{RS}^{l}_{t+j+1} - \xi_{HL}\widehat{RS}^{l}_{t+j}\right] \\
&\quad + \frac{1-\beta(1-\delta_{RS})}{\chi_{RS}(1+\beta)}\sum_{j=0}^{\infty}\beta^j(1-\delta_{RS})^j E_t\boldsymbol{\zeta}^{hl,l}_{t+j+1} \\
&\quad + \frac{1}{\chi_{RS}(1+\beta)}\sum_{j=0}^{\infty}\left[1-\beta^j(1-\delta_{RS})^j\right] E_t\epsilon^{C}_{t+j} \ . \qquad (1.43)
\end{aligned}
$$

This equation shows that we can also express the evolution of investment in residential structures as a function of the expected paths of the different interest rates, household financial wealth, the CPI inflation rate, and the stock of housing relative to its long-run desired level, as well as the first leads and lags of residential structures investment and its contemporaneous relative price. This equation also shows that the interest rate elasticity of housing demand can differ from that of consumption in equation (1.41), depending on the size of the adjustment cost, habit persistence and the discount factor. The elasticity of housing demand with respect to financial wealth also differs from the one of consumption. For instance, investment in residential structures does not respond to changes in financial wealth, whereas consumption does. In general, given that the discount factor β is very close to one and the depreciation rate on housing is very close to zero, the coefficients in front of the expected path of financial wealth in the housing demand equation tend to be close to zero. This suggests that consumption is more sensitive than investment in residential structures to changes in financial wealth.

1.5 Wage setting

We assume that the economy has a continuum of unions, each representing one type of differentiated labour. Each union sets the wage rate for its members. Inside each union, there are two types of members: a fraction s_u are unrestricted lifetime-income consumers and a fraction $1 - s_u$ are restricted lifetime-income consumers. The current-income consumers receive the aggregate wage rate W_t. Firms allocate labour demand uniformly across different workers of type h, independently of their household type. Given this assumption, it follows that $N_t^{ul}(h) = N_t^{rl}(h) = N_t^{ci}(h) = N_t(h)$.

We assume the existence of two different types of unions: a measure Ω of the rule-of-thumb (RT) unions and a measure $1-\Omega$ of forward-looking (FL) unions. For each type of union, there are two different types of wage setting. With probability θ_w, all unions index their own wage to the inflation target. With probability $1-\theta_w$, RT unions and FL unions have different wage-setting rules.

More specifically, with probability $1-\theta_w$, RT unions adjust their wages according to the rule

$$w_t^b = w_{t-1} + \gamma_w \pi_{t-1}^w + (1 - \gamma_w)\overline{\pi_t} + \Theta \mu_t^w \ , \tag{1.44}$$

where w_t^b is the wage set by RT unions, w_t is the average wage level, $\overline{\pi_t}$ denotes the inflation target in period t, π_t^w is nominal wage inflation, μ_t^w is a wage markup shock, and γ_w is the fraction of RT unions that index to lagged inflation, with the remaining fraction $1 - \gamma_w$ of RT unions indexing to the inflation target.

In turn, with probability $1 - \theta_w$, FL unions choose the optimal wage $W_t^*(h)$ to maximize the total present value of its members' current and future period utility levels,

$$F_t = \mathbf{E}_t \sum_{s=t}^{\infty} \boldsymbol{\beta}_{t,s} (\theta_w)^{s-t} \left\{ \ s_u U_s^{ul} + (1 - s_u) U_s^{rl} \ \right\} \ , \tag{1.45}$$

subject to the budget constraints for its members and the labour demand schedule

$$N_s(h) = \left(\frac{W_t^*(h)}{W_s} \Pi_{s,t}^w \right)^{-\epsilon_w} H_t \ , \tag{1.46}$$

with $\Pi^w_{s,t}$ defined by

$$\Pi^w_{s,t} = \begin{cases} \mathbf{E}_t \left(\prod_{j=t}^{s-1} (1 + \overline{\pi}_j) \right) & s > t \\ = 1 & s = t \ . \end{cases} \tag{1.47}$$

Following Erceg, Henderson and Levin (2000), we assume that there is full consumption risk-sharing across lifetime households. Therefore, each FL union chooses the same wage w^*_t (expressed relative to the core consumption price index), which satisfies the first-order condition

$$\begin{aligned} w^*_t - w_t &= \beta\theta_w E_t \left\{ w^*_{t+1} - w_{t+1} \right\} - \frac{\{1 - \beta\theta_w\}\,\eta}{\eta + \epsilon_w} \left[\widehat{w}_t - \widehat{p^{tot}_{c,t}} - \widehat{\tau_{w,t}} - \widehat{mrs_t} - \mu^w_t \right] \\ &\quad + \beta\theta_w E_t \left\{ \pi^w_{t+1} - \overline{\pi_{t+1}} \right\} \ , \end{aligned} \tag{1.48}$$

where the marginal rate of substitution is given by

$$\widehat{mrs_t} = \frac{1}{1-\xi}(\widehat{C}^l_t - \xi \widehat{C}^l_{t-1}) + \frac{1}{\eta}\widehat{L}^l_t + \widehat{E}^l_t \ . \tag{1.49}$$

The average wage for those unions that do not index their own wage to the inflation target is

$$\widetilde{w}_t - w_t = \Omega(w^b_t - w_t) + (1-\Omega)(w^*_t - w_t) \ . \tag{1.50}$$

The relation between $\widetilde{w}_t$ and the aggregate wage level is

$$\widetilde{w}_t - w_t = \frac{\theta_w}{1-\theta_w} \left\{ \pi^w_t - \overline{\pi_t} \right\} \ . \tag{1.51}$$

Finally, by combining equations (1.44), (1.48), (1.50) and (1.51), we obtain a wage-inflation equation

$$\pi^w_t = (1-\theta_w)\,\gamma_w \Omega \phi_w^{-1} \pi^w_{t-1} + \beta\theta_w \phi_w^{-1} E_t \left\{ \pi^w_{t+1} \right\} - \widetilde{\lambda}_w \left[\widehat{w}_t - \widehat{p^{tot}_{c,t}} - \widehat{\tau_{w,t}} - \widehat{mrs_t} \right] + \varepsilon^w_t \ , \tag{1.52}$$

where ε^w_t is a linear combination of the inflation target and the wage markup shock.

ϕ_w and $\widetilde{\lambda}_w$ are given by

$$\begin{aligned}\phi_w &\equiv \theta_w + \Omega\left(1-\theta_w\right)\left(1+\gamma_w \beta \theta_w\right) \\ \widetilde{\lambda}_w &\equiv \left(1-\Omega\right)\left(1-\theta_w\right)\left(1-\beta\theta_w\right)\frac{\eta}{\eta+\epsilon_w}\phi_w^{-1} \ .\end{aligned}$$

As with our pricing specification, the introduction of RT wage setters decreases the effect of expected net real wages and of the marginal rate of substitution on wage inflation. Moreover, expected net real wages and marginal rates of substitution in the distant future receive smaller weights relative to near-term conditions (i.e., there is overdiscounting), when compared to the standard Calvo/indexation set-up used in ToTEM, in which those weights were determined by the household's discount factor β.

1.6 Foreign links

Foreign links are introduced in the same way as in ToTEM. Aggregate rest-of-world demand in ToTEM II is affected by short- and long-term foreign interest rates in a way that is analogous to the one used in the domestic block, and also features a fully specified term structure that relates foreign short- and long-term rates. This structure allows the study of unconventional monetary policies in the rest of the world and, in particular, their effects on the foreign term premium and the foreign aggregate demand. Inflation in the rest of the world is driven by the same Calvo/indexation set-up as in ToTEM, and we do not consider the possibility of overdiscounting.

Canadian exporters of finished products sell a good that has some degree of market power as a result of its differentiation relative to its global competitors, as in the first version of the model. In ToTEM II, the export demand function is the same as the domestic demand function for imports. The inclusion of adjustment costs means that the first lead and the first lag of exports and relevant foreign output enter the linearized export demand function,

$$\widehat{X}_t = \widehat{Y}_t^* - \vartheta \widehat{p}_{m,t}^* + \vartheta_2\left(\widehat{X}_{t-1} - \widehat{Y}_{t-1}^*\right) + \vartheta_3 E_t\left(\widehat{X}_{t+1} - \widehat{Y}_{t+1}^*\right) \ , \qquad (1.53)$$

where $\widehat{p}_{m,t}^*$ is the log deviation of the relative foreign dollar price of Canada's non-commodity exports $\left(P_{m,t}^*/P_t^*\right)$, and ϑ is the price elasticity of export demand. $\widehat{Y}_t^*$

is the log deviation of the foreign activity measure relative to its steady state.[25] The specification of the export demand function is motivated by a simplifying assumption that, given the close trade links between the United States and Canada, U.S. economic activity matters more for Canada's non-commodity exports. The rest-of-world aggregate output affects Canadian non-commodity exports only to the extent that it affects U.S. economic activity. The existence of adjustment costs is associated with high inertia in manufactured exports and low sensitivity to contemporaneous foreign relative-price movements.

Given the resource constraint for the commodity sector, commodities produced in Canada must either be used for the domestic production of finished goods, consumed directly by households or exported. Therefore, commodity exports are determined as a residual between total commodity production and domestic commodity demand.

1.7 Monetary policy

We assume that the monetary authority sets the short-term risk-free interest rate $R_{RF,t}$ and chooses the sequences of those rates according to an augmented Taylor rule that depends on the output gap $\widehat{y_t}$, deviations of expected two-quarter-ahead core inflation from the inflation target π_t^* and the interest rate smoothing term

$$R_{RF,t} = \Theta_R R_{RF,t-1} + (1 - \Theta_R)(\overline{r} + \pi_t^* + \Theta_\pi(\pi_{t+2} - \pi_t^*) + \Theta_y \widehat{y_t}) \ , \tag{1.54}$$

where $\overline{r}$ is the steady-state real interest rate, Θ_R is the interest rate smoothing parameter, Θ_π is the sensitivity of the short-term risk-free interest rate to core inflation deviation from the target and Θ_y is the sensitivity of the short-term risk-free interest rate to the output gap.

1.8 Fiscal policy

The government in ToTEM II is specified in the same way as in ToTEM. The function of the government in ToTEM II is to: (i) purchase goods and services

[25] Morel (2012) constructs a measure of foreign activity that takes into account the composition of foreign demand for Canadian exports. Given the close trade linkages between the United States and Canada, the composition of demand in the United States has an 87 per cent weight in the foreign activity measure relative to a weight of 13 per cent of economic activity outside the United States. We use this foreign activity measure as a proxy for foreign demand for Canadian exports.

for the government from local producers who face imperfect competition and some degree of price rigidity; (ii) distribute transfers to households; (iii) collect taxes on labour income and consumption; (iv) issue nominal government bonds to domestic and foreign households. The government budget constraint is specified as

$$\frac{B_{g,t}}{(1+R_{RF,t})}+\frac{B^{20}_{g,t}}{(1+R^{20}_{RF,t})^{20}}=B_{g,t-1}+B^{20}_{g,t-1}+P^g_tG_t+TF_t-TX_t\ , \quad (1.55)$$

where $B_{g,t}$ and $B^{20}_{g,t}$ are the nominal short-term and long-term government bonds, respectively. TF_t is the level of nominal transfers, and TX_t is the sum of consumption and income tax revenues:

$$TX_t = TX^w_t + TX^c_t\ , \quad (1.56)$$

$$TX^w_t = \tau_{w,t}W_tH_t\ , \quad (1.57)$$

$$TX^c_t = \tau_{c,t}\left(\frac{P^{tot}_{c,t}C^{tot}_t}{1+\tau_{c,t}}\right)\ , \quad (1.58)$$

where $\tau_{w,t}$ is the direct tax rate on labour income, and $\tau_{c,t}$ is the indirect tax rate on consumer expenditure following a simple autoregressive stochastic process of the form

$$\tau_{c,t} = \rho_{\tau_c}\tau_{c,t-1}+\varepsilon^{\tau_c}_t, \quad \text{where } \varepsilon^{\tau_c}_t \sim iid(0,\sigma^2_{\tau_c}). \quad (1.59)$$

Government spending on goods and services is characterized by the following equation:

$$\frac{P^g_tG_t}{P_tY_t} = \rho_g\frac{P^g_{t-1}G_{t-1}}{P_{t-1}Y_{t-1}}+(1-\rho_g)\left(\overline{\frac{P^gG}{PY}}\right)+\mu^g_t\ , \quad (1.60)$$

where μ^g_t follows the autoregressive process

$$\mu^g_t = \rho_g\mu^g_{t-1}+\varepsilon^g_t, \quad \text{where } \varepsilon^g_t \sim iid(0,\sigma^2_g). \quad (1.61)$$

Similarly, the share of transfers to nominal GDP is determined according to

$$\frac{TF_t}{P_tY_t} = \rho_{tf}\frac{TF_{t-1}}{P_{t-1}Y_{t-1}}+(1-\rho_{tf})\left(\overline{\frac{TF}{PY}}\right)+\varepsilon^{tf}_t, \quad \text{where } \varepsilon^{tf}_t \sim iid(0,\sigma^2_{tf}). \quad (1.62)$$

The government in ToTEM II finances its spending and transfers by levying direct and indirect taxes to maintain a desired debt-to-GDP ratio, $\left(\overline{\frac{B_{g,t-1}+B^{20}_{g,t-1}}{P_{t-1}Y_{t-1}}}\right)$, over the medium term. It is assumed that fiscal policy satisfies equations (1.55)–(1.62) as well as the fiscal policy rule for the labour income tax rate, $\tau_{w,t}$, specified as

$$\tau_{w,t} = \rho^1_{\tau_w}\tau_{w,t-1} + (1-\rho^1_{\tau_w})\left(\tau_w + \rho^2_{\tau_w}\left(\frac{B_{g,t}+B^{20}_{g,t}}{P_tY_t} - \overline{\frac{B_{g,t-1}+B^{20}_{g,t-1}}{P_{t-1}Y_{t-1}}}\right)\right) + \mu^{\tau_w}_t \ , \tag{1.63}$$

where τ_w is the steady-state value of the labour income tax rate, and $\mu^{\tau_w}_t$ is a temporary tax shock following an AR(1) process,

$$\mu^{\tau_w}_t = \rho_{\tau_w}\mu^{\tau_w}_{t-1} + \varepsilon^{\tau_w}_t, \varepsilon^{\tau_w}_t \sim iid(0, \sigma^2_{\tau_w}). \tag{1.64}$$

The government in ToTEM II exhibits non-Ricardian behaviour due to the hand-to-mouth consumers and distortionary consumption and labour income taxation. The main implication of the non-Ricardian behaviour is that the level of government debt has real implications for the economy.

Table B.1 in Appendix B summarizes the main changes in ToTEM II.

Chapter 2

Model Estimation

2.1 Why estimation?

To improve the goodness of fit and forecasting accuracy of ToTEM II, we estimate a sizable subset of its parameters; the remaining parameters are calibrated such that the model matches some of the key moments in the Canadian data. Since estimation involves a large number of parameters that enter the likelihood non-linearly, we use Covariance Matrix Adaptation Evolution Strategy (CMA-ES), which is a genetic evolutionary heuristic algorithm (Hansen and Ostermeier 2001) to select parameters that maximize the likelihood (Amemiya 1985). We construct the likelihood using a method that is numerically equivalent to the Kalman filter for well-behaved (i.e., non-stochastically singular) systems based on the state-space representation of the model's rational expectations solution (Sims 2001). In this chapter, we describe the data used in the estimation, outline the calibration strategy, discuss estimated parameters and summarize the goodness of fit.

2.2 Data

For calibration and estimation, we use quarterly data for Canada from 1980Q1 to 2012Q2. Table 2.1 shows the observed variables used for estimation. We obtain National Accounts variables that enter the GDP identity equation, including: consumption (personal expenditures), residential investment (residential construction), business investment, inventory investment (total business inventories), government spending (government expenditure), exports and imports. Relative prices in sectors are constructed using their respective deflators; the relative price of consumption, for example, is the ratio of nominal personal consumption expenditures to real personal consumption expenditures. The quarterly log-difference in the core consumer price index (CPI excluding eight of the most volatile components) is our measure of core inflation in the consumption sector. We use the 90-day risk-free bank rate as our measure of the nominal short-term risk-free rate.

Fiscal variables include real direct labour income tax revenues, real indirect consumption tax revenues and current transfers to persons. Nominal government debt is included as a ratio to nominal GDP, and comprises the sum of the net financial assets of the federal, provincial and local governments, in addition to social security funds. Average hours worked are included as the ratio of total hours to total employment.

We include some of the foreign variables: foreign economic activity measure, rest-of-world inflation, and short- and long-term interest rate.[25]

We also include multiple interest rates as observables. The long-term risk-free rate is included as the 5-year yield on Government of Canada marketable bonds. Short-term household risk premia are included as the spread between 90-day chartered bank administered interest rates and the 90-day risk-free rate. Long-term household risk premia are included as the spread between the 5-year mortgage rate and the 5-year risk-free rate. Short-term firm risk premia are included as the spread between the 90-day prime corporate paper rate and the 90-day risk-free rate. Long-term firm risk premia are included as the spread between the average of the 3- to 5-year and 5- to 7-year Merrill Lynch corporate paper indices and the 5-year risk-free rate. For the rest-of-world block, the short-term rate is included as a trade-weighted composite and the long-term rate is the U.S. 5-year bond yield.

Net foreign assets are included as Canada's net international investment position as a share of nominal GDP. The real effective exchange rate is included as the trade-weighted average composite. The current account balance is included as a ratio to nominal GDP, and is defined as nominal exports less nominal imports, plus investment income from net foreign assets and net transfers abroad.

Most of the data are detrended based on cointegration relationships or the LRX filter (see Berg, Karam and Laxton 2006 for a description). The estimation is performed on the stationary series.

[25] We use U.S. short-term and long-term interest rates as proxies of foreign rates. The foreign aggregate output gap is used in a reduced-form relationship to affect the consumption preference shock in section 1.4.

Table 2.1: **Observed variables in ToTEM II estimation**

Category	Variables
National Accounts	Consumption, Residential investment, Business investment, Government expenditures, Imports
Prices	Relative price of government goods, Relative price of imports, Core CPI inflation
Labour Market	Labour income tax revenue, Nominal wage
Fiscal Variables	Transfers to persons, Consumption tax revenue, Government debt-to-GDP ratio
Foreign Variables	Rest-of-world output gap, Rest-of-world inflation, U.S. activity measure, Foreign short-term (ST) interest rate, Foreign long-term (LT) interest rate
Interest Rates	Domestic ST and LT interest rate, ST and LT corporate risk premium, ST and LT household risk premium
Commodity Sector	Energy commodity price, Non-energy commodity price
Other	Net foreign assets, Current account-to-GDP ratio, Canada/ROW real effective exchange rate

2.3 Calibration

We partition the calibrated parameters into two groups: steady-state targets and steady-state parameters.[26] The first group includes the discount factor β, the value of which corresponds to $\beta = (1+r)^{-1}$ given a choice for the real quarterly interest rate of 0.8 per cent. The depreciation rate d is set at 0.032, as estimated in studies of capital prices in secondary markets (Statistics Canada 2007).

In the latter group, values of parameters affecting the steady state are chosen primarily such that the model's steady state is consistent with historical averages in the Canadian data over the sample period considered for estimation. For example, the share of capital in the production function is chosen to replicate the historical nominal investment-to-GDP ratio. For each sector, steady-state productivity is chosen to match steady-state relative prices in the data; this means that sectors with low relative prices are assumed to be very productive. In the commodities sector, the fixed factor of production $LAND$ is chosen to match the historical commodity production-to-GDP ratio. Marginal tax rates τ^c and τ^n are chosen to match historical consumption tax revenue relative to GDP and labour income tax

[26] There is no growth in the steady state, unlike in ToTEM.

Table 2.2: **Calibration Targets**

Target description	Value
Share of imported goods used in consumption production	0.50
Share of imported goods used in investment production	0.18
Share of imported goods used in exports production	0.33
Ratio of residential structures investment to GDP	0.06
Government expenditures-to-GDP ratio	0.25
Inventory investment-to-GDP ratio	0.004
Commodity production-to-GDP ratio	0.17
Commodity exports-to-GDP ratio	0.14
Investment-to-GDP ratio	0.12
Imports-to-GDP ratio	0.29
Exports-to-GDP ratio	0.31
Relative price of government goods	0.92
Relative price of investment goods	1.26
Relative price of imported goods	1.28
Relative price of exported goods	1.15
Consumption tax revenue-to-GDP ratio	0.18
Labour income tax revenue-to-GDP ratio	0.18
Transfers revenue-to-GDP ratio	0.09
Government debt-to-GDP ratio	0.49
Exchange rate	0.78
Nominal labour income-to-GDP ratio	0.55

revenue relative to GDP, respectively. The share of imports used in production for each sector is chosen to match the historical share of goods imported by that sector. The total supply of bonds is chosen to match the government debt-to-GDP ratio with a portion allocated to short-term bonds and a portion allocated to long-term bonds. The share of core consumption in total consumption can be calculated directly from the data. Table 2.2 summarizes the calibration.

2.4 Parameter estimates

The estimated parameters are summarized in Tables 2.3 and 2.4. There is a high degree of habit formation in consumption (0.94), suggesting a delayed peak response of consumption to movements in real interest rates and fiscal changes. With an intertemporal elasticity of substitution (IES) estimated at 0.88, consumption is less sensitive to changes in interest rates in ToTEM II.

Estimates for firms' pricing behaviour vary across sectors. For the consumption sector, the estimated Phillips curve is almost entirely forward looking, very flat, with a high degree of price stickiness and low marginal cost pass-through. Including the recent financial crisis period data also makes the Phillips curve flatter. The following linearized equations for core CPI inflation illustrate those differences in ToTEM II and ToTEM:

$$\pi_t^c = 0.008\pi_{t-1}^c + 1.0068 E_t \sum_{i=0}^{\infty} 0.8559^i \left\{ 0.0151 \widehat{rmc}_{t+i}^c + 0.1324 \widehat{\mu}_{t+i}^c \right\} \text{ , (ToTEM II)} \tag{2.1}$$

$$\pi_t^c = 0.18\pi_{t-1}^c + 0.12 E_t \sum_{i=0}^{\infty} 0.993^i \left\{ \widehat{rmc}_{t+i}^c + \widehat{\mu}_{t+i}^c \right\} \text{ , (ToTEM)} \tag{2.2}$$

where π_t is the quarterly core consumption inflation rate, $\widehat{rmc}_{t+i}^c$ is the real marginal cost in the core consumption good production and $\widehat{\mu}_{t+i}^c$ is an exogenous disturbance.

Note that to minimize the number of estimated parameters, the share of the rule-of-thumb firms is fixed at 0.3 in all sectors except consumption, where it is estimated to be 0.48. The imports sector has greater short-run marginal-cost pass-through, but still features moderately sticky prices. Commodity prices have the most infrequently changing prices, according to the estimates, with Calvo parameters surprisingly close to 1.0. We also estimate moderately high wage stickiness around 0.6, although this is much lower than the value in the previous version of the model.

Estimated monetary policy parameters indicate a high degree of interest rate smoothing, at a value of 0.9. The response to core inflation is 2.14, consistent with previous estimates (see Hofmann and Bogdanova 2012 for a review) but much lower than the optimized parameter in ToTEM (at a value of 20). We also estimate a weak response to the output gap at 0.076.

The share of households that are unrestricted in their access to financial markets is estimated to be 0.12, suggesting a substantial degree of asset market segmentation in Canada. The share of current-income households is estimated to be around 8 per cent, less than half of the calibrated share (20 per cent) in ToTEM, implying lower sensitivity of consumption to labour income movements in ToTEM II. The lower share of current-income households also implies lower sensitivity of

Table 2.3: **Estimated Phillips Curve Parameters**

Parameter	Description	Value
θ_E	Calvo - energy	0.95
θ_{NE}	Calvo - non-energy	0.93
θ_{C^*}	Calvo - ROW C	0.81
θ_{X^*}	Calvo - ROW X	0.01
θ_G	Calvo - G	0.78
θ_I	Calvo - I	0.40
θ_M	Calvo - M	0.59
θ_X	Calvo - X	0.45
θ_W	Calvo - wage	0.59
θ_C	Calvo - C	0.75
γ_E	Indexation - energy	0.58
γ_{NE}	Indexation - non-energy	0.57
γ_{C^*}	Indexation - ROW C	0.53
γ_{X^*}	Indexation - ROW X	0.77
γ_G	Indexation - G	0.49
γ_I	Indexation - I	0.56
γ_M	Indexation - M	0.74
γ_X	Indexation - X	0.01
γ_W	Indexation - wage	0.11
γ_C	Indexation - C	0.06
ω_-	RT share - not C	0.3
ω_C	RT share - C	0.48

consumption to government spending. The estimated weight on the lagged real exchange rate in the uncovered interest parity (UIP) condition is 0.16, suggesting lower persistence in the real exchange rate movement than in ToTEM (with a weight of 0.475 on the lagged real exchange rate). This estimate also helps to yield more realistic dynamics (see Berg, Karam and Laxton 2006).

2.4.1 Rule-of-thumb price and wage setters

Price and wage setting in ToTEM are not fully forward-looking given price/wage indexation to lagged inflation, leading to a Phillips curve in which inflation is equal

Table 2.4: **Other Estimated Parameters**

Parameter	Description	Value
μ	Consumption IES[1]	0.88
η	Wage elasticity of labour supply	0.07
μ_{HL}	Residential structures IES	0.9
ξ	Consumption habit	0.94
s_u	Share of unrestricted households	0.12
ψ	Sensitivity of discount rate to financial wealth	0.01
$\varkappa$	Weight on lagged real exchange rate in UIP	0.16
Θ_R	Policy rule - smoothing	0.9
Θ_π	Policy rule - inflation	2.14
Θ_y	Policy rule - output gap	0.08

[1] IES: Intertemporal elasticity of substitution

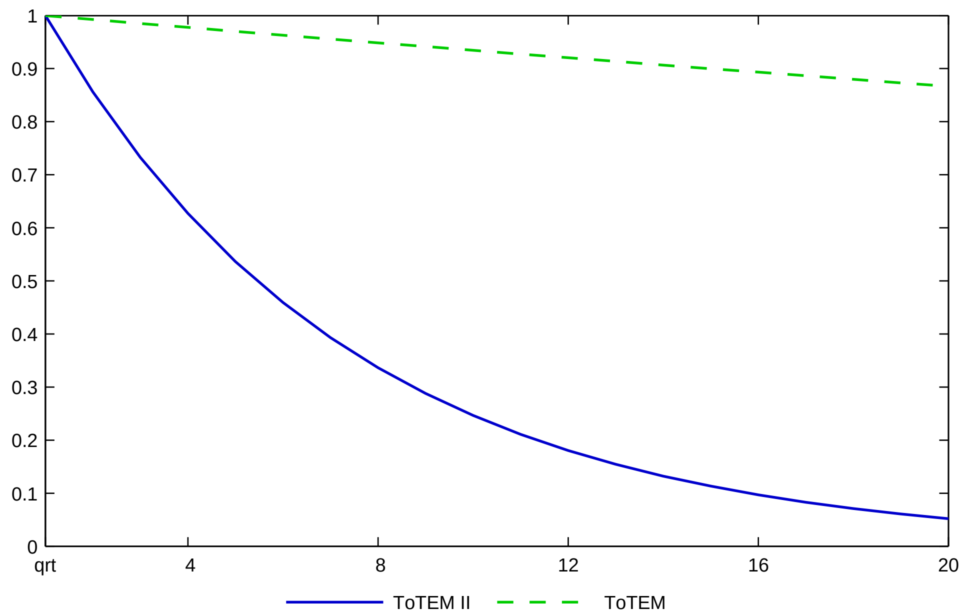
to a sum of real marginal cost in future periods discounted at the household's discount rate (see equation (2.29) in Murchison and Rennison 2006). ToTEM II also features a micro-founded model of forward-looking price and wage determination, but abandons the assumption that all households and firms set prices and wages in an optimal manner. A portion of the firms and households that reset their prices and wages do so according to a rule of thumb, so that expected marginal cost in the distant future receives less weight in determining current inflation than in ToTEM. Figure 2.1 plots the weight that price setters assign to future expected marginal costs in the two models to illustrate the effects of overdiscounting. Because of rule-of-thumb pricing, the new specification in ToTEM II does not nest the ToTEM specification. We show, however, that estimation reveals a significant role for rule-of-thumb behaviour at both the firm and household levels.

2.5 Goodness of fit

The estimated version of ToTEM II dominates ToTEM in its ability to predict every observable series common to the two models. This occurs in part because of estimation and in part because of the richer structure in the newer version of the model. Table 2.5 provides a summary of the improvement in one-quarter-ahead forecast errors for key variables in ToTEM, using the sample from 1980Q1 to 2012Q2.

Some variables, such as inflation, are difficult to forecast because they are not

Figure 2.1: Weight assigned to future marginal cost



sufficiently persistent or lack enough co-movement with other observables. Estimation has a limited ability to improve forecasts for series like this. Nonetheless, the prediction error for inflation is reduced slightly by 0.04 percentage points to 0.28 per cent. Investment is already predicted well (relative to its volatility) by ToTEM, but improves modestly from 3.11 per cent to 2.73 per cent, and improvements in imports and exports are similarly modest. There is a 28 per cent improvement in the one-quarter-ahead forecast for GDP as the prediction error falls from 0.8 per cent to 0.58 per cent. Finally, the hours worked forecast error standard deviations improve similarly, from 0.72 per cent to 0.57 per cent.

Forecasting performance for other key variables is improved significantly. For example, the one-step-ahead forecast for net foreign assets falls by nearly 60 per cent from 2.7 per cent to 1.13 per cent. The nominal exchange rate forecast is also improved substantially, falling over 50 per cent from 6.54 per cent to 3.07 per cent, an improvement that results in part from our ability to account for the influence of long-term interest rates on exchange rate movements. To compare consumption goodness of fit, we construct a consumption composite in ToTEM II that includes consumption, residential investment and inventories, as in ToTEM. The forecast error for this composite decreases by 68 per cent from 3.48 per cent to 1.10 per cent.

Table 2.5: **Comparison of in-sample goodness of fit**

Series	Standard deviation of detrended data (in per cent)	Root mean square one-quarter-ahead prediction error (in per cent)	
		ToTEM	ToTEM II
Quarterly core inflation	0.28	0.32	0.28
Nominal exchange rate	6.41	6.54	3.07
Government expenditures	3.59	0.85	0.80
Business investment	9.25	3.11	2.73
Labour hours worked	2.51	0.72	0.57
Imports	8.04	2.59	2.38
Nominal wage	1.95	0.84	0.65
Exports	5.61	2.67	2.23
Output	2.44	0.80	0.58
Consumption+res inv+inventories	4.17	3.48	1.10
Net foreign assets	2.32	2.70	1.13
Domestic short-term interest rate	0.46	0.32	0.32

Chapter 3

Changes to Model Properties

This chapter assesses the quantitative importance of changes in the model discussed in Chapter 1. Specifically, we examine some noteworthy exogenous disturbances to the model and discuss their implications for the paths of endogenous variables, emphasizing differences from ToTEM. We consider: (i) a house price shock, (ii) an exchange rate shock, (iii) a commodity-price shock and a foreign demand shock, (iv) a term premium shock, (v) a monetary policy shock, and (vi) a productivity shock.

In all simulations considered below, we focus on the implications of changes to the domestic economy of the model. To this end, the monetary policy rule, equations in the rest-of-world block and serially correlated error processes are unchanged. In comparing the impulse-response functions between ToTEM II and ToTEM, in the case of transitory shocks, we scale the shocks to match the cumulative impact of the associated variable over the first four quarters after the shocks; in the case of permanent shocks, we rescale to match the long-run impact of the associated observable. For example, for a monetary policy (transitory) shock, the size of the shock is scaled such that the impact on nominal interest rates is the same in both models over the first four quarters after the shock. For a productivity (permanent) shock, the size of the shock is chosen such that the long-run impact on output is the same in both models.

3.1 House price shock

As discussed in Chapter 1, households are allowed to borrow at an exogenous foreign interest rate, and this makes their net foreign asset (NFA) position nonstationary, even after transitory shocks. To obtain stationarity in this case, in ToTEM, the country risk premium is an increasing function of Canada's net foreign asset position (NFA) relative to its steady state. In turn, in ToTEM II stationarity of the ratio is obtained by assuming that the household discount rate is a decreasing function of the financial wealth-to-disposable-income ratio relative to its (exogenous) desired level. Thus low household financial wealth makes households more patient and increases incentives to save; in turn, high financial wealth makes households more impatient and willing to spend. Household net financial wealth includes the value of the stock of housing, holdings of government debt, stock market wealth and net claims on foreign assets. This modification of the model creates a direct link between household financial variables and consumption behaviour.

One new useful property in ToTEM II is the positive co-movement between the nominal stock of housing and consumption. The impact of movements in house prices on consumption depends on the persistence of those movements. Figure 3.1 illustrates the significance of house price persistence, comparing an estimated process for house prices (dashed green line) to a nearly permanent one (solid blue line). Even though the fall in house prices over the first year is the same, the effect on consumption of the persistent shock is much greater.

The quantitative relationship between house price shocks and other macroeconomic variables, such as consumption or residential investment, should be treated with caution, since the model does not account for all the channels that affect the transmission of house price shocks. For instance, the model does not account for the effect that a house price shock may have on financial intermediation.

3.2 Exchange rate shock

In ToTEM II, the elasticity of exports with respect to the exchange rate is significantly lower than in ToTEM. Figure 3.2 shows a much more muted response of manufactured exports to a temporary exogenous exchange rate disturbance.[27]

[27] An increase in the real exchange rate corresponds to a depreciation.

Figure 3.1: Response to a 10 per cent house price decline (%)

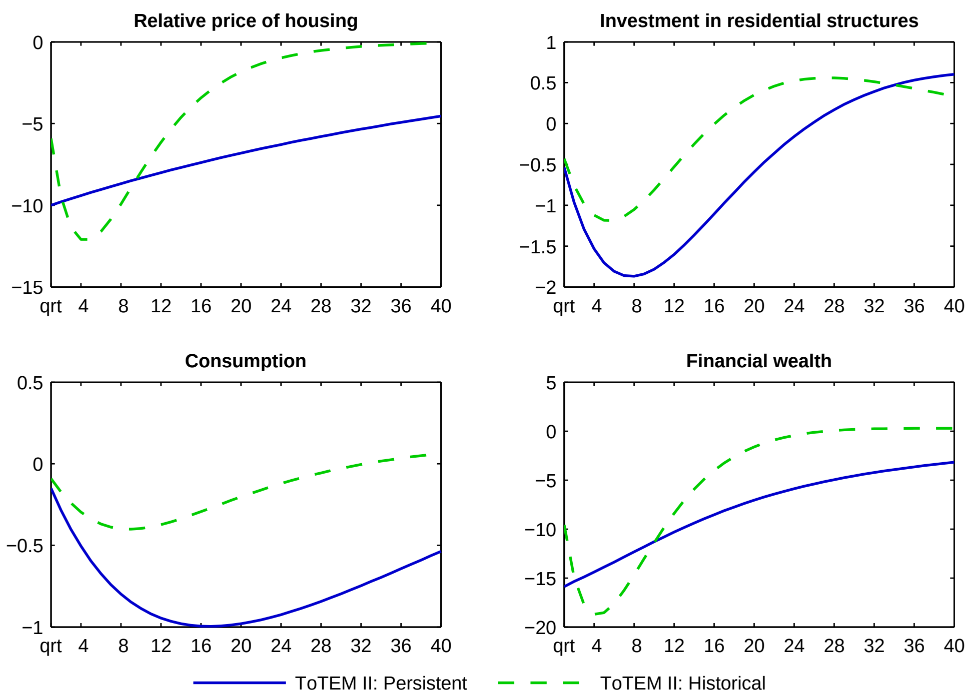
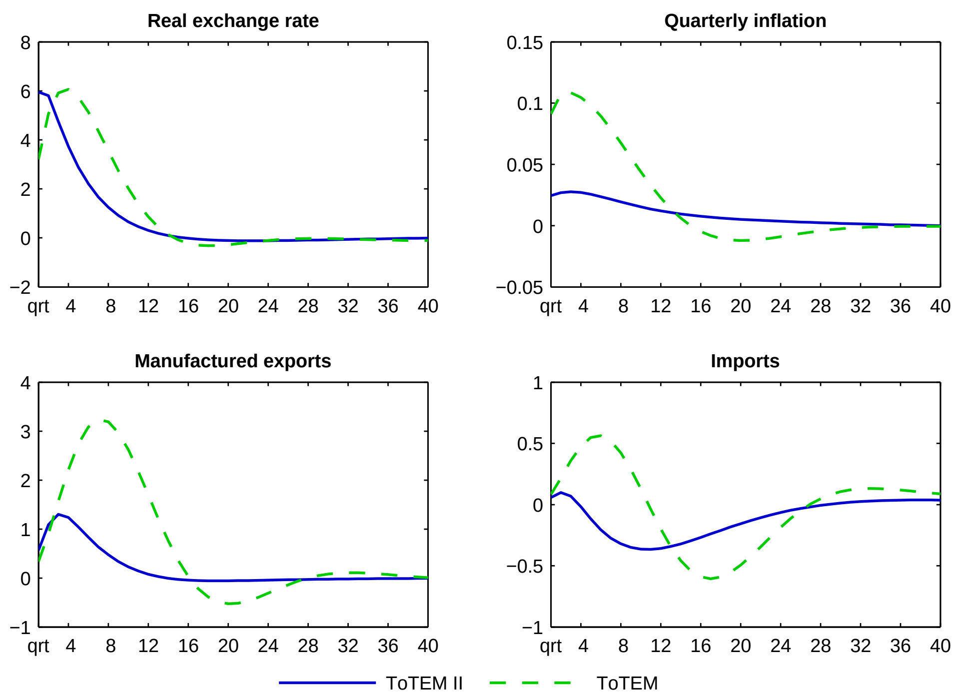


Figure 3.2: Response to an exchange rate shock (%)



Manufactured exports and imports in ToTEM covary positively because the production of exports uses a large share of imported goods. However, since the export response is more muted in ToTEM II, the export sector demand for imported goods is lower.

3.3 Commodity-price shock and foreign demand shock

The estimated version of ToTEM II reveals that, for a given monetary policy rule, the peak response of inflation is smaller than in ToTEM for nearly all shocks, and there is no secondary cycle. This change reflects the lower sensitivity of inflation to the marginal cost associated with a flatter estimated Phillips curve (as discussed in section 3.1), as well as the lower sensitivity of the marginal cost itself to some shocks. Insofar as inflation is the critical determinant of policy in the Taylor rule, this suggests a more muted policy response to most shocks, and accordingly less policy rate volatility.

Figure 3.3 illustrates these effects by simulating a decline in rest-of-world commodity prices similar to that following the financial crisis in 2008–09. In ToTEM, the decline in commodity prices produces disinflation over the first year and the subsequent secondary cycle as inflation overshoots the target three years ahead. By contrast, in ToTEM II the response of inflation is nearly zero once the impact of the exchange rate is considered, due to several factors. First, the commodity-price pass-through is lower than in ToTEM due to the higher estimated price stickiness in commodity distribution. Second, inflation is now much less sensitive to exogenous exchange rate disturbances (see Figure 3.2).

Exports are less sensitive to exchange rate movements regardless of the cause. Figure 3.4 illustrates an increase in rest-of-world demand that induces an exchange rate depreciation and a surge in exports, although the exports movement in ToTEM II is much smaller. The relatively lower demand for exports in ToTEM II accordingly puts less upward pressure on the price level, and both the inflation movement and accompanying policy response are more muted.

Figure 3.3: Response to a permanent commodity-price decline (%)

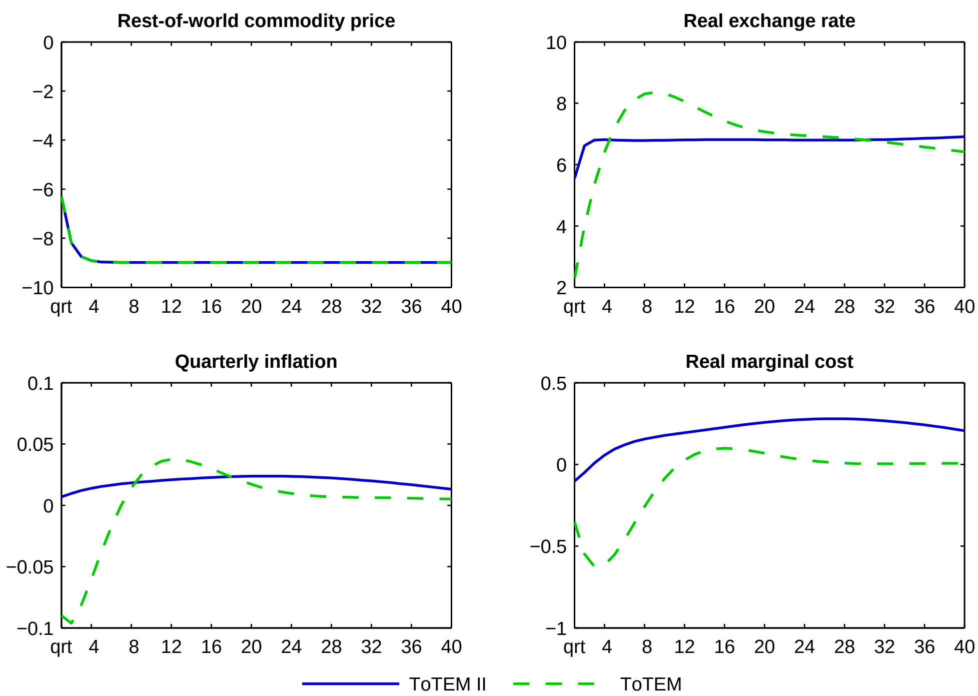


Figure 3.4: Response to a rest-of-world demand shock (%)

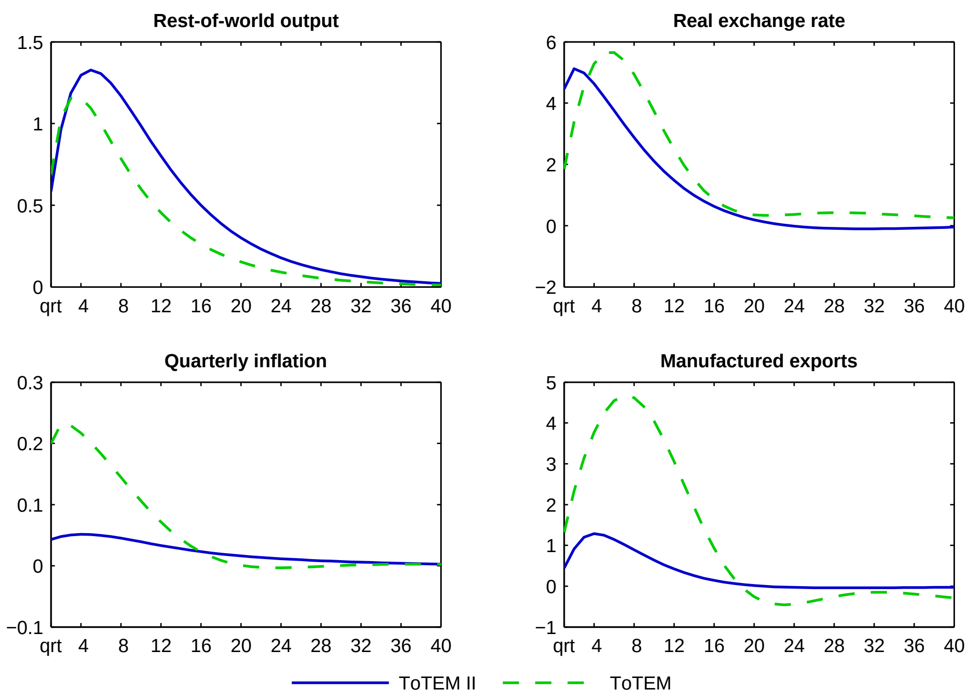
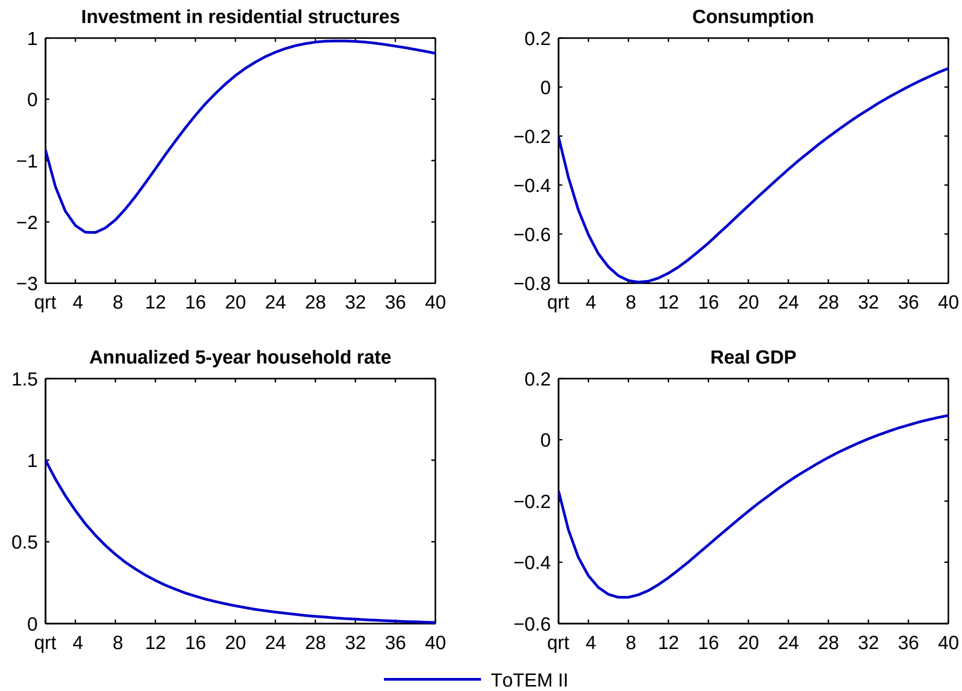


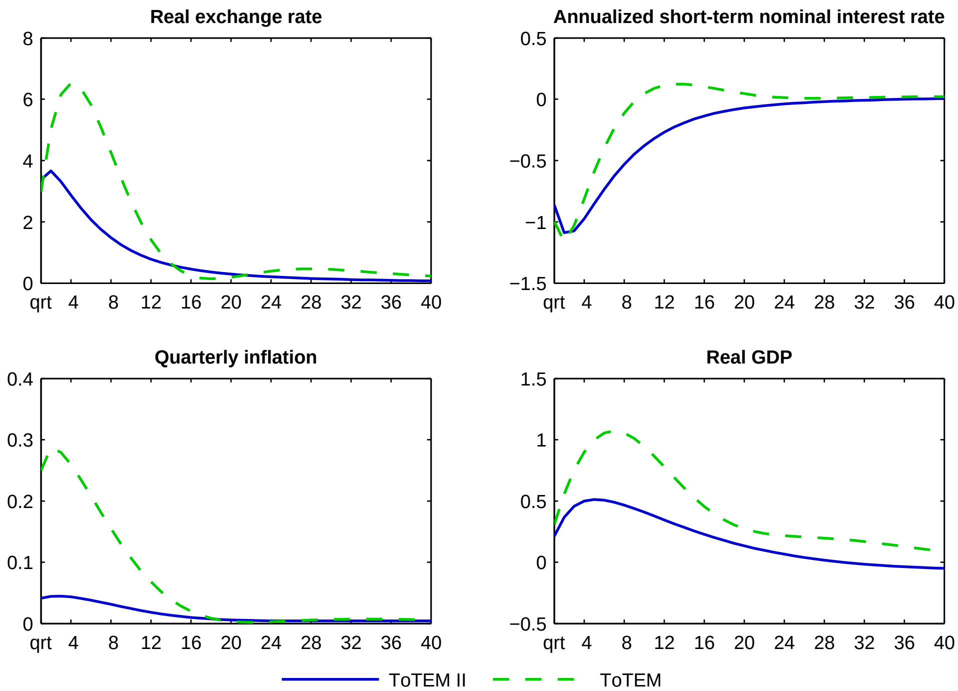
Figure 3.5: Response to a 1 per cent increase in the 5-year household rate (%)



3.4 Term premium shock

ToTEM II allows the interest rate elasticities of consumption and investment in residential structures to differ from one another, and estimation results confirm that residential structure investment has a much higher interest rate elasticity than does consumption (see section 3.5). Figure 3.5 illustrates this by comparing the responses of consumption and investment in residential structures to a 1 per cent increase in the 5-year household rate due to a long-term premium shock. At the trough, the decline in investment in residential structures is roughly three times larger than the decline in consumption. Moreover, at the trough, real GDP declines by 0.51 per cent.

Figure 3.6: Response to a monetary shock (%)

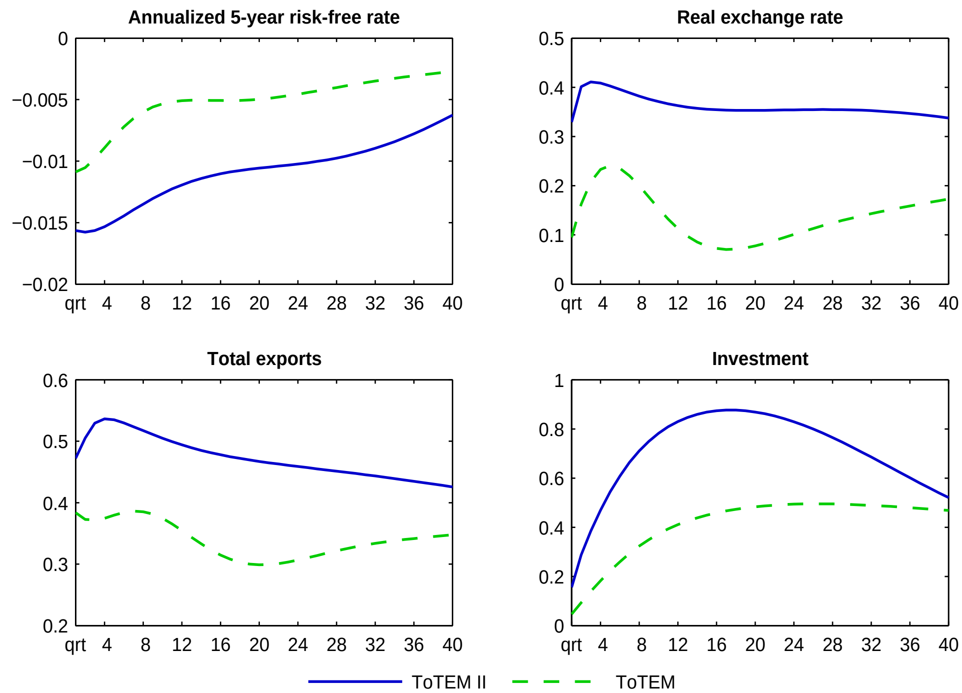


3.5 Monetary policy shock

The sensitivity of the exchange rate to a monetary policy shock is lower in ToTEM II, as illustrated in Figure 3.6. While the initial impact is nearly identical, the peak response in ToTEM II occurs one quarter after the policy shock, and returns to steady state thereafter. In contrast, the peak response in ToTEM is not reached until a year after the policy shock, and the magnitude of the response is more than double that in ToTEM II. These results are consistent with a flatter estimated Phillips curve and the inclusion of the rule-of-thumb price setters discussed in Chapter 2.

In ToTEM II, movements in the policy rate are also transmitted to the 5-year household rate. During the first year, this rate goes down by 31 basis points, on average, after a decrease of 100 basis points in the policy rate.

Figure 3.7: Response to a permanent productivity increase (%)



3.6 Productivity shock

In response to a 1 per cent permanent productivity shock (see Figure 3.7), consumption adjusts to its new steady state more slowly in ToTEM II as a result of higher estimated habit persistence. This puts downward pressure on real interest rates and, accordingly, induces a larger exchange rate depreciation, increasing demand from the rest-of-world for both manufactured exports and commodity exports. With lower estimated nominal rigidities in the investment sector in ToTEM II, the price level moves more quickly to offset movements in the demand for investment goods. Despite this fact, investment production increases more in ToTEM II in order to satisfy higher demand for capital in commodity production and manufactured exports production.

Chapter 4

Policy Applications

In this chapter, we consider two applications of ToTEM II to policy questions on recent economic topics. In particular, we review the implications of shocks for risk spreads during the financial crisis,[28] and evaluate how different types of shocks to the supply and demand for commodities impact the Canadian economy.

4.1 Impact of shocks to risk spreads during a crisis

The first version of ToTEM, like most linearized DSGE models, has a trivial term structure implied by perfect arbitrage between the long- and short-term bonds, so that long-term interest rates are equal to the average of expected future short-term rates. As discussed in Chapter 1, long-term and short-term bonds in ToTEM II are imperfect substitutes for all households, and some households trade only in long-term bonds; this creates a role for long-term interest rates in determining aggregate demand (see section 1.4). Using the new feature of multiple interest rates in ToTEM II, we conduct simulations to examine the impact of shocks on interest rate spreads during a financial crisis.

In the wake of the recent global financial crisis, many developed economies experienced a large and persistent tightening of credit market conditions (e.g., for Canada, see the Bank of Canada's January 2009 *Monetary Policy Report*; for the

[28]For a detailed analysis of this application of ToTEM II, see Dorich, Mendes and Zhang (2011).

United States, see Brunnermeier 2009; Coulibaly, Sapriza and Zlate 2011). This motivates an important question concerning the contribution of worsening credit conditions to the decline in Canadian business and residential investment during the crisis. We explore this question by using counterfactual simulations to back out structural shock series that replicate the historical observations. We then ask how much of the observed decline in residential and business investment can be explained by shocks to credit spreads alone.

Figure 4.1 shows that widening credit spreads since the beginning of 2008 have caused business investment to decline 3.0 per cent below the trend before beginning to recover, a number much smaller in magnitude than the 20 per cent trend deviation observed in the data. Therefore, our counterfactual simulation suggests that less than one-fifth of the decline in business investment can be attributed to higher spreads. Figure 4.2 shows that the contribution of higher spreads to the decline in residential investment is also small. During the recession, residential investment fell below the trend by more than 16 per cent at the trough, while the estimated impact of higher spreads on residential investment is only 1.5 per cent. Even though business and residential investment are the components of aggregate demand most sensitive to interest rates, higher interest rate spreads alone explain relatively little of the observed decline over the recession. Such small importance of the rise in spreads on business and residential investment during the recession is partly due to that rise being short-lived.

By examining the contribution of other shocks in a similar manner, we find that ToTEM II attributes most of the decline in residential and business investment to lower foreign economic activity and domestic demand shocks. First, the sharp contraction of the global economy caused a deterioration in Canadian net exports and terms of trade, which, in turn, propagated to residential and business investment through household financial wealth and weaker capital input demands. Second, domestic demand shocks, modelled as disturbances in the household discount factor, are also important in explaining the contraction in investment. While it is difficult to interpret preference shocks, they can be considered to represent weak consumer confidence and elevated uncertainty surrounding the global economy.

Counterfactual simulations such as this are useful, but should be treated with caution, since their interpretations rest on the assumption that the model is correctly specified. In fact, many potentially important linkages between the financial sector and the real economy are not explicitly modelled. For example, there is no banking sector and no possibility for non-price conditions of credit. Moreover, quantity restrictions on credit are not explicitly modelled in ToTEM II and could

Figure 4.1: Business investment (deviation from trend)

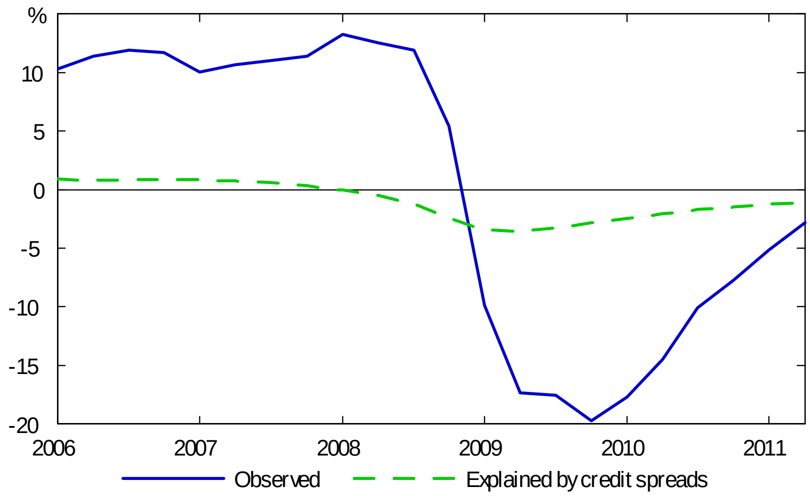
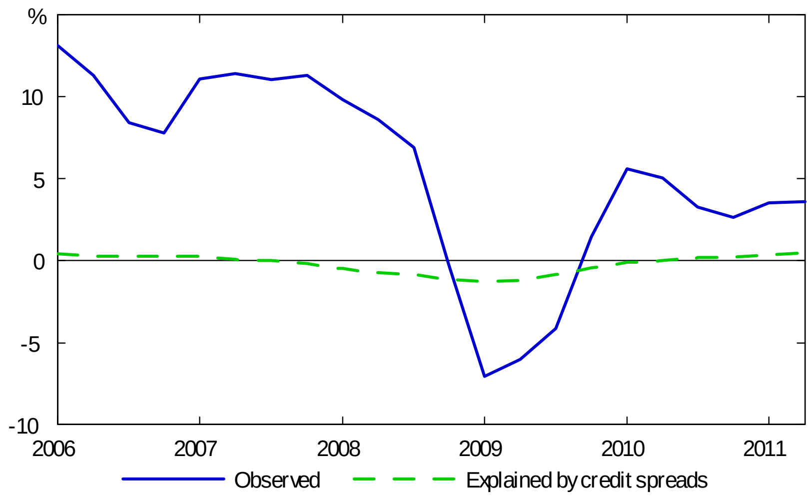


Figure 4.2: Residential investment (deviation from trend)



be a contributing factor to the identified negative shocks to domestic demand.

4.2 Implications of higher commodity prices for Canada

Commodity prices are an important determinant of economic activity in general, especially for a country such as Canada, which has a large commodity endowment. For a net commodity exporter, higher commodity prices are often associated with real exchange rate appreciation, and this may adversely affect manufacturers to the extent that their goods also become more expensive in the global market. It has been alleged that Canada suffers from "Dutch Disease," and that the persistent decline in domestic manufacturing is the result of a commodity-export-driven real exchange rate appreciation.[29] We examine several alternative causes for commodity-price increases and discuss the implications that they have for Canadian GDP.

In our simulation, an increase in commodity prices leads to an improvement in the terms of trade as export prices rise relative to import prices. Accompanying the commodity-price increase, the Canadian dollar also experiences an appreciation in real terms, making manufactured exports less competitive globally. We consider three different sources of commodity-price increases, each of which is normalized to produce a 20 per cent increase in world energy prices (the approximate size of the increase that occurred between mid-2010 and 2011), see Figure 4.3.

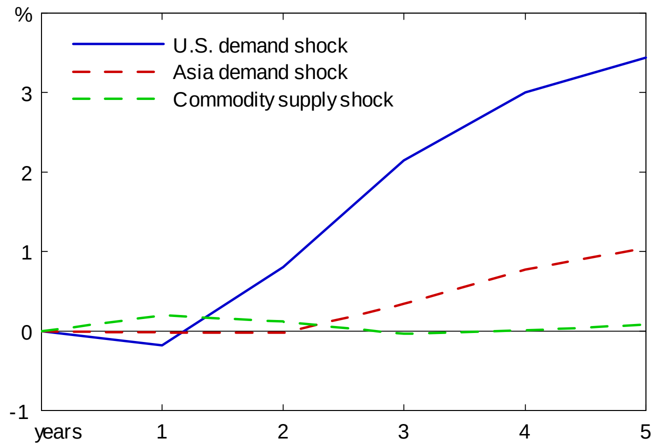
In the first case, we consider a transitory negative commodity supply shock, which raises global production costs (for example, due to weather or geopolitical developments). Since Canada's primary export markets are net commodity importers, their lower economic activity depresses demand for Canadian non-commodity goods and reinforces the effects of real exchange rate appreciation. On net, the terms-of-trade effect dominates the appreciation effect from the commodity supply shock, leading to a very small increase in the Canadian GDP. We find that this shock has very modest implications for Canada: at trough, non-commodity exports fall by 0.1 per cent and GDP rises by 0.2 per cent.

In the second case, we consider a positive global demand shock that originates in the United States, a country that has strong trade linkages with Canada.[30]

[29]The term "Dutch Disease" was first coined by the *Economist* in 1977 after the poor performance of Holland's economy following a major natural gas discovery.

[30]Recall that the Canadian non-commodity exports demand function is modelled assuming

Figure 4.3: Response of domestic GDP to 20 per cent increase in world energy prices (deviation in GDP from baseline)



Not surprisingly, this produces the largest response in GDP, which rises 3 per cent after five years. Even domestic manufacturing benefits, since the positive effect of stronger U.S. demand dominates the adverse effect of Canadian-dollar appreciation, and after five years non-commodity exports are 7.3 per cent higher.

Finally, we consider a positive global demand shock originating in a non-U.S. part of the world (in this case, Asia), with which Canada has less-direct trade linkages. The net impact on Canadian GDP is still positive, but relatively modest. After five years, GDP is 1 per cent higher, about one-third the magnitude of the U.S. demand case, and non-commodity exports rise by only 1.1 per cent.

Overall, our simulation suggests that, regardless of the source of the commodity-price shock, the effects are, on net, positive, since gross domestic income, wealth and GDP all rise. In all cases, the Canadian dollar appreciates, but its adverse impact on manufacturing exports is partially offset by the reduced costs of imported production inputs. In another words, higher commodity prices are unambiguously good for Canada in ToTEM II for the shocks considered (Carney 2012).

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that the U.S. component in foreign economic activity is the most important one for Canada's non-commodity exports.

Chapter 5

Conclusion

Since the inception of ToTEM in 2005 as the Bank's main projection and policy analysis model, the staff have made considerable effort in further improving its ability to explain Canadian macroeconomic data as well as expanding its capacity to address a growing variety of pertinent policy questions. This work culminated in the development of the updated version of the model, ToTEM II, which replaced ToTEM in June 2011. This report describes the main changes to the model structure, summarizes the model estimation and demonstrates key implications of the new changes for model predictions.

The key changes in ToTEM II include: (i) multiple interest rates, (ii) separate demands for consumption, housing investment and inventory investment, (iii) a role for net wealth in household consumption, and (iv) rule-of-thumb price and wage setters. These new features remove some of the restrictions on model dynamics implied by assumptions in ToTEM, making ToTEM II more general and flexible than its predecessor. Moreover, the new estimation has significantly improved the model's forecasting behaviour.

ToTEM II has proven to be a useful tool for addressing a broader array of policy questions. For example, the richer interest rate structure in ToTEM II has allowed Bank staff to include short-term and long-term risk spreads in the quarterly projection analysis.[31] As another example, staff recently employed the model to assess the macroeconomic impact of higher requirements for bank capital and liquidity.[32] Staff also examined several alternative causes for commodity-price

[31]Dorich, Mendes and Zhang (2011).

[32]Dorich and Zhang (2010).

increases and discussed their implications for the Canadian GDP.[33]

Future work on developing the Bank's projection and policy analysis model will concentrate on improving its empirical performance as well as its theoretical specification. Bank staff are currently exploring avenues to further enhance the linkages between the financial developments and the real economy. In the medium term, the staff plan to review the theoretical aspects of the labour market in the model, with the aim to expand the mechanisms through which labour market developments affect the macroeconomy.

[33]This work contributed to Carney (2012).

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Appendix A

The Non-Linear Model

A.1 Finished-products sector

A.1.1 First stage of production

A firm in the consumption sector owns the technology for producing a core consumption finished good. This technology is characterized by a nested CES production function given by functions $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$:

$$\mathcal{F}(\mathcal{H}(\cdot), M_t^c \xi_t^{M,c}) = [(\alpha_{c,3})^{1/\psi} \mathcal{H}(\cdot)^{(\psi-1)/\psi} + (1-\alpha_{c,3})^{1/\psi} (M_t^c \xi_t^{M,c})^{(\psi-1)/\psi}]^{\psi/(\psi-1)},$$

$$\mathcal{H}(\mathcal{G}(\cdot), COM_t^c \xi_t^{COM,c}) = [(\alpha_{c,2})^{1/\rho} \mathcal{G}(\cdot)^{(\rho-1)/\rho} + (1-\alpha_{c,2})^{1/\rho} (COM_t^c \xi_t^{COM,c})^{(\rho-1)/\rho}]^{\rho/(\rho-1)},$$

$$\mathcal{G}(A_t E_t^c H_t^c \xi_t^{H,c}, u_t^c K_t^c \xi_t^{K,c}) = [(\alpha_{c,1})^{1/\sigma} (A_t E_t^c H_t^c \xi_t^{H,c})^{(\sigma-1)/\sigma} + (1-\alpha_{c,1})^{1/\sigma} (u_t^c K_t^c \xi_t^{K,c})^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)},$$

where α's are the share parameters, and ψ, ρ, σ are the elasticities of substitutions.

Such a firm faces constraints imposed by the capital accumulation equation, whose associated Lagrangian will be represented by Q_t^c; the gross output production equation, whose associated Lagrangian will be represented by Λ_t^c; and the constraint equating supply with demand, whose associated Lagrangian will be represented by Λ_t^{ca}. Thus, the firm's problem is to choose $Y_t^{c,no}$, $Y_t^{c,g}$, H_t^c, L_t^c, K_{t+1}^c,

I_t^c, u_t^c, COM_t^c and M_t^c to maximize in expectation the following:

$$\mathcal{L}_t = \mathbf{E}_t \sum_{s=t}^{\infty} \mathcal{R}_{t,s} \left\{ \begin{array}{c} P_s^{c,no} Y_s^{c,no} - W_s H_s^c - P_s^{com} COM_s^c - P_s^I I_s^c - P_s^M M_s^c \\ -Q_s^c \left(K_{s+1}^c - (1 - d\left(u_t^c\right)) K_s^c - I_s^c \right) \\ -\Lambda_s^c \left(Y_s^{c,g} - \mathcal{F}\left(\mathcal{H}_s, M_s^c\right) \right) \\ +\Lambda_s^{no} \left(-Y_s^{c,no} + Y_s^{c,g} - \frac{\chi_I}{2} \left(\frac{I_s^c}{I_{s-1}^c} - 1 \right)^2 I_s^c \right) \end{array} \right\}. \quad \text{(A.1)}$$

The production of other sectors is analogous to core consumption goods production.

First-order conditions

These conditions are written in real terms, deflated by the core price index.

Capital $K_{t+1}^c : \frac{\partial \mathcal{L}_t}{\partial K_{t+1}^c} = 0$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial K_{t+1}^c} &= -q_t^c + \lambda_t^c \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M_t^c\right)}{\partial K_{t+1}^c} \right) \\ &\quad + \mathcal{R}_{t,t+1} \left((1 - d\left(u_{t+1}^c\right))(1 + \pi_{t+1}^c) q_{t+1}^c + \lambda_{t+1}^c (1 + \pi_{t+1}^c) \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M_{t+1}^c\right)}{\partial K_{t+1}^c} \right) \right), \end{aligned}$$

$$\begin{aligned} q_t^c &= \lambda_t^c \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M_t^c\right)}{\partial K_{t+1}^c} \right) \qquad \text{(A.2)} \\ &\quad + \left(\frac{1}{1 + R_t} \right) \left((1 - d\left(u_{t+1}^c\right))(1 + \pi_{t+1}^c) q_{t+1}^c + \lambda_{t+1}^c (1 + \pi_{t+1}^c) \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M_{t+1}^c\right)}{\partial K_{t+1}^c} \right) \right), \end{aligned}$$

where

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M_t^c\right)}{\partial K_{t+1}^c} = \left(\frac{\alpha_{c,3} \mathcal{F}_t}{\mathcal{H}_t} \right)^{\frac{1}{\varphi}} \left(\frac{\alpha_{c,2} \mathcal{H}_t}{\mathcal{G}_t} \right)^{\frac{1}{\varrho}} \left(\frac{(1 - \alpha_{c,1}) \mathcal{G}_t}{u_t^c K_t^c \xi_t^{K,c}} \right)^{\frac{1}{\sigma}} u_t^c K_t^c \frac{\partial \xi_t^{K,c}}{\partial K_{t+1}^c}$$

$$\frac{\partial \xi_t^{K,c}}{\partial K_{t+1}^c} = -\chi^K \left(\frac{K_{t+1}^c / Y_t^{c,ig}}{K_t^c / Y_{t-1}^{c,ig}} - 1 \right) \left(\frac{K_{t+1}^c / Y_t^{c,g}}{K_t^c / Y_{t-1}^{c,ig}} \right) \left(\frac{1}{K_{t+1}^c} \right)$$

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M^c_{t+1}\right)}{\partial K^c_{t+1}} = \left(\frac{\alpha_{c,3}\mathcal{F}_{t+1}}{\mathcal{H}_{t+1}}\right)^{\frac{1}{\varphi}} \left(\frac{\alpha_{c,2}\mathcal{H}_{t+1}}{\mathcal{G}_{t+1}}\right)^{\frac{1}{\varrho}} \left(\frac{(1-\alpha_{c,1})\mathcal{G}_{t+1}}{u^c_{t+1}K^c_{t+1}\xi^{K,c}_{t+1}}\right)^{\frac{1}{\sigma}} u^c_{t+1} \left\{\xi^{K,c}_{t+1} + K^c_{t+1}\frac{\partial \xi^{K,c}_{t+1}}{\partial K^c_{t+1}}\right\}$$

$$\frac{\partial \xi^{K,c}_{t+1}}{\partial K^c_{t+1}} = \left(\chi^K \left(\frac{K^c_{t+2}/Y^{c,g}_{t+1}}{K^c_{t+1}/Y^{c,g}_t} - 1\right)\left(\frac{K^c_{t+2}/Y^{c,g}_{t+1}}{K^c_{t+1}/Y^{c,g}_t}\right)\left(\frac{1}{K^c_{t+1}}\right)\right).$$

Investment I^c_t : $\frac{\partial \mathcal{L}_t}{\partial I^c_t} = 0$

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial I^c_t} &= 0 = -p^I_t + q^c_t \\ &-\lambda^{no}_t\left(\chi_I\left(\frac{I^c_t}{I^c_{t-1}} - 1\right)\left(\frac{I^c_t}{I^c_{t-1}}\right) + \frac{\chi_I}{2}\left(\frac{I^c_t}{I^c_{t-1}} - 1\right)^2\right) \\ &+\mathcal{R}_{t,t+1}\lambda^{no}_{t+1}(1+\pi^c_{t+1})\chi_I\left(\frac{I^c_{t+1}}{I^c_t} - 1\right)\left(\frac{I^c_{t+1}}{I^c_t}\right)^2.\end{aligned} \tag{A.3}$$

Hours worked H^c_t : $\frac{\partial \mathcal{L}_t}{\partial H^c_t} = 0$:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial H^c_t} &= 0 = -w_t + \lambda^c_t\left(\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M^c_t\right)}{\partial H^c_t}\right) \\ &+\mathcal{R}_{t,t+1}\lambda^c_{t+1}(1+\pi^c_{t+1})\left(\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M^c_{t+1}\right)}{\partial H^c_t}\right),\end{aligned} \tag{A.4}$$

where

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M^c_t\right)}{\partial H^c_t} = \left(\frac{\alpha_{c,3}\mathcal{F}_t}{\mathcal{H}_t}\right)^{\frac{1}{\varphi}}\left(\frac{\alpha_{c,2}\mathcal{H}_t}{\mathcal{G}_t}\right)^{\frac{1}{\varrho}}\left(\frac{\alpha_{c,1}\mathcal{G}_t}{A_t L^c_t \xi^{H,c}_t}\right)^{\frac{1}{\sigma}} A_t\left\{E^c_t\xi^{H,c}_t + L^c_t\frac{\partial \xi^{H,c}_t}{\partial H^c_t}\right\}$$

$$\frac{\partial \xi^{H,c}_t}{\partial H^c_t} = -\chi^H\left(\frac{H^c_t}{H^c_{t-1}} - 1\right)\left(\frac{1}{H^c_{t-1}}\right)$$

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M^c_{t+1}\right)}{\partial H^c_t} = \left(\frac{\alpha_{c,3}\mathcal{F}_{t+1}}{\mathcal{H}_{t+1}}\right)^{\frac{1}{\varphi}}\left(\frac{\alpha_{c,2}\mathcal{H}_{t+1}}{\mathcal{G}_{t+1}}\right)^{\frac{1}{\varrho}}\left(\frac{\alpha_{c,1}\mathcal{G}_{t+1}}{A_{t+1}L^c_{t+1}\xi^{H,c}_{t+1}}\right)^{\frac{1}{\sigma}} A_{t+1}L^c_{t+1}\frac{\partial \xi^{H,c}_{t+1}}{\partial H^c_t}$$

$$\frac{\partial \xi^{H,c}_{t+1}}{\partial H^c_t} = \chi^H \left(\frac{H^c_{t+1}}{H^c_t} - 1 \right) \left(\frac{H^c_{t+1}}{H^c_t} \right) \left(\frac{1}{H^c_t} \right).$$

Capital utilization $u^c_t : \frac{\partial \mathcal{L}_t}{\partial u^c_t} = 0$

$$\frac{\partial \mathcal{L}_t}{\partial u^c_t} = 0 = -q^c_t K^c_t \rho^c \overline{d} e^{\rho^c (u^c_t - 1)} + \lambda^c_t \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M^c_t\right)}{\partial u^c_t} \right) , \tag{A.5}$$

where

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M^c_t\right)}{\partial u^c_t} = \left(\frac{\alpha_{c,3} \mathcal{F}_t}{\mathcal{H}_t} \right)^{\frac{1}{\varphi}} \left(\frac{\alpha_{c,2} \mathcal{H}_t}{\mathcal{G}_t} \right)^{\frac{1}{\varrho}} \left(\frac{(1-\alpha_{c,1}) \mathcal{G}_t}{u^c_t K^c_t \xi^{K,c}_t} \right)^{\frac{1}{\sigma}} K^c_t \xi^{K,c}_t.$$

Effective employment $L^c_t : \frac{\partial \mathcal{F}(\cdot)}{\partial L^c_t} = \frac{w_t}{\lambda^{cg}_t}$

$$\frac{w_t}{\lambda^c_t} = \left(\frac{\alpha_{c,3} \mathcal{F}_t}{\mathcal{H}_t} \right)^{\frac{1}{\varphi}} \left(\frac{\alpha_{c,2} \mathcal{H}_t}{\mathcal{G}_t} \right)^{\frac{1}{\varrho}} \left(\frac{\alpha_{c,1} \mathcal{G}_t}{A_t L^c_t \xi^{H,c}_t} \right)^{\frac{1}{\sigma}} A_t \xi^{H,c}_t. \tag{A.6}$$

Gross output $Y^{c,g}_t : \frac{\partial \mathcal{L}_t}{\partial Y^{c,g}_s} = 0$

$$\frac{\partial \mathcal{L}_t}{\partial Y^{c,g}_s} = 0 = -\lambda^c_t + \lambda^{no}_t. \tag{A.7}$$

Intermediate output $Y^{c,no}_t$:

$$\lambda^{no}_t = p^{c,no}_t. \tag{A.8}$$

Commodities COM^c_t :

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial COM^c_t} &= 0 = -p^{com}_t + \lambda^c_t \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M^c_t\right)}{\partial COM^c_t} \right) \\ &\quad + \mathcal{R}_{t,t+1} \lambda^c_{t+1} (1 + \pi^c_{t+1}) \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M^c_{t+1}\right)}{\partial COM^c_t} \right) , \end{aligned} \tag{A.9}$$

where

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M_t^c\right)}{\partial COM_t^c} = \left(\frac{\alpha_{c,3}\mathcal{F}_t}{\mathcal{H}_t}\right)^{\frac{1}{\varphi}} \left(\frac{(1-\alpha_{c,2})\mathcal{H}_t}{COM_t^c \xi_t^{COM,c}}\right)^{\frac{1}{\varrho}} \left\{\xi_t^{COM,c} + COM_t^c \frac{\partial \xi_t^{COM,c}}{\partial COM_t^c}\right\}$$

$$\frac{\partial \xi_t^{COM,c}}{\partial COM_t^c} = -\chi^{COM}\left(\frac{COM_t^c}{COM_{t-1}^c}\frac{Y_{t-1}^{c,g}}{Y_t^{c,g}} - 1\right)\left(\frac{Y_{t-1}^{c,g}}{Y_t^{c,g}}\frac{1}{COM_{t-1}^c}\right)$$

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M_{t+1}^c\right)}{\partial COM_t^c} = \left(\frac{\alpha_{c,3}\mathcal{F}_{t+1}}{\mathcal{H}_{t+1}}\right)^{\frac{1}{\varphi}} \left(\frac{(1-\alpha_{c,2})\mathcal{H}_{t+1}}{COM_{t+1}^c \xi_{t+1}^{COM,c}}\right)^{\frac{1}{\varrho}} \frac{COM_{t+1}^c \partial \xi_{t+1}^{COM,c}}{\partial COM_t^c}$$

$$\frac{\partial \xi_{t+1}^{H,c}}{\partial COM_t^c} = \chi^{COM}\left(\frac{COM_{t+1}^c}{COM_t^c}\frac{Y_t^{c,g}}{Y_{t+1}^{c,g}} - 1\right)\left(\frac{Y_t^{c,g}}{Y_{t+1}^{c,g}}\frac{COM_{t+1}^c}{COM_t^c}\right)\frac{1}{COM_t^c}.$$

Imports M_t^c :

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial M_t^c} &= 0 = -p_t^m + \lambda_t^c \left(\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M_t^c\right)}{\partial M_t^c}\right) \\ &+\mathcal{R}_{t,t+1}\lambda_{t+1}^c(1+\pi_{t+1}^c)\left(\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M_{t+1}^c\right)}{\partial M_t^c}\right),\end{aligned} \tag{A.10}$$

where

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_t, M_t^c\right)}{\partial M_t^c} = \left(\frac{(1-\alpha_{c,3})\mathcal{F}_t}{M_t^c \xi_t^{M,c}}\right)^{\frac{1}{\varphi}} \left\{\xi_t^{M,c} + M_t^c \frac{\partial \xi_t^{M,c}}{\partial M_t^c}\right\}$$

$$\frac{\partial \xi_t^{M,c}}{\partial M_t^c} = -\chi^{M}\left(\frac{M_t^c}{M_{t-1}^c}\frac{Y_{t-1}^{c,g}}{Y_t^{c,g}} - 1\right)\left(\frac{Y_{t-1}^{c,g}}{Y_t^{c,g}}\frac{1}{M_{t-1}^c}\right)$$

$$\frac{\partial \mathcal{F}\left(\mathcal{H}_{t+1}, M_{t+1}^c\right)}{\partial M_t^c} = \left(\frac{(1-\alpha_{c,3})\mathcal{F}_{t+1}}{M_{t+1}^c \xi_{t+1}^{M,c}}\right)^{\frac{1}{\varphi}} \frac{M_{t+1}^c \partial \xi_{t+1}^{M,c}}{\partial M_t^c}$$

$$\frac{\partial \xi_{t+1}^{H,c}}{\partial M_t^c} = \chi^{COM}\left(\frac{M_{t+1}^c}{M_t^c}\frac{Y_t^{c,g}}{Y_{t+1}^{c,g}} - 1\right)\left(\frac{Y_t^{c,g}}{Y_{t+1}^{c,g}}\frac{M_{t+1}^c}{M_t^c}\right)\frac{1}{M_t^c}.$$

A.2 Unrestricted lifetime-income households

By choosing C_t^{ul}, B_t^{ul}, B_t^{ul*}, B_t^{20ul}, RS_{t+1}^{ul}, INV_{t+1}^{ul}, $I_{RS,t}^{ul}$ and $I_{INVT,t}^{ul}$, the unrestricted lifetime-income household seeks to maximize the following objective function:

$$\mathbf{E}_t \sum_{s=t}^{\infty} \boldsymbol{\beta}_{t,s} \mathbb{U}_s^{ul} \ ,$$

with $\boldsymbol{\beta}_{t,s} \equiv \prod_{v=t}^{s-1} \beta_v$, $\boldsymbol{\beta}_{t,t} \equiv 1$ and $\beta_v = \beta \left(\frac{FW_v}{\overline{FW}} \frac{\overline{YD}}{YD_v} \right)^{-\psi} \left(\epsilon_v^C\right)^{-1} = (fw_t)^{-\psi} \left(\epsilon_v^C\right)^{-1}$ subject to the budget constraint and the laws of motion for residential structures and inventories. FW is the aggregate net wealth, YD is the aggregate disposable income and ϵ_v^C is a positive consumption preference shock. This optimization problem yields the first-order conditions that characterize unrestricted lifetime-income agents' economic decisions, except for the optimal wage-setting decision, which is described in detail later.

A.2.1 First-order conditions

These conditions are written in real terms, deflated by the core consumer price index.

$\mathbf{C}_t^{ul}$(external habits)**:**

$$\lambda_t^{ul} = (C_t^{ul} - \xi C_{t-1}^{ul})^{\frac{-1}{\mu}} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \int_0^1 \left(E_t N_{ht}^{ul}\right)^{\frac{1+\eta}{\eta}} dh \right) \ , \tag{A.11}$$

B_t^{ul}:

$$\frac{(1+\pi_{t+1}^c)\lambda_t^{ul}}{p_{c,t}^{tot}} = \frac{\lambda_{t+1}^{ul}}{p_{c,t+1}^{tot}} \beta_{t,t+1}(1+R_t) \ , \tag{A.12}$$

B_t^{ul*}:

$$\frac{q_t p_t \lambda_t^{ul}}{p_{c,t}^{tot}} = \frac{q_{t+1} p_{t+1} \lambda_{t+1}^{ul}}{p_{c,t+1}^{tot}} \beta_{t,t+1} \frac{(1+R_t^*)}{(1+\pi_{t+1}^*)} \ , \tag{A.13}$$

where q_t is the real exchange rate, p_t is the GDP deflator deflated by the core consumer price index and π_t^* is foreign inflation.

B_t^{Lul}:

$$-\frac{1+\phi_t}{(1+R_t^{20})^{20}}\frac{\lambda_t^{ul}}{p_{c,t}^{tot}}+\beta_{t,t+20}\frac{\lambda_{t+20}^{ul}}{p_{c,t+20}^{tot}\prod_{j=1}^{20}(1+\pi_{t+j}^{c})}=0\ , \tag{A.14}$$

RS_{t+1}^{ul}:

$$\Psi_t^{RSul}=\beta_{t,t+1}\left\{\zeta_{t+1}^{hl,ul}(RS_{t+1}^{ul}-\xi_{HL}RS_t^{ul})^{\frac{-1}{\mu_{HL}}}+\Psi_{t+1}^{RSul}(1-\delta_{RS})\right\}\ , \tag{A.15}$$

$I_{RS,t}^{ul}$:

$$-\frac{\lambda_t^{ul}}{p_{c,t}^{tot}}+\Psi_t^{RSul}\left\{1-\frac{\chi_{RS}}{2}\left(\frac{I_{RS,t}^{ul}}{I_{RS,t-1}^{ul}}-1\right)^2-\chi_{RS}\frac{I_{RS,t}^{ul}}{I_{RS,t-1}^{ul}}\left(\frac{I_{RS,t}^{ul}}{I_{RS,t-1}^{ul}}-1\right)\right\}$$
$$+\Psi_{t+1}^{RSul}\beta_{t,t+1}\chi_{RS}\left(\frac{I_{RS,t+1}^{ul}}{I_{RS,t}^{ul}}\right)^2\left(\frac{I_{RS,t+1}^{ul}}{I_{RS,t}^{ul}}-1\right)=0\ , \tag{A.16}$$

INV_{t+1}^{ul}:

$$\Psi_t^{INVul}=\beta_{t,t+1}\left\{\zeta_{t+1}^{inv,ul}(INV_{t+1}^{ul}-\xi_{INV}INV_t^{ul})^{\frac{-1}{\mu_{HL}}}+\Psi_{t+1}^{INVul}(1-\delta_{INVT})\right\}\ , \tag{A.17}$$

$I_{INVT,t}^{ul}$:

$$-\frac{\lambda_t^{ul}}{p_{c,t}^{tot}}+\Psi_t^{INVul}\left\{1-\frac{\chi_{INVT}}{2}\left(\frac{I_{INVT,t}^{ul}}{I_{INVT,t-1}^{ul}}-1\right)^2-\chi_{INVT}\frac{I_{INVT,t}^{ul}}{I_{INVT,t-1}^{ul}}\left(\frac{I_{INVT,t}^{ul}}{I_{INVT,t-1}^{ul}}-1\right)\right\}$$
$$+\Psi_{t+1}^{INVul}\beta_{t,t+1}\chi_{INVT}\left(\frac{I_{INVT,t+1}^{ul}}{I_{INVT,t}^{ul}}\right)^2\left(\frac{I_{INVT,t+1}^{ul}}{I_{INVT,t}^{ul}}-1\right)=0. \tag{A.18}$$

A.3 Restricted lifetime-income consumers

The optimization problem for these agents can be seen as a particular case of the one solved for unrestricted agents. In fact, all of the first-order conditions for restricted agents are the same, except for those associated with long-term bonds. Moreover, notice that there are no first-order conditions associated with short-term bonds.

The first-order condition associated with domestic long-term bonds is given by B_t^{20rl}:

$$-\frac{1}{(1+R_t^{20})^{20}}\frac{\lambda_t^{rl}}{p_{c,t}^{tot}}+\beta_{t,t+20}\frac{\lambda_{t+20}^{rl}}{p_{c,t+20}^{tot}\prod_{j=1}^{20}(1+\pi_{t+j}^{c})}=0. \quad \text{(A.19)}$$

A.4 Optimal wage-setting decision

With probability $1-\theta_w$, forward-looking unions choose the optimal wage W_{ht}^* to maximize:

$$\begin{aligned} F_t &= \mathbf{E}_t\sum_{s=t}^{\infty}\boldsymbol{\beta}_{t,s}(\theta_w)^{s-t}\left\{s_u\mathbb{V}_s^{ul}+(1-s_u)\mathbb{V}_s^{rl}+\Lambda\lambda_s^{ul}\left[-C_s^{ul}+(1-\tau_{w,t})\int_0^1 N_{hs}\frac{W_{hs}^*}{P_{c,t}^{tot}}\Pi_{s,t}^{w}dh+...\right]\right. \\ &\left.+(1-\Lambda)\lambda_s^{rl}\left[-C_s^{rl}+(1-\tau_{w,t})\int_0^1 N_{hs}\frac{W_{hs}^*}{P_{c,t}^{tot}}\Pi_{s,t}^{w}dh+...\right]\right\}, \end{aligned} \quad \text{(A.20)}$$

subject to the following labour demand schedule:

$$N_{hs}=\left(\frac{W_{ht}^*}{W_s}\Pi_{s,t}^{w}\right)^{-\epsilon_{w,t}}H_t\ , \quad \text{(A.21)}$$

where

$$\mathbb{V}_s^i=\frac{\mu}{\mu-1}(C_s^i-\xi C_{s-1}^i)^{\frac{\mu-1}{\mu}}\exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)}\int_0^1\left(E_sN_{hs}^i\right)^{\frac{1+\eta}{\eta}}dh\right) \text{ for } i=ul,rl$$

$$\begin{aligned}\Pi^w_{s,t} &= \left(\frac{W_{s-1}}{W_{t-1}}\right)^{\gamma_w} \mathbf{E}_t \left(\prod_{j=t}^{s-1} (1+\overline{\pi}_j)\right)^{1-\gamma_w} \quad s > t. && \text{(A.22)}\\ &= 1 \quad s = t && \text{(A.23)}\end{aligned}$$

The first-order condition of this problem is given by

$$\frac{w^*_{ht}}{p^{tot}_{c,t}} = \mathbf{E}_t \frac{\frac{\mu-1}{\mu}\sum_{s=t}^{\infty} \beta_{t,s}(\theta_w)^{s-t}\left\{\Lambda \mathbb{V}^{ul}_s + (1-\Lambda)\mathbb{V}^{rl}_s\right\}(E_s N_{hs})^{\frac{1+\eta}{\eta}} \epsilon_{w,s}}{\sum_{s=t}^{\infty} \beta_{t,s}(\theta_w)^{s-t}\left\{\Lambda\lambda^{ul}_s + (1-\Lambda)\lambda^{rl}_s\right\}(1-\tau_{w,s})\frac{P^{tot}_{c,t}}{P^{tot}_{c,t+s}}\Pi^w_{s,t} N_{hs}(\epsilon_{w,s}-1)}. \tag{A.24}$$

We can rewrite (A.24) as follows:

$$\frac{w^*_{ht}}{p^{tot}_{c,t}} = \mathbf{E}_t \frac{\frac{\mu-1}{\mu}\Delta^{W1}_t}{\Delta^{W0}_t}\ , \tag{A.25}$$

where

$$\Delta^{W1}_t = \left\{\Lambda \mathbb{V}^{ul}_t + (1-\Lambda)\mathbb{V}^{rl}_t\right\}(E_t N_{ht})^{\frac{1+\eta}{\eta}} \epsilon_{w,s} + \beta\theta_w \Delta^{W1}_{t+1}\ , \tag{A.26}$$

and

$$\begin{aligned}\Delta^{W0}_t &= \left\{\Lambda\lambda^{ul}_t + (1-\Lambda)\lambda^{rl}_t\right\}(1-\tau_{w,t})N_{ht}(\epsilon_{w,t}-1) && \text{(A.27)}\\ &\quad + \beta\theta_w \frac{p^{tot}_{c,t}}{p^{tot}_{c,t+1}} \frac{1}{(1+\pi^c_{t+1})}(1+\pi^w_t)^{\gamma_w}\left((1+\overline{\pi}_t)\, g_t\right)^{1-\gamma_w} \Delta^{W0}_{t+1}.\end{aligned}$$

A.5 Financial asset accumulation

Define aggregate variables:

$$C_t = C^{ul}_t + C^{rl}_t + C^{ci}_t, \tag{A.28}$$

$$I_{RS,t} = I^{ul}_{RS,t} + I^{rl}_{RS,t} + I^{ci}_{RS,t} \tag{A.29}$$

$$I_{INV,t} = I^{ul}_{INV,t} + I^{rl}_{INV,t} + I^{ci}_{INV,t} \tag{A.30}$$

$$B_{g,t} = B_t^{ul} + B_{g,t}^{ul} \tag{A.31}$$

$$B_t^* = B_t^{ul*} \tag{A.32}$$

$$B_{g,t}^{20} = B_t^{20ul} + B_{g,t}^{20ul} + B_t^{20rl} + B_{g,t}^{20rl}. \tag{A.33}$$

The aggregate household budget constraint is

$$\begin{aligned} & P_{c,t}^{tot} C_t + P_t^c I_{RS,t} + P_t^c I_{INV} + \frac{B_{g,t}}{(1+R_{RF,t})} + \frac{e_t B_t^*}{(1+R_t^*)} + \frac{B_{g,t}^{20}}{(1+R_{RF,t}^{20})^{20}} \\ & = B_{g,t-1} + e_t B_{t-1}^* + B_{g,t-20}^{20} + (1-\tau_{w,t}) \int_0^1 N_t(h) W_t(h) dh + TF_t + \Pi_t \ , \end{aligned} \tag{A.34}$$

where $\int_0^1 N(h)_t W(h)_t dh$ and TF_t are the wage income and government transfer at the aggregate level. The laws of motion for the stock of residential structures is given by

$$RS_{t+1} = (1-\delta_{RS}) RS_t + I_{RS,t} - \frac{\chi_{RS}}{2} I_{RS,t} \left(\frac{I_{RS,t}}{I_{RS,t-1}} - 1 \right)^2 . \tag{A.35}$$

For simplicity, we assume that UIP holds, $\frac{e_t(1+R_{RF,t})}{(1+R_t^*)} = e_{t+1}$, which allows us to express the budget constraint equation in terms of the domestic interest rate only. We also assume that the long-term bonds are in zero net supply. We can thus rewrite the budget constraint as

$$\begin{aligned} & B_{g,t} + e_{t+1} B_t^* \\ & = (1+R_{RF,t}) \left[\begin{array}{c} B_{g,t-1} + e_t B_{t-1}^* + (1-\tau_{w,t}) \int_0^1 N(h)_t W(h)_t dh + TF_t \\ +\Pi_t \ - P_{c,t}^{tot} C_t - P_t^c I_{RS,t} - P_t^c I_{INV,t} \end{array} \right] . \end{aligned} \tag{A.36}$$

We define $B_{f,t} = e_{t+1} B_t^*$ as the nominal value of net foreign assets held by Canadians denominated in Canadian dollars, and we can rewrite equation (A.36) as

$$\begin{aligned} & B_{g,t} + B_{f,t} \\ & = (1+R_{RF,t}) \left[\begin{array}{c} B_{g,t-1} + B_{f,t-1} + (1-\tau_{w,t}) \int_0^1 N(h)_t W(h)_t dh \\ +TF_t + \Pi_t \ - P_{c,t}^{tot} C_t - P_t^c I_{RS,t} - P_t^c I_{INV,t} \end{array} \right] . \end{aligned} \tag{A.37}$$

Assuming zero adjustment costs and substituting equation (A.35) into equation (A.37), we obtain

$$\begin{aligned}
&B_{g,t} + B_{f,t} \\
&= (1 + R_{RF,t}) \left[B_{g,t-1} + B_{f,t-1} + (1 - \tau_{w,t}) \int_0^1 N(h)_t W(h)_t dh + TF_t + \Pi_t \right] \\
&-(1 + R_{RF,t}) \left[P^{tot}_{c,t} C_t + \frac{P^c_t}{P^c_{t+1}} P^c_{t+1} RS_{t+1} - P^c_t (1 - \delta_{RS}) RS_t + P^c_t I_{INV,t} \right].
\end{aligned}$$

Let $B_{rs,t} = P^c_{t+1} RS_{t+1}$ be the nominal value of residential structural assets at the end of period t. We then have

$$\begin{aligned}
&B_{g,t} + B_{f,t} + (1 + R_{RF,t}) \frac{P^c_t}{P^c_{t+1}} B_{rs,t} \\
&= (1 + R_{RF,t}) \left[\begin{array}{c} B_{g,t-1} + B_{f,t-1} + (1 - \tau_{w,t}) \int_0^1 N(h)_t W(h)_t dh + TF_t + \Pi_t \\ -P^{tot}_{c,t} C_t \ + B_{rs,t-1} - \delta_{RS} B_{rs,t-1} - P^c_t I_{INV,t} \end{array} \right].
\end{aligned}$$

We further define $(1 + r_t) = (1 + R_{RF,t}) \frac{P^c_t}{P^c_{t+1}}$, where r_t is the real interest rate. Thus we have

$$\begin{aligned}
&B_{g,t} + B_{f,t} + B_{rs,t} \\
&= (1 + R_{RF,t}) \left[\begin{array}{c} B_{g,t-1} + B_{f,t-1} + (1 - \tau_{w,t}) \int_0^1 N(h)_t W(h)_t dh + TF_t \\ +\Pi_t - P^{tot}_{c,t} C_t - P^c_t I_{INV,t} + B_{rs,t-1} - \delta_{RS} B_{rs,t-1} - \frac{r_t}{1+R_{RF,t}} B_{rs,t} \end{array} \right].
\end{aligned} \tag{A.38}$$

We define economy-wide accounting profits Π_t as corporate profit (Π^{corp}_t) plus inventory accumulation less new investment:

$$\Pi_t = \Pi^{corp}_t + P^c_t I_{INV,t} - P_{I,t} I_t, \tag{A.39}$$

where Π_t is computed by sales less wage costs. The law of motion of capital accumulation is

$$K_{t+1} = (1 - d\,(u_t)) K_t + I_t \ . \tag{A.40}$$

In the absence of adjustment costs, the price of investment equals nominal q_t, which is the value in terms of discounted future profits of a marginal increase in the capital stock. Thus the nature measure of stock market wealth is $q_t K_t$.

Substituting equation (A.40) into equation (A.39) and replacing $P_{I,t}$ with q_t and rearranging equation (A.38) yields

$$B_{g,t} + B_{f,t} + B_{rs,t} + (1 + R_{RF,t})\frac{q_t}{q_{t+1}}q_{t+1}K_{t+1}$$
$$= (1 + R_{RF,t})\left[\begin{array}{c} B_{g,t-1} + B_{f,t-1} + B_{rs,t-1} - P_{c,t}^{tot}C_t \\ +(1-\tau_{w,t})\int_0^1 N(h)_t W(h)_t dh + TF_t + \Pi_t^{corp} + q_t K_t - d(u_t)\, q_t K_t \\ -\delta_{RS}B_{rs,t-1} - \frac{r_t}{1+R_{RF,t}}B_{rs,t} \end{array}\right].$$

Defining the real interest rate on capital to be $(1 + r_{k,t}) = (1 + R_{RF,t})\frac{q_t}{q_{t+1}} - 1$ and rearranging the above equation to isolate $B_{k,t} = q_{t+1}K_{t+1}$, the stock market wealth at the end of period t, we have

$$B_{g,t} + B_{f,t} + B_{rs,t} + B_{k,t}$$
$$= (1 + R_{RF,t})\left[\begin{array}{c} B_{g,t-1} + B_{f,t-1} + B_{rs,t-1} + B_{k,t-1} - P_{c,t}^{tot}C_t \\ +(1-\tau_{w,t})\int_0^1 N(h)_t W(h)_t dh + TF_t + \Pi_t^{corp} - d(u_t)\, B_{k,t-1} - \frac{r_{k,t}}{(1+R_t)}B_{k,t} \\ -\delta_{RS}B_{rs,t-1} - \frac{r_t}{1+R_{RF,t}}B_{rs,t} \end{array}\right].$$

Finally, defining financial wealth FW_t to be the sum of domestic and foreign asset holdings plus housing and the market value of the capital stock,

$$FW_t = B_{g,t-1} + B_{f,t-1} + B_{rs,t-1} + B_{k,t-1},$$

and defining household savings to be after-tax labour income plus corporate profits plus total government transfers less consumption expenditures,

$$S_t = (1-\tau_{w,t})\int_0^1 N(h)_t W(h)_t dh + TF_t + \Pi_t^{corp} - P_{c,t}^{tot}C_t,$$

we have

$$FW_{t+1} = (1 + R_{RF,t})(FW_t + S_t - \epsilon_t),$$

where ϵ_t is defined as the sum of the depreciation, interest on the existing capital and housing:

$$\epsilon_t = d(u_t)\, B_{k,t-1} + \frac{r_{k,t}}{(1 + R_{RF,t})}B_{k,t} + \delta_{RS}B_{rs,t-1} + \frac{r_t}{1 + R_{RF,t}}B_{rs,t}.$$

Appendix B

Summary of Model Changes

Table B.1: Overview of Model Changes in ToTEM II

		ToTEM II	ToTEM I
Firms	Production	1. Adjustment costs on changes to input shares for all inputs	1. Adjustment costs are resource costs
		2. Capital utilization costs are measured in terms of depreciation	2. They are measured in terms of net output
		3. Strategic complementarities are achieved using multiple stages of production	3. It is achieved by firm-specific capital
		4. Fixed costs in the second-stage production	-
		5. Produced goods used as material inputs for second-stage production	-
	Price setting	Two groups of price setters:	Forward-looking price setter:
		1. Forward-looking: Calvo with indexation	Calvo setting with indexation
		2. Rule of thumb: rule of thumb and indexation	-
	Commodity	Separate energy and non-energy goods production	No distinction between energy and non-energy
Households	Preferences	1. Three types of household with asset market segmentation and imperfect asset substitution	1. Two types of household:
		a. Unrestricted lifetime: access to both short-term and long-term bonds	a. Lifetime income: access to short-term bonds
		b. Restricted lifetime income: access only to long-term bonds	b. Current income: no access to credit markets
		c. Current income: no access to credit markets	-
		2. Separate demand function for national account consumption, residential structures and inventories	2. Consumption composite includes national account consumption, residential structures and inventories
	Wage setting	Two groups of wage setters:	Forward-looking wage setter:
		1. Forward-looking: Calvo with indexation	Calvo setting with indexation
		2. Rule of thumb: rule of thumb and indexation	-
	UIP	1. Country risk premium affects UIP through exchange rate shock	1. It is a function of net foreign asset position
		2. Weighting factor applies to both exchange rate and interest differential	2. It applies only to exchange rate
Foreign links		Rest of the world demand affected by short-term and long-term foreign interest rates	It is only affected by short-term rates
Model closure condition		Household wealth closure condition	Country-specific premium depends on net foreign asset position gap