Real and nominal frictions within the firm: How lumpy investment matters for price adjustment*

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July 29, 2011

Abstract

Real rigidities are an important feature of modern sticky price models and are policy-relevant because of their welfare consequences, but cannot be structurally identified from time series. I evaluate the plausibility of capital specificity as a source of real rigidities using a two-dimensional generalized (s,S) model calibrated to micro evidence. Capital lumpiness reduces price stickiness as endogenous fluctuations in the marginal cost of output increase willingness to pay menu costs (an extensive effect), but increases price stickiness through complementarities (an intensive effect). The extensive effect warrants higher menu costs to match evidence on price changes, and the effects of complementarities prevail.

JEL Classifications: E12, E22, E31

Keywords: New Keynesian, menu costs, generalized (s,S), firm-specific investment, attached factors

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*This is a revision of the first chapter of my dissertation at Boston University. I am grateful to Robert King for guidance and support throughout this project. I have also benefited from conversations with Ruediger Bachmann, Marco Del Negro, Jose Dorich, Simon Gilchrist, Marvin Goodfriend, Francois Gourio, Brian Griffin, Chun-Yu Ho, Chris House, Oleksiy Kryvtsov, Yang Lu, Rhys Mendes, Stephen Murchison, and Lutz Weinke. Additional comments and corrections are welcome.

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1 Introduction

Many product prices change infrequently, and this has motivated a lot of research on sticky prices as a means for generating real effects from monetary disturbances. An important strand of this research is concerned with connecting nominal frictions to real frictions, typically in the context of infrequent price adjustment and costly capital adjustment (Kimball (1995), Altig, Christiano, Eichenbaum, Linde (2005), and Woodford (2005)). The consensus is that costly capital increases the real effects of monetary disturbances by effectively making prices more sticky. This is useful in models designed for policy analysis because it can reconcile frequently changing prices with price level persistence (Smets and Wouters (2007), Altig et. al. (2005), and many others). However, these studies model capital in a way that is counterfactual because establishment-level investment dynamics are too smooth. I attempt to evaluate the extent to which this abstraction from reality matters by constructing a two-dimensional generalized (s,S) model with endogenous sticky prices and lumpy investment.

The infrequent adjustment of prices and capital at the establishment level motivates the development of a model in which these features are replicated and used to study real and nominal frictions. It requires a model where firms face costly price and capital adjustment, but none has so far been developed because of an intrinsic complexity: one must track a joint distribution of capital and prices, as well as determining how price adjustment and capital adjustment incentives at the firm level translate into the time series behavior of aggregates. I develop such a model, which nests as special cases two prominent models of costly adjustment: Dotsey, King and Wolman’s (1999) analysis of state dependent price dynamics and Thomas’s (2002) analysis of state dependent capital dynamics. The model allows for endogenous coordination between the implementation of nominal and real adjustments. Further, the model permits the analysis of how the optimal time pattern of price adjustment depends on the state of the firm’s capital stock, with the timing of capital adjustments also chosen optimally.

The model is calibrated to match some features of the micro data, such as the average age of a price in CPI data, and the cross sectional distribution of investment rates in the LRD. I find that capital rigidities are a useful and reasonable means of generating price level persistence, even if those rigidities are somewhat lumpy. Specifically, holding the average age of a price fixed (as in Calvo-based models), firm-specific capital distributes the real effects of monetary shocks forward in time relative to a model with frictionless
capital. While investment has an important lumpy component, these large investment episodes occur infrequently relative to price adjustments (Doms and Dunne (1998)). From the perspective of pricing behavior, the assumption that capital is approximately fixed in the short run seems quantitatively reasonable.

Section 3 describes the model, beginning with the description of the household, proceeds to describe the problem of the firm, and concludes with equilibrium conditions that close the model. Section 4 describes the calibration procedure. Section 5 presents the results, considering first the aggregate effects of changing the capital structure of a model while holding the nominal side of the model fixed, and concluding with the implications that the capital structure modification for the calibration of the nominal side. Section 6 concludes.

2 Literature

Inflation is sufficiently persistent and insensitive to marginal cost movements to imply average price durations of at least 5.9 quarters (Gali and Gertler (1999)) with evidence that this estimate is biased in favor of price flexibility (Linde (2005)). Yet the prices of individual products change frequently with high estimates around 3.0 quarters (Steinsson and Nakamura (2008)) and others are much lower (Klenow and Kryvtsov (2008)). The tension between the observation that prices of individual goods change frequently and the duration of prices implied by the persistence of inflation is successfully mitigated through the introduction real rigidities (Kimball (1995), Sbordone (1999), Gali et. al. (2001)).

Interpretation of these real rigidities is ambiguous, however, because a number of plausible structural explanations exist, including variable elasticity of demand (Dotsey and King (2005)), heterogeneity in nominal rigidities across sectors (Carvalho (2007)), and merely increasing short-run marginal cost directly (Woodford (1996)). None of these explanations are identified in macroeconomic studies because the time series can only identify the reduced form parameter.\footnote{Gali (2005) points this out very clearly in a discussion of Altig, et. al. (2005).} This matters a great deal in terms of the conduct of monetary policy because the source of real rigidities has strong implications for model welfare properties (Levin et. al. (2007)).

Firm-specific capital plays an important role in a number of important recent studies, including Smets and Wouters (2007), Murchison and Rennisson (2006), Altig et. al. (2005), Woodford (2005), Sveen and Weinke (2007), and many others, including policy models at
many central banks. Its introduction is justified by the widely shared idea that capital is not freely transferable between firms each period, and it is one of the most widely adopted cases in which interesting dynamics obtain from the interaction of nominal and real rigidities (Ball and Romer (1990)). In each of these models, simulated plant-level investment patterns are counterfactual. Monetary models with capital specificity typically have very smooth firm-level investment rates in which depreciated capital is perpetually replaced each period. Some of these studies use convex adjustment costs (Altig et. al. (2005), Woodford (2005)), and others rely on ad hoc decision rules or complete maintenance investment (Sveen and Weinke (2007)). In contrast, while regular capital maintenance makes up about half of the investment activity in the U.S., the majority of manufacturing plants only partially maintain their equipment in a typical year and compensate for obsolescence and maintenance shortcomings with large but infrequent investment episodes (Doms and Dunne (1998)). Although large investment episodes occur infrequently, there are relatively few cases in which very small investment rates are observed as well. In the ASM, only 18.5% of firms have investment rates below 1% per annum. Matching plant-level stylized facts necessitates both convex and non-convex investment, which I implement by allowing maintenance on broken machines in conjunction with larger capital changes to compensate for non-repairable depreciation and technological obsolescence.

Because firm-specific capital in previous studies is fixed in the current period (or is exogenously determined), the focus in these studies was on the intensive effect. Even if returns to scale are constant in the long run, fixing one factor (like capital) means that returns are diminishing in the short run (Woodford (1996)). By making the short run marginal cost schedule slope up, the magnitude of price increases is constrained; the higher the price, the lower will be demand, the lower will be marginal cost, and the lower will be the desired price (Gali et. al. (2001)). My model also has an extensive margin in that variability in firm capital stocks (either through depreciation or infrequent investment) shifts the marginal cost schedule for each firm over time. A sticky price model with firm-specific productivity fluctuations (Golsov and Lucas (2008)) shows exactly this; and the reasoning is somewhat similar in my model, except that changes in the marginal cost of output are endogenous.

If one fixes the distribution of menu costs, then firm-specific capital appears to mitigate the real effects of monetary shocks through the extensive effect. Variability in the marginal cost schedule creates incentives to adjust price which don’t exist in an economy without micro-founded capital frictions. This is the effect documented in previous versions of this
paper (Johnston (2007)) and other studies have since found similar results (Reiter, Sveen and Weinke (2009)). Note, however, that the introduction of capital frictions also lowers the average age of a price. Higher menu costs can restore the average age of a price to its empirical level and also restore the dominance of the intensive effect.

Heterogeneous agents models are notoriously difficult to solve. Solution methods for these economies can be broadly classified into those which approximate the true distribution (Krusell and Smith (1998), Khan and Thomas (2007)) and those which construct economies where the true distribution is a tractable (Dotsey, King, and Wolman (1999), Thomas (2002), Khan and Thomas (2007)). I use the second approach. The idiosyncratic shocks which induce heterogeneity are i.i.d. so the joint distribution of prices and capital is discrete at each point in time. A few other studies have examined general equilibrium models with more than one dimension of heterogeneity (Golosov and Lucas (2007), Dotsey, King, and Wolman (2009), Khan and Thomas (2008), Midrigan (2008)), although this is one of the first models in which the additional dimension of heterogeneity is endogenous.

I build and solve a theoretical model with several key features that enable me to more closely match microeconomic evidence as I reexamine the extent to which firm-specific capital might be useful as an endogenous propagation mechanism. First, prices change infrequently because of menu costs. Second, large investment episodes occur infrequently because of fixed installation costs. Finally, capital maintenance allows machine failures to be repaired without installation costs, but is not sufficient to fully compensate for degradation and obsolescence over time. I calibrate my model to match key moments from the price panels for the U.S. Consumer Price Index and from the investment panels in the Longitudinal Research Database. The question I wish to answer is whether the menu cost model with firm-specific capital generates more endogenous persistence than a model with frictionless capital, holding the average age of a price fixed.

3 Model

Households have preferences over consumption and leisure and invest in homogeneous capital goods. In the subsequent period, firms are able to purchase capital goods from households for installation at their location, at which point the capital becomes firm-specific. Firms can purchase capital whenever they want, but must pay fixed installation costs. Similarly, firms can adjust prices whenever they want, but must pay fixed menu costs.
3.1 Preferences

Households have preferences over consumption and leisure and maximize the present discounted value of lifetime utility. The household lifetime utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln (C_t) - \chi N_t]$$

(1)

where $C_t$ and $N_t$ are aggregates constructed from consumption of differentiated products and hours worked at individual firms.

In addition to labor income $W_t N_t$, households own shares in firms and receive profits $Z_t$ earned through the extraction of monopoly rents and the real return on installed capital. After imposition of a CIA constraint, the household intertemporal budget constraint is

$$C_t + I_t + B_t \leq \Pi_t^{-1} (1 + R_{t-1}) B_{t-1} + W_t N_t + Z_t + Q_t I_{t-1}.$$  

(2)

Intertemporal transfer of wealth can occur through purchases $B_t$ of nominal bonds which promise a net nominal return $R_t$ or through use of an investment technology which converts output into capital with a one period delay. After output in period $t$ is converted to capital in period $t + 1$, the homogeneous capital good is sold to firms at the market clearing price $Q_{t+1}$, the gross real rate of return on investment. The supply curve for investment goods which is upward sloping one period in advance but is vertical at time $t$. Capital becomes firm-specific when it is purchased and installed by firms. Modeling capital markets in this way works well for things like automobiles and copy machines, but not for made-to-order capital goods or specialized production machinery. I selected this structure for tractability, but it seems well-motivated and has been used in prior studies to some extent (Abel, Dixit, Eberly and Pindyck (1996)).

The direct interpretation of this setup is that households produce capital. Alternatively, if a perfectly competitive sector purchases consumption goods and stores them for one period to create capital, the result is the same. In the absence of capital frictions at the firm level, this model reduces to a neoclassical model of investment with pricing frictions. Euler equations determine the demand for bonds and the supply of capital investment goods to firms. If $\Lambda_t$ is the Lagrange multiplier on the budget constraint (2), then first-order necessary conditions from this setup are
\[ 1 = \Lambda_t C_t \]
\[ W_t \Lambda_t = \chi \]
\[ \Lambda_t = \beta \Theta \Lambda_t E_t \Pi_t^{-1} (1 + R_t) \Lambda_{t+1} \]
\[ \Lambda_t = \beta \Theta \Lambda_t E_t Q_{t+1} \Lambda_{t+1} \]

where $\Theta$ refers to the growth rate of $\Lambda_t$ induced through technological progress, to be discussed later. Finally, let the stochastic discount factor be defined as $S_{t,t+h} \equiv (\beta \Theta \Lambda)^h E_t \Lambda_{t+h}$. Consumers have isoelastic preferences over a continuum of products $s \in [0, 1]$ such that relative price of a product determines its relative demand,

\[ y_t(s) = Y_t p_t(s)^{-\varepsilon} \quad (3) \]

where $\varepsilon$ is the elasticity of demand. The output aggregate is composed of a basket of these goods (Dixit and Stiglitz (1977)).

Money demand comes from a constant velocity CIA constraint, $m_t = P_t Y_t$. Money growth is stationary at a rate consistent with steady state inflation, and cyclical fluctuations in money growth follow a first-order autoregressive process with coefficient $\rho_M$.

### 3.2 Technology

The problem of the firm is to choose its current price and capital stock given the current states, including its fixed idiosyncratic capital installation costs and menu costs. Economies with heterogeneous agents are notably more intractable, largely because the state variables and policies of other firms impact future prices. Policies depend on the joint distribution of prices and capital stocks and its evolution.

Dotsey, King and Wolman (1999) have a clever way of solving the model while tracking the heterogeneity. They introduce heterogeneity via i.i.d. menu costs so that the menu cost realization is not a state variable, and there is a uniform action principle in the sense that all firms who choose to change price select the same price. This means that the age of a price, or its vintage $(j)$, is a sufficient statistic for state variables. They limit the number of vintages by placing an upper bound on the menu cost distribution, and assuming a positive
inflation rate.

I would like to use a similar trick in my environment, except with two vintages: one for price \((j)\) and one for capital \((h)\). For this to work I need all firms who last adjusted their capital stock \(h\) periods ago to have the same capital stock \(k_h\), so that all firms who adjust their capital stock today must choose the same capital stock \(k_0\). This must not be a function of any historical prices, so I assume that price changes must accompany major investment projects. This seems reasonable since major investments create large shifts in the short run marginal cost schedule and accordingly provide incentives for price adjustment. Each firm with capital stock \(k_h\) which changes its price today will choose a price \(p_{0h}\) which is a function of its capital stock \(k_h\) so that \(p_{jh}\) is the price of a firm with a capital stock of vintage \(h\) and a price of vintage \(j\). Again, i.i.d. menu costs ensure that their realizations are not state variables. Positive depreciation and inflation, coupled with an upper bound on both menu costs and capital installation costs ensure a finite memory property.

At the beginning of each period capital breaks down with probability \(\psi\) and needs a capital investment that is distributed Uniform over \([0,2\theta]\) to work properly again. This is as in Gourio and Kashyap (2007), except that the repair cost is random in my model. Allowing for some non-convex investment is important in replicating the empirical cross sectional distribution of investment rates, as will be shown later.

After any breakdowns occur and are repaired, idiosyncratic adjustment costs for changing prices and installing capital are revealed, any adjustments or installations occur, and production takes place. Let the production function be Cobb-Douglas in labour and capital

\[
y_{jht} = A_t n_{jht}^\rho k_{ht}^\sigma, \quad \text{where } A_t \text{ grows at rate } \Theta_A.\]

Cyclical fluctuations in productivity follow a first-order autoregressive process with coefficient \(\rho_A\). Profits in the current period are

\[
z_{jht} = Y_t x_{jht} p_{jht} - W_{t+1} n_{jht}.\]

Assume for the moment that the probability of adjusting price and capital together \(\alpha_{jht}^{PK}\) is known, that the probability of adjusting price only \(\alpha_{jht}^P\) is also known, and that total expected cost of these changes \(\Xi_{jht}\) is known. Both capital installation costs and menu costs are denominated in labour units.

The value function is then

\[
v_{jht} = z_{jht} + E_t S_{t,t+1} \left\{ \left( \frac{1 - \alpha_{j+1,h+1,t+1}^P - \alpha_{j+1,h+1,t+1}^{PK}}{\alpha_{j+1,h+1,t+1}^{PK}} \right) v_{j+1,h+1,t+1} + \alpha_{j+1,h+1,t+1}^P \left( v_{0,h+1,t+1} \right) - W_{t+1} \Xi_{j+1,h+1,t+1} - \theta \psi k_{h+1,t+1} \right\}
\]
The marginal value of an additional unit of capital for firm \((j, h)\) comes from the marginal reduction in future labour costs (Woodford (2005)), and higher capital purchases today marginally lower anticipated future purchases. These benefits are partially offset by the requirement that installed capital be maintained.

\[
\frac{\partial v_{jht}}{\partial k_{ht}} = W_t \left( \frac{\gamma}{\nu} \right) \frac{n_{jht}}{k_{ht}} + \left( \frac{1 - \delta}{\Theta K} \right) E_t S_{t+1} \left[ \left( 1 - \alpha_{j+1, h+1, t+1} P - \alpha_{j+1, h+1, t+1} P K \right) \frac{\partial v_{j+1, h+1, t+1}}{\partial k_{j+1, h+1, t+1}} \right]
\]

The optimal reset capital stock \(k_{0t}\) must equate this value with the cost per unit \(Q_t\),

\[
Q_t = \frac{\partial v_{00t}}{\partial k_{0t}}.
\]

When the firm does not install new capital, its existing stock of capital depreciates,

\[
k_{t+1, h+1} = 1 - \delta \frac{1}{\Theta K} k_{ht}.
\]

The marginal value of a slightly higher price for firm \((j, h)\) is,

\[
\frac{\partial v_{jht}}{\partial p_{jht}} = \frac{\partial z_{jht}}{\partial p_{jht}} + E_t \frac{S_{t+1}}{\Pi_{t+1}} \left[ \left( 1 - \alpha_{j+1, h+1, t+1} P - \alpha_{j+1, h+1, t+1} P K \right) \frac{\partial v_{j+1, h+1, t+1}}{\partial p_{j+1, h+1, t+1}} \right]
\]

and optimal reset prices conditional on adjustment \(\{p_{0ht}\}\) are determined through the value maximizing conditions,

\[
0 = \frac{\partial v_{0ht}}{\partial p_{0ht}}.
\]

When the firm does not change price, its historical price erodes stochastically with inflation,

\[
p_{jht} = \left( \frac{1}{1 + \pi_t} \right) p_{j-1, h-1, t-1}.
\]

### 3.2.1 Adjustment

Recall that in Dotsey, King and Wolman (1999), price adjustment policies were found by determining the menu cost which would make a firm indifferent between adjustment and non-adjustment. The proportion of firms adjusting price is then the proportion of menu cost realizations below the breakeven value. In my model, the approach is similar, except...
that my problem is two-dimensional, and I have three breakeven conditions to ensure that any adjustment is not only value improving but value maximizing. To obtain analytic results, I assume menu costs $\xi^P$ and capital installation costs $\xi^K$ are independent and uniformly distributed. Let the menu cost $\xi^P$ be distributed Uniform $[0, B_p]$. Let the capital installation cost $\xi^K$ be distributed Uniform $[B_l, B_h]$.

Price adjustment (only) is optimal if and only if

$$v_{0ht} - v_{jht} > W_t \xi^P_t$$
$$v_{0ht} > v_{00t} - Q_t i_{ht} - \xi^K_t W_t$$

where the first condition ensures that adjustment is value improving, and the second condition ensures price adjustment (only) is not dominated by simultaneous adjustment of price and capital.

Combined price adjustment and capital investment is optimal if and only if

$$v_{00t} - Q_t i_{ht} - v_{jht} - (\xi^K_t + \xi^P_t) W_t > 0$$
$$v_{00t} - Q_t i_{ht} - \xi^K_t W_t > v_{0ht}$$

where the first condition ensures that adjustment is value improving, and the second condition ensures simultaneous price adjustment and capital investment is not dominated by a simple price change only.

It is useful to define

$$A^0_{jht} (\xi^P_t, \xi^K_t) = 1_{[v_{00t} - Q_t i_{ht} - v_{jht} - (\xi^K_t + \xi^P_t) W_t > 0]}$$
$$A^1_{jht} (\xi^P_t, \xi^K_t) = 1_{[v_{00t} - Q_t i_{ht} - \xi^K_t W_t > v_{0ht}]}$$
$$A^2_{jht} (\xi^P_t, \xi^K_t) = 1_{[v_{0ht} - v_{jht} > W_t \xi^P_t]}$$

so that the adjustment probabilities are,
Figure 1: **Figure 1**: Calculation of adjustment proportions. Two conditions are necessary for adjustment: (1) the discrete decision must yield weakly positive net value, and (2) must not be dominated by another discrete decision.

\[ P_{jht}^{PK} = \int \int A_{jht}^0 (\xi_t^P, \xi_t^K) A_{jht}^1 (\xi_t^P, \xi_t^K) dF (\xi_t^P, \xi_t^K) \]

\[ P_{jht}^{PK} = \int \int A_{jht}^2 (\xi_t^P, \xi_t^K) [1 - A_{jht}^1 (\xi_t^P, \xi_t^K)] dF (\xi_t^P, \xi_t^K) \]

and similarly integral expressions define the expected costs of adjustment as well. These can be found either through integration or through simple geometry.

Figure 1 provides an illustration of the case in which \( B_l = 0 \), and is especially useful for thinking about the model when adjustment costs are bivariate uniform, as I assume; in this case each adjustment probability is just the ratio of the darkly shaded region to the domain.

Large investment projects accompanied by small price changes occur whenever the associated net change in expected value exceeds the idiosyncratic costs and the change is not dominated by simple price adjustment.

\[ P_{jht}^{PK} = \hat{a}^{PK} \left( \frac{v_{00t} - Q_{ijht} - v_{jht}}{W_t}, \frac{v_{00t} - Q_{ijht} - v_{0ht}}{W_t} \right) \]

(4)

where
\[ \hat{a}^P(x_P, x_D) = \frac{(\max(\min(x_D, B_h), B_l) - B_l) \max(\min(x_P, B_p), 0)}{B_h - B_l} \]  

Small price changes absent large investment projects occur whenever the net change in expected value exceeds the idiosyncratic costs and the change is not dominated by a change that incorporates a large capital investment project.

\[ \alpha^P_{jht} = \hat{a}^P \left( \frac{v_{0ht} - v_{jht}}{W_t}, B_h - \frac{v_{00t} - Q_{t iht} - v_{0ht}}{W_t} \right) \]  

where

\[ \hat{a}^{PK}(x_{PK}, x_D) = \frac{1}{(B_h - B_l) B_p} \left\{ \int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK}) dz_2 \right\} - \frac{1}{2} \left( \min(B_h, x_D)^2 - (B_l)^2 \right) \]

and the integral in the expression above simplifies to \( \frac{1}{2} \left( \min(B_h, x_D)^2 - (B_l)^2 \right) + B_p (\min(B_h, x_D) - B_l) \) when \( \min(B_h, x_D) + B_p \leq x_{PK} \), to \( x_{PK} (\min(B_h, x_D) - B_l) \) for \( x_{PK} \leq B_l + B_p \), and is

\[ \frac{1}{2} \left( (x_{PK} - B_p)^2 - (B_l)^2 \right) + B_p (x_{PK} - B_p - B_l) + x_{PK} (\min(B_h, x_D) - (x_{PK} - B_p)) \]

otherwise. Expected adjustment costs are computed found analogously, but are omitted from the body for the sake of brevity. Derivations are in one of the technical appendices.

### 3.2.2 Distribution evolution

Adjustment policy likelihoods and the restrictions below provide all of the information necessary to exactly describe the evolution of the discrete density of prices and capital. The end of period density is \( \theta_{jht} \). Firms either inherit their price and capital states as they evolve from the previous period,

\[ \theta_{j+1,h+1,t+1} = (1 - \alpha^P_{jht} - \alpha^{PK}_{jht}) \theta_{jht} \]

or change prices,

\[ \theta_{1,h+1,t+1} = \sum_j \alpha^P_{jht} \theta_{jht} \]

and total firm mass is normalized to unity.
\[ 1 = \sum_j \sum_h \theta_{j+1,h+1,t+1} \]

### 3.3 Market clearing

Aggregate resources in the current period are split between consumption and investment and therefore must obey the resource constraint, \( C_t + I_t = Y_t \). Investment is an aggregation of demands for capital goods across firms,

\[
I_{t-1} = \sum_h (k_{0t} - k_{ht}) \sum_j \theta_{jht} \alpha_{jht}^{PK} + \psi \sum_h k_{ht} \sum_j \theta_{jht} \tag{9}
\]

which includes lumpy investment, \( i(k_t, M_t) \alpha^{PK} (k_t, p_t, M_t) \) as the product of conditional investment demand and the likelihood of drawing low enough fixed costs, and maintenance on existing capital.

Labor market clearing requires labor supply meet the aggregated labor demand,

\[
N_t = \sum_h \sum_j (\theta_{jht} \Xi_{jht} + \theta_{j+1,h+1,t+1} n_{jht}) \tag{10}
\]

where \( n_{jht} \) is labor for production and \( \Xi_{jht} \) is labor for adjusting prices and installing capital, conditional on survival.

### 3.4 Numerical procedure

I find the non-stochastic steady state of my model by solving the general equilibrium problem into a sequence of partial equilibrium problems, each of which is solved using standard policy function iteration techniques. In each of the partial equilibrium problems the functional contraction mapping theorems apply and assure convergence. I check market clearing conditions for each partial equilibrium problem and use a Newton-Raphson method to find aggregate prices at which markets clear. A dynamic solution to the perturbation of the model equations around the non-stochastic steady state is acquired using standard rational expectations techniques (Sims (2002)).
4 Calibration

A number of parameters are set to match post-war business trends. The discount rate is set to induce an average real interest rate of 4% per year. Technological progress occurs at the geometric rate of 1.6% per year. The autocorrelation coefficient in the process for the log-deviation of aggregate productivity from trend is 0.97. The autocorrelation coefficient on the process for the deviation of the money growth rate from its stationary point is 0.5, a value which allows predictable but not excessively protracted nominal demand movements. I use this simple money supply rule as a diagnostic for judging the behavior of my models, as is common in the New Keynesian literature, and not as a description of actual monetary policy. Returns to scale in production are slightly diminishing at 0.9, a choice consistent with the evidence in Basu and Fernald (1997), and the share of labor in output is 0.64 as in Prescott (1986). The parameter on the disutility of labor in the utility function is selected to generate steady state labor supply of 0.2.

New Keynesian studies have a long tradition of using very high elasticities of demand. I follow microeconometric evidence which indicates that demand elasticities are, at most, one third of the conventional values. For example, Bijmolt, Van Heerde and Pieters (2005) survey more than a thousand microeconometric studies and find an average absolute price elasticity of demand of 2.62, and virtually no microeconometric studies obtain demand elasticity estimates above the value of 12 that is commonly used. I follow empirical evidence and Midrigan (2007) in selecting a demand elasticity of 3.0. Because this value increases the market power of firms and demand is, by definition, less sensitive to price movements in a way that serves to mitigate the impact of firm-specific marginal cost movements on profits. This calibration will, if anything, cause the results I present to be tempered.
Parameters of governing the maintenance, installation, and depreciation of capital are selected to match moments from plant level data in the Annual Survey of Manufacturers (ASM) in the Longitudinal Research Database (LRD) as reported by Cooper and Haltiwanger (2006). First, I match the average investment to capital ratio of 12.2% per year, a number that depends on the depreciation rate, obsolescence rate, the probability of machine failures, and the cost of machine repair in the case of a failure. Higher maintenance investment, either through higher repair costs or lower machine reliability, requires lower depreciation rates to match the observed investment spike rates. Matching this is critical as more effective capital maintenance decreases lumpiness and the depreciation rate net of
<table>
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<th>Parameter</th>
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<th>LS</th>
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<tr>
<td>Steady-state labor supply</td>
<td>$\bar{n}$</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money growth persistence</td>
<td>$\rho_M$</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.9902 per qtr., 0.9615 per yr.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>$\pi$</td>
<td>0.61% per qtr., 2.46% per yr.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology growth rate</td>
<td>$\Theta_A$</td>
<td>0.4% per qtr., 1.6% per yr.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of labor in output</td>
<td></td>
<td>0.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Returns to scale</td>
<td></td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: MLS: the most elaborate model with maintenance investment, lumpiness in investment, and sticky prices. LS: lumpy investment and sticky prices without opportunities for capital maintenance. S: a simple sticky price model with capital.

repairs, an important parameter in this class of models (House (2008)). Second, I minimize the weighted sum of squared differences between the model CDF and the CDF and the cross-sectional plant-level investment rates in the Longitudinal Research Database, where the weights are the proportion of observations found at each investment rate.² Figure 2 provides a visual comparison of the cross sectional distribution of plant-level investment rates and compares them with the data.

Finally, I match the median duration of non-sale price changes (without substitutions) in the panels underlying the U.S. Consumer Price Index from 1998 through 2005, as reported in Steinsson and Nakamura (2008); at 7.4 months it is the largest estimate of the elapsed time between price changes found in the U.S. Consumer Price Index over that period. Average inflation is set to 2.46% per year, the average rate in the U.S. between 1998 and 2005, the period included in the most recently analyzed panel of prices from the U.S. Consumer Price Index.

²I am grateful to John Haltiwanger and Russell Cooper for providing me with this information.
5 Results

In my discussion of the results, I will focus primarily on the model that best fits the micro-economic evidence: it features both large but infrequent capital investment episodes and regular maintenance of capital breakdowns, in addition to the nominal rigidities introduced through menu costs of price adjustment. The adjustment probabilities implied by the generalized (s,S) policies found earlier are shown in Figure 3, where dark regions correspond to high adjustment probabilities and light regions to low adjustment probabilities. Note how unlikely it is for the firm to undertake a major investment project without waiting several years, a consequence of the lower bound on the fixed cost component associated with major investment undertakings and the effectiveness of capital maintenance activities.

Figure 3: Hazard rates for price changes and large investment projects in steady state for the fully articulated model with capital maintenance.
I begin the discussion of the dynamic implications by holding the nominal frictions constant across all models. Specifically, the results in Figure 4 obtain from a persistent shock to the money supply growth rate when the average duration of product prices is held constant at the Steinsson and Nakamura (2008) levels, and the distribution of menu costs (parameterized by $B_p$) is adjusted as required. Both of the models with real rigidities have larger and more protracted dynamic responses relative to the simple menu cost model. In the simple (s,S) menu cost model, the upper bound on the support for the distribution of adjustment costs is $B_p = 0.003$, and the average menu cost paid is fairly close to zero. By contrast, the upper bound on the support for the distribution of menu costs is $B_p = 0.045$ in the exclusively lumpy model. The model with capital maintenance has a lower effective depreciation rate (and hence less cross-sectional variation in the marginal cost of output) and requires only $B_p = 0.024$ to match the duration of prices in the U.S. Consumer Price Index.

The dynamic responses confirm, in a more nuanced environment, a well-known propagation mechanism that was first introduced by Woodford (1996), and was subsequently empirically investigated and found to be relevant in studies like Sbordone (1999) and Gali, Gertler, and Lopez-Salido (2001). In a positive inflation environment, the average price change is naturally a price increase, and the size of the increase can be effectively constrained when the marginal cost schedule the firm faces is upward sloping. Price increases lower output, and because of the upward sloping short-run marginal cost schedule, lower marginal cost, providing an incentive for the firm to select a price lower than it otherwise would. Holding the frequency of price changes constant, the aggregate series should behave somewhat more sluggishly when the average change is smaller. While returns to scale in both factors are close to constant (0.9), the inability of the firm to undertake major capital restructuring projects without incurring a fixed cost leaves one factor of production effectively fixed in the short run; the marginal cost of output is determined by the number of workers required to generate an additional unit of output.
Figure 4: Dynamic responses in output, investment, and inflation, to a persistent shock to the growth rate of the money supply. For this comparison, the support for the distribution of menu costs is varied to hold the average frequency of price changes constant at the level observed in the U.S. Consumer Price Index as reported by Steinsson and Nakamura (2008).

I now examine aggregate dynamics, holding the level of nominal frictions (i.e., the menu cost distribution as parameterized by $B_p$) constant, as opposed to varying menu costs to hold price durations constant. I fix the distribution at one of the intermediate calibrations found previously and examine the implications of altering the capital structure, holding constant the distribution of nominal frictions. Figure 5 shows that the effect of nominal disturbances on the simple menu cost model are much larger than in the case just considered. Output is almost zero on average over the first year following impact in Figure 4, as discussed earlier, and is large and positive in Figure 5, below.
Figure 5: Dynamic responses in output, investment, and inflation, to a persistent shock to the growth rate of the money supply. For this comparison, the distribution of menu costs is fixed across models at the level implied by the calibration of the model with both (S,s) decisions in prices and capital and maintenance of some capital breakdowns.

The cross sectional variation in the marginal cost of output that results from investment lumpiness increases the relative willingness of the firm to pay menu costs of price adjustment, relative to the simple (s,S) sticky price model I consider. Each period, a portion of each firms’ effective capital stock decreases, either by obsolescence or depreciation; some of these effects are offset through maintenance, but not all. The marginal cost of output rises with each quarter since the last major capital restructuring project, raising the value to the firm of paying menu costs. Intuitively, one can think about the typical (s,S) story in which proximity to one of the bounds either deterministically or stochastically triggers
a discrete decision in which the firm returns the control state in question to its target. As capital structures wear out and become old, workers are increasingly unproductive: this raises the target price that would be chosen under flexibility – a markup over marginal cost – and can be thought of as moving the (s,S) bands and target, making the target more variable and the bounds more frequently encountered.

6 Conclusion

Is firm specific capital, when calibrated to match key properties of U.S. data, capable of increasing the protracted sensitivity of real variables to nominal disturbances? I answer yes, based on results from a two-dimensional generalized (s,S) model which features capital lumpiness and infrequent price changes. While the economic story is somewhat more nuanced than previously – firm-specific capital both motivates price changes, and changes the prices chosen – the idea in the literature that this is a means of producing an upward sloping short run marginal cost schedule (and hence smaller price changes) is robust.

Yet the magnitude to which this mechanism is typically used in the literature is excessive, and the transitive nature of even the most persistent dynamics I present shows that capital specificity can only be one part of the story. There is clearly evidence of sectoral price heterogeneity (Dhyne et. al. (2005)) and theoretical work that supports its efficacy (Carvalho (2007), Steinsson and Nakamura (2009)). So far no there is limited evidence on the variable price elasticity of demand explanation (Dotsey and King (2005)), but the story is intuitively appealing and will hopefully be more closely examined in future research.

References


Midrigan, Virgiliu. “Menu Costs, Multi-Product Firms, and Aggregate Fluctuations,” working paper, Ohio State University, January 2006.


Midrigan, Virgiliu. “Menu Costs, Multi-Product Firms, and Aggregate Fluctuations,” working paper, Ohio State University, January 2006.


7 Appendix A: Adjustment hazards

7.1 Price and capital likelihood

\[ \hat{a}_{PK} (x_{PK}, x_D) = (B_h - B_l)^{-1} (B_p)^{-1} \int_{-\infty}^{x_D} \int_{-\infty}^{x_{PK}} 1_{[B_l \leq z_2 \leq B_h]} 1_{[z_2 \leq z_1 \leq z_2 + B_p]} dz_1 dz_2 \]
\[ = (B_h - B_l)^{-1} (B_p)^{-1} \int_{B_l}^{\min(B_h, x_D)} \int_{z_2}^{\min(z_2 + B_p, x_{PK})} dz_1 dz_2 \]
\[ = (B_h - B_l)^{-1} (B_p)^{-1} \left\{ \int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK}) dz_2 \right\} \]
\[ = \frac{1}{2} \left( \min(B_h, x_D)^2 - (B_l)^2 \right) + B_p \left( \min(B_h, x_D) - B_l \right) \]

Obviously, \( \hat{a}_{PK} (x_{PK}, x_D) = 0 \) if either \( \min(B_h, x_D) < B_l \) or \( x_{PK} < B_l \). Otherwise, the result can be easily derived in pieces. If \( \min(B_h, x_D) + B_p \leq x_{PK} \),

\[ \int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK}) dz_2 \]
\[ = \frac{1}{2} \left( \min(B_h, x_D)^2 - (B_l)^2 \right) + B_p \left( \min(B_h, x_D) - B_l \right) \]

and if \( x_{PK} \leq B_l + B_p \),

\[ \int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK}) dz_2 \]
\[ = x_{PK} \left( \min(B_h, x_D) - B_l \right) \]

and for the values of \( a \) in between,

\[ \int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK}) dz_2 = \int_{B_l}^{x_{PK} - B_p} (z_2 + B_p) dz_2 + \int_{x_{PK} - B_p}^{\min(B_h, x_D)} x_{PK} dz_2 \]
\[ = \frac{1}{2} \left( (x_{PK} - B_p)^2 - (B_l)^2 \right) + B_p (x_{PK} - B_p - B_l) + x_{PK} \left( \min(B_h, x_D) - (x_{PK} - B_p) \right) \]
7.2 Price likelihood

\[ \hat{a}^P(x_P, x_D) = (B_h - B_l)^{-1} (B_p)^{-1} \int_{-\infty}^{x_P} \int_{-\infty}^{x_D} 1_{[B_l \leq z_2 \leq B_h]} 1_{[0 \leq z_1 \leq B_p]} dz_1 dz_2 \]

\[ = (B_h - B_l)^{-1} (B_p)^{-1} \int_{\min(B_h,x_D)}^{\min(B_p,x_P)} \int_{0}^{\min(B_p,x_P)} dz_1 dz_2 \]

\[ = (B_h - B_l)^{-1} (B_p)^{-1} \left( \max \left( \min(x_D, B_h) - B_l \right) \right) \max \left( \min(x_P, B_p), 0 \right) \]

7.3 Expected cost

Note: this is not the same \( \Xi(\cdot) \) function used in the body of the paper.

\[ \Xi(x_{PK}, x_P, x_D) = (B_h - B_l)^{-1} (B_p)^{-1} \left\{ \int_{-\infty}^{x_D} \int_{-\infty}^{x_{PK}} z_1 1_{[B_l \leq z_2 \leq B_h]} 1_{[0 \leq z_1 \leq B_p]} dz_1 dz_2 \right\} \]

\[ + \int_{-\infty}^{x_D} \int_{-\infty}^{x_{PK}} z_1 1_{[B_l \leq z_2 \leq B_h]} 1_{[0 \leq z_1 \leq B_p]} dz_1 dz_2 \]

\[ = (B_h - B_l)^{-1} (B_p)^{-1} \left\{ \int_{B_l}^{\min(B_h,x_D)} \int_{\min(B_p,x_{PK})}^{\min(z_2+B_p,x_{PK})} z_1 dz_1 dz_2 \right\} \]

\[ + \int_{B_l}^{\min(B_h,x_D)} \int_{0}^{\min(B_p,x_P)} z_1 dz_1 dz_2 \]

\[ = (B_h - B_l)^{-1} (B_p)^{-1} \left\{ \int_{B_l}^{\min(B_h,x_D)} \frac{1}{2} \left( \min(z_2 + B_p, x_{PK})^2 - z_2^2 \right) dz_2 \right\} \]

\[ + \frac{1}{2} \min(B_p, x_P)^2 \left( \min(B_h, x_D) - B_l \right) \]

Because \( \hat{a}^{PK}(x_{PK}, x_D) = 0 \) if either \( \min(B_h, x_D) < B_l \) or \( x_{PK} < B_l \), it is also the case that the first component in the expression above is zero under the same conditions. Otherwise, the result can be easily derived in pieces. If \( \min(B_h, x_D) + B_p \leq x_{PK} \),

\[ \int_{B_l}^{\min(B_h,x_D)} \frac{1}{2} \left( (z_2 + B_p)^2 - z_2^2 \right) dz_2 \]

\[ = \int_{B_l}^{\min(B_h,x_D)} \frac{1}{2} (2z_2B_p + B_p^2) dz_2 \]

\[ = \frac{1}{2} \left( B_p \left( \min(B_h, x_D)^2 - B_l^2 \right) + B_p^2 \left( \min(B_h, x_D) - B_l \right) \right) \]

and if \( x_{PK} \leq B_l + B_p \),

27
\[
\int_{B_l}^{\min(B_h,x_D)} \frac{1}{2} (x_{PK}^2 - z_2^2) \, dz_2 \\
= \frac{1}{2} \left( x_{PK}^2 \left( \min(B_h,x_D) - B_l \right) \right) - \frac{1}{6} \left( \min(B_h,x_D)^3 - B_l^3 \right)
\]

and for the values of \( a \) in between,

\[
\int_{B_l}^{\min(B_h,x_D)} \frac{1}{2} \left( \min(z_2 + B_p, x_{PK})^2 - z_2^2 \right) \\
= \int_{B_l}^{x_{PK} - B_p} \frac{1}{2} \left( 2z_2B_p + B_p^2 \right) \, dz_2 + \int_{x_{PK} - B_p}^{\min(B_h,x_D)} \frac{1}{2} \left( x_{PK}^2 - z_2^2 \right) \, dz_2 \\
= \frac{1}{2} \left( B_p \left( (x_{PK} - B_p)^2 - B_l^2 \right) + B_p^2 \left( x_{PK} - B_p - B_l \right) \right) \\
+ \frac{1}{2} \left( x_{PK}^2 \left( \min(B_h,x_D) - (x_{PK} - B_p) \right) \right) - \frac{1}{6} \left( \min(B_h,x_D)^3 - (x_{PK} - B_p)^3 \right)
\]
8 Appendix B: Steady state

Variables in this section are the stationary equivalents to the balanced growth path variables of the body of the paper.

8.1 Introduction

I use an algorithm that transforms the general equilibrium steady state problem into a sequence of partial equilibrium problems. For a vector of aggregate prices $M$, I solve the functional fixed point program that is the problem of the firm. Firm policies imply a distribution $\theta(p,k)$ from which I am able to evaluate market clearing conditions. I search over $M$ to find a value satisfying both the implied market clearing conditions $g(M) = 0$.

8.2 Market clearing conditions

I solve for $M \equiv \begin{bmatrix} Y & \Lambda & W \end{bmatrix}$ to satisfy $g(M) = 0$ for

$$g(M) = \begin{bmatrix} N - \int_{\mathbb{R}_+ \times \mathbb{R}_+} (n(p) + \Xi(p,k)) \theta(dp,dk) \\ Y - \int_{\mathbb{R}_+ \times \mathbb{R}_+} i(k) \alpha^{PK}(p,k) \theta(dp,dk) - \Lambda^{-1} \\ Y - \int_{\mathbb{R}_+ \times \mathbb{R}_+} py(p) \theta(dp,dk) \end{bmatrix}$$

8.3 Price reset targets

Given a capital stock, a new price $\tilde{p}$ is independent of its cost of implementation, and satisfies

$$\frac{\partial V(\tilde{p},k,M)}{\partial \tilde{p}} = 0$$

for

$$\frac{\partial V(p,k,M)}{\partial p} = \frac{\partial Z(p,k,M)}{\partial p}$$

$$+ S \left[ \Pi^{-1} (1 - \alpha^P(p',k',M') - \alpha^{PK}(p',k',M')) \frac{\partial V(p',k',M')}{\partial p'} \right]$$

where conditions $\delta > 0$ and $\pi \neq 0$ guarantee this recursion is finite.
8.4 Capital reset targets

Capital $\dot{k}$, chosen in conjunction with price, equates marginal value to marginal cost

$$Q = \frac{\partial V^{PK}(M)}{\partial k}$$

where

$$\frac{\partial V(p, k, M)}{\partial k} = \frac{\partial z(p, k, M)}{\partial k} + (1 - \delta) \left( (1 - \alpha^P(\cdot) - \alpha^{PK}(\cdot)) \frac{\partial V^{NA}(p', k', M')}{\partial k'} + \alpha^P(\cdot) \frac{\partial V^P(k', M)}{\partial k'} + \alpha^{PK}(\cdot) Q' - \psi \theta \right)$$

and $\delta > 0$.

8.5 Stationary distribution

The stationary distribution of firms across capital and price vintages is determined by

$$\theta(p', k') = (1 - \alpha^P(p, k) - \alpha^{PK}(p, k)) \theta(p, k)$$

$$1 = \int_{\mathbb{R}_+ \times \mathbb{R}_+} \theta(dp, dk)$$

$$\theta(\bar{p}(k'), k') = \int_{\mathbb{R}_+} \alpha^P(p, k, M) \theta(dp, k)$$

which is linear in $\theta$ over the known $(p, k)$ grid implied by

$$p' = \Pi^{-1}p$$

$$k' = (1 - \delta) k$$

and the targets determined previously.

8.6 Adjustment policies

See Appendix A.
9 Appendix C: Model equations

9.1 Growth rates

Variables in this section are not identical to those in the body of the paper because they are detrended. Technological progress in the steady state grows at rate $\Theta_A$ where the production function is $Y = R_y(n) j dj = A n (j)^\gamma k (j)^\gamma dj$ so that both $Y$ and $y (j)$ grow at rate $\Theta_Y$. The growth rate of output must satisfy $\Theta_Y = \Theta_A (\Theta_K)^\gamma$. From the resource constraints $C + I = Y$ and $I = \int [k_t (j) - (1 - \delta) k_{t-1} (j)] dj$, it must be the case that $C$, $I$, $Y$, $K$ and $k (j)$ must grow at the same rate $\Theta_Y$. It must also therefore be the case that $\Theta_Y = (\Theta_A)^{\frac{1}{\gamma}}$. From the first order household condition $C^{-1} = \Lambda$ we know $\Theta_\Lambda = (\Theta_C)^{-1}$.

9.2 Household

The representative consumer maximizes the expected present discounted value of lifetime utility, where preferences are separable in time, consumption and disutility of labor.

$$\sup_{\{C_t, N_t, B_{t+1}, I_t\}_{t=0}^\infty} E_t \left\{ \sum_{t=0}^\infty \beta^t [\ln (C_t) - \chi N_t] \right\}$$  (11)

subject to

$$C_t + I_t + B_{t+1} \leq \Pi_t^{-1} (1 + R_{t-1}) B_t + W_t N_t + Z_t + Q_t I_{t-1}$$  (12)

where

$$C_t = \left[ \sum_{h=0}^{H-1} \min(h, J-1) \sum_{j=0}^{\theta_{j+1, h+1, t+1} (c_{jht})^{(\frac{\theta_{jht}}{\epsilon})}} \right]^{\frac{\epsilon}{\epsilon-1}}$$  (13)

$$Z_t = \sum_{h=0}^{H-1} \min(h, J-1) \sum_{j=0}^{\theta_{j+1, h+1, t+1} (Y_t x_j h t P_j h t - W_t n_{jht})}$$  (14)

$$N_t = \sum_{h=0}^{H-1} \min(h, J-1) \sum_{j=0}^{\theta_{j+1, h+1, t+1} n_{jht}} + \sum_{h=1}^{H} \min(h, J) \sum_{j=1}^{\theta_{jht} n_{jht}}$$  (15)

and $B_t$ represents real consumer bond holdings, $R_t$ is the nominal interest rate and $Z_t$ contains profits from the firms which the household owns. The household first-order neces-
sary conditions are included in what follows in addition to other aggregate definitions and constraints.

\[
\sum_{t=0}^{\infty} \beta^t [\ln (C_t) - \chi N_t]
\]

\[
+ \sum_{t=0}^{\infty} \beta^t (\Theta T)^t \Lambda_t \left[ \Pi_{t-1}^{-1} (1 + R_{t-1}) B_t + W_t N_t + Z_t + Q_t I_{t-1} - C_t - I_t - B_{t+1} \right]
\]

\[
1 = C_t \Lambda_t
\]

\[
\chi N_t^\rho = W_t
\]

\[
\Lambda_t = \beta \Theta A E_t \Pi_{t+1}^{-1} (1 + R_t) \Lambda_{t+1}
\]

\[
\Lambda_t = Q_{t+1} \Theta A \beta \Lambda_{t+1}
\]

9.3 Block 1: Aggregate variables and restrictions

1.A: Marginal utility of consumption

\[
1 = \Lambda_t C_t
\]

1.B: Labor supply

\[
W_t \Lambda_t = \chi
\]

1.C: Consumption Euler equation

\[
\Lambda_t = \beta E_t \Pi_{t+1}^{-1} (1 + R_t) \Theta A \Lambda_{t+1}
\]

1.D: Aggregate investment

\[
I_t^L = \sum_{h=1}^{H} (k_{ht} - k_{ht}) \sum_{j=1}^{\min(h,J)} \theta_j \eta_j \alpha_{jht}^{PK} + \psi \sum_{h=1}^{H} k_{ht} \sum_{j=1}^{\min(h,J)} \theta_j
\]

1.E: Aggregate resource constraint
\[ C_t + I_t = Y_t \]

1.F: Cash in advance constraint

\[ m_t = P_t Y_t \]

1.G: Exogenous monetary rule

\[ \Delta \ln m_{t+1} = \rho_2 \Delta \ln m_t + e_{m,t+1} \]

1.H: Net inflation

\[ \Pi_t = \frac{P_t}{P_{t-1}} \]

1.I: Lagged price level

\[ \Pi P_{t+1}^L = P_t \]

1.J: Total labour

\[ N_t = \sum_{h=1}^H \sum_{j=1}^{\min(h,J)} \theta_{jht} e_{jht} + \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} n_{jht} \]

1.K: Price level

\[ 1 = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} P_{jht} x_{jht} \]

1.L: The aggregator constraint

This equation is viewed as implicitly defining \( \zeta_t \).

\[ 1 = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} \left[ 1 + \frac{\varepsilon}{1 + \kappa} - \frac{\varepsilon}{1 + \kappa} \left( 1 + \frac{\kappa}{\varepsilon} - \frac{\kappa}{\varepsilon} x_{jht} \right)^{\frac{1+\kappa}{\kappa}} \right] \]

1.M: Investment arbitrage constraint

Investors purchase capital at price 1 and sell it for \( Q_{t+1} \). As these investors are perfectly competitive and have access to the bond with gross return \( (1 + R_t) \Pi_{t+1}^{-1} \) it must be the
case that the following holds. This equation is viewed as defining $Q_{t+1}$.

$$
\Lambda_t = \beta E_t Q_{t+1} \Theta_A \Lambda_{t+1}
$$

1.N: Lagged investment

$$
I_{t+1}^L = I_t
$$

1.O: SDF

$$
S_{t,t+1} = \beta \Theta_Y \Theta_A E_t \frac{\Lambda_{t+1}}{\Lambda_t}
$$

9.4 Block 2: The evolution of firm-level heterogeneity, and firm technology and demand

2.A: Distribution evolution

The mass of firms in vintage $(j+1, h+1)$ at time $t+1$ is the mass of firms in vintage $(j, h)$ at time $t$ who do not choose to adjust capital or price.

$$
\theta_{j+1,h+1,t+1} = (1 - \alpha_{jht}^P - \alpha_{jht}^{PK}) \theta_{jht}
$$

for $h = 1, \ldots, H - 1$; $j = 1, \ldots, \min(h, J - 1)$

The mass of firms choosing to adjust price, but not capital, defines all but one of the remaining $\{\theta_{jht}\}$ variables.

$$
\theta_{1,h+1,t+1} = \sum_{j=1}^{\min(h,J)} \alpha_{jht}^P \theta_{jht}
$$

for $h = 1, \ldots, H - 1$

2.C: Restriction on firm mass

The total mass of the firms must sum to a constant which, in this case, we pick to be unity without loss of generality.
\[ 1 = \sum_{h=0}^{H-1} \min(h+1,J-1) \sum_{j=0}^{\theta_{j+1,h+1,t+1}} \]

2.D: Firm-specific capital evolution

Firms which do not adjust their capital stocks see their capital depreciate at rate \( \delta \) net of maintenance.

\[ k_{h+1,t+1} = \frac{(1 - \delta)}{\Theta K} k_{ht} \]
for \( h = 0, \ldots, H - 1 \)

2.E: Lagged prices

\[ p_{j_{ht},t+1} = p_{jht} \]
for \( h = 0, \ldots, H - 2; \ j = 0, \ldots, \min(h, J - 2) \)

2.F: Current relative prices

The marginal value recursions imply the \( H \) prices \( p_{0ht} \) for \( h = 0, \ldots, H - 1 \). Non-reset prices are restricted by this equation.

\[ p_{jht} = \left( \frac{1}{1 + \pi_t} \right) p_{j-1,h-1,t} \]
for \( h = 1, \ldots, H - 1; \ j = 1, \ldots, \min(h, J - 1) \)

9.5 Block 3: Firm output and demand

3A: Production

Production functions are Cobb-Douglas. This equation implicitly defines labour demand \( n_{jht} \) since capital is fixed and firms are required to meet demand given their current relative price.
\[ Y_t x_{jht} = A_t n_{jht}^{v} k_{ht}^{j} \]

for \( h = 0, \ldots, H - 1; \ j = 0, \ldots, \min (h, J - 1) \)

### 3B: Demand

Demand is as in Dotsey and King (2005) and Dotsey, King and Wolman (2007). The parameter \( \varepsilon \) is the local demand elasticity when \( x = 1, \ p = 1, \ \zeta = 1 \) and \( \kappa \) is a shape parameter. If \( \kappa = -\varepsilon \) this reduces to the standard Dixit-Stiglitz aggregator \( x_{jht} = \left( \frac{p_{jht}}{\zeta_{t}} \right)^{-\varepsilon} \).

\[ x_{jht} = \left( 1 + \frac{\varepsilon}{\kappa} \right) - \varepsilon \left( \frac{p_{jht}}{\zeta_{t}} \right)^{\kappa} \]

for \( h = 0, \ldots, H - 1; \ j = 0, \ldots, \min (h, J - 1) \)

#### 9.5.1 Block 4: Value function recursions

\[ v_{jht} = Y_t x_{jht} p_{jht} - W_t n_{jht} \]

\[ + E_t S_{t,t+1} \left[ \left( 1 - \alpha_{j+1,h+1,t+1}^{P} - \alpha_{j+1,h+1,t+1}^{PK} \right) v_{j+1,h+1,t+1} + \alpha_{j+1,h+1,t+1}^{PK} (v_{0,h+1,t+1} - Q_{t+1} (k_{0,t+1} - k_{H,t+1})) + \alpha_{j+1,h+1,t+1}^{P} v_{0,h+1,t+1} - W_{t+1} \Xi_{j+1,h+1,t+1} - \psi_{0} k_{t+1} \right] \]

for \( h = 0, \ldots, H - 2; \ j = 0, \ldots, \min (h, J - 2) \)

\[ v_{J-1,h,t} = Y_t x_{J-1,h,t} p_{J-1,h,t} - W_t n_{J-1,h,t} \]

\[ + E_t S_{t,t+1} \left[ \alpha_{J,h+1,t+1}^{PK} (v_{0,h+1,t+1} - Q_{t+1} (k_{0,t+1} - k_{H,t+1})) + \alpha_{J,h+1,t+1}^{P} v_{0,h+1,t+1} - W_{t+1} \Xi_{J,h+1,t+1} - \psi_{0} k_{t+1} \right] \]

for \( h = J - 1, \ldots, H - 2 \)
\[
v_{j,H-1,t} = (Y_j x_{j,H-1,t} p_j, H-1, t - W_t n_{j,H-1,t}) \\
+ E_t S_{t,t+1} \left[ v_{00,t+1} - Q_{t+1} (k_{0,t+1} - k_{H,t+1}) \\
- W_{t+1} z_{j+1,H,t+1} - \psi g k_{t+1} \right] \\
for j = 0, \ldots \min(h, J - 1)
\]

### 9.5.2 Block 5: Marginal value recursions

Marginal values with respect to capital are given by

\[
\frac{\partial v_{jht}}{\partial k_{ht}} = W_t \left( \frac{\gamma}{\nu} \right) n_{jht} \frac{k_{ht}}{k_{ht}} + \left( 1 - \frac{\delta}{\Theta K} \right) E_t S_{t,t+1} \left[ \left( 1 - \alpha_{j+1,h+1,t+1}^P - \alpha_{j+1,h+1,t+1}^P K \right) \frac{\partial v_{j+1,h+1,t+1}}{\partial k_{j+1,h+1,t+1}} \\
+ \alpha_{j+1,h+1,t+1}^P Q_{t+1} + \alpha_{j+1,h+1,t+1}^P \frac{\partial v_{j+1,h+1,t+1}}{\partial k_{j+1,h+1,t+1}} - \psi Q \right] \\
for h = 1, \ldots H - 2; \ j = 0, \ldots \min(h, J - 1)
\]

\[
\frac{\partial v_{J-1,h,t}}{\partial k_{h,t}} = W_t \left( \frac{\gamma}{\nu} \right) n_{J-1,h,t} \frac{k_{h,t}}{k_{h,t}} + \left( 1 - \frac{\delta}{\Theta K} \right) E_t S_{t,t+1} \left[ \alpha_{J,h+1,t+1}^P Q_{t+1} + \alpha_{J,h+1,t+1}^P \frac{\partial v_{J,h+1,t+1}}{\partial k_{J,h+1,t+1}} - \psi Q \right] \\
for h = J - 1, \ldots H - 2
\]

\[
\frac{\partial v_{j,H-1,t}}{\partial k_{H-1,t}} = W_t \left( \frac{\gamma}{\nu} \right) n_{j,H-1,t} \frac{k_{H-1,t}}{k_{H-1,t}} + \left( 1 - \frac{\delta}{\Theta K} \right) E_t S_{t,t+1} \left[ Q_{t+1} - \psi Q \right] \\
for j = 0, \ldots J - 1
\]

\[
Q_t = W_t \left( \frac{\gamma}{\nu} \right) n_{00t} \frac{k_{0t}}{k_{0t}} + \left( 1 - \frac{\delta}{\Theta K} \right) E_t S_{t,t+1} \left[ \left( 1 - \alpha_{11,t+1}^P - \alpha_{11,t+1}^P K \right) \frac{\partial v_{11,t+1}}{\partial k_{11,t+1}} \\
+ \alpha_{11,t+1}^P Q_{t+1} + \alpha_{11,t+1}^P \frac{\partial v_{11,t+1}}{\partial k_{11,t+1}} - \psi Q \right] \\
\]

Marginal values with respect to price are given by
\[
\frac{\partial v_{jht}}{\partial p_{jht}} = Y_t x_{jht} - \left[ Y_t p_{jht} - W_t \left( \frac{1}{\nu} \left( \frac{n_{jht}}{x_{jht}} \right) \right) \right] \left( \varepsilon (\zeta_t)^{-\kappa} (p_{jht})^{\kappa-1} \right) \\
+ E_t \Pi_{t+1}^{-1} S_{t,t+1} \left[ (1 - \alpha_{j+1,h+1,t+1} - \alpha_{j+1,h+1,t+1}^{PK}) \frac{\partial v_{j+1,h+1,t+1}}{\partial p_{j+1,h+1,t+1}} \right]
\]

for \( h = 0, \ldots, H - 2; \ j = 0, \ldots, \min(h, J - 2) \)

\[
0 = Y_t x_{0ht} - \left[ Y_t p_{0ht} - W_t \left( \frac{1}{\nu} \left( \frac{n_{0ht}}{x_{0ht}} \right) \right) \right] \left( \varepsilon (\zeta_t)^{-\kappa} (p_{0ht})^{\kappa-1} \right) \\
+ E_t \Pi_{t+1}^{-1} S_{t,t+1} \left[ (1 - \alpha_{11,t+1}^{PK} - \alpha_{11,t+1}^{P}) \frac{\partial v_{1,h+1,t+1}}{\partial p_{1,h+1,t+1}} \right]
\]

for \( h = 0, \ldots, H - 1 \)

**5G: Price marginal values (terminal H and terminal J vintages)**

Marginal values with respect to price for terminal \( H \) and terminal \( J \) firms are

\[
\frac{\partial v_{jht}}{\partial p_{jht}} = Y_t x_{jht} - \left[ Y_t p_{jht} - W_t \left( \frac{1}{\nu} \left( \frac{n_{jht}}{x_{jht}} \right) \right) \right] \left( \varepsilon (\zeta_t)^{-\kappa} (p_{jht})^{\kappa-1} \right)
\]

for \( j = J - 1; \ h = J, \ldots, H - 2; \)

and for \( h = H - 1; \ j = 1, \ldots, J - 1. \)

### 9.6 Block 6: Adjustment policies and costs

See Appendix A.