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Main results

- Does transparency affect traders’ behaviors? Yes
- Does transparency affect market outcomes? No
- Interesting results with regulatory implications on dark trading
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- Does transparency affect traders’ behaviors? Yes
- Does transparency affect market outcomes? No
- Interesting results with regulatory implications on dark trading
- These results are also natural through the lens of a model.
  - A simple trading model in the form of a batch double auction, e.g. an open auction, or a close auction. Adapted from Du and Zhu (2012) “Ex Post Equilibria…”
  - Model predicts the sign of comp stat, but not statistical significance.
  - “Revenue equivalence theorem” in auction theory typically applies to independent values.
A trading model

- $n$ symmetric traders. Trader $i$:
  - receives a private signal $s_i$ and values the asset at
    \[ v_i = \alpha s_i + \beta \sum_{j \neq i} s_j, \quad \text{where } \alpha + (n - 1)\beta = 1. \]
  - has the ex post utility
    \[ U(q_i, p^*; v_i) = (v_i - p^*)q_i - \frac{1}{2} \lambda q_i^2, \quad \text{where } q_i = \text{quantity and } p^* = \text{price}. \]
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- The market is organized as a uniform-price double auction, with observed liquidity supply \( S \).
  - Each trader \( i \) submits a demand schedule \( x_i(p) \).
    Trader \( i \) is willing to buy \( x_i(p) \) units at the price of \( p \).
  - The market-clearing price \( p^* \) satisfies
    \[ \sum_{i=1}^{n} x_i(p^*) = S. \]
An ex post equilibrium

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Proposition

Suppose that $n\alpha > 2$. There exists an ex post equilibrium in which trader $i$ submits the demand schedule

$$x_i(p) = \frac{n\alpha - 2}{\lambda(n-1)} (s_i - p) + \frac{1 - \alpha}{n-1} S,$$

and the equilibrium price is

$$p^* = \frac{1}{n} \sum_{i=1}^{n} s_i - \frac{(n\alpha - 1)}{n(n\alpha - 2)} \lambda S.$$ 

Ex post optimality is much stronger than Bayesian optimality.
An ex post equilibrium and interpretation

- In the ex post equilibrium, the distribution of signals \((s_1, s_2, ..., s_n)\) is irrelevant. If transparency only affects (beliefs about) distribution of signals, then the (ex post) equilibrium outcome is independent of transparency regimes.
An ex post equilibrium and interpretation

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- But transparency may change \(\lambda\).
  e.g. \(\lambda = a \text{Var}(v_i \mid s_i, \text{display}) + b\) funding costs.

  - How much information is displayed: Visible ≥ Iceberg ≥ Hidden
  - Uncertainty about \(\{v_i\}\): Hidden ≥ Iceberg ≥ Visible
  - \(\lambda\) is weakly increasing in opacity: \(\lambda_{hid} \geq \lambda_{ice} \geq \lambda_{lit}\).
  - If \(\lambda_{hid} \approx \lambda_{ice} \approx \lambda_{lit}\), then there is no observable difference across transparency regimes.
3.4 Market quality: Information Efficiency

**Model:** The equilibrium price

\[
p^* = \frac{1}{n} \sum_{i=1}^{n} s_i - \frac{(n\alpha - 1)}{n(n\alpha - 2)} \lambda S.
\]

If \( \mathbb{E}(S) = 0 \), then \( \mathbb{E}(p^*) = \sum_i s_i / n \) in all three regimes.

**Experiment:** Info efficiency is similar across transparency regimes.

**Consistent**
3.5 Trading profits

**Model:** Liquidity orders have a price impact proportional to $\lambda S$. Their loss is proportional to $\lambda S^2$. More opaque, more uninformed loss.
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**Experiment:** Profits(hidden) $>$ profits(visible) for high-value info, otherwise not significantly different.

Consistent, up to statistical power.
3.3 Market quality: Liquidity

**Model:** Recall $x_i(p; s_i) = \frac{n^{\alpha-2}}{\lambda(n-1)} (s_i - p) + \frac{1-\alpha}{n-1} S$. Aggregate depth at price $p$ is

$$\left| \frac{\partial \sum_i x_i(p)}{\partial p} \right| = \frac{n(n\alpha - 2)}{(n-1)} \cdot \lambda^{-1}.$$ 

Price impact, $\partial p/\partial x_i$, is proportional to $\lambda$.

Effective bid-ask spread is proportional to price impact (and to $\lambda$).

More opacity, higher $\lambda$, higher effective spread, and higher price impact.
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**Experiment:** Bid-ask spread and price impact do not vary with opacity. Consistent, up to statistical power.

Depths are larger in opaque markets. Inconsistent, but how about “serious”, closer-to-mid depth in the data? (If I can manage orders as price moves, I don’t have to post many orders away from the mid.)
3.1 Who use nondisplayed orders?
3.2 Who supplies or demands liquidity?

The model has limited predictions for these questions.

**Experiment:**
- Informed traders respond more to changes in transparency.
- Total number of limit orders submitted does not vary significantly with transparency.
- Submission rates, fill rates, and taking rates vary (mildly) with transparency.
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**Questions:**
- What determines traders’ choices between lit/dark orders? e.g. Why would informed submit any visible order in the hidden regime?
- Plot the schedule of limit orders by informed and uninformed traders?
Summary

- Conclusion: Market outcomes vary little with transparency regimes.
- Provocative (and natural) results, fresh insights on dark trading.
- A model that “explains” the results?
- Would richer order types “offset” market structure changes?