MARKET ORDER FLOWS, LIMIT ORDER FLOWS AND EXCHANGE RATE DYNAMICS

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Abstract

We extend the 'portfolio shifts' model of Evans and Lyons (2002) to allow FX dealers to use both limit and market orders in order to exploit private information in inter-dealer trading. Both market and limit orders contain information on customer-dealer flows. Consequently, equilibrium exchange rates depend on both types of order flows. Limit orders have lower price impact than market orders because they do not have to be absorbed by ultimate customers. We test these predictions using 2 years data for GBP/USD, EUR/USD and EUR/GBP from an order-driven inter-dealer FX trading venue. Our empirical analysis gives strong support to the predictions of the model at macroeconomically relevant sampling frequencies. Empirically, the price impact of market order flows is substantially increased by the inclusion of limit orders in the regression indicating that the omission of a relevant explanatory variable seriously understates the importance of the Evans-Lyons result.

1. Introduction

In recent years, the microstructure approach to exchange rate determination has become an important part of international finance. This approach is, at its core, based on a single, robust empirical finding: order flow in the inter-dealer segment of the FX market has strong explanatory power for exchange rate returns at both high (e.g. minutely) and low (macroeconomically relevant) sampling frequencies (Lyons 1995, Evans and Lyons 2002, Payne 2003, Bjönnes and Rime 2005, Killeen, Lyons and Moore 2006).[1] Order flow in an interval is defined as net signed trading activity i.e. the number of aggressive inter-dealer buy trades less the number of aggressive inter-dealer sells.

Subsequent to this empirical regularity being established, several theoretical frameworks have been proposed which can be used to explain the positive correlation between order flows and returns. Perraudin and Vitale (1996) argue that FX dealers obtain private information regarding macroeconomic fundamentals from their (opaque) trading with customers, and this is then exploited in the inter-dealer market. As in standard market microstructure models, the existence of private information leads to a positive relationship between inter-dealer flows and returns. An alternative explanation is provided by Breedon and Vitale (2010). They argue that the relationship between returns and flows is driven by the temporary market impact of flows on liquidity and that flows contain no fundamental macroeconomic information. Their view is challenged empirically by Moore and Payne (2011). Bacchetta and van Wincoop (2006) consider a model in which agents trading FX have differing information about FX fundamentals and also heterogeneous exchange rate exposures through their non-financial income. This model provides a close relationship between order flow and exchange rate changes at all sampling frequencies. There is, however, a disconnect between exchange rates and fundamentals at high frequencies, while at longer horizons the relationship between exchange rates and macroeconomic fundamentals re-asserts itself.

Perhaps the most widely used model for explaining the flow/return relationship is the 'portfolio shifts' model of Evans and Lyons (2002). Customer orders are useful for forecasting exchange rates, as in a subsequent trading round, dealers pass the aggregate inventory derived

[1] There is some evidence, however, that at extremely low sampling frequencies, e.g. monthly, the relationship between order flow and returns weakens. See Berger, Chaboud, Chernenko, Howorka, and Wright (2008). However, this is contested by Chinn and Moore (2011).

from customer trading back to the same group of clients. As customers are risk averse, they must be induced to hold this inventory by a change in the exchange rate and as each individual dealer's customer trade is a noisy signal of the aggregate inventory passed to customers, dealers can speculate on this information in the inter-dealer market. Again, a positive relationship between inter-dealer order flows and exchange rate changes emerges. For a description of the original portfolio shifts model, including some extensions, see the detailed treatment in Chapter 6 of Evans (2011).

In common with most of the theoretical market microstructure literature, the portfolio shifts model forces individuals endowed with private information to exploit that information through the use of market orders i.e. aggressive trades. However, in recent times several contributions to microstructure theory have emerged in which informed traders optimally choose to use limit orders instead of or as well as market orders in their attempts to profit from private information. See, for examples, Kaniel and Liu (2006), Bloomfield, Saar and O'Hara (2005) and Rosu (2009). A limit order is a price contingent order, specifying a maximum quantity to trade, but which does not execute if the price condition is not met. In order driven markets, such as the dominant venues for inter-dealer trade in FX, limit orders represent the supply of liquidity and can be seen as earning the bid-ask spread if they execute whereas market orders consume liquidity and pay the bid-ask spread[2]. Thus the choice between market and limit orders comes down to a choice between costly market orders with guaranteed execution versus cheaper limit orders that are associated with execution risk. Early theoretical contributions which model limit order markets include Foucault (1999), Parlour (1998) and Foucault, Kadan and Kandel (2005), although, unlike the references above, these papers do not allow informational differences between financial market participants.[3]

On an empirical level, several recent papers have shown that order book imbalances (the difference in the quantity of outstanding limit buys less the quantity of outstanding limit sells) and flows of limit orders (the aggregate quantity of newly submitted limit buy orders less the aggregate quantity of new limit sells) have predictive power for security returns (Harris and Panchapagesan, 2005, Cao, Hansch and Wang, 2009, Kozhan and Salmon, 2012, Latza and

[2] See Bank for International Settlements (2010)

[3] See Parlour and Seppi (2008) for a review of the literature on limit order markets.

Payne, 2011). Thus there is empirical support for the notion that some information is transmitted to prices through limit orders.

This paper embeds limit order trading by dealers into the standard portfolio shifts model. In reality, the inter-dealer segment of FX is largely order driven and our model allows dealers to choose both limit and market orders. In order to do this, we split the population of dealers into two types. The first is the standard Evans-Lyons dealer who receives customer order flow and attempts to infer the future value of the exchange rate from that customer flow. These dealers trade amongst themselves using market orders and also post limit orders at which others can trade (say on EBS or Reuters' Dealing systems). From here on we will just call such agents 'dealers'. The second class of dealer (and only they) can execute against these limit orders. We think of these traders as 'hedgers' and will refer to them in this way from now on. Hedgers are dealers who do not have a significant customer base and thus do not trade for informational reasons, but who are trading a currency pair to mitigate a risk that they are exposed to. For example, a dealer who is running a triangular arbitrage strategy on EUR-JPY-USD may take a position in EUR/JPY as he believes it is mis-priced and then lock in a profit on that position through offsetting trades in EUR/USD and USD/JPY. Note that none of his trades are driven by underlying customer flows. Moreover, our hedger is likely to demand liquidity as, in our example, once he has traded the cross-rate, he needs to execute quickly in the liquid rates in order to fix his arbitrage profit (see Kozhan and Tham 2012).

Moore and Payne (2011) present empirical work that validates the existence of our two types of dealer. They show that informed traders in liquid exchange rates (e.g. EUR/USD or USD/JPY) tend to specialise in a particular rate and also tend to work in large institutions, i.e. those with larger customer bases. They also show that dealers in cross-rates (e.g. EUR/JPY) take positions in liquid pairs (e.g. EUR/USD and USD/JPY) in order to eliminate currency risk and that such dealers possess information relevant to the evolution of cross rates, but not liquid rates.[4]

Aside from this modification, the setup of the portfolio shifts model is unchanged. Our model, with dealers optimising over both market and limit orders in forming their inter-dealer

[4] Bjonnes, Osler and Rime (2012) also provide empirical evidence to support the view that larger institutions have better information than do smaller banks and also that larger banks limit orders may be informative.

trading strategy and with limit orders facing execution risk is related to the theoretical and empirical work in Engle and Ferstenberg (2007).

We solve for the Perfect Bayesian Nash Equilibrium of the modified model. We show that, in general, it is optimal for a dealer with private information from customers to use both limit and market orders. Thus, limit orders contain private information and that is they move exchange rates, just like market orders in the standard Evans-Lyons model. The model has the following implications: (1) the price impacts of market order flow and limit order flow are both positive and (2) the effect of market order flows on exchange rate changes is greater than that of limit order flows. These results require that our FX dealers are significantly more risk averse than end-users in the FX market. This is consistent with the fact that most FX dealers like to finish the trading day with a small or zero position[5].

We evaluate these theoretical predictions using a two-year span of data on GBP/USD, EUR/USD and EUR/GBP from a major inter-dealer trading platform (Reuters Dealing 3000). As in Evans-Lyons, we use a daily sampling frequency and in all cases the main results of our model are verified.[6] First of all, limit order flows add significant explanatory power to regressions of returns on market order flows. The coefficients on limit order flows tend to be positive and in all cases are smaller than those on corresponding market flows. Secondly, cancelled limit orders also have very significant explanatory power and have a price impact which is economically though not always statistically insignificant from the price impact of new limit order submissions. Thus we conclude that the ability of FX dealers to exploit information via limit orders has a strong influence on the manner in which exchange rates are determined. Both our model and our empirical work suggest that dealers' limit order submissions are used to speculate on private information and are thus informative about exchange rate movements. From a purely econometric perspective, regressions which explain exchange rate changes with market order flows only are subject to misspecification if limit order flows are neglected.

The rest of the paper is structured as follows. In Section 2 we set out our model and describe its equilibrium. Section 3 introduces our data. Section 4 presents our empirical work and in

[5] The standard portfolio shifts model assumes that dealers and their customers share the same risk aversion parameter.

[6] Our results also hold when we use an hourly sampling frequency.

Section 5 we make some concluding remarks. An Appendix contains proofs of the propositions laid out in Section 2.

2. Model

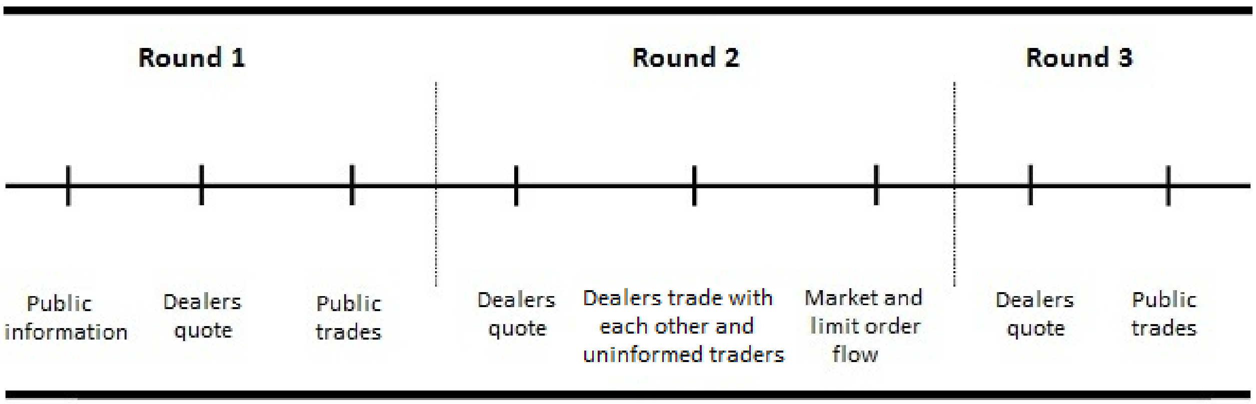
We extend the portfolio shifts models of Evans and Lyons (1999, 2002) and Killeen, Lyons and Moore (2006) by allowing for trading using limit orders as well as market orders in the inter-dealer market. Though limit orders are not included explicitly in the portfolio shifts model, our specification of trading within each day is similar to earlier specifications so our exposition below is fullest where the models differ.

Consider an infinitely lived, pure-exchange economy with two assets, one riskless and one with stochastic payoffs (foreign exchange). At the beginning of each day t, foreign exchange earns a payoff R_t, which is composed of a series of increments, so that $R_t = \sum_{\tau=1}^{t-1} \Delta R_\tau$. The increment ΔR_t is observed publicly on day t before trading. These realized increments represent innovations over time in public macroeconomic information and are i.i.d. normal variables with zero mean and σ_R^2 variance.

The foreign-exchange market is organized as a dealership market with N dealers, indexed by i, a group of hedgers, and a continuum of non-dealer customers (the public). The hedgers are participants in the inter-dealer market who do not have a significant customer base and trade for non-informational reasons. The mass of both customers and hedgers on [0,1] is large (in a convergence sense) relative to the N dealers. This assumption will drive both the model's overnight risk-sharing features and the treatment of limit orders. Dealers are quadratic utility (mean-variance) maximizers with parameter θ_d.

Within each day there are three trading rounds. In the first round, dealers trade with the public. In the second round, dealers trade among themselves (to share the resulting inventory risk) using market orders. They are also allowed to submit limit orders that can be hit or taken by hedgers. There is an exogenous probability of limit order execution that is equal to q. Unexecuted limit orders expire at the end of the second trading round. In the third round, dealers trade again with the public (to share inventory risk throughout the economy). Figure 1 provides an overview of the model's timing.

Figure 1: Intra-Day Sequence of Trading



Each day begins with payment and public observation of the payoff R_t. Then each dealer quotes a scalar price to his customers at which he agrees to buy and sell any amount (quoting is simultaneous). We denote this round 1 price of dealer i on day t as $P_{1,t}$. Each dealer then receives a customer-order realization $c^i_{1,t}$ that is executed at his quoted price $P_{1,t}$. Let $c^i_{1,t} < 0$ denote net customer selling (dealer i buying). The individual $c^i_{1,t}$ are distributed normally with mean zero and variance σ_c^2. They are uncorrelated across dealers and uncorrelated with the payoff R_t at all leads and lags. Define the aggregate public demand in round 1 as the sum of customer demands over the N dealers $c_{1,t} = \sum_{i=1}^{N} c^i_{1,t}$. In aggregate, these orders represent an exogenous portfolio shift of the non-dealer public. Their realizations are not publicly observable.[7]

[7] In Evans (2011), customer trades are not exogenous. In that version of the portfolio shifts model, each customer receives FX income which is private information to each investor and generates an optimal hedging motive for customer orders in round 1 trading. Evans acknowledges that, at a *daily* frequency, the dynamics of the spot rate and its relation to order flow are unaffected by this modification.

In round 2, dealers quote a scalar price $P_{2,t}$ to other dealers at which they agree to buy and sell any amount. These quotes are posted simultaneously so that they cannot be conditioned on one another. Moreover, they are observable and available to all dealers. Each dealer then trades on other dealers' quotes. Trades are also simultaneous so that they cannot be conditioned on one another. Orders at a given price are split evenly across dealers quoting that price. Let M_t^i denote the (net) interdealer market order trade initiated by dealer i in round 2 (we denote M_t^i as negative for dealer i net selling).[8]

At the same time, at the beginning of round 2, dealers submit limit buy (sell) orders at price $P_{2,t} - s$ ($P_{2,t} + s$, respectively), where $2s$ is an exogenously given bid-ask spread. These limit orders will be filled by hedgers with probability q. The hedgers are new to the portfolio shifts model. They can be thought of as a subset of the dealer community who do not trade for informational reasons, as they lack the significant customer base that would endow them with information. Instead, they are trading for risk-reduction purposes, perhaps hedging positions taken on via trades in other currency pairs. As discussed above, a triangular arbitrageur who has noticed a trading opportunity through the mis-pricing of a cross-rate, will trade the relevant liquid rates in order hedge the risk of the cross position and lock in an arbitrage profit. Note that we restrict hegders to trade aggressively. They do not wish to run the risk of non-execution that passive trading would bring as this would leave them exposed to the risk they are trying to remove. This assumption is easy to justify in our triangular arbitrage example. Unexecuted limit orders are automatically cancelled at the end of the trading round. Let L_t^i denote the (net) interdealer limit order initiated by dealer i in round 2 (L_t^i is negative for dealer i net selling).

How does the dealer allocate his or her trades between limit and market orders? The dealer treats them as two separate assets with different risk/return characteristics and treats the decision as a mean variance optimisation problem. Limit orders have higher return (because the dealer gains the spread) than market orders but a higher variance (because execution is uncertain). At

[8] Since dealers cannot hit or take other dealers' limit orders, market orders can be interpreted as marketable limit orders.

the close of round 2, all dealers observe the interdealer market order flow M_t from that day defined as $M_t = \sum_i M_t^i$ and limit order flow $L_t = \sum_i L_t^i$.

In round 3 of each day, dealers share overnight risk with the non-dealer public. Unlike round 1, the public's motive for trading in round 3 is non-stochastic and purely speculative. Initially, each dealer quotes a scalar price $P_{3,t}$ at which he agrees to buy and sell any amount (effected simultaneously). These quotes are observable and available to the public. We assume that aggregate public demand for the risky asset in round 3, denoted $c_{3,t}$, is less than infinitely elastic. With the earlier assumptions, this allows us to write public demand as a linear function of expected return:

$$c_{3,t} = \frac{1}{\theta_c \sigma^2}\left[E(P_{3,t+1} - P_{3,t} \mid \Omega_t^3)\right] = \frac{\Delta}{\mu}\left[E(P_{3,t+1} - P_{3,t} \mid \Omega_t^3)\right], \tag{1}$$

where σ^2 is the variance of price changes, θ_c is the public's risk aversion parameter, $\mu = \theta_d \sigma^2$ and $\Delta = \dfrac{\theta_d}{\theta_c}$ is the ratio of dealers' and public's risk aversion parameters. Note also that we have allowed dealers and customers to have different levels of risk aversion.[9] The information in Ω_t^3 is that which is available to the public at the time of trading in round 3 of day t. Importantly, because we have assumed that dealers' collective risk-bearing capacity is small relative to that of the public, equilibrium prices in round 3 will adjust such that all risky positions are held by the public overnight.

Since dealers end the day with no net expected position, market clearing requires that aggregate customer demand absorbs the total dealer inventory, which consists of the negative of customers' orders from the first round $c_{1,t}^i$ plus limit orders executed against hedgers. Therefore, a market clearing condition at period 3 is:

$$\Delta c_{3,t} = \sum_{i=1}^{N}\left(-c_{1,t}^i + qD_{l,t}^i\right) = -c_{1,t} + qL_t \tag{2}$$

where $\Delta c_{3,t}$ is the change in the aggregate customer position in period 3.

[9] This is a departure from other versions of the portfolio shifts model and is essential for our results as will be clear below.

2.1. Equilibrium

The equilibrium relation between price adjustment and interdealer market order flow and dealer - hedger limit order flow is a generalisation of results established for the simultaneous trade model of Lyons (1997). An equilibrium in this model comprises (1) a set of FX orders by customers in round 3; (2) a set of FX price quotes by dealers in rounds 1-3; (3) a set of dealer trading decisions in respect of market orders and limit orders in round 2 and (4) a set of dealer trading decisions in round 3.

Propositions 1 and 2 of Lyons (1997) show that, to prevent arbitrage, in both rounds 1 and 2, all dealers quote a common price. It follows that this price is conditioned on common information only. Aggregate market and limit order flows are not observed until the end of round 2. It is only in round 3 that the price reflects information from order flows. This again applies here and is adapted as follows:

$$\begin{aligned} P_{1,t} &= P_{2,t} = P_{3,t-1} + R_t, \\ P_{3,t} &= P_{2,t} + \beta_m M_t + \beta_l L_t. \end{aligned} \tag{3}$$

where β_m and β_l are parameters and are price impacts of aggregate net market and aggregate net limit orders respectively.

Proposition 1:

(i) Given the quoting strategy described above, the following trading strategy is optimal and corresponds to symmetric linear equilibrium:

$$\begin{aligned} M_t^i &= \alpha_m c_{1,t}^i + \omega_m s, \\ L_t^i &= \alpha_l c_{1,t}^i + \omega_l s. \end{aligned} \tag{4}$$

(ii) The values of the parameters $\alpha_m, \alpha_l, \omega_m$ and ω_l are given by Equation (A.3) in the Appendix.

Proof: By the method of undetermined coefficients. See the Appendix.

As in Evans and Lyons (2002) and Killeen, Lyons and Moore (2006), the equilibrium is of the Bayesian Nash variety. The dealer receives a private customer flow and uses this to generate an optimal speculative demand for both market and limit orders. However the dealer takes account of the price impact of aggregate order flow, β_m and β_l in her optimisation problem. The optimisation problem is discussed in detail in the Appendix but for convenience, Appendix Equation (A.4) is repeated here:

$$\max_{\mathrm{D}_t}\left\{\mathrm{D}_t^{'}\left(\begin{pmatrix}\beta_m\\ q\beta_m\end{pmatrix}c_{1,t}^i+\begin{pmatrix}\beta_m & \beta_l\\ q\beta_m & q\beta_l\end{pmatrix}\mathrm{D}_t-\begin{pmatrix}1\\0\end{pmatrix}s\right)-\frac{1}{2}\theta_d\mathrm{D}_t^{'}\Sigma\mathrm{D}_t\right\}$$

where $\mathrm{D}_{\mathbf{t}}'=\left(D_{m,t},D_{l,t}\right)$ is the vector of speculative demands for market and limit orders respectively. This leads to Proposition 2:

Proposition 2: There is unique global maximum to the dealer's optimisation problem with a unique vector of price impacts (β_m,β_l).

Proof: See the Appendix.

The expressions for β_m and β_l are given in Appendix Equation (A.6) and are determined as the intersection of two equilibrium locus functions. For convenience, these are repeated here. The first one is a limit order equilibrium locus function (LEL) defined in Appendix Equation (A.4):

$$\beta_l=\frac{\beta_m q\Delta+\mu q(\Delta-1)-\sqrt{q\left(\beta_m^2 q\Delta^2+\mu^2 q\left(\Delta+1\right)^2-2\beta_m\mu\Delta\left(q\left(\Delta+3\right)+2\Delta\left(1-q\right)\left(1+z^2\right)\right)\right)}}{2\Delta}$$

where z is defined as the Sharpe ratio. The limit orders equilibrium locus (LEL) is the locus of combinations of liquidity supply by dealers in the form of round 2 limit orders and the exogenous demand for liquidity by hedgers. It is easy to show that this curve is upward sloping in $\left(\beta_m,\beta_l\right)$ space. The intuition for the upward slope is straightforward. As β_l rises, the profitability of limit order submission declines leading to a fall in the supply of liquidity. By contrast, as β_m rises, dealers substitute out of market orders into liquidity provision. So β_m and β_l must move in the same direction ensure that the supply of limit orders remains at its optimal level.

Similarly, MEL is the locus of combinations of β_m and β_l that are consistent with equilibrium between supply (by dealers) and the demand (by customers in round 3) of market orders, for which markets clear in round 3. It is given in Appendix Equation (A.5).

$$\beta_l = \mu q\left(1+\frac{\beta_m}{\beta_m\Delta-\mu}\right)$$

This is downward sloping in (β_m,β_l) space. The intuitive reason why MEL is downward sloping goes to the heart of the model. As β_m rises, the supply of market orders falls because of reduced profitability. As β_l there are two competing effects. A substitution effect increases market orders but this is dominated by the fact that any increase in either price impact makes market orders less profitable because they have to be absorbed by ultimate customers in contrast to limit orders. So β_m and β_l must move in opposite directions to ensure equilibrium in the supply and demand for market orders.

Figure 2: Equilibrium

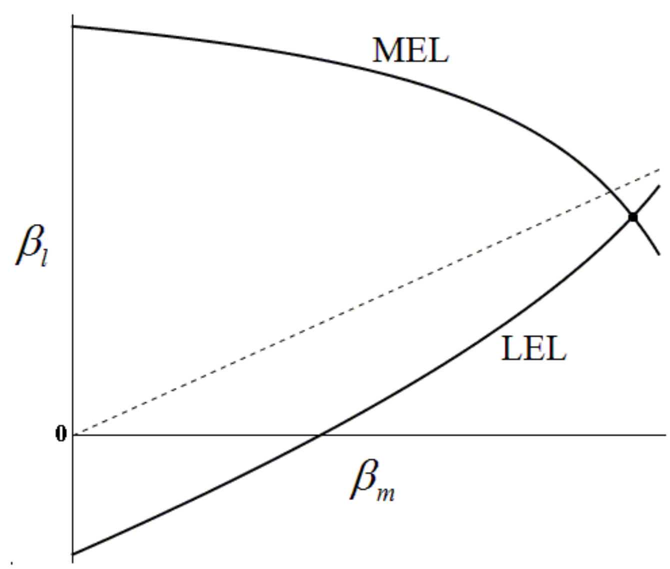


Figure 2 illustrates both MEL and LEL along with the 45 degree line. The intersection of the MEL and LEL occurs below the line. In fact this embodies Proposition 3.

Proposition 3: Whenever $\frac{z^2}{1+z^2} < q < \frac{1+z^2}{2+z^2}$ and $\Delta > \frac{q-\left(1-q^2\right)\left(1+z^2\right)}{q\left(1-q\right)\left(1+z^2\right)\left[\left(1-q\right)z^2-q\right]}$ we have that $\beta_m > \beta_l$.

Proof: See the Appendix.

The intuition is straightforward: limit orders are informative but are partially absorbed by hedgers not ultimate customers and hence have lower price impact. The first condition given in Proposition 3 is easily satisfied empirically for our sample period. For example, in order to violate this condition, given that the average Sharpe ratio for GBP/USD exchange rate is 0.059 (see Table 1), limit orders should be executed with probability smaller than 0.0035. This value is two orders of magnitude smaller than the value of 0.361 observed in the data (see Table 1). The second condition sets restrictions on the unobservable ratio of risk aversion parameters. For the price impact of market orders to be larger than the price impact of limit orders, the ratio of dealers' to public's risk aversion parameter should be larger than a certain threshold value above 1. This suggests that dealers should be on aggregate more risk averse than customer traders. This is consistent with the reluctance of foreign exchange dealers to hold inventories (Bjönnes and Rime, 2005).

3. Data Sources and Variable Definition

Our sample includes tick by tick data from the Reuters trading system Dealing 3000 for three currency pairs: US dollar per euro, US dollar per pound sterling, pound sterling per euro (hereafter EUR/USD, GBP/USD, and EUR/GBP, respectively) and the sample period runs from January 2, 2003 to December 30, 2004. The Bank for International Settlement (BIS, 2004) estimates that trades in these currencies constitute up to 60 percent of the FX spot transactions, 53 percent of which are interdealer trades during the sample period. Thus, our data represents a substantial part of the FX market.

The data analyzed consists of continuously recorded transactions and orders (including those overnight). For each limit order, the data set reports the currency pair, unique order identifier, price, order quantity, hidden quantity (D3000 function), quantity traded, order type, transaction identifier of order entered or removed, status of market order, entry type of orders, removal reason, time of orders entered and removed. The data time stamps are accurate to one-hundredth of a second. The minimum trade size in Reuters trading system Dealing 3000 is 1 million units of the base currency. This extremely detailed data set makes it easier for us to track all types of orders submitted throughout the day and to update the limit order book for all entries, removals, amendments, and trade executions. It is important to note that, while Reuters is the platform where most of the GBP trades take place, EBS has the highest share of trades in EUR/USD. We exclude weekends (between 21:00 GMT Friday until 21:00 GMT Sunday) from the sample. We aggregate the data to the daily and hourly level.

We denote by p_t the log of the closing level of exchange rate at 21:00 GMT on day t for daily data and the last traded price during the corresponding hour for hourly data. The exchange rate return, Δp_t, is calculated as the difference between the log midpoint exchange rate at time t and $t-1$, expressed in basis points.

We construct three different order flow variables. Net market order flow mo_t from period $t-1$ to t is measured as the aggregated difference between the trade quantities initiated by sellers and the trade quantities initiated by buyers (offer minus bid) for the foreign (base) currency over the corresponding period of time. We define the net new limit order flow lo_t as the sum of all bid order quantities arrived between time $t-1$ and t at the best price (either price improving or best price matching) minus the sum of all offer order quantities over the corresponding period. We define the net cancellation order flow co_t as the sum of all offer order cancellations between time $t-1$ and t at the best price minus the sum of all bid order quantities over the corresponding period.

We measure the bid-ask spread s as the difference between the closing level of the best ask and the best bid exchange rate for the corresponding period. We compute the probability of limit order execution as the ratio of the total quantity of limit orders executed during a period to the total quantity of limit orders submitted during the same period. We use σ for the realized volatility of the exchange rate returns defined as the square root of the sum of squared five minutes mid-quote returns during the day and measured in basis points. We denote by z the Sharpe ratio defined as the ratio of daily return of the exchange rate and the daily realized volatility.

Insert Table 1 about here

Table 1 contains descriptive statistics on exchange rate returns and order flow variables for our data at daily frequency. The average change in log exchange rates is 1.625 bp for EUR/GBP, 3.570 bp for GBP/USD and 5.204 bp for EUR/USD exchange rate. EUR/USD was the most volatile during the sample period while EUR/GBP exhibited the lowest variation. The highest average probability of limit order execution was 0.361 for the GBP/USD exchange rate and the smallest was for EUR/USD. The average daily Sharpe ratios were quite small ranging from 0.027 (for EUR/GBP) to 0.064 (for EUR/USD). The EUR/GBP was the most liquid rate according to bid-ask spread (4.157 bp) while EUR/USD was the least liquid (with mean spread equal to 7.153 bp). Average market order flow for all three currency pairs was positive while mean limit order flow and cancellations were negative (except the mean cancellation order flow for EUR/USD).

Insert Table 2 about here

Table 2 shows that there is positive statistically significant correlation between changes in log exchange rates and both market and limit order flow variables for all three currency pairs while log returns negatively correlate with cancelation order flow. Correlation between market and limit order flow is negative but statistically insignificant (except the case of EUR/USD). Cancellation order flow negatively correlates with the other two order flow variables for all three exchange rates.

4. Empirical Results

The theory of section 2 provides us with two testable hypotheses. These are:

Hypothesis 1: Price impacts of market and limit orders are positive and statistically significant.

Hypothesis 2: Price impact of market order flow is larger than the price impact of limit order flow.

4.1. Testing Hypothesis 1

We start testing the hypotheses with estimating the following regression models:

$$\Delta p_t = \beta_0 + \beta_{mo} mo_t + \beta_{lo} lo_t + \beta_{co} co_t + u_t \,. \tag{5}$$

It is worth noting that our theoretical model only allows limit orders to live for one trading period and thus contains no cancellations. This obviously does not correspond with reality where limit orders may live for hours or days and where liquidity suppliers may strategically cancel their orders. Thus, although our model cannot make any predictions about flows of limit order cancellations, we include cancellation flow in the empirical model. We expect cancellation flow to behave much like a negative counterpart of new limit order flow. Excess cancellation of limit buys over sells is expected to be accompanied by price drops and vice versa.

Insert Table 3 about here

Table 3 contains the estimation results of the above model for three different exchange rates using daily data. We start with estimating the standard Evans-Lyons type regression as a benchmark and the price impact of market order flow is positive and statistically significant as expected. In the extended regression with included limit and cancelation order flow variables, all coefficients for three order flow variables are positive and statistically significant for each of the exchange rate as predicted by the model. An interesting fact is that the price impact of market orders increases substantially when limit order flow is included in the model. This suggests that a regression in which exchange rates are determined by market order flow only is misspecified and, due to an omitted variable bias, seriously underestimates the price impact of market order flow. Explanatory power as measured by $\bar{R}^2$ increases substantially and this increase is statistically significant.

Our regression might suffer from endogeneity problems. Within an interval, flows of new and cancelled limit orders and price changes are likely to be simultaneously determined. Mechanically, at the highest frequencies price changes can only occur due to the entry of new limit orders or the removal of limit orders. Evans and Lyons (2002) argue that market orders do not suffer from such problem, however. In order to correct for potential endogeneity we re-estimate the model using instrumental variables. We identify instruments by estimating first stage regressions of limit and cancellation order flows on lags (up to 5th order) of all three types of order flow and prices changes to all three exchange rates. We keep a variable as an instrument if it is significant at a 5% level in at least one of the two first stage regressions.

As shown in Table 3 the estimation results are qualitatively similar to the OLS results. All three price impact coefficients are statistically significant at the 5% level for each of the exchange rates. Price impacts of market orders are again larger when limit and cancellation orders are included in the regression.

We repeat the same exercise at an hourly frequency. Table 5 contains the estimation results. All the above conclusions remain unchanged.

4.2.Testing Hypothesis 2.

We can see from the previous results that price impact of market order flows is larger than the price impact of limit order flow for all three exchange rates. We proceed to test if the differences between price impacts are statistically significant using a set of Wald tests. Specifically, we test the following two hypotheses:

$$H_0 : \beta_{mo} = \beta_{lo} = \beta_{co} \quad \text{vs} \quad H_1 : H_0 \text{ is not true}$$

$$H_0 : \beta_{mo} = \beta_{lo} \quad \text{vs} \quad H_1 : \beta_{mo} \neq \beta_{lo}$$

Insert Table 4 about here

Results are given in Table 4. For the GBP/USD and EUR/USD exchange rates, the difference between price impacts of market order flow and limit order flow is statistically significant at 5% level. The regression for EUR/GBP produces insignificant differences for the second hypothesis. Given that the instrumental variable approach substantially reduces the efficiency of the estimators we are unable to reject the hypotheses at 5% level when using the IV GMM method.

In Table 4, we also provide tests of the relationships between the price impact of cancellations and the price impacts of market orders and new limit orders. For all exchange rates, the cancellation price impact is smaller than the price impact of market orders and is economically though not always statistically insignificantly different from the price impact of limit orders.

Table 6 contains the values of Wald test statistics and the corresponding p-values for the estimations using the hourly data. In this case, we do reject the hypotheses that the market order flow price impact is the same as that for new limit and cancellation flows. Given that the efficiency of the estimators at the hourly frequency is increased due to the increased number of observations, we can also reject the hypotheses even when using the instrumental variable approach. Thus, empirical evidence supports the predictions we derive from our theoretical model.

5. Conclusions

We extend the ‘portfolio shifts’ model of Evans and Lyons (2002) to allow FX dealers to trade both via limit and market order in the inter-dealer segment of the market. In the original formulation of the model, dealers were only allowed to trade via market orders. We introduce an inter-dealer limit order trading round, where dealers with significant customer flows (i.e. informed dealers) supply liquidity to those wishing to hedge currency risk. This generalisation has important effects on the model equilibrium. Informed dealers can now exploit any advantage they possess via both order types and thus equilibrium exchange rate changes depend on both market order flows and limit order flows. The equilibrium is such that the price impacts of both market and limit orders are positive and the price impact of market orders is greater than that of limit orders. The latter result is driven by the fact that the portion of aggregate inventory executed in inter-dealer limit order trading does not need to be passed back to customers.

We test these propositions using 2 years of inter-dealer FX microstructure data drawn from the Reuters Dealing system for GBP/USD, EUR/USD and EUR/GBP. We are unable to reject all of the main results of the model. Coefficients on limit order flows are always positive but always smaller than those on market order flows. Limit order flows add significant explanatory power to all of our regressions. Thus, there is robust support for our model in respect of all the exchange rate pairs in the data.

From a microstructure perspective, this paper can be viewed as supporting the recent literature which argues that, under certain conditions, traders use limit orders to exploit private information (Kaniel and Liu, 2006, Bloomfield, Saar and O'Hara, 2005, Rosu, 2009) and reinforcing empirical work which investigates the information content of limit order data (Harris and Panchapagesan, 2005, Cao, Hansch and Wang, 2009, Kozhan and Salmon, 2012, Latza and Payne, 2011). From an international finance/macroeconomics perspective, the paper provides robust evidence that another key microstructure variable, limit order flow, can explain exchange rates at the relatively low frequencies that are typically of most interest to macroeconomists. Finally, we show that the omission of limit orders has the effect of very significantly understating the importance of market orders in exchange rate determination.

Tables

Table 1: Descriptive Statistics

This table presents descriptive statistics for on exchange rate returns and order flow variables for three currency pairs GBP/USD/ EUR/USD and EUR/GBP. Δp is the change in log exchange rates measured in basis points, σ is the realized volatility of the exchange rate returns defined as the square root of the sum of squared five minutes mid-quote returns during the day and measured in basis points. SR is the Sharpe ratio defined as the ratio of daily return of the exchange rate and the daily realized volatility. s is the average daily bid-ask spread measured in basis points. q is the probability of limit order execution defined as the ratio of the total quantity of limit orders executed during the day to the total quantity of limit orders submitted during the same day. Net market order flow mo is the aggregated difference between the trade quantities initiated by sellers and the of trade quantities initiated by buyers for the foreign (base) currency over the day. Net new limit order flow lo is the sum of all bid order quantities arrived during the day at the best price minus the sum of all offer order quantities. Net cancellation order flow co is the sum of all offer order cancellations at the best price minus the sum of all bid order quantities. All order flow variables are measured in billions of the base currency. The sample period is from January 2, 2003 to December 30, 2004.

	GBP/USD					EUR/USD					EUR/GBP				
Variable	**Mean**	**Std. Dev.**	**Q1**	**Median**	**Q3**	**Mean**	**Std. Dev.**	**Q1**	**Median**	**Q3**	**Mean**	**Std. Dev.**	**Q1**	**Median**	**Q3**
Δp	3.57	55.87	-31.32	7.001	40.54	5.204	67.36	-32.08	6.199	47.08	1.625	45.31	-25.06	-1.42	31.44
σ	77.40	21.64	62.87	73.15	87.30	97.42	35.07	75.68	90.90	112.22	69.33	18.64	57.42	67.34	79.38
SR	0.059	0.688	-0.424	0.103	0.555	0.064	0.668	-0.363	0.068	0.521	0.027	0.629	-0.399	-0.017	0.489
s	4.293	1.345	3.747	4.068	4.508	7.153	2.785	5.126	6.727	9.504	4.157	1.797	2.995	3.815	4.913
q	0.361	0.027	0.344	0.362	0.380	0.117	0.042	0.076	0.109	0.139	0.275	0.059	0.231	0.267	0.326
mo	0.075	0.282	-0.106	0.081	0.276	0.029	0.338	-0.11	0.016	0.167	0.042	0.266	-0.114	0.026	0.189
lo	-0.042	0.714	-0.494	-0.035	0.405	-0.068	1.245	-0.772	-0.07	0.753	-0.004	0.708	-0.404	-0.017	0.377
co	-0.032	0.554	-0.385	-0.015	0.301	0.141	1.13	-0.586	0.14	0.779	-0.092	0.602	-0.391	-0.048	0.249

Table 2: Correlations

This table presents correlations among log change of exchange rates, market, limit and order flow variables for each of the currency pair GBP/USD/ EUR/USD and EUR/GBP. Δp is the change in log exchange rates. Net market order flow mo is the aggregated difference between the trade quantities initiated by sellers and the of trade quantities initiated by buyers (offer minus bid) for the foreign (base) currency over the day. Net new limit order flow lo is the sum of all bid order quantities arrived during the day at the best price minus the sum of all offer order quantities. Net cancellation order flow co is the sum of all offer order cancellations at the best price minus the sum of all bid order quantities. Bold values are statistically significant at 5% level. The sample period is from January 2, 2003 to December 30, 2004.

	Variable	Δp	mo	lo	co
GBP/USD	Δp	1	**0.383**	**0.483**	**-0.327**
	mo		1	-0.035	**-0.177**
	lo			1	**-0.879**
	co				1
EUR/USD	Δp	1	**0.286**	**0.411**	**-0.429**
	mo		1	**-0.088**	**-0.136**
	lo			1	**-0.956**
	co				1
EUR/GBP	Δp	1	**0.277**	**0.426**	**-0.282**
	mo		1	-0.080	**-0.178**
	lo			1	**-0.898**
	co				1

Table 3: In-Sample Fit of Portfolio Shift Model: Daily Data

This table presents estimation results of the portfolio shift model equation $\Delta p_t = \beta_0 + \beta_{mo} mo_t + \beta_{lo} lo_t + \beta_{co} co_t + u_t$ based on daily data along with Evans-Lyons type equation $\Delta p_t = \beta_0 + \beta_{mo} mo_t + u_t$ for three exchange rates. OLS panel reports the estimation results based on the OLS method. IV GMM reports the estimation results using the instrumental variable GMM approach. We use up to 5 lags of market, limit and cancellation order flows from all 3 markets which are statistically significant in one of the first-stage regression at 5% level. T-statistics are given in parentheses. Standard errors are adjusted for autocorrelation. The sample period is from January 2, 2003 to December 30, 2004.

	GBP/USD				EUR/USD				EUR/GBP			
	mo	lo	co	$\bar{R}^2$	mo	lo	co	$\bar{R}^2$	mo	lo	co	$\bar{R}^2$
OLS	76.12 (9.44)			14.5	57.22 (6.80)			8.1	47.51 (6.56)			7.5
OLS	121.46 (19.2)	108.73 (21.0)	101.36 (15.0)	57.6	109.94 (9.77)	77.03 (7.44)	60.05 (5.24)	30.8	113.26 (17.9)	110.02 (20.8)	103.92 (16.4)	52.6
IV GMM	121.94 (6.34)	106.80 (3.24)	103.66 (2.41)	57.4	211.33 (3.69)	196.59 (2.73)	202.35 (2.61)	8.55	118.04 (4.24)	108.61 (2.40)	115.14 (2.57)	49.7

Table 4: Test Restrictions: Daily Data

This table presents Wald test statistics and p-values (in parentheses) for the coefficient restrictions based on daily data. OLS panel contains the results for the test based on the OLS estimation while IV GMM reports the test statistics and p-values based on the instrumental variable approach. We use up to 5 lags of market, limit and cancellation order flows from all 3 markets which are statistically significant in one of the first-stage regression at 5% level. The sample period is from January 2, 2003 to December 30, 2004.

	Hypothesis	GBP/USD	EUR/USD	EUR/GBP
OLS	$H_0: \beta_{mo} = \beta_{lo} = \beta_{co}$ vs $H_1: H_0$ is not true	5.08 (0.0065)	35.95 (0.0000)	3.61 (0.0276)
	$H_0: \beta_{mo} = \beta_{lo}$ vs $H_1: \beta_{mo} \neq \beta_{lo}$	4.08 (0.0440)	18.83 (0.0001)	0.34 (0.5607)
	$H_0: \beta_{lo} = \beta_{co}$ vs $H_1: \beta_{lo} \neq \beta_{co}$	5.76 (0.0167)	51.1 (0.0001)	6.59 (0.0106)
IV GMM	$H_0: \beta_{mo} = \beta_{lo} = \beta_{co}$ vs $H_1: H_0$ is not true	0.38 (0.6789)	1.35 (0.2595)	0.31 (0.7361)
	$H_0: \beta_{mo} = \beta_{lo}$ vs $H_1: \beta_{mo} \neq \beta_{lo}$	0.78 (0.3791)	0.67 (0.4142)	0.23 (0.6285)
	$H_0: \beta_{lo} = \beta_{co}$ vs $H_1: \beta_{lo} \neq \beta_{co}$	0.0388 (0.8439)	0.64 (0.4245)	0.56 (0.4553)

Table 5: In-Sample Fit of Portfolio Shift Model: Hourly Data

This table presents estimation results of the portfolio shift model equation $\Delta p_t = \beta_0 + \beta_{mo} mo_t + \beta_{lo} lo_t + \beta_{co} co_t + u_t$ based on hourly data along with Evans-Lyons type equation $\Delta p_t = \beta_0 + \beta_{mo} mo_t + u_t$ for three exchange rates. OLS panel reports the estimation results based on the OLS method. IV GMM reports the estimation results using the instrumental variable GMM approach. We use up to 5 lags of market, limit and cancellation order flows from all 3 markets which are statistically significant in one of the first-stage regression at 5% level. T-statistics are given in parentheses. Standard errors are adjusted for autocorrelation. The sample period is from January 2, 2003 to December 30, 2004.

	GBP/USD				EUR/USD				EUR/GBP			
	mo	lo	co	$\bar{R}^2$	mo	lo	co	$\bar{R}^2$	mo	lo	co	$\bar{R}^2$
OLS	89.70 (50.9)			17.1	61.91 (29.7)			6.6	69.83 (41.7)			12.2
OLS	137.42 (94.7)	110.88 (92.4)	100.48 (63.2)	53.7	122.60 (47.3)	100.95 (41.9)	82.07 (29.7)	26.4	124.43 (75.2)	99.92 (74.2)	89.93 (53.4)	42.2
IV GMM	121.57 (8.53)	65.93 (2.36)	69.88 (2.01)	45.6	140.93 (6.97)	109.62 (3.67)	106.23 (3.68)	23.9	118.40 (4.79)	83.94 (2.17)	83.08 (1.97)	40.4

Table 6: Test Restrictions: Hourly Data

This table presents Wald test statistics and p-values (in parentheses) for the coefficient restrictions based on hourly data. OLS panel contains the results for the test based on the OLS estimation while IV GMM reports the test statistics and p-values based on the instrumental variable approach. We use up to 5 lags of market, limit and cancellation order flows from all 3 markets which are statistically significant in one of the first-stage regression at 5% level. The sample period is from January 2, 2003 to December 30, 2004.

	Hypothesis	GBP/USD	EUR/USD	EUR/GBP
OLS	$H_0: \beta_{mo} = \beta_{lo} = \beta_{co}$ vs $H_1: H_0$ is not true	253.1 (0.0000)	373.9 (0.0000)	266.8 (0.0000)
	$H_0: \beta_{mo} = \beta_{lo}$ vs $H_1: \beta_{mo} \neq \beta_{lo}$	328.7 (0.0000)	118.9 (0.0000)	275.3 (0.0000)
	$H_0: \beta_{lo} = \beta_{co}$ vs $H_1: \beta_{lo} \neq \beta_{co}$	173.7 (0.0000)	592.0 (0.0000)	178.3 (0.0000)
IV GMM	$H_0: \beta_{mo} = \beta_{lo} = \beta_{co}$ vs $H_1: H_0$ is not true	10.87 (0.0000)	5.01 (0.0067)	3.13 (0.0438)
	$H_0: \beta_{mo} = \beta_{lo}$ vs $H_1: \beta_{mo} \neq \beta_{lo}$	3.89 (0.0001)	2.37 (0.0178)	5.54 (0.0186)
	$H_0: \beta_{lo} = \beta_{co}$ vs $H_1: \beta_{lo} \neq \beta_{co}$	0.16 (0.6916)	1.08 (0.2870)	0.02 (0.8822)

Appendix

Proof of Proposition 1: Equation (4) implies that given $\alpha_m \omega_l - \alpha_l \omega_m \neq 0$

$$c_{1,t}^i = \frac{\omega_l}{\alpha_m \omega_l - \alpha_l \omega_m} M_t^i - \frac{\omega_m}{\alpha_m \omega_l - \alpha_l \omega_m} L_t^i$$

$$\sum_i c_{1,t}^i = c_{1,t} = \frac{\omega_l}{\alpha_m \omega_l - \alpha_l \omega_m} M_t - \frac{\omega_m}{\alpha_m \omega_l - \alpha_l \omega_m} L_t$$

Given the market clearing condition (2) and the customers' demand (1) we have that the change in the aggregate customer holding, $\Delta c_{3,t}$, is $c_{3,t}$ minus previous accumulated holdings:

$$\Delta c_{3,t} = \frac{\Delta}{\mu}\left[E\left(\Delta P_{3,t+1} + \beta_0 R_{t+1} \right) \right] - \left(\sum_{\tau=1}^{t-1} -c_{1,\tau} \right) - q\left(\sum_{\tau=1}^{t-1} L_\tau \right)$$

where R_{t+1} is the public signal. This implies a market clearing round 3 price of

$$P_{3,t} = E\left(P_{3,t+1} + \beta_0 R_{t+1} \right) + \frac{\mu}{\Delta} \sum_{\tau=1}^{t} \left(\frac{\omega_l}{\alpha_m \omega_l - \alpha_l \omega_m} M_\tau - \frac{\omega_m}{\alpha_m \omega_l - \alpha_l \omega_m} L_\tau - qL_\tau \right)$$

Taking the first difference we get

$$\Delta P_{3,t} = \beta_0 \Delta R_{t+1} + \beta_m M_t + \beta_l L_t$$

with

$$\beta_m = \frac{\mu}{\Delta} \frac{\omega_l}{\left(\alpha_m \omega_l - \alpha_l \omega_m \right)},$$
$$\beta_l = -\frac{\mu}{\Delta} \frac{\omega_m}{\left(\alpha_m \omega_l - \alpha_l \omega_m \right)} - \frac{q\mu}{\Delta} = -\frac{\beta_m \omega_m}{\omega_l} - \frac{q\mu}{\Delta}. \tag{A.1}$$

Let $E\left(\Delta P_{3,t} \right) = \gamma - s$ and $Var\left(\Delta P_{3,t} \right) = \sigma^2$ denote expected return and variance from a **buy** market order[10]. Denote by Y the gain from submitting a limit order. If $\gamma - s > 0$ then, the expected gain from a limit order is $q\gamma$. This captures the idea that a submitter of a market order foregoes the spread while a submitter of a limit order benefits from the spread but suffers execution risk. We neglect any opportunity cost from non-execution and also assume that there is no uncertainty about the price at which a limit order is executed. That is, an individual limit order either executes instantly or not all and dealer never has to "walk up the book".

[10] The model of section 2 is parameterised differently. There the return on a market order is defined as $\Delta P_{3,t}$ with the return on a limit order given as $\Delta P_{3,t} + s$. There is, of course, no loss of generality but the presentation here in the Appendix is expositionally less cumbersome.

The variance of the gain on a limit order is derived as follows. Random variable Y takes on the value $\Delta P_{3,t}$ with probability q and 0 with the probability $(1-q)$. The second moment of Y is $E(Y^2) = qE\left[\Delta P_{3,t}^2\right] = q(\sigma^2 + \gamma^2)$. Given that $E(Y) = q\gamma$ we have

$$Var(Y) = E(Y^2) - E^2(Y) = q(\sigma^2 + \gamma^2) - q^2\gamma^2 = q\left[\sigma^2 + (1-q)\gamma^2\right].$$

The covariance of the gains on the two types of order σ_{12} is derived in a similar fashion:

$$\begin{aligned}\sigma_{12} &= E\left[(\Delta P_{3,t} - s)Y\right] - E(\Delta P_{3,t} - s)E(Y) = \left[qE(\Delta P_{3,t}^2) - q\gamma s\right] - \left[q\gamma^2 - q\gamma s\right] \\ &= q\left(\left[\sigma^2 + \gamma^2 - \gamma s\right] - \left[\gamma^2 - \gamma s\right]\right) = q\sigma^2\end{aligned}.$$

Therefore $\boldsymbol{\Sigma} = \begin{bmatrix}\sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22}\end{bmatrix} = \begin{bmatrix}\sigma^2 & q\sigma^2 \\ q\sigma^2 & q\left[\sigma^2 + (1-q)\gamma^2\right]\end{bmatrix} = \begin{bmatrix}\sigma^2 & q\sigma^2 \\ q\sigma^2 & q\sigma^2\left[1 + (1-q)z^2\right]\end{bmatrix}$

where $z = \dfrac{\gamma}{\sigma}$, the Sharpe ratio.

Taking into account Equation (3), the expected gain from both market and limit orders is given as the vector:

$$\begin{pmatrix}\gamma - s \\ q\gamma\end{pmatrix} = \begin{pmatrix}\beta_m M_t^i + \beta_l L_t^i - s \\ q\left(\beta_m M_t^i + \beta_l L_t^i\right)\end{pmatrix} = \begin{pmatrix}\beta_m\left(D_{m,t}^i + c_{1t}^i\right) + \beta_l D_{l,t}^i - s \\ q\left(\beta_m\left(D_{m,t}^i + c_{1t}^i\right) + \beta_l D_{l,t}^i\right)\end{pmatrix}$$

Denote the vector $\mathrm{D}_{\mathbf{t}}' = \left(D_{m,t}, D_{l,t}\right)$. This is the 'hot potato' element of inter dealer trading for both limit and market orders[11]. They now have to be optimally chosen. We can express the dealer's optimisation problem now as follows

$$\max_{\mathrm{D_t}}\left\{\mathrm{D_t'}\begin{pmatrix}\beta_m D_{m,t}^i + \beta_l D_{l,t}^i + \beta_m c_{1,t}^i - s \\ q\beta_m D_{m,t}^i + q\beta_l D_{l,t}^i + q\beta_m c_{1,t}^i\end{pmatrix} - \frac{1}{2}\theta_d \mathrm{D_t'}\Sigma\mathrm{D_t}\right\},$$

or in full matrix form as

$$\max_{\mathrm{D_t}}\left\{\mathrm{D_t'}\left(\begin{pmatrix}\beta_m \\ q\beta_m\end{pmatrix}c_{1,t}^i + \begin{pmatrix}\beta_m & \beta_l \\ q\beta_m & q\beta_l\end{pmatrix}\mathrm{D_t} - \begin{pmatrix}1 \\ 0\end{pmatrix}s\right) - \frac{1}{2}\theta_d \mathrm{D_t'}\Sigma\mathrm{D_t}\right\} \qquad \text{(A.2)}$$

The first order condition is[12]:

$$\begin{pmatrix}\beta_m \\ q\beta_m\end{pmatrix}c_{1,t}^i + \begin{pmatrix}2\beta_m & \beta_l + q\beta_m \\ \beta_l + q\beta_m & 2q\beta_l\end{pmatrix}\mathrm{D_t} - \begin{pmatrix}1 \\ 0\end{pmatrix}s - \theta_d\Sigma\mathrm{D_t} = 0,$$

[11] Note that, as in the standard portfolio shifts model, all customer orders are executed as market orders. This does not, of course, mean that limit orders are not informed by customer flows as will be clear below.

[12] We are invoking the matrix differentiation rule $\dfrac{\partial x' A x}{\partial x'} = (A + A')x$

which yields

$$\mathrm{D_t} = \begin{pmatrix} D^i_{m,t} \\ D^i_{l,t} \end{pmatrix} = \begin{pmatrix} \theta_d\sigma_{11} - 2\beta_m & \theta_d\sigma_{12} - \beta_l - q\beta_m \\ \theta_d\sigma_{12} - \beta_l - q\beta_m & \theta_d\sigma_{22} - 2q\beta_l \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \beta_m \\ q\beta_m \end{pmatrix} c^i_{1,t} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} s \right\}.$$

It is clear that market and limit orders are linear combinations of customer inflows and the spread and that the trial solution of Equation (4) is valid with parameters[13]

$$\begin{aligned}
\alpha_m &= 1 + \frac{q\beta_l\beta_m - q\beta_m\left[(1-q)\mu\left(z^2+1\right) + q\beta_m\right]}{\left(\beta_l - q\beta_m\right)^2 - (1-q)q\mu\left(z^2+1\right)\left(\mu - 2\beta_m\right)} \\
\omega_m &= \frac{q\mu\left((1-q)z^2+1\right) - 2q\beta_l}{\left(\beta_l - q\beta_m\right)^2 - (1-q)q\mu\left(z^2+1\right)\left(\mu - 2\beta_m\right)} \\
\alpha_l &= \frac{q\beta_m{}^2 - \beta_l\beta_m}{\left(\beta_l - q\beta_m\right)^2 - (1-q)q\mu\left(z^2+1\right)\left(\mu - 2\beta_m\right)} \\
\omega_l &= \frac{\beta_l + q\beta_m - q\mu}{\left(\beta_l - q\beta_m\right)^2 - (1-q)q\mu\left(z^2+1\right)\left(\mu - 2\beta_m\right)}.
\end{aligned} \tag{A.3}$$

We do not consider the case when $\alpha_m\omega_l - \alpha_l\omega_m = 0$ because this makes β_m and β_l indeterminate. Q.E.D.

Proof of Proposition 2: In order to solve for β_m and β_l we substitute the expressions for α_m , w_m, α_l and w_l from Equation (A.3) into Equation (A.1). The following two solutions for β_l as a function of β_m follow from the relation $\beta_l = -\frac{\beta_m\omega_m}{\omega_l} - \frac{q\mu}{\Delta}$:

$$LEL:\ \beta_l = \frac{\beta_m q\Delta + \mu q(\Delta-1) - \sqrt{q\left(\beta_m^2 q\Delta^2 + \mu^2 q\left(\Delta+1\right)^2 - 2\beta_m\mu\Delta\left(q\left(\Delta+3\right) + 2\Delta\left(1-q\right)\left(1+z^2\right)\right)\right)}}{2\Delta} \tag{A.4}$$

$$\tilde{L}EL:\ \beta_l = \frac{\beta_m q\Delta + \mu q(\Delta-1) + \sqrt{q\left(\beta_m^2 q\Delta^2 + \mu^2 q\left(\Delta+1\right)^2 - 2\beta_m\mu\Delta\left(q\left(\Delta+3\right) + 2\Delta\left(1-q\right)\left(1+z^2\right)\right)\right)}}{2\Delta}.$$

[13] Recall that $M^i_t = D^i_{m,t} + c^i_{1t}$. This is why unity has to be added to the coefficient on c^i_{1t} in the expression for $D^i_{m,t}$ in the solution to the optimisation problem to obtain the expression for α_m in Equation (10)

The equation for price impact of market orders $\beta_m = \frac{\mu}{\Delta} \frac{\omega_l}{(\alpha_m \omega_l - \alpha_l \omega_m)}$ gives us a unique solution for β_l as a function of β_m:

$$MEL: \qquad \beta_l = \mu q \left(1 + \frac{\beta_m}{\beta_m \Delta - \mu} \right) \tag{A.5}$$

The intersections of *MEL* with *LEL* and *MEL* with $\tilde{L}EL$ produce two pairs of possible solutions for price impact parameters (See Mathematica file Equilibrium_A2.nb):

$$\begin{aligned} \beta_m &= \frac{\mu\left(2\Delta(1-q)(1+z^2) - q(\Delta+1) + \sqrt{q\left(q(\Delta+1)^2 - 4\Delta(1-q)(1+z^2)\right)}\right)}{2\Delta^2(1-q)(1+z^2)} \\ \beta_l &= \frac{\mu\left(q(\Delta+1) - \sqrt{q\left(q(\Delta+1)^2 - 4\Delta(1-q)(1+z^2)\right)}\right)}{2\Delta} \end{aligned} \tag{A.6}$$

and

$$\begin{aligned} \tilde{\beta}_m &= \frac{\mu\left(2\Delta(1-q)(1+z^2) - q(\Delta+1) - \sqrt{q\left(q(\Delta+1)^2 - 4\Delta(1-q)(1+z^2)\right)}\right)}{2\Delta^2(1-q)(1+z^2)} \\ \tilde{\beta}_l &= \frac{\mu\left(q(\Delta+1) + \sqrt{q\left(q(\Delta+1)^2 - 4\Delta(1-q)(1+z^2)\right)}\right)}{2\Delta} \end{aligned} \tag{A.7}$$

Note that both expressions exist (are real numbers) if

(i) $0 < q \le \frac{1+z^2}{2+z^2}$ and $\Delta \ge \bar{\Delta} = \frac{2(1-q)(1+z^2) + 2\sqrt{(1-q)(1+z^2)(1-2q+(1-q)z^2)}}{q} - 1$ or

(ii) $q > \frac{1+z^2}{2+z^2}$.

We will further concentrate our attention on the (i) as it corresponds to the empirical range of the parameter values (see Table 1 for the empirical values of the parameters).

Under condition (i) the Hessian $\begin{pmatrix} 2\beta_m - \mu & \beta_l + q\beta_m - q\mu \\ \beta_l + q\beta_m - q\mu & 2q\beta_l - q\mu\left[1 + (1-q)z^2\right] \end{pmatrix}$ is negative-definite for both solutions (see Mathematica file Hessian_A3.nb) implying that both solutions are local maxima of the utility function. It is easy to see however, that for any $\mu > 0$, q and Δ

satisfying condition (i) the first pair of solution for β_m and β_l given in Equations (A.6) forms the global maximum (see Mathematica file CompareMax_A4.nb).

Q.E.D.

Proof of Proposition 3:

The proof is given in Mathematica file RelationOfBetas_A5.nb.

Q.E.D.

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