# Interchange fees and innovation in payment systems

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**Abstract**

In this paper, we analyze the impact of interchange fees on consumers' and merchants' incentives to adopt an innovative payment instrument, in a setting where two issuing banks compete to attract consumers on the Hotelling line, while exerting some market power over an installed base of consumers on their hinterländer. We show that the relationship between consumer adoption and interchange fees is non monotonic, when there are adoption externalities between consumers and merchants. We also compare the issuers' incentives to innovate when they cooperate and when they make their innovation decisions separately.

*Keywords*: Payment systems; Innovation; Cooperation.

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# 1 Introduction

Innovations in retail payment systems have proliferated on the market over the last ten years. Contactless payments, mobile payments, P2P payments are currently mentioned as the future payment media of a "cashless society". However, paradoxically, the adoption of innovations in retail payment systems is slow. As in other network industries, firms have to attract a critical mass of users for the market to take-off. Up to now, banks have relied on the business model of interchange fees in payment card systems to provide consumers with incentives to use their cards. This business model is increasingly challenged in various countries and jurisdictions (e.g. in the US in 2011)[1], giving rise to regulatory uncertainty for innovative payment solutions.

This paper adresses two important issues for competition and regulatory authorities. First, do interchange fees increase the adoption of innovations in retail payment systems? Second, does cooperation between issuing banks lead to more innovation? The main contribution of our paper is to show that positive interchange fees favor consumer adoption of innovations only if merchants exert low externalities on consumers.

To study the impact of interchange fees on innovation, we build a theoretical model in which two issuing banks compete to offer an innovative payment instrument to consumers. We model innovation as an improvement in the quality of the payment instrument, which increases its value for consumers. The acquiring side of the market is assumed to be perfectly competitive. On the issuing side, we assume that both issuers compete on the Hotelling line. Each issuer has also an installed based of consumers, which is located on its hinterland. The consumers that are located on a bank's hinterland can only buy the payment instrument from their bank, whereas the consumers that are located on the linear city can choose between the two firms. The issuers charge fixed fees for the adoption of the payment instrument and do not charge transaction fees. Once a consumer has adopted the innovative payment instrument, we assume that he makes one transaction at each merchant's who accepts it. Our model takes into account adoption externalities, since the value of adopting the payment instrument for a consumer increases with the number of merchants who accept it. On the merchant side, merchants decide whether or not to adopt the payment instrument based on their net transactional benefit and on the number of transactions that are made by consumers, since they need to cover their fixed adoption costs.

[1] In the United-States, in 2011, the Federal Reserve Board, mandated by the Dodd-Frank Wall Street Reform and Consumer Protection Act, decided to set a price cap for debit card interchange fees.

In our baseline model, quality levels are exogenous. We focus on the impact of interchange fees on consumers' and merchants' incentives to adopt the innovation. Banks set an interchange fee to maximize their joint profit at the first stage of the game. At the second stage, the issuers compete in prices. Finally, consumers and merchants decide whether or not to adopt the payment instrument. Our results show that the relationship between interchange fees and consumer adoption is non monotonic. This conclusion is in sharp contradiction with the widespread intuition that positive interchange fees benefit consumer adoption. We show that this may be the case only if consumers do not value a lot the presence of merchants on the platform, that is, if the degree of externality is low. If consumers value the presence of merchants on the plaform, the interchange fee that maximises consumer adoption may be equal to zero. Our result can be explained by the complex impact of interchange fees on consumer adoption. In our framework, two effects are at play when the issuers increase interchange fees: a price effect and a quality degradation effect. In the payment cards literature literature, the price effect is always positive, since the issuers pass through higher interchange fee to consumers through lower prices. In our paper, the price effect may be negative in some cases, as the issuers charge only a lump-sum fee for consumer adoption. Therefore, when the interchange fee increases, fewer merchants adopt the innovation, which increases the perceived marginal cost of an issuer, as interchange fee revenues per consumer are reduced. Furthermore, if the degree of externality is high, higher interchange fees reduce the utility that consumers obtain when they adopt the innovation, as the number of adopting merchants decreases. This quality degradation effect is always negative, and may even compensate the price effect if the latter is positive. By contrast, we show that, in our linear setting, merchant adoption decreases with the interchange fee if banks are sufficiently differentiated. Finally, we obtain intuitive results about profit-maximising interchange fees. When consumers do not value the presence of merchants on the platform, while the reverse is true, the platform sets an interchange fee that favors consumer adoption over merchant adoption. By contrast, if consumers value merchant adoption, the platform sets an interchange fee that favors merchant adoption. We also prove that the profit-maximising interchange fee is suboptimally high compared to the welfare-maximising interchange if issuers' prices are more sensitive to a higher interchange fee than the number of merchants.

The model is then extended to the case of endogenous investments in quality. The game that we study is as follows. At the first stage, banks set an interchange fee to maximize their joint-profit. At the second stage, banks choose cooperatively or non cooperatively the quality of the innovation that they offer to their consumers. At the second stage, they compete in prices. Then, at the last stage, consumers and merchants make their adoption decisions, based on the price and the quality of the payment instrument. Our aim is to compare two compare two competitive equilibria: the equilibrium in which banks cooperate to invest in innovations and the equilibrium in which banks do not cooperate. When quality levels are endogenous, the main difference with respect to our baseline model is that issuing banks now exert externalities on each other when they choose how much to innovate. If an issuer decides to innovate more, this generates more adoption from its consumers, which increases merchants' adoption because of adoption externalities. In turn, this generates more adoption for its competitor, who benefits from a higher number of merchants. [**To be written**]

While the impact of interchange fees on innovation in retail payment systems has generated rich policy debate, few academic papers have adressed this issue. Our paper is among the first theoretical contributions to analyze whether interchange fees increase issuers' incentives to innovate and consumers' incentives to adopt innovative payment solutions.

In the literature on payment card systems, the adoption decisions are generally overlooked by considering that consumers' and merchants' decisions to use and to adopt an electronic payment instrument are equivalent (See Chakravorti (2010) or Verdier (2011) for a survey). One exception is the paper by Bedre and Calvano (2011), who model adoption and usage decisions as being distinct choices. They show that a monopolistic issuer chooses interchange fees that exceed the social optimum because merchants cannot refuse payment cards once they have accepted them. Our paper departs from their work in several directions. In their paper, as in ours, the number of cardholders depends on the number of merchants who adopt, because consumers value the expected benefit of being able to pay by card. However, the number of merchants who adopt does not depend on the number of cardholders because there are no fixed adoption costs. It follows that, in their paper, merchants accept cards when their net benefit of being paid by card is positive. By contrast, our paper captures the fact that merchants accept innovative payment instruments when they expect a sufficiently high volume of transactions to cover their fixed costs. There is another important difference between our paper and theirs. We consider, in line with most of the empirical observations, that consumers pay a fixed membership fee, while they do not pay transaction fees. And finally, we consider the quality of the innovation as endogenous.

Our paper also contributes to the scarce literature on investments in two-sided markets. Most papers in this literature deal with different issues. For instance, Peitz and Belleflamme (2010) study the effect of the intermediation mode (for-profit competing platforms versus free access) on sellers' investment incentives, in a model where sellers' investment increase the buyers' utility of belonging to the platform. They show that for-profit intermediation may lead to overinvestment when innovations increase buyers' surplus, because competing intermediaries react by lowering the access fees on the seller side. The perspective of our work is different, as we consider the intermediation mode as given, and we consider the case of a four-party payment platform, which corresponds to the specific case of the retail payments industry. In our paper, competition takes place at the bank level and not at the platform level as in Peitz and Belleflamme (2010). Hagiu (2009) analyzes platform strategies when users differ across their average quality. He finds that, in this context, platforms may find it profitable to exclude low quality users on one side, even though some would be willing to pay the platform access price. Our paper considers that all merchants exert the same degree of externality on consumers. Though we think it would be another interesting research question, we do not assume that some merchants bring more value to consumers than other. Finally, in the payment systems literature, Verdier (2010) studies the impact of interchange fees on monopolistic banks' incentives to invest in quality of a payment card system. In her framework, investment decisions occur on both sides of the market. She finds that a reduction of interchange fees is socially desirable if the acquirers contribute a lot to investments in quality and if consumers benefit more than merchants from quality investments. Our paper deals with a different case, since investment decisions are made by firms who compete on the same side of the market (the issuing side).

Finally, our paper extends the analysis of cooperation strategies to platform markets. The literature has shown that cooperation between competitors can be socially beneficial when the degree of spillovers at the R&D stage is sufficiently high (see, for example, Katz (1986) and d'Aspremont and Jacquemin (1988)). In our setting, the externality between consumers and merchants plays a similar role as spillovers: an issuer's R&D investment benefits the rival issuer through the link with the other side of the market (the merchants' side). We find that cooperation on one side of the market is socially beneficial if the externality between the two sides is sufficiently strong.

The paper is organized as follows. In Section 2, we introduce our model. In Section 3, we analyze the impact of interchange fees on consumers' and merchants' adoption decisions. In Section 4, we endogenize the choice of quality levels. We compare two equilibria: an equilibrium in which banks choose the quality of the payment instrument non cooperatively, and an equilibrium in which banks cooperate on the choice of quality. Finally, we conclude.

## 2 The Model

We build a model which enables us to study how interchange fees impact users' incentives to adopt an innovative payment instrument. In our framework, two issuers offer a new electronic payment instrument to consumers, such as mobile payments. Consumers' incentives to adopt the electronic payment instrument depend on the number of merchants who adopt the new technology, and vice versa.

**Banks:** We consider two issuers that are located at the extremities of a linear city of length one, and that compete by offering electronic payment instruments to consumers. The first issuer, that we denote by issuer 1, is located at point 0, and the second issuer, that we denote by issuer 2, is located at point 1.

There is a mass $\gamma \geq 0$ of consumers that are uniformly distributed along the city (the Hotelling line). On each side of the city, we add an hinterland in which there is a mass $\beta \geq 0$ of consumers. The firm's hinterland goes from one of the extremes of the city to infinity.[2] All consumers hold cash to pay for their expenses, but they can decide to adopt the electronic payment instrument offered by the issuers. The consumers located on firm $i$'s hinterland can only buy the payment instrument from firm $i$, whereas the consumers that are located on the linear city can choose between the two firms.

Issuing banks compete in prices to attract consumers. They charge consumers with fixed fees for adopting the innovation but transactions are free.[3] The marginal cost of offering the electronic payment instrument to customers is denoted by $k_I$, whereas the marginal cost of transactions is normalized to zero.[4] Issuer $i = 1, 2$ offers a quality $\theta_i$ to consumers.[5] Though the two firms can offer electronic payment instruments of different qualities, we assume that their payment technologies are compatible on the merchants' side.[6]

[2] Each hinterland represents a base of captive customers for one of the issuers. The existence of a captive base of customers for each bank could be explained by the presence of switching costs in the banking industry (for deposits, across products and areas). For estimates of the switching costs in the banking industry, see for instance Kiser (2002), Shy (2002), or Kim & al. (2003).

[3] Our aim is to model consumer adoption of innovative payment solutions. Therefore, consistent with market practices (for Internet Payments, for debit cards), it seemed to us relevant to use fixed adoption fees on the consumer side and free transactions.

[4] We consider that there is no transaction price, which corresponds to the standard pricing scheme for debit cards. For simplicity, we also normalize the marginal cost of transactions to zero.

[5] In the baseline model, we assume that quality levels are exogenous. In Section 4, we allow the issuers to decide, cooperatively or non-cooperatively, on the quality levels.

[6] This means that merchants who have adopted the electronic payment instrument can accept payments from either of the two issuers.

The acquirers are assumed to be perfectly competitive. They charge a fee $m$ to merchants for each electronic transaction. Without loss of generality, their marginal cost for transactions is normalized to zero.[7]

Each time a consumer uses the electronic payment instrument, the acquirer pays an interchange fee $a$ to the issuer of the consumer's payment instrument.[8]

**Consumers:** Consumers have to decide whether or not to adopt the electronic payment instrument that is provided by the two issuers.[9] Once the consumer has adopted the electronic payment instrument, he makes one transaction with each merchant equipped with the new payment technology. The consumer's choice of a payment instrument depends on whether the consumer is located on the Hotelling line or on one of the firms' hinterländer.

The utility of a consumer located at point $x$ on the linear city who purchases the payment instrument offered by issuer $i$, located at point $\xi_i$, is given by:

$$u_B^i = v_B + \theta_i + \alpha_B n_S - p_i - t\left|\xi_i - x\right|,$$

where $v_B > 0$ is the surplus of adopting the electronic payment instrument, $\theta_i$ is the quality of issuer $i$'s payment service (e.g., applications that help consumers manage their payments), $p_i$ is the fixed adoption fee paid to the issuer , $t$ is the transportation cost, $\alpha_B \geq 0$ represents the transactional net benefit from using the electronic payment instrument at merchants', and $n_S$ is the number of merchants equipped with the technology.[10] We refer to $\alpha_B$ as the "degree of externality," as it measures the externality exerted by merchants on consumers. In our framework, $B$ stands for buyer, and $S$ for seller.

With this formulation, the utility obtained by a consumer when he adopts the electronic payment

[7] We assume a linear tariff on the merchant side. However, since the acquirers are perfectly competitive, it would be equivalent to assume a two-part tariff on the merchant side.

[8] This assumption is consistent with the industry practices in innovative markets such as Internet payments, mobile payments... However, one could think of other business models for innovative payment instruments.

[9] In our framework, we assume that it is never in the consumer's interest to buy the electronic payment instrument from both issuers, and hence, each consumer chooses between purchasing the payment instrument from issuer 1 or issuer 2, or not purchasing it (i.e., single-homes).

[10] Strictly speaking, we should write here the *expected* number of merchants. However, consistent with the two-sided market literature (e.g., Caillaud and Jullien, 2003), we assume that consumers have rational responsive expectations. This means that consumers form their expectations after firms have chosen their qualities and prices. Therefore, to clarify the exposition, we will only refer to the actual number of merchants. The same remark applies for the number of consumers. Other forms of expectations in the network economics literature include the concept of fulfilled passive expectations which has been introduced by Katz and Shapiro (1985). See Hurkens and López (2010) for a discussion and comparison of rational responsive expectations and fulfilled passive expectations.

instrument increases with the number of merchants that are equipped with the technology and with the quality of service provided by the issuer. Note that, as the issuers' payment technologies are assumed to be compatible, the consumer's utility depends on the *total* number of merchants who adopt the electronic payment instrument.[11]

We assume that $v_B$ is sufficiently large such that the market is covered in equilibrium. This assumption means that the consumers who are located on the linear city never pay cash, and trade off between buying the electronic payment instrument from bank 1 or from bank 2.

The utility of a consumer located at a distance $y$ from firm $i$ on firm $i$'s hinterland is given by

$$u_B^i = v_B + \theta_i + \alpha_B n_S - p_i - ty.$$

We assume that firm $i$'s hinterland is sufficiently large such that it is not covered in equilibrium. That is, in equilibrium, some consumers do not adopt the electronic payment instrument. The total number of consumers of bank $i$ who use the electronic payment instrument is denoted by $D_B^i$.

**Merchants:** There is a mass 1 of monopolistic merchants,[12] who have to decide whether or not to adopt the new payment technology. Merchants are heterogenous with respect to the fixed cost $F_S$ of adopting the new payment technology.[13] The fixed cost of adopting the new technology is uniformly distributed on $[0, 1]$.[14] The merchants all have the same transaction benefit $b_S > 0$. Each time a consumer pays with the new payment instrument, the merchant pays a merchant fee $m$ to the acquirer.

Finally, we make the following assumptions.

**Assumption 1:** $v_B > k_I$.

**Assumption 2:**

[11] In our framework, if $n_S = 0$, consumers still derive a positive utility from adoption, $v_B + \theta_i$, because for example the new payment instrument can be used at ATMs or abroad.

[12] This normalization is done without loss of generality, as the mass of merchants affects only the externality term in the utility function of consumers. Therefore, what matters is the product of $\alpha_B$ by the number of merchants. An increase in the mass of merchants can be interpreted as an increase in the externality parameter, $\alpha_B$.

[13] The adoption cost includes, for example, the cost of the new equipment, the cost of training staff for the new payment methods, etc. Fixed costs might differ, for example, because merchants have different skills in using payment systems, or different opportunity costs in using an electronic payment instrument instead of cash (e.g., due to different levels of fraud).

[14] With a general distribution, there are no simple conditions that ensure that issuer prices are strategic complements and that, at the same time, the second-order condition for profit maximization for the issuer prices is satisfied.

(i) $t > 4\beta\alpha_B b_S$;

(ii) $t > 2\beta b_S (b_S + \alpha_B)$;

(iii) $t > 2\beta(1 + \beta/\gamma)\alpha_B b_S$.

Assumption 1 ensures that an issuer has a strictly positive demand when it sets its price at marginal cost. Assumption 2(i) implies that $n_S$ is uniquely defined in equilibrium. Assumption 2(ii) ensures that the second-order condition for the issuers' profit maximization with respect to prices is satisfied. Finally, Assumption 2(iii) implies that an issuer's demand is increasing with the rival issuer's price.

**Timing of the game:**

1. The interchange fee is set either by a regulator, or by the payment association.
2. The issuers choose the price of the electronic payment instrument, and the acquirers choose the merchant fee.
3. Consumers and merchants decide whether or not to adopt the new payment technology.

We look for the subgame perfect equilibrium of this game, and solve the game by backward induction.

# 3 Equilibrium

In this section, we study how the interchange fee affects the adoption of the new technology, for given quality levels.

## 3.1 Stage 3: Adoption of the electronic payment instrument

**Consumer adoption.** We start by studying the consumers' adoption decision. All consumers who are located on the linear city trade off between buying the electronic payment instrument from issuer 1 or from issuer 2.[15] A consumer who is located at point $x$ buys from issuer 1 rather than issuer 2 if and only if:

$$v_B + \theta_1 + \alpha_B n_S - p_1 - tx \geq v_B + \theta_2 + \alpha_B n_S - p_2 - t(1 - x),$$

[15] Since we have assumed that in equilibrium the market is covered on the Hotelling line, we ignore the outside option of not adopting the new payment instrument when a consumer lies on the linear city.

that is, if

$$x \leq \frac{1}{2} + \frac{1}{2t}(\theta_1 - \theta_2 + p_2 - p_1).$$

On issuer $i$'s hinterland, a consumer located at $y \geq 0$ buys the electronic payment instrument if

$$v_B + \theta_i + \alpha_B n_S - p_i - ty \geq 0,$$

that is, if $y \leq (v_B + \theta_i + \alpha_B n_S - p_i)/t$.

The demand of issuer $i$ for the electronic payment instrument is given by the sum of its demand on the linear city and on its hinterland, that is,

$$D_B^i(\Theta, P, n_S) = \gamma \left[ \frac{1}{2} + \frac{1}{2t}(\theta_i - \theta_j + p_j - p_i) \right] + \beta \left( \frac{v_B + \theta_i + \alpha_B n_S - p_i}{t} \right), \tag{1}$$

for all $i \in \{1, 2\}$, where $\Theta = (\theta_1, \theta_2)$, and $P = (p_1, p_2)$. The total demand for the electronic payment instrument is given by:

$$D_B(\Theta, P, n_S) = D_B^1 + D_B^2 = \gamma + \beta \left( \frac{2v_B + \theta_1 + \theta_2 - p_1 - p_2 + 2\alpha_B n_S}{t} \right). \tag{2}$$

Note that $D_B$ increases with the number of merchants who accept the electronic payment instrument, and that its sensitivity to the number of merchants depends on a term, $2\alpha_B \beta / t$, which increases with the degree of externality $\alpha_B$.

**Merchant adoption.** We consider now the merchant's decision of whether or not to adopt the electronic payment instrument. A merchant obtains a net transactional benefit $\pi(m) = b_S - m$ when a consumer pays with the electronic payment instrument. Given that $D_B$ consumers are using the electronic payment instrument at each merchant's, a merchant with a fixed adoption cost $F_S$ decides to adopt the electronic payment technology if and only if

$$\pi(m) D_B(\Theta, P, n_S) - F_S \geq 0.$$

Note that it must be that $\pi(m) \geq 0$, otherwise no merchant would adopt the electronic payment technology. In the rest of the analysis, we assume that this condition holds.

Since $F_S$ is distributed uniformly over $[0, 1]$, the mass of merchants who adopt the electronic payment instrument is implicitly defined by

$$\widetilde{n}_S = \pi(m) D_B(\Theta, P, \widetilde{n}_S). \tag{3}$$

Under Assumption 2(i), $\widetilde{n}_S$ is uniquely defined and belongs to $[0,1]$. Using (2) and (3), the mass of merchants who adopt the innovative payment instrument is $\widetilde{n}_S(\Theta, P, m) = \mu(m)\, \widetilde{n}_S^0(\Theta, P, m)$,[16] where

$$\widetilde{n}_S^0(\Theta, P, m) = \pi(m)\left[\gamma + \frac{\beta}{t}(2v_B + \theta_1 + \theta_2 - p_1 - p_2)\right] \tag{4}$$

represents the merchants' demand where there are no externalities (i.e., when $\alpha_B = 0$), and

$$\mu(m) = \frac{1}{1 - 2\pi(m)\alpha_B \beta / t}. \tag{5}$$

Since $\mu(m)$ is strictly positive[17] and increases with the degree of externalities $\alpha_B$, it can be interpreted as a multiplier effect of the externalities on merchants' adoption.[18] To explain this multiplier effect, assume that a unit mass of consumers decides to adopt the electronic payment instrument. This increases the number of merchants who adopt the electronic payment instrument by $\pi(m)$. In turn, this generates more adoption on both issuers' hinterländer. From (2), the increase in the total number of consumers is equal to $2\alpha_B\beta\pi(m)/t$. For a similar reason, the number of merchants is raised by $2\alpha_B\beta(\pi(m))^2/t$, which in turn increases the number of consumers of both firms by $(2\alpha_B\beta\pi(m)/t)^2$. At the end of the day, repeating this reasoning an infinite number of times, we find that the number of consumers is increased by $\mu(m)$, which is given by (5).

Similarly, let $\widetilde{D}_B(\Theta, P, m) \equiv D_B(\Theta, P, \widetilde{n}_S(\Theta, P, m))$ and $\widetilde{D}_B^i(\Theta, P, m) \equiv D_B^i(\Theta, P, \widetilde{n}_S(\Theta, P, m))$. The number of consumers who adopt the electronic payment instrument can be written as $\widetilde{D}_B = \mu \widetilde{D}_B^0$, where

$$\widetilde{D}_B^0(\Theta, P) = \gamma + \frac{\beta}{t}(2v_B + \theta_1 + \theta_2 - p_1 - p_2) \tag{6}$$

is the consumers' demand for the electronic payment instrument when there are no externalities (i.e., $\alpha_B = 0$). Finally, from equation (1), the mass of consumers of issuer $i = 1, 2$ is

$$\widetilde{D}_B^i(\Theta, P, m) = \gamma\left[\frac{1}{2} + \frac{1}{2t}(\theta_i - \theta_j + p_j - p_i)\right] + \frac{\beta}{t}\left(v_B - p_i + \theta_i + \alpha_B \mu \widetilde{n}_S^0(\Theta, P, m)\right). \tag{7}$$

[16] If $\mu\widetilde{n}_S^0 < 0$, no merchant adopts the electronic payment instrument (i.e., $\widetilde{n}_S = 0$), whereas if $\mu\widetilde{n}_S^0 > 1$, they all adopt (i.e., $\widetilde{n}_S = 1$).

[17] Indeed, from Assumption 2(i), $t > 4b_S\alpha_B\beta$ implies that $t > 2\pi(m)\alpha_B\beta$.

[18] This multiplier effect is similar to the one found in Jeon, Jullien and Klimenko (2011).

**Comparative statics.** From now on, we restrict our analysis to the case of an interior solution, where $\widetilde{n}_S \in (0,1)$. We have the following comparative statics.

**Lemma 1** *The mass of merchants who adopt the electronic payment instrument decreases with the prices chosen by the issuers and the acquirers, and it increases with the levels of quality. It also increases with the degree of externality, $\alpha_B$, and the size of the hinterländer, $\beta$.*

**Proof.** See Appendix A1. ■

Higher prices for consumers or lower quality levels decrease total consumer demand, and thus merchant adoption, due to the externality.

**Lemma 2** *The mass of consumers who adopt the electronic payment instrument decreases with the prices chosen by the issuers and the acquirers, and it increases with the levels of quality. It also increases with the degree of externality, $\alpha_B$, and the size of the hinterländer, $\beta$.*

**Proof.** See Appendix A2. ■

When an issuer increases its price (resp., decreases its quality), the total consumer demand is reduced because of two effects. First, this price increase (resp., quality reduction) has a negative direct effect on consumer utility. Second, it has an negative indirect effect on the number of merchants who decide to adopt the electronic payment instrument. Note also that, because of the adoption externality, the consumer demand decreases with the merchant fee, as a higher merchant fee reduces the number of merchants, which in turn lowers consumer demand.

Finally, we study how an issuer's demand is affected by its own price (resp., quality) and by its rival's price (resp., quality). Define $\delta(m) \equiv \partial \widetilde{D}_B^i / \partial p_i$. For issuer $i = 1, 2$, we have

$$\delta(m) = \frac{\partial D_B^i}{\partial p_i} + \left. \frac{\partial D_B^i}{\partial n_S} \right|_{n_S = \widetilde{n}_S} \frac{\partial \widetilde{n}_S}{\partial p_i} = -\frac{\gamma}{2t} - \frac{\beta}{t} - \frac{\beta^2}{t^2} \alpha_B \mu(m) \pi(m). \tag{8}$$

Since $\delta(m) < 0$, we have the standard effect that an issuer's demand decreases with its own price, that is, $\partial \widetilde{D}_B^i / \partial p_i < 0$, for $i = 1, 2$. Similarly, we find that $\partial \widetilde{D}_B^i / \partial \theta_i = -\delta(m) > 0$.

However, due to the externality between the consumer side and the merchant side, the effect on an issuer's demand of the price or the quality level that is chosen by the other firm is less clear-cut.

Indeed, for $i, j = 1, 2$, and $j \neq i$, we have

$$\begin{aligned}\Delta(m) &\equiv \frac{\partial \widetilde{D}_B^i}{\partial p_j} = \frac{\partial D_B^i}{\partial p_j} + \left.\frac{\partial D_B^i}{\partial n_S}\right|_{n_S = \widetilde{n}_S} \frac{\partial \widetilde{n}_S}{\partial p_j} \\ &= \frac{\gamma}{2t} - \frac{\beta^2}{t^2} \alpha_B \mu(m) \pi(m), \end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{\partial \widetilde{D}_B^i}{\partial \theta_j} &= \frac{\partial D_B^i}{\partial \theta_j} + \left.\frac{\partial D_B^i}{\partial n_S}\right|_{n_S = \widetilde{n}_S} \frac{\partial \widetilde{n}_S}{\partial \theta_j} \\ &= -\frac{\gamma}{2t} + \frac{\beta^2}{t^2} \alpha_B \mu(m) \pi(m) = -\Delta(m). \end{aligned} \tag{10}$$

Equations (9) and (10) show that competition between the two issuers with respect to prices or qualities gives rise to two conflicting effects. On the one hand, if the rival firm decreases its price (resp., increases its level of quality), a firm faces a reduction of its demand, which corresponds to the standard effect of competition (the first term in equations (9) and (10)). On the other hand, a lower price from the rival firm (resp., a higher level of quality) attracts more merchants due to the externality effect, which in turn increases the consumer demand for the firm. This effect corresponds to the second term in equations (9) and (10).[19]

In a pure Hotelling setting with $\beta = 0$, only the competition effect would be present, whereas in a pure local monopolies setting with $\gamma = 0$, only the externality effect would be operational.

In our framework, Assumption 2(iii) ensures that the first effect always dominates the second effect, and, therefore, an issuer's demand increases with the price chosen by its rival (i.e., $\Delta(m) > 0$). Lemma 3 summarizes the impact of prices and quality levels on an issuer's demand.

**Lemma 3** *An issuer's demand decreases with its own price (resp., increases with its level of quality), whereas it increases with the rival issuer's price (resp., decreases with its rival issuer's level of quality).*

**Proof.** See Appendix A3. ■

[19] These effects are also present in the adoption model of Armstrong (2006), with a monopolistic platform. However, there is a difference in our setting. Armstrong (2006) looks at the impact on consumer demand of the price that is chosen by the platform for merchants. Our focus is different, as we study the pricing strategies of two players that are on the same side of the market (the issuing side), and are both affected by the same externality, that is, the number of merchants who adopt the electronic payment instrument.

### 3.2 Stage 2: Consumers' and merchants' prices

At this stage, banks compete in prices. As the acquirers are perfectly competitive, the merchant fee is equal to $m^* = a$ in the equilibrium of the subgame, where $a$ denotes the interchange fee paid by the acquirers to the issuers. We now proceed by determining the equilibrium for issuers' prices. To simplify the exposition, in what follows, we replace $m$ for $m^* = a$.

**Price equilibrium.** Each issuer $i = 1, 2$ chooses the price $p_i$ that maximizes its profit,

$$\pi_i = (p_i + a\widetilde{n}_S - k_I)\widetilde{D}_B^i - C(\theta_i).$$

The issuer receives a price $p_i$ from each consumer who adopts its electronic payment instrument, and an interchange fee revenue $a\widetilde{n}_S\widetilde{D}_B^i$ from acquirers.

We denote $M_i = p_i + a\widetilde{n}_S - k_I$ issuer $i$'s margin, and

$$\phi\left(a\right) \equiv -\frac{\partial M_i}{\partial p_j} = \left(\beta/t\right) a\mu\left(a\right)\pi\left(a\right) \geq 0, \tag{11}$$

the sensitivity of an issuer's margin to the price of its rival.[20] The first-order condition is then

$$\frac{d\pi_i}{dp_i} = \left(1 - \phi(a)\right)\widetilde{D}_B^i + \delta(a) M_i = 0, \tag{12}$$

whereas the second-order condition always holds as[21]

$$\frac{d^2\pi_i}{dp_i^2} = 2\left(1 - \phi(a)\right)\delta(a) \leq 0. \tag{13}$$

The first-order condition (12) shows that the issuer trades off between increasing its margin and increasing the volume of transactions. Note that, due to interchange fee revenues and indirect externalities, the marginal effect of a price increase on the issuer's margin ($\phi$) is lower than one, that is, lower than in the standard Hotelling model.

In Appendix B2, we prove that, under Assumption 2, prices are strategic complements. We denote $x_i$ and $y_i$ the mass of consumers who adopt issuer $i$'s electronic payment instrument on the

[20] An issuer's margin is sensitive to the rival issuer's price because issuers receive interchange fee revenues, which depend on merchant adoption. When the rival issuer increases its price, all things being equal, an issuer's margin decreases. Note that $\phi$ is equal to zero when there are no interchange fees, and that $\partial M_j/\partial p_i = \partial M_i/\partial p_j$.

[21] See Appendix B1 for more details.

Hotelling line and on issuer $i$'s hinterland, respectively, when both issuers set their prices to zero. Finally, we denote $d_i = x_i + y_i$. We can now characterize the price equilibrium.

**Lemma 4** *In equilibrium, issuer $i$ charges a price*

$$p_i^* = \frac{2\delta\left(1-\phi\right)\left[\left(k_I - a\overline{n}_S\right)\delta - \left(1-\phi\right)d_i\right] + \left[\left(k_I - a\overline{n}_S\right)\delta - \left(1-\phi\right)d_j\right]\left[\delta\phi - \left(1-\phi\right)\Delta\right]}{4\delta^2\left(1-\phi\right)^2 - \left[\delta\phi - \left(1-\phi\right)\Delta\right]^2},$$

*where* $\overline{n}_S = \widetilde{n}_S|_{P=(0,0)}$.

**Proof.** See Appendix B3. ■

In the symmetric case, where the two issuers offer the same quality of service $\theta$, the equilibrium issuer price simplifies to

$$p^* = \frac{\left(1-\phi\left(a\right)\right)d - \left(k_I - a\overline{n}_S\left(a\right)\right)\delta\left(a\right)}{\delta\left(a\right)\left(3\phi\left(a\right)-2\right) - \left(1-\phi\left(a\right)\right)\Delta\left(a\right)},$$

where $d = x_1 + y_1 = x_2 + y_2$. Since the denominator of $p^*$ is always strictly positive,[22] everything else equal, the issuer price is higher when the "market size" $d$ is higher and when the number of merchant adopters when issuers' prices are equal to zero, $\overline{n}_S$, is higher.

In the pure Hotelling setting, where $\beta = 0$, the symmetric equilibrium price becomes $p^* = t + k_I - a\overline{n}_S\left(a\right)$. The price is equal to the transportation cost of consumers minus the perceived marginal cost of an issuer, which depends on interchange fee revenues per consumer.

**Comparative statics.** The symmetric equilibrium price $p^*$ can be a priori either increasing or decreasing with the degree of externalities $\alpha_B$.[23] On the one hand, when the externalities are stronger, the value of adopting the innovation increases for consumers, which provides an issuer with incentives to extract more rents by charging higher adoption prices. Since prices are strategic complements, the other issuer reacts by increasing its price. On the other hand, the issuer's margin increases through higher interchange fee revenues, which amounts to a reduction of its perceived marginal cost. This effect provides the issuer with the incentive to lower its price.

We denote by $n_S^*\left(\Theta, a\right) = \widetilde{n}_S\left(\Theta, P^*, m^*\right)$ and $D_B^*\left(\Theta, a\right) = \widetilde{D}_B\left(\Theta, P^*, m^*\right)$ the mass of merchants who adopt the new technology and the consumer demand, respectively, at the equilibrium of the price-setting subgame.

[22] See Appendix B4 for the proof.
[23] See Appendix B5.

### 3.3 Stage 1: Interchange fee

We now analyze the impact of the interchange fee on users' decisions to adopt the new payment technology. We start by studying how the interchange fee affects the price of the electronic payment instrument for consumers, and the adoption decisions of consumers and merchants. Then, we determine the profit-maximizing interchange fee.

**Impact of the interchange fee on the issuers' prices.** An important question is whether raising the interchange fee leads to lower prices for consumers. In Appendix C1, we show that $dp_i^*/da$ has the same sign as $\partial^2\pi_i/\partial p_i \partial a$. From the first-order condition (12), we have

$$\left.\frac{\partial^2 \pi_i}{\partial p_i \partial a}\right|_{p_1^*,p_2^*} = \underbrace{(1-\phi(a))\frac{\partial \widetilde{D}_B^i}{\partial a}}_{(-)} + \underbrace{\delta'(a)\, M_i}_{(+)} + \underbrace{\delta(a)\frac{\partial M_i}{\partial a} - \phi'(a)\,\widetilde{D}_B^i}_{(+)\text{ or }(-)}. \tag{14}$$

An increase in the interchange fee has therefore different and opposite effects on issuer $i$'s price. First, it lowers the number of consumers who adopt the EPI, which reduces the issuer's incentive to increase its price. This corresponds to the first term of equation (14), which is negative as $\partial \widetilde{D}_B^i/\partial a = (\beta/t)\alpha_B \partial \widetilde{n}_S/\partial a < 0$.

Second, the sensitivity of consumer demand to the issuer's price decreases, and therefore, the issuer's incentives to lower its price are reduced–this corresponds to the second term of equation (14), which is positive as $\delta'(a) = (\beta\mu(a)/t)^2\alpha_B > 0$. Finally, the sum of the third and fourth terms of (14) can be written as

$$\delta(a)(\partial M_i/\partial a) - \phi'(a)\,\widetilde{D}_B^i = \mu(a)\left[a\mu(a) - \pi(a)\right]\left[(\beta/t)\widetilde{D}_B^i - \delta(a)\,\widetilde{D}_B^0\right].$$

Since $\delta < 0$ from (8) and $\mu > 0$, this expression has the sign of $a\mu(a) - \pi(a)$, which is negative for low values of the interchange fee, and positive otherwise.[24] On the one hand, the revenue per transaction becomes higher when the interchange fee increases, which provides the issuer with an incentive to increase its price. On the other hand, the number of merchants who adopt the EPI is reduced, which gives the issuer a countervailing incentive.

To sum up, the impact of the interchange fee on the issuer's price is a priori ambiguous. We can however show the following result for the symmetric equilibrium price.

[24] See Appendix C2.

**Proposition 1** *If $b_S/\alpha_B$ is sufficiently high (i.e., higher than $2\beta(\gamma+\beta)/((2\beta+\gamma)(3\beta+\gamma))$), the symmetric equilibrium issuer's price varies non-monotonically with the interchange fee; it is decreasing for low values of the interchange fee, and increasing otherwise. For lower values of $b_S/\alpha_B$, the issuer's price always decreases with the interchange fee.*

**Proof.** See Appendix C3. ■

This non-monotonic relationship between the issuer's price and the interchange fee is in contrast with the literature (e.g., Rochet and Tirole (2002 and 2003), Wright (2002 and 2004)), which usually assumes that the interchange fee corresponds to a reduction of the issuer's marginal cost that is passed through to consumers.

In our framework, two effects are at play, an interchange revenue effect and a quality degradation effect. First, the equilibrium issuers' prices are affected by the interchange fee revenues, which depend on the number of transactions, $\widetilde{n}_S$. As the interchange revenue ($a\widetilde{n}_S$) is increasing with the interchange fee for low values of the interchange fee, and decreasing otherwise, the price offered to consumers for the electronic payment instrument tends to decrease for low values of the interchange fee, and to increase for high values of it. In the literature, this effect is not present, because issuers charge per-transaction prices, whereas in our framework, they charge only a lump-sum adoption fee.

Second, since consumers value the presence of merchants, an increase in the interchange fee can be interpreted as a lower quality of service for consumers, as it leads to a lower number of merchants. Issuers react to this quality degradation effect by lowering their prices.

When $b_S/\alpha_B$ is low, interchange fee revenues are of a low magnitude as merchant adoption is low, and the quality degradation effect dominates the interchange fee revenue effect. We then have the standard effect that the issuers' prices decrease with the interchange fee. By contrast, when $b_S/\alpha_B$ is sufficiently high, the interchange fee revenue effect dominates the quality degradation effect, and the issuers' prices vary non-monotonically with the interchange fee.

In the pure Hotelling setting, where $\beta = 0$, the prices charged by the issuers decrease with the interchange fee for $a \leq b_S/2$, and then increase with the interchange fee.

**Effect of the interchange fee on the adoption of the innovation.** We now study the effect of the interchange fee on the adoption of the new technology by consumers and merchants.

Since $D_B^* = \mu(a)\widetilde{D}_B^0(p_1^*, p_2^*)$, the effect of the interchange fee on consumer adoption is given by

$$\begin{aligned} \frac{dD_B^*}{da} &= \frac{\partial \mu}{\partial a}\widetilde{D}_B^0(p_1^*, p_2^*) + \mu\left[\frac{\partial \widetilde{D}_B^0}{\partial p_1}\frac{\partial p_1^*}{\partial a} + \frac{\partial \widetilde{D}_B^0}{\partial p_2}\frac{\partial p_2^*}{\partial a}\right] \\ &= \underbrace{\frac{-\beta\alpha_B\mu^2}{t}\widetilde{D}_B^0(p_1^*, p_2^*)}_{(-)} - \frac{\beta\mu}{t}\underbrace{\left[\frac{\partial p_1^*}{\partial a} + \frac{\partial p_2^*}{\partial a}\right]}_{(+)\text{ or }(-)}. \end{aligned} \tag{15}$$

A higher interchange fee has two different effects on consumer adoption. The first effect is the quality degradation effect, which is negative and represented by the first term in the above equation; a higher interchange fee reduces the number of merchants, which lowers the adoption benefit for consumers. The second effect is a price effect, which is ambiguous as shown in Proposition 1.

If $dp_i^*/da \geq 0$, we have $dD_B^*/da \leq 0$, that is, a higher interchange fee reduces adoption by consumers. If $dp_i^*/da \leq 0$, the sign of $dD_B^*/da$ is indeterminate. On the one hand, a higher interchange fee decreases the fees charged to consumers for the electronic payment instrument, which increases consumer demand. On the other hand, it increases the merchant fee, which reduces the number of merchant who adopt the innovation. This, in turn, decreases the number of consumers who adopt the innovation.

The following Proposition shows that consumer adoption can vary non-monotonically with the interchange fee.

**Proposition 2** *If the degree of externality $\alpha_B$ is high and if $b_S$ is low, consumer adoption decreases with the interchange fee for low values and high values of it. In all other cases, consumer adoption increases with the interchange fee for low values of it and decreases for high values of it.*

**Proof.** See Appendix C4. ■

This result shows that, due to the externality, consumer adoption is strongly linked to merchant adoption. As a consequence, to ensure adoption on the consumers' side, the interchange fee should be set at a level that will also foster adoption on the merchants' side, that is, not too high. At the extreme, the value of the interchange fee that maximizes consumer adoption can even be zero.

The effect of the interchange fee on the merchants' adoption is given by

$$\frac{dn_S^*}{da} = -D_B^* + \frac{dD_B^*}{da}\pi(a).$$

As on the consumers' side, merchant adoption is strongly linked to consumer adoption. If consumer adoption decreases with the interchange fee, that is, if $dD_B^*/da \leq 0$, the number of merchants who adopt the new payment technology also decreases with the interchange fee. This is because merchants have to pay a higher price to use the new payment technology, while a smaller share of consumers adopts the innovation when the interchange fee increases.

By contrast, if $dD_B^*/da \geq 0$, raising the interchange fee causes ambiguous effects on merchants' adoption. On the one hand, a higher interchange fee increases the merchant fee, which reduces the profitability of merchants, and hence, the number of merchants who adopt the new technology. On the other hand, a higher interchange fee yields to a higher volume of transactions for merchants, as it decreases the price of the new technology for consumers (since $dD_B^*/da \geq 0$ implies that $dp_i^*/da \leq 0$). This effect provides merchants with higher incentives to adopt the new technology. We prove that, under our assumptions, if the transportation cost is sufficiently high, the former effect always dominates the latter, that is, $dn_S^*/da \leq 0$.

**Proposition 3** *If the transportation cost $t$ is sufficiently high, merchant adoption decreases with the interchange fee.*

**Proof.** See Appendix C5. ■

As we show in Appendix C5, in our setting, merchant adoption can increase with the interchange fee only in some particular cases, where the transportation cost is very low and close to the minimal value imposed by Assumption 2. Otherwise, merchant adoption is reduced when the interchange fee becomes higher.

Figures 1a and 1b below illustrate the two different cases that occur in most cases, according to our results in Propositions 2 and 3.[25] In both figures, merchant adoption is higher when the interchange fee is higher. In Figure 1a, the degree of externality if low, and consumer adoption varies non-monotically with the interchange fee. By contrast, in Figure 1b, where the degree of externality is higher, consumer adoption decreases with the interchange fee. In this latter case,

[25] The two figures are drawn for the following parameter values, which satisfy our Assumptions 1 and 2: $t = 3$, $v_B = 4$, $\beta = 0.5$, $\gamma = 0.5$, $k_I = 0$, $b_S = 0.5$, and $\theta_1 = \theta_2 = 0.5$.

users' adoption is maximized by setting a zero interchange fee.

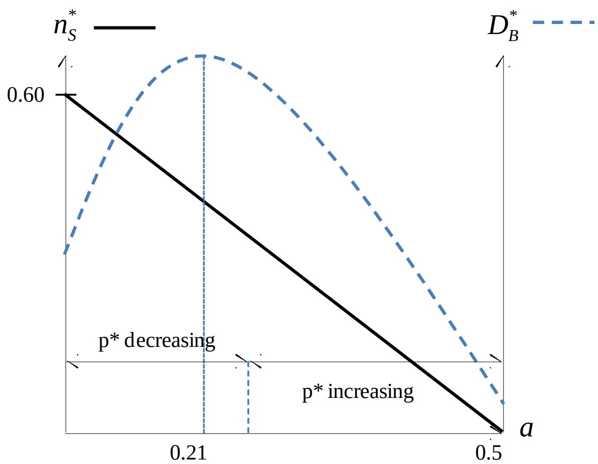


Fig. 1a: Impact of interchange fee on adoption (low degrees of externality, $\alpha_B = 0.1$)

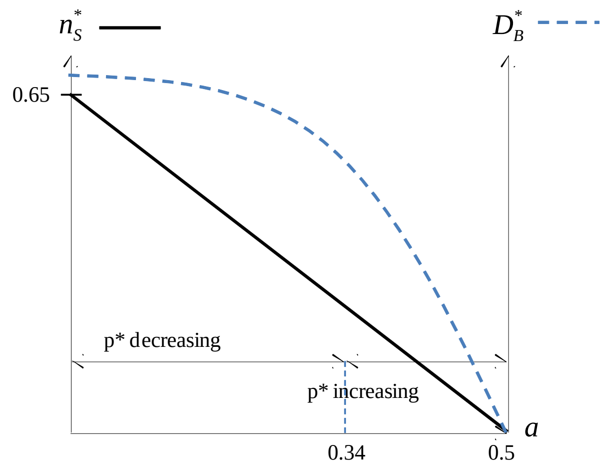


Fig. 1b: Impact of interchange fee on adoption (high degrees of externality, $\alpha_B = 0.6$)

**Profit-maximizing interchange fee.** The payment platform sets the interchange fee so as to maximize banks' joint profits. As the acquirers are perfectly competitive, this amounts to maximizing the issuers' profits,

$$\pi_1 + \pi_2 = (p_1^*(a) + an_S^* - k_I)\,\widetilde{D}_B^1(P^*(a), m^*(a)) + (p_2^*(a) + an_S^* - k_I)\,\widetilde{D}_B^2(P^*(a), m^*(a)).$$

We start by analyzing the effect of an increase in the interchange fee on issuer $i$'s profit, which is given by

$$\frac{d\pi_i}{da} = \left.\frac{\partial \pi_i}{\partial a}\right|_{P^*} + \left.\frac{\partial \pi_i}{\partial m}\right|_{P^*}\frac{dm^*(a)}{da} + \left.\frac{\partial \pi_i}{\partial p_i}\right|_{P^*}\frac{dp_i^*(a)}{da} + \left.\frac{\partial \pi_i}{\partial p_j}\right|_{P^*}\frac{dp_j^*(a)}{da}.$$

From the envelop theorem, we have $\partial\pi_i/\partial p_i|_{P^*} = 0$. Furthermore, $dm^*(a)/da = 1$. Therefore,

$$\frac{d\pi_i}{da} = \underbrace{n_S^*\widetilde{D}_B^i(P^*, m^*)}_{\text{Direct effect: }(+)} + \underbrace{\left.\frac{\partial \pi_i}{\partial m}\right|_{P^*}}_{\text{Indirect effect n°1: }(+)} + \underbrace{\frac{dp_j^*(a)}{da}\left.\frac{\partial \pi_i}{\partial p_j}\right|_{P^*}}_{\text{Indirect effect n°2: }(+)\text{ or }(-)}.$$

The interchange fee impacts issuer $i$'s profit through a direct effect and two indirect effects, which depend respectively on the merchant fee and on the price that is chosen by the rival issuer at the price competition stage of the game. The direct effect is positive, as $n_S^*\widetilde{D}_B^i > 0$. This is because a higher interchange fee enables the issuer to obtain higher revenues per transaction. The first indirect effect is negative, as $\partial \pi_i/\partial m < 0$. A higher merchant fee lowers issuer $i$'s profit as the number of consumers and merchants who adopt the innovation decreases, and hence, the number of transactions is reduced. The sign of the second indirect effect can be either positive or negative. As shown in Proposition 1, depending on the value of $b_S$ and on the interchange fee, the price chosen by the issuer can either increase or decrease with the interchange fee. Furthermore, it can be shown that $\partial \pi_i/\partial p_j|_{P*} > 0$ if $t$ is sufficiently high.[26]

The payment platform chooses an interchange fee which reflects a trade-off between increasing the interchange fee revenues, lowering the number of adopting merchants, and thus consumer demand, and impacting the price competition that takes place at stage 2.

We denote by $a^S$, $a^B$, $a^V$ and $a^\pi$ the interchange fee that maximizes merchant adoption, consumer adoption, the volume of transactions, and the issuers' profits, respectively, in a symmetric equilibrium. From Proposition 2, $a^B$ can either be zero or an intermediate value, that is, $a^B \in [0, b_S)$, as consumer adoption may vary non-monotonically with the interchange fee.[27] From Proposition 3, if the transportation cost is sufficiently high, $a^S = 0$, as merchant adoption decreases with the interchange fee.

We now proceed by comparing $a^S$, $a^B$, $a^V$ and $a^\pi$ in two simple benchmark cases. First, consider that there is no externality on the consumers' side, that is, $\alpha_B = 0$. We find that $a^\pi = a^B = b_S/2$ and that $a^V \in [0, b_S/2)$. Therefore, $a^S \leq a^V < a^B = a^\pi$. In other words, the profit-maximizing platform sets the interchange fee at a level that maximizes consumer adoption, but which is too high to maximize merchant adoption or the volume of transactions. Intuitively, since consumers do not value the presence of merchants on the platform, whereas the reverse is true, the platform favors consumer adoption over merchant adoption.

Second, consider the pure Hotelling case, where $\beta = 0$. As the total demand is constant and equal to $\gamma$, consumer adoption is insensitive to the interchange fee, while the number of adopting merchants is equal to $\gamma\pi(a)$, and hence, independent of consumer adoption. Thus, the pure Hotelling case can be interpreted as a situation where there is no externality on the merchants' side. We find that $a^S = a^V = 0$, whereas $a^B \in [0, b_S]$ and $a^\pi \in [0, b_S]$. In this case, as consumers value merchant adoption, whereas the reverse is not true, the platform may favor merchant adoption over consumer adoption.

In the general case, where $\beta > 0$ and $\alpha_B > 0$, there are two different cases. First, if consumer

[26] See Appendix C6.

[27] We have $a^B < b_S$ as $dD^*_B/da|_{a=b_S} < 0$.

and merchant adoption decrease with the interchange fee, we have $a^B = a^S = a^V = 0$.[28] From Proposition 2, this happens if the degree of externality $\alpha_B$ is high and if $b_S$ is low. A sufficient condition for the profit-maximizing interchange fee to be equal to zero is that the issuers' margin decreases with the interchange fee at $a^B$. Otherwise, if $n_S^*$ is sufficiently high at $a^B$, the profit-maximizing interchange fee can a priori be greater than zero.

Second, if consumer adoption varies non-monotically with the interchange fee, we prove in Appendix C7 that $a^\pi \leq a^B$ if and only if $a^B \geq b_S/2 - \alpha_B/4$. If this condition is verified, since merchant adoption decreases with the interchange fee, we have $a^S = 0 \leq a^\pi \leq a^B$. We have $a^S \leq a^V \leq a^B$. Besides, we have $a^V \leq a^\pi$ if the marginal revenue obtained from an increase of the interchange fee, taking the volumes of transactions as fixed, is positive at $a^V$. In other cases, we can either have $a^V \leq a^\pi$ or the reverse.

**Welfare-maximizing interchange fee.** Now, we consider the case where the interchange fee is set by the regulator. The regulator chooses the interchange fee to maximize social welfare, which is defined as the sum of users' surplus and issuers' profits. We denote by $a^W$ the welfare-maximizing interchange fee, and define $u^* = v_B + \theta + \alpha_B n_S^* - p^*$. We find that $a^\pi \geq a^W$ if

$$\left.\frac{dn_S^*}{da}\right|_{a=a^\pi} \left(\alpha_B + \frac{n_S^*(a^\pi)}{\gamma + \beta u^*(a^\pi)/(2t)}\right) \geq \left.\frac{dp^*}{da}\right|_{a=a^\pi}, \tag{16}$$

otherwise, we have $a^\pi \leq a^W$.[29]

In other words, given that $dn_S^*/da < 0$, the profit-maximizing interchange fee is suboptimally high if at $a = a^\pi$, prices are more sensitive to a higher interchange fee than the number of merchants. In particular, we have $a^\pi \leq a^W$ if $dp^*/da|_{a=a^\pi} > 0$.

**Numerical example.** We consider the same parameter values as for Figure 1: $\beta = 0.5$, $\gamma = 0.5$, $b_S = 0.5$, $k_I = 0$, $\theta_1 = \theta_2 = 0.5$, $t = 3$ and $v_B = 4$. When the degree of externality is low ($\alpha_B = 0.1$), we find that $a^S = a^V = a^W = 0 < a^\pi = 0.1986 < a^B = 0.2040$. When the degree of externality is high ($\alpha_B = 0.6$), we have $a^S = a^B = a^V = a^\pi = a^W = 0$.

[28] Similarly, if $b_S$ is close to zero, we also find that $a^B = a^S = a^V = 0$.
[29] See Appendix C8 for the detailed analysis.

## 4 Investment decisions

In this Section, we extend our baseline model to study issuers' incentives to upgrade the quality of the electronic payment instrument, and the effect of the interchange fee on investment incentives. We compare two different scenarios: (i) the issuers set the levels of quality non-cooperatively, and (ii) the issuers cooperate to improve the quality level of the electronic payment instrument. We assume that when the issuers cooperate to develop the electronic payment instrument, they set the levels of quality jointly, but do not share the investment costs.[30] Offering a level of quality $\theta_i$ to consumers costs $C(\theta_i)$ to firm $i$, with $C'(\theta_i) \geq 0$ and $C''(\theta_i) \geq 0$.

The timing of the game is modified as follows:

1. The interchange fee is set either by a regulator, or by the payment association.
2. The issuers choose cooperatively or non cooperatively the levels of quality.
3. The issuers choose the price of the electronic payment instrument, and the acquirers choose the merchant fee.
4. Consumers and merchants decide whether or not to adopt the new payment technology.

The analysis of stage 3 and stage 4 is similar to the baseline model. We start by analyzing the effect of a change in the quality levels on equilibrium prices, then we determine the equilibrium with and without cooperation.

### 4.1 The effect of quality levels on equilibrium prices

With the following Lemma, we characterize the effect of the quality levels on the equilibrium prices.

**Lemma 5** *The price chosen by issuer $i = 1, 2$ increases with its level of quality, $\theta_i$, and decreases with the level of quality chosen by the rival issuer, $\theta_j$.*

**Proof.** See Appendix D1. ■

When firm $i = 1, 2$ increases its quality, $\theta_i$, from Lemma 2, for given prices, its consumer demand increases, as a result of both a direct and an indirect effect. First, firm $i$ can charge a higher price

[30] This is referred to as R&D cartelization in the R&D cooperation literature (see Kamien & al. (1992)). In all cases, we assume that the payment technologies that are developed by the issuers are compatible on the merchants' side.

to its consumers due to the additional surplus provided by a higher quality level to its consumers and the increase in merchant adoption. Second, the rival issuer can also charge a higher price to its own consumers when the number of merchants increases. Since prices are strategic complements, issuer $i$ reacts by increasing its price.

By contrast, the incidence of the quality that is chosen by firm $j$ on firm $i$'s price reflects two conflicting effects. First, there is a *competition effect*. If firm $j$ increases its quality level, it becomes more attractive to consumers. Firm $i$ then reacts by reducing its price. However, this reaction is offset by firm $i$'s trade-off between competing with firm $j$ on the linear city and extracting consumer surplus on its hinterland. Firm $i$'s reaction is all the more aggressive as the size of the hinterland is small. Second, there is an *externality effect*. When firm $j$ increases its quality, from Lemma 1, this increases the demand of merchants, which has a positive impact on firm $i$'s attractiveness for consumers. Therefore, firm $i$ can increase its price.[31] In our setting, due in particular to the uniform distribution for $F_S$, the competition effect always dominates the externality effect.

## 4.2 No cooperation

### 4.2.1 Stage 2: investment decisions

We now study the choice of quality levels for the payment instrument when banks do not cooperate. Let $\widetilde{\pi}_i(\theta_1,\theta_2) = (p_i^*(\theta_1,\theta_2) + a\widetilde{n}_S(\Theta, P^*(\Theta)) - k_I)\widetilde{D}_B^i(\Theta, P^*(\Theta))$ denote issuer $i$'s profit, gross of investment cost, at the equilibrium of stage 3. At stage 2, each issuer $i = 1,2$ sets its level of quality $\theta_i$ so as to maximize its profit,

$$\pi_i(\theta_1,\theta_2) = \widetilde{\pi}_i(\theta_1,\theta_2) - C(\theta_i),$$

where $P^* = (p_1^*, p_2^*)$. From the envelop theorem, the first order condition is

$$\frac{d\pi_i}{d\theta_i} = \underbrace{\left.\frac{\partial\widetilde{\pi}_i}{\partial\theta_i}\right|_{P^*}}_{(+)} + \underbrace{\left.\frac{\partial\widetilde{\pi}_i}{\partial p_j}\right|_{P^*}\frac{\partial p_j^*}{\partial\theta_i}}_{(+)\text{ or }(-)} - C'(\theta_i) = 0.$$

The second order condition for a local maximum holds if $C(\cdot)$ is sufficiently convex, and we assume that it is the case (see Appendix D2 for a discussion - A REFAIRE).

[31] Note that the magnitude of this externality effect is higher when $\beta$ or $\alpha_B$ is higher. It follows that, when $\beta$ is high, the externality effect dominates the competition effect, whereas the reverse is true when $\beta$ is low.

The derivative of the issuer's profit with respect to the level of quality is expressed as a sum of a direct and an indirect effect. The direct effect on the issuer's profit, gross of investment cost, can be written as[32]

$$\left.\frac{\partial \widetilde{\pi}_i}{\partial \theta_i}\right|_{P^*} = \phi \widetilde{D}_B^i - \delta M_i. \tag{17}$$

Since $\delta < 0$ and $\phi \geq 0$, the direct effect of the level of quality on the issuer's gross profit is always positive. Taking prices as constant, a higher quality implies a higher demand, which benefits the firm; this corresponds to the first term in (17). Furthermore, a higher quality increases merchant adoption, which in turn raises interchange fee revenues –see the second term in (17).

The indirect effect is given by

$$\left.\frac{\partial \widetilde{\pi}_i}{\partial p_j}\right|_{P^*} \frac{\partial p_j^*}{\partial \theta_i} = \underbrace{\left[\Delta M_i - \phi \widetilde{D}_B^i\right]}_{\text{(+) or (-)}} \underbrace{\frac{\partial p_j^*}{\partial \theta_i}}_{\text{(-)}}. \tag{18}$$

To understand the sign of the indirect effect, we start by assuming that the interchange fee is equal to zero, which implies that $\phi = 0$. The first term into brackets into equation (18) is then positive, as Lemma 3 shows that $\Delta > 0$. As $\partial p_j^*/\partial \theta_i < 0$ from Lemma 5, the indirect effect is negative if there is no interchange fee. The intuition is that, if firm $i$ invests in quality, it generates an aggressive reaction from its rival in terms of lower prices, which hurts the firm's profit.

When the interchange fee increases, the reaction of the rival can be offset by higher interchange fee revenues, as the number of merchants who adopt the electronic payment instrument increases when the rival issuer reduces its price. Therefore, when the interchange fee is high, the indirect effect may become positive (to check: is it possible under our assumptions and conditions).

Finally, if the interchange fee becomes close to its maximum value, that is $a = b_S$, the merchants' transaction benefit, $\pi(a)$, is equal to zero. In this case, the indirect effect is negative, as $\phi = 0$ if $\pi(a) = 0$.

The following table summarizes the sign of the direct and the indirect effect in the non-cooperative case.

[32] Since $\partial \widetilde{D}_B^i/\partial \theta_i = -\delta(a)$ and $\partial M_i/\partial \theta_i = a \partial \widetilde{n}_S/\partial \theta_i = \phi(a)$.

| No cooperation | Direct effect | Indirect effect |
|---|---|---|
| Interchange small | (+) | (-)<br>Investments increase price competition. |
| Interchange intermediate | (+) | (-) or (+)<br>... |
| Interchange very high | (+) | (-) |
| | | ... |

Numerical examples; figures.

Est-ce qu'on peut avoir une expression pour theta*?

Effet de l'IF sur theta*?

## 4.3 Cooperation

We now proceed by analyzing quality investment decisions at stage 2 when the issuers cooperate on the quality choices.

### 4.3.1 Stage 2: investment decisions

The issuers choose cooperatively the levels of quality $\theta_1$ and $\theta_2$ so as to maximize their joint profit,

$$\begin{aligned}\pi_1+\pi_2 &= (p_1^*(\theta_1,\theta_2)+a\widetilde{n}_S-k_I)\widetilde{D}_B^1(\Theta,P^*(\Theta))+(p_2^*(\theta_1,\theta_2)+a\widetilde{n}_S-k_I)\widetilde{D}_B^2(\Theta,P^*(\Theta)) \\ &\quad -C(\theta_1)-C(\theta_2).\end{aligned}$$

Differentiating the joint profit with respect to $\theta_i$, and using the envelop theorem, we have

$$\frac{d(\pi_1+\pi_2)}{d\theta_i}=\frac{\partial(\pi_1+\pi_2)}{\partial\theta_i}+\frac{\partial\pi_i}{\partial p_j}\frac{\partial p_j^*}{\partial\theta_i}+\frac{\partial\pi_j}{\partial p_i}\frac{\partial p_i^*}{\partial\theta_i}. \tag{19}$$

The direct effect of the level of quality of firm $i$ on joint profit, gross of investment costs, is

$$\frac{\partial(\pi_1+\pi_2)}{\partial\theta_i}=-\delta M_i-\delta M_j+\phi\widetilde{D}_B.$$

At the symmetric equilibrium, the direct effect can be rewritten as

$$\frac{\partial(\pi_1+\pi_2)}{\partial\theta_i}=-2\delta M_i+\phi\widetilde{D}_B. \tag{20}$$

We can compare the direct effect with cooperation to the direct effect without cooperation, which is given by equation (17).

Note that there are two differences. First, when the issuers cooperate, they internalize the competition for market shares which is embedded in quality decisions. Therefore, the direct effect is reduced when there is cooperation, and hence, issuers have less incentives to invest. This can be seen by the fact that the term $\frac{1}{2t}$ in equation (17) is absent in equation (20). Second, when the firms cooperate, they internalize the effect of each issuer's quality choice on the hinterland demand of the other issuer, which increases through a rise of the number of merchants who adopt the innovation. This gives issuers incentives to invest more, when they cooperate. Note that, when $\beta$ or $\alpha_B$ are very low and if the interchange fee is low, the first effect dominates, and hence, the direct effect is reduced. Whereas, if $\beta$ or $\alpha_B$ are sufficiently high, or if the interchange fee is high, the second effect can dominate, and the direct effect is then increased. [other effect: internalize all IF revenues]

The indirect effect is given by

$$\frac{\partial \pi_i}{\partial p_j}\frac{\partial p_j^*}{\partial \theta_i} + \frac{\partial \pi_j}{\partial p_i}\frac{\partial p_i^*}{\partial \theta_i} = \left[\Delta M_i - \phi \widetilde{D}_B^i\right]\frac{\partial p_j^*}{\partial \theta_i} + \left[\Delta M_j - \phi \widetilde{D}_B^j\right]\frac{\partial p_i^*}{\partial \theta_i}. \tag{21}$$

Compared to the non cooperative case, the two issuers internalize the indirect effect of $\theta_i$ not only on firm $i$'s profit but also on firm $j$'s profit (see the second term of (21)).

We start by assuming that the interchange fee is equal to zero. The second term (21) is positive as $\partial p_i^*/\partial \theta_i > 0$ from Lemma 5 and $\Delta = \partial \widetilde{D}_B^i/\partial p_j > 0$, which increases investment incentives compared to the non-cooperative case. When the interchange fee increases, the second term in equation (21) can become negative, as $\partial \widetilde{n}_S \ /\partial p_i < 0$ from Lemma 1, which means that quality investment incentives may be reduced relative to the non cooperative situation.

The comparison between the quality levels in the cooperative and non cooperative cases depends on how firms trade off between the direct and the indirect effect of the quality choice on firms' profits. If the interchange fee is sufficiently small, cooperation tends to reduce the direct effect, which decreases firms' incentives to invest in quality. However, cooperation tends to increase the indirect effect, as a higher $\theta_i$ implies a higher price for firm $i$, which increases the demand and hence the profit of firm $j$. Whether cooperation increases or decreases investments in quality depends on the variation of each of these two effects with respect to the cooperative case. On the contrary, if the interchange fee is high, cooperation tends to increase the direct effect, which provides firms with higher incentives to invest in quality. However, in this case, it decreases the indirect effect.

Similarly, cooperation may either increase or decrease firms' investments in quality.

Therefore, the variation of the quality levels in the cooperation case with respect to the non cooperative case may be very similar with low and high, but for very different reasons.

| Cooperation | Direct effect | Indirect effect |
|---|---|---|
| Interchange small | (+) | (?)<br>Investments increased compared to cooperation. |
| Interchange high | (+) | (?)<br>Investments decreased compared to cooperation.<br>Firm i's higher price reduces firm j's profit. |

# 5 Conclusion

In this paper, we analyze the impact of interchange fees on consumers' and merchants' incentives to adopt an innovative payment instrument, in a setting where two issuing banks compete to attract consumers on the Hotelling line, while exerting some market power over an installed base of consumers on their hinterländer.

We show that the relationship between consumer adoption and interchange fees is non monotonic, when there are adoption externalities between consumers and merchants. Our results contradict the widespread intuition that high interchange fees favor consumer adoption over merchant adoption. We show that this may the case only if merchants do not exert strong externalities on consumer adoption, under the assumption that merchants incur fixed adoption costs. When the degree of externalities is high and when the merchant adoption benefit is low, the interchange fee that maximises consumer and merchant adoption is equal to zero. The profit-maximising interchange fee exceeds the welfare-maximising interchange fee when issuers' prices are more sensitive to a higher interchange fee than the number of merchants at the profit-maximing interchange fee.

We also compare the issuers' incentives to innovate when they cooperate and when they make their innovation decisions separately. [**To be written**]

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# Appendix

## Appendix A: Stage 3 (Adoption of the EPI)

### Appendix A1: Proof of Lemma 1

Since $\widetilde{n}_S(\Theta, P, m) = \mu(m)\, \widetilde{n}_S^0(\Theta, P, m)$, from (4),

$$\frac{\partial \widetilde{n}_S}{\partial p_i} = -\frac{\beta}{t}\mu(m)\pi(m). \tag{22}$$

Hence, the number of merchants who adopt the electronic payment instrument decreases with the issuer's price, as $\pi(m) \geq 0$ and $\mu(m) > 0$. Similarly, we find that $\partial \widetilde{n}_S / \partial m < 0$, that is, the number of merchants who adopt the electronic payment instrument decreases with the merchant fee. We also have

$$\frac{\partial \widetilde{n}_S}{\partial \theta_i} = \frac{\beta}{t}\mu(m)\pi(m) \geq 0.$$

Hence, the number of merchants who adopt the electronic payment instrument increases with the quality provided by the issuers. Finally, we can prove in a similar way that $\partial \widetilde{n}_S / \partial \alpha_B > 0$ and $\partial \widetilde{n}_S / \partial \beta > 0$. ■

### Appendix A2: Proof of Lemma 2

From equation (2), we have $d\widetilde{D}_B/dp_i = -(\beta/t)\mu(m) < 0$. We also have

$$\frac{d\widetilde{D}_B}{dm} = \frac{2\alpha_B\beta}{t}\frac{\partial\widetilde{n}_S}{\partial m} < 0, \tag{23}$$

as $\partial\widetilde{n}_S\ /\partial m < 0$. Similarly, we have $d\widetilde{D}_B/d\theta_i = (\beta/t)\mu(m) > 0$. Finally, we have

$$\frac{d\widetilde{D}_B}{d\alpha_B} = \frac{2\beta}{t}\left(\widetilde{n}_S + \alpha_B\frac{\partial\widetilde{n}_S}{\partial\alpha_B}\right) > 0, \tag{24}$$

as $\partial\widetilde{n}_S\ /\partial\alpha_B > 0$. We also have

$$\frac{d\widetilde{D}_B}{d\beta} = (\widetilde{y}_1 + \widetilde{y}_2) + \frac{2\alpha_B\beta}{t}\frac{\partial\widetilde{n}_S}{\partial\beta} > 0, \tag{25}$$

as $\widetilde{y}_i = (v_B + \theta_i + \alpha_B\widetilde{n}_S - p_i)/t > 0$ and $\partial\widetilde{n}_S\ /\partial\beta > 0$. ■

### Appendix A3: Proof of Lemma 3

From (7), we have

$$\frac{d\widetilde{D}^i_B}{dp_j} = \frac{\gamma}{2t} + \frac{\beta\alpha_B}{t}\frac{\partial\widetilde{n}_S}{\partial p_j} = \frac{\mu(m)}{2t^2}\left(t\gamma - 2\beta(\gamma+\beta)\alpha_B\pi\left(m\right)\right).$$

Since $t\gamma - 2\beta\left(\gamma+\beta\right)\left(b_S - m\right)\alpha_B \geq t\gamma - 2\beta\left(\gamma+\beta\right)b_S\alpha_B$, and since from Assumption 2(iii), $t\gamma - 2\beta\left(\gamma+\beta\right)b_S\alpha_B > 0$, we have $t\gamma - 2\beta\left(\gamma+\beta\right)\left(b_S - m\right)\alpha_B > 0$. Since $\mu(m) > 0$, we conclude that $d\widetilde{D}^i_B/dp_j > 0$. Besides, we have

$$\frac{d\widetilde{D}^i_B}{d\theta_j} = -\frac{d\widetilde{D}^i_B}{dp_j},$$

which implies that $d\widetilde{D}^i_B/d\theta_j > 0$. ■

## Appendix B: Stage 2 (Prices)

### Appendix B1: Second-order condition

We have $1 - \phi\left(a\right) = \mu\left(t - \pi(a)\left(a + 2\alpha_B\right)\beta\right)$. Since $t - \pi(a)\left(a+2\alpha_B\right)\beta \geq 0$ under Assumption 2(ii), and since $\mu(a) > 0$, we have $1 - \phi\left(a\right) \geq 0$. Since $\delta(a) < 0$ from Lemma 3 and $1 - \phi(a) \geq 0$, we have $\delta(a)(1 - \phi(a)) \leq 0$.

## Appendix B2: Prices are strategic complements

**Proof.** From the first-order condition (12), we have

$$\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = (1-\phi(a))\Delta(a) + a\delta(a)\frac{\partial \widetilde{n}_S}{\partial p_j}. \tag{26}$$

We now prove that $\partial^2\pi_i/\partial p_i \partial p_j > 0$. From Lemma 1, we have $\partial \widetilde{n}_S/\partial p_j \leq 0$. From Lemma 3, $\phi(a) > 0$, $\Delta(a) > 0$, and $\delta(a) < 0$. From Assumption 2(ii), $1-\phi(a) > 0$. Hence, $\partial^2\pi_i/\partial p_i\partial p_j > 0$. From the implicit function theorem, since $\partial^2\pi_i/\partial p_i^2 < 0$, prices are strategic complements, that is, $\partial p_i^{BR}/p_j > 0$, where $p_i^{BR}$ denotes the best-response of issuer $i$. ■

## Appendix B3: Proof of Lemma 4

From the FOC (12), the price chosen by issuer $i$ at the equilibrium of stage 2 solves

$$(1-\phi(a))\,\widetilde{D}_B^i + \delta(a)M_i = 0, \tag{27}$$

for $i = 1, 2$. Since $\widetilde{D}_B^i = x_i + y_i + \delta(a)p_i + \Delta(a)p_j$ and $M_i = (1-\phi(a))p_1 - \phi(a)p_2 + a\overline{n}_S - k_I$, solving for $p_i$ and $p_j$ in (27), we obtain that the price chosen by issuer $i$ at the equilibrium of the subgame is

$$p_i^* = \frac{2\delta\,(1-\phi)\left[(k_I - a\overline{n}_S)\,\delta - (1-\phi)\,d_i\right] + \left[(k_I - a\overline{n}_S)\,\delta - (1-\phi)\,d_i\right]\left[\delta\phi - (1-\phi)\,\Delta\right]}{4\delta^2\,(1-\phi)^2 - \left[\delta\phi - (1-\phi)\,\Delta\right]^2},$$

for $i = 1, 2$. ■

## Appendix B4: Proof that the denominator of $p^*$ is strictly positive

Let $E \equiv \delta\,(3\phi - 2) - (1-\phi)\,\Delta$ denote the denominator of $p^*$. Replacing for $\delta$, $\Delta$ and $\phi$, we find that

$$E = \frac{t^2\,(\gamma + 4\beta) - 2t\beta\,(a\gamma + 3a\beta + \alpha_B\,(2\gamma + 5\beta))\,\pi + 4\alpha_B\,(a + \alpha_B)\,\beta^2\,(\gamma + \beta)\,\pi^2}{2t\,(t - 2\alpha_B\beta\pi)^2}.$$

Since under Assumption 2(iii),

$$\frac{\partial E}{\partial a} = -\frac{\beta\pi}{t}\frac{\left[t(\gamma + 3\beta) - 2\alpha_B\beta\,(\gamma+\beta)\,\pi\right]}{(t - 2\alpha_B\beta\pi)^2} < 0,$$

then $E$ decreases with $a$. We also find that $E\left(a=a^{\max}=b_S\right)=(\gamma+4\beta)/(2t)>0$, and therefore, $E>0$ for all $a\leq a^{\max}$. ■

**Appendix B5: Sign of $dp^*/d\alpha_B$**

We have $\partial\delta/\partial\alpha_B=\partial\Delta/\partial\alpha_B=-\beta^2\left(\mu\left(a\right)\right)^2\pi(a)/t^2<0$, $\partial\phi/\partial\alpha_B=2a(\beta\mu\left(a\right)\pi(a)/t)^2>0$, $\partial d/\partial\alpha_B>0$ and $\partial\overline{n}_S/\partial\alpha_B>0$. Therefore,

$$\frac{dp^*}{d\alpha_B}=\underbrace{\frac{\partial\delta}{\partial\alpha_B}}_{(-)}\underbrace{(\frac{\partial p^*}{\partial\delta}+\frac{\partial p^*}{\partial\Delta})}_{(+)\text{ or }(-)}+\underbrace{\frac{\partial\phi}{\partial\alpha_B}}_{(+)}\underbrace{\frac{\partial p^*}{\partial\phi}}_{(-)}+\underbrace{\frac{\partial d}{\partial\alpha_B}}_{(+)}\underbrace{\frac{\partial p^*}{\partial d}}_{(+)}+\underbrace{\frac{\partial\overline{n}_S}{\partial\alpha_B}}_{(+)}\underbrace{\frac{\partial p^*}{\partial\overline{n}_S}}_{(-)},$$

where

$$\begin{aligned}
\frac{\partial p^*}{\partial\delta} &= \frac{(1-\phi)\left(d(2-3\phi)+(k_I-a\overline{n}_S)\Delta\right)}{\left[(1-\phi)\Delta+\delta\left(2-3\phi\right)\right]^2},\\
\frac{\partial p^*}{\partial\Delta} &= \frac{(1-\phi)\left(-d(1-\phi)+(k_I-a\overline{n}_S)\delta\right)}{\left[(1-\phi)\Delta+\delta\left(2-3\phi\right)\right]^2},\\
\frac{\partial p^*}{\partial\phi} &= \frac{-\delta\left(d(1-\phi)-(k_I-a\overline{n}_S)(3\delta+\Delta)\right)}{\left[(1-\phi)\Delta+\delta\left(2-3\phi\right)\right]^2}.
\end{aligned}$$

Since $1-\phi>0$, $d>0$, and $\delta<0$, we have $\partial p^*/\partial\Delta<0$ if $k_I-a\overline{n}_S>0$. Since $3\delta+\Delta=(-\gamma/t)-3\beta/t-(4\beta^2\alpha_B\mu\pi(m)/t^2)<0$, we have $\partial p^*/\partial\phi>0$ if $k_I-a\overline{n}_S>0$. The sign of $\partial p^*/\partial\delta$ is ambiguous.

## Appendix C: Stage 1 (Interchange fee)

**Appendix C1:**

In equilibrium, we have

$$R_i(\theta_i,\theta_j,R_j(\theta_i,\theta_j,p_i^*))=p_i^*\ \text{ and }\ R_j(\theta_i,\theta_j,R_i(\theta_i,\theta_j,p_j^*))=p_j^*,$$

where $R_i$ and $R_j$ denote the best-response functions of firm $i$ and firm $j$, respectively. We have

$$\frac{\partial p_i^*}{\partial a}=\left.\frac{\partial R_i}{\partial a}\right|_{P^*}+\left.\frac{\partial R_i}{\partial p_j}\right|_{P^*}\left(\left.\frac{\partial R_j}{\partial a}\right|_{P^*}+\left.\frac{\partial R_j}{\partial p_i}\right|_{P^*}\frac{\partial p_i^*}{\partial a}\right),$$

which simplifies to

$$\frac{dp_i^*}{da}\left[1-\left.\frac{\partial R_j}{\partial p_i}\right|_{P^*}\left.\frac{\partial R_i}{\partial p_j}\right|_{P^*}\right]=\left.\frac{\partial R_i}{\partial a}\right|_{P^*}+\left.\frac{\partial R_i}{\partial p_j}\right|_{P^*}\left.\frac{\partial R_j}{\partial a}\right|_{P^*}. \tag{C1-1}$$

We start by proving the following result.

**Lemma 6**

$$1-\left.\frac{\partial R_i}{\partial p_j}\right|_{p^*}\left.\frac{\partial R_j}{\partial p_i}\right|_{p^*}>0.$$

**Proof.** Using the implicit function theorem, we have

$$1-\left.\frac{\partial R_i}{\partial p_j}\right|_{P^*}\left.\frac{\partial R_j}{\partial p_i}\right|_{P^*}=\left(\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{P^*}\right)^{-1}\left(\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{P^*}-\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_i\partial p_j}\right|_{P^*}\right).$$

As the second-order condition of profit maximization is verified at $P^*$, we have

$$\left(\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{P^*}\right)^{-1}\geq 0.$$

We now prove that

$$\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{P^*}-\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_i\partial p_j}\right|_{P^*}\geq 0.$$

From (13),

$$\frac{\partial^2\pi_i}{\partial p_i^2}=\frac{\partial^2\pi_j}{\partial p_j^2}=2\left(1-\phi(a)\right)\delta(a). \tag{C1-2}$$

Besides, we have

$$\frac{\partial^2\pi_j}{\partial p_i\partial p_j}=\frac{\partial^2\pi_i}{\partial p_i\partial p_j}=\left(1-\phi(a)\right)\Delta(a)-\delta(a)\phi(a). \tag{C1-3}$$

Since $\Delta(a)=-\delta(a)-\beta/t$, we have

$$\frac{\partial^2\pi_j}{\partial p_i\partial p_j}=\frac{\partial^2\pi_i}{\partial p_i\partial p_j}=\left(1-\phi(a)\right)\left(-\delta(a)-\beta/t\right)-\delta(a)\phi(a).$$

From (C1-2) and (**??**), we have

$$\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{P^*}-\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{P^*}\left.\frac{\partial^2\pi_j}{\partial p_i\partial p_j}\right|_{P^*}=\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*}^2-\left.\frac{\partial^2\pi_j}{\partial p_i\partial p_j}\right|_{P^*}^2=XY, \tag{C1-4}$$

where

$$X = \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*} + \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{p^*} \quad \text{and} \quad Y = \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*} - \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{p^*}.$$

From (C1-2) and (**??**), we have

$$X = \delta(a)\,(1 - 2\phi(a)) - \frac{\beta}{t}(1 - \phi(a)) \quad \text{and} \quad Y = \delta(a)\,(2 - \phi(a)) - (1 - \phi(a))\,\Delta(a).$$

From (11), we have

$$1 - 2\phi(a) = \frac{t - 2\beta\pi(a)(\alpha_B + a)}{t}\mu(a).$$

Since $a \leq b_S$ (otherwise, no merchant would adopt the electronic payment instrument), we have $t - 2\beta\pi(a)(\alpha_B + a) \geq t - 2\beta b_S(\alpha_B + b_S)$. From Assumption 2(ii), $t - 2\beta b_S(\alpha_B + b_S) > 0$. Since $\mu(a) > 0$, it follows that $1 - 2\phi(a) > 0$.

Since $\Delta(a) > 0$, $\delta(a) < 0$, $1 - 2\phi(a) > 0$ and $1 - 2\phi(a) > 0$, we have $X \leq 0$ and $Y \leq 0$. As $XY \geq 0$, we can conclude that

$$\left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{p^*} - \left.\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|_{p^*} \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{p^*} \geq 0,$$

which proves the Lemma. ■

From (C1-1) and Lemma 6, $dp_i^*/da$ has the same sign as

$$\left.\frac{\partial R_i}{\partial a}\right|_{P^*} + \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial a}\right|_{P^*}. \tag{28}$$

From the implicit function theorem, we have

$$\left.\frac{\partial R_i}{\partial a}\right|_{P^*} = - \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*}^{-1} \left.\frac{\partial^2 \pi_i}{\partial p_i \partial a}\right|_{p^*}.$$

Replacing for $\left.\frac{\partial R_i}{\partial a}\right|_{P^*}$, $\left.\frac{\partial R_i}{\partial p_j}\right|_{P^*}$ and $\left.\frac{\partial R_j}{\partial a}\right|_{P^*}$ in (28), we obtain that $dp_i^*/da$ has the same sign as

$$- \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{p^*}^{-1} \left.\frac{\partial^2 \pi_i}{\partial p_i \partial a}\right|_{p^*} + \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*}^{-1} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{p^*}^{-1} \left.\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|_{p^*} \left.\frac{\partial^2 \pi_j}{\partial p_j \partial a}\right|_{p^*}.$$

Since

$$\left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{p^*} = \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*},$$

it follows that $dp_i^*/da$ has the same sign as

$$\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}^{-2}\left.\frac{\partial^2\pi_i}{\partial p_i\partial a}\right|_{p^*}\left[-\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}+\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{p^*}\right].$$

Since $\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}<0$ from (C1-2) and since $\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{p^*}>0$ from (C1-3), it follows that $dp_i^*/da$ has the same sign as $\left.\frac{\partial^2\pi_i}{\partial p_i\partial a}\right|_{p^*}$.

**Appendix C2: Sign of $a\mu(a)-\pi(a)$**

Let $h(a)\equiv a\mu(a)-\pi(a)$. We have

$$h'(a)=a\mu'(a)+\mu(a)+1=\frac{2\left(t^2-(3b_S-2a)\,\alpha_B\beta t+2\,(b_S-a)^2\,\alpha_B^2\beta^2\right)}{\left(t-2\,(b_S-a)\,\alpha_B\beta\right)^2}. \qquad \text{(C2-1)}$$

The denominator of (C2-1) is strictly positive. The numerator of (C2-1) has an inverted bell curve, and it attains its minimum in $t$ at $t=-a\alpha_B\beta+(3/2)\,\alpha_B\beta b_S$, which is lower than the minimum value of $t$ imposed by Assumption 2(i), that is, $t=4\alpha_B\beta b_S$. We find that

$$\left.h'(a)\right|_{t=4\alpha_B\beta b_S}=\frac{a^2+2ab_S+3b_S^2}{(b_S+a)^2}>0.$$

Therefore, for all values of $t$ that satisfy Assumption 2, and for all $a$, we have $h'(a)>0$. Besides, $h(0)<0$ and $h(a^{\max})>0$. Hence, $h(a)$ is negative for low values of $a$, and positive otherwise.

**Appendix C3: Proof of Lemma 1**

First, we compute the derivative of the issuer's price in the symmetric equilibrium at $a=0$. We have

$$\left.\frac{\partial p^*}{\partial a}\right|_{a=0}=-\frac{2(-c_I+\theta+v_B)\beta+t\gamma}{t(t(4\beta+\gamma)-2b_S\alpha_B\beta)^2}\mu(0)P(t),$$

where $P(t)=2\alpha_B\beta(\gamma+\beta)+b_S(2\beta+\gamma)(3\beta+\gamma)t^2-2\alpha_B\beta(\gamma+\beta)(5b_S\beta+2\alpha_B\beta+2b_S\gamma)b_St+4b_S^3\alpha_B^2\beta^2(\gamma+\beta)^2$. Therefore, $\left.\partial p^*/\partial a\right|_{a=0}<0$ if and only if $P(t)>0$. As the coefficient of the $t^2$ factor is positive, this second-order polynomial is U-shaped, it and has two positive roots. The highest root is

$$t^{+}=\frac{b_S\alpha_B\beta\left(\gamma+\beta\right)\left(5b_S\beta+2\alpha_B\beta+2b_S\gamma\right)+\sqrt{b_S^2\alpha_B^2\beta^4\left(\gamma+\beta\right)^2\left(b_S^2+12\alpha_B b_S+4\alpha_B^2\right)}}{2\alpha_B\beta\left(\gamma+\beta\right)+b_S\left(2\beta+\gamma\right)\left(3\beta+\gamma\right)}.$$

Since $\sqrt{b_S^2\alpha_B^2\beta^4\left(\gamma+\beta\right)^2\left(b_S^2+12\alpha_B b_S+4\alpha_B^2\right)} \leq \sqrt{b_S^2\alpha_B^2\beta^4\left(\alpha+\beta\right)^2\left(9b_S^2+12\alpha_B b_S+4\alpha_B^2\right)}$ and $\sqrt{b_S^2\alpha_B^2\beta^4\left(\alpha+\beta\right)^2\left(9b_S^2+12\alpha_B b_S+4\alpha_B^2\right)}=b_S\alpha_B\beta^2\left(\gamma+\beta\right)\left(3b_S+2\alpha_B\right)$, we have

$$t^{+}<\frac{2b_S\alpha_B\beta\left(\gamma+\beta\right)\left(b_S\gamma+4b_S\beta+2\alpha_B\beta\right)}{2\alpha_B\beta\left(\gamma+\beta\right)+b_S\left(2\beta+\gamma\right)\left(3\beta+\gamma\right)}\equiv\bar{t}^{+}.$$

We find that $\bar{t}^{+}<4\alpha_B\beta b_S$, and therefore from Assumption 2(i), we always have $t>\bar{t}^{+}>t^{+}$. It follows that $\partial p^*/\partial a|_{a=0}<0$.

Second, since $v_B>k_I$ from Assumption 1,

$$\left.\frac{\partial p^*}{\partial a}\right|_{a=b_S}=\frac{\left[\gamma t+2\beta\left(v_B+\theta-k_I\right)\right]\left[b_S\left(2\beta+\gamma\right)\left(3\beta+\gamma\right)-2\alpha_B\beta\left(\gamma+\beta\right)\right]}{t\left(\gamma+4\beta\right)^2}>0$$

if and only if $b_S\left(2\beta+\gamma\right)\left(3\beta+\gamma\right)>2\alpha_B\beta\left(\gamma+\beta\right)$. ■

### Appendix C4: Proof of Proposition 2

For all parameter values, consumer adoption decreases with $a$ at $a=b_S$, as

$$\left.\frac{dD_B^*}{da}\right|_{a=b_S}=-\frac{2\beta\left[\gamma t+\beta\left(2v_B+\theta_1+\theta_2-2k_I\right)\right]\left[6\beta^2\left(b_S+\alpha_B\right)+\left(5b_S+4\alpha_B\right)\gamma\beta+\left(b_S+\alpha_B\right)\gamma^2\right]}{t^2(4\beta+\gamma)^2}<0.$$

We now compute $dD_B^*/da$ at $a=0$ for low and high values of $\alpha_B$. First, we have

$$\left.\frac{dD_B^*}{da}\right|_{a=0,\ \alpha_B=0}=\frac{2b_S\beta\left(2\beta+\gamma\right)\left(3\beta+\gamma\right)\left[\gamma t+\beta\left(2v_B+\theta_1+\theta_2-2k_I\right)\right]}{t^2(4\beta+\gamma)^2}>0,$$

and hence, $D_B^*$ is increasing with $a$ at $a=0$ for low values of $\alpha_B$. Second, we compute $dD_B^*/da$ at $a=0$ at the maximum value of $\alpha_B$ that is given by Assumption 2(i). We have

$$\left.\frac{dD_B^*}{da}\right|_{a=0,\ \alpha_B=t/(4\beta b_S)}=\frac{2\left(\gamma t+\left(2v_B+\theta_1+\theta_2-2k_I\right)\right)\left[4(b_S)^2\beta\left(3\beta+\gamma\right)\left(5\beta+\gamma\right)-t(17\beta^2+6\beta\gamma+\gamma^2)\right]}{b_S t^2(7\beta+\gamma)^2}.$$

This expression has the sign of $4(b_S)^2\beta\left(3\beta+\gamma\right)\left(5\beta+\gamma\right)-t(17\beta^2+6\beta\gamma+\gamma^2)$, which is positive for high values of $b_S$, and negative otherwise. To sum up, if $\alpha_B$ is high and $b_S$ is low, $D_B^*$ is decreasing both at $a = 0$ and at $a = b_S$. Otherwise, $D_B^*$ is increasing at $a = 0$ and decreasing at $a = b_S$. ■

**Appendix C5: Proof of Proposition 3**

We have

$$\frac{dn_S^*}{da} = \frac{-K}{\left(4(b_S - a)^2\alpha_B(a+\alpha_B)\beta^2(\gamma+\beta) + t^2(\gamma+4\beta) + 2(a-b_S)t\beta(a\gamma+3a\beta+\alpha_B(2\gamma+5\beta))\right)^2},$$

and hence, $dn_S^*/da$ has the sign opposite to that of $K$. We find that $K = (t\gamma + (2(v_B - k_I) + \theta_1 + \theta_2)\beta)K_1$, and therefore, from Assumption 1, $K$ has the sign of $K_1$. $K_1$ is a polynomial of degree 2 of $\alpha_B$ and it has an inverted bell curve, as the coefficient in $(\alpha_B)^2$ is positive, since

$$K_1 = 4(b_S - a)^2\beta^2(\beta+\gamma)\left[t(3\beta+\gamma) - 2(b_S - a)^2\beta(\beta+\gamma)\right](\alpha_B)^2 + ...,$$

and $t(3\beta+\gamma) > 2(b_S - a)^2\beta(\beta+\gamma)$ from Assumption 2(ii). We compute the discriminant of $K_1$ and show that it is strictly negative if $t$ is sufficiently high, which proves that $dn_S^*/da > 0$.

The discriminant of $K_1$ is equal to $\Delta_{K_1} = -16(b_S - a)^2 t^2\beta^4(\beta+\gamma)\times g$, and hence, it has the opposite sign to that of $g$. We have $g = 2(4\beta+\gamma)t^2 - 2(b_S - a)(2b_S\beta(3\beta+\gamma) - a(2\beta+\gamma)(5\beta+\gamma))t - (b_S - a)^2\beta(\beta+\gamma)(4a(2\beta+\gamma)(b_S - a) + b_S^2\beta)$. Note that $g|_{t=0} < 0$. However, the first two terms of $g$ can rearranged so that

$$g = 2t\left[t(4\beta+\gamma) - (b_S - a)(2b_S\beta(3\beta+\gamma) - a(2\beta+\gamma)(5\beta+\gamma))\right] + g|_{t=0}.$$

Since the first term in the equation above is increasing in $t$, then $g$ is positive for sufficiently high values of $t$. A sufficient condition is

$$2t\left[t(4\beta+\gamma) - 2\beta b_S^2(\beta+\gamma)\right] > \beta b_S^4(\beta+\gamma)(7\beta+4\gamma),$$

which shows that $dn_S^*/da < 0$ for most parameter values. ■

**Appendix C6:** $\partial\pi_i/\partial p_j|_{P^*} > 0$

We find that

$$\left.\frac{\partial\pi_i}{\partial p_j}\right|_{P^*} = \frac{(t\gamma + 2(\theta + v_B - c_I)\beta)(t\gamma - 2(b_S - a)(a+\alpha_B)\beta(\beta+\gamma))}{2\left(t^2(4\beta+\gamma) - 2(b_S - a)\beta t(3a\beta + 5\alpha_B\beta + a\gamma + 2\alpha_B\gamma) + 4(b_S - a)^2(a+\alpha_B)\beta^2(\beta+\gamma)\alpha_B\right)}.$$

As $v_B > c_I$ from Assumption 1, the numerator is strictly positive if $t > 2\beta b_S(b_S + \alpha_B)(1 + \beta/\gamma)$, which is a slightly stronger condition than Assumptions 2(ii) and 2(iii). The denominator is positive if $t > 2\beta b_S \left[(3\beta + \gamma)\, b_S + (5\beta + 2\gamma)\, \alpha_B\right] / \left(4\beta + \gamma\right)$. All in all, it means that $\partial\pi_i/\partial p_j|_{P^*} > 0$ if $t$ is sufficiently high. ■

**Appendix C7: Comparison of $a^\pi$, $a^B$, $a^S$ and $a^V$**

There are two cases. Either the interchange fee that maximizes consumer adoption is equal to zero, or the interchange fee that maximizes consumer adoption is an interior solution. Our aim is to compare $a^B$, $a^S$ and $a^\pi$ in the second case. We have

$$\left.\frac{dD_B^*}{da}\right|_{a=a^B} = 0,$$

which from (15) is equivalent to

$$\frac{-\beta\alpha_B\mu^2}{t}\widetilde{D}_B^0(p_1^*(a^B), p_2^*(a^B)) = \frac{2\beta\mu}{t}\left.\frac{\partial p^*}{\partial a}\right|_{a=a^B}$$

in a symmetric equilibrium. Simplifying this expression by $\beta\mu/t$, we obtain that, in a symmetric equilibrium,

$$\left.\frac{dp^*}{da}\right|_{a=a^B} = \frac{-\alpha_B}{2}D_B^*(a^B).$$

Since $n_S^* = \pi(a)D_B^*$, we have

$$\frac{dn_S^*}{da} = -D_B^* + (b_S - a)\frac{dD_B^*}{da}.$$

It follows that

$$\left.\frac{dn_S^*}{da}\right|_{a=a^B} = -D_B^*(a^B).$$

We denote the transaction volume by $V = n_S^* D_B^*$. We have

$$\left.\frac{dV}{da}\right|_{a=a^B} = -(D_B^*(a^B))^2 \leq 0.$$

Since $V$ is concave in $a$, we have $a^B \geq a^V$. We denote by $M^* = p^* + an_S^* - k_I$ the margin of an issuer in a symmetric equilibrium. Since

$$\frac{dM^*}{da} = \frac{dp^*}{da} + n_S^* + \frac{dn_S^*}{da},$$

and since $n_S^* = \pi(a)D_B^*$, we have

$$\left.\frac{dM^*}{da}\right|_{a=a^B} = (b_S - \frac{\alpha_B}{2} - 2a^B)D_B^*(a^B).$$

We denote by $\pi^*$ the profit of an issuer in a symmetric equilibrium. Since

$$\frac{d\pi^*}{da} = \frac{dM^*}{da}D_B^* + \frac{dD_B^*}{da}M^*,$$

we have

$$\left.\frac{d\pi^*}{da}\right|_{a=a^B} = (b_S - \frac{\alpha_B}{2} - 2a^B)(D_B^*(a^B))^2.$$

Since $\pi^*$ is concave in $a$, (**For Marc: can this be shown or is this an assumption?**) a sufficient condition for $a^B$ to be higher than $a^\pi$ is that $b_S - (\alpha_B/2) - 2a^B \leq 0$, that is $a^B \geq (b_S/2) - (\alpha_B/4)$.

We now proceed by comparing $a^\pi$ and $a^V$. We have:

$$\left.\frac{d\pi^*}{da}\right|_{a=a^V} = \left(\left.\frac{dp^*}{da}\right|_{a=a^V} + n_S^* + p^*(a^V) - k_I\right) D_B^*(a^V).$$

Since $\pi^*$is concave in $a$, a sufficient condition for $a^V \leq a^\pi$ is that

$$\left.\frac{dp^*}{da}\right|_{a=a^V} + n_S^* + p^*(a^V) - k_I \geq 0.$$

This condition can be interpreted as follows: the marginal revenue obtained from an increase of the interchange fee, taking the volume of transactions as fixed, is positive at $a = a^V$.

### Appendix C8: Welfare analysis

The merchants' surplus is

$$MS = \int_0^{\pi(a)D_B^*} (\pi(a)D_B^* - F_S)dF_S = \frac{(\pi(a)D_B^*)^2}{2}.$$

Since $\pi(a)D_B^* = n_S^*$, we have

$$\frac{dMS}{da} = \frac{dn_S^*}{da}n_S^*.$$

Define $u^* = v_B + \theta + \alpha_B n_S^* - p^*$. In a symmetric equilibrium, the total consumer surplus is

$$
\begin{aligned}
CS &= 2\gamma\int_0^{1/2}(v_B + \theta + \alpha_B n_S^* - p^* - tx)dx + 2\beta\int_0^{u^*/t}(v_B + \theta + \alpha_B n_S^* - p^* - ty)dy \\
&= \gamma\left(u^* - \frac{t}{4}\right) + \beta\frac{(u^*)^2}{t}.
\end{aligned}
$$

We have

$$
\frac{dCS}{da} = \left(\gamma + \frac{\beta u^*}{2t}\right)\frac{du^*}{da} = \left(\gamma + \frac{\beta u^*}{2t}\right)\left(\alpha_B\frac{dn_S^*}{da} - \frac{dp^*}{da}\right).
$$

The variations of the total user surplus, $TUS = MS + CS$, with the interchange fee are

$$
\frac{dTUS}{da} = \frac{dn_S^*}{da}\left(n_S^* + \left(\gamma + \frac{\beta u^*}{2t}\right)\alpha_B\right) - \frac{dp^*}{da}\left(\gamma + \frac{\beta u^*}{2t}\right).
$$

Under the assumption that $TUS$ is concave in $a$, if

$$
\left.\frac{dTUS}{da}\right|_{a=a^\pi} < 0,
$$

we have $a^\pi > a^W$, where $a^W$ denotes the interchange fee that maximizes social welfare. We have

$$
\left.\frac{dTUS}{da}\right|_{a=a^\pi} = \left.\frac{dn_S^*}{da}\right|_{a=a^\pi}\left(n_S^*(a^\pi) + \left(\gamma + \frac{\beta u^*(a^\pi)}{2t}\right)\alpha_B\right) - \left.\frac{dp^*}{da}\right|_{a=a^\pi}\left(\gamma + \frac{\beta u^*(a^\pi)}{2t}\right).
$$

If

$$
\left.\frac{dn_S^*}{da}\right|_{a=a^\pi}\left(\alpha_B + \frac{n_S^*(a^\pi)}{\gamma + \beta u^*(a^\pi)/(2t)}\right) \geq \left.\frac{dp^*}{da}\right|_{a=a^\pi},
$$

we have $a^\pi \geq a^W$. Otherwise, we have $a^\pi \leq a^W$.

### Appendix D1: Proof of Lemma 5

In equilibrium, we have

$$
R_i(\theta_i, \theta_j, R_j(\theta_i, \theta_j, p_i^*)) = p_i^*,
$$

and

$$
R_j(\theta_i, \theta_j, R_i(\theta_i, \theta_j, p_j^*)) = p_j^*,
$$

where $R_i$ and $R_j$ denote the best response functions of firm $i$ and firm $j$, respectively.

Let $P^* = (p_1^*, p_2^*)$. We have

$$\frac{\partial p_i^*}{\partial \theta_i} = \left.\frac{\partial R_i}{\partial \theta_i}\right|_{P^*} + \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left( \left.\frac{\partial R_j}{\partial \theta_i}\right|_{P^*} + \left.\frac{\partial R_j}{\partial p_i}\right|_{P^*} \frac{\partial p_i^*}{\partial \theta_i} \right). \tag{F1}$$

It follows that

$$\frac{\partial p_i^*}{\partial \theta_i} \left[ 1 - \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial p_i}\right|_{P^*} \right] = \left.\frac{\partial R_i}{\partial \theta_i}\right|_{P^*} + \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial \theta_i}\right|_{P^*}. \tag{F2}$$

We start by showing that

$$1 - \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial p_i}\right|_{P^*} \geq 0. \tag{F3}$$

Using the implicit function theorem, we have

$$1 - \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial p_i}\right|_{P^*} = \left( \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{P^*} \right)^{-1} \left( \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{P^*} - \left.\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{P^*} \right).$$

As the second-order condition of profit maximization is verified at $P^*$, we have

$$\left( \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{P^*} \right)^{-1} \geq 0.$$

We now prove that

$$\left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{P^*} - \left.\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{P^*} \geq 0.$$

From the second-order condition (13), since $\partial \widetilde{n}_S / \partial p_i = \partial \widetilde{n}_S / \partial p_j$ and $\partial \widetilde{D}_B^i / \partial p_i = \partial \widetilde{D}_B^j / \partial p_j$, we have

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{\partial^2 \pi_j}{\partial p_j^2} = 2 \frac{\partial \widetilde{D}_B^i}{\partial p_i} \left( 1 + a \frac{\partial \widetilde{n}_S}{\partial p_i} \right). \tag{F4}$$

Besides, since $\partial \widetilde{n}_S / \partial p_i = \partial \widetilde{n}_S / \partial p_j$ and $\partial \widetilde{D}_B^i / \partial p_i = \partial \widetilde{D}_B^j / \partial p_j$, from (26) in Appendix E, we have

$$\frac{\partial^2 \pi_j}{\partial p_i \partial p_j} = \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \frac{\partial \widetilde{D}_B^i}{\partial p_j} \left( 1 + a \frac{\partial \widetilde{n}_S}{\partial p_i} \right) + a \frac{\partial \widetilde{n}_S}{\partial p_j} \frac{\partial \widetilde{D}_B^i}{\partial p_i}. \tag{F5}$$

From (F4) and (F5), we have

$$\left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{P^*} - \left.\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|_{P^*} \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{P^*} = \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{P^*}^2 - \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{P^*}^2 = XY, \tag{F6}$$

where

$$X = \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*} + \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{p^*} \quad \text{and} \quad Y = \left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*} - \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{p^*}.$$

From (F4) and (F5), we have

$$X = 2\frac{\partial \widetilde{D}_B^i}{\partial p_i}(1 + a\frac{\partial \widetilde{n}_S}{\partial p_i}) + \frac{\partial \widetilde{D}_B^i}{\partial p_j}(1 + a\frac{\partial \widetilde{n}_S}{\partial p_i}) + a\frac{\partial \widetilde{n}_S}{\partial p_i}\frac{\partial \widetilde{D}_B^i}{\partial p_i},$$

and

$$Y = \frac{\partial \widetilde{D}_B^i}{\partial p_i}(2 + a\frac{\partial \widetilde{n}_S}{\partial p_i}) - \frac{\partial \widetilde{D}_B^i}{\partial p_j}\left(1 + a\frac{\partial \widetilde{n}_S}{\partial p_i}\right).$$

From issuer $i$'s demand, we find that

$$\frac{\partial \widetilde{D}_B^i}{\partial p_j} = -\frac{\partial \widetilde{D}_B^i}{\partial p_i} - \frac{\beta}{t} + \frac{2\alpha_B \beta}{t}\frac{\partial \widetilde{n}_S}{\partial p_i},$$

we have

$$X = \frac{\partial \widetilde{D}_B^i}{\partial p_i}(1 + 2a\frac{\partial \widetilde{n}_S}{\partial p_i}) + \left(-\frac{\beta}{t} + \frac{2\alpha_B \beta}{t}\frac{\partial \widetilde{n}_S}{\partial p_i}\right)(1 + a\frac{\partial \widetilde{n}_S}{\partial p_i}).$$

**Proof.**

**Lemma 7** $1 + 2a\frac{\partial \widetilde{n}_S}{\partial p_i} > 0$.

■

**Proof.** From (22) in Appendix B, we have

$$1 + 2a\frac{\partial \widetilde{n}_S}{\partial p_i} = \frac{t - 2\beta(b_S - m)(\alpha_B + a)}{t - 2\beta(b_S - m)\alpha_B}.$$

Since $a \le b_S - c_A$ (otherwise, no merchant would adopt the electronic payment instrument), we have $t - 2\beta(b_S - m)(\alpha_B + a) \ge t - 2\beta(b_S - c_A)(\alpha_B + b_S - c_A)$. From Assumption 2, $t - 2\beta(b_S - c_A)(\alpha_B + b_S - c_A) > 0$. As $t - 2\beta(b_S - m)\alpha_B > 0$ for all $a \le b_S - c_A$, it follows that $1 + 2a\,(\partial \widetilde{n}_S / \partial p_i) > 0$. ■

From Lemma 7, we have

$$1 + a\frac{\partial \widetilde{n}_S}{\partial p_i} > 1 + 2a\frac{\partial \widetilde{n}_S}{\partial p_i} > 0.$$

Since $\partial \widetilde{n}_S / \partial p_i \le 0$, $\partial \widetilde{D}_B^i / \partial p_i \le 0$ and $\partial \widetilde{D}_B^i / \partial p_j \ge 0$ from Lemma 1 and Lemma 3, it follows that $X \le 0$ and $Y \le 0$. As $XY \ge 0$, we can conclude that

$$\left.\frac{\partial^2 \pi_i}{\partial p_i^2}\right|_{p^*} \left.\frac{\partial^2 \pi_j}{\partial p_j^2}\right|_{p^*} - \left.\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|_{p^*} \left.\frac{\partial^2 \pi_j}{\partial p_i \partial p_j}\right|_{p^*} \ge 0,$$

which shows that the inequality (F3) is always satisfied.

From equation (F2), $\partial p_i^*/\partial\theta_i$ has the same sign as

$$\left.\frac{\partial R_i}{\partial\theta_i}\right|_{P^*} + \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial\theta_i}\right|_{P^*}.$$

From the implicit function theorem, $\partial R_i/\partial\theta_i$ has the same sign as $\partial^2\pi_i/\partial p_i\partial\theta_i$. We have

$$\frac{\partial^2\pi_i}{\partial p_i\partial\theta_i} = \left(1 + a\frac{\partial\widetilde{n}_S}{\partial p_i}\right)\frac{\partial\widetilde{D}_B^i}{\partial\theta_i} + \frac{a}{t}\frac{\partial\widetilde{n}_S}{\partial\theta_i}(-\frac{1}{2} - \beta + \alpha_B\beta\frac{\partial\widetilde{n}_S}{\partial p_i}).$$

Since

$$\frac{\partial\widetilde{n}_S}{\partial p_i} = -\frac{\partial\widetilde{n}_S}{\partial\theta_i},$$

and since

$$\frac{\partial\widetilde{D}_B^i}{\partial\theta_i} = \frac{1}{2t} + \frac{\beta}{t} - \frac{\beta\alpha_B}{t}\frac{\partial\widetilde{n}_S}{\partial p_i} = -\frac{\partial\widetilde{D}_B^i}{\partial p_i},$$

we have

$$\frac{\partial^2\pi_i}{\partial p_i\partial\theta_i} = \frac{\partial\widetilde{D}_B^i}{\partial\theta_i}\left(1 + 2a\frac{\partial\widetilde{n}_S}{\partial p_i}\right).$$

From Lemma 7 and since $\partial\widetilde{D}_B^i/\partial\theta_i \geq 0$, we have $\partial^2\pi_i/\partial p_i\partial\theta_i \geq 0$. This implies that $\partial R_i/\partial\theta_i \geq 0$. Similarly, $\partial R_i/\partial\theta_j$ has the same sign as $\partial^2\pi_i/\partial p_i\partial\theta_j$. Using the first-order condition (12)

$$\frac{\partial^2\pi_i}{\partial p_i\partial\theta_j} = -\frac{\partial^2\pi_i}{\partial p_i\partial p_j} = \left(1 + a\frac{\partial\widetilde{n}_S}{\partial p_i}\right)\frac{\partial\widetilde{D}_B^i}{\partial\theta_j} - a\frac{\partial\widetilde{n}_S}{\partial\theta_j}\frac{\partial\widetilde{D}_B^i}{\partial\theta_i} \leq 0.$$

Since $\partial\widetilde{D}_B^i/\partial\theta_j \leq 0$, $\partial\widetilde{n}_S/\partial\theta_j \geq 0$, $1 + a(\partial\widetilde{n}_S/\partial p_i) > 0$ and $\partial\widetilde{D}_B^i/\partial\theta_i \geq 0$, we conclude that $\partial^2\pi_i/\partial p_i\partial\theta_j \leq 0$. This implies that $\partial R_i/\partial\theta_j \leq 0$. We proved that $\partial p_i^*/\partial\theta_i$ has the same sign as $A$, where

$$A = \left.\frac{\partial R_i}{\partial\theta_i}\right|_{P^*} + \left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \left.\frac{\partial R_j}{\partial\theta_i}\right|_{P^*}.$$

Since $\left.\frac{\partial R_i}{\partial\theta_i}\right|_{P^*} \geq 0$, $\left.\frac{\partial R_i}{\partial p_j}\right|_{P^*} \geq 0$ from Appendix E, and $\left.\frac{\partial R_j}{\partial\theta_i}\right|_{P^*} \leq 0$, the sign of $A$ is ambiguous. From the implicit function theorem, we have

$$A = \left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}^{-1} \left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{p^*}^{-1} \left[\left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_j}\right|_{p^*} \left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{p^*} - \left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*} \left.\frac{\partial^2\pi_j}{\partial p_i\partial\theta_i}\right|_{p^*}\right].$$

Since, from the second-order conditions,

$$\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}^{-1}\left.\frac{\partial^2\pi_j}{\partial p_j^2}\right|_{p^*}^{-1}\geq 0,$$

the sign of A is identical to the sign of

$$B=\left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_j}\right|_{p^*}\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{p^*}-\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}\left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_i}\right|_{p^*}.$$

We have

$$\left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_j}\right|_{p^*}\left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{p^*}=-\left[\frac{\partial\widetilde{D}_B^i}{\partial p_j}\left(1+a\frac{\partial\widetilde{n_S}}{\partial p_i}\right)+a\frac{\partial\widetilde{n_S}}{\partial p_j}\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right]^2.$$

We also have

$$\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{p^*}\left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_i}\right|_{p^*}=-2\left(\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right)^2\left(1+a\frac{\partial\widetilde{n_S}}{\partial p_i}\right)\left(1+2a\frac{\partial\widetilde{n_S}}{\partial p_i}\right).$$

We denote by

$$x=\frac{\partial\widetilde{n_S}}{\partial p_i}\quad\text{and}\quad y=\frac{\partial\widetilde{D}_B^i}{\partial p_i}.$$

We have

$$\frac{\partial\widetilde{D}_B^i}{\partial p_j}=-y-\frac{\beta}{t}+\frac{2\alpha_B\beta x}{t}.$$

Replacing for $x$ and $y$ in $B$, we obtain that

$$\begin{aligned}B\quad&=\quad\frac{1}{t^2}\left[t^2y^2\left(1+2ax+2ax(1+ax)\right)-t\beta y\left(2+ax(1-ax)-2\alpha_Bx(2+ax)\right)\right]\\&\quad+\frac{\beta^2}{t^2}\left[(-1+4\alpha_Bx)(1+ax)-4(1-ax)(\alpha_Bx)^2\right].\end{aligned}$$

Since $x<0$, $1+2ax>0$ and $1+ax>0$ by Assumption 2, we have

$$1+2ax+2ax(1+ax)>0,$$

and

$$(-1+4\alpha_Bx)(1+ax)-4(1-ax)(\alpha_Bx)^2\leq 0.$$

As $1 + ax > 0$, we have $2 + ax(1 - ax) > 2(1 + ax) > 0$. Hence,

$$(2 + ax(1 - ax) - 2\alpha_B x(2 + ax)) \geq 0.$$

This implies that $B$ is positive for low values of $\beta$ and negative for high values of $\beta$ (as $B$ is strictly decreasing in $\beta$).

Similarly, $\partial p_i^*/\partial\theta_j$ has the same sign as $C$, where

$$C = \left.\frac{\partial^2\pi_j}{\partial p_j\partial\theta_j}\right|_{P^*} \left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{P^*} - \left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_j}\right|_{P^*}.$$

Since $\partial\widetilde{D}_B^i/\partial p_i = -\partial\widetilde{D}_B^i/\partial\theta_i$, we have

$$\left.\frac{\partial^2\pi_j}{\partial p_j\partial\theta_j}\right|_{P^*} \left.\frac{\partial^2\pi_i}{\partial p_i\partial p_j}\right|_{P^*} = -\frac{\partial\widetilde{D}_B^i}{\partial p_i}\left(1 + 2a\frac{\partial\widetilde{n}_S}{\partial p_i}\right)\left[\frac{\partial\widetilde{D}_B^i}{\partial p_j}\left(1 + a\frac{\partial\widetilde{n}_S}{\partial p_i}\right) + a\frac{\partial\widetilde{n}_S}{\partial p_j}\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right],$$

and

$$\left.\frac{\partial^2\pi_i}{\partial p_i^2}\right|_{P^*} \left.\frac{\partial^2\pi_i}{\partial p_i\partial\theta_j}\right|_{P^*} = -2\left(\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right)\left(1 + a\frac{\partial\widetilde{n}_S}{\partial p_i}\right)\left[\frac{\partial\widetilde{D}_B^i}{\partial p_j}\left(1 + a\frac{\partial\widetilde{n}_S}{\partial p_i}\right) + a\frac{\partial\widetilde{n}_S}{\partial p_j}\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right].$$

Therefore,

$$C = \left(\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right)\left[\left(1 + a\frac{\partial\widetilde{n}_S}{\partial p_i}\right)\frac{\partial\widetilde{D}_B^i}{\partial p_j} + a\frac{\partial\widetilde{n}_S}{\partial p_i}\frac{\partial\widetilde{D}_B^i}{\partial p_i}\right] \leq 0.$$

Since $C \leq 0$, we conclude that $\partial p_i^*/\partial\theta_j \leq 0$. ■

**Appendix D2:**

(à écrire)