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by Hajime Tomura

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Abstract

This paper presents a model of an over-the-counter bond market in which bond dealers and cash investors arrange repurchase agreements (repos) endogenously. If cash investors buy bonds to store their cash, then they suffer an endogenous bond-liquidation cost because they must sell their bonds before the scheduled times of their cash payments. This cost provides incentive for both dealers and cash investors to arrange repos with endogenous margins. As part of multiple equilibria, the bond-liquidation cost also gives rise to another equilibrium in which cash investors stop transacting with dealers all at once. Credit market interventions block this equilibrium.

JEL classification: G24
Bank classification: Payment, clearing, and settlement systems; Financial markets; Financial stability

Résumé

L'auteur présente le modèle d'un marché obligataire de gré à gré sur lequel des investisseurs disposant de liquidités et des courtiers concluent de façon endogène des opérations de pension. Si les investisseurs achètent des obligations pour y conserver leurs liquidités, ils s'exposent à un coût de liquidation, car ils seront forcés de les vendre avant l'échéance des paiements qu'ils doivent régler comptant. Ce coût endogène incite tant les courtiers que les acquéreurs d'obligations à conclure des pensions assorties de marges endogènes. Le coût de liquidation des obligations permet l'apparition d'un second équilibre, dans lequel tous les investisseurs cessent en même temps de traiter avec les courtiers. Cet équilibre est entravé par des interventions sur les marchés de crédit.

Classification JEL : G24
Classification de la Banque : Systèmes de paiement, de compensation et de règlement; Marchés financiers; Stabilité financière

1 Introduction

Repurchase agreements, or repos, are one of the primary instruments in the money market. In a repo, a cash investor buys bonds with a promise that the seller of the bonds, typically a bond dealer, will buy back the bonds at a later date. A question arises from this observation regarding why cash investors need repos when they can simply buy and resell bonds in a series of spot transactions. In this paper, I present a model to show that cash investors arrange repos with bond dealers because of an endogenous bond-liquidation cost in an over-the-counter (OTC) bond market. This result is consistent with the fact that almost all bond markets are OTC markets in practice (Harris 2003). Furthermore, the bond-liquidation cost makes repos exist in tandem with a possibility of a repo-market collapse. This result provides an explanation as to why a repo market with safe repo collateral, such as the U.S. tri-party repo market, can collapse, as concerned during the recent financial crisis.

In the model, cash investors buy long-term bonds to store their cash, and resell them when they need to pay out cash. In each bond transaction, the seller and the buyer bargain over the terms of trade bilaterally. Thus, the bond market is an OTC market. When a cash investor resells bonds, the buyer of the bonds can negotiate down the bond price because a cash investor needs to obtain cash before the scheduled time of the investor's cash payment. This ex-post price discount on cash investors' bonds induces dealers to sell bonds with repos to cash investors ex-ante. In these transactions, a sufficiently low ask price makes cash investors willing to buy bonds despite an ex-post price discount for their bonds. Dealers can offer such a low ask price because repos allow dealers to repurchase cash investors' bonds at a discounted price later. The ask price of bonds with repos becomes so low that dealers must finance part of the cost to acquire bonds for cash investors by their own cash. Thus, a repo margin emerges endogenously.

This equilibrium with repos is part of multiple equilibria. There exists another equilib-

rium in which cash investors stop transacting with dealers all at once. In this equilibrium, a disappearance of cash investors entering into repos with dealers causes aggregate cash shortage for dealers. As a result, dealers run short of cash to repurchase bonds from cash investors who entered into repos before and need cash now. In response, these cash investors sell their bonds to other cash investors in search of market liquidity. This bond liquidation justifies the disappearance of cash investors entering into repos with dealers, because cash investors holding cash can instead buy the liquidated bonds at a discounted price due to the sellers' imminent need for cash.

This result is consistent with the concern over a collapse of the U.S. tri-party repo market in the run-up to the Bear Stearns' collapse in March 2008. As will be described in Section 2, a perhaps puzzling feature of this concern was that most of the bonds in the market was safe long-term bonds, such as Treasury securities and agency debt. The result of the model, however, shows that a repo market with safe bonds can collapse. For policy implications, the model shows that a central-bank facility for lending cash to dealers like the Primary Dealer Credit Facility, which was introduced by the Federal Reserve in March 2008, prevents a repo-market collapse if dealers have a sufficiently high time discount factor. Alternatively, the central bank can prevent a repo-market collapse by committing to a bond purchase within a certain range of prices in the interdealer market.

1.1 Related literature

This paper is related to several strands of the literature. Duffie, Gârleanu and Pedersen (2005) show that bid-ask spreads appear when asset dealers set their prices in light of their clients' outside options in an OTC market.[1] They derive this result without inventory risk

[1]Lagos and Rocheteau (2010) extend Duffie, Gârleanu and Pedersen's model to introduce unrestricted asset holdings by dealers' clients. They show that the adjustments of asset holdings have important effects on trade volume, bid-ask spreads, and trading delays.

to dealers or asymmetric information.[2] In this paper, similar bid-ask spreads arise, because the difficulty for cash investors to postpone their cash payments lowers the value of their outside options in OTC bond transactions. Based on this result, I show that bid-ask spreads provide incentive to arrange repos for both dealers and cash investors.

The analysis of fragility of a repo market also adds to Duffie, Gârleanu and Pedersen's work. In their model, dealers never suffer aggregate cash shortage because, in aggregate, dealers are always matched with a fixed number of investors through random matching at each time point. In this paper, aggregate cash shortage for dealers, and hence a repo-market collapse, can occur, because a cash investor can choose either to transact with a dealer or to try to be matched with another investor. In this regard, this paper is related to Miao's (2006) model, in which investors choose between a decentralized market among investors and a centralized market intermediated by dealers. Miao focuses on spot asset trade and analyzes an equilibrium in which both markets are active.

Hart and Moore (1994) and Kiyotaki and Moore (1997) explain the existence of secured loans in practice by considering renegotiations of debt by borrowers given their intangible human capital for production. In this paper, repo margins emerge because of a possibility of renegotiations of repos by dealers given cash investors' need for cash. This result is consistent with the fact that repos are regarded as secured loans in practice, because, if a dealer could commit to repurchasing bonds at an arbitrary price without any possibility of a renegotiation, then a repo would be just a non-secured loan.

The existence of endogenous repo margins in this paper is also related to Geanakoplos' (2009) model. His model features an endogenous borrowing constraint due to a no-default condition imposed by agents, given existence of a stochastic shock to the value of borrowers' assets in the future. The reason behind endogenous repo margins is different from his model,

[2]This result contrasts with an earlier literature that explains bid-ask spreads by inventory risk to dealers (Garman 1976, Amihud and Mendelson 1980, and Ho and Stoll 1981) or asymmetric information (Bagehot 1971, Glosten and Milgrom 1985, and Kyle 1985).

because I feature safe bonds as repo collateral.

Martin, Skeie and von Thadden (2010) present a model to analyze the role of clearing banks in causing the fragility of the U.S. tri-party repo market. Before the market reform in 2010, clearing banks returned cash to cash investors in tri-party repos during each daytime. Martin, Skeie and von Thadden show that this feature of the U.S. tri-party repo market leads to a possibility of unexpected runs on dealers, given an exogenous asset-liquidation cost. In this paper, I illustrate another mechanism of a repo-market collapse by featuring an endogenous bond-liquidation cost in an OTC bond market.

Monnet and Narajabad (2011) present a generic model of repos to analyze co-existence of repos and spot trade of assets in an OTC market. They show that investors arrange repos between them if each investor has uncertainty about the future use of assets. This paper differs from their analysis in analyzing repos between bond dealers and cash investors.

Finally, there is a strand of literature on special repo rates due to security lending to short-sellers, such as Duffie (1996) and Vayanos and Weill (2008), for example. In this paper, I focus on general repos by considering homogeneous bonds in the model.

2 The concern over a collapse of the U.S. tri-party repo market during the recent financial crisis

In this section, I briefly summarize the concern over a collapse of the U.S. tri-party repo market during the recent financial crisis as an empirical motivation for this paper.

A repo is a spot sale of securities combined with an agreement to repurchase the securities later. This transaction is akin to a secured loan, since the provider of cash, called a cash lender, can liquidate securities if the provider of the securities, called a cash borrower, fails to repurchase the securities. Thus, the securities are, in effect, collateral. A cash lender usually lends cash to a cash borrower only up to a fraction of the market value of the securities

received as repo collateral. The remaining value of repo collateral is called a margin, which must be financed by the cash borrower's own cash. A cash lender earns interest through a higher repurchase price than the initial price of repo collateral that the cash lender pays.

In the U.S., the main repo market is the tri-party repo market. In this market, typical cash borrowers and lenders are bond dealers and institutional cash investors, respectively.[3] Also, most of the repo collateral is safe long-term bonds. Analyzing data from the two clearing banks involved with tri-party repos (The Bank of New York Mellon and JPMorgan Chase), Copeland, Martin and Walker (2010) report that around 85% of collateral for tri-party repos was Treasury securities and agency debt over the sample period between July 2008 and January 2010.[4] Also, Krishnamurthy, Nagel and Orlov (2011) gather data from the quarterly filings of MMFs to the Securities and Exchange Commission (SEC) to show that Treasury securities and agency debt accounted for more than around 70% of repo collateral held by a sample of MMFs between the first quarters of 2007 and 2010. MMFs are among main cash lenders in repos and typically use the tri-party repo market for their repo transactions.

Despite the protection of cash lenders by safe long-term bonds as repo collateral, however, policy makers had serious concern over a collapse of the tri-party repo market in the run-up to the Bear Stearns' collapse in March 14, 2008. For example, the Financial Crisis Inquiry Commission (2011) notes the concern expressed by Federal Reserve Board Chairman Ben Bernanke, such that:

[3]The majority of dealers in the market are securities broker-dealers called primary dealers, who can trade directly with the Federal Reserve Bank of New York in its open market operations. The two large groups of cash investors in the market are money market funds (MMFs) and securities lenders, who receive cash collateral from short-sellers in exchange for lending securities. Municipalities and non-financial firms are also among cash investors in the market. See Copeland, Martin and Walker (2010) for more details.

[4]Note that agency debt was deemed safe between 2007 and 2010. Moody's long-term ratings for Fannie Mae and Freddie Mac, the two government-sponsored enterprises (GSEs) that guarantee agency mortgage-backed securities (agency MBS), remained at Aaa between 2007 and 2010 because of government guarantees. Agency MBS accounted for most of the agency debt in the tri-party repo market, as reported by Copeland, Martin and Walker (2010).

> The $2.8 trillion tri-party repo market had "really [begun] to break down," Bernanke said. "As the fear increased," short-term lenders began demanding more collateral, "which was making it more and more difficult for the financial firms to finance themselves and creating more and more liquidity pressure on them. And, it was heading sort of to a black hole." He saw the collapse of Bear Stearns as threatening to freeze the tri-party repo market, leaving the short-term lenders with collateral they would try to "dump on the market. You would have a big crunch in asset prices." (pp. 290-291)

Also, in accordance with Bernanke's comment, Adrian, Burke and McAndrews (2009, page 2) at the Federal Reserve Bank of New York note an increase in haircuts (the ratios of margins to the values of repo collateral) for repos backed by Treasury securities and agency debt before the Bear Stearns' collapse.

These observations by the Fed officials are puzzling, because cash lenders in the tri-party repo market were protected by safe long-term bonds as repo collateral. Supporting the Fed officials' concern, however, the model in this paper illustrates that a repo market can collapse despite having safe repo collateral. Also, the model implies that a central-bank facility for lending cash to dealers like the Primary Dealer Credit Facility (PDCF), which was introduced by the Federal Reserve in March 17, 2008, prevents a repo-market collapse if dealers have a sufficiently high time discount factor. This result offers an explanation as to why the quarterly SEC filings of MMFs, whose figures are as of different month-ends throughout each quarter, did not show any significant increase in the average haircut for repos backed by Treasury securities and agency debt at the end of March 2008, as uncovered by Krishnamurthy, Nagel and Orlov (2011). If the PDCF had stabilized the tri-party repo market before the end of March 2008, then the observations by the Fed officials do not contradict the finding by Krishnamurthy, Nagel and Orlov.

3 A basic model of a bond market with cash investors

I start from a set-up without dealers. This set-up is the basic market structure in this paper, into which dealers will be introduced later.

3.1 The set-up

Time is discrete and its horizon is infinite. In each period, a $[0, 1]$ continuum of risk-neutral investors are born with a fixed amount e_I (> 0) of a cash endowment for each. They live for two periods and consume cash in the last period of their lives. I call investors in their first period "young" and those in their last period "old". Each investor is indexed by $i \in \mathbb{Z} \times [0, 1]$, which is the pair of the integer denoting the period when the investor is born and the real number assigned to the investor on the unit continuum of the cohort.

Young investors can invest their cash in two instruments. One is safe short-term bills that return a fixed amount $1 + r$ (≥ 1) of cash in the next period for each unit of cash invested. I call this instrument "T-bills". The other instrument is safe long-term bonds which generate a fixed amount d (> 0) of cash dividends for the holders of bonds at the beginning of every period. I call this instrument "bonds". Bonds are divisible and their supply is fixed to unity. Thus, bonds are Lucas trees. An investor can store cash by buying bonds when young and reselling them when old.[5]

Investors can trade bonds in a brokered OTC market. In each period, each young investor is matched with an old investor, and vice versa, through pairwise random matching. Implicitly, the matching can be interpreted as arranged by brokers. The terms of bond trade in each match are determined by Nash bargaining with equal bargaining powers for both parties in the match. See Table 1 for a summary of the bond market structure.

[5]If an old investor does not resell bonds, then that investor keeps holding the bonds even after exiting from the economy. Nobody gains utility from the cash dividends of the bonds after the investor's exit in this case. This case does not occur in equilibrium as shown below.

3.2 An endogenous bond-liquidation cost for investors

The finite time horizon for each investor's cash consumption represents the difficulty for cash investors to postpone their cash payments when they need to pay out cash in practice. Now I show that this difficulty leads to an endogenous bond-liquidation cost.

I consider the case in which investors are homogeneous. In this case, each old investor holds a unit of bonds at the beginning of each period. Each old investor sells the investor's whole bond to a young investor in the brokered market, because an old investor can consume cash only in the current period. Thus, the cash consumption of an old investor equals the sum of gross returns on the investor's bond and T-bills:

$$c_{i,t} = d + p_{BR,t} + (1 + r)(e_I - p_{BR,t-1}), \tag{1}$$

where i is the index for an old investor, t denotes the time period, $c_{i,t}$ is the investor's consumption, and $p_{BR,t}$ is the price of a unit of bonds in the brokered market. The value of $p_{BR,t}$ becomes identical for each investor as shown below.

Suppose that the cash endowment for each young investor, e_I, is arbitrarily large, so that young investors always have a residual of cash to invest in T-bills at the end of each period. Given Eq. (1), the Nash-bargaining problem over bond trade between a young and an old investor can be written as:

$$\max_{p_{BR,t}} (p_{BR,t} - 0)^{0.5}[E_t(d + p_{BR,t+1}) + (1 + r)(e_I - p_{BR,t}) - (1 + r)e_I]^{0.5}, \tag{2}$$

where the left parenthesis and the right square bracket represent the gains from trade for an old and a young investor, respectively. In the left parenthesis, there appears a zero as the outside option value of keeping holding a unit of bonds for an old investor, because the old investor needs to consume cash now. In the right square bracket, $(1 + r)e_I$ is the expected

consumption of an young investor when the investor does not buy any bond, in case of which the investor invests all of the investor's cash in T-bills.

The solution to the bargaining problem is:

$$p_{BR,t} = \frac{0.5E_t(d + p_{BR,t+1})}{1 + r}, \tag{3}$$

which implies that a young investor can buy a bond at a price lower than the indifference price for a young investor, $E_t(d + p_{BR,t+1})/(1 + r)$. Intuitively speaking, a young investor can negotiate down the price of an old investor's bond because an old investor must obtain cash to consume by the end of the current period. This price discount is a bond-liquidation cost from an old investor's perspective.

4 A model of a bond market with dealers and cash investors

Now I introduce dealers into the basic model. In addition to investors described above, there exists a $[0, 1]$ continuum of infinite-lived risk-neutral dealers maximizing the expected discounted consumption of cash:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} c_{j,s}, \tag{4}$$

where β ($\in (0, 1)$) is the time discount factor for dealers, j ($\in [0, 1]$) is the index for a dealer, and $c_{j,t}$ is the consumption of cash.

In the presence of dealers, an investor has two choices for the investor's bond trade in each period. One is to enter the brokered market to trade bonds with another investor as in the basic model. The other is to trade bonds with a dealer. An investor cannot choose both options within a period because it takes time to find a trading counterparty in each option. See Table 2 for a summary of the bond market structure with dealers.

4.1 The brokered market for investors

Because investors do not necessarily enter the brokered market, I redefine the matching probabilities in the market. As in the basic model, young investors entering the brokered market are matched with old investors in the market, and vice versa, through pairwise random matching. The matching probabilities are:

$$\mu_{BR,Y,t} \equiv \min\left\{1,\ \frac{\theta_{BR,O,t}}{\theta_{BR,Y,t}}\right\}, \quad \mu_{BR,O,t} \equiv \min\left\{1,\ \frac{\theta_{BR,Y,t}}{\theta_{BR,O,t}}\right\}, \tag{5}$$

where: $\mu_{BR,Y,t}$ and $\mu_{BR,O,t}$ denote the matching probabilities for a young and an old investor in the market, respectively; and $\theta_{BR,Y,t}$ and $\theta_{BR,O,t}$ are the fractions of young and old investors entering the market, respectively. Thus, the short side of the market matches with probability one.[6] If $\theta_{BR,Y,t} = \theta_{BR,O,t} = 0$, then $\mu_{BR,Y,t} = \mu_{BR,O,t} = 0$. Once matched, a young and an old investor trade bonds as in the basic model.

4.2 The dealer market for investors

As the alternative to the brokered market, an investor can enter the dealer market. If a young investor enters the dealer market, then the investor is always randomly matched with one of the dealers. The probability for a dealer to meet with a young investor equals the fraction of young investors entering the dealer market, $1 - \theta_{BR,Y,t}$. A dealer cannot be matched with more than one young investor in each period. These matching probabilities are similar to those in the brokered market, with which the short side matches with probability one.[7]

A matched pair of a young investor and a dealer bargain over bond trade. The terms of

[6] In the basic model described above, every investor enters the brokered market in each period because there is no alternative way to trade bonds. Thus, $\theta_{BR,Y,t} = \theta_{BR,O,t} = 1$ for all t. Accordingly, every young investor meets with an old investor, and vice versa, in each period.

[7] Note that the populations of young investors and dealers are unity in each period. Thus, $1 - \theta_{BR,Y,t}$ is the ratio of young investors to dealers in the dealer market. Because the ratio of dealers to young investors in the dealer market, $1/(1 - \theta_{BR,Y,t})$, is equal to, or greater than, one, the matching probability for a young investor is always one.

trade are the price and the quantity of bonds that the young investor buys and whether to arrange a repo or not. If an investor buys bonds with a repo from a dealer when young and enters the dealer market when old, then that investor is matched with the same dealer again when old.[8] In this case, either party to the repo can initiate a renegotiation of the terms of the repo when they meet again. Expecting the possibility of an ex-post renegotiation, a dealer and a young investor arranging a repo set the contracted repurchase price to the expected value of a renegotiated repurchase price, so that a renegotiation will be unnecessary in the next period. This assumption reflects the fact that repos are regarded as secured loans in practice. If a dealer could commit to paying an arbitrary repurchase price to an investor in the next period, then a repo would be just a non-secured loan to a dealer.[9]

An old investor entering the dealer market without a repo is always, but randomly, matched with one of the dealers. Thus, arranging a repo is necessary for a dealer to combine a spot sale and a repurchase of bonds for an investor. The probability for a dealer to be matched with an old investor without a repo equals the fraction of old investors entering the dealer market without repos.[10] A dealer cannot be matched with more than one old investor without a repo in each period. In a match between a dealer and an old investor without a repo, the pair negotiate the price and the quantity of bonds that the old investor sells.

Overall, a dealer can meet with three investors at maximum in each period: a young investor, an old investor with a repo, and an old investor without a repo. Without loss of generality, each dealer is matched with investors in this order in each period. The result of the model is insensitive to the order of the matches (see Appendix B). The outcome of each

[8]Without loss of generality, I assume that an old investor with a repo never meets with a dealer other than the dealer for the repo. This behaviour is weakly optimal for an old investor because a dealer offers the same terms of trade for an old investor's bonds regardless of existence of a repo, as shown below.

[9]As will be shown below, the renegotiation-proof repurchase price equals the spot price that a dealer would offer to an old investor who holds a bond without a repo. Thus, the amount of cash that a dealer can commit to paying to an investor in the next period is tied to a spot bond price for the investor in the next period, as if bonds are collateral.

[10]The fraction can be written as $1 - \theta_{BR,O,t} - \theta_{RP,O,t}$, where $\theta_{RP,O,t}$ denotes the fraction of old investors who return to the dealers for the investors' repos in period t.

match is determined by Nash bargaining with equal bargaining powers for both parties in the match, as in the brokered market. See Table 3 for a summary of the structure of the dealer market.

4.3 The interdealer markets and settlements

After meeting with investors, dealers can trade bonds with other dealers in a competitive interdealer bond market. This assumption is based on the feature of the interdealer market for U.S. Treasury securities in practice, in which interdealer brokers allow dealers to trade in size anonymously and distribute the best bid and ask prices to dealers. See Huang, Cai and Wang (2002) and Fleming and Mizrach (2009) for more details. Also, dealers can borrow cash from other dealers overnight at a competitive interest rate in an interdealer loan market. This assumption makes it tractable to solve the Nash-bargaining problem for each match between an investor and a dealer.[11]

The settlements of transactions in each period take place only at the end of the period.[12] Hence dealers can settle transactions with investors after trading in the interdealer markets in the period. After the settlements, young investors can invest the residual of their cash in T-bills, T-bills return cash to old investors, and dealers and old investors can consume cash.

Dealers and investors take as given the competitive interdealer bond price and interest rate. An equilibrium is such that these two competitive interdealer prices clear the interdealer markets in each period. See Appendix A for an analytical definition of the market clearing conditions.

[11] With this assumption, the marginal shadow values of bonds and cash for a dealer become independent of the dealer's state variables. See Appendix B for more details.

[12] This assumption reflects the fact that the settlement date of an asset transaction is typically set to a few days after the transaction date in practice.

5 Existence of a repo market

In this section, I show the existence of a symmetric stationary equilibrium in which all investors transact with homogeneous dealers. I assume that:

$$\beta(1+r) < 1, \tag{6}$$

which implies that the rate of return that dealers require for their investments, β^{-1}, is higher than the rate of return on T-bills, $1+r$. Also, the cash endowment for each young investor, e_I, is arbitrarily large, as assumed in the basic model.

In this equilibrium, no entry of investors into the brokered market leads to a zero matching probability in the brokered market. Thus every investor enters the dealer market. Given no entry of investors into the brokered market, I solve the events in each period backward. See Appendix B for the solution and its proof. Here, I sketch the solution with its intuition.

5.1 The pay-off for each dealer at the end of each period

At the end of each period, a dealer can trade bonds and arrange loans in the interdealer markets, and can also consume cash. The highest marginal return from these three options determines the shadow value of cash for a dealer at the end of each period:

$$1+\eta_t = \max\left\{1,\ \beta(1+rr_t)E_t(1+\eta_{t+1}),\ \frac{\beta E_t[(d+p_{ID,t+1})(1+\eta_{t+1})]}{p_{ID,t}}\right\}, \tag{7}$$

where $1+\eta_t$ denotes the shadow value of a unit of cash at the end of period t for each dealer, rr_t the competitive interdealer interest rate, and $p_{ID,t}$ the competitive interdealer bond price.[13] Note that a dealer takes as given all of the variables on the right-hand side of

[13]The variable η_t is the Lagrange multiplier for the flow-of-funds constraint for a dealer. See Appendix B for the flow-of-funds constraint.

Eq. (7). Hence the value of $1+\eta_t$ is exogenous for every dealer.

The shadow value of cash, $1+\eta_t$, can be used to summarize each dealer's expected discounted utility at the end of each period by the following value function, $V_{j,t}$:

$$V_{j,t} = (1+\eta_t)\left[(d+p_{ID,t})a_{j,t-1} + (p_{Y,j,t}-p_{ID,t})q_{Y,j,t} + \left(p'_{Y,j,t}-p_{ID,t}\right)x_{j,t} \right.$$
$$\left. + (p_{ID,t}-p_{O,j,t})q_{O,j,t} + (p_{ID,t}-p_{RP,j,t})q_{RP,j,t}\right] + \beta E_t\left(V^*_{x,t+1}x_{j,t} + f_{j,t+1}\right), \quad (8)$$

where j and t denote the indices for a dealer and the time period, respectively. The terms inside the square bracket on the right-hand side of Eq. (8) are cash flows due to bond transactions with investors in the same period. See Table 4 for the notation of the variables. These cash flows are evaluated by the shadow value of cash, $1+\eta_t$.

The variable $x_{j,t}$ denotes the amount of bonds that the dealer sells to a young investor with a repo in the period. The value of $V^*_{x,t+1}$ equals the marginal return for the dealer from repurchasing bonds from the investor and reselling the bonds in the interdealer market in the next period:

$$V^*_{x,t+1} = (1+\eta_{t+1})(p_{ID,t+1}-p_{RP,j,t+1}), \quad (9)$$

where $p_{RP,j,t+1}$ is the repurchase price. Thus, $E_tV^*_{x,t+1}x_{j,t}$ is the expected return from arranging a repo for the dealer. The variable $f_{j,t}$ is a residual exogenous component of the value function for the dealer.

5.2 An endogenous bond-liquidation cost in bilateral bargaining between a dealer and an old investor

Moving backward, I solve the Nash-bargaining problem for a match between a dealer and an old investor without a repo in the dealer market. As in Eq. (1), the consumption of an

old investor without a repo equals the gross returns on bonds and T-bills:

$$c_{i,t} = (d + p_{O,j,t})q_{Y,j',t-1} + (1 + r)(e_I - p_{Y,j',t-1}q_{Y,j',t-1}), \tag{10}$$

where i is the index for an old investor without a repo, j is the index for the dealer matched with the old investor, $p_{O,j,t}$ is the bid price of bonds for the old investor, $q_{Y,j',t-1}$ is the amount of bonds that the old investor bought from some dealer j' without a repo in the previous period, and $p_{Y,j',t-1}$ is the ask price of bonds for the old investor then.[14] Note that every investor buys bonds from a dealer when young, given no entry of investors into the brokered market.

Because an old investor needs to sell all of the investor's bonds to consume cash within the current period, the Nash-bargaining problem for a match between dealer j and old investor i without a repo can be written as:

$$\max_{p_{O,j,t}} \left(p_{O,j,t}q_{Y,j',t-1} - 0\right)^{0.5} \left[(p_{ID,j,t} - p_{O,j,t})q_{Y,j',t-1}\right]^{0.5}, \tag{11}$$

where $p_{ID,t}$ is the interdealer bond price. In the left parenthesis, a zero appears as the outside option value of keeping holding bonds for the old investor, because the investor does not gain any utility from keeping holding bonds after the current period. In the right square bracket, $(p_{ID,j,t} - p_{O,j,t})q_{Y,j',t-1}$ is the net profit for dealer j from buying the old investor's bonds and reselling them in the interdealer bond market. The solution for the problem is:

$$p_{O,j,t} = 0.5p_{ID,t}. \tag{12}$$

The Nash-bargaining problem for a renegotiation of a repo between a dealer and an old investor takes a similar form to Eq. (11), because existence of a repo does not alter the events

[14] Precisely speaking, the index j' is a function of i in equilibrium.

following a match between a dealer and an old investor. As a result, the Nash-bargaining problem implies that the renegotiation-proof repurchase price in a repo equals the spot bond price for an old investor without a repo:

$$p_{RP,j,t} = 0.5 p_{ID,t}. \tag{13}$$

Eqs. (12) and (13) imply that a dealer can make a profit by buying an old investor's bonds and reselling them in the interdealer bond market (i.e., $V^*_{x,t} > 0$), because an old investor's imminent need for cash allows a dealer to negotiate down the price of the investor's bonds. This price discount is of the same nature as the endogenous bond-liquidation cost for investors described in Section 3. Also, the existence of a repo does not alter the price of bonds resold by an old investor. Thus, a repo does not provide any exogenous pricing power for a dealer, but just combines a spot sale and a repurchase of bonds by a dealer.

5.3 Emergence of a repo in bilateral bargaining between a dealer and a young investor

Finally, I describe matches between dealers and young investors. Given no entry of investors into the brokered market, every young investor enters the dealer market. As a result, every dealer meets with a young investor with probability one, and vice versa, given the matching probabilities assumed above. The Nash-bargaining problem for each match between a dealer and a young investor can be written as:

$$\max_{\{p_{Y,j,t},\, q_{Y,j,t},\, p'_{Y,j,t},\, x_{j,t}\}} \left\{ (p_{Y,j,t} - p_{ID,t}) q_{Y,j,t} + \left[p'_{Y,j,t} - p_{ID,t} + \frac{\beta E_t V^*_{j,t+1}}{1+\eta_t} \right] x_{j,t} \right\}^{0.5}$$

$$\cdot \left\{ \left[d + E_t p_{O,j',t+1} - (1+r) p_{Y,j,t} \right] q_{Y,j,t} + \left[d + E_t p_{RP,j,t+1} - (1+r) p'_{Y,j,t} \right] x_{j,t} \right\}^{0.5}, \tag{14}$$

$$\text{s.t.} \quad p_{Y,j,t} q_{Y,j,t} + p'_{Y,j,t} x_{j,t} \leq e_I, \tag{15}$$

where: j is the index for the dealer; $(p_{Y,j,t},\ q_{Y,j,t})$ and $(p'_{Y,j,t},\ x_{j,t})$ are the pairs of the ask price and the quantity of bonds for the young investor without and with a repo, respectively; and $p_{O,j',t+1}$ is the resale bond price for the young investor in the next period if the investor buys bonds without a repo now. In this case, the index j' denotes a randomly matched dealer who buys bonds from the investor in the next period. Eq. (15) is the budget constraint for the young investor.[15]

The left and the right curly brackets in Eq. (14) show the gains from trade for the dealer and the young investor, respectively. In the left curly bracket, the first term, $(p_{Y,j,t} - p_{ID,t})q_{Y,j,t}$, is the dealer's net profit from selling bonds to the young investor without a repo. The competitive interdealer bond price, $p_{ID,t}$, appears as the marginal bond acquisition cost for the dealer, because a dealer can buy any amount of bonds at that price.

The second term in the left curly bracket, $[p'_{Y,j,t} - p_{ID,t} + \beta E_t V^*_{j,t+1}/(1+\eta_t)]x_{j,t}$, is the dealer's expected net profit from selling bonds with a repo. The key difference between the first and the second terms is in the expected discounted ex-post profit from a repo for the dealer, $\beta E_t V^*_{j,t+1}/(1+\eta_t)$.[16] This term appears only in the second term because the young investor will be randomly matched with one of the dealers in the next period if the dealer does not arrange a repo now. In this case, the probability for a rematch is zero, because each dealer has a zero measure on the unit continuum of dealers.[17]

In the right curly bracket in Eq. (14), the first term, $[(d + E_t p_{O,j',t+1} - (1+r)p_{Y,j,t}]q_{Y,j,t}$, is the gain for the young investor from buying bonds without a repo. This term includes the opportunity cost of paying the bond price for the young investor, $(1+r)p_{Y,j,t}$. The second

[15]Precisely speaking, $q_{Y,j,t}$ or $x_{j,t}$ must equal 0 given the assumption that a young investor chooses whether to arrange a repo for all of her bonds or not. I omit this constraint here because it is satisfied endogenously as shown below.

[16]This term is discounted by $\beta/(1+\eta_t)$, which is the effective time discount factor for dealers. This factor takes into account the shadow value of cash, $1+\eta_t$, in the current period.

[17]Note that every dealer has capacity to serve both old investors with and without a repo in each period, as assumed in the previous section. Given the probability for a dealer to be matched with an old investor without a repo in the next period, a dealer just loses the chance to meet with an investor again if the dealer does not arrange a repo when the investor is young.

term, $\left[d + E_t p_{RP,j,t+1} - (1+r)p'_{Y,j,t}\right] x_{j,t}$, is the gain for the young investor from buying bonds with a repo. Because $p_{O,j',t+1} = p_{RP,j,t+1}$ as shown in Eqs. (12) and (13), the joint gain from trade for the dealer and the young investor is larger if they arrange a repo. The dealer can induce the young investor to enter into a repo by lowering the ask price of bonds for the investor, $p'_{Y,j,t}$. Hence, the young investor buys bonds with a repo, i.e., $q_{Y,j,t} = 0$.

Substituting $q_{Y,j,t} = 0$ into the Nash-bargaining problem (14) implies that, if there exist any gains from trade for the dealer and the young investor, then the investor would buy an arbitrarily large amount of bonds with a repo (i.e., $x_{j,t}$ would be arbitrarily large) with an arbitrarily large cash endowment, e_I. Since such a large demand for bonds from every young investor exceeds the fixed supply of bonds in the economy, the competitive interdealer bond price, $p_{ID,t}$, takes such a value that there are no gains from trade for dealers and young investors in equilibrium:

$$p_{ID,t} = \frac{d + E_t p_{RP,j,t+1}}{1+r} + \frac{\beta E_t V^*_{j,t+1}}{1+\eta_t}. \tag{16}$$

Given this value of $p_{ID,t}$, each pair of a dealer and a young investor choose:

$$p'_{Y,j,t} = \frac{d + E_t p_{RP,j,t+1}}{1+r}, \tag{17}$$

with which both the left and the right curly brackets in Eq. (14) equal zero.[18]

I can show that the value of $p_{ID,t}$ is too high for dealers to buy bonds in the interdealer market, given a low time discount factor for dealers assumed in Condition (6). Thus, every bond is sold to a young investor with a repo in each period (i.e., $x_{j,t} = 1$ for all j and t).[19]

[18]Eq. (17) implies that the rate of return on a repo for a young investor, $E_t(d + p_{RP,j,t+1})/p'_{Y,j,t}$, equals the rate of return on T-bills, $1+r$. In this equilibrium, a young investor needs to arrange a repo to earn the T-bill rate on the investor's bonds, because the investor suffers a price discount when reselling the bonds in the next period.

[19]Note that both the supply of bonds and the population of young investors in each period are unity.

5.4 An endogenous repo margin

Eqs. (16) and (17) imply that the ask price of bonds with a repo, $p'_{Y,j,t}$, is lower than the interdealer bond price, $p_{ID,t}$:

$$p_{ID,t} - p'_{Y,j,t} = \frac{\beta E_t V^*_{j,t+1}}{1+\eta_t} > 0, \tag{18}$$

where the last inequality follows from Eqs. (7), (9) and (13). The difference between $p'_{Y,j,t}$ and $p_{ID,t}$ is a repo margin, because the interdealer bond price, $p_{ID,t}$, is the marginal bond acquisition cost for dealers. Eq. (18) implies that the expected discounted gain for a dealer from repurchasing bonds in the next period offsets the dealer's loss from financing the margin in the current period.

Each dealer finances the repo margin by the profit that the dealer earns from repurchasing bonds from an old investor and reselling the bonds in the interdealer market: $(p_{ID,t} - p_{RP,j,t})x_{j,t-1}$. Dealers consume the residual of the profit, given that they buy no bond in the interdealer market for their own holding as described above:

$$c_{j,t} = (p_{ID,t} - p_{RP,j,t})x_{j,t-1} - (p_{ID,t} - p'_{Y,j,t})x_{j,t} > 0, \tag{19}$$

where the last inequality holds in the stationary equilibrium.[20]

Finally, the interdealer interest rate, rr_t, satisfies:

$$\beta(1+rr_t) = 1, \tag{20}$$

in the stationary equilibrium. The interdealer loan market can clear only at this interest

[20]To confirm the inequality, derive the stationary equilibrium value of $p_{ID,t}$ from Eqs. (9), (13) and (16). Then, substitute Eqs. (13) and (17) and $x_{j,t} = 1$ for all j and t into Eq. (19). The inequality implies that $\eta_t = 0$, because the shadow value of cash equals the marginal utility from consumption, which is unity.

rate, with which homogeneous dealers do not take or provide interdealer loans.

5.5 The role of the possibility of a renegotiation in an endogenous repo margin

Before moving to the next section, I discuss the role of a renegotiation of a repo in the model. Even if a dealer and an investor could commit to not renegotiating a repo, they would still arrange a repo. In this case, a repo would allow a dealer to commit to not negotiating down the price of an old investor's bonds, so that the dealer can raise the ask price of bonds for a young investor. Increasing the current revenue through a higher ask price is attractive for a dealer, given a low time discount factor for dealers as assumed in Condition (6). Thus, the existence of a repo does not depend on the assumption that a repo can be renegotiated. Instead, the possibility of a renegotiation is crucial for an endogenous repo margin: If a dealer could commit to not negotiating down the price of an old investor's bonds, then the dealer would not have any incentive to incur a loss to finance a repo margin when arranging a repo.

6 Fragility of a repo market

The stationary equilibrium with repos described above is part of multiple equilibria. There exists another equilibrium in which, given the stationary equilibrium with repos in period $t-1$, every investor enters the brokered market unexpectedly in period t. Thus, the repo market can collapse despite having safe long-term bonds as repo collateral.[21] The economy returns to the stationary equilibrium with repos from period $t+1$ onward in this equilibrium.[22]

[21] Regarding the interpretation of this result, note that MMFs, a main group of cash lenders in the U.S. tri-party repo market, cannot hold long-term securities by regulation. Implicitly, a repo-market collapse shown below can be interpreted as ultimate cash investors, such as corporate treasuries, stop putting their cash on MMFs and instead buy bonds directly from other cash investors who liquidate repo collateral.

[22] If e_I and β satisfy certain conditions tighter than Conditions (21) and (22) and if r is sufficiently close to 0, then there exists an equilibrium in which every investor enters the brokered market from period t to period T for an arbitrarily large integer T. See Appendix D for more details.

In this section, I set two conditions on the cash endowment for each young investor, e_I, and the time discount factor for dealers, β:

$$e_I \in \left(p_{ID,SS},\ \frac{(d+0.5p_{ID,SS})(d+0.75p_{ID,SS})}{0.5(1+r)p_{ID,SS}} \right], \tag{21}$$

$$\beta(1+r) < \min\left\{ 1,\ \frac{4+6r+(1+2r)\sqrt{33+32r}}{5+\sqrt{33+32r}} \right\}, \tag{22}$$

where $p_{ID,SS}$ denotes the value of the interdealer bond price, $p_{ID,t}$, in the stationary equilibrium with repos:

$$p_{ID,SS} = \frac{d}{0.5[1-\beta(1+r)]+r}. \tag{23}$$

Eq. (23) can be derived from Eqs. (9), (13) and (16). Condition (21) indicates that e_I is not extremely large, but large enough for each young investor to buy a bond at $p_{ID,SS}$, so that there exists the stationary equilibrium with repos described above.[23] Condition (22) incorporates Condition (6) assumed above and also ensures that the range for e_I in Condition (21) is a non-empty set. Given Conditions (21) and (22), I guess and verify the existence of an equilibrium with a repo-market collapse with the features described above.

6.1 The existence of the stationary equilibrium with repos from period $t+1$ onward

Suppose that each young investor buys a bond in the brokered market in period t, as verified below. Thus, each old investor has a bond without a repo at the beginning of period $t+1$. All of the results described in Section 5 hold for period $t+1$ and later, except that each old investor in period $t+1$ is randomly matched with a dealer, and vice versa.[24]

[23]The condition $e_I > p_{ID,SS}$ ensures that $p_{ID,t} = p_{ID,SS}$ in the stationary equilibrium with repos. If $p_{ID,t} < p_{ID,SS}$, then young investors would put all of their cash on repos. As a result, dealers would need to obtain an amount $\int (e_I/p_{Y',j,t})dj$ of bonds in total. Because $p_{Y',j,t} \leq p_{ID,t}$, the bond demand would exceed the unit bond supply in the economy, which would violate the market clearing condition.

[24]For the existence of the stationary equilibrium with repos, it is not necessary that old investors in the initial period have repos. The key condition for the existence is that no investor enters the brokered market,

6.2 Bilateral bargaining between a young and an old investor in the brokered market in period t

Now I describe events in period t. When every investor enters the brokered market in period t, each young investor is matched with an old investor, and vice versa, given the matching probabilities in the brokered market assumed above. Since each young investor in period $t-1$ buys a bond with a repo in the stationary equilibrium with repos (i.e., $x_{j,t-1}=1$ for all j), each old investor in period t has a bond at the beginning of the period. Hence, each young investor buys a bond from an old investor in the brokered market in period t.[25]

The Nash-bargaining problem for each match between a young and an old investor in the brokered market takes the same form as Eq. (2), except that the bid price of bonds in the dealer market, $p_{O,j,t+1}$, replaces the bond price in the brokered market, $p_{BR,t+1}$, as the resale bond price for the young investor in the next period. This modification is necessary because the economy will return to the stationary equilibrium with repos from the next period onward. The solution to the Nash-bargaining problem yields:

$$p_{BR,t}=\frac{0.5(d+E_t p_{O,j,t+1})}{1+r}. \tag{24}$$

Eq. (24) implies that, as in the basic model, a young investor can buy an old investor's bonds at a price lower than the indifference price for a young investor, $(d+E_t p_{O,j,t+1})/(1+r)$, because the old investor's imminent need for cash allows the young investor to negotiate down the bond price.

because young investors arrange repos with dealers once they enter the dealer market.

[25]Old investors in period t cause settlement fails, i.e., failures to deliver securities to the buyers, to the dealers for their repos. I assume no punishment for settlement fails because a settlement fail is not regarded as default in practice. See the article by Fleming and Garbade (2005) for more details on settlement fails.

6.3 The dominance of the brokered market for old investors in period t

For each old investor in period t, the alternative to entering the brokered market is to return to the dealer for the investor's repo. The repurchase price offered by the dealer, $p_{RP,j,t}$, would equal $0.5p_{ID,t}$ as shown in Eq. (13), because the Nash-bargaining problem for a renegotiation of a repo takes the same form as in the stationary equilibrium with repos.

Entering the brokered market is a dominant choice for an old investor in period t if the bond price in the brokered market is equal to, or higher than, the repurchase price offered by dealers. Accordingly, suppose that:

$$p_{BR,t} = p_{RP,j,t} = 0.5p_{ID,t}, \tag{25}$$

in period t, so that old investors in the period are indifferent between entering the brokered market and returning to the dealers for their repos.

Given Eq. (25), the interdealer bond price, $p_{ID,t}$, takes the following value in period t:

$$p_{ID,t} = \frac{(1 - 0.5\beta)d}{0.5[1 - \beta(1 + r)] + r}. \tag{26}$$

This equation is derived from Eqs. (12), (23) and (24), given that the economy returns to the stationary equilibrium with repos from period $t + 1$ onward. Thus, the interdealer bond price drops in period t, because $p_{ID,t-1} = p_{ID,SS}$ in the stationary equilibrium with repos in period $t - 1$.[26]

The result that $p_{ID,t} < p_{ID,SS}$ in period t implies that the contracted repurchase price set in the previous period, $E_{t-1}p_{RP,j,t}$, exceeds the renegotiated repurchase price in period t, $0.5p_{ID,t}$, because investors and dealers in period $t-1$ expect that the stationary equilibrium with repos continues in period t (i.e., $E_{t-1}p_{RP,j,t} = 0.5p_{ID,SS}$ given Eq. 13). Hence, a dealer

[26]Because $p_{ID,t} < p_{ID,SS}$ in period t, Condition (21) ensures that each young investor has a enough cash endowment, e_I, to pay $p_{BR,t}$ for a bond in the brokered market in period t, given Eq. (25).

would default on, and renegotiate, a repo in period t if the dealer met with the old investor holding the repo.

6.4 The dominance of the brokered market for young investors in period t

A young investor in period t is faced with the following trade-off. On one hand, the investor can buy an old investor's bond at a discounted price in the brokered market, as shown above. On the other hand, the investor can buy more than a unit of bonds in the dealer market, because a dealer has access to the competitive interdealer bond market.

In the second case, the investor would arrange a repo with a dealer, because the investor would be able to commit to returning to the dealer, given that the stationary equilibrium with repos resumes from the next period onward. The Nash-bargaining problem for a match between a young investor and a dealer in period t would yield:

$$(p'_{Y,j,t},\ x_{j,t}) = \left(0.5\left[p_{ID,t} - \frac{E_t V^*_{x,t+1}}{1+rr_t}\right] + 0.5\left(\frac{d + E_t p_{RP,j,t+1}}{1+r}\right),\ \frac{e_I}{p'_{Y,j,t}}\right). \tag{27}$$

See Appendix C for the proof for this solution to the Nash-bargaining problem. The ask price of bonds with a repo, $p'_{Y,j,t}$, would equal the average of the indifference price for the dealer (the left term) and the one for the young investor (the right term) given the equal bargaining powers for the dealer and the investor.[27] Because Eqs. (25) and (26) imply that the indifference price for the dealer is strictly less than the one for the young investor, there would be strictly positive gains from trade for the dealer and the young investor. As a result, the young investor would put all of the investor's cash on the repo (i.e., $x_{j,t} = e_I/p'_{Y,j,t}$) to earn a higher rate of return than the rate of return on T-bills, $1 + r$.

Given Condition (21) and Eq. (26), the cash endowment for a young investor, e_I, is so

[27]Note that if $p'_{Y,j,t} = p_{ID,t} - E_t V^*_{x,t+1}/(1 + rr_t)$, then the rate of return on the repo for the dealer, $E_t V^*_{x,t+1}/(p'_{Y,j,t} - p_{ID,t})$, equals the rate of return on interdealer loans, $1 + rr_t$. Similarly, if $p'_{Y,j,t} = (d + E_t p_{RP,j,t+1})/(1 + r)$, then the rate of return on the repo for the young investor, $(d + E_t p_{RP,j,t+1})/p'_{Y,j,t}$, equals the rate of return on T-bills, $1 + r$.

small that the expected consumption of a young investor in period t is higher if the investor enters the brokered market rather than the dealer market:[28]

$$d + E_t p_{O,j,t+1} + (1+r)(e_I - p_{BR,t}) \geq (d + E_t p_{RP,j,t+1}) x_{j,t}. \tag{28}$$

Thus, young investors enter the brokered market in period t.

6.5 Aggregate cash shortage in the interdealer markets in period t

Now I only need to verify that the interdealer markets clear with Eq. (26) in period t. As implied by comparison between Eqs. (23) and (26), the interdealer bond price drops in period t. As a result, the rate of return on bonds in the interdealer market rises in period t:[29]

$$\frac{d + E_t p_{ID,t+1}}{p_{ID,t}} = 1 + r + \frac{1}{2-\beta}. \tag{29}$$

Given Condition (22), the rate of return on bonds for dealers exceeds the intertemporal marginal rate of substitution for dealers, β^{-1}, if $\beta(1+r)$ is sufficiently close to 1. In this case, dealers would buy bonds in the interdealer market if they had their own cash or could borrow cash at a sufficiently low cost. However, dealers become cashless as investors shift to the brokered market all at once in period t.[30] Also, a resulting zero cash supply in the interdealer loan market makes the interdealer interest rate, rr_t, equal the rate of return on bonds in the interdealer market, so that dealers are discouraged from taking interdealer

[28]The equality holds if e_I equals the upper bond shown in Condition (21). In this case, entering the brokered market is weakly optimal for young investors.

[29]Given Eqs. (16) and (26), substitute $p_{ID,t+1} = p_{ID,SS}$ to derive Eq. (29). To see the rise in the rate of return on bonds, note that the only change from the stationary equilibrium with repos in period $t-1$ comes through a decline in $p_{ID,t}$.

[30]Dealers own no bond in the stationary equilibrium with repos in period $t-1$ as described in Section 5. Thus, dealers do not receive any cash dividend from bonds at the beginning of period t.

loans.[31]

If the rate of return on bonds in the interdealer market is smaller than the intertemporal marginal rate of substitution for dealers, β^{-1}, then dealers are unwilling to buy bonds for their own.[32] In this case, $1+rr_t$ remains equal to β^{-1} so that dealers do not take or provide interdealer loans. Overall, the interdealer markets clear with Eq. (26) and the interdealer interest rate, rr_t, satisfying:

$$1 + rr_t = \max\left\{\frac{d + E_t p_{ID,t+1}}{p_{ID,t}},\ \beta^{-1}\right\}. \tag{30}$$

in period t.[33]

6.6 The effects of credit market interventions

Given the existence of multiple equilibria, I discuss the effects of credit market interventions on a repo-market collapse. Suppose that there exists a central bank which can commit to providing interdealer loans at a rate less than $r + (2-\beta)^{-1}$, but more than $\beta^{-1} - 1$. This central-bank loan facility for dealers is akin to the Primary Dealer Credit Facility (PDCF) introduced in March 2008, because it involves a penalty rate above the normal interdealer interest rate prevailing in the stationary equilibrium with repos, $\beta^{-1} - 1$.[34] The results described above imply that, if the intertemporal marginal rate of substitution for dealers, β^{-1}, is less than $1+r+(2-\beta)^{-1}$, then the central-bank loan facility eliminates the equilibrium with a repo-market collapse. If there occurred the equilibrium with a repo-market collapse in this case, then dealers would take interdealer loans to buy bonds in the interdealer market.

[31] Because dealers are cashless, they cannot provide interdealer loans despite that $\beta(1 + rr_t) > 1$.

[32] This case can be consistent with Condition (22).

[33] Even though each old investor is indifferent between transacting with a dealer and entering the brokered market with Eq. (25), the aggregate cash shortage for dealers makes it necessary that every old investor enters the brokered market. If a positive measure of old investors entered the dealer market, then the zero cash supply in the interdealer markets would prevent dealers from obtaining cash to pay for old investors' bonds by selling the bonds in the interdealer bond market or taking interdealer loans.

[34] See Eq. (20).

A resulting rise in the interdealer bond price, $p_{ID,t}$, would enable dealers to repurchase bonds from old investors at a sufficiently high price, which would prevent old investors from leaving for the brokered market. No liquidation of bonds by old investors would induce young investors to enter the dealer market and thus arrange repos with dealers.

This result, however, does not hold if $\beta^{-1} \geq 1+r+(2-\beta)^{-1}$. In this case, dealers require too high a rate of return for their investments. Accordingly, they would not borrow from the central bank to buy bonds in the interdealer market, unless the interest rate offered by the central bank were below the normal interdealer interest rate, $\beta^{-1}-1$. But such a policy would block both the stationary equilibrium with repos and the equilibrium with a repo-market collapse. To eliminate only the latter equilibrium, the central bank must commit to a direct bond purchase in the interdealer market at a price higher than the one prevailing during a repo-market collapse (see Eq. 26), but not more than the one prevailing in the stationary equilibrium with repos (see Eq. 16).

Finally, note that preventing a repo-market collapse is not Pareto-improving. Young investors in period t are better off in the equilibrium with a repo-market collapse, because they can earn a higher rate of return on bonds than in the stationary equilibrium with repos. Thus, a repo-market collapse benefits investors holding cash. These investors' gains come from the losses for dealers and old investors in period t: the old investors, who hold bonds at the beginning of period t, suffer a drop in the price for their bonds; and dealers lose the profit from trading with investors, which is shown in Eq. (19), in period t. Investors born in period $t+1$ or later are indifferent.

7 Conclusions

I have presented a model featuring bond dealers and cash investors in an OTC bond market. The model illustrates that bilateral bargaining over bond trade leads to an endogenous bond-

liquidation cost for cash investors. This cost explains both the existence of repos and the possibility of an unexpected repo-market collapse. Using this model, I have discussed the conditions under which a central-bank loan facility for dealers, like the PDCF, and a direct bond purchase by the central bank prevent a repo-market collapse.

In this paper, I take as given the OTC bond market structure. Thus, a question remains regarding the optimal market design, such as whether to introduce a centralized bond market or a set-up to ensure anonymity of cash investors. Also, the empirical implications of the model are yet to be tested. One of the testable implications is that a repo margin is increasing in the difference between the interdealer bond price and the repurchase bond price. Another implication is that spot transactions in a brokered bond market increase if a repo market collapses. Addressing these issues are left for future research.

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Appendices

A The market clearing conditions in the model of a bond market with dealers and cash investors

The market clearing conditions for the interdealer bond price, $p_{ID,t}$, and interest rate, rr_t, are:

$$\int a_{j,t} + x_{j,t} + q_{Y,j,t}\, dj + \int_{i \in I_t} q_{BR,i,t}\, di = 1, \tag{A.1}$$

$$\int b_{j,t}\, dj = 0, \tag{A.2}$$

where: $a_{j,t}$ is the amount of bonds owned by dealer j at the end of period t; $x_{j,t}$ and $q_{Y,j,t}$ are the amounts of bonds sold by dealer j to young investors with and without a repo, respectively; $q_{BR,i,t}$ is the amount of bonds that old investor i sells to a young investor in the brokered market; I_t is the set of the indices for old investors in period t; and $b_{j,t}$ is the net balance of interdealer loans for dealer j. The left-hand side of Eq. (A.1) is the sum of bonds held by dealers and young investors at the end of period t, and the right-hand side is the unit supply of bonds in the economy.

B The proof for the existence of a repo market in Section 5

In the following, I prove the existence of the symmetric stationary equilibrium in which every investor transacts with a dealer. No entry of investors into the brokered market is self-fulfilling, as described in Section 5. Thus, $q_{BR,i,t} = 0$ for all i and t. Given this result, I solve each event in each period backward.

B.1 Utility maximization by dealers in the interdealer markets

The maximization problem for dealer j in the interdealer markets can be written as:

$$
\begin{aligned}
&V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t}) = \\
&\max_{\{c_{j,t},\ q_{ID,j,t},\ b_{j,t}\}} c_{j,t} + \beta E_t V_{t+1}(S_{j,t},\ Z^*_{Y,j,t+1}(S_{j,t}),\ Z^*_{O,j,t+1}(S_{j,t}),\ Z^*_{RP,j,t+1}(S_{j,t})), \qquad \text{(B.1)} \\
&\text{s.t. } c_{j,t} + p_{ID,t} q_{ID,j,t} + \frac{b_{j,t}}{1+rr_t} \\
&\qquad = da_{j,t-1} + b_{j,t-1} + p_{Y,j,t} q_{Y,j,t} + p'_{Y,j,t} x_{j,t} - p_{O,j,t} q_{O,j,t} - p_{RP,j,t} q_{RP,j,t}, \qquad \text{(B.2)} \\
&\qquad a_{j,t} = q_{ID,j,t} + q_{O,j,t} + q_{RP,j,t} - q_{Y,j,t} - x_{j,t} + a_{j,t-1}, \qquad \text{(B.3)} \\
&\qquad c_{j,t},\ a_{j,t} \geq 0, \qquad \text{(B.4)}
\end{aligned}
$$

where V_t is the value function for each dealer's expected discounted consumption of cash. See Table 4 for the notation of variables.

In this problem, a dealer chooses the current consumption, $c_{j,t}$, the net balance of bonds to trade in the interdealer bond market, $q_{ID,j,t}$, and the net balance of interdealer loans, $b_{j,t}$, given the competitive interdealer bond price, $p_{ID,j,t}$ and interest rate, rr_t. Eq. (B.2) is the flow of funds constraint, the right-hand side of which records the cash flows before the interdealer markets and the dealer's consumption in the period. Eq. (B.3) is the law of motion for bonds owned by dealer j at the end of the period, $a_{j,t}$. Eq. (B.4) is the non-negativity constraint on $c_{j,t}$, and $a_{j,t}$. The non-negativity constraint on $a_{j,t}$ is introduced without loss of generality, just to avoid indeterminacy between short-selling bonds and taking inter-dealer loans.

The vector $S_{j,t-1}$ stores the state variables for dealer j at the beginning of period t: the amount of bonds owned by dealer j at the end of the previous period, $a_{j,t-1}$; the amount of bonds sold to a young investor with a repo in the previous period, $x_{j,t-1}$; and the balance of interdealer loans at the end of the previous period, $b_{j,t-1}$. The other pre-determined variables

for the dealer in the interdealer markets are $Z_{Y,j,t}$, $Z_{O,j,t}$, and $Z_{RP,j,t}$, which store the terms of bond trade with a young investor, an old investor without a repo, and an old investor with a repo, in order. The functions $Z^*_{Y,j,t+1}(S_{j,t})$, $Z^*_{O,j,t+1}(S_{j,t})$ and $Z^*_{RP,j,t+1}(S_{j,t})$ denote the terms of bond trade with investors in the next period, given $S_{j,t}$.

I denote the derivatives of the value function, V_{t+1}, with respect to the state variables stored in $S_{j,t}$ ($\equiv [a_{j,t},\ b_{j,t},\ x_{j,t}]$) by:

$$\begin{bmatrix} V^*_{a,t+1} \\ V^*_{b,t+1} \\ V^*_{x,t+1} \end{bmatrix} \equiv \frac{dV_{t+1}(S_{j,t}, Z^*_{Y,j,t+1}(S_{j,t}), Z^*_{O,j,t+1}(S_{j,t}), Z^*_{RP,j,t+1}(S_{j,t}))}{dS_{j,t}}. \tag{B.5}$$

I guess and verify that dealers and cash investors take as given the values of $V^*_{a,t+1}$, $V^*_{b,t+1}$ and $V^*_{x,t+1}$.

Given this conjecture, the solution to the maximization problem yields:

$$1 + \eta_t = \beta(1 + rr_t)E_t V^*_{b,t+1} \geq 1, \tag{B.6}$$

$$\eta_t c_{j,t} = 0, \tag{B.7}$$

$$[(1 + \eta_t)p_{ID,t} - \beta E_t V^*_{a,t+1}]a_{j,t} = 0, \tag{B.8}$$

where η_t is the Lagrange multiplier for $c_{j,t} \geq 0$ in Eq. (B.4). Eq. (B.6) implies that the shadow value of a unit of cash for a dealer at the end of the period, $1 + \eta_t$, is pinned down by the discounted internal rate of return on interdealer loans, $\beta(1 + rr_t)E_t V^*_{b,t+1}$. Thus, dealers and investors take the value of η_t as given. This result is because a dealer can use the interdealer loan market as a buffer for the dealer's excess cash or cash shortage at a competitive interest rate, rr_t. In equilibrium, the cost of inter-bank loans (i.e., $\beta(1 + rr_t)E_t V^*_{b,t+1}$) must be equal to, or greater than, the marginal utility from consumption (i.e.,

1), because otherwise every dealer would take interdealer loans. Such behavior of dealers would violate the market clearing condition (A.2). To confirm Eq. (B.7), note that $c_{j,t} = 0$ if $\beta(1 + rr_t)E_t V^*_{b,t+1} > 1$, because dealers are better off by postponing consumption in this case. Eq. (B.8) follows from the first-order condition with respect to $q_{ID,j,t}$, which implies that:

$$a_{j,t} \begin{cases} = 0, & \text{if } (1+\eta_t)p_{ID,t} > \beta E_t V^*_{a,t+1}, \\ = \infty, & \text{if } (1+\eta_t)p_{ID,t} < \beta E_t V_{a,t+1}, \\ \in [0,\infty) & \text{if } (1+\eta_t)p_{ID,t} = \beta E_t V^*_{a,t+1}. \end{cases} \tag{B.9}$$

Since $a_{j,t} = \infty$ for all j would violate the market clearing condition (A.1), Eq. (B.8) must hold.

Substituting Eqs. (B.2), (B.3) and (B.6)-(B.8) into Eq. (B.1) yields:

$$\begin{aligned} &V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t}) \\ &= (1+\eta_t)\left[(d + p_{ID,t})a_{j,t-1} + (p_{Y,j,t} - p_{ID,t})q_{Y,j,t} + \left(p'_{Y,j,t} - p_{ID,t}\right)x_{j,t} + (p_{ID,t} - p_{O,j,t})q_{O,j,t}\right. \\ &\qquad \left. + (p_{ID,t} - p_{RP,j,t})q_{RP,j,t}\right] + \beta E_t\left(V^*_{x,t+1}x_{j,t} + f_{j,t+1}\right), \end{aligned} \tag{B.10}$$

where:

$$f_{j,t+1} \equiv V_{t+1}(S_{j,t}, Z^*_{Y,j,t+1}, Z^*_{O,j,t+1}, Z^*_{RP,j,t+1}) - V^*_{a,t+1}a_{j,t} - V^*_{b,t+1}b_{j,t} - V^*_{x,t+1}x_{j,t}, \tag{B.11}$$

which is the residual component of V_{t+1} that does not depend on the state variables for dealer j. Thus, dealers and investors take $f_{j,t}$ as given for all j and t.

B.2 Bilateral bargaining between a dealer and an old investor

Given no entry of investors into the brokered market, the consumption of old investor i without a repo equals:

$$dq_{Y,j',t-1} + p_{O,j,t}q_{O,j,t} + (1+r)s_{i,t-1}, \tag{B.12}$$

where $q_{Y,j',t-1}$ is the amount of bonds that the investor bought from a randomly matched dealer j' without a repo in the previous period, $(p_{O,j,t},\ q_{O,j,t})$ is the pair of the price and the quantity of bonds that the investor sells to a randomly matched dealer j in the dealer market, and $s_{i,t-1}$ is the amount of cash invested in T-bills by the investor in the previous period.

Given Eq. (B.12), the Nash-bargaining problem for a match between dealer j and old investor i without a repo can be written as:

$$\begin{aligned}
&\max_{Z_{O,j,t}} \left[V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t}) - V_t(S_{j,t-1},\ Z_{Y,j,t},\ \mathbf{0}, Z_{RP,j,t})\right]^{0.5} \\
&\qquad \cdot \left\{dq_{Y,j',t-1} + p_{O,j,t}q_{O,j,t} + (1+r)s_{i,t-1} - [dq_{Y,j',t-1} + (1+r)s_{i,t-1}]\right\}^{0.5}, \qquad \text{(B.13)} \\
&\text{s.t. } q_{O,j,t} \leq q_{Y,j',t-1}. \qquad \text{(B.14)}
\end{aligned}$$

The solution is $Z_{O,j,t} = [0.5p_{ID,t},\ q_{Y,j',t-1}]$. Thus:

$$Z^*_{O,j,t} = \begin{cases} [0.5p_{ID,t},\ q_{Y,j',t-1}], & \text{if dealer } j \text{ meets with old investor } i \text{ without a repo,} \\ \mathbf{0}, & \text{if dealer } j \text{ meets no old investor without a repo,} \end{cases} \tag{B.15}$$

where $Z^*_{O,j,t}$ denotes the value of $Z_{O,j,t}$ $(\equiv [p_{O,j,t},\ q_{O,j,t}])$ given $S_{j,t-1}$. Note that $q_{Y,j',t-1} = 0$ is equivalent to no matching for dealer j in the dealer market.

The Nash-bargaining problem for a renegotiation of a repo between a dealer and an

old investor with a repo is similar to the bargaining problem described above, except that: $(p_{O,j,t},\ q_{O,j,t})$ is replaced with the pair of the price and the quantity of bonds repurchased by the dealer from the investor with a repo, $(p_{RP,j,t},\ q_{RP,j,t})$; $q_{Y,j',t-1}$ is replaced by $x_{j,t-1}$; and $Z^*_{O,j,t}$ is substituted into $Z_{O,j,t}$. Because the variables contained in $Z^*_{O,j,t}$ are exogenous to dealers and investors, solving the Nash-bargaining problem for $Z_{RP,j,t}$ $(\equiv [p_{RP,j,t},\ q_{RP,j,t}])$ yields:

$$Z^*_{RP,j,t} = [0.5p_{ID,t},\ x_{j,t-1}], \tag{B.16}$$

where $Z^*_{RP,j,t}$ denotes the value of $Z_{RP,j,t}$ given $S_{j,t-1}$.

B.3 Bilateral bargaining between a dealer and a young investor

Given no entry of investors into the brokered market, every young investor enters the dealer market. Thus, each dealer meets with a young investor, and vice versa, given the matching probabilities assumed in Section 4. The Nash-bargaining problem for a match between dealer j and young investor i can be written as:

$$\begin{aligned}
&\max_{Z_{Y,j,t}} \left[V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z^*_{O,j,t},\ Z^*_{RP,j,t}) - V_t(S_{j,t-1},\ \mathbf{0},\ Z^*_{O,j,t},\ Z^*_{RP,j,t})\right]^{0.5} \\
&\qquad \cdot \{E_t[(d + p_{O,j',t+1})q_{Y,j,t} + (d + p_{RP,j,t+1})x_{j,t}] + (1+r)s_{i,t} - (1+r)e_I\}^{0.5}, \qquad \text{(B.17)} \\
&\text{s.t. } s_{i,t} = e_I - (p_{Y,j,t}q_{Y,j,t} + p'_{Y,j,t}x_{j,t}) \geq 0. \qquad \text{(B.18)}
\end{aligned}$$

The subscript j' denotes the index for a dealer randomly matched with the young investor in the next period in case that the investor buys bonds without a repo in the current period. Also, $(p_{Y,j,t},\ q_{Y,j,t})$ and $(p'_{Y,j,t},\ x_{j,t})$ are the pairs of the price and the quantity of bonds when the dealer sells the bonds without and with a repo, respectively. Formally speaking, there should be such a constraint that $q_{Y,j,t}$ or $x_{j,t}$ must equal 0, because of the assumption that

a young investor must choose whether to buy all of their bonds with a repo or not. This constraint is satisfied endogenously, as shown below.

Now suppose that the terms of trade between a dealer and a young investor in the next period, $Z^*_{Y,j,t+1}$, is independent of the dealer's state variables, $S_{j,t}$, as will be verified later. Given this conjecture, Eqs. (B.10), (B.15) and (B.16) imply that:

$$V^*_{a,t+1} = (1+\eta_{t+1})(d + p_{ID,t+1}), \tag{B.19}$$

$$V^*_{b,t+1} = (1+\eta_{t+1}), \tag{B.20}$$

$$V^*_{x,t+1} = (1+\eta_{t+1})(p_{ID,t+1} - p_{RP,j,t+1}) = (1+\eta_{t+1})0.5p_{ID,t+1}, \tag{B.21}$$

which verifies the initial conjecture that dealers and investors take as given the values of $V^*_{a,t+1}$, $V^*_{b,t+1}$, and $V^*_{x,t+1}$.

The value of $V^*_{x,t+1} > 0$ is positive because the interdealer bond price, $p_{ID,t+1}$, is positive in equilibrium. Also, Eqs. (B.15) and (B.16) imply that $E_t p_{O,j',t+1} = E_t p_{RP,t+1}$. Given $V^*_{x,t+1} > 0$ and $E_t p_{O,j',t+1} = E_t p_{RP,t+1}$, the Nash-bargaining problem implies that $p_{Y,j,t} = q_{Y,j,t} = 0$, that is, the young investor and the dealer arranges a repo. Thus, $Z^*_{O,j,t} = \mathbf{0}$ for all j because there is no old investor without a repo in each period. This result, in turn, leads to $f_{j,t} = 0$, because substituting $q_{Y,j,t} = q_{O,j,t} = 0$ and $q_{RP,j,t} = x_{j,t-1}$ into Eq. (B.10) implies that all terms of the value function, V_t, are linear to $a_{j,t-1}$ and $x_{j,t-1}$.

Substituting $p_{Y,j,t} = q_{Y,j,t} = 0$ into the Nash-bargaining problem (B.17) reduces the problem to:

$$\max_{\{p'_{Y,j,t},\, x_{j,t}\}} \left(p'_{Y,j,t} - p_{ID,t} + \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} \right)^{0.5} \left[d + E_t p_{RP,j,t+1} - (1+r) p'_{Y,j,t} \right]^{0.5} x_{j,t}, \tag{B.22}$$

$$\text{s.t. } e_I - p'_{Y,j,t} x_{j,t} \geq 0. \tag{B.23}$$

Thus, $x_{j,t} = 0$ if $p_{ID,t} - \beta E_t V^*_{x,t+1}/(1+\eta_t) > (d + E_t p_{RP,j,t+1})/(1+r)$, and $x_{j,t}$ is arbitrarily

large if $p_{ID,t} - \beta E_t V^*_{x,t+1}/(1+\eta_t) < (d + E_t p_{RP,j,t+1})/(1+r)$, given an arbitrary large cash endowment for each young investor, e_I. The interdealer bond market would not clear in either case. In the first case, $p_{ID,t} > \beta(d + p_{ID,t+1}) = \beta V^*_{a,t+1}$ given Condition (6) and Eqs. (B.6) and (B.19). Thus, $a_{j,t} = 0$ for all j as implied by Eq. (B.9). This result would violate the market clearing condition (A.1), because $x_{j,t} = q_{Y,j,t} = 0$ for all j and $q_{BR,i,t} = 0$ for all i. In the second case, an arbitrary large value of $x_{j,t}$ for all j would violate the market clearing condition (A.1), given the fixed supply of bonds. Hence:

$$p_{ID,t} = \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} + \frac{d + E_t p_{RP,j,t+1}}{1+r}. \tag{B.24}$$

Accordingly, the ask price of bonds with a repo for a young investor, $p'_{Y,j,t}$, satisfies:

$$p'_{Y,j,t} = \frac{d + E_t p_{RP,j,t+1}}{1+r}, \tag{B.25}$$

for any positive value of $x_{j,t}$.

Suppose that:

$$\beta(1 + rr_t) = 1, \quad \eta_t = 0, \tag{B.26}$$

for all t, which is consistent with Eqs. (B.6) and (B.20). Then, Eqs. (B.19), (B.21) and (B.24) and $\eta_{t+1} = 0$ jointly imply that:

$$p_{ID,t} = \frac{d}{0.5[1 - \beta(1+r)] + r}, \tag{B.27}$$

$$\frac{E_t V^*_{a,t+1}}{p_{ID,t}} = 1.5 + r - 0.5\beta(1+r) < \beta^{-1}, \tag{B.28}$$

in the stationary equilibrium, in which $p_{ID,t}$ is constant for all t. The last inequality in Eq. (B.27) follows from Condition (6). Given Eq. (B.9), Eq. (B.28) implies that dealers do not

own bonds at the end of each period, i.e., $a_{j,t} = 0$. Thus, given that $a_{j,t} = q_{Y,j,t} = 0$ for all j and $q_{BR,i,t} = 0$ for all i, the market clearing condition (A.1) implies that $x_{j,t} = 1$ in the symmetric stationary equilibrium, in which $x_{j,t}$ takes the same value for all j.

Overall, the solution to the Nash-bargaining problem (B.17) is:

$$Z^*_{Y,j,t} = \left[0,\ 0,\ \frac{d + 0.5E_t p_{ID,t+1}}{1+r},\ 1\right]. \tag{B.29}$$

Backward induction with Eq. (B.29) confirms the conjecture that $Z^*_{Y,j,t+1}$ is independent of $S_{j,t}$.

Also, note that $Z^*_{O,j,t}$, $Z^*_{RP,j,t}$ and $Z^*_{Y,j,t}$ are independent of one another in equilibrium. Thus, the result of the model is insensitive to the order of the matches.

B.4 Dealers' consumption of cash

Eq. (B.26) implies that dealers are indifferent to interdealer loans. Thus, $b_{j,t} = 0$ for all j to satisfy the market clearing condition (A.2). Substituting Eqs. (B.15), (B.16) and (B.29) and $a_{j,t} = b_{j,t} = 0$ into Eq. (B.2) yields:

$$c_{j,t} = (p_{ID,t} - p_{RP,j,t})x_{j,t-1} - (p_{ID,t} - p'_{Y,j,t})x_{j,t} = \frac{0.5(1-\beta)d}{0.5[1-\beta(1+r)]+r} > 0, \tag{B.30}$$

in the stationary equilibrium, where the last inequality follows from Condition (6). Given Eq. (B.7), $c_{j,t} > 0$ confirms Eq. (B.26) as conjectured.

C The Nash-bargaining problem between a dealer and a young investor in the equilibrium with a repo-market collapse in Section 6

To incorporate the possibility that an old investor with a repo does not return to the dealer for the repo, I introduce the following shock to $x_{j,t-1}$: substitute zero into $x_{j,t-1}$ for dealer j at the

beginning of period t if an investor who arranges a repo with the dealer in period $t-1$ does not return to the dealer in period t. Given this shock to $x_{j,t-1}$ in period t, Eqs. (B.6)-(B.9) remain correct. A dealer's value function at the end of the period, $V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t})$, is also the same as in Eq. (B.10). Lastly, I can guess and verify Eqs. (B.19)-(B.21), as shown in Appendix B.3.

Accordingly, the Nash-bargaining problem for a match between a dealer and a young investor is identical to the one defined by Eqs. (B.17) and (B.18). Then, the Nash-bargaining problem is reduced to the one defined by Eqs. (B.22) and (B.23), because the dealer and the young investor arranges a repo (i.e., $p_{Y,j,t} = q_{Y,j,t} = 0$) as described in Appendix B.3. The Nash-bargaining problem implies that the price of bonds with a repo, $p'_{Y,j,t}$, falls between the following range:

$$p'_{Y,j,t} \in \left[p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t},\ \frac{d + E_t p_{RP,j,t+1}}{1+r} \right]. \tag{C.1}$$

If Eq. (26) holds, then:

$$p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} < \frac{d + E_t p_{RP,j,t+1}}{1+r}. \tag{C.2}$$

Thus, there exists such a value of $p'_{Y,j,t}$ that both the young investor and the dealer are strictly better off than their outside options. Accordingly, the young investor puts all of their cash, e_I, on the repo that the investor arranges with the dealer. Hence:

$$x_{j,t} = \frac{e_I}{p'_{Y,j,t}}. \tag{C.3}$$

Given this value of $x_{j,t}$, the Nash-bargaining problem yields:

$$p'_{Y,j,t} = 0.5\left(p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t}\right) + 0.5\left(\frac{d + E_t p_{RP,j,t+1}}{1+r}\right). \tag{C.4}$$

Substituting Eqs. (B.6) and (B.19)-(B.21) and $\eta_{t+1} = 0$ into Eqs. (C.3) and (C.4) yields Eq. (27).

D An equilibrium with a prolonged repo-market collapse

Suppose that the economy was in the symmetric stationary equilibrium with repos in period $t-1$. Given Condition (6), I describe the sufficient condition for the existence of such an equilibrium that every investor enters the brokered market unexpectedly from period t to period T and that the economy returns to the stationary equilibrium with repos from period $T+1$ onward.

From period t to period T, each investor buys a bond from an old investor in the previous cohort when young, and resells the bond to a young investor in the next cohort when old. Thus, the initial condition of the economy in each period between t and T is as same as in the case described in Section 6. Accordingly, the condition for each investor to choose to enter the brokered market in period T is as same as Eqs. (24)-(28) described in Section 6.

For each period between t and $T-1$, its difference from period T is only in that every investor enters the brokered market in the next period. Thus, Eq. (3) holds for the bond price in the brokered market, $p_{BR,t}$, instead of Eq. (24). This change does not affect any result because $p_{O,j,t} = p_{RP,j,t}$, as described in Appendix B. Eq. (25) also remains the same. Note that each young investor would be able to commit to a repo if the investor entered the dealer market, given Eq. (25) in the next period. Accordingly, Eq. (27) is unchanged. Hence Eq. (28) must hold so that young investors enter the brokered market in the period.

Regarding Eq. (26), it is changed to:

$$p_{ID,s} = \frac{d + 0.5E_s p_{ID,s+1}}{1+r}, \tag{D.1}$$

for $s = t, t+1, ..., T$, which is implied by Eqs. (3) and (25). Overall, Eqs. (28) and (D.1) must be satisfied in each period between t and T.

Eq. (D.1) holds with Eq. (30) in the interdealer markets. The reason is the same as the one described in Section 6. Eq. (28) can be rewritten as:

$$e_I \leq f(s) \equiv \frac{(d + 0.5E_s P_{ID,s+1})(d + 0.75E_s P_{ID,s+1})}{0.5(1+r)E_s P_{ID,s+1}}. \tag{D.2}$$

Thus, the cash endowment for each young investor, e_I, must satisfies:

$$e_I \in \left(p_{ID,SS}, \min_{s=t,...,T} f(s) \right], \tag{D.3}$$

to ensure the existence of the stationary equilibrium with repos as well as Eq. (28) for each period between t and T.

If the T-bill rate, r, satisfies:

$$8r^2 + 8r - 1 < 0, \tag{D.4}$$

then I can show that:

$$f(s) < f(s+1) \tag{D.5}$$

for $s = t, t+1, ...$ Then, Eq. (D.1) implies that:

$$\inf_{T=t,t+1,...} \min_{s=t,t+1,...,T} f(s) = \frac{(d+0.5\hat{P})(d+0.75\hat{P})}{0.5(1+r)\hat{P}} \tag{D.6}$$

where $\hat{P}$ is the limit of $p_{ID,t}$ with $T \to \infty$:

$$\hat{P} \equiv \frac{d}{0.5+r}. \tag{D.7}$$

Substituting Eqs. (D.6) and (D.7) into Condition (D.3) yields that, if r and e_I satisfy Condition (D.4) and:

$$e_I \in \left(p_{ID,SS},\ \frac{(d+0.5\hat{P})(d+0.75\hat{P})}{0.5(1+r)\hat{P}} \right], \tag{D.8}$$

then T can be an arbitrary integer not less than t. The range for e_I in Condition (D.8) is non-empty if:

$$\beta(1+r) < \min\left\{1,\ \frac{(1+2r)(3+4r)}{5+4r}\right\}, \tag{D.9}$$

given that $\beta(1+r) < 1$ as assumed in Condition (6).

Table 1: The bond market structure in the basic model

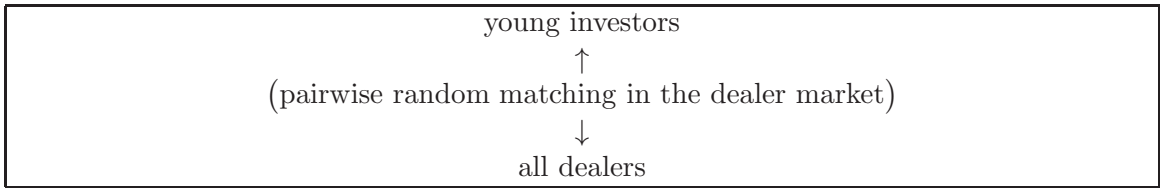
	Period t	$t+1$ ...
Cohort-$(t-1)$ investors:	sell bonds and consume cash ("old")	
	bonds $\downarrow\uparrow$ cash $\cdots$ bilateral (Nash) bargaining	
Cohort-t investors:	born with cash and buy bonds ("young")	sell bonds and consume cash ("old")
		bonds $\downarrow\uparrow$ cash
Cohort-$(t+1)$ investors:		born with cash and buy bonds ("young")

Table 2: The bond market structure in the extended model

	Period t		$t+1$ ...
Cohort-$(t-1)$ investors:	sell bonds and consume cash ("old")		
	(option 1) $\swarrow$	$\searrow$ (option 2)	
		bonds $\downarrow\uparrow$ cash	$\cdots$ bilateral
		dealers	
		bonds $\downarrow\uparrow$ cash	
bilateral $\cdots$	bonds $\downarrow\uparrow$ cash	interdealer market	$\cdots$ competitive
		bonds $\downarrow\uparrow$ cash	
		dealers	
		bonds $\downarrow\uparrow$ cash	$\cdots$ bilateral
	(option 1) $\nwarrow$	$\nearrow$ (option 2)	
Cohort-t investors:	born with cash and buy bonds ("young")		sell bonds and consume cash ("old")

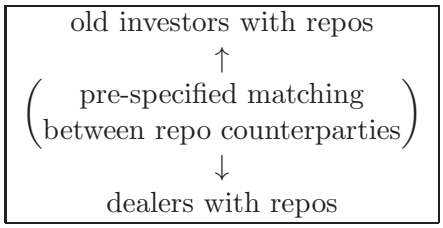
Table 3: Details on bilateral matches between dealers and investors in Table 2

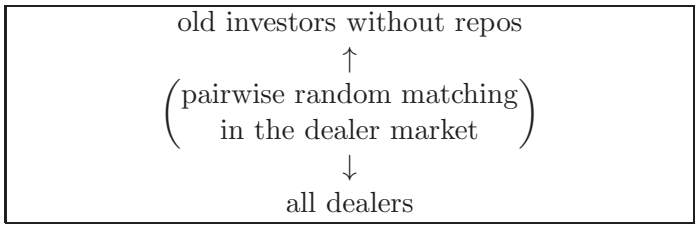
Young investors choosing option 2



Terms of trade in each match: the price and the quantity of bonds, and whether to arrange a repo.

Old investors choosing option 2





Term of trade in each match: the price and the quantity of the old investor's bonds.

Table 4: The notation of variables in period t

Variables	Definitions
	(Parameters)
β	the time discount factor for dealers
r	the interest rate on T-bills
d	dividends per bond every period
e_I	the cash endowment for each young investor
	(State variables for dealer j)
$a_{j,t}$	the amount of bonds the dealer owns at the end of period t
$b_{j,t}$	the net balance in interdealer loans at the end of period t
$x_{j,t}$	the amount of bonds sold to a young investor with a repo
	(Terms of trade between dealer j and an investor)
$(p_{Y,j,t},\ q_{Y,j,t})$	the price-quantity pair for bonds sold to a young investor without a repo
$(p'_{Y,j,t},\ x_{j,t})$	the price-quantity pair for bonds sold to a young investor with a repo
$(p_{O,j,t},\ q_{O,j,t})$	the price-quantity pair for bonds bought from an old investor without a repo
$(p_{RP,j,t}, q_{RP,j,t})$	the price-quantity pair for bonds repurchased from an old investor with a repo after a renegotiation
	(Other variables for dealer j)
$p_{ID,t}$	the competitive interdealer bond price
$q_{ID,j,t}$	the net balance of bonds traded in the interdealer market
rr_t	the competitive interdealer interest rate
$c_{j,t}$	consumption of cash
	(Variables for investor i)
$p_{BR,t}$	the bond price in the brokered market (endogenously identical for all i in t)
$q_{BR,i,t}$	the quantity of bonds sold in the brokered market when old
$s_{i,t}$	the amount of cash invested in T-bills when young
$a_{i,t}$	the amount of bonds held at the end of period t when young
$c_{i,t}$	consumption of cash when old
	(Pre-determined variables for dealer j in the interdealer markets)
$S_{j,t}$	$S_{j,t} \equiv [a_{j,t},\ b_{j,t},\ x_{j,t}]$ (the endogenous state variables)
$Z_{Y,j,t}$	$Z_{Y,j,t} \equiv [p_{Y,j,t},\ q_{Y,j,t},\ p'_{Y,j,t},\ x_{j,t}]$ (the terms of trade with a young investor)
$Z_{O,j,t}$	$Z_{O,j,t} \equiv [p_{O,j,t},\ q_{O,j,t}]$ (the terms of trade with an old investor without a repo)
$Z_{RP,j,t}$	$Z_{RP,j,t} \equiv [p_{RP,j,t},\ q_{RP,j,t}]$ (the terms of trade with an old investor with a repo)
	(Functions for dealer j)
$Z^*_{k,j,t+1}(S_{j,t})$ for $k = Y, O, RP$	the value of $Z_{k,j,t+1}$ given $S_{j,t}$