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by Francisco Rivadeneyra

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Abstract

The author describes the construction of the U.S.-dollar-denominated zero-coupon curve for the supranational asset class from 1995 to 2010. He uses yield data from a cross-section of bonds issued by AAA-rated supranational entities to fit the Svensson (1995) term-structure model. Results show the expected pattern of interest rates over the U.S. business cycle. The author computes the spreads relative to the U.S. Treasury zero-coupon yields data of Gürkaynak, Sack and Wright (2007). The average spread for this period is equal to 44 basis points; it increases during recessions and narrows during expansions. Also, the slope of the term structure of spreads shows a countercyclical pattern.

JEL classification: G12, G15
Bank classification: Financial markets; Asset pricing

Résumé

L’auteur explique la façon dont il a construit la courbe de rendement coupon zéro des obligations supranationales libellées en dollars É.-U. pour la période 1995-2010 – soit en estimant le modèle de structure par terme de Svensson (1995) au moyen de données sur les rendements d’un éventail d’obligations émanant d’entités supranationales notées AAA. Ses résultats sont conformes au profil attendu des taux d’intérêt au cours du cycle économique américain. L’auteur calcule les écarts de crédit par rapport aux rendements coupon zéro des titres du Trésor américain établis par Gürkaynak, Sack et Wright (2007). L’écart moyen est de 44 points de base sur la période étudiée; l’écart se creuse en période de récession et se rétrécit en période d’expansion. En outre, la pente de la structure par terme des écarts de crédit affiche un comportement contracyclique.

Classification JEL : G12, G15
Classification de la Banque : Marchés financiers; Évaluation des actifs

1 Introduction

This paper estimates the historical zero-coupon constant-maturity yield curve for the supranational asset class denominated in U.S. dollars between 1995 and 2010. Supranationals are entities whose capital is provided by one or more sovereigns and are explicitly or implicitly guaranteed by them. We focus on the multilateral development banks, which are financial institutions whose objective is to spur economic development and trade. This subset of supranationals is generally treated by investors (and covered by analysts) as a homogeneous asset class. To finance their lending activities, supranational entities borrow using a wide array of instruments, mostly bonds and notes. We use a large cross-section of U.S.-dollar-denominated bonds issued by a sample of AAA-rated supranational entities to fit the Svensson (1995) term-structure model for this period.

Accurate estimates of the term structure of interest rates of supranationals are important to the Bank of Canada. First, as manager of the international reserves portfolio, the Bank of Canada allocates some of its U.S.-dollar-denominated investments to this asset class. To our knowledge, there is no publicly available supranational zero-coupon curve. These estimates may also be useful to other researchers working on term-structure and credit-risk models. We make available the data as well as the code used in the estimation.

Specifically, we estimate the zero-coupon curve using the term-structure model of Svensson (1995), which is itself an extension of the model of Nelson and Siegel (1987). The model is a function of a small number of parameters that we estimate from January 1995 until September 2010 using a total of 1,107 bonds. Our model is estimated period-by-period without dynamic restrictions, as is customary in the estimation of zero-coupon yields from securities data. We restrict ourselves to the 19950–2010 period because in this subsample we have a sufficient amount of bonds distributed along the whole cross-section of maturities. Theoretically, the model can be used to compute the yield of a bond of any maturity, but we focus on obtaining robust estimates of zero-coupon yields of bonds with maturities between 1 and 10 years, because this is the asset allocation horizon of the international reserves portfolio. Therefore, in the estimation, we use the observed yields of bonds that have between 3 months and 20 years to maturity. We exclude bonds of shorter maturity, since, close to their maturity date, we observe large price movements that occur due to illiquidity. On the very long end of the curve, our selection excludes just a few bonds.

The results show that the U.S.-dollar supranational zero-coupon yields follow closely the dynamics of the U.S. Treasury yields. Supranational yields follow the secular downward trend of U.S. Treasuries during this period. We compute spreads relative to the U.S. Treasury zero-coupon yields fitted by Gürkaynak, Sack and Wright (2007). On average, supranational yields are 44 basis points above the U.S. Treasury yields. The term structure of spreads is, on average, upward sloping. The 10-year to 1-year slope is, on average, 15 basis points during this sample period. Spreads are, on average, countercyclical, rising during recessions and falling during expansions, with long-term spreads increasing more than short-term spreads.[1] Given that all

[1]Notice that our sample includes the 2001 and the 2008–09 U.S. recessions. This cyclical behaviour has also been documented

the supranational entities in our sample are AAA-rated, we argue that the dynamics of spreads is consistent with the flight-to-liquidity hypothesis of Longstaff (2004).

The exception to the countercyclical pattern of spreads occurred during the 2008–09 financial crisis. Our computed spreads are unresponsive to the drastic changes in the federal funds rate during the four weeks following the bankruptcy of Lehman Brothers. The results show that the 10-year spread narrowed after the collapse of Lehman in September while the rest, as expected by the flight-to-liquidity behaviour, widened. This result is robust to the sample of issuers, maturities and weighting methods used in the computation of our supranational curve. Analyzing the weekly movement of yields, we observe that after the lowering of the federal funds rate, short-term U.S. Treasury yields fell while long-term yields increased substantially. This substantial increase in the slope of the U.S. Treasury curve is not observed in our computed supranational yields. After the middle of September 2008, short-term supranational yields fall slightly and long-term yields increase modestly. We conclude that using dealer quotes (instead of transaction prices) can be the source of the slow response of the supranational term structure, particularly for this period, when these bonds became less actively traded.

This paper is related to the methodological literature on the term structure of interest rates. McCulloch (1975) proposed using cubic splines to fit the term structure using ordinary least squares. Nelson and Siegel (1987) used a parametric function, and Svensson (1995) extended their methodology. Later, Bliss (1997) compared empirically the methodologies available at the time.[2] More recently, Diebold and Li (2006) and Christensen, Diebold and Rudebusch (2011) addressed the issues with previous methodologies that neither link temporarily the term structure of interest rates nor rule out arbitrage opportunities. An important zero-coupon curve is the one derived from U.S. Treasury yields. Fama and Bliss (1987) estimated this curve from bond prices of what would become the CRSP bond file. More recently Gürkaynak, Sack and Wright (2007) re-estimated the U.S. Treasury yield curve for a wider set of maturities that encompass the CRSP data.

As with fitting methods used to compute the U.S. Treasury curve, we assume no default risk. Therefore, in our setting, movements of spreads have to be explained by liquidity factors, interest rate risk and other sources of time-varying risk premia. For example, Longstaff (2004) estimated the flight-to-liquidity premium in U.S. yields by comparing the bond prices of two sets of U.S. government debt with identical risk of default but different liquidity characteristics. Finally, and specifically related to supranational bonds, Kan (1998) estimated the yield spread between supranational bonds and the government bonds of France, Germany, Italy and the United Kingdom for 1996. In this paper we limit ourselves to estimating the curve and describing the dynamics of spreads. An explanation of the spread dynamics is left for future research.

The rest of this paper is organized as follows. Section 2 reviews the institutional details of supranational debt and section 3 describes the data. Section 4 describes the model used to compute the curve. Section 5

for corporate bonds (as in Duffee 1998 and Duffie and Singleton 2003) using indexes of U.S. corporate bond yields.

[2] See also Waggoner (1997).

discusses the estimation of the model and Section 6 shows the results. Section 7 concludes.

2 Institutional Details of Supranational Debt

This section describes the institutional details of supranational entities and their debt. We describe in some detail the capital structure of the four largest issuers in our sample: the European Investment Bank (EIB), the Inter-American Development Bank (IDB), the International Finance Corporation (IFC) and the International Bank for Reconstruction and Development (IBRD). We also mention the legal framework of their U.S.-dollar-denominated debt instruments, their lending activities and risk-management practices. Finally, we briefly describe the activities of the remaining seven supranational entities in our sample.

In general, a supranational is an entity formed by a group of two or more countries through international treaties. We focus on multilateral development banks whose objective is to promote trade or economic development by extending loans, offering guarantees, implementing development programs or providing technical assistance. To carry out these activities, supranationals generally finance themselves by issuing debt in their respective member countries, through global bonds or other debt programs. Global bonds are large syndicated placements, usually to institutional investors. Other programs are medium-term notes, plain vanilla or structured, and discount notes for retail investors. Given the nature of their lending activities, supranationals borrow in a wide variety of currencies, and therefore are active users of currency swaps for hedging purposes. In particular, the EIB, IDB, IFC and IBRD are all regular issuers in the major currencies and design their funding strategies to maintain benchmark placements that remain liquid in secondary markets. The debt instruments of the IBRD, EIB and IDB all qualify for a 0 per cent risk weighting under Basel II and are eligible collateral for the Federal Reserve repo transactions.

The debt issued by supranational entities is understood to be backed by the credit of the sovereigns supporting each entity. At the same time, their respective charters give supranationals extensive legal capacity. Moreover, the legal details of the credit backing of the sovereigns are not entirely clear, since they are not explicitly stated in the founding documents of the entities and have not been tested in practice. For example, the charter of the EIB states that "The EIB is separate from the EU institutions and it has its own governing bodies, sources of revenues and financial operations and is solely responsible for its indebtedness." The charters of the IDB, IFC and IBRD state that "The Bank [Corporation] shall possess full juridical personality." The law that applies to the entities depends on the member countries and the location of their borrowing and lending activities. Regarding specific debt issues, for example, the prospectuses of U.S.-dollar-denominated global bonds of the four entities identified above indicate that New York State law applies to these securities.[3]

[3] See http://idbdocs.iadb.org/wsdocs/getdocument.aspx?docnum=781584 and http://go.worldbank.org/WAUZA5KF90 for the charters, and http://idbdocs.iadb.org/wsdocs/getdocument.aspx?docnum=2254738 for an example of the detailed prospectus of U.S.-dollar global bonds from the IDB.

The first buffer against losses is the liquidity position and the equity of each entity. The risk-bearing capacity of these entities is determined by the amount of callable capital and retained profits relative to their loan portfolio. For the EIB, the amount of callable capital was equal to 95 per cent of the Bank's subscribed capital or close to 221 billion euros as of 2010. The IBRD callable capital was equal to US$182 billion or 93 per cent of subscribed capital. The IFC, on the other hand, relies more on retained earnings since all of its subscribed capital of US$2.4 billion has been paid in. Most of the callable capital is subscribed by highly rated sovereigns. For the EIB, Germany, France and the United Kingdom represent 49 per cent of the capital. For the IDB, 50 per cent of the subscribed capital belongs to non-borrowing members (the United States and Japan represent 35 per cent). Similarly, the IFC and the IBRD have the United States as their largest shareholder, with 24.03 and 16.83 per cent of the capital, respectively.

The risk exposures of these entities arise mainly from their lending activities. The IBRD lends only to sovereigns or sovereign-guaranteed projects and programs, while for the IDB this equals 95 per cent of its loan portfolio. In addition to sovereign-guaranteed lending, the EIB lends a large portion directly to projects without explicit guarantees. Finally, the IFC lends only to private sector companies, without any sovereign guarantees. To mitigate risk, strict concentration limits and close monitoring of the programs are key. The IDB has the highest concentration of lending in a few countries, with Brazil, Mexico and Argentina representing close to 34.6 per cent of its outstanding portfolio of loans in 2011.

These entities have preferred lender status, so they usually are first in seniority relative to other lenders. An indication of this status is the amount of writeoffs. The EIB, IBRD and IDB have never written off a sovereign-guaranteed loan and restructurings have been exceedingly rare. On the other hand, the IFC, which lends directly to private firms, has a relatively low ratio of writeoffs to total cumulative lending (6.4 per cent).

The rest of the supranational entities in our sample are the African and Asian development banks (AFDB and ADB, respectively), the Council of Europe Social Development Fund (COE), the European Bank for Reconstruction and Development (EBRD), the European Company for the Financing of Railroad Rolling Stock (EUROFIMA), and the Nordic Investment Bank (NIB). These supranationals strive to spur development in the member countries as well as provide policy advice and technical assistance to support development efforts. Some of the European entities have very specific objectives, as in the case of EUROFIMA, which supports the development of rail transportation in Europe. The EBRD provides loan and equity finance to new and existing businesses in member countries. The COE strives to strengthen European integration among EU member countries.

The only supranational in our sample that is not a development bank is the Bank for International Settlements (BIS). Its objective is to foster central bank co-operation and in particular to manage foreign exchange flows by accepting fixed-term deposits, as well as extend short-term collateralized credit and provide asset-management services to its members and clients. In the course of these activities, the BIS issues marketable short-term instruments (called FIXBIS) and medium-term instruments (MTIs).

The supranational debt market is well established, with all the major credit-rating agencies (Moody's, Standard and Poor's, Fitch, DBRS) regularly reporting ratings for most supranational entities. The most salient common feature of our sample of entities is their AAA credit rating with Standard and Poor's (S&P) and other agencies. As described above, the credit rating of supranational entities depends to a large extent on the credit rating of their supporting countries (given the amount of callable capital) but also on their risk-bearing capacity and risk-management practices. The next section describes the instruments used to compute the curve.

3 The Data

This section describes the data used to estimate the supranational zero curve. Regarding the sample of entities, we select the 11 largest issuers by volume in U.S. dollars. Table 1 summarizes the list of selected supranational entities. All the entities selected have the highest credit rating by S&P (AAA). In terms of geographical location, half of the issuers in our sample are based—and conduct most of their lending—in Europe (EIB, COE, EBRD, EUROFIMA, NIB). The IFC and IBRD belong to the World Bank Group and are global both in their membership and loan activities. Finally there are the African, Inter-American and Asian development banks (AFDB, IDB and ADB, respectively). In terms of borrowing, the IBRD is the largest, with an average issuance per year of close to US$80 billion, followed by the EIB with US$30 billion per year.

We use Bloomberg to retrieve the time series of the par yield of all bonds issued by any of the entities listed in Table 1. We obtain end-of-week quotes or transaction prices of all available U.S.-dollar-denominated fixed-rate straight coupon, zero-coupon or stripped bonds for each issuer.[4] We discard bonds with any type of embedded optionality. All bonds in our sample, except for the zero-coupon or stripped bonds, pay semi-annual coupons. The data are retrieved from their earliest point available in Bloomberg, which is 1987 until September 2010. For each bond, we retrieve its coupon, maturity date and volume (issuance amount at par value). We confirm that our bond prices calculated from the yield to maturity and the coupons are exactly equal to the prices quoted in Bloomberg. The final cross-section of bonds consists of 1,107 bonds of varying terms to maturity, from 1 month to 40 years, with the bulk being between 1 and 10 years. Table 1 shows the distribution of bonds across the issuers. Each bond has a different length in the time series between March 1987 and September 2010. It is important to notice that, prior to 1995, most of the bonds in our sample have maturities exceeding five years. Figure 1 shows this graphically by plotting the term-to-maturity of each bond and following the bond through its life until maturity.

We examine the properties of the outstanding volume of our sample of bonds. In estimating the zero-

[4] All supranational bonds are traded over the counter. However, Bloomberg's sources of prices are dealer quotes, or in some instances, transaction prices from the trading platform that executed the trade.

Table 1: List of issuers and number of bonds in our sample. All issuers had AAA credit-rating by Standard & Poor's at the end of 2011.

Issuer	No. of bonds	Zeros/Strip	Abbreviation
African Development Bank	33	0	AFDB
Asian Development Bank	67	1	ADB
Bank for International Settlements	110	0	BIS
Council of Europe Social Development Fund	39	1	COE
European Bank for Reconstruction and Development	29	2	EBRD
European Investment Bank	203	23	EIB
European Co. for Financing of Railroad Rolling Stock	14	0	EUROFIMA
Inter-American Development Bank	132	27	IDB
Int. Bank for Reconstruction and Development	388	94	IBRD
International Finance Corporation	56	0	IFC
Nordic Investment Bank	36	0	NIB

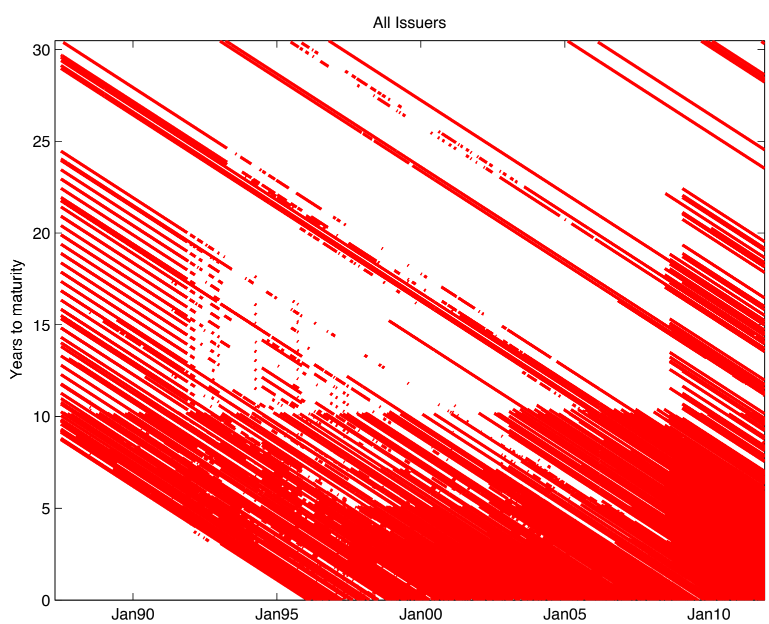

Figure 1: Term-to-maturity plot (TTM) of all the bonds in our sample. We compute the TTM of each bond at every date and plot it against the time series. Notice that before 1995, few bonds with maturities of less than five years are available. This plot also shows that, at inception, most bonds are 10-year bonds.

coupon curve, the number of bonds or their outstanding volume is not taken into consideration, since only their prices are used. Figure 2 shows the shares by issuer in our sample. The first panel shows the number of bonds by issuer as a share of the total number of bonds outstanding at every point in time. The second panel shows the share by issuer, but weighted by the volume of each bond as a share of the total volume of bonds using their par value at the time of issuance. These panels show that the number of bonds issued by each of the supranational entities is relatively stable, with the exception of the IBRD, which increased the number of issues but decreased their volume after the financial crisis of 2008. Also, we can see that the EIB and the IBRD are the most important issuers by volume and number of bonds, due to the frequency of issuance and their size.

Plotting the bond yield data shows many clear outliers and some others that require more careful examination. In general, during the last three months of the life of a given bond, yield movements tend to be large, since bonds close to their maturity date become illiquid. Anecdotal evidence suggests that reduced liquidity for those issues can come from the inelastic demand from particular investors looking to match particular positions in their books. Therefore, we discard all bonds with a term-to-maturity of less than three months. This is an arbitrary choice, but justified, since we are mostly interested in the 1- to 10-year zero-coupon rates. This procedure removes a large amount of outliers. Other outliers are removed on a case-by-case basis, commonly instances in which yields appear unchanged for more than three observations. Although it is theoretically possible to have a bond trade at the same price for three weeks in a row, it seems unlikely that information that prompted a trade on a bond would be perfectly offset so that the price does not change.

4 The Model

This section describes the Svensson (1995) term-structure model. We start with some definitions. Assume that a default-free zero-coupon bond pays one unit of account at maturity. Let the price of that bond at time t maturing at time τ be given by $P(t,\tau)$. By no arbitrage, the price of this security at maturity is equal to its payoff, $P(\tau,\tau) = 1$. A forward contract is an agreement today, t, for a loan between any two dates in the future, say from s to τ. Evidently, $t < s < \tau$. This contract can be thought of as a long position of a $(\tau - t)$-period bond, $P(t,\tau)$, today, together with a short position of a $(s-t)$-period bond, $P(t,s)$. In terms of continuously compounded rates, the forward rate from s to τ at time t, $f(t;s,\tau)$, solves

$$e^{-f(t;s,\tau)(\tau-s)} = \frac{P(t,\tau)}{P(t,s)}, \tag{1}$$

and taking logs

$$f(t;s,\tau) = -\frac{\log P(t,\tau) - \log P(t,s)}{\tau - s}. \tag{2}$$

The last equation is useful because we can derive two important rates from it. First, the time t instantaneous forward rate of maturity τ, denoted by $f(t,\tau)$, is the forward rate when the investment horizon varies

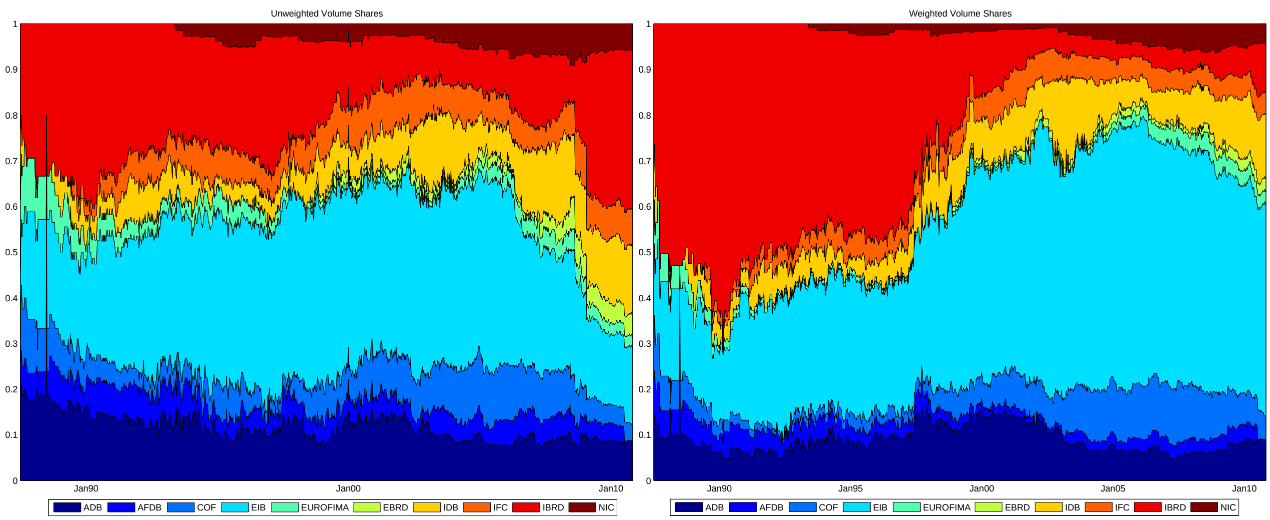

Figure 2: Volume of bonds by issuer. The first panel shows the number of bonds by issuer as a share of the total amount of bonds outstanding at every point in time. The second panel shows the share by issuer, but weighted by the volume of each bond. These panels show that the number of bonds issued by each of the supranational entities is relatively stable, with the exception of the IBRD, which increased the number of issues but decreased their volume after the financial crisis of 2008. Plots are computed only with those bonds that are quoted at a particular date regardless of them being outstanding. BIS bonds are not included in these plots, because no volume data are available.

infinitesimally around maturity date τ:

$$f(t,\tau) = -\frac{\partial \log P(t,\tau)}{\partial \tau}. \tag{3}$$

Second, instead of varying the length of the investment horizon, one can vary the distance between contracting and the start of the investment horizon. The yield to maturity, denoted by $y(t,\tau)$, is the forward rate when the time of contracting coincides with the start of the interval over which the contract is effective; i.e., $t = s$ in (2). Solving gives us

$$y(t,\tau) = -\frac{\log P(t,\tau)}{\tau - t}, \tag{4}$$

using the fact that $P(t,t) = 1$.

A yield or forward curve, or its term structure, is a graph of bond yields or forward rates plotted against their maturities. Using the relationship between equations (2) and (4), one can calculate the yield curve given a range of forward rates, or, alternatively, given the yield curve, one can calculate the forward curve.

Note that all derivations above are for zero-coupon bonds; i.e., those whose only payoff occurs at the terminal date. However, most of the bonds in our data pay regular coupons and therefore we need to do some transformations. In terms of coupon bonds, the yield to maturity at time t of a bond maturing at date τ, given by $y(t,\tau)$, is the single rate that makes the discounted value of future bond payments equal to today's price. Using the yields, one can discount the cash flow of any coupon-paying bond denoted by $P_c(t,\tau)$ using the no-arbitrage condition:

$$P_c(t,\tau) = \sum_{n=1}^{\tau-t} c \exp\left(-y(t,n)(n-t)\right) + \exp\left(-y(t,\tau)(\tau-t)\right), \tag{5}$$

where the par or redemption payment is again assumed to be one unit of account, c is the fixed coupon rate and $\tau - t$ is the number of discrete interest periods.

To estimate forward rates from coupon bond yield data, we are required to impose some minimal structure on the temporal structure of rates. Spline methods are a popular way to do so. For the historical supranational U.S.-dollar yield curve, we choose the Svensson (1995) model, which is an extension of the Nelson and Siegel (1987) (NS) model. NS propose to fit the term structure of interest rates using a flexible, smooth parametric function. Fitting the term structure implies minimizing some criterion of pricing errors between the observed bond yields and the theoretical zero-coupon rates. Therefore, flexibility in the parametric function can come at the cost of difficulty in identifying the parameters. Although the NS model is capable of capturing many of the observed shapes of the yield curve, it cannot capture convexity in the long end of the curve. A popular extension that addresses this caveat is the six-factor Svensson model. Another reason to choose the Svensson model is that it enables us to compare our results with the U.S. Treasury curve. Gürkaynak, Sack and Wright (2007) use the NS and Svensson model to estimate the U.S. Treasury yield and forward curves. In fact, a large proportion of central banks use either the Svensson or NS model. The Bank for International Settlements

(2005) reports that, currently, nine out of thirteen central banks which report their curve estimation methods use the NS or the Svensson model to construct zero-coupon yield curves.

Specifically, the NS model is a smooth function of four parameters. This model assumes that instantaneous forward rates n years ahead are characterized by the function

$$f(t,n) = \beta_0 + \beta_1 \exp\left(-\frac{n}{\tau_1}\right) + \beta_2 \left(\frac{n}{\tau_1}\right) \exp\left(-\frac{n}{\tau_1}\right),$$

where n is the term to maturity, β_0 is the long-run level of interest rates, β_1 is the short-term component, β_2 is the medium-term component and τ_1 is the decay factor. Notice the different notation compared with equation (3), where the second argument of the function is the date and not the horizon of the forward rate. This is done to preserve the notation of NS. It is easy to see that instantaneous forward rates equal $\beta_0 + \beta_1$ at maturity zero and asymptote toward β_0 for large n. The shape of the curve does not need to be monotonic. In fact, between the short and the long end, forward rates can have a hump determined by β_2, while its location is determined by τ_1. In effect, τ_1 dictates where β_2 achieves its maximum. For small values of τ_1 the rate of decay of the curve is slow, while large values of τ_1 produce faster decay. (See Figure 1a of Svensson 1995 for a decomposition of the components of the forward curve.) The estimation of the model involves obtaining measures of the parameters $\beta_0, \beta_1, \beta_2$ and τ_1 from securities data.

The convexity bias tends to pull down the yields on longer-term securities, giving the yield curve a concave shape at longer maturities. Consequently, the extension of Svensson (1995) allows long-dated convexity correction by adding two additional parameters. Instantaneous forward rates in the Svensson model are

$$f(t,n) = \beta_0 + \beta_1 \exp\left(-\frac{n}{\tau_1}\right) + \beta_2 \left(\frac{n}{\tau_1}\right) \exp\left(-\frac{n}{\tau_1}\right) + \beta_3 \left(\frac{n}{\tau_2}\right) \exp\left(-\frac{n}{\tau_2}\right),$$

where $\beta_{3,}\tau_2$ are the additional parameters to be estimated. The two new terms allow for a second hump in the forward rate curve. Note that when β_3 is equal to zero, this equation reduces back to the NS equation.

In keeping with convention, we will express the curve in terms of yields. The spot rates (equivalent to yields for zero-coupon bonds) can be obtained using equation (3):

$$y(t,n) = \beta_0 + \beta_1 \left(\frac{1-\exp(-\frac{n}{\tau_1})}{\frac{n}{\tau_1}}\right) + \beta_2 \left(\left(\frac{1-\exp(-\frac{n}{\tau_1})}{\frac{n}{\tau_1}}\right) - \exp\left(-\frac{n}{\tau_1}\right)\right) + \beta_3 \left(\frac{1-\exp(-\frac{n}{\tau_2})}{\frac{n}{\tau_2}} - \exp\left(-\frac{n}{\tau_2}\right)\right), \tag{6}$$

where n is the term to maturity and t is the trade date. Note that we can have a different set of parameters $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ for every trade date. Thus, for a given set of parameters, the Svensson (1995) specification characterizes the yield curve at all maturities, accounting for convexity in the curve at both long and short maturities.

The Svensson (1995) model does not link intertemporally the parameters of the term structure. Empirically, this provides flexibility for the estimation at the cost of parameter estimates that vary too much. Theoretically, without some time structure in the parameters, the model may allow arbitrage opportunities. Diebold and Li

(2006) and Christensen, Diebold and Rudebusch (2011) address these issues, but the cross-equation restrictions come with assumptions about risk-premium dynamics. Despite these problems, the Svensson (1995) model is very popular precisely for its flexibility in fitting securities data.

5 Estimation

In the estimation of the model, all issuers and corresponding bonds are treated as equivalent. That is, we do not weight one supranational issuer over another, although our sample is dominated by a couple of issuers. This may be an issue, compared to other yield curves estimated for a single issuer. We address this issue later in the results. We convert the yield-to-maturity data from Bloomberg to their clean bond prices. We use the MATLAB fixed-income toolbox to estimate equation (6) using the IRFUNCTIONCURVE command.[5] What the toolbox actually does is run a non-linear least-squares optimization problem to solve for the six Svensson (1995) parameters. We solve for the parameters at each date in our sample. Obviously, we have to restrict the estimation to dates at which we have more than six observed bonds.

One important caveat is that the availability of short-dated bonds before 1995 hinders considerably the estimation of the model for that period. Not surprisingly, the parameters are very unstable when no bonds of less than five years to maturity are available. Given this problem, we report our estimates only after 1995.

We have to be careful with the optimization problem, because the objective function is not strictly convex and thus may have multiple local optima. The optimization problem is a minimization of the sum of squared errors for each date t. The problem is

$$\min_{\theta_t} \sum_{j=1}^{J} \left(y(t, n_j; \theta_t) - y^M_{t,j} \right)^2 , \tag{7}$$

where θ_t are the date t parameters $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ of equation (6), $y(t, n_j; \theta_t)$ represents the date t predicted zero-coupon rates of bond of maturity n_j, and j is the index of available securities at that date. Finally, $y^M_{t,j}$ represents the date t observed market rates and J stands for the number of securities available on date t.

As in any other non-linear least-squares problem, the weighting of the pricing errors is important. Three versions are commonly used in the literature: the duration, yield and price weights. Duration are the weights that result from computing the inverse duration of the bond. The price weights are the ones resulting from converting the bond yield into prices.[6] We discard the price weighting because it creates larger pricing errors in the short end of the curve. We report only the results from the duration weights but, in practical terms, the duration and yield weights give almost identical results.

Gilli, Große and Schumann (2010) show that parameter identification is only possible when some of the parameters are restricted to certain ranges. Following their work, we set the parameter constraints as follows:

[5] See Ferstl and Hayden (2010) for an implementation in R.

[6] See De Pooter (2007) and Ferstl and Hayden (2010) for more details on the weighting and their implications.

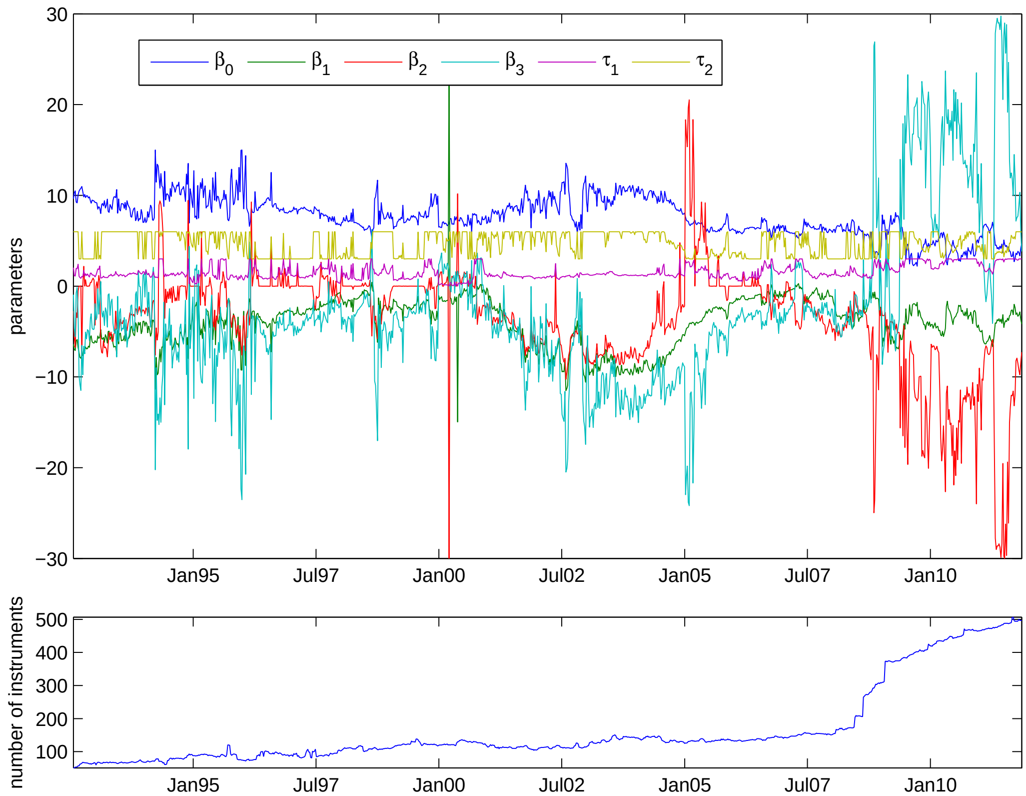

Figure 3: Parameter values over time and number of bonds. The top plot shows the parameter values estimated at every date in the sample. The lower plot shows the number of bonds used at every date, J in equation (7). During most of the sample, the number of bonds used is between 70 and 150. After mid-2007, the number of bonds available increases sharply to close to 500 bonds.

$0 < \beta_0 < 15$, $-15 < \beta_1 < 30$, $-30 < \beta_2 < 30$, $-30 < \beta_{3<}30$, $0 < \tau_1 < 30$, $0 < \tau_2 < 30$.[7] Restricting the parameter space in the optimization problem significantly improves the stability of our results. Figure 3 shows the time series of the estimated parameters on the left y-axis and the number of instruments in black on the right y-axis. Looking at the parameters, we can see how they vary significantly over time. This is not an uncommon feature of this model and can be seen, for example, in the parameter estimates of the U.S. Treasury curve published online by Gürkaynak, Sack and Wright (2007).

Our estimation procedure does not weigh by the number of bonds or outstanding amount of a particular issuer. Therefore, we check that illiquid issuers, which tend to be the smaller supranationals, do not have a disproportionate effect on the results. This could be problematic, particularly if issuers have different characteristics regarding the market liquidity of their bonds. To do so, we compute the curve for the two largest issuers, IBRD and EIB. We group IFC and IBRD together and call this group the World Bank. We

[7]See Bliss (1997) and De Pooter (2007) for other suggestions for identification restrictions.

Table 2: Statistics of pricing errors. Root mean square error (RMSE) and mean absolute deviation (MAD) are calculated weekly for each section of the yield curve and then averaged over the entire sample period. This exercise is done for the complete set of issuers and separately for the World Bank (IFC and IBRD) and EIB, relative to their own estimated zero-coupon curve. The last column reports the average number of bonds used to calculate the statistics at every point in the sample.

Maturity (years)	RMSE	MAD	Avg. number of bonds
All issuers			
0-2	15.75	11.35	43.54
2-5	16.18	12.07	70.10
5-7	18.39	14.27	26.23
7-10	19.98	16.07	27.62
World Bank			
0-2	16.16	12.40	13.96
2-5	19.46	14.86	19.48
5-7	18.07	14.96	13.41
7-10	18.84	15.62	12.41
EIB			
0-2	8.26	6.64	11.53
2-5	9.84	7.64	15.82
5-7	13.76	12.01	7.62
7-10	16.49	14.89	6.98

compute the zero-coupon curve for these two groups and report pricing errors for each in Table 2. Pricing errors of the World Bank zero-coupon curve measured by the root mean square error and the mean absolute deviation do not differ significantly from the curve estimated with the whole sample of issuers. The pricing errors of the EIB curve are smaller for the short end of the curve, while the long end are similar in magnitude. As expected, our fitting errors relative to Gürkaynak, Sack and Wright (2007) are significantly larger.[8]

6 Results

We organize our results into two parts. The first describes the results focusing on the level of supranational yields, while the second describes the unconditional moments and dynamics of the spreads of supranational

[8]Another robustness check is to use different weights in the optimization. Using the duration weights in the optimization procedure reduces this problem, because illiquid bonds tend to have larger price movements than liquid ones, particularly closer to their maturity date. These results are not reported in the paper but are available upon request.

Table 3: Autocorrelation structure of supranational zero-coupon yields

	1 month	6 months	1 year
UST 1-year	0.9987	0.9341	0.8568
UST 5-year	0.9915	0.9498	0.8971
UST 10-year	0.9917	0.9520	0.9020
Supra 1-year	0.9876	0.9038	0.7224
Supra 5-year	0.9698	0.8626	0.7386
Supra 10-year	0.9614	0.8257	0.7109

yields. We do not focus on discussing the results of the forward rates, although they are available online. We report online the time-varying parameter estimates, $\hat{\theta}_t$, which allow the computation of yields at any maturity, as well as forward rates. At each point in time, the curve is computed with yields of bonds that have between 3 months and 20 years to maturity. The exact number of bonds for every period is reported online.

6.1 Level of yields

The left panel of Figure 4 shows the time-series plot of the supranational zero-coupon constant-maturity curve denominated in U.S. dollars from 1995 to September 2010 for the 1-, 5-, 7- and 10-year maturities. The right panel shows the cross-sectional evolution of yields. As expected, we see a very similar cyclical pattern, as in the U.S. Treasury yield curve, which follows the U.S. business cycle. Supranational yields also follow the secular downward trend of U.S. Treasuries during this period. Later, we analyze the behaviour of spreads more closely. Figure 5 shows the time series of the yields of supranationals and U.S. Treasuries for the 1-, 5-, 7- and 10-year maturities.

The basic stylized facts of the supranational curve are interesting. First, notice the average term structure. Supranational entities have a yield that is, on average, 44 basis points above the U.S. Treasuries. Also note that the differences across maturities are increasing with tenor, which suggests that most of the term structure of spreads can be explained by level and slope factors. Second, and surprisingly, the volatility of yields measured as the standard deviation of weekly yield changes shows a mixed picture. In the short end of the curve, supranationals are as volatile as U.S. Treasuries, while in the rest of the curve they seem much less volatile (Figure 6). Third, comparing the autocorrelation structure shows that U.S. Treasuries and supranationals share a similar time dependence. At every lag and maturity, supranationals show a faster decay than U.S. Treasuries, and this behaviour is more pronounced at longer horizons (Table 3).

As suggested by Litterman and Scheinkman (1991), decomposing the cross-sectional variation of yields into their principal components has been very useful to understand the term structure of yields. Table 4 shows the cumulative percentage of variation explained by the first k principal components of yields and yield

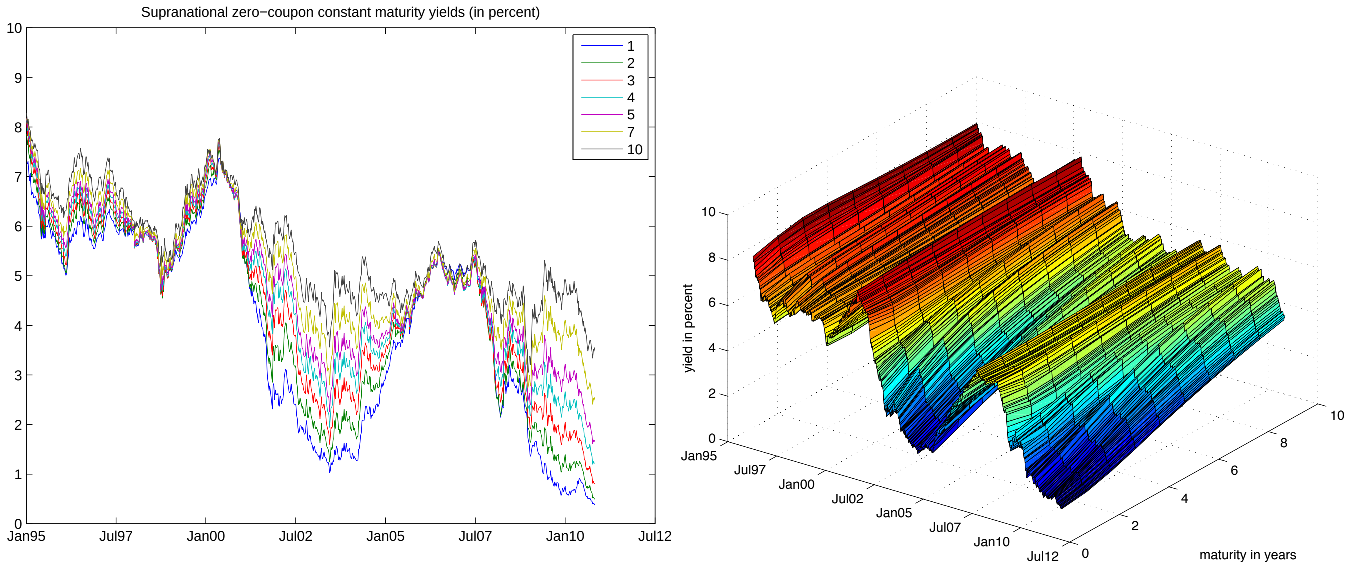

Figure 4: Time series (left) and evolution of the term structure (right) of the supranational U.S.-dollar zero-coupon constant-maturity curve for selected maturities between 1995 and 2010. The expected pattern of yields follows the U.S. business cycle.

changes for each curve. As is well known, the first three principal components explain close to 99.9 per cent of the variation of yields and yield changes in U.S. Treasuries (see for example Ang and Piazzesi 2003 for a calculation in monthly frequency and Litterman and Scheinkman 1991 for weekly frequency). More interesting is to find that these results carry over to the supranational curve. In the calculation, we use ten yields from the 1- to the 10-year maturity from 1995 to 2010. Figure 7 shows the loadings of the principal components for each maturity for the supranational yields. As expected, they show similar patterns to the U.S. Treasury loadings (see Piazzesi 2010).

Table 4: Percentage variation of yields and yield changes explained by the first k principal components (weekly sample from 1995–2010 using 1- to 10-year maturities)

	U.S. Treasuries		Supranationals	
k	Y_t	ΔY_t	Y_t	ΔY_t
1	97.3	91.2	97.1	91.1
2	99.8	98.6	99.8	98.4
3	99.9	99.8	99.9	99.6
4	100	100	100	100

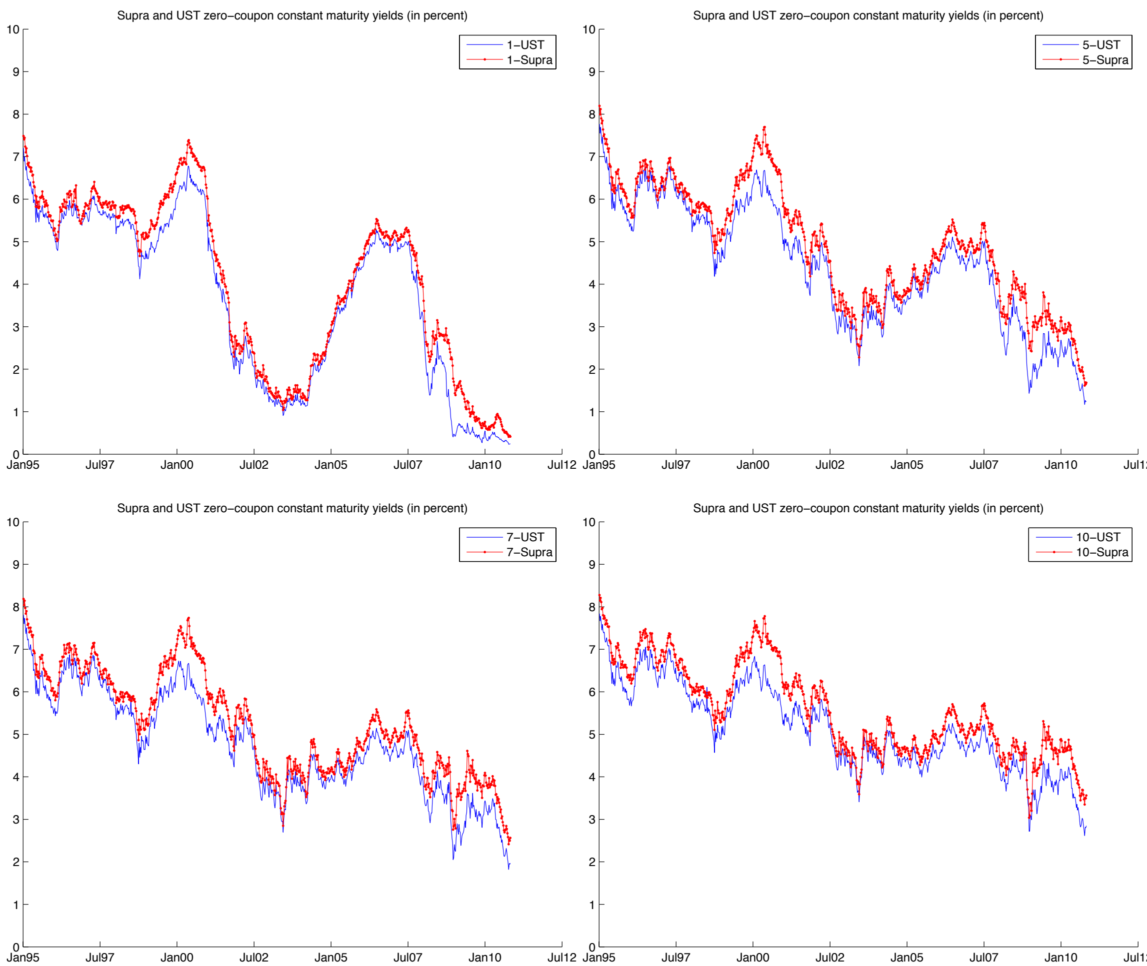

Figure 5: Supra and U.S. Treasury zero-coupon constant-maturity curve for the 1-, 5-, 7- and 10-year bonds. Note that the difference between the 10-year supranational and the 10-year U.S. Treasury bond narrows during September 2009.

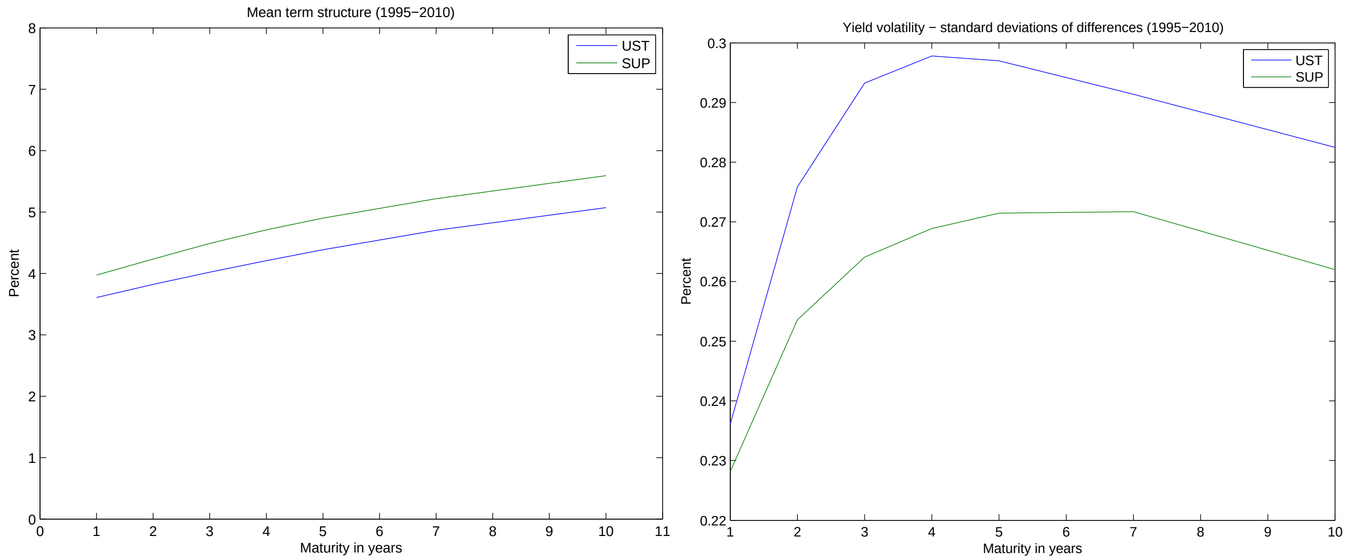

Figure 6: Average yield curve (left) and standard deviation of yields (right) computed from 1995 to 2010

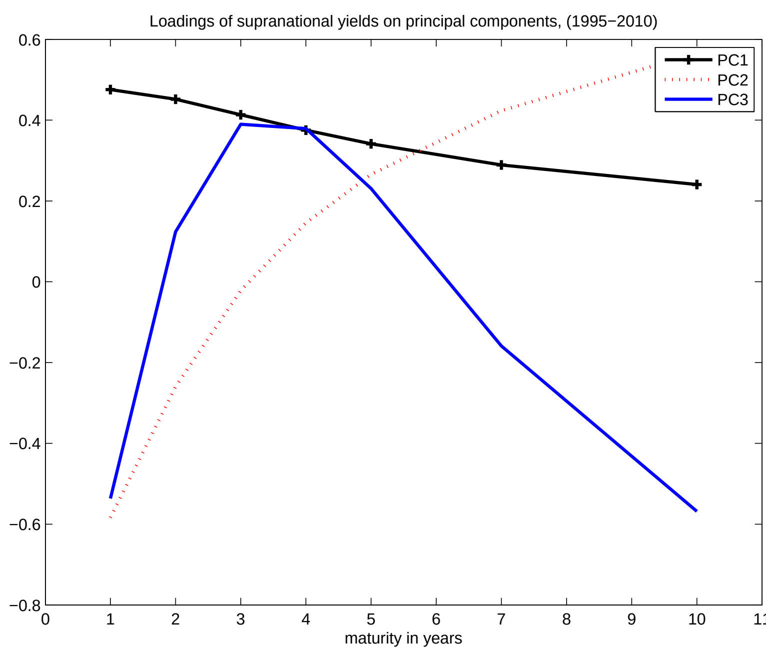

Figure 7: Principal-component loadings for the supranational yields computed from 1995 to 2010. The traditional pattern of level, slope and curvature emerges. Similar loadings and relative magnitudes are observed in the principal-component decomposition of U.S. Treasury yields.

6.2 Spreads

The left panel of Figure 8 shows the time series of the supranational spreads for the 2-, 5-, 7- and 10-year maturities. The right panel shows the cross-sectional evolution of spreads. With this plot we can see the sharp increase in short-term spreads during the financial crisis, which was much larger than the previous peak in spreads during the 2001 recession. The individual plots in Figure 9 show the zero-coupon constant-maturity spreads for the 1-, 5-, 7- and 10-year bonds relative to the U.S. Treasury yields. The 1- and 5-year spreads increased significantly during the 2008–09 financial crisis, which we interpret as flight-to-quality as well as flight-to-liquidity.

On average, supranational yields are 44 basis points above the U.S. Treasury yields. The term structure of spreads is, on average, upward sloping. The 10-year to 1-year slope is, on average, 15 basis points during this sample period. Spreads are, on average, countercyclical, rising during recessions and falling during expansions, with long-term spreads increasing more than short-term spreads. Notice that our sample includes only two U.S. recessions. Given that all the supranational entities in our sample are AAA-rated, we argue that the dynamics of spreads is consistent with the flight-to-liquidity hypothesis (Longstaff 2004).

The exception to the countercyclical pattern of spreads occurred during the 2008–09 financial crisis. Our computed spreads are unresponsive to the drastic changes in the federal funds rate during the four weeks following the bankruptcy of Lehman Brothers. Spreads of 6-year and longer maturities moved in the opposite direction, falling during 4 weeks after 15 September. This result is robust to the selection of different issuers. To test this result, we recompute the curves using two subsets of issuers, the first including only the European issuers and another including only the World Bank (IFC, IBRD). Both subsets have a large set of bonds along the sample period. The same pattern of spreads appears in both subsets in which the 10-year spread narrows during the Lehman episode (results available upon request). One final examination regarding this reversal of the long-term spread is to plot the yield curve around the dates of the Lehman bankruptcy. Figure 10 shows the term structure of the U.S. Treasury yields and our computation of the supranational curve for selected dates around this event. This shows that the main cause of the reversal of long-term spreads, in our estimation, is the slow steepening of the supranational yield curve, which, in contrast, occurred immediately to the U.S. Treasury curve after 15 September 2008. A look at the individual bond prices confirms this fact, showing that supranational bonds did not respond immediately to the lowering of the federal funds rate. We speculate that these bonds were less actively traded relative to their normal trading levels during the weeks following the Lehman bankruptcy. Therefore, our data from dealer quotes can be misleading if they do not reflect actual transaction prices. We plan to explore this issue in future research.

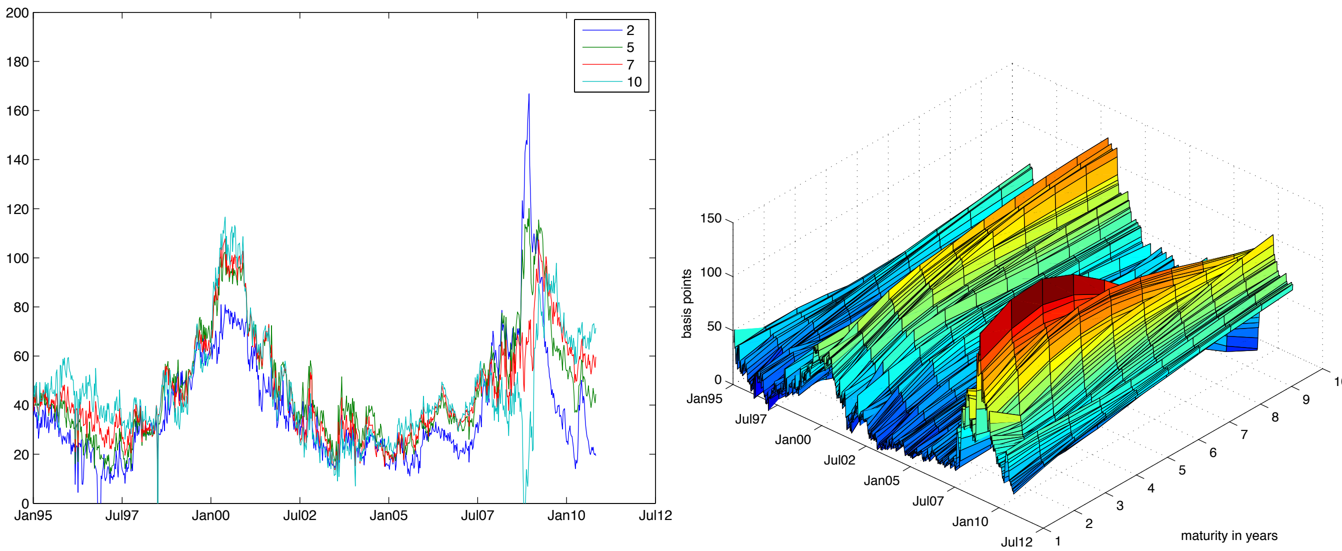

Figure 8: Time series (left) and evolution of the term structure (right) of the supranational spreads computed as the difference between the supranational yields and the U.S. Treasury yields. Spreads are, on average, countercyclical; however, during the financial crisis of 2008–09, the long-term spreads increased due to a slow response of our estimated supranational yields in comparison with the U.S. Treasury yields.

7 Concluding Remarks

This paper shows the construction of a supranational zero-coupon constant-maturity curve. Using the parameterization of Svensson (1995), we construct the curve using a cross-section of 1,107 bonds with selected maturities between 3 months and 20 years from 1995 to 2010. We compare our results with the U.S. Treasury zero-coupon curve constructed by Gürkaynak, Sack and Wright (2007). During this period, supranational yields follow the downward trend of the U.S. Treasury yields. On average, supranational yields are 44 basis points above U.S. Treasury yields and their mean term structure is upward sloping. On average, spreads increase during recessions or crises and fall during expansions. This is consistent with the countercyclical movements of credit spreads found in the credit-spreads literature. The only exception to this pattern is the reversal of the 10-year spread during September 2008 at the height of the financial crisis. All other spreads widen, as predicted by the traditional flight-to-liquidity hypothesis during periods of market stress. The slope of the U.S. Treasury curve increased rapidly in this period, while the slope of the supranational curve was less responsive. We speculate that illiquidity factors could be the source of such a slow response.

Future research involves exploring in more detail the causes of the reversal of spreads during the financial crisis. Also we will explore the determinants of spreads to measure liquidity, credit or other sources of risk premia that may have forecasting power for excess bond returns. Finally, we will explore the existence of global and local factors in bond risk premia using the geographical subsets of supranational entities.

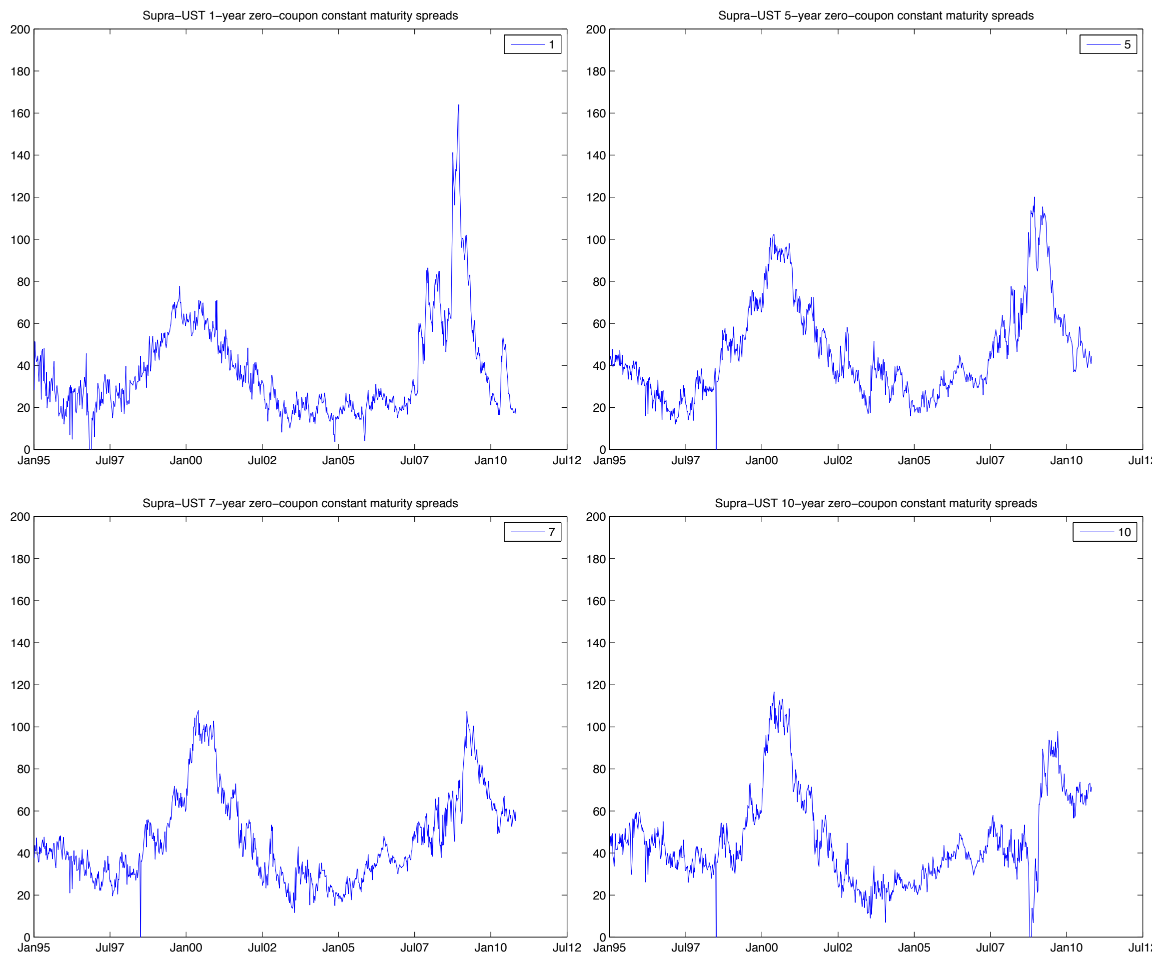

Figure 9: Spreads of the 1-, 5-, 7- and 10-year zero-coupon constant-maturity yields relative to the corresponding U.S. Treasury yield. Notice that the 10-year spread narrows during September 2008. In the rest of the sample, spreads behave as expected, widening during recessions and becoming narrower during expansions.

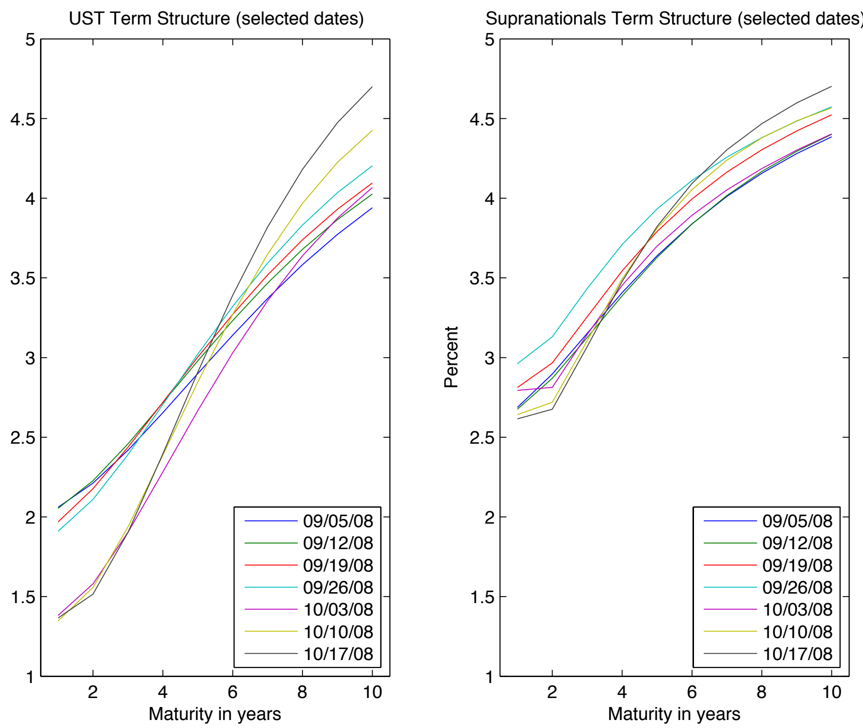

Figure 10: U.S. Treasury and supranational yield curves for selected dates around the Lehman collapse of 15 September 2008. This plot shows how the steepening of the Treasury curve occurs immediately, while the supranational does not. The response of the Treasury curve follows the aggressive reduction in the federal funds rate.

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A Description of the Supranational Dataset

This appendix describes the dataset with the results publicly available on the web. The MATLAB file `TermStructureSupranationals.mat` includes all relevant variables. `ZeroYields` is the 10x824 matrix of the supranational zero-coupon constant-maturity yields for the 1- to 10-year maturities from 20 January 1995 to 29 October 2010 computed using the inverse-duration weights and restricted parameter space. `Parameters` is the 6x824 matrix of the estimated parameters of the Svensson model for each date. The parameters are in the following order of columns: $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$. The vector `numberObs` is the total number of supranational bonds used at each date to estimate the parameters. The matrix `Forwards` is the term structure of supranational forwards computed using the estimated parameters. `mats` is the 1x10 row vector of maturities. `Dates` is the vector of dates in MATLAB serial format. Most dates are Friday except when it is a U.S. holiday, in which case the date for that week is Thursday.