An Approach to Stress Testing the Canadian Mortgage Portfolio

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In Canada, residential mortgage loans account for close to 47 per cent of the total loan portfolio of commercial banks. Despite this large exposure to the housing and mortgage markets, most of the risk of default rests with the mortgage insurers rather than with the banks.[1] Currently, the default rate on mortgage loans is near its historical low. Nevertheless, it is important to improve our ability to assess the risks to the financial system associated with the housing and mortgage markets.

For example, it would be helpful to have tools to assess how a slowdown in the Canadian housing market of a magnitude similar to that observed in Canada during the early 1990s, or to that currently under way in the United States,[2] would increase the overall risk of default on mortgage loans, particularly for highly leveraged loans.

The objective of this work is to present an option-pricing approach to assessing the vulnerabilities of the Canadian mortgage market. This approach is confined to analyzing only financially motivated defaults. It is based on the microeconomic principle that default can be a rational response to adverse changes in the housing market. It does not take into account involuntary defaults caused by income constraints, such as those caused by job loss.

To illustrate how this approach could be applied, we evaluate the overall risk of default on Canadian mortgages under a scenario in which house prices are falling. This is done using the empirical distribution of loan-to-value (LTV) ratios in 2006, obtained from the Canadian Financial Monitor (CFM), a survey conducted by Ipsos Reid Canada.[3]

The Model

There is a growing body of literature on mortgage default risk and how it relates to house prices and interest rates. One strand of this literature, which is motivated by option theory, maintains that, in a perfectly competitive market, mortgage borrowers can increase their wealth by defaulting when the market value of the mortgage equals or exceeds the market value of the collateral, which depends on the price of the house.

Indeed, under conditions of limited liability, negligible transactions costs, and no exogenous reasons for residential mobility, default can be seen as a financial decision that can be separated from the real (housing) decision, and the Merton theory of the pricing of corporate debt can be applied.[4]

Here, we analyze the homeowner's decision to default based on this criterion. We use a standard two-factor theoretical contingent-claims pricing model. This model, which was initially

1. In Canada, mortgages with a down payment of less than 20 per cent must be insured. In 2006Q4, insured mortgages represented more than 45 per cent of total residential mortgage balances outstanding at chartered banks. Uninsured mortgages are associated with a low risk of default, because they are backed by a relatively large amount of collateral.
2. In the United States, the largest recent decrease in house prices was the cumulative decline in the nominal median selling price of existing houses of 8.1 per cent between June 2005 and October 2006 (NSA-National Association of Realtors). Note that this measure of house prices has increased since October 2006.
3. For details on CFM data, see Faruqui, Lai, and Traclet (2006).
4. For a more detailed discussion, see Deng, Quigley, and Van Order (2000) and Kau et al. (1995).

developed to evaluate mortgage contracts,[5] generates all the information needed to compute the probability of default on any fixed-rate mortgage contract.[6]

The first factor in the model is the price of housing, which is assumed to follow the standard geometric Brownian motion, the equivalent in continuous time of a random walk with a drift. The return from owning a house consists of price appreciation and the flow of services that the owner incurs by living in the house.

The second factor is the short-term interest rate. We assume that it follows a mean-reverting process. This process assumes that the interest rate reverts to its long-term value at a certain speed, but that this pattern is constantly disturbed by stochastic events.

In our model, for every possible outcome for house prices and interest rates over the length of the contract, the borrower faces three options: making the required payment, defaulting, or prepaying the mortgage.

The opportunity to default is treated as a put option, since it enables the borrower to sell the property to the mortgagee at a price equal to the loan's outstanding balance. This opportunity has value if the expected present value of the remaining payments becomes higher than the market price of the house.

The mortgagor also has the opportunity to prepay the mortgage loan.[7] Prepayment can be viewed as refinancing. We treat the opportunity to prepay as a call option, in that it allows the borrower to buy all future obligations remaining under the mortgage at a price equal to the loan's outstanding balance. Prepayment has value if interest rates fall below the fixed rate of the mortgage to the extent that the expected present value of the remaining payments becomes higher than the unpaid balance of the mortgage.

Note that closed mortgages generally cannot be paid off before maturity without a penalty. Prepayment penalties in Canada are frequently calculated as the greater of three months' interest or the interest differential applied to the outstanding balance. For simplicity, we use the former.

These options are "embedded" in the sense that they give the mortgagor not only the opportunity to default or prepay now, but also the opportunity to postpone the default or the prepayment by at least one period to see if it will provide additional value.

Hence, at every period, the borrower solves a dynamic problem wherein today's options are considered, as well as the potential options over the rest of the contract. At any time, the borrower observes the current values of the house price and the interest rate. Given these values and the assumed processes for how these variables evolve over time, the homeowner evaluates ex ante the possible values of the house price, the interest rate in the next period, and their respective probabilities. Based on these values, the borrower assesses whether it is less costly to default, to prepay, or to make the scheduled payment.

Caveats

Several caveats apply to our approach:

- Limited liability is assumed. This assumption may lead to an exaggerated measure of the risk of default on mortgages because, in Canada, borrowers remain liable for the unpaid balance of the mortgage loan over and above the current value of the house.
- As noted earlier, income constraints are not taken into account within this methodology.
- Costs associated with the loss of reputation for a defaulting borrower are not considered here. These costs can be significant (Kau, Keenan, and Kim 1994). The decision to default can make it more difficult for the individual to obtain credit in the future. This creates an upward bias in our estimated probability of default. These costs could be incorporated into the default decision by adding a cost term to the outstanding balance at the time of default.
- As mentioned above, prepayment can be viewed as refinancing. Although refinancing, like prepayment, implies termination of the

5. As was pointed out in Chatterjee, Edmister, and Flatfield (1998), the two-factor model is efficient in predicting market mortgage values.
6. In this work, we focus exclusively on fixed-rate mortgage loans, which account for about 75 per cent of total mortgage loans outstanding in Canada.
7. As suggested in Deng and Gabriel (2006) and Deng, Quigley, and Van Order (2000), one cannot accurately calculate the economic value of the default option without simultaneously considering the financial incentive for prepayment.

Table 1

Base-Case Parameters for Numerical Modelling

Parameters	Base case
Mortgage term	5 years
Amortization period	25 years
Contract mortgage rate at origination	r_c = 5.70%
Expected rate of appreciation of nominal house price	α = 6.50%
Original 1-month interest rate	r_0 = 3.00%
Transaction cost of prepayment (three months' interest, dollar amount)	1% of the mortgage balance

Note: Values of other parameters related to the stochastic behaviour of house prices and the interest rate are chosen as follows. The standard deviation of stochastic disturbances to change in house prices is estimated (over the 2001–06 period) at 4 per cent per year. The standard deviation of stochastic disturbances to interest rates and the reversion parameter, which measures the speed of return to the mean interest rate, are set equal to 10 basis points and 25 per cent per year, respectively. These values are within the range of those reported in previous works by McManus and Watt (1999) and Bolder (2001).

current mortgage contract, it also implies the origination of a new mortgage loan on which the borrower may default. This is not modelled in this study because of its complexity. Consequently, the probability of default that we compute at a given time is specific to the original mortgage contract. This leads to a downward bias in our estimated probability of default, since refinanced mortgages are assumed not to default.

The Simulations

These simulations illustrate how this model could be used to analyze the impact of decreasing house prices on mortgage defaults. [8]

We measure the *overall* default rate using a two-step default analysis. First, the probabilities of default for different loan-to-value ratios are estimated using an option-pricing model as described above. The overall default rate is then estimated by applying these probabilities to the empirical LTV distribution, which we construct from the CFM database.

Parameters of the simulations

We consider a representative homeowner who has taken out a 5-year mortgage contract with a 25-year amortization period.

To illustrate how the model works, we calibrated the parameters of our model so that they reflect as closely as possible the economic situation in Canada over the 2001Q1–2006Q1 period. This is our base case. In fact, we used the average values over the period of the 5-year discounted mortgage rate, the rate of nominal appreciation in house prices, and the 1-month treasury bill rate. The latter is used for both the original discounting rate and the rate to which it reverts over the given 5-year period. We also assume that some transactions costs are charged in the case of a prepayment. The chosen parameters are summarized in Table 1.

After valuing the probability of default for different LTV ratios at origination in the base case, we repeat the exercise, assuming other scenarios for the evolution of house prices.

8. The same method could be used to examine the potential impact of a change in interest rates.

In the first of three further scenarios considered in this illustration, the moderate case, we assume that house prices are expected to increase moderately at an annual rate of 2.5 per cent. The second scenario is the extreme case in which nominal house prices decline at an annual rate of -2 per cent (the rate of decline observed over the 1990Q1–1995Q1 period). In the third scenario, the very extreme case, nominal house prices decrease at an annual rate of -5 per cent. This value reflects a real decrease in house prices equivalent to that observed in the early 1990s. All other parameters are equal to those in the base case.[9]

Table 2

Probability of Default at Maturity Date

Per cent

LTV ratios						Overall default rate
40%	75%	80%	90%	95%	100%	
Base case (α = 6.50%)						
0.00	0.05	0.36	1.39	2.62	3.80	0.31
Moderate case (α = 2.5%)						
0.00	0.19	1.08	2.51	5.10	6.98	0.63
Extreme case (α = -2%)						
0.00	0.77	2.89	5.53	9.11	12.10	1.35
Very extreme case (α = -5%)						
0.00	2.01	5.96	8.13	12.47	16.22	2.25

Results

The results of our simulations are summarized in Table 2. The first six columns indicate the cumulative probabilities of default over the five years of the loan for mortgages with different LTV ratios.

As expected, the probability of default is greater the higher the LTV ratio[10] and the lower the rate of increase in house prices. For example, as shown in Table 2, under the base-case scenario (house price increase), a loan with a 75 per cent LTV has a 0.05 per cent chance of reaching a point where it is optimal to default. This probability is higher, 0.77 per cent, in the extreme scenario. In the case of a 100 per cent LTV ratio, these probabilities increase to 3.8 per cent (base case) and 12.1 per cent (extreme case).

For a given LTV ratio, the cumulative probabilities of default over the five years of the contract can be interpreted as the proportion of default in the pool of current mortgages that share the same LTV ratio and were signed five years earlier.

The overall default rate is a weighted average calculated by multiplying these cumulative probabilities by the weights given by the empirical distribution of LTV ratios. For simplicity,

9. To better reflect the current interest rate environment, we also simulated the outcomes of these scenarios using 4.5 per cent as the value of the original discounting rate and the rate to which it reverts over the coming 5-year period. Our results did not change significantly.
10. The insurance premium paid by a mortgagor whose down payment is less than 20 per cent increases with the LTV ratio. This is consistent with our results showing that probabilities of default increase with LTV ratios (at origination).

Table 3

Distribution of Mortgages in 2006 by LTV Ratio

As a percentage of asset values

LTV ratios	Frequency
Less than 75	79.45
75 to 80	5.34
80 to 90	8.81
90 to 95	1.53
95 to 100	0.00
100 and > 100	4.87

we used the 2006 distribution, as shown in Table 3, in our examples.

In what follows, we compare our estimated overall default rate with actual default rates. The simulated default rates differ from observed rates, because we consider only fixed-rate mortgages in our model, while actual default rates reflect defaults on both fixed-rate and variable-rate mortgages. Defaults on variable-rate mortgages may be more sensitive to changes in interest rates than defaults on fixed-rate mortgages. This comparison is intended to provide only a rough test of whether our estimates are in the general range of historical experience.

Our estimated rates of default appear reasonable and broadly within the range of historically observed default rates. The overall rate of default estimated for the base case (0.31 per cent) is slightly higher than the actual rate of default observed in 2006 (0.23 per cent).

Also, our results suggest that the rate of default would reach 1.35 per cent following a persistent decrease in house prices similar to the one observed over the 1990Q1–1995Q1 period. This rate is higher than the peak observed in Canada in 1992Q1 (0.62 per cent).[11] This is because, as mentioned in the caveats, the assumption of limited liability may lead to an exaggerated measure of the risk of default, particularly in scenarios where defaults are more likely to happen (i.e., decreasing house prices). The rate of default is still much higher in the very extreme scenario (2.25 per cent).

These rates do not reflect actual losses to banks and mortgage insurers, because the loss-given-default on mortgages is considerably less than 100 per cent of the balance of the mortgage. Anecdotal evidence suggests that the loss-given-default on mortgages may be around 10 per cent.

These comparisons should, however, be interpreted with caution, given the caveats mentioned above. Nevertheless, they suggest that the methodology applied here can be useful for stress testing the portfolio of Canadian mortgage loans.

11. The 0.62 per cent rate is measured as a percentage of the number of mortgage loans in arrears three months or more. Data on default rates as a percentage of asset values are not available before 1997.

Conclusions

This work applies a contingent-claims model of mortgage default to analyze the impact of changes in house prices on the decision to default.

This approach has limitations. In particular, it is technically very difficult to introduce additional factors into this framework to take into account other important aspects of the default decision, such as the risk of income loss. Also, we do not explicitly model the fact that, besides the options to default and to prepay, the mortgagor can choose to refinance his loan at a new mortgage rate. This would require the introduction of a third stochastic variable, which would make the solution of the model extremely complex.

On the whole, however, this work appears helpful in gauging the risk of default on mortgage loans under different scenarios and assumptions regarding the evolution of the distribution of LTV ratios.

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