Collateral Portfolios and Adverse Dependence

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As financial markets and their supporting infrastructure continue to develop over time, banking professionals and regulators are taking steps to make them safer. Many of those steps involve the use of collateral to manage financial risks.[1] But collateral itself may consist of risky assets whose value can change over time. Consequently, a pledge of collateral must be large enough to adequately cover all losses in the event of a counterparty default. Thus, the initial value of collateral is discounted. In other words, the amount of collateral pledged must be larger than the amount owing. This discount, often referred to as the "haircut," lowers the risk associated with a transaction. But because collateral is costly to pledge, the framework established for setting haircuts must recognize the inherent trade-off between costs and risks associated with collateral. This framework could also provide useful information to determine the desirable allocation of the portfolio of collateral.

This article summarizes the second of two papers that explore a framework for calculating haircuts for different assets. The first, García and Gençay (2006), proposed a framework for comparing different methods for computing haircuts for individual assets. Particular attention was paid to selecting a method that would accomplish two goals. First, it would provide sufficient collateral in the case of low-probability events (large unexpected declines in asset prices) that might affect the stability of the financial system. Second, it would take into account the cost of pledging collateral. The second paper, García and Gençay (2007), examines how haircuts should be calculated in situations where a variety of assets are pledged as collateral. Here, the focus is on the relationship among the prices of the different assets pledged as collateral and, in particular, how this relationship can change when markets are under stress. We refer to this change as a change in the dependence structure,[2] which is caused by an event that changes the relationship between the returns on the assets in the pool of collateral. For example, during normal market conditions, a given pool of collateral may exhibit diversification benefits. However, during extreme market conditions, few, if any, such benefits may be evident for the same pool.

Financial Risks during Extreme Events

When collateral consists of a variety of assets, note should be taken of two effects generally associated with extreme events. The first is associated with the individual security, and the second with the portfolio as a whole. The former is referred to as the *individual effect*, the latter as the *portfolio effect*. The individual effect occurs when there is a negative return on an asset pledged as collateral, but the dependence structure of the portfolio does not change significantly. The portfolio effect occurs when there is a change in the relationship among the various assets pledged as collateral; that is, the dependence structure between the assets changes and exhibits smaller diversification benefits than observed historically.[3] To illustrate the portfolio effect, consider two hypothetical securities, x and y, that are pledged as collateral; and two states of the world:

1. According to Khan (2007), the use of collateral to mitigate counterparty credit risk has increased substantially.

2. This is usually referred to as a change in correlation, but this is not always correct, since there can be a change in the dependence structure without a change in the correlation.

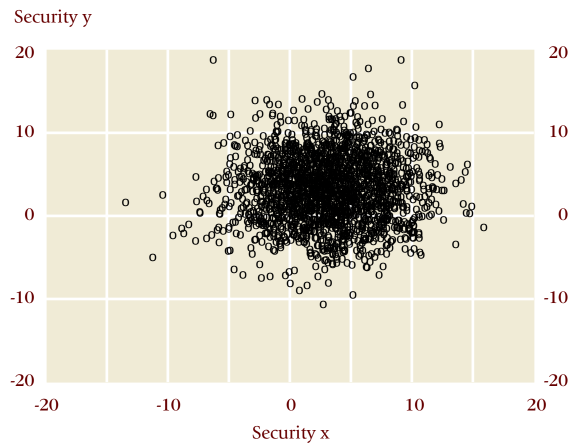
3. Chan et al. (2005) refer to this as a "phase-locking" effect. The authors offer an explanation for these effects from a financial-engineering perspective.

a normal state and an extreme state. During the normal state, the scatter plot of per cent losses for x and y may be represented by Chart 1; during the extreme state, it can be represented by Chart 2.[4] In this example, we assume that the distribution of returns for each asset was the same in both states, but that the dependence structure between the assets changes.

Chart 1 illustrates that, in a normal state, there are many instances in which a large loss for one asset does not coincide with a large loss for the other. In contrast, as Chart 2 shows, in an extreme event, the diversification benefits are significantly reduced compared with those observed in normal periods. Chart 2 shows a greater degree of positive dependence, that is, large losses in one asset coincide with large losses for the other asset.[5]

Chart 1 Scatter Plot of Losses during Normal Periods

Per cent



Scatter plot of 2,000 points representing losses from two hypothetical securities with normal marginals N(3,4) and the dependence structure of a Normal copula with correlation 0.1.

Chart 2 Scatter Plot of Losses during Extreme Events

Per cent

Scatter plot of 2,000 points representing losses from two hypothetical securities with normal marginals N(3,4) and the dependence structure of a Gumbel copula G(4).

Managing Portfolio Effects

To manage the financial risks associated with the portfolio effect, the dependence among assets must be modelled in a way that reflects what could happen if there were an extreme event. We accomplish this using copulas—multivariate distributions that are very useful in financial-engineering problems involving modelling two or more random variables. Because copulas allow the multivariate distribution of returns for the portfolio to have a wide range of marginals (i.e., the distribution of returns for each asset) and dependence structures, they allow us to separate the behaviour of the dependence structure from the behaviour of individual asset prices. This separation is not possible with traditional representations of multivariate distribution functions and may lead to a misspecification of the multivariate distribution. The use of copulas thus facilitates the aggregation of risk across securities that may have different return distributions.

We use the copula-based method to determine whether a collateral pool contains assets that

4. Charts 1 and 2 represent losses as positive values and profits as negative values. This is a standard convention in statistics, since actuarial risk theory is a theory of positive random variables.
5. Note that other outcomes are possible during extreme events. For example, if the portfolio is composed of a risk-free asset and a risky asset, the result could be a more negative dependence.

have a low probability of joint losses.[6] This is done to assess whether, during extreme events, the returns of the assets in the portfolio are likely to continue to be as diversified as they were historically.

Note that while a receiver of collateral would prefer to have a collateral portfolio with large diversification benefits, the collateral pledgor would normally decide which assets will be pledged, subject to the rules of the collateral-pledging agreement. A copula-based methodology could be an input in determining maximum limits for classes of assets that can be pledged as collateral (i.e., sector limits), creating incentives for those pledging collateral to supply a diversified pool. In the event of a counterparty default, having a diversified pool of collateral could reduce the costs associated with selling (liquidating) the collateral portfolio, because it may be easier to find counterparties to take those assets that still exhibit diversification benefits. In contrast, a portfolio with lower diversification benefits may require a significant discount in order to sell the assets in time to cover the losses.

Stress Testing Portfolio Dependence

García and Gençay (2007) also present a methodology for examining the performance of the portfolio in the face of an event that negatively affects the dependence structure. The collateral pool in question is subjected to stress tests in which the dependence is changed by (i) using a comprehensive set of copula families that represent different dependence structures and (ii) increasing the degree of positive dependence for each copula. When conducting stress tests, we assume that the characteristics of the individual assets in the portfolio do not change, only their dependence on each other. We estimate the distributions for each asset based on historical data, and, for the dependence structure, we start with a scenario based on historical observations of the dependence. This approach provides a range of the possible adverse dependencies (and their associated losses) that may occur during extreme events. For example, using various copula models to capture the dependence between the price changes of two Canadian investment-grade assets, we observe that the portfolio losses (negative returns) can vary by as much as half of a percentage point. This result, coupled with the substantial size of collateral portfolios, may translate into a large discrepancy between the different models in dollar terms.[7]

6. Based on Carmona (2004) and Zivot and Wang (2006), our copula-based method uses a semi-parametric approach to model the marginals and a copula to model the dependence.

Conclusion

This work, together with García and Gençay (2006), proposes: (i) a framework for calculating haircuts for individual assets, (ii) a method for monitoring changes in the dependence structure of assets, and (iii) a method for stress testing and measuring the possible effects of adverse dependence structures on portfolio value.

This research has two policy implications. First, there is a need for caution when considering the extent to which a haircut should be reduced to take account of diversification benefits in a collateral pool. While those benefits may be evident in normal situations, they may decline significantly during extreme events. This could lead to uncollateralized exposures, or even losses, if collateral has to be liquidated in a period when markets are under stress. Second, when the number of assets accepted as collateral increases, it is important to consider not only the individual characteristics of the asset in question, but also its effect on the overall dependence structure of the portfolio of collateral.

References

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7. Note that this result is specific to the portfolio examined.

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