Using No-Arbitrage Models to Predict Exchange Rates

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Exchange rate predictions have many important applications. Risk managers use exchange rate predictions when deciding if (and when) to hedge currency movements. Portfolio managers use exchange rate predictions to obtain expected returns on foreign assets. Academics test their models of exchange rate determination on the basis of their ability to predict exchange rate movements.

Central bankers are also interested in having accurate models of exchange rate determination. For example, it is important to understand the forces that are driving currency movements, because different causes will have different implications for the economy. Ultimately, they may even require a different monetary policy response (Bailliu and King 2005; Ragan 2005). An assessment of international financial market stability and contagion also depends on the ability to understand large movements in currency markets.[1]

Predicting currency movements is, however, a difficult task. Despite the large body of research on exchange rate modelling, a key stylized fact in international finance is that the best prediction for tomorrow's exchange rate is today's rate (known as the "random-walk forecast").[2] This result was first discovered by Meese and Rogoff (1983a, b) and, even 25 years later, few models can do better than this one.[3] A related result, also found in the literature starting in the early 1980s, is that the forward rate does not provide the best prediction for tomorrow's exchange rate.[4] Thus, Clarida et al. (2003) note that "from the early 1980s onwards, exchange rate forecasting in general came increasingly to be seen as a hazardous occupation, and this remains largely the case."

This article summarizes a working paper (Diez de los Rios 2006) that proposes an arbitrage-free model of the joint behaviour of interest rates and exchange rates that provides exchange rate forecasts with improved predictive power when compared with the current set of foreign exchange rate models that do not impose these no-arbitrage restrictions.

No Arbitrage

It is hard to believe that exchange rates move independently of, for example, interest rates. The reason for such a skeptical statement is the concept of arbitrage in financial markets. If the prices of two related securities differ by a great amount, then an investor will have an incentive to buy the undervalued asset and sell the overvalued one to make a profit.[5] Thus, in an efficient market, arbitrage ensures that the prices of both assets do not move independently. For example, spot, forward, and Eurocurrency interest rates are mutually dependent through the familiar covered interest parity condition.[6]

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1. See Berg, Borensztein, and Pattillo (2004) for a review on early-warning systems for currency crises.
2. Similarly, the best prediction at the one-month or one-year horizon is also today's exchange rate.
3. See Bailliu and King (2005) for a review of these successful models (including the Bank of Canada's Exchange Rate Equation).
4. Finance theory suggests that a risk-neutral investor should be indifferent between buying a one-month forward contract for a foreign currency or waiting one month and buying the currency directly in the spot market. This theory, known as "uncovered interest rate parity," implies that the best prediction for the future exchange rate is its forward counterpart (see Hansen and Hodrick 1980).
5. The technical definition of the absence of arbitrage states that it is impossible to obtain a portfolio that might provide a positive payoff (and never incur losses) without cost (see Cochrane 2001).
6. See Mark (2001) for more details on the covered interest parity condition.

A similar argument applies to domestic and foreign bonds. These assets are essentially imperfect substitutes with different levels of exchange rate risk. For instance, a Canadian investor who buys a one-year bond in the United Kingdom will know how many pounds sterling he will get in the future, but not how many Canadian dollars. Therefore, a Canadian investor will demand compensation for bearing the exchange rate risk. In other words, he will expect compensation for holding an asset that, from his point of view, is not perfectly risk free. If the rate of return (in Canadian dollars) of this British bond does not reflect this compensation, then the prices of British and Canadian bonds, as well as the bilateral exchange rate, should adjust until any arbitrage opportunities disappear. Therefore, the absence of arbitrage opportunities links the way in which interest rates and exchange rates can move over time.[7]

Overall, these so-called "no-arbitrage restrictions" provide useful information on how to model exchange rate movements and, therefore, how to improve exchange rate predictions.[8]

Model and Methodology

Motivated by the above arguments, Diez de los Rios (2006) uses a two-country affine term-structure model[9] to predict currency movements. The model leverages the no-arbitrage relationship between interest rates and exchange rates, itself a generalized version of the covered interest rate parity relation described above. In this model, the yield curve and the expected rate of depreciation of a currency are functions of the same set of state variables: domestic and foreign short-term interest rates.

The model is estimated for two different currency pairs: U.S. dollar–pound sterling and U.S. dollar–Canadian dollar. The dataset consists of monthly rates of depreciation[10] of these two currency pairs over the period January 1976 to December 2004, along with monthly observations of the corresponding U.S., British, and Canadian Eurocurrency interest rates for maturities of one, three, six, and twelve months. These Eurocurrency deposits are essentially zero-coupon bonds whose payoffs at maturity are the principal plus the interest payment.

The estimations are carried out using data over the period January 1976 to December 1997 in order to reserve the last seven years for an out-of-sample forecasting exercise. The exchange rate forecasts, in particular, are computed according to a recursive procedure: at each month t, the model is re-estimated using data up to and including that month, and then forecasts of the spot exchange rate, up to one year ahead, are obtained.

A "horse race" is conducted between the forecasts obtained using this no-arbitrage model and those generated by three alternative benchmarks: a random walk, a vector autoregression on the forward premiums and the rate of depreciation, and the forward-premium regression. A comparison of the author's forecasts with those produced by the random-walk model is motivated by the fact that the random-walk model is considered to be the usual metric by which to evaluate exchange rate forecasts since the original work of Meese and Rogoff (1983a, b). However, Clarida and Taylor (1997) show that if one uses a vector autoregression (VAR) on the forward premiums and the rate of depreciation, it is possible to obtain out-of-sample forecasts of spot exchange rates that beat the random-walk model. Therefore, a VAR model is also included as a second benchmark. Finally, and for completeness, the author also includes the forecasts produced by a standard ordinary least-squares regression of the rate of depreciation onto a constant and the lagged forward premium (the forward-premium regression).

The forecasts produced by the term-structure model, as well as those of the three competing models, are evaluated in terms of two widely used criteria: the root-mean-square error (RMSE) and the mean-absolute error (MAE). The smaller these criteria are, the better the performance of the model.

7. The absence of arbitrage opportunities will not only restrict the way in which interest rates and exchange rates move, but will also restrict how interest rates at different maturities move together.
8. In fact, there is empirical evidence that one can also improve interest rate predictions if such no-arbitrage restrictions are exploited (Duffee 2002; Ang and Piazzesi 2003).
9. For a review of affine term-structure models and their applications, see Piazzesi (2003).
10. Note that a negative rate of depreciation would imply an appreciation in the currency.

Results

The author finds that using no-arbitrage restrictions reduces, for example, the RMSE in forecasting the spot U.S. dollar–pound sterling rate by about 35 per cent at the one-year forecast horizon relative to the VAR approach, and by about 15 per cent for the U.S. dollar–Canadian dollar rate. The gains from using a VAR model over a random-walk model are negligible. For example, the gain at the one-year horizon for the U.S. dollar–pound sterling pair is only 2.4 per cent (versus the 40 per cent reported by Clarida and Taylor 1997). Similar results are obtained when using the MAE criteria.

Conclusions

Overall, these results support the use of no-arbitrage methods to generate more accurate exchange rate predictions. The success of this approach provides indirect support for the assumption that markets are efficient, since it is based on a generalization of covered interest rate parity. Still, more work can be done in this direction. The predictions in these models are based exclusively on the information contained in interest rates, while one would also like to use the information contained in other macroeconomic variables (such as output growth, inflation, or even commodity prices) to obtain even better predictions. Developing a no-arbitrage model of the joint behaviour of macroeconomic variables, interest rates, and exchange rates that, at the same time, is able to deliver good exchange rate forecasts is a new challenge that is left for further research.

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