Using the Contingent Claims Approach to Assess Credit Risk in the Canadian Business Sector

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In analyzing the financial system, central banks are interested in systemic risk. This can generally be taken to include risks that may lead to substantial problems for the financial system and ultimately result in a significant decline in real GDP. Hence, monitoring the risks facing Canadian financial and non-financial corporate sectors is an important part of overall financial system surveillance.

Risk in the corporate sector can be assessed in different ways. A large body of literature links risk to balance sheet ratios of profitability, liquidity, and leverage (Aaron and Hogg 2005; Altman 1983; Vlieghe 2001). Other approaches use financial market information to assess risk.

This report explores one such method, the contingent claims approach (CCA), which relies on both market information (including a measure of risk stemming from the volatility of market prices) and balance sheet information to model corporate credit risk.

Although the CCA is an interesting modelling tool for analyzing credit risk, it is data and computationally intensive. It can also be difficult to implement, since it requires matching different types of data—usually obtained from different sources—for a large number of companies. Hence, judgment has to be exercised in balancing the surveillance requirements with the cost of data gathering and integration.

This report uses the Canadian non-financial corporate sector and the banking sector to explore the implementation of the CCA for macrofinancial surveillance. It begins with a brief overview of the methodology, together with the issues that arise in applying CCA at a sectoral level. Next, CCA-based risk indicators are presented for some industry sectors and for the entire non-financial corporate sector. This is followed by an application to the Canadian banking sector. The report concludes with an evaluation of the CCA for macrofinancial surveillance, and outlines further avenues of research.

The CCA: Merton-Type Models

Distance-to-default measure

The CCA is a method that uses Black-Scholes option-pricing techniques to calculate the likelihood of corporate default. It is an extension of the Merton (1974) model based on the insight that a shareholder has an implicit call option on the value of the assets of the firm. The CCA uses both historical balance sheet data (leverage ratio) and timely and forward-looking equity market information (volatility of returns) to calculate a measure called distance to default (DD).

Distance to default represents the number of standard deviations that the market value of a firm's assets is away from the level of its liabilities. A higher DD (which means that the level of a firm's assets is expected to be farther away from the level of its liabilities) is interpreted as a lower risk of default. This could be caused by an improving leverage ratio, better asset returns, lower asset volatility, or any combination of these.[1]

Market-based indicators derived from Merton models have several advantages over indicators that rely primarily on accounting data. Market indicators are forward looking, they are available at a higher frequency, and the methods for extracting risk measures are broadly accepted.[2] On the other hand, market prices may reflect changes in attributes that could be unrelated to

1. A brief overview of the Merton model is presented in the Appendix.
2. European Central Bank (2005); Sveriges Riksbank (2005); Danmarks Nationalbank (2005); Persson and Blåvarg (2003).

financial stability. For example, an increase in market prices would be reflected in a higher DD (lower default risk), even though the price increase was due to market overreaction to good news or herding behaviour, rather than being the result of improved fundamentals. Nevertheless, market-based indicators have been shown to have leading information on corporate distress (Chan-Lau and Gravelle 2005; Chan-Lau, Jobert, and Kong 2004; Dionne et al. 2006; Tudela and Young 2003; and Gropp, Vesala, and Vulpes 2002).

Assessing sector-level risk

The CCA can also be used for sector analysis. This can be done by applying the CCA to each firm in the sector and aggregating the results into a sector measure. This approach has the advantage of providing information on the distribution of individual DD measures, which allows the analysis to focus on the vulnerable tails of these distributions.[3] The disadvantage is in the cost of data integration, which can be substantial for frequent surveillance.

An alternative approach is to apply the CCA to sector-level data (Gapen et al. 2004). This approach treats each sector as a single firm by aggregating firm-level debt and equity information for all companies in a particular sector. Aggregating firm-level debt and equity information requires less computation and is easier to update regularly. Also, in aggregating the market values of equity and calculating its volatility, we implicitly take into account the individual volatilities and their correlations. This application of the CCA to sector-level data explicitly gives more weight to larger firms. Hence, these aggregate measures should be sensitive to systemic vulnerabilities arising from the deteriorating financial condition of a large firm or that of a critical mass of smaller firms.[4]

Regardless of the approach taken, it is important to recognize that extending Merton-type models to sector-wide analysis requires a different interpretation of the DD measure. It may not be appropriate to interpret a sector-level DD measure as a risk of "sector default." But since the sector-level DD will reflect the risks of the underlying firms, it should reflect the overall vulnerability of the sector.

This report uses both approaches. For the non-financial sector, where it is unlikely that any single non-financial corporation is systemically important, the CCA is applied to the sector-level aggregation.[5] For the major Canadian banks, which could be systemically important, the CCA is applied at both the individual and sector-level aggregation.

Methodology and data

All market data are from Thompson Financial Datastream. The balance-sheet data for the public non-financial companies are from the *Globe and Mail* database.[6] The balance sheet information for the Canadian banks was obtained from the monthly returns filed by the banks with the Office of the Superintendent of Financial Institutions. The distance-to-default measures were estimated using the method set out in Chan-Lau, Jobert, and Kong (2004).[7]

Corporate bond defaults are measured by the number of public companies that defaulted in a given year as a proportion of all companies in an industry rated by Standard & Poor's.[8] Because of

3. Aaron and Hogg (2005) follow this route, using different balance-sheet ratios to construct an indicator of vulnerability in the corporate sector.
4. Sector-level aggregation may mask the weak firms, since it implicitly assumes that the assets of one firm can be used to back up the liabilities of another firm, which is not strictly true. But a similar masking issue would arise if firm-level DD measures were averaged.
5. There are over 1,500 non-financial public companies in Canada.
6. The public companies in the *Globe and Mail* database represent 55 per cent of total assets of all companies (public and private) in the non-financial business sector in 2004, as reported by Statistics Canada, and the coverage varies by industry. For example, for the forestry industry, the share of assets of public companies in the *Globe and Mail* database represents 45 per cent of total assets (private and public companies) in the industry.
7. For non-financial companies, annual balance-sheet information was used to calculate the default barrier by adding current liabilities and half of long-term debt for all companies in an industry. Taking half of long-term debt is arbitrary and follows the practice presented in other studies. Total liabilities were used for the banks. Annualized equity volatilities were calculated at the beginning of every month, using a one-year rolling window of daily market values of equity. The monthly DD values were calculated following the procedure outlined in the Appendix.
8. Not all of the companies in the *Globe and Mail* database are rated, and, therefore, data on bond defaults might not include the defaults of all companies in the *Globe and Mail* database.

Table 1

Correlation Between Distance to Default and Bond Defaults

Distance to default	Bond defaults
Forestry (lagged)	-0.658
Forestry (contemporaneous)	-0.550
Manufacturing (lagged)	-0.146
Manufacturing (contemporaneous)	-0.524

Chart 1 Distance to Default and Bond Defaults

Forestry

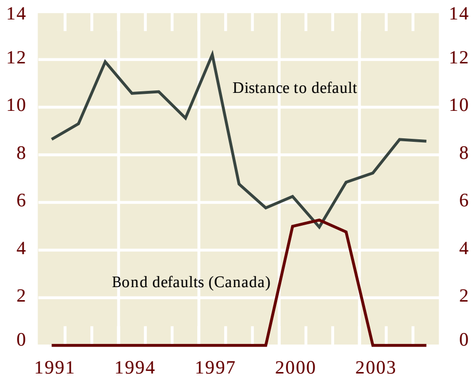
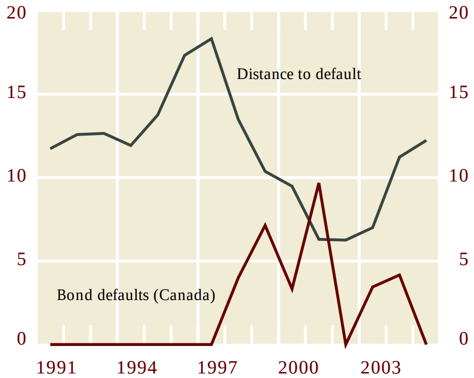


Chart 2 Distance to Default and Bond Defaults

Manufacturing



data limitations, the sample period for the analysis of the non-financial sector is 1991–2005.

Assessing Risks in the Non-Financial Corporate Sector

To assess the usefulness of the CCA for macrofinancial surveillance, we applied the CCA to the major non-financial corporate sectors. Each sector underwent a preliminary examination of the leading-indicator properties of DD for corporate bond defaults.

Industry-level risk measure

Charts 1 and 2 show DD for the forestry and manufacturing sectors. In both sectors, DD began to decrease in 1997 and reached a trough in 2001. Since 2001, DD has shown an upward trend, suggesting that risk in these sectors has decreased.

The correlations between DD (and DD lagged one year) and bond defaults (Table 1) support the expected negative relationship.[9] The high correlation in the forestry sector suggests that DD has some leading-indicator properties for corporate bond defaults, which is desirable for financial-stability surveillance. For the manufacturing sector, contemporaneous correlation is also high, but one-year lagged correlation is rather low. Charts 1 and 2 suggest that DD may, indeed, have some leading-indicator properties for the sectors examined.

Risk measures for the overall corporate sector

Increased vulnerabilities in a small sector are likely to have a smaller risk of systemic impact than vulnerabilities in a larger sector. But a sector's size or its share of GDP or bank loans are not the only factors affecting its contribution to systemic risk. It is also important to take the correlation of risks among sectors into account. In this section, we propose two different ways to measure risk in the overall corporate sector.

The first approach is to aggregate the balance-sheet and equity information of all companies and then calculate DD for the aggregate corporate

9. Note that the correlations should be interpreted carefully, since the relationship between DD and bond defaults is not linear, and only 14 years of annual data were studied.

sector. An alternative approach uses the market value of assets, one of the main outputs from the CCA. Since the whole corporate sector can be viewed as a portfolio containing the assets (in market value) of all the companies in the corporate sector, we propose the variance of the return on this portfolio as a proxy for the risk in the overall corporate sector.

The resulting DD for the aggregate corporate sector seems to have some leading-indicator properties for bond defaults (Chart 3). The correlation between bond defaults and a DD lagged one year is high (-0.74) and is still significant using a two-year lagged variance (-0.56). Even though the analysis covers a short period, this suggests that the corporate sector DD has some leading-indicator properties for credit risk.

The variance of the corporate sector portfolio also seems to have some leading-indicator properties for bond defaults (Chart 4). The correlation between one-year lagged variance and bond defaults is very strong (0.84) and is still high using a two-year lagged variance (0.69), supporting the leading-indicator properties of the variance measure for bond defaults.[10]

Thus, both measures of aggregate credit risk seem to have some leading-indicator properties for bond defaults.[11] As expected, there is overlap in the information content of these two measures, which are highly correlated (-0.79).

Chart 3 Distance to Default and Bond Defaults for the Aggregate Corporate Sector

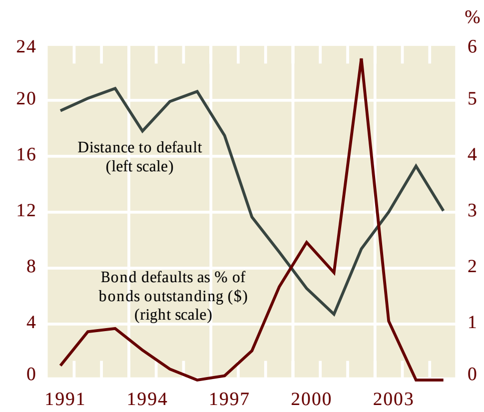
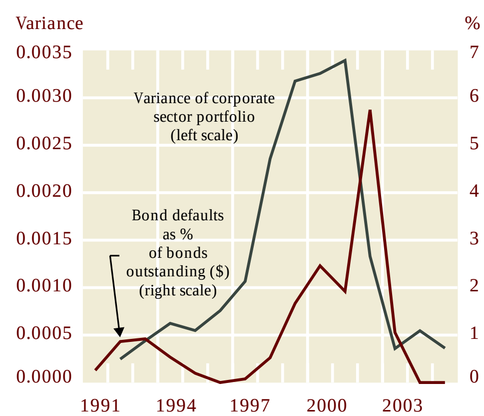


Chart 4 Variance of Corporate Sector Portfolio and Bond Defaults for the Corporate Sector

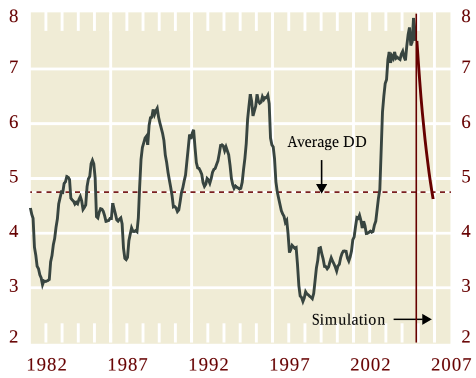


Assessing Risks in the Banking Sector

In this section, the DD measure is used to assess the overall financial health of Canadian banks. The Canadian banking sector is proxied here by the six largest Canadian Banks (major banks). This is justified by the high concentration of Canada's banking sector, where the major banks held approximately 91 per cent of the banking assets in Canada, as of January 2006.

10. In comparison, the microdata indicator developed in Aaron and Hogg (2005) had a one-year lagged correlation of 0.46. See also Box 2 on page 11 of this issue.
11. A similar correlation exercise with impaired business loans for banks gave much weaker results.

Chart 5 Average Distance to Default for Major Banks



Historical evolution of the risk measure

The average DD for the major banks during the period 1982–2005 is presented in Chart 5.[12] During this period, there have been important changes in the business practices of the major banks and in risk-management and risk-mitigation techniques.[13]

Movements in DD can be broadly related to major credit developments at the banks. For example, the measure fell sharply in the early 1980s, when many developing countries were encountering difficulties in servicing their debt, and was marginally below the mean in 1990 before the 1991 recession. Distance to default was also low following the crash in the technology sector in 2000–01 and the associated concerns about the exposure of some major banks to the telecom and cable sector. But there were also major declines around 1997–98, the period of extreme market volatility triggered by the 1997 Asian crisis and the 1998 Russian default/LTCM events, which are not thought to be particularly stressful for the major banks except, perhaps, for their market operations. Hence, these linkages must be interpreted cautiously, since changes in DD during the periods mentioned could be caused primarily by broader movements in the markets that might be only tangentially related to the risk exposure of Canadian banks.

The underlying drivers of DD (assets/liabilities and asset volatility) have subsequently improved, which has resulted in the observed decrease in risk (increase in DD) since that time. Of most interest is the strong increase in DD in 2003–04. Although all DD drivers improved during those years, the main driver was a strong decrease in asset volatility. This could emanate from a number of sources, such as a fundamental improvement in the riskiness of major

12. The average DD is the asset-weighted average of each individual bank's DD, computed using the procedure outlined in the Appendix. Although some information is lost in the aggregation process, it should provide a good indication of important changes in the risks of major banks.

13. For example, in the early 1990s, there was a major shift towards reliance on fee income at the expense of interest income, and the trading book expanded much more rapidly than the banking book. Moreover, since the mid-1980s, residential mortgage lending has risen at the expense of business lending.

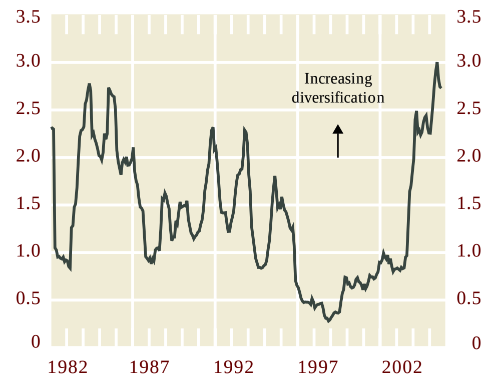
banks, or the banks may simply have benefitted from the low volatility of the stock market as a whole. To see if the latter is the case, a simulation was done using a scenario in which the volatility of the major banks' equity returns to its sample mean.[14] Chart 5 indicates that, should this occur, the recent improvement in the DD measure would be substantially reduced but DD would still be at the historical average.

Assessing risk diversification in the banking sector

The average DD measure analyzed above does not explicitly account for diversification of risk among the major banks, which requires the incorporation of correlations among these institutions. Calculating DD for a "representative bank" is one way to measure this benefit.[15]

As with the methodology used above for the non-financial corporations, DD for the representative bank is calculated by aggregating the major banks into a single entity. This procedure accounts for the correlation among the major banks and, hence, should include a measure of the diversification benefits.[16] Distance to default for the representative bank will be higher than the average DD because of diversification, and the difference between the two measures should reflect this benefit.[17] The lower the correlation among institutions, the more the system as a whole will benefit from "diversification" effects, and the larger the difference between the representative bank DD and average DD will be. The results are shown in Chart 6. This difference reached a peak recently, indicating good diversification across major

Chart 6 DD for Representative Bank Minus Average DD



14. This simulation assumes that all input parameters are fixed except for the volatility of major banks' equity, which returns to its sample average linearly over one year. The correlation between market value of equity and volatility is not significant, suggesting that this assumption is reasonable. A scenario where the volatility of the major banks' equity returns to its 10-year average gave similar results.
15. This approach has been used by the International Monetary Fund in its Article IV reports.
16. The aggregate market capitalization of the major banks and the volatility of their equity, which are used as inputs into the model calculations, will, by definition, include the correlations among the equity-price movements of the major banks.
17. In addition to the diversification effects, the difference may also reflect the effects of aggregation.

banks and that the "sector" is expected to be resilient to shocks. Note, however, that the profile for this measure follows the profile for the average DD (Chart 5). This implies that the diversification benefits seem to be reduced in times of greater stress (lower average DD).[18] Hence, this diversification benefit should not be overstated. In addition, although the DD for the sector incorporates the correlations, it does not account for second-round or network effects, which arise from the linkages between the constituent banks, except to the extent that movements in market prices incorporate such effects.

Conclusion

The CCA has advantages for macrofinancial surveillance over financial accounts measures, since it uses more timely and forward-looking information. These measures are gaining acceptance among many central banks and international institutions as tools for monitoring systemic risks.

The work summarized here shows that the CCA can be useful for analyzing systemic risks in the non-financial and financial corporate sector. Depending on the surveillance requirements, it can be applied at the firm level or at the aggregate sector level.

Additional research is being done to better understand the value of this tool. For example, Gropp, Vesala, and Vulpes (2002) suggest that DD leads downgrades of European banks by six to eighteen months. This result is being assessed for Canadian financial institutions. Research using simulations is also being conducted to quantify the impacts of aggregation in applying the CCA to sector-level analysis. Lastly, measures from the CCA are being incorporated into studies that are investigating the links between corporate vulnerabilities and macroeconomic variables.

18. It is well known that in bad times, not only does the likelihood of defaults increase, but also the correlation of defaults. The underlying causes of this behaviour and the methodologies to distinguish between them are still not well understood (Forbes and Rigobon 2002).

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Appendix

The Merton Model

The methodology followed here is Merton's option-based model of credit risk. The details of this methodology are explained in Chan-Lau, Jobert, and Kong (2004). The Merton model of credit risk treats the equity of a firm as a call option on the underlying assets of the firm. This formulation allows the calculation of an expected distance to default (DD), which can be taken as a measure of the probability that the market value of the assets will be equal to or less than the liabilities (also known as the default barrier) over the chosen time horizon, which is taken here to be one year.

More formally, the Merton equations for the pricing of a call option are:

$$E = AN(d_1) - Le^{-rT}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{A}{L}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}, d_2 = d_1 - \sigma_A\sqrt{T}, \qquad (1)$$

where

- E = market value of equity
- A = market value of assets
- N = the cumulative density function of the standard normal distribution
- L = value of liabilities
- r = 1-year treasury bill rate
- T = the chosen time horizon
- σ_A = asset volatility
- σ_E = volatility of equity.

The Merton framework also links equity volatility and asset volatility through the following relationship:

$$\sigma_E E = N(d_1)\sigma_A A. \qquad (2)$$

Hence, given the book value of debt, the maturity, the firm's equity value, and its volatility, the implied market value of its assets, and the asset volatility can be calculated by solving equations (1) and (2) simultaneously. Now, using the known values of the liabilities and the calculated values of assets and asset volatility from above, the distance to default, which is a measure of the firm's credit risk, can be calculated as:

$$DD = \frac{\ln\left(\frac{A}{L}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}. \qquad (3)$$

Note that a large DD is consistent with low risk, since the firm is a greater number of standard deviations away from the default threshold, and vice versa.

Given the assumptions of a standard normal distribution for DD, the probability of default is calculated as follows:

$$P_{def} = N(-DD). \qquad (4)$$

In practice, the probabilities of default calculated from Merton-type models do not map exactly into observed probabilities for firm default because they rely on risk-neutral pricing, which overstates the true probability of default. Hence, although this measure has been shown to be a complete and unbiased indicator of firm vulnerability, it is appropriate to think of it as a default-likelihood indicator (Gapen et al. 2004; Vassalou and Xing 2004). Commercial vendors such as Moody's KMV use historical data to map these calculated probabilities into estimated default frequencies.