Using High-Frequency Data to Model Volatility Dynamics

*Gregory H. Bauer**

The covariance matrix of asset returns is important for a wide range of individuals.[1] Academics use estimates of the covariance matrix to test asset-pricing theories. Portfolio managers use the covariance matrix in designing tracking strategies where the return on their portfolio is designed to closely follow the return on a benchmark portfolio. Risk managers use the matrix to construct measures such as "value at risk." Corporate managers require accurate measures of covariances for hedging strategies.

Central bankers also have a profound interest in this concept. An assessment of financial market stability and contagion depends on measuring the time-varying variances and covariances that make up the matrix. For example, research has shown that there is an "excess" comovement of international equity markets during market downturns (e.g., Connolly and Wang 2003; Ribeiro and Veronesi 2002). Whether this is a rational response to current economic conditions or the result of irrational "contagion" remains an open question.

It is a key stylized fact in empirical finance that the variances and covariances of asset returns fluctuate over time.[2] Central bankers and others, therefore, require a model of a time-varying or "conditional" covariance matrix.[3] Several distinct methods for estimating a conditional covariance matrix have evolved in the literature, but since an asset's true volatility cannot be observed, researchers must treat the elements of the covariance matrix as non-observed or "latent" processes. This greatly complicates the modelling of the covariance matrix. If the actual matrix could be observed, the causes of time-varying market volatilities and correlations could be measured more accurately.

Realized Volatility

The concept of "realized volatility" has recently been developed to provide more precise estimates of the volatility of a single asset or index. Assets such as stocks and bonds trade second by second throughout the day. These high-frequency data can be recorded and aggregated to yield a relatively precise estimate of the daily volatility of the asset. The resulting realized volatility is not latent, but observed, which results in more accurate forecasts.[4] While most papers have focused on estimates of the volatility of a single asset, it would be interesting to see whether a better estimator of the entire conditional covariance matrix could be created in this way.

In "Multivariate Realized Stock Market Volatility," Gregory Bauer (Bank of Canada) and Keith Vorkink (MIT) introduce a new model of the conditional covariance matrix. High-frequency data for a number of stocks are recorded during

1. A *covariance* measures how the price of one asset moves over time in relation to the price of another. A *covariance matrix* is a mathematical concept that measures how several asset prices move together over time. It is composed of the variances of the individual assets and the covariances between them.
2. For a comprehensive survey of the literature on volatility modelling and forecasting, see Andersen et al. (2005).
3. "Conditional" refers to market participants using current information to make optimal forecasts.

* This summary is based on Bauer and Vorkink (2006).

4. Andersen and Bollerslev (1998) introduced the idea of using high-frequency data to construct estimates of the daily realized volatility of a single asset. Andersen et al. (2003) formalized the definition, which was applied to equity markets in Andersen, Bollerslev, Diebold, and Ebens (2001) and exchange rates in Andersen, Bollerslev, Diebold, and Labys (2001). Constructing realized volatilities requires care because of the institutional trading features present in high-frequency data.

the day. Once aggregated, the data can be combined to construct estimates of the daily conditional covariance matrix. By using this approach, the variances and covariances of a number of assets can be treated as being observed. As a result, more accurate estimates of the factors driving the conditional covariance matrix can be found.

Bauer and Vorkink apply their new approach to the cross-section of size-sorted U.S. stock portfolios. While earlier papers have examined asset-price volatility in the cross-section of small and large firms,[5] they used existing models of latent volatility to capture the variation in the covariances. In contrast, Bauer and Vorkink use high-frequency data to construct daily measures of the realized covariance matrix of small and large firm return indexes over the 1988 to 2002 period. Their measures of volatility are more precise than those in previous work and allow for a more detailed examination of the causes of conditional covariances.

Once the matrix of realized variances and covariances has been constructed, a new factor model is used to capture its dynamics.[6] The factors are functions of past volatilities and other variables that can help forecast future volatility. A number of possible sets of variables from the academic finance literature are then examined to see how well they forecast the covariance matrix. The authors note that while researchers have examined different variables for their ability to forecast stock market returns, there is much less evidence that the variables forecast stock market volatility.[7]

Results

Bauer and Vorkink evaluate their model of the daily conditional covariance matrix in two ways. First, they use a set of standard statistical tests and find that, in general, the factor model performs well in describing how the volatility matrix changes each day. Surprisingly, however, there does not appear to be a lot of difference between the alternative forecasting variables used to construct the factors: one set of variables appears to forecast the covariance matrix just as well as another set. This is because a single dominant factor drives the volatilities of all of the different-sized stocks: if the overall market is volatile, then the prices of all stocks on that day are volatile. As long as the forecasting variables are able to capture the dynamics of aggregate market volatility, they will also capture the dynamics of the size-sorted stocks.

The second and more informative method of evaluating the model is to see how well it constructs optimal stock market portfolios. In particular, the authors examine how the model can be used to construct a daily “tracking-error” portfolio.[8] The covariance matrix of the size-sorted stocks is modelled, and the indexes are used to track the portfolio of “value” stocks (i.e., those with high book-to-market ratios). Including variables that forecast stock returns (such as dividend yields) along with lagged volatility factors is found to produce portfolios with superior tracking performance. In other words, variables that forecast returns also forecast risks (i.e., volatility) in the market.

The authors hope to use this method to explore the time-varying relationship among other asset markets and to determine how well alternative variables are able to forecast large movements in market prices. The model can also be used to examine the covariances among international assets with a view to better understanding the transmission of shocks from one country to another, especially during times of market stress.

5. See Conrad, Gultekin, and Kaul (1991); Kroner and Ng (1998); Chan, Karceski, and Lakonishok (1999); and Moskowitz (2003).
6. In the factor model, the variances and covariances of a large number of assets are explained by a small number of variables.
7. For example, there is evidence that a stock market’s dividend yield (the dividend-to-price ratio of the index) may help predict the average return on the index, but whether it predicts the volatility of returns (from holding the index) is unknown.
8. A tracking-error portfolio is one in which the portfolio manager uses a small set of assets to “track” or closely follow the performance of the target portfolio. The idea is to minimize the difference between the returns on the tracking and target portfolios. For example, fund managers may combine a number of stocks and derivative products to match the performance of a broad equity market index, such as the TSX composite index. The manager may thus trade in only a few assets to follow the returns on many stocks, which would greatly reduce transactions costs. Because the tracking-error portfolio test is based on the difference between the volatilities on the tracking and target portfolios, it is less influenced by moves in aggregate market volatility that affect both portfolios.

References

Andersen, T.G. and T. Bollerslev. 1998. "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts." *International Economic Review* 39: 885–905.

Andersen, T.G., T. Bollerslev, P.F. Christoffersen, and F.X. Diebold. 2005. "Volatility Forecasting." National Bureau of Economic Research Working Paper No.11188.

Andersen, T.G., T. Bollerslev, F.X. Diebold, and H. Ebens. 2001. "The Distribution of Realized Stock Return Volatility."*Journal of Financial Economics* 61: 43–76.

Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys. 2001. "The Distribution of Realized Exchange Rate Volatility." *Journal of the American Statistical Association* 96: 42–55.

———. 2003. "Modeling and Forecasting Realized Volatility." *Econometrica* 71: 579–625.

Bauer, G.H. and K. Vorkink. 2006. "Multivariate Realized Stock Market Volatility." Bank of Canada. Photocopy.

Chan, L.K.C., J. Karceski, and J. Lakonishok. 1999. "On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model." *The Review of Financial Studies* 12: 937–74.

Connolly, R.A. and F.A. Wang. 2003. "International Equity Market Co-Movements: Economic Fundamentals or Contagion?" *Pacific-Basin Finance Journal* 11: 23–43.

Conrad, J., M.N. Gultekin, and G. Kaul. 1991. "Asymmetric Predictability of Conditional Variances." *The Review of Financial Studies* 4: 597–622.

Kroner, K.F. and V.K. Ng. 1998. "Modeling Asymmetric Comovements of Asset Returns." *The Review of Financial Studies* 11: 817–44.

Moskowitz, T.J. 2003. "An Analysis of Covariance Risk and Pricing Anomalies." *The Review of Financial Studies* 16: 417–57.

Ribeiro, R. and P. Veronesi. 2002. "The Excess Comovement of International Stock Markets in Bad Times: A Rational Expectations Equilibrium Model." Graduate School of Business, University of Chicago Working Paper.