Optimum Structure of a Central Counterparty*

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Abstract

A CCP novates all trade by its members, in effect standing as one side of all trades. Therefore, it must design membership requirements and collateralization of risk exposures to meet potential obligations that arise from an eventual default of one of its member. These constraints reduce systemic risk (i.e., CCP default risk) but they also increase the market power and revenues of its member. We model the strategic behavior of dealers that intermediate between hedgers in the presence of a CCP. We find that monopolistic incentives do not align with risk considerations, that dealers prefer higher level of systemic risk associated with higher level of revenues, and that regulation of CCP structure is justified. We find that Hedgers are typically better off when competition is more intense but they may trade-off between lesser competition and higher systemic risk in some cases.

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Introduction

Clearing houses and Central Counter-Parties (CCPs) have for long played a key role in securities markets, especially those exchange-based financial markets (Kroszner 1999). A CCP stands between the two parties in a trade and guarantees that each party fulfill their obligations. In practice, the CCP novates all trades and becomes the buyer to every seller and the seller to every buyer in a given market. In so doing, it reduces counterparty credit risk (counterparty risk, hereafter) and vastly simplifies the security market structures. Nonetheless, the use of CCPs in over-the-counter markets has been much more limited; until recently that is. The crisis of 2007-2009 revealed a complex network of interconnections plagued with uncertainty as every financial institutions not only struggled to assess its exposure to direct counterparties (see Allen and Babus 2008 for a review) but, also, its counterparties' exposures to others (Caballero and Simsek, 2009). In response, regulators have been since pursuing the increased use of CCPs to enhance the resilience of OTC markets.[1] The need for a CCP can also be justified theoretically (Acharya and Bisin 2009; Koeppl and Monnet 2010). In any case, we take the existence of a CCP as given.

A CCP concentrates counterparty risk. Consequently, the CCP collects initial and variation margins on cleared contracts, obtains default fund contributions from members, and imposes position limits on cleared portfolios. A CCP typically imposes stringent membership requirements as well. This is illustrated by the following quote:

> Bank of New York Mellon has been trying to become a so-called clearing member since early this year. But three of the four main clearinghouses told the bank that its derivatives operation has too little capital, and thus potentially poses too much risk to the overall market.
>
> *A Secretive Banking Elite Rules Trading in Derivatives*, New York Times, 2010.

For example, LCH.Clearnet's SwapClear facility for interest rate swaps requires that members have \$5 billion in equity capital and a \$1 trillion derivatives book.[2] The

[1] The leaders of the G-20 group of nations stated in September 2009 that all standardized OTC derivatives contracts should be cleared through CCPs. Norman (2010) discusses how CCPs effectively managed counterparty risk in some markets during the crisis. Duffie et al. (2010) discuss the policy response to problems in the OTC derivatives market during the crisis.

[2] Membership requirements for other major OTC derivatives CCPs such as ICE and CME Clearing are similar. The US CFTC has recently proposed rules that would cap minimum capital requirements at \$50 million.

result is that few dealers are eligible to clear directly at SwapClear. Incumbent CCP members claim that high membership requirements are essential to control risk. Unsurprisingly, not all market participants agree. Many argue that incumbents influence CCPs access and risk management policies to maintain and capture the economic rent associated with clearing services. In particular, Pirrong (2011) argue that "it cannot be ruled out that CCPs will utilize membership requirements for strategic, competitive purposes." Moreover, these restrictions may not address the most salient sources of risk. In particular, Perignon and Jones (2009) argue that proprietary trading on the part of members impose substantial systemic risk on the CME clearinghouse. Still, the empirical literature remains thin, in large part due limited access to data.

This paper is the first to a analyze the interactions between the structure of a CCP (e.g, number of firms, capital requirements) and its members' market power and profit, the equilibrium trading strategies, market prices and quantities, and the CCP residual default risk. We consider a trading environment where continuum of risk-averse Hedgers meet with a small number of Dealers to trade a financial contract and reduce their exposure. Dealers intermediate between long and short hedgers but a CCP novates all their trades. The CCP controls membership, levies default fund contributions and imposes restrictions on the amount of risk each dealer can carry. In returns, the CCP covers losses in the event of a Dealer default. The CCP may default if its resources are insufficient and, in this case, some Hedgers incur losses.[3]

We first analyze the costs and benefits from restricting membership. We find that the relationship between competition and CCP default risk is not monotonous. Starting from the case of a monopoly, competition initially decreases Dealers revenues and increases the CCP default risk. However, as additional Dealers become clearing members, leverage decreases and the CCP default risk decreases, even as revenues continue to decline. Note that Dealers unambiguously prefer a lower level of competition, irrespective of its effect on risk. On the other hand, Hedgers unambiguously prefer a higher level of competition. However, the preference of Hedgers differs if the marginal dealers have a decreasing level of capitalization. In other words, when the size of the market is large relative to the availability of capital in the economy. In this case, the Hedgers may prefer an intermediate level of competition where monopoly

[3]On the other hand, we assume that Hedgers do not account for the possibility that the CCP may default. For example, the CCP may be thought as too big to fail and government intervention may be anticipated.

power implies lower default probability.

The following two important normative implications follow. Limiting competition may benefit Hedgers only if this decreases the default probability of existing Dealers and, incidentally, that of the CCP. This arises naturally in our model since Dealers' only possible trade is trade with Hedgers and, therefore, they cannot leverage the extra resources from the increased monopoly power to take more risk elsewhere. The other implications is that competition is unambiguously good for Hedgers if the marginal Dealers are subject to a sufficient minimum capital requirement. In other words, decreasing the average level of leverage among clearing members eventually decreases CCP default risk.

Second, we analyze the effect of varying the quantity of risk each Dealer can carry. Dealers are risk-neutral but have limited liability. In particular, their utility function is not linear and the choice of price and quantity in the long market interacts with the choice in the short market. When the individual dealer default probablity constraints are not binding, the effect of limited liability does not change how competition affects prices and CCP default risk. This is not true when the default probability constraint binds, however. In this case, limited liability introduces a wedge between the prices in the long and short markets for any level of competition. An important normative implication is that Hedgers would prefer to decrease CCP risk via a higher number of well-capitalized Dealers rather than via more restrictive individual risk controls.

Access and clearing configuration will have an increasing impact on market outcomes as CCPs become mandatory in many jurisdictions. The literature on the emergence of CCPs include Koeppl and Monnet (2010), who study the benefits of novation and mutualization of losses, and Acharya and Bisin (2009), who study the case where dealers cannot credibly commit to maintain their current level of risk in future transaction (with other counterparties). There is a nascent literature discussing the effects of different CCP configurations but none consider the effects on the competitive environment. Pirrong (2002) and Pirrong (2011) discuss market power in CCPs. Haene and Sturm (2009) model the optimal balance between default fund and margin contributions in capitalizing a CCP. They discuss how this choice will affect the incentives of CCP users. Rausser et al. (2010) suggest that a CCP is not capable of internalizing all the benefits it creates and therefore should be run as a public-private partnership. Jackson and Manning (2007) use simulations to examine alternative CCP configurations and argue that tiered membership may be optimal. Renault (2010) models the optimal number of CCPs for a market. Duffie and Zhu (2011) considers the effect of inefficient netting by CCPs on the counterparty risk of members. Finally, this paper is related to an important literature assessing the interaction between competition and stability in the banking industry, where competition sometimes reduces stability (Keeley 1990) but sometimes promotes stability (Boyd and De Nicolo 2005).

1 Model

The economy consists of a continuum of Hedgers and a small number of Dealers. Hedgers cannot trade with each others but Dealers can intermediate between types of hedgers. However, Dealers must post collateral at a CCP that novates all of their trades. The CCP controls membership, default fund contribution and collateralization rules which apply to its members. The following section details the problem of each agent in turn.

1.1 Hedgers

There is a continuum of risk-averse Hedgers of two different types that consume from a random endowment, $x^i(\omega)$, where $\omega \in \Omega$ is a random variable. Hedgers are either "long" or "short", in a sense that we make clear below. However, Hedgers cannot trade directly with each other to smooth future consumption. Dealers, trade with contingent contract with payoff $f(\omega)$ with long and short hedgers.

Long and short hedgers can be interpreted as investors in money centers and borrowers in rural areas, farmers and bakers, agents in two different industries with opposite correlation with an aggregate shock, or investors in different countries. The inability to trades may arise because of spatial separation, because hedgers are unable to credibly commit or because they cannot access the required trading technology. Before trading with Dealer j, a Hedger must incur a fixed costs c_j depending on its relationship with that dealer. In effect, Dealers offer differentiated services and we use the circle model of price differentiation from Salop (1979) to represent the relationships between Dealers and Hedgers. A mass L of long Hedgers form a circle where J dealers are located at equal distance around the circle. A mass S of short Hedgers form a separate circle served by J other dealers. We label these circles the long and short city, respectively. Figure 1(a) illustrates the circle model and Figure 1(b) summarizes the environment. For any given Hedger, the distance on the circle between him and Dealer j, given by d_j, represents how remote is their relationship and the transaction costs he face to trade with that dealer is proportional to the distance, $c_j = td_j$. This representation nests a competitive structure à la Bertrand when $t = 0$.

Hedgers in the long city have that $cov(x_L, f) > 0$ and could benefit from selling one contract. Hedgers in the short city have that $cov(x_S, f) < 0$ and could benefit from buying one contract. Hedgers can buy or sell one contract, or do nothing. Dealers post a price p_j in each city to buy or sell a contract with payoff f. For simplicity, we take $E[f] = 0$. For short hedgers, and assuming mean-variance utility, trading with dealer j yields,

$$\begin{aligned} EU_j &= EU(x_S + 1 \cdot (f - p_j) - d \cdot t) \\ &= E(x_S) - p_j - d \cdot t - (r/2) \cdot [var(x_S) + var(f) + 2cov(x_S, f)] \end{aligned} \tag{1}$$

where r is the risk aversion parameter. This Hedger will trade only if this makes him better off. That is,

$$\begin{gathered} EU_j \geq E(x_S) - (r/2)var(x_S) \\ \Leftrightarrow \\ p_j + c_j \cdot t + (r/2)var(f) \leq -r \cdot cov(x_S, f), \end{gathered} \tag{2}$$

where $cov(x_S, f) < 0$ for short Hedgers by definition. Therefore, a short hedger is willing to pay a premium (i.e., $p > E[f] = 0$) to buy the contract if the payoff, f, is sufficiently negatively correlated with its endowment and if transaction costs, c_j, are not too high. We restrict our attention to this case where the outside options is dominated by trade with (at least) one dealer. In other words, the market is covered in each city. Next, consider dealer $j - 1$ located contiguously to dealer j and at a distance $\frac{S}{n} - d$ from the hedger. He posts a price p_{j-1} and trading with him yields,

$$EU_{j-1} = E(x_S) - p_{j-1} - \left(\frac{S}{n} - d\right) \cdot t - (r/2) \cdot [var(x_S) + var(f) + 2cov(x_S, f)]\,. \tag{3}$$

The hedger will prefer to buy from dealer j if

$$p_j \leq p_{j-1} + \left(\frac{S}{n} - 2d\right) t, \tag{4}$$

and, similarly on the other side, he will prefer to buy from dealer j instead of dealer $j+1$ if

$$p_j \leq p_{j+1} + \left(\frac{S}{n} - 2d\right) t. \tag{5}$$

As in Salop (1979), we will consider symmetric equilibrium where all firms charge the same price. First consider the case where all firms but firm j charge the same price, $\overline{p}$. In that case, standard results show that the demand faced by dealer j is

$$D_S(p_n, \overline{p}) = \frac{S}{n} + \frac{-p_j + \overline{p}}{t}$$

in the short city. Then, in the symmetric equilibrium were everybody sets the same price, we have that $D_S(\overline{p}) = S/n$. In an symmetric equilibrium, every dealer is trading with hedgers located on the half-line between him and contiguous dealers.

For Dealers trading with long Hedgers, we have that

$$D_L(p_n, \overline{p}) = -\frac{L}{n} + \frac{p_j - \overline{p}}{t}$$

and $D_L(\overline{p}) = -L/n$. Long hedgers are willing to receive less than the zero expected value of the payoff and, therefore, they sell at a discount. This generate positive expected profit to the Dealer.

The total utility of hedgers in the short city is

$$\frac{1}{S}\left(E(x_S) - \overline{p}^S - \frac{1}{4n} \cdot t - (r/2) \cdot [var(x_S) + var(f) + 2cov(x_S, f)]\right).$$

and, for the long city,

$$\frac{1}{L}\left(E(x_L) + \overline{p}^L - \frac{1}{4n} \cdot t - (r/2) \cdot [var(x_L) + var(f) - 2cov(x_L, f)]\right)$$

Note that total utility increases in each city when transaction costs are lower or when the number of dealers increases.

1.2 Dealers

Dealers in each city trade y^S and y^L contracts at price at price p^S and p^L in the short city and the long city, respectively. Dealers are risk-neutral, they have limited liability and they are endowed with K units of capital. They maximize,

$$EU(p^L, p^S, y^L, y^S) \equiv E\left[\max(y^L(f-p^L) + y^S(f-p^S), -K)\right], \tag{6}$$

where the *max* operator arises due to limited liability. The distribution of the dealer's wealth is censored from below at $-K$. We take as given the presence of a CCP that constrains each dealer positions to control the probability of a default. We discuss the role of the CCP in the next section. Formally, dealers choice are subject to the the following constraint,

$$PD(p^L, p^S, y^L, y^S) \equiv \Pr\left(y^L(f-p^L) + y^S(f-p^S) + K < 0\right) \le \alpha \tag{7}$$

where PD is the Probability of Dealer defaulting and α is controlled by the CCP.

Consider the case where f follows a standard normal distribution, then the distribution of wealth corresponds to a Normal distribution with mean $K - (y^L p^L + y^S p^S)$ and variance $(y^L + y^S)^2\sigma^2$ but censored at zero. Then, ignoring the effect of other dealers, the probability PD decreases with the revenues $(y^L p^L + y^S p^S)$ and capital K but increases with the risk $(\sigma(y^L + y^S))^2$. In particular, given that quantity traded in each city is S/n and $-L/n$, entry by other dealers affects the probability of default via lower revenue (the mean) and via lower risk (the variance). The net effect is ambiguous.

1.3 The Central Counterparty

Acharya and Bisin (2009) emphasize the role of a Central Counterparty [CCP] to eliminate the counterparty risk externality that arise if Hedgers cannot assess the position of a Dealers. In other words, each dealers has an incentive to take too much risk in the presence of limited liability. The CCP novates all contracts by Dealers and collects collateral to control the probability that Dealers will default on their obligations.[4] The CCP also has access to a default fund, F, to settle its obligation

[4] We do not consider explicitly the possibility that Hedgers may default. We assume the costs of hedgers default is reflected in the transactions costs c_j.

when a dealer defaults. Nevertheless, the default fund may be insufficient to cover all the CCP's obligation. With n dealers, the probability of a CCP default is given by,

$$PCCP \equiv \Pr\left(F + \sum_{i=1}^{n} \min\left(0, y_i^L(f - p_i^L) + y_i^S(f - p_i^S) + K\right) < 0\right), \tag{8}$$

where the CCP loss is a random variable, dependent on the realization of f, created by the sum of n censored distributions.

2 Equilibrium

2.1 Symmetric Equilibrium

We consider a symmetric Equilibrium in each Hedgers' market. Dealers in the long market charge the same price and the market clears (i.e. contract are in zero net supply). The same hold in the short market but possibly with a different price. Each dealers take into account the demand of Hedgers and the price set by all other dealers when setting its own price. The market-clearing conditions are then given by:

$$\begin{aligned} -y^{S*} &= D_S(p_j^{S*}, \overline{p}) = \frac{S}{n} - \frac{p_j^{S*} + \overline{p}}{t} \\ -y^{L*} &= D_L(p_j^{L*}, \overline{p}) = -\frac{L}{n} - \frac{p_j^{L*} + \overline{p}}{t}, \end{aligned} \tag{9}$$

where $D_L(p_j^{L*}, \overline{p})$ and $D_S(p_j^{S*}, \overline{p})$ are the demand functions that solve the hedgers problem in each city, respectively, and, finally, the Kuhn-Tucker first-order conditions for constrained optimization from one dealer are given by:

$$\left(\frac{\partial EU}{\partial p^L} - \frac{\partial EU}{\partial y^L}\frac{\partial D_L(p^{L*})}{\partial p^L}\right) = \lambda\left(\frac{\partial PD_L^*}{\partial p^L} - \frac{\partial PD_L^*}{\partial y^L}\frac{\partial D_L(p^{L*})}{\partial p^L}\right) \tag{10}$$

$$\left(\frac{\partial EU}{\partial p^S} - \frac{\partial EU}{\partial y^S}\frac{\partial D_S(p^{S*})}{\partial p^S}\right) = \lambda\left(\frac{\partial PD_S^*}{\partial p^S} - \frac{\partial PD_S^*}{\partial y^S}\frac{\partial D_S(p^{S*})}{\partial p^S}\right) \tag{11}$$

$$\lambda(PD - \alpha) = 0, \tag{12}$$

subject to feasibility and complementary slackness conditions, and where each of the partial derivatives are available in closed-form (See Appendix A.1).

2.2 Balanced Symmetric Equilibrium

In general, the solution for equilibrium price is not available in closed-form. However, the case where each city has the same size provides a useful benchmark. The properties of this Balanced Symmetric Equilibrium are given in Proposition 1. The proposition shows that in a balanced equilibrium, where there is an equal mass of long hedgers and of short hedgers, the dealers offset the demand of short hedgers with the supply of long hedgers. Then, the default probability constraint is not binding and the equilibrium reduces to the standard Salop case. Importantly, the CCP is irrelevant.

Proposition 1 Balanced Symmetric Equilibrium

When $L = S$, i.e., long hedgers and short hedger have equal mass, then the symmetric equilibrium has the following property,

1. *The equilibrium prices identical in the long hedgers' market and the short hedgers' market, i.e., $p^L = p^S$.*
2. *Dealers have zero net position, i.e., $y^L = y^S$.*
3. *Probability of a dealer's default is zero. The probability of a CCP default is zero.*

2.3 CCP Default Probability

All Dealers are identical in a symmetric equilibrium and they are perfectly correlated in default. They either all default, or none of them default. As a consequence, the default probability of the CCP simplifies to the default probability of every dealer if the default fund is zero,

$$
\begin{aligned}
&\Pr\left(\sum_{i=1}^{n} \min\left(0, y_i^{L*}(f - p^{L*}) + y_i^{S*}(f - p^{S*}) + K\right) < 0\right) \\
&= \Pr\left(n \cdot \min\left(0, \frac{-L}{n}(f - p^{L*}) + \frac{S}{n}(f - p^{S*}) + K\right) < 0\right) \\
&= \Pr\left(\min\left(0, \frac{-L}{n}(f - p^{L*}) + \frac{S}{n}(f - p^{S*}) + K\right) < 0\right) \\
&= \Pr\left(\frac{-L}{n}(f - p^{L*}) + \frac{S}{n}(f - p^{S*}) + K < 0\right) \leq \alpha,
\end{aligned}
$$

and, therefore, the default probability of the CCP is less than or equal to α. It will be strictly less than α if the CCP can access a default fund to cover losses arising from a dealer's default.

3 Competition and CCP Default Probability

We use numerical methods to find prices and quantities that solve the conditions for a symmetric equilibrium (see Section 2). Table 1 gives the parameters values that we use. In particular, we consider a case where the Long city is twice as large as the Short city. These are the economically important cases since the CCP has no risk otherwise (see Proposition 1). Moreover, we first consider the case where the default fund is zero.

Table 1: **Benchmark Parameter Values**

S	L	α	F	t	K	σ
1.0	2.0	0.5%	0	35	0.03	1.5

3.1 Competition, price, and Dealers' net positions

Figure 2 shows the equilibrium net positions of dealers starting with the case with $n = 2$ clearing members. For any value of n, the sum of dealer net positions is the difference between each city's size. This follows directly from our assumption that no hedger chooses the no-trade outside option. Then, net position per dealers decreases with $1/n$. Next, Figure 3 shows that the equilibrium discount offered by long hedgers when they sell to the dealers is larger than the premium paid by short hedgers when they buy from the dealers. (Recall that the contract has zero expected payoff) In other words, the dealers enjoy a stronger monopolistic position in the bigger city. Nonetheless the dealers revenues from these discount and premium decrease with the number of clearing member, due to higher competition.

3.2 CCP default probability

Therefore, higher competition decreases net position of dealers (i.e., risk) and decreases revenues. The interaction between lower net positions, lower revenues and dealers default probability implies that the effect of higher competition on CCP default probability is ambiguous. Figure 4 shows the CCP default probability as the competition increases. It is initially close to zero for low values of n. This reflects the high degree of differentiation between dealers ($t = 35$) and, therefore, the high level of revenues. The probability of default initially increases with competition as the premium, or discount, charged to Hedgers decreases quickly. However, the probability of default decreases for large values of n, and eventually reaches zero. This reflects the continuous decline of net positions relative to the fixed level of capital per Dealer.

3.3 The relative effect of differentiation and capital

To see the effect of decreasing revenues, Figure 5(a) displays the CCP default probability as t goes to zero. This shows that as differentiation disappears the probability of default increases. This is due to lower revenues but effect is not symmetric. Decreasing the number of dealers, and increasing their monopoly power, is not always sufficient to eventually generate enough revenue and lowers the default probability. Indeed, for low level of differentiation, the default probability is always binding as we decrease n.

On the other hand, adding more dealers eventually leads to decreasing default probability even for low level of differentiation. This arises because capital per dealer is kept constant. To see the effect of decreasing level of capital per dealer, we consider a case where the capital of marginal dealers is not constant but decreases with n. This may arise, for example, if the size of the market is large relative to the rest of the economy and where capital is scarce. Figure 5(b) displays the CCP default probability in an extreme case where K decreases close to zero for low values of n. Then, as competition increases, and revenue decreases, the default probability eventually binds.

3.4 Objective Functions

Figure 6(a) shows the expected utility of a Dealer multiplied by the probability of no CCP default. It increases when the number of clearing member decreases and competition increases. This is true for any positive level of differentiation although the benefit of a monopolistic position may be small when the level of differentiation is small. Dealers always prefer less competition. On the other hand, the expected utility of Hedgers, conditional on the CCP not defaulting on its obligations, increases monotonically with competition. This follows directly from decreasing monopoly rent (Figure 3), due to higher competition, and from lower CCP default probability (Figure 5(a)), due to higher total capital of members. This results hinges crucially, however, on keeping constant the marginal dealer capital level.

Figure 5(b) makes clear that the probability of default remains constant as we increase competition when the marginal level of capital is relatively low, or decreasing. Then, the expected utility of Hedgers, conditional on the CCP not defaulting on its obligations, does not vary monotonically with competition. Hedgers may prefer an intermediate level of competition where the monopoly power of Dealer is somewhat higher but the CCP default probability is lower. Importantly, this arises in circumstances where the default probability is not binding. Figure 6(a) shows the expected utility of Hedgers, when capital is scarce, and for different level of differentiation. The corresponding CCP default probabilities are given in Figure 5(b). We see that in all case where Dealers are unconstrained, then Hedgers utility is hump-shaped and possesses a local optima when the number of clearing members is relatively low. A similar competitive optimum may be available only for very large values of n, especially if differentiation is strong.

The following two important normative implications follow. First, limiting competition can benefit Hedgers only if this decreases the default probability of existing Dealers. This arises naturally in this model since Dealers only possible trade is to match the two cities and, therefore, they cannot leverage the extra resources from this trade to take risk elsewhere. It is unlikely to arise if real-world Dealers are left unchecked since they have many alternate use of capital. Second, if the marginal Dealers are subject to a sufficient minimum capital requirement, then increasing competition is unambiguously good. Decreasing the average level of leverage among clearing members eventually decreases CCP default risk.

3.5 Constrained prices

The equilibrium prices are qualitatively different when the default probability constraint of Dealers is binding. When the constraint is not binding, the Dealers buys at a discount from the long Hedgers and sell at a premium to the short Hedgers. They keep a net position on their balance sheet. As the differentiation level or the capital level per Dealer decreases, and the constraint becomes binding, the shadow price of the constraint becomes non-negative and this alter the concavity of dealers objective function (See Equation 10).

The Dealers then choose to increase the monopoly rent they extract from the (larger) long city to satisfy the constraint. Then, controlling the default probability counteract the effect of competition in the long Hedgers market when the marginal Dealer is insufficiently capitalized. The effect increases as we make the default probability safer. On the other hand, the Dealers decreases the rent they extract from the short city. Hence the net effects on Hedgers utility ambiguous.

Limited liability makes the utility function of dealers non-separable with respect to prices in each city. Risk-neutral dealers would maximize with respect to price and quantity in each city independently since the revenue from each city would enter linearly the objective function. This non-separability does not alter the direction of the effect of varying competition or differentiation on prices and quantities when the constraint is not binding. This does not hold when the constraint is binding and significant distortions may arise.

An important normative implication from this observation is that varying the level of default probability is not a perfect substitute to increasing capital per Dealers. Hedgers would prefer to reduce the CCP default via a greater number of well-capitalized Dealers rather than via more restrictive default probability constraints. In contrast, Dealers would prefer to reduce risk via tighter default probability constraint.

4 Conclusion

We consider a trading environment where continuum of risk-averse Hedgers meet with a small number of Dealers to trade a financial contract and reduce their exposure. Dealers intermediate between long and short hedgers but a CCP novates all their trades. We first analyze the costs and benefits from restricting membership. We find conditions for which competition increases the CCP default probability and conditions for which competition decreases the CCP default probability. Importantly, the compositions of the pool of marginal clearing member is a key variable driving the normative implications from the model. We also analyze the effect of imposing tighter risk controls on the clearing members. Work in progress considers several extensions. Does a wider membership induce diversification effect and add to the increased competition in lowering CCP default risk? Can the Dealers or the Hedgers internalize costs from a CCP default? Does relaxing the assumption of a covered market, and allowing dealers to vary the quantities as well as the price in each city, affect the results?

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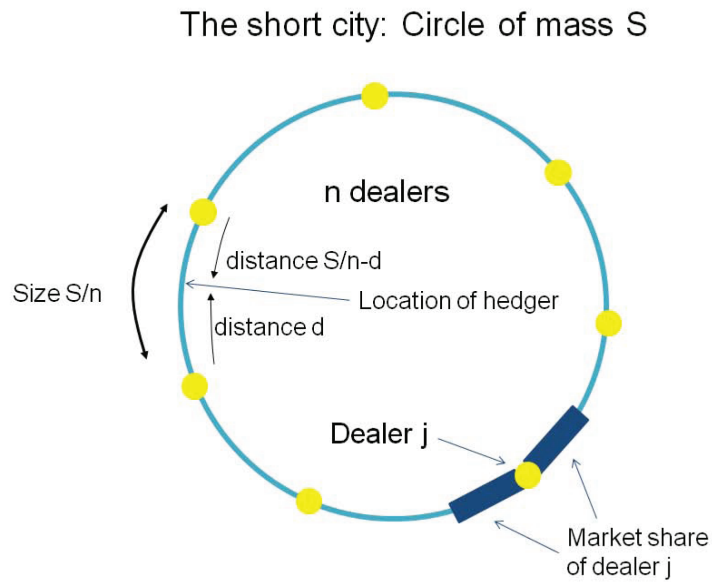
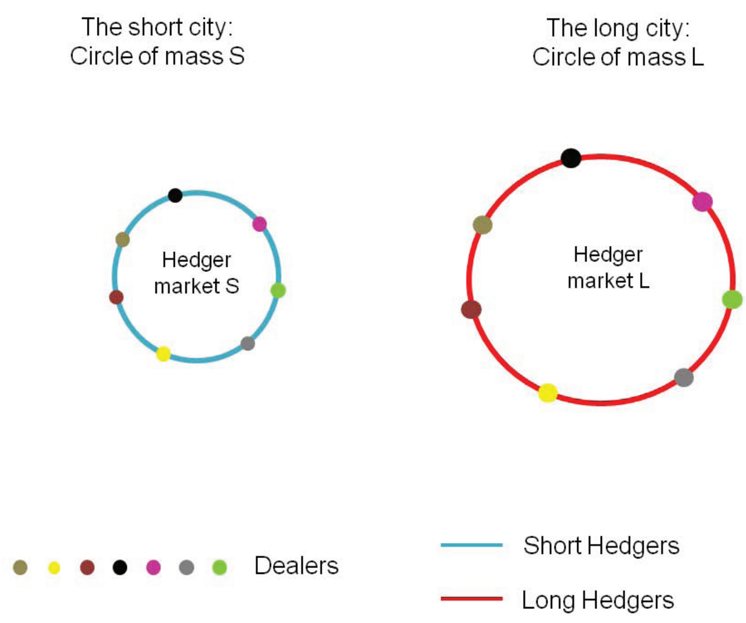


(a) Circle Model



(b) Hedgers and Dealers

Figure 1: **Environment**

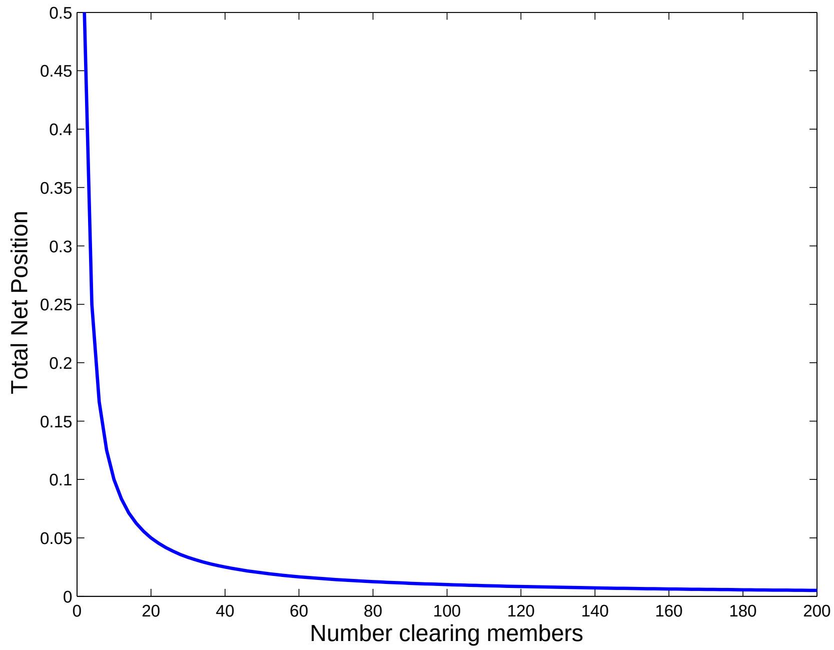
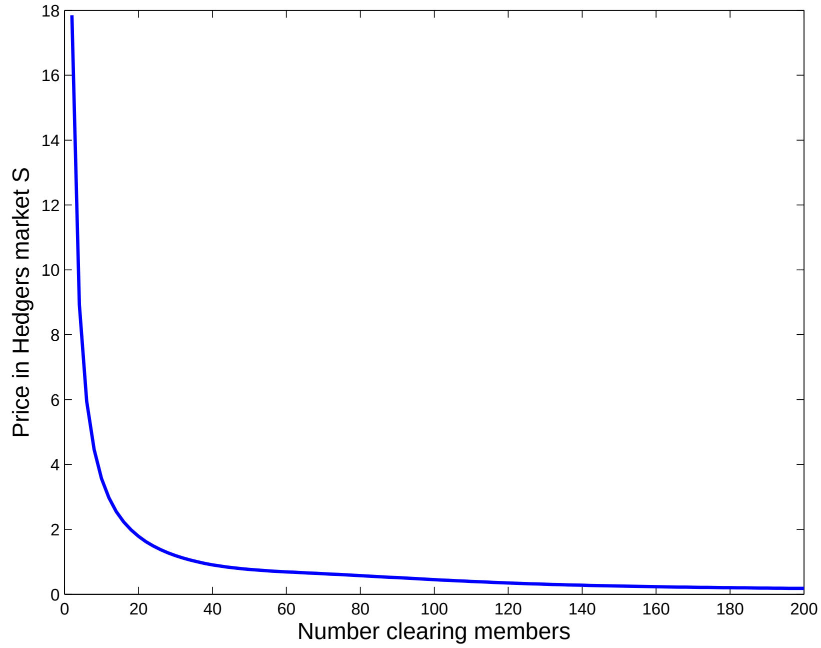
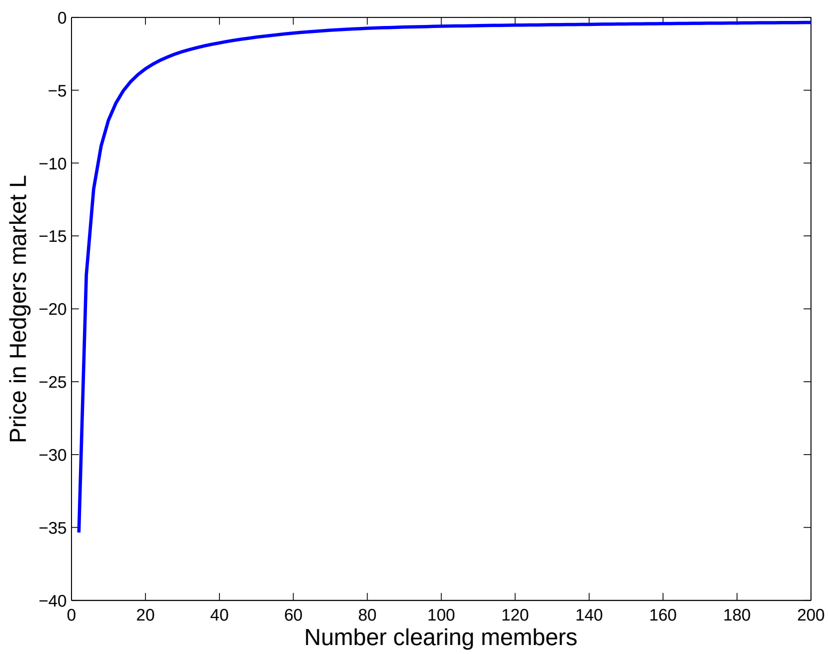


Figure 2: **Net Position of Dealers**

Equilibrium default probability of the CCP for different number, n, of clearing members. Parameter values are given in Table 1.



(a) Short city



(b) Long city

Figure 3: **Prices in the long city and in the short city**
Equilibrium price in each city for different number, n, of clearing members. Parameter values are given in Table 1.

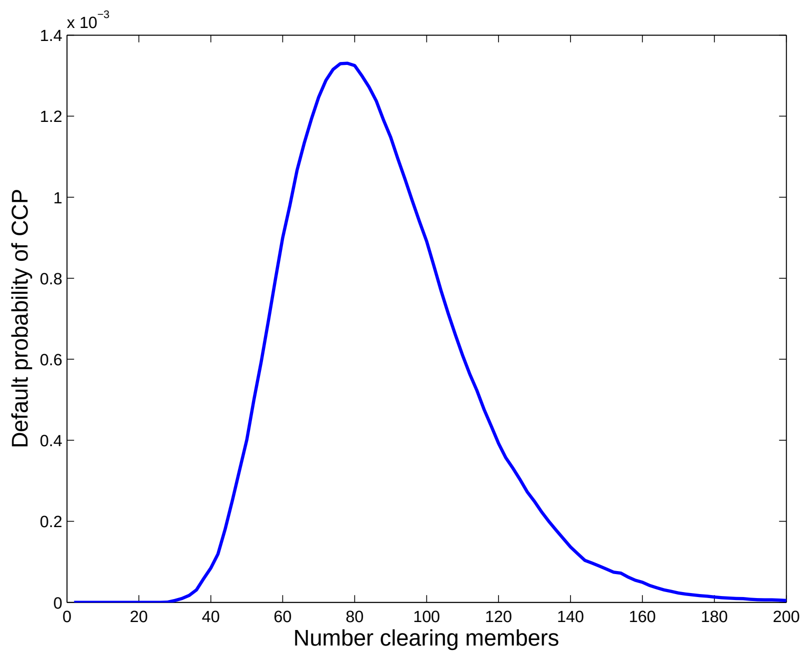
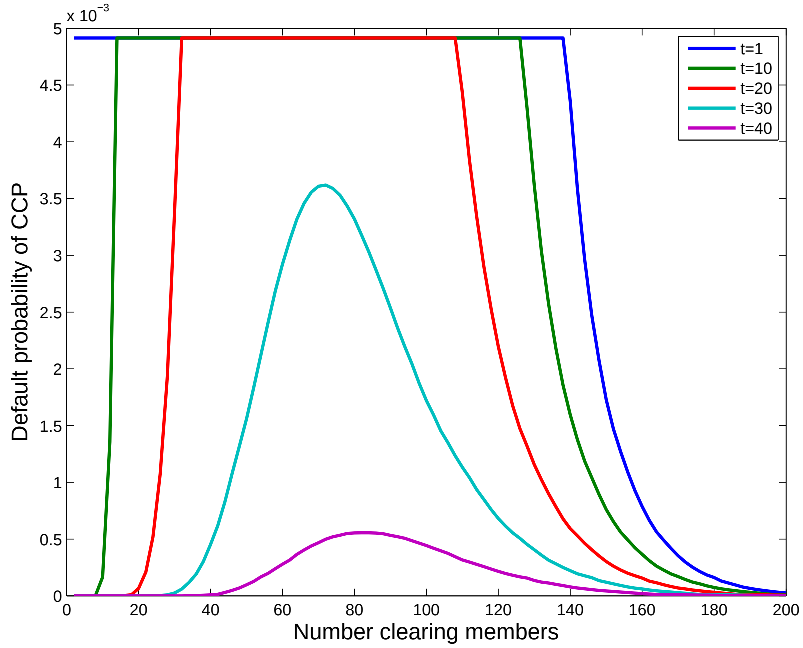
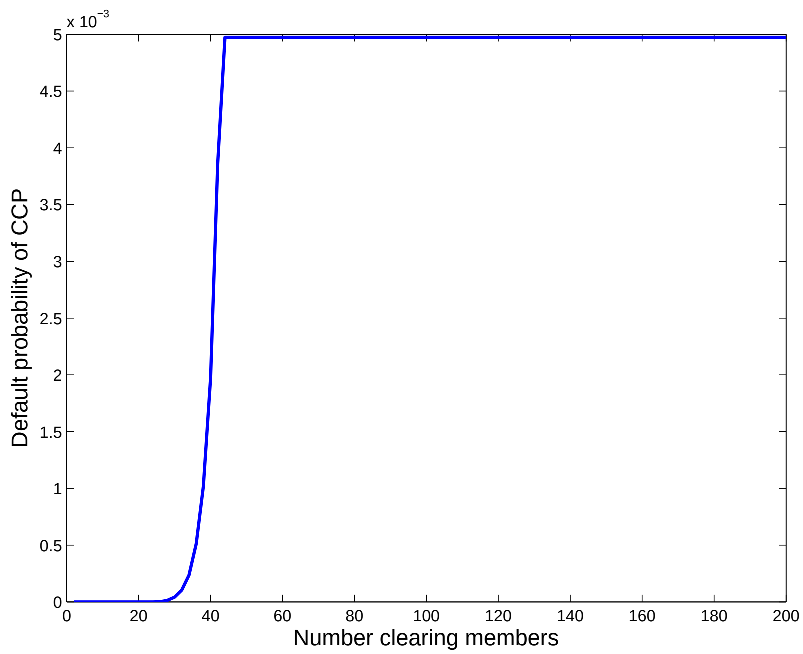


Figure 4: **CCP Default Probability**

Equilibrium default probability of the CCP for different number, n, of clearing members. Parameter values are given in Table 1.

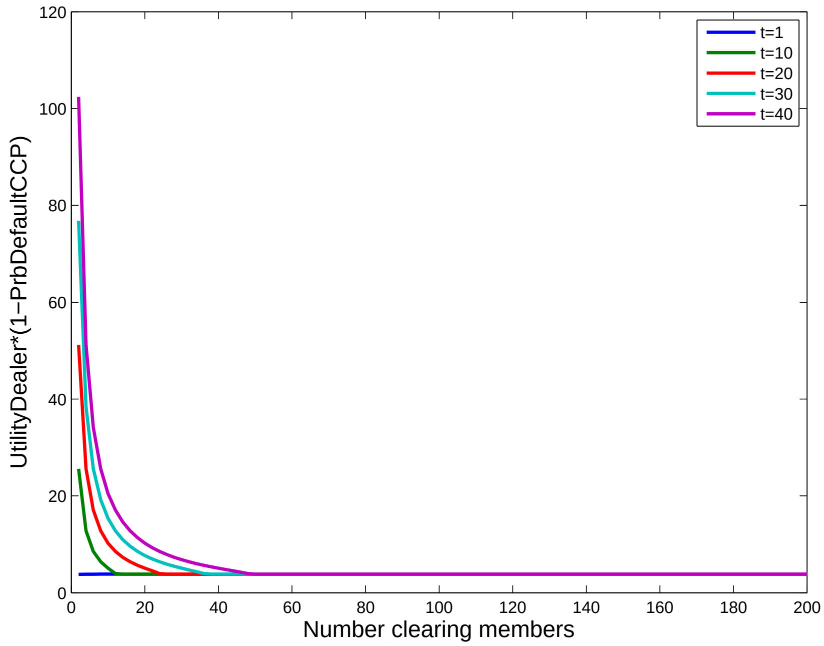


(a) Short city

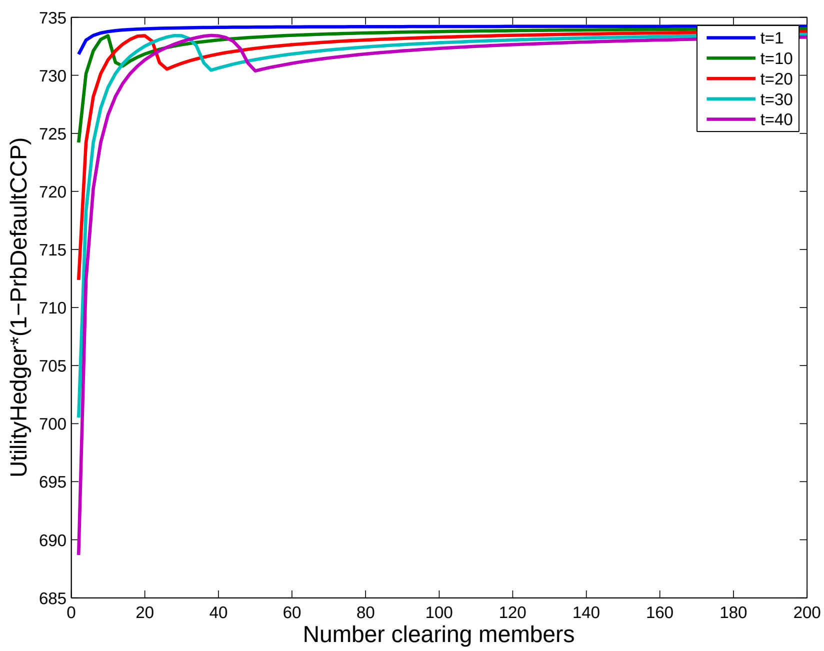


(b) Long city

Figure 5: **CCP Default Probability - Alternate calibration**
Equilibrium default probability of the CCP for different number, n, of clearing members. Parameter values are given in Table 1. Panel 5(a) varies the differentiation level, t. Panel 5(b) varies the level of capital for dealers.



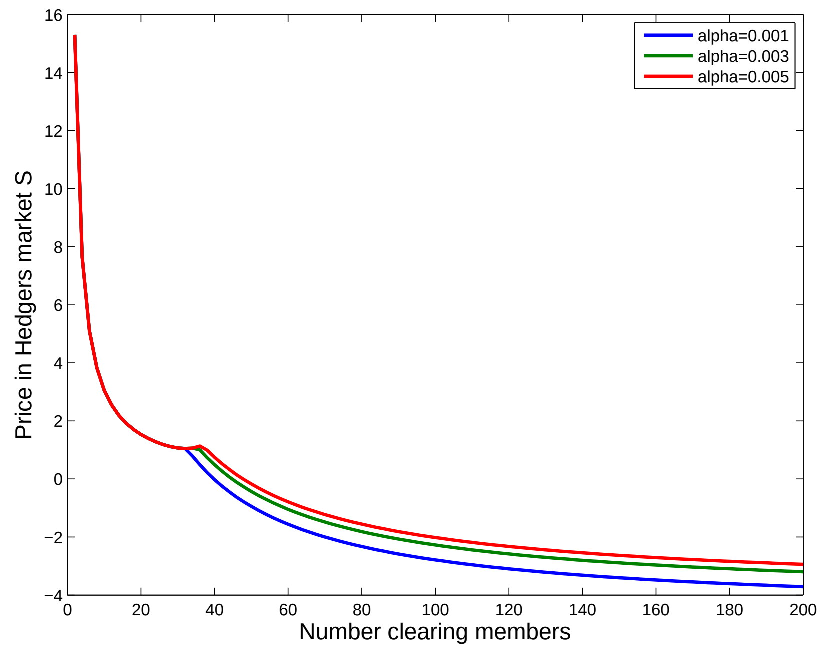
(a) Dealers



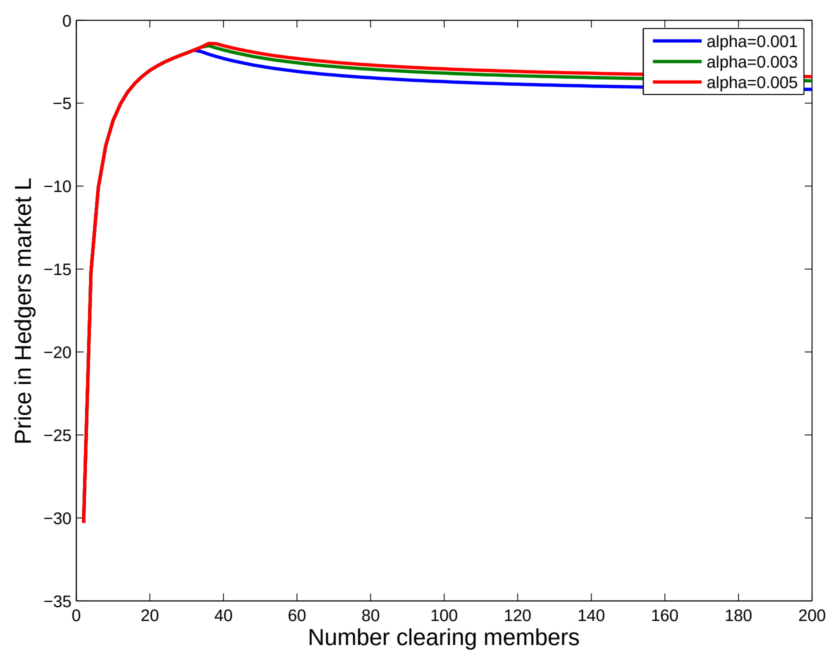
(b) Hedgers

Figure 6: **Objective Functions**

Equilibrium prices in hedgers market for different number, n, of clearing members. Parameter values are given in Table 1 but the differentiation level, t, and the level of capital per dealer varies. Panel 7(a) shows objective function of Hedgers. Panel 7(b) shows the objective function of Dealers.



(a) Short city



(b) Long city

Figure 7: **Hedgers Market Prices**

Equilibrium prices in hedgers market for different number, n, of clearing members. Parameter values are given in Table 1 but for different values of α and where the level of capital per dealer decreases quickly with n. Panel 7(a) shows the price in the short Hedgers market. Panel 7(b) shows the price in the long Hedgers market.

A Appendix

A.1 First-Order Conditions

The Kuhn-Tucker first-order conditions for constrained optimization from one dealer are given by:

$$\left(\frac{\partial EU}{\partial p^L} - \frac{\partial EU}{\partial y^L}\frac{\partial D_L(p^{L*})}{\partial p^L}\right) = \lambda\left(\frac{\partial PD_L^*}{\partial p^L} - \frac{\partial PD_L^*}{\partial y^L}\frac{\partial D_L(p^{L*})}{\partial p^L}\right)$$
$$\left(\frac{\partial EU}{\partial p^S} - \frac{\partial EU}{\partial y^S}\frac{\partial D_S(p^{S*})}{\partial p^S}\right) = \lambda\left(\frac{\partial PD_S^*}{\partial p^S} - \frac{\partial PD_S^*}{\partial y^S}\frac{\partial D_S(p^{S*})}{\partial p^S}\right)$$
$$\lambda(PD - \alpha) = 0.$$

If $y^L + y^S > 0$, the derivative of the default probability are

$$\frac{\partial PD}{\partial y^L} = \phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S z)}) \cdot \frac{p^L(y^L + y^S) - (p^L y^L + p^S y^S - K)}{(y^L + y^S)^2\sigma}$$
$$\frac{\partial PD}{\partial y^S} = \phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S z)}) \cdot \frac{p^S(y^L + y^S) - (p^L y^L + p^S y^S - K)}{(y^L + y^S)^2\sigma},$$

$$\frac{\partial PD}{\partial p^L} = \phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S)}) \cdot \frac{y^L}{(y^L + y^S)\sigma}$$

$$\frac{\partial PD}{\partial p^S} = \phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S)}) \cdot \frac{y^S}{(y^L + y^S)\sigma}$$

If $y^L + y^S < 0$, the derivative of this probability can be written as

$$\frac{\partial PD}{\partial y^L} = -\phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S)}) \cdot \frac{p^L(y^L + y^S) - (p^L y^L + p^S y^S - K)}{(y^L + y^S)^2\sigma}$$

$$\frac{\partial PD}{\partial y^S} = -\phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S)}) \cdot \frac{p^S(y^L + y^S) - (p^L y^L + p^S y^S - K)}{(y^L + y^S)^2\sigma}$$
$$\frac{\partial PD}{\partial p^L} = -\phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S)}) \cdot \frac{y^L}{(y^L + y^S)\sigma}$$
$$\frac{\partial PD}{\partial p^S} = -\phi(\frac{p^L y^L + p^S y^S - K}{\sigma(y^L + y^S)}) \cdot \frac{y^S}{(y^L + y^S)\sigma},$$

but these derivatives are zero if $y^L + y^S = 0$. A dealer's objective function, $EU(p^L, p^S, y^L, y^S)$, can be rewritten in two terms,

$$EU = EU_1 \cdot PR_1 - K \cdot PR_2,$$

and from here, the derivatives are given by:

$$\frac{\partial EU}{\partial y^L} = \frac{\partial EU_1}{\partial y^L} \cdot PR_1 + \frac{\partial PR_1}{\partial y^L} \cdot EU_1 - K \cdot \frac{\partial PR_2}{\partial y^L}$$
$$\frac{\partial EU}{\partial y^S} = \frac{\partial EU_1}{\partial y^S} \cdot PR_1 + \frac{\partial PR_1}{\partial y^S} \cdot EU_1 - K \cdot \frac{\partial PR_2}{\partial y^S}$$
$$\frac{\partial EU}{\partial p^L} = \frac{\partial EU_1}{\partial p^L} \cdot PR_1 + \frac{\partial PR_1}{\partial p^L} \cdot EU_1 - K \cdot \frac{\partial PR_2}{\partial p^L}$$
$$\frac{\partial EU}{\partial p^S} = \frac{\partial EU_1}{\partial p^S} \cdot PR_1 + \frac{\partial PR_1}{\partial p^S} \cdot EU_1 - K \cdot \frac{\partial PR_2}{\partial p^S}$$

Consider the case of the Normal distribution. Note that $y^L(f-p^L)+y^S(f-p^S)$ follows a normal distribution $N(-y^Lp^L - y^Sp^S, (y^L+y^S)^2\sigma^2)$. By the properties of truncated normal distributions, we have

$$EU_1 \equiv E[y^L(f-p^L)+y^S(f-p^S)/y^L(f-p^L)+y^S(f-p^S) \geq -K] =$$
$$\begin{array}{ll} -y^Lp^L - y^Sp^S + \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})}{1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})}(y^L+y^S)\sigma & if \quad y^L+y^S>0 \\ -y^Lp^L - y^Sp^S - \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})}{1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})}(y^L+y^S)\sigma & if \quad y^L+y^S<0 \\ -y^Lp^L - y^Sp^S & if \quad y^L+y^S=0 \end{array}$$

Note that in the last case there is not uncertainty, that is why we have the simpler expression. The derivatives of this term are for $y^L+y^S>0$

$$\frac{\partial EU_1}{\partial y^L} = -p^L + \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})}{1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})}\sigma +$$
$$+ (y^L+y^S)\sigma \frac{\phi'(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{p^L(y^L+y^S)-(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)}$$
$$+ (y^L+y^S)\sigma \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\frac{p^L(y^L+y^S)-(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)^2}$$

$$\frac{\partial EU_1}{\partial y^S} = -p^S + \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})}{1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})}\sigma +$$

$$+ (y^L+y^S)\sigma \frac{\phi'(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{p^S(y^L+y^S)-(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)}$$

$$+ (y^L+y^S)\sigma \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\frac{p^S(y^L+y^S)-(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)^2}$$

$$\frac{\partial EU_1}{\partial p^L} = -y^L + (y^L+y^S)\sigma \frac{\phi'(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{y^L}{(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)}$$

$$+ (y^L+y^S)\sigma \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\frac{y^L}{(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)^2}$$

$$\frac{\partial EU_1}{\partial p^S} = -y^S + (y^L+y^S)\sigma \frac{\phi'(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{y^S}{(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)}$$

$$+ (y^L+y^S)\sigma \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\frac{y^S}{(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{(y^L+y^S)\sigma})\right)^2}$$

The derivatives of this term are for $y + z < 0$

$$\frac{\partial EU_1}{\partial y^L} = -p^L - \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})}{1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})}\sigma +$$

$$- (y^L+y^S)\sigma \frac{\phi'(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\frac{-p^L(y^L+y^S)+(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)}$$

$$- (y^L+y^S)\sigma \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\frac{-p^L(y^L+y^S)+(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)^2}$$

$$\frac{\partial EU_1}{\partial y^S} = -p^S - \frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})}{1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})}\sigma +$$

$$-(y^L+y^S)\sigma\frac{\phi'(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\frac{-p^S(y^L+y^S)+(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)}$$

$$-(y^L+y^S)\sigma\frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\frac{-p^S(y^L+y^S)+(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)^2}$$

$$\frac{\partial EU_1}{\partial p^L} = -y^L - (y^L+y^S)\sigma\frac{-\phi'(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\frac{y^L}{-(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)}$$

$$-(y^L+y^S)\sigma\frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\frac{y^L}{-(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)^2}$$

$$\frac{\partial EU_1}{\partial p^S} = -y^S - (y^L+y^S)\sigma\frac{-\phi'(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\frac{y^S}{-(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)}$$

$$-(y^L+y^S)\sigma\frac{\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\frac{y^S}{-(y^L+y^S)\sigma}}{\left(1-\Phi(\frac{p^Ly^L+p^Sy^S-K}{-(y^L+y^S)\sigma})\right)^2}$$

And the derivatives when $y^L + y^S = 0$ are

$$\frac{\partial EU_1}{\partial y^L} = -p^L$$
$$\frac{\partial EU_1}{\partial y^L} = -p^S$$
$$\frac{\partial EU_1}{\partial p^L} = -y^L$$
$$\frac{\partial EU_1}{\partial p^S} = -y^S$$

And the probabilities are just,

$$PR_1 \equiv \Pr(y^L(f-p^L)+y^S(f-p^S) \geq -K) = \begin{cases} 1-\Phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma}) & if \quad y^L+y^S>0 \\ 1-\Phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma}) & if \quad y^L+y^S<0 \\ 1 & if \quad y^L+y^S=0 \text{ and } -yp^H-zp^T \geq -K \\ 0 & if \quad y^L+y^S=0 \text{ and } -yp^H-zp^T < -K \end{cases}$$

$$PR_2 \equiv \Pr(y^L(f-p^L)+y^S(f-p^S) < -K) = \begin{cases} \Phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma}) & if \quad y^L+y^S>0 \\ \Phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma}) & if \quad y^L+y^S<0 \\ 0 & if \quad y^L+y^S=0 \text{ and } -yp^H-zp^T \geq -K \\ 1 & if \quad y^L+y^S=0 \text{ and } -yp^H-zp^T < -K \end{cases}$$

The derivatives of PR_1 when $y^L+y^S>0$ are

$$\frac{\partial PR_1}{\partial y} = -\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{p^H(y+z)-(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}$$

$$\frac{\partial PR_1}{\partial z} = -\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{p^T(y+z)-(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}$$

$$\frac{\partial PR_1}{\partial p^H} = -\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{y}{(y^L+y^S)\sigma}$$

$$\frac{\partial PR_1}{\partial p^T} = -\phi(\frac{-K+p^Ly^L+p^Sy^S}{(y^L+y^S)\sigma})\frac{z}{(y^L+y^S)\sigma}$$

The derivatives of PR_1 when $y^L+y^S<0$ are

$$\frac{\partial PR_1}{\partial y} = -\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\frac{-p^H(y+z)+(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}$$

$$\frac{\partial PR_1}{\partial z} = -\phi(\frac{-K+p^Ly^L+p^Sy^S}{-(y^L+y^S)\sigma})\frac{-p^T(y+z)+(-K+p^Ly^L+p^Sy^S)}{(y^L+y^S)^2\sigma}$$

$$\frac{\partial PR_1}{\partial p^H} = -\phi(\frac{-K + p^L y^L + p^S y^S}{-(y^L + y^S)\sigma})\frac{y}{-(y^L + y^S)\sigma}$$

$$\frac{\partial PR_1}{\partial p^T} = -\phi(\frac{-K + p^L y^L + p^S y^S}{-(y^L + y^S)\sigma})\frac{z}{-(y^L + y^S)\sigma}$$

When $y^L + y^S = 0$ these derivatives are just zero.
The derivatives of PR_2 when $y^L + y^S > 0$ are

$$\frac{\partial PR_2}{\partial y} = \phi(\frac{-K + p^L y^L + p^S y^S}{(y^L + y^S)\sigma})\frac{p^H(y+z) - (-K + p^L y^L + p^S y^S)}{(y^L + y^S)^2\sigma}$$

$$\frac{\partial PR_2}{\partial z} = \phi(\frac{-K + p^L y^L + p^S y^S}{(y^L + y^S)\sigma})\frac{p^T(y+z) - (-K + p^L y^L + p^S y^S)}{(y^L + y^S)^2\sigma}$$

$$\frac{\partial PR_2}{\partial p^H} = \phi(\frac{-K + p^L y^L + p^S y^S}{(y^L + y^S)\sigma})\frac{y}{(y^L + y^S)\sigma}$$

$$\frac{\partial PR_2}{\partial p^T} = \phi(\frac{-K + p^L y^L + p^S y^S}{(y^L + y^S)\sigma})\frac{z}{(y+z)\sigma}$$

The derivatives of PR_2 when $y^L + y^S < 0$ are

$$\frac{\partial PR_2}{\partial y} = \phi(\frac{-K + p^L y^L + p^S y^S}{-(y^L + y^S)\sigma})\frac{-p^H(y^L + y^S) + (-K + p^L y^L + p^S y^S)}{(y^L + y^S)^2\sigma}$$

$$\frac{\partial PR_2}{\partial z} = \phi(\frac{-K + p^L y^L + p^S y^S}{-(y^L + y^S)\sigma})\frac{-p^T(y^L + y^S) + (-K + p^L y^L + p^S y^S)}{(y^L + y^S)^2\sigma}$$

$$\frac{\partial PR_1}{\partial p^H} = \phi(\frac{-K + p^L y^L + p^S y^S}{-(y^L + y^S)\sigma})\frac{y}{-(y^L + y^S)\sigma}$$

$$\frac{\partial PR_1}{\partial p^T} = \phi(\frac{-K + p^L y^L + p^S y^S}{-(y^L + y^S)\sigma})\frac{z}{-(y^L + y^S)\sigma}$$

When $y + z = 0$ these derivatives are just zero.
Let denote this expected utility function as $EU(p^H, p^T, y_i, z_i)$

$$\frac{\partial EU}{\partial y} = \frac{\partial EU_1}{\partial y} \cdot PR_1 + EU_1 \cdot \frac{\partial PR_1}{\partial y} - K \cdot \frac{\partial PR_2}{\partial y}$$

$$\frac{\partial EU}{\partial z} = \frac{\partial EU_1}{\partial z} \cdot PR_1 + EU_1 \cdot \frac{\partial PR_1}{\partial z} - K \cdot \frac{\partial PR_2}{\partial z}$$

$$\frac{\partial EU}{\partial p^H} = \frac{\partial EU_1}{\partial p^H} \cdot PR_1 + EU_1 \cdot \frac{\partial PR_1}{\partial p^H} - K \cdot \frac{\partial PR_2}{\partial p^H}$$

$$\frac{\partial EU}{\partial p^T} = \frac{\partial EU_1}{\partial p^T} \cdot PR_1 + EU_1 \cdot \frac{\partial PR_1}{\partial p^T} - K \cdot \frac{\partial PR_2}{\partial p^T}$$

Another way of writing these derivates is the following. By the properties of censored normal, we have

$$\frac{\partial EU}{\partial \widetilde{\sigma}} = \phi(\frac{-K-\mu}{\widetilde{\sigma}}) > 0$$
$$\frac{\partial EU}{\partial \mu} = 1 - \Phi(\frac{-K-\mu}{\widetilde{\sigma}}) > 0$$

where as denoted before $\mu = -y^L p^L - y^S p^S$ and $\widetilde{\sigma}^2 = (y^L + y^S)^2 \sigma^2$. Given this

$$\widetilde{\sigma} = \begin{cases} (y^L + y^S)\sigma & if \quad y^L > -y^S \\ -(y^L + y^S)\sigma & if \quad y^L < -y^S \end{cases}$$

and therefore

$$\frac{\partial \mu}{\partial p^L} = -y^L$$
$$\frac{\partial \mu}{\partial y^L} = -p^L$$

$$\frac{\partial \widetilde{\sigma}}{\partial y^L} = \begin{cases} \sigma & if \quad y^L > -y^S \\ -\sigma & if \quad y^L < -y^S \end{cases}$$

Therefore,

$$\frac{dEU}{dp^L} = \frac{\partial EU}{\partial \mu} \frac{\partial \mu}{\partial y^L} \frac{\partial(-D^L)}{\partial p^L} + \frac{\partial EU}{\partial \mu} \frac{\partial \mu}{\partial p^L} + \frac{\partial EU}{\partial \widetilde{\sigma}} \frac{\partial \widetilde{\sigma}}{\partial y^L} \frac{\partial(-D^L)}{\partial p^L} =$$
$$=$$
$$= \begin{cases} \left(1 - \Phi(\frac{-K-\mu}{\widetilde{\sigma}})\right)\left(-p^L \frac{1}{t} - y^L\right) + \phi(\frac{-K-\mu}{\widetilde{\sigma}})\sigma \frac{1}{t} & if \quad y^L > -y^S \\ \left(1 - \Phi(\frac{-K-\mu}{\widetilde{\sigma}})\right)(-p^T) - \phi(\frac{-K-\mu}{\widetilde{\sigma}})\sigma & if \quad y^L < -y^S \end{cases}$$