Collateral Requirements and Asset Prices

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March 15, 2011

Abstract

In this paper we examine the effect of collateral requirements on the prices of long-lived assets. We consider a Lucas-style infinite-horizon exchange economy with heterogeneous agents and collateral constraints. There are two trees in the economy which can be used as collateral for short-term loans. For the first tree the collateral requirement is determined endogenously while the collateral requirement for loans on the second tree is exogenously regulated. We show that the presence of collateral constraints and the endogenous margin requirements for the first tree lead to large excess price-volatility of the second tree. Changes in the regulated margin requirements for the second tree have large effects on the volatility of both trees. While tightening margins for loans on the second tree always decreases the price volatility of the first tree, price volatility of the second tree might very well increase with this change. In our calibration we allow for the possibility of disaster states. This leads to very large quantitative effects of collateral requirements and to realistic equity risk premia. We show that our qualitative results are robust to the actual parametrization of the economy.

Keywords: Collateral constraints, leverage, heterogeneous agents, endogenous margins, regulated collateral.

JEL Classification Codes: D53, G11, G12.

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1 Introduction

The vast majority of debt, especially if it extends over a long period of time, is guaranteed by tangible assets called collateral. For example, residential homes serve as collateral for short- and long-term loans to households, and investors can borrow money to establish a position in stocks, using these as collateral. The margin requirement dictates how much collateral one has to hold in order to borrow one dollar. Clearly these margin requirements will have important implications for the price of collateral. In the recent financial crisis it was argued that excessively low margin requirements were part of the cause of the crisis. In this paper, we conduct a quantitative study on the effect of margins requirements on asset prices.

Many previous papers have formalized the idea that borrowing on collateral might give rise to cyclical fluctuations in real activity and enhance volatility of prices (see e.g. Geanakoplos (1997), Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999)). In these models, it is possible to have substantial departures of the market price from the corresponding price under frictionless markets. These results have led researchers to suggest that by managing leverage (or the amount of collateralized borrowing), a central bank can reduce aggregate fluctuations (see e.g. Ashcroft et al. (2010) or Geanakoplos (2010)). However, establishing the quantitative importance of collateral requirements as a source of excess volatility has been a challenge in the literature (see Kocherlakota (2000) or Cordoba and Ripoll (2004)). Moreover, so far, there have been few quantitative studies that take into account that a household can use several different assets as collateral, and that regulated margin requirements for loans on one asset might have important effects on the volatility of other assets in the economy.

In this paper we consider a Lucas (1978) style exchange economy with heterogeneous agents and collateral constraints. We assume that agents can only take short positions if they hold an infinitely-lived asset (a Lucas tree) as a long position. This model was first analyzed by Kubler and Schmedders (2003) and subsequently used by Cao (2009) and Brumm and Grill (2010). As in Kubler and Schmedders (2003) we assume that agents can default on a negative bond position at any time without any utility penalties or loss of reputation. Financial securities are therefore only traded if the promises associated with these securities are backed by collateral. Our main focus is on an economy with two trees which can be used as collateral for short-term loans. For the first tree the collateral requirement is determined endogenously while the collateral requirement for loans on the second tree is exogenously regulated. We show that the presence of collateral constraints and the endogenous margin requirements for the first tree lead to large excess price-volatility of the second tree. Changes in the regulated margin requirements for the second tree have large effects on the volatility of both trees. While tightening margins for loans on the second tree always decreases the price volatility of the first tree, price volatility of the second tree might very well increase with this change. In our calibration we allow for the possibility of

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disaster states. This leads to very large quantitative effects of collateral requirements and to realistic equity risk premia.

Margin requirements are a crucial feature of our model. They determine with how much leverage agents can invest in risky assets. Following Geanakoplos (1997) and Geanakoplos and Zame (2002), we endogenize the margin requirements by introducing a menu of financial securities. All securities promise the same payoff, but they distinguish themselves by their respective margin requirement. In equilibrium only some of them are traded, thereby determining an endogenous margin requirement. This implies, of course, that for many bonds and many next period's shocks, the face value of the debt falls below the value of the collateral. As a result there is default in equilibrium. However, in an extension of the model we allow for costly default by introducing a real cost to the lender. We examine the impact of such default costs on equilibrium trading volume and prices. As an alternative to endogenous margin requirements, we also consider regulated margin requirements. In particular, our two-tree economy allows us to compare a tree with endogenous margins to a tree with regulated margins.[1]

In our calibration of the model there are two heterogeneous agents with Epstein-Zin utility. They have identical elasticities of substitution (IES) but distinguish themselves by their risk-aversion (RA). The agent with the low risk aversion is the natural buyer of risky assets and takes on leverage to finance these investments. The agent with the high risk aversion has a strong insurance motive against bad shocks and, therefore, is a natural buyer of safe bonds and a natural seller of risky assets. The idea behind this model setup is as follows. When the economy is hit with a negative shock, the collateral constraint forces the leveraged agent to reduce consumption or to even sell risky assets to the risk-averse agent, thereby resulting in substantial changes in the wealth distribution which in turn affect agents' portfolios and asset prices.

We start our analysis with an economy with a single Lucas tree that can be used as collateral. In this baseline model we exogenously assume that collateral requirements are set to the lowest possible level that still ensures that there is never default in equilibrium. To obtain a sizable market price of risk, we follow the specification in Barro and Jin (2009) and introduce the possibility of 'disaster shocks' into the otherwise standard calibration. In this model, the effect of scarce collateral on the volatility of the tree is quantitatively large. We then allow agents to choose from a menu of bonds with different margin requirements

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[1]Depending on the asset that is used as collateral, market forces might play an important role in establishing margin requirements. For stocks the situation is not obvious: The Federal Reserve Board sets minimum margin requirements for broker-dealer loans, using what is called Regulation T. In fact, until 1974, the Fed considered initial margin percentages as an active component of monetary policy and changed them fairly often (see Willen and Kubler, 2006). In the US housing market, there are no such regulations and margins can be arbitrarily small.

which are determined in equilibrium. Agents do trade bonds that have a positive probability of default. However, as soon as we introduce moderate default cost, trade in these default bonds is shut down.

The main contribution of the paper is the analysis of an economy with two trees which have identical cash-flows but distinguish themselves by their 'collateralizability'. We first analyze a specification of the model in which only the first tree can be used as collateral. In this specification, the return volatility of the collateralizable tree is significantly smaller than that of the single tree in the baseline model. However, the volatility of the second tree, which cannot be used as collateral, is comparable. A possible interpretation of these findings is to identify the collateralizable tree with housing and the non-collateralizable tree with the aggregate stock market. Using stocks as collateral is subject to many regulations and often very costly, while individuals can easily use houses. Volatility and excess returns for houses is much smaller than for stocks, which is in line with our findings.

We then relax the assumption of the non-collaterizability of the second tree. We assume that a regulating agency sets an exogenous margin requirement for this tree. We find that regulation of the second tree has a strong impact on the volatility of the first tree. In particular, a tightening of margin requirements for the regulated tree uniformly decreases volatility of the unregulated tree. For the regulated tree, tighter margins initially increase the price volatility but then decrease it once margins become very large. We further show how the regulation of margin requirements only in times when the economy exhibits strong growth can substantially decrease volatility compared to the case of uniform regulation of margin requirements. This result holds true both for the baseline model with a single tree as well as the two-tree economy and suggests a strong policy recommendation for counter-cyclical margin requirements.

Finally, we conduct a thorough sensitivity analysis and show that our qualitative results are robust to the actual parametrization of the economy. In particular, we document that the key effects for the two-tree economy are robust to changes in the magnitude of the disaster shocks.

The remainder of this paper is organized as follows. We introduce the model in Section 2. In Section 3 we discuss results for economies with a single tree. Section 4 focuses on economies with two trees. In Section 5 we consider extensions and sensitivity analysis. Section 6 concludes.

2   The economic model

We examine a model of an exchange economy that extends over an infinite time horizon and is populated by infinitely-lived heterogeneous agents.

2.1 Infinite-horizon economy

This section describes the details of the infinite-horizon economy.

The Physical Economy

Time is indexed by $t = 0, 1, 2, \ldots$. A time-homogeneous Markov chain of exogenous shocks (s_t) takes values in the finite set $\mathcal{S} = \{1, \ldots, S\}$. The $S \times S$ Markov transition matrix is denoted by π. We represent the evolution of time and shocks in the economy by a countably infinite event tree Σ. The root node of the tree represents the initial shock s_0. Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \ldots, s_t)$ and is also called date-event. We use the symbols σ and s^t interchangeably. To indicate that $s^{t'}$ is a successor of s^t (or s^t itself) we write $s^{t'} \succeq s^t$. We use the notation s^{-1} to refer to the initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$ there is a single perishable consumption good. The economy is populated by H agents, $h \in \mathcal{H} = \{1, 2, \ldots, H\}$. Agent h receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. In addition, at $t = 0$ the agent owns shares in Lucas trees. We interpret these Lucas trees to be physical assets such as firms, machines, land or houses. There are A different such assets, $a \in \mathcal{A} = \{1, 2, \ldots, A\}$. At the beginning of period 0, each agent h owns initial holdings $\theta_a^h(s^{-1}) \geq 0$ of tree a. We normalize aggregate holdings in each Lucas tree, that is, $\sum_{h \in \mathcal{H}} \theta_a^h(s^{-1}) = 1$ for all $a \in \mathcal{A}$. At date-event σ, we denote agent h's (end-of-period) holding of Lucas tree a by $\theta_a^h(\sigma)$.

The Lucas trees pay positive dividends $d_a(\sigma)$ in units of the consumption good at all date-events. We denote aggregate endowments in the economy by

$$\bar{e}(\sigma) = \sum_{h \in \mathcal{H}} e^h(\sigma) + \sum_{a \in \mathcal{A}} d_a(\sigma).$$

The agents have preferences over consumption streams representable by the following recursive utility function, see Epstein and Zin (1989),

$$U^h(c, s^t) = \left\{ \left[c^h(s^t) \right]^{\rho^h} + \beta \left[\sum_{s_{t+1}} \pi(s_{t+1}|s_t) \left(U^h(c, s^{t+1}) \right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{\rho^h}},$$

where $\frac{1}{1-\rho^h}$ is the intertemporal elasticity of substitution (IES) and $1 - \alpha^h$ is the relative risk aversion of the agent.

Security Markets

At each date-event agents can engage in security trading. Agent h can buy $\theta_a^h(\sigma) \geq 0$ shares of tree a at node σ for a price $q_a(\sigma)$. Agents cannot assume short positions of the

Lucas trees. Therefore, the agents make no promises of future payments when they trade shares of physical assets and thus there is no possibility of default.

In addition to the physical assets, there are J financial securities, $j \in \mathcal{J} = \{1, 2, \ldots, J\}$, available for trade. These assets are one-period securities in zero-net supply. Security j traded at node s^t promises a payoff of one unit of the consumption good at each immediate successor node s^{t+1}. We denote agent h's (end-of-period) portfolio of financial securities at date-event σ by $\phi^h(\sigma) \in \mathbb{R}^J$ and denote the price of security j at this date-event by $p_j(\sigma)$. Whenever an agent assumes a short position in a financial security j, $\phi_j^h(\sigma) < 0$, she promises a payment in the next period. In our economy such promises must be backed up by collateral holdings.

Collateral and Default

At each node σ, we associate with each financial security $j \in \mathcal{J}$ a tree $a(j) \in \mathcal{A}$ and a collateral requirement $k_{a(j)}^j(\sigma) > 0$. If an agent sells one unit of security j, then she is required to hold $k_{a(j)}^j(\sigma)$ units of tree $a(j)$ as collateral. If an asset a can be used as collateral for different financial securities, the agent is required to buy $k_{a(j)}^j(\sigma)$ shares for each security $j \in \mathcal{J}_a$, where $\mathcal{J}_a \subset \mathcal{J}$ denotes the set of financial securities collateralized by the same tree a. In the next period, the agent can default on her earlier promise. In this case the agent loses the collateral she had to put up. In turn, the buyer of the financial security receives this collateral associated with the initial promise.[2]

Since there are no penalties for default, a seller of security j at date-event s^{t-1} defaults on her promise at a successor node s^t whenever the initial promise exceeds the current value of the collateral, that is, whenever

$$1 > k_{a(j)}^j(s^{t-1}) \left(q_{a(j)}(s^t) + d_{a(j)}(s^t) \right).$$

The payment by a borrower of security j at node s^t is, therefore, always given by

$$f_j(s^t) = \min \left\{ 1, k_{a(j)}^j(s^{t-1}) \left(q_{a(j)}(s^t) + d_{a(j)}(s^t) \right) \right\}.$$

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[2]Following Geanakoplos and Zame (2002) we make the strong assumption that an agent can default on individual promises without declaring personal bankruptcy and giving up all the assets he owns. There are no penalties for default and a borrower always defaults once the value of the debt is above the value of the collateral. Since this implies that the decision to default on a promise is independent of the debtor, we do not need to consider pooling of contracts as in Dubey et al. (2000), even though there may be default in equilibrium. This treatment of default is somewhat unconvincing since default does not affect a household's ability to borrow in the future and it does not lead to any direct reduction in consumption at the time of default. Moreover, declaring personal bankruptcy typically results in a loss of all assets, and it is rarely possible to default on some loans while keeping the collateral for others. However, there do exist laws for collateralizable borrowing where default is possible without declaring bankruptcy. Examples include pawn shops and the housing market in many US states, in which households are allowed to default on their mortgages without defaulting on other debt. It is certainly true that the recent 2008 housing crises makes this assumption look much better.

Our model includes the possibility of costly default. This feature of the model is meant to capture default costs such as legal cost or the physical deterioration of the collateral asset. For example, it is well known that housing properties in foreclosure deteriorate because of moral hazard, destruction, or simple neglect. We model such costs by assuming that part of the collateral value is lost and thus the payment received by the lender is smaller than the value of the borrower's collateral. Specifically, the loss is proportional to the difference between the face value of the debt and the value of collateral, that is, the loss is

$$l_j(s^t) = \lambda \left(1 - k_{a(j)}^j(s^{t-1}) \left(q_{a(j)}(s^t) + d_{a(j)}(s^t)\right)\right)$$

for some parameter $\lambda \geq 0$. The resulting payment to the lender of the loan in security j when $f_j(s^t) < 1$ is thus given by

$$r_j(s^t) = \max\left\{0, f_j(s^t) - l_j(s^t)\right\} = \max\left\{0, (1+\lambda)k_{a(j)}^j(s^{t-1})(q_{a(j)}(s^t) + d_{a(j)}(s^t)) - \lambda\right\}.$$

If $f_j(s^t) = 1$ then $r_j(s^t) = f_j(s^t) = 1$. This repayment function does not capture all costs associated with default. For example, it does not allow for fixed costs which are independent of how much the collateral value falls short of the repayment obligation. However, our functional form offers the advantage that the resulting model remains tractable since the repayment function is continuous in the value of the collateral.

The specification of the collateral requirements $k_a^j(s^t)$ for bond j, tree a and across date-events s^t has important implications for equilibrium prices and allocations. The collateral levels $k_a^j(s^t)$ are endogenously determined in equilibrium. In this paper we examine two different rules for the endogenous determination of collateral levels. The first rule determines endogenous collateral requirements along the lines of Geanakoplos and Zame (2002). The second rule assumes exogenously regulated capital-to-value ratios which in turn lead to endogenous collateral requirements.

Default and Endogenous Collateral Requirements

One of the contributions of this paper is to endogenize collateral requirements in an infinite-horizon dynamic general equilibrium model. For this purpose, our first collateral rule follows Geanakoplos (1997) and Geanakoplos and Zame (2002) who suggest a simple and tractable way to endogenize collateral requirements. They assume that, in principle, financial securities with any collateral requirement could be traded in equilibrium. Only the scarcity of available collateral leads to equilibrium trade in only a small number of such securities. Our first rule follows this approach.

Recall that the S direct successors of a node s^t are denoted $(s^t, 1), \ldots, (s^t, S)$ and that $\mathcal{J}_a$ denotes the set of bonds collateralized by the same tree a. We define endogenous margin requirements for bonds $j \in \mathcal{J}_a$ collateralized by the same tree $a \in \mathcal{A}$ as follows. For each shock next period, $s' \in \mathcal{S}$, there is at at least one bond which satisfies $k_{a(j)}^j(s^t)\left(q_{a(j)}(s^t, s') + d_{a(j)}(s^t, s')\right) = 1$. For each bond in the set $\mathcal{J}_a$ the promised payoff

is equal to the collateral in (generically) exactly a single state. Generically the set $\mathcal{J}_a$ thus contain exactly S bonds, however the bond with the lowest collateral requirement is redundant in our model because its payoff vector is collinear with the tree's dividend vector. (Therefore, we consider only models with at most $S-1$ bonds in our numerical analysis of the model.) The arguments in Araujo et al. (2010) show that adding additional bonds with other collateral requirements (also only using tree a as collateral) do not change the equilibrium allocation. In the presence of S bonds as specified above, any bond with an intermediate collateral requirement can be replicated by holding a portfolio of the existing bonds using the same amount of collateral.

We begin our model examinations always with economies with a single bond, $J = 1$, on which agents cannot default. That is, the collateral requirements are endogenously set to the lowest possible value which still ensures no default in the subsequent period (this specification is similar to the collateral requirements in Kiyotaki and Moore, 1997). Formally, the resulting condition for the collateral requirement $k^1_{a(1)}(s^t)$ of this bond is

$$k^1_{a(1)}(s^t) \left(\min_{s^{t+1} \succ s^t} \left(q_{a(1)}(s^{t+1}) + d_{a(1)}(s^{t+1}) \right) \right) = 1.$$

We refer to this bond as the 'risk-free' or 'no-default' bond.

To simplify the discussion of models with several bonds, it is useful to refer to the different bonds by the number of states in which they default, respectively. In our model specifications below, the set $\mathcal{J}_a$ always contains a no-default bond. In models with several bonds, the second bond defaults in precisely one state, the third bond in precisely two states, and so on. Hence we refer to these additional bonds as the 1-default bond, the 2-default bond etc. In the absence of default costs, some of these bonds will typically be traded in equilibrium. However, we see below that, in our calibration, rather moderate default costs generally suffice to shut down trade in these bonds.

Financial Markets Equilibrium with Collateral

We are now in the position to formally define the notion of a financial markets equilibrium. To simplify the statement of the definition, we assume that for a set of trees $\hat{\mathcal{A}} \subset \mathcal{A}$ collateral requirements are endogenous, that is for each $\hat{a} \in \hat{\mathcal{A}}$, there exist a set $\mathcal{J}_{\hat{a}}$ of S bonds for which this tree can be used as collateral. It is helpful to define the terms $[\phi^h_j]^+ = \max(0, \phi^h_j)$ and $[\phi^h_j]^- = \min(0, \phi^h_j)$. We denote equilibrium values of a variable x by $\bar{x}$.

DEFINITION 1 *A financial markets equilibrium for an economy with initial tree holdings* $(\theta^h(s^{-1}))_{h \in \mathcal{H}}$ *and initial shock s_0 is a collection of agents' portfolio holdings and consumption allocations as well as security prices and collateral requirements for all trees* $\hat{a} \in \hat{\mathcal{A}} \subset \mathcal{A}$

$$\left(\left(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma) \right)_{h \in \mathcal{H}}; \left(\bar{q}_a(\sigma) \right)_{a \in \mathcal{A}}, \left(\bar{p}_1(\sigma) \right)_{j \in \mathcal{J}}, \left(\bar{k}^j_{\hat{a}}(\sigma) \right)_{j \in \mathcal{J}_{\hat{a}}, \hat{a} \in \hat{\mathcal{A}}} \right)_{\sigma \in \Sigma}$$

satisfying the following conditions:

(1) Markets clear:

$$\sum_{h \in \mathcal{H}} \bar{\theta}^h(\sigma) = 1 \quad \text{and} \quad \sum_{h \in \mathcal{H}} \bar{\phi}^h(\sigma) = 0 \quad \text{for all } \sigma \in \Sigma.$$

(2) For each agent h, the choices $\left(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\right)$ solve the agent's utility maximization problem,

$$\max_{\theta \geq 0, \phi, c \geq 0} U_h(c) \quad \text{s.t.} \quad \text{for all } s^t \in \Sigma$$

$$
\begin{aligned}
c(s^t) &= e^h(s^t) + \sum_{j \in \mathcal{J}} \left([\phi_j(s^{t-1})]^+ r_j(s^t) + [\phi_j(s^{t-1})]^- f_j(s^t) \right) + \\
&\quad \theta^h(s^{t-1}) \cdot \left(\bar{q}(s^t) + d(s^t) \right) - \theta^h(s^t) \cdot \bar{q}(s^t) - \phi^h(s^t) \cdot \bar{p}(s^t) \\
0 &\leq \theta_{\hat{a}}^h(s^t) + \sum_{j \in \mathcal{J}_{\hat{a}}} \bar{k}_{\hat{a}}^j(s^t) [\phi_j^h(s^t)]^-, \quad \text{for all } \hat{a} \in \hat{\mathcal{A}}.
\end{aligned}
$$

(3) For all s^t and for each $\hat{a} \in \hat{\mathcal{A}}$, there exists for each state $s' \in \mathcal{S}$ a financial security j such that $\hat{a} = a(j)$ and

$$\bar{k}_{\hat{a}}^j(s^t) \left(\bar{q}_{\hat{a}}(s^t, s') + d_{\hat{a}}(s^t, s') \right) = 1.$$

The approach in Kubler and Schmedders (2003) can be used to prove existence. The only non-standard part—besides the assumption of recursive utility, which can be handled easily—is the assumption of default costs. Note, however, that our specification of these costs still leaves us with a convex problem and standard arguments for continuity of best responses go through.

To approximate equilibrium numerically, we use the algorithm in Brumm and Grill (2010). In Appendix A, we describe the computations and the numerical error analysis in detail. For the interpretation of the results to follow it is useful to understand the recursive formulation of the model. The natural endogenous state-space of this economy consists of all agents' beginning of period financial wealth as a fraction of total financial wealth (i.e. value of the trees cum dividends) in the economy. That is, we keep track of the current shock s_t and of

$$\omega^h(s^t) = \frac{\sum_{j \in \mathcal{J}} \left([\phi_j^h(s^{t-1})]^+ r_j(s^t) + [\phi_j^h(s^{t-1})]^- f_j(s^t) \right) + \theta^h(s^{t-1}) \cdot \left(\bar{q}(s^t) + d(s^t) \right)}{\sum_{a \in \mathcal{A}} q_a(s^t) + d(s^t)},$$

across all agents $h \in \mathcal{H}$. As in Kubler and Schmedders (2003) we assume that a recursive equilibrium on this state space exists and compute prices, portfolios and individual consumptions as a function of the exogenous shock and the distribution of financial wealth. In our calibration we assume that shocks are iid and that these shocks only affect the aggregate growth rate. In this case, policy- and pricing functions are independent of the exogenous

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shock, thus depend on the wealth distribution only, and our results can be easily interpreted in terms of these functions.

<p style="text-align:center">Regulated Collateral Requirements</p>

The second rule for setting collateral requirements relies on regulated capital-to-value ratios. An agent selling one unit of bond j with price $p_j(s^t)$ must hold collateral with a value of at least $k^j_{a(j)}(s^t)q_{a(j)}(s^t)$. We can interpret the difference between the value of the collateral holding and the debt as the amount of capital an agent must put up to obtain the loan in form of a short position in the financial security. A (not further modeled) regulating agency now requires debtors to hold a certain minimal amount of capital relative to the value of the collateral they hold. Put differently, the regulator imposes a lower bound $m^j_{a(j)}(s^t)$ on this capital-to-value ratio,

$$m^j_{a(j)}(s^t) = \frac{k^j_{a(j)}(s^t)q_{a(j)}(s^t) - p_j(s^t)}{k^j_{a(j)}(s^t)q_{a(j)}(s^t)}.$$

Using language from financial markets we also call these bounds margin requirements. If the margin requirement is regulated to be $m^j_{a(j)}(s)$ in shock $s \in \mathcal{S}$ and constant over time, then the collateral requirement at each node s^t is

$$k^j_{a(j)}(s^t) = \frac{p_j(s^t)}{q_{a(j)}(s^t)(1 - m^j_{a(j)}(s_t))}.$$

Note that, contrary to the exogenous margin requirement, the resulting collateral requirement is endogenous since it depends on equilibrium prices. For economies with regulated margins, condition (3) of the definition of a financial markets equilibrium must be replaced by the following condition.

(3') *For all s^t and for each $\hat{a} \in \hat{\mathcal{A}}$, the collateral requirement $\bar{k}^j_{\hat{a}}(s^t)$ of the unique bond j with $\hat{a} = a(j)$ and the given margin requirement $m^j_{\hat{a}}(s_t)$ satisfies*

$$\bar{k}^j_{\hat{a}}(s^t) = \frac{\bar{p}_j(s^t)}{\bar{q}_{\hat{a}}(s^t)(1 - m^j_{\hat{a}}(s_t))}.$$

Sometimes people use the term margin requirement for the capital-to-loan ratio,

$$\frac{k^j_a q_a(s^t) - p_j(s^t)}{p_j(s^t)},$$

which does not have a natural normalization and can be larger than one. On the contrary, the margin requirement $m^j_{a(j)}(s_t)$ as defined above has a natural normalization since it is bounded above by one.

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2.2 Calibration

This section discusses the calibration of the model's exogenous parameters.

2.2.1 Growth rates

We consider a growth economy with stochastic growth rates. The aggregate endowment at date-event s^t grows at the stochastic rate $g(s_{t+1})$ which (if no default cost are incurred) only depends on the new shock $s_{t+1} \in \mathcal{S}$, that is, if either $\lambda = 0$ or $f_j(s_{t+1}) = 1$ for all $j \in \mathcal{J}$, then

$$\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1})$$

for all date-events $s^t \in \Sigma$. If there is default in s_{t+1}, then the endowment $\bar{e}(s_{t+1})$ is reduced by the costs of default and the growth rate is reduced respectively.

There are $S = 6$ exogenous shocks. We declare the first three of them, $s = 1, 2, 3$, to be "disasters". We calibrate the disaster shocks to match the first three moments of the distribution of disasters in Barro and Jin (2009). Also following Barro and Jin, we choose transition probabilities such that the six exogenous shocks are i.i.d. The non-disaster shocks, $s = 4, 5, 6$, are then calibrated such that their standard deviation matches "normal" business cycle fluctuations with a standard deviation of 2 percent and an average growth rate of 2.5 percent, which results in an overall average growth rate of about 2 percent. We sometimes find it convenient to call shock $s = 4$ a "recession" since $g(4) = 0.966$ indicates a moderate decrease in aggregate endowments. Table 1 provides the resulting growth rates and probability distribution for the six exogenous shocks of the economy.

Shock s	1	2	3	4	5	6
$g(s)$	0.566	0.717	0.867	0.966	1.025	1.089
$\pi(s)$	0.005	0.005	0.024	0.065	0.836	0.065

Table 1: Growth rates and distribution of exogenous shocks

In our results sections below we report that collateral requirements have quantitatively strong effects on equilibrium prices. Obviously, the question arises what portion of these effects is due to the large magnitude of the disaster shocks. We address this issue in the discussion of our results. In addition, Section 5 examines the equilibrium effects of collateral requirements for an economy with less severe disaster shocks.

2.2.2 Endowments and dividends

There are $H = 2$ types of agents in the economy, the first type, $h = 1$, being less risk-averse than the second. Each agent h receives a fixed share of aggregate endowments as individual endowments, that is, $e^h(s^t) = \eta^h \bar{e}(s^t)$. We assume that $\eta^1 = 0.092$, $\eta^2 = 0.828$. Agent

1 receives 10 percent of all individual endowments, and agent 2 receives the remaining 90 percent of all individual endowments. The remaining part of aggregate endowments enters the economy as dividends of Lucas trees, that is, $d_a(s^t) = \delta_a(s_t)\bar{e}(s^t)$ and $\sum_a \delta_a(s) = 0.08$ for all $s \in \mathcal{S}$.

Several comments on the distribution of the aggregate endowment are in order. First, we abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks for a model with two types of agents. We conjecture that our effects would likely be larger if we considered a model with a continuum of agents receiving i.i.d. idiosyncratic shocks. Second, a dividend share of 8 percent may appear a little too low if one interprets the tree as consisting of both the aggregate stock market as well as housing wealth. However, this number is in line with Chien and Lustig (2009) who base their calibration on NIPA data. We conduct some sensitivity analysis below and, in particular, report results for the case $\sum_a \delta_a(s) = 0.15$ and thus $\eta^1 = 0.085$, $\eta^2 = 0.765$. Third, for simplicity we do not model trees' and other assets' dividends to have different stochastic characteristics as aggregate consumption. Fourth, in Section 4 we examine an economy with two Lucas trees. For such economies, we want to interpret the first tree as aggregate housing and its dividends as housing services while we interpret the second tree as the aggregate stock market. Following Cecchetti et al. (1993), we calibrate dividends to be 4 percent of aggregate consumption which leaves housing services to be of the same size. In order to focus on the effects of collateral and margin requirements, we assume that the two trees have the exact same dividend payments, that is, in the absence of collateral constraints these two trees would be identical assets. Therefore, this calibration allows for a careful examination of the impact of different collateral properties of the two trees.

2.2.3 Utility parameters

The choice of an appropriate value for the IES is rather difficult. On the one hand, several studies that rely on micro-data find values of about 0.2 – 0.8, see, for example, Attanasio and Weber (1995). On the other hand, Vissing-Jorgensen and Attanasio (2003) use data on stock owners only and conclude that the IES for such investors is likely to be above one. Barro (2009) finds that for a successful calibration of a representative-agent asset-pricing model the IES needs to be larger than one.

In our benchmark calibration both agents have identical IES of 1.5, that is, $\rho^1 = \rho^2 = 1/3$. In our sensitivity analysis we also consider the case of both agents having an IES of 0.5. For this specification the quantitative results slightly change compared to the benchmark calibration, but the qualitative insights remain intact.

Agent 1 has a risk aversion of 0.5 and so $\alpha^1 = 0.5$ while agent 2's risk aversion is 6 and thus $\alpha^2 = -5$. Recall the weights for the two agents in the benchmark calibration, $\eta^1 = 0.092$ and $\eta^2 = 0.828$. The majority of the population is therefore very risk-averse, while 10 percent of households have low risk aversion. This heterogeneity of the risk aversion

among the agents is the main driving force for volatility in the model. (Agent 1 wants to hold the risky assets in the economy and leverages to do so. In a bad shock, his de-leveraging leads to excess volatility.) In the equilibria of our model, the risky assets are mostly held by agent 1, but there are extended periods of time where also agent 2 holds part of the asset. Loosely speaking, we therefore choose the fraction of very risk-averse agents to match observed stock-market participation.

Finally, we set $\beta^h = 0.95$ for both $h = 1, 2$, which turns out to give us a good match for the annual risk-free rate.

3   Economies with a single Lucas tree

We first consider economies with a single Lucas tree available as collateral. We show that scarce collateral has a large effect on the price volatility of this tree and examine how the magnitude of this effect depends on the specification of margin requirements. This section sets the stage for our analysis of economies with two trees in Section 4.

3.1   Collateral and volatility with a single risk-free bond

The starting point of our analysis is an economy with a single Lucas tree and a single bond. We assume that the collateral requirements on the single bond ensure that there is no default in equilibrium and so the bond is risk-free. We calibrate this baseline model according to the parameters presented above.

For an evaluation of the quantitative effects of scarce collateral, we benchmark our results against those for two much simpler models. The model *B1: No bonds* is an economy with a single tree and no bond. Thus, agents in this economy cannot borrow. The model *B2: Unconstrained* is an economy in which agents can use their entire endowment as collateral. This model is equivalent to a model with natural borrowing constraints. Table 2 reports four statistics for each of the three economies. (See Appendix A for a description of the estimation procedure.) Throughout the paper we measure tree-price volatility by the average standard deviation of tree returns over a long horizon. Another meaningful measure is the average one-period-ahead conditional price volatility. These two measures are closely correlated for our models. In Table 2 we report both measures but omit the second one in the remainder of the paper. We also report average interest rates and equity premia. While our paper does not focus on an analysis of these measures, we do check them because we want to ensure that our calibration delivers reasonable values for these measures.

Recall that in our calibration agents of type 1 are much less risk averse than type 2 agents. And, therefore, in the long run agent 1 holds the entire Lucas tree in model *B1* with no borrowing and agent 2 effectively lives in autarchy. As a result the tree price is determined entirely by the Euler equation of agent 1, and so the price volatility is as low as

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Model	Std returns	1-period price vol.	Risk-free rate	EP
B1: No bonds	5.33	4.98	n/a	n/a
B2: Unconstrained	5.38	5.05	5.88	0.55
Scarce Collateral	8.14	7.54	1.10	3.86

Table 2: Three Economies with a single tree (all figures in percent)

in the model with a representative agent whose preferences exhibit very low risk aversion. The wealth distribution remains constant across all date-events. In the second benchmark model *B2* the less risk-averse agent 1 holds the entire tree during the vast majority of time periods. A bad shock to the economy leads to shifts in the wealth distribution and a decrease of the tree price. However, since in our calibration shocks are iid, these shifts in the wealth distribution have generally small effects on prices (except in the very low-probability case of several consecutive disaster shocks). The resulting price volatility in model *B2* is of similar magnitude as the volatility in *B1*. Moreover, in the model *B2* the risk-free rate is high and the equity premium is very low. Despite the presence of disaster shocks, the market price of risk is low because it is borne almost entirely by agent 1 who has very low risk aversion.

Table 2 shows that both first and second moments show substantial differences when we compare models without collateral requirements to a model with tight collateral constraints. The perhaps most striking result reported in Table 2 is that volatility in our baseline economy is about 50 percent larger than in the two benchmark models without borrowing (*B1: No bonds*) and with natural borrowing constraints (*B2: Unconstrained*), respectively. The standard deviation of returns is 8.14 percent in the baseline economy but only 5.33 percent and 5.38 percent for the benchmark models *B1* and *B2*, respectively.[3]

Collateral constraints drastically increase the volatility in the standard incomplete markets model. Figure 1 shows the typical behavior of four variables in the long run during a simulation for a time window of 200 periods. The first graph displays agent 1's holding of the Lucas tree. The second graph shows the normalized tree price, that is, the equilibrium price of the tree divided by aggregate consumption in the economy. The last two graphs show the price and agent 1's holding of the risk-free bond, respectively. In the sample displayed in Figure 1, the disaster shock $s = 3$ (smallest disaster with a drop of aggregate

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[3]The stock return volatility in our baseline economy is considerably smaller than the volatility in U.S. data. For comparison, Lettau and Uhlig (2002) report that the quarterly standard deviation of returns of S&P-500 stocks in post-war US data is about 7.5 percent. Similarly, Fei et al. (2008) report an annual volatility of about 14.8 percent for the period January 1987 to May 2008. However, it is important to note that we want to interpret the aggregate tree as a mix of stocks and housing assets. The volatility of housing prices is U.S. data is much lower. Fei et al. (2008) report an annual volatility of the Case/Shiller housing price index of less than 3 percent (for January 1987 to May 2008). A similar comment applies to the equity premium. While the average risk-free rate roughly matches U.S. data, the equity premium is substantially lower than in the data. We discuss this point in more detail in Section 4 for an economy with two trees.

consumption of 13.3 percent) occurs in periods 71 and 155 while disaster shock 2 occurs in period 168 and disaster shock 1 (worst disaster) hits the economy in period 50.
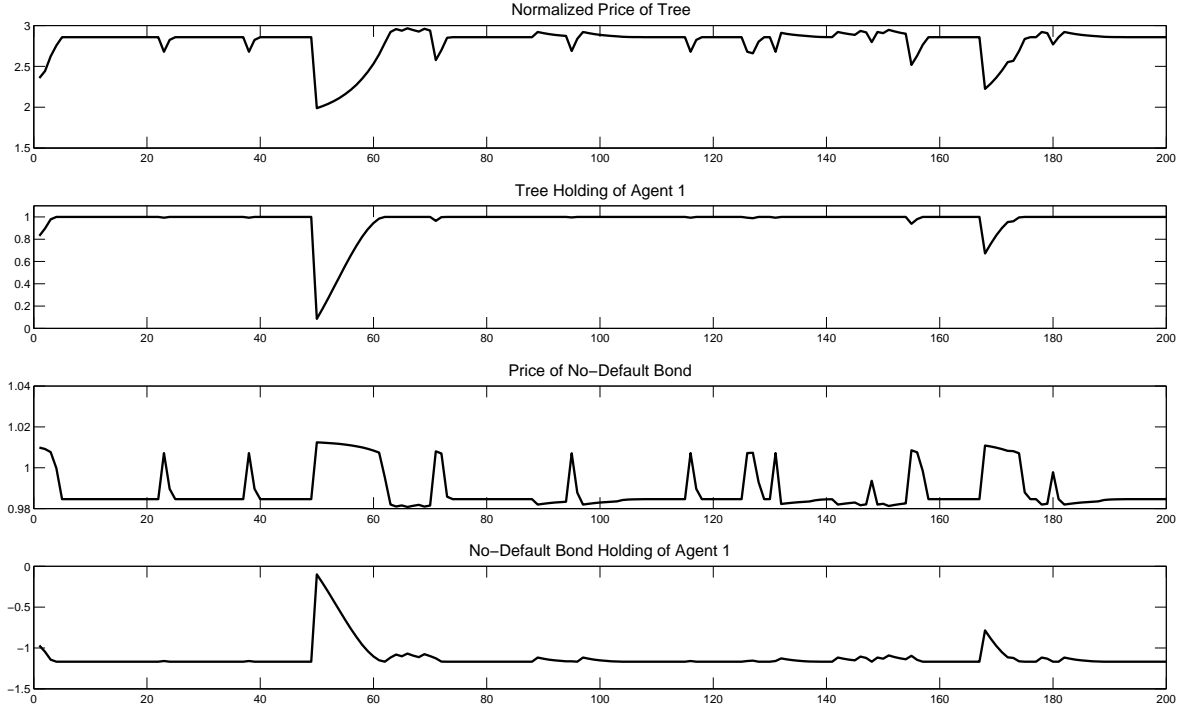

Figure 1: Snapshot from a simulation of the baseline model

When a bad shock occurs, both the current dividend and the expected net present value of all future dividends of the tree decrease. As a result the price of the tree drops, but in the absence of further effects, the normalized price should remain the same since shocks are iid. (That's exactly what happens in the benchmark model *B1*.) In our baseline economy with collateral constraints, however, additional effects occur in equilibrium. First, note that agent 1 is typically leveraged, that is, when a bad shock occurs his beginning-of-period financial wealth falls relative to the financial wealth of agent 2. This effect is the strongest when the worst disaster shock 1 occurs. If agent 1 was fully leveraged in the previous period then her wealth decreases to zero because shock 1 always determines the collateral requirement $k^j_{a(j)}$.

High leverage leads to large changes in the wealth distribution when bad shocks occur. The fact that collateral is scarce in our economy now implies that these changes in the wealth distribution strongly affect equilibrium portfolios and prices. Since agent 1 cannot borrow against her future labor income, she can only afford to buy a small portion of the tree if her financial wealth is low. In equilibrium, therefore, the price has to be sufficiently low to induce the much more risk-averse agent 2 to buy a substantial portion of the tree. On top of that within-period effect, there is a dynamic effect at work. As agent 1 is poorer

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today, she will also be poorer tomorrow (at least in shocks 2-6) implying that the price of the tree tomorrow is depressed as well. This further reduces the price that agent 2 is willing to pay for the tree today. Clearly, this dynamic effect is active not only for one but for several periods ahead, which is displayed in Figure 1 by the slow recovery of the normalized price of the tree after bad shocks. Figure 1 shows that the total impact of the above described effects is very strong for shock $s = 1$ but also large for shock 2. Note that the prices are normalized prices, so the drop of the actual tree price is much larger than displayed in the figure. In disaster shock 1, agent 1 is forced to sell almost the entire tree and the normalized price drops by almost 30 percent (the actual price drops by approximately 60 percent). In shock 2 she sells less than half of the tree but the price effect is still substantial. In shock 3 the effect is still clearly visible, although the agent has to sell only very little of her tree.

While the effects of collateral and leverage on volatility are very large, it is important to note that in the baseline specification of our model with a single tree and a single bond there is no *financial accelerator*. Kiyotaki and Moore (1997), Aiyagari and Gertler (1999) and others highlight the idea that in the presence of collateral constraint the fact that the price of collateral decreases might make it more difficult for the borrower to maintain his debt position because collateral requirements increase in anticipation of a value of the collateral in the next period which is now lower than if the shock had not happend. In the baseline case, this effect is absent for two reasons. First, whenever agent 1 is constrained, the collateral requirement $k_{a(j)}^j$ is independent of today's price of the collateral, it is in fact constant. This is because the collateral requirement is determined by tomorrow's tree price (plus dividend) in case of the worst shock. If this shock occurs and agent 1 is constrained today, he has to use his entire tree holding to repay his debt. Hence, no matter how large agent 1's tree holding is today, he ends up with zero financial wealth tomorrow. This implies a specific price for the tree tomorrow in shock 1 which is independent of today's price (as long as agent 1 is constrained) and consequently a specific collateral requirement today[4]. Second, an examination of the bond price in Figure 1 reveals an important general equilibrium effect in our economy that counteracts an increase of the margin requirement. When a bad shock occurs and the share of financial wealth of agent 1 decreases, then the demand of the now relatively richer agent 2 for the risk-free asset increases the bond price substantially. In fact, occasionally the interest rate even becomes negative. As a result of the constant collateral requirement, the increase in the bond price and the decrease in the tree price the equilibrium margin requirement actually decreases substantially in a bad shock.

In sum, scarce collateral plays an important role for the volatility of the tree price because it leads to large price drops in bad shocks since agent 1 cannot borrow against future labor income. As we would expect, this effect depends on the amount of available collateral in the economy. Figure 2 illustrates this point. The figure depicts the tree's

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[4]If we assume that the tree's dividends cannot be used as collateral, this argument is no longer correct. However, for our calibration the effects of this assumption are quantitatively negligable.

average return volatility and the fraction of times the collateral constraint is binding for agent 1 (i.e. the probability of constraint being binding) as a function of the dividend share δ in the economy.
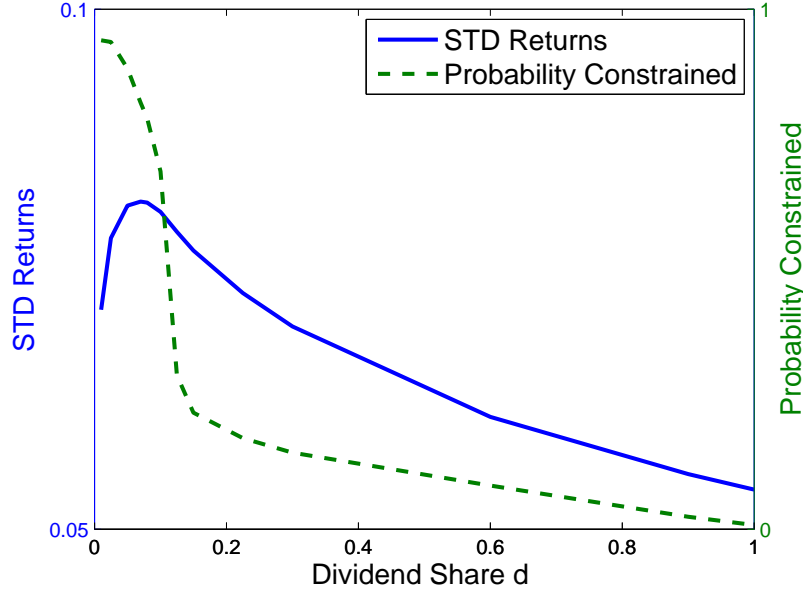

Figure 2: Volatility as a function of the dividend share

For very small values of δ, there is only little collateral in the economy and so the collateral constraint is almost always binding. However, the stock is so small that agent 1 does not have to sell the stock even if the economy is hit by an extremely bad aggregate shock. The resulting return volatility is relatively small. As δ increases the probability of the collateral constraint being binding decreases rapidly but the effects of it being binding become larger. There is an interior maximum for the stock-return volatility around $\delta = 0.07$. Although the constraint is much less often binding than for a smaller tree, the trade-off between agent 1 being forced to sell the tree and agent 1 getting into this situation leads to maximal volatility. As δ increases further, the constraint becomes binding much less frequently and eventually at $\delta = 1$ the stock return volatility is very low, simply because the collateral constraint never binds and so collateral plays no role. This situation is identical to the case of natural borrowing constraints where a binding constraint would imply zero consumption for the borrower.

3.2 Collateral and several bonds

In the economy with a single tree and a single bond, equilibrium margin requirements are sufficiently high to ensure that there is no default. The bond is risk-free and always pays its face value. We now examine whether the observed results are just a consequence of this

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restrictive assumption. In the enhanced model a menu of bonds is available for trade and the accompanying collateral requirements are endogenously determined in equilibrium.

3.2.1 Full set of bonds without costly default

Our calibrated model with $S = 6$ exogenous states allows the analysis of economies with five bonds. As explained above, these bonds are characterized by the shocks in which they are on the 'verge of default' and so we call them no-default bond, 1-default bond, 2-default bond, etc. Figure 3 shows the portfolio holdings of agent 1 as well as the normalized tree price along the same simulated series of shocks as in Figure 1 above.
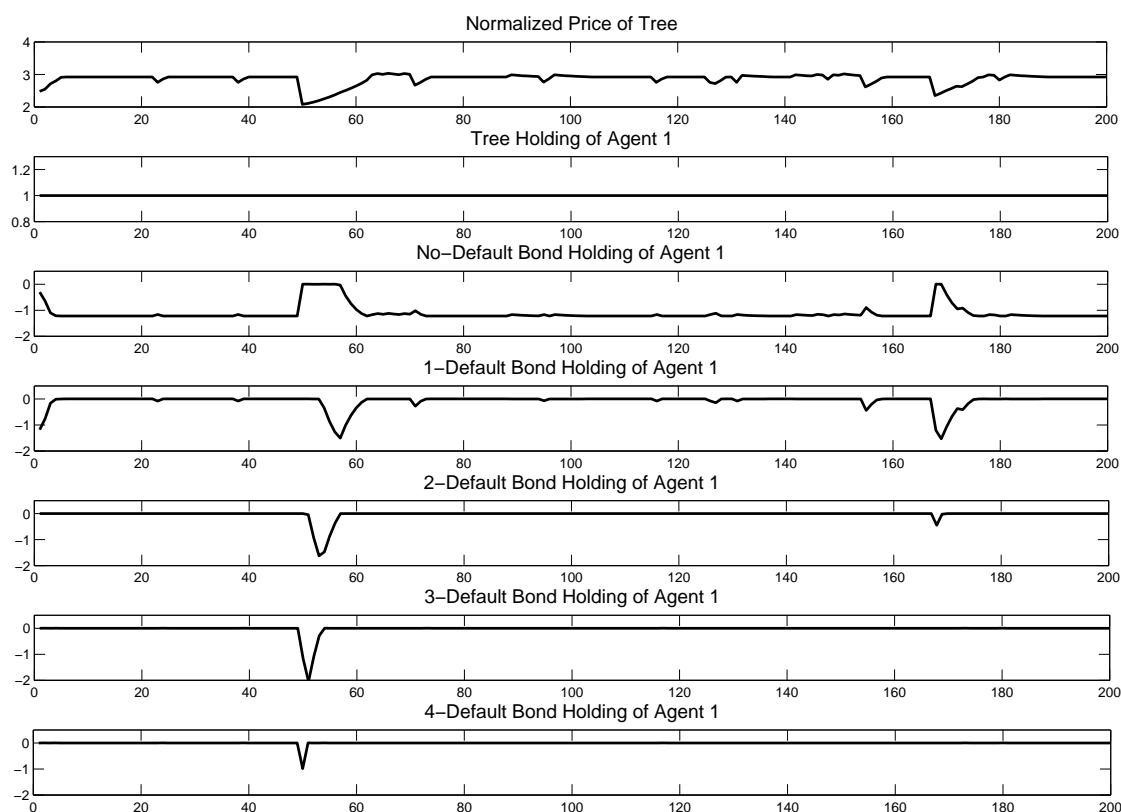

Figure 3: Snapshot from a simulation of the model with 1 tree and 5 bonds

During "normal times" (that is, if the last disaster shock occurred sufficiently long ago) only the no-default bond is traded in equilibrium. (There is a tiny amount of trade in the 1-default bond in recessions, shock 4, which is quantitatively negligible.) In normal times the agents' portfolios resemble those in an economy with a single risk-free bond. The risk-averse agent 2 holds the risk-free bond while agent 1 holds the risky tree and is short in the bond.

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Disaster shocks are the only reason for equilibrium trade in default bonds. In our economy, the risk-averse agent 2 always seeks to buy an asset that insures him against bad aggregate shocks — only the risk-free bond can play this role. However, the risky default bonds play an important role once a disaster shock occurs. Agent 1 no longer needs to sell the stock but is now able to raise additional funds by selling default bonds to agent 2. Such a trade shifts some of the tree's risk to agent 2 who demands a high interest rate for assuming such risk. But the default bonds are still less risky than the tree and thus preferred by the risk-averse agent. In fact, the presence of the default bonds enables agent 1 to always hold the entire tree. Figure 3 shows that after an occurrence of the worst disaster shock 1, which happens in period 50, agent 1 is able to hold on to the entire tree and to sell the 4-default and the 3-default bond to agent 2. As the economy recovers, agent 1 sells the 1-default bond to agent 2 and holds a short-position in this bond for approximately 10 periods until her wealth has recovered sufficiently so that she is able to leverage exclusively in the default free bond.

Despite the fact that the leveraged agent 1 no longer has to sell the tree after bad shocks, such shocks continue to have a strong impact on asset prices. Figure 3 shows that the normalized tree price decreases in all three disaster shocks as well as in recessions, just as in an economy with a single risk-free bond, see Figure 1. By selling the default bonds to the risk-averse agent 2, agent 1 shifts the tree's (tail) risk to agent 2. This circumstance must be reflected in the equilibrium price. This reasoning becomes clear if we considered the case of identical dividends in shocks 5 and 6. Under this scenario, the tree and the 4-default bond have identical payoffs and hence it should be irrelevant for the price of the tree who holds it, that is, whether agent 1 holds it financed by a short position in the 4-default bond or agent 2 holds it directly.

Moreover, unlike in the previous model with one bond, the financial accelerator now plays a role. A lower tree holding of agent 1 in this period reduces the price of the tree in the next period in shocks 2-6 and hence makes it more difficult for agent 1 to hold default bonds.

Table 3 reports the tree-return volatility for economies with $1, 2, \ldots, 5$ bonds, respectively. The presence of a bond that defaults only in shock 1 (when the economy shrinks by 43.4 percent) leads to a decrease in the volatility of the tree price. A third bond that defaults in shocks 1 and 2 leads to an additional small reduction of volatility. The impact of additional bonds is negligible. This fact is not surprising since we observed that these bonds are rarely traded.

	One bond	Two bonds	Three bonds	Four bonds	All bonds
Std returns	8.14	7.87	7.84	7.84	7.84

Table 3: The effect of endogenous margins on return volatility

Unfortunately, the fact that investors only trade bonds with a high probability of default during bad times seems counterfactual. Several features of our model may lead to this result. Clearly bad times are often persistent and not iid as in our calibration. More importantly, default is typically costly. We next show that fairly small default costs eliminate trade in default bonds.

3.2.2 Costly default

Until now our treatment of default is somewhat unsatisfactory since it neglects both private and social costs of default. We now introduce default costs as described in Section 2.1 above. Table 4 shows how the trading volume of the default bonds changes as a function of the cost parameter λ. The reported trading volume is the average absolute bond holding of agent 1 (which is the same as that of agent 2) over the simulation path.

	$\lambda = 0$	$\lambda = 0.01$	$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.2$	$\lambda = 0.25$
Std dev tree return	7.84	7.87	7.98	8.12	8.15	8.14
Total trading	1.260	1.236	1.183	1.161	1.126	1.123
No-default bond	1.110	1.099	1.076	1.076	1.099	1.123
1-default bond	0.084	0.080	0.075	0.085	0.027	0
2-default bond	0.034	0.034	0.032	0	0	0
3-default bond	0.026	0.023	0	0	0	0
4-default bond	0.006	0	0	0	0	0

Table 4: The effect of default costs on tree-return volatility and bond trading volume

In the absence of default costs ($\lambda = 0$), the average trading volume of all bonds is nonzero. As we observed in the previous section, it is substantial for the no-default and 1-default bond and rather small for the remaining bonds. Proportional default cost of as low as 10 percent ($\lambda = 0.1$) result in zero trade for the bonds defaulting in two or more states. For default costs of 25 percent, trade in any type of default bond ceases to exist. Only the risk-free bond is traded and the resulting equilibrium prices and allocations are identical to our baseline economy above.

Recall from the description in Section 2.1 that the cost is proportional to the difference of the face value of the bond and the value of the underlying collateral. Therefore, a proportional cost of 25 percent means a much smaller cost as a fraction of the underlying collateral. Campbell et al. (2010) find an average 'foreclosure discount' of 27 percent for foreclosures in Massachusetts from 1988 until 2008. This discount is measured as a percentage of the total value of the house. As a percentage of the difference between the house value and face value of the debt this figure would be substantially larger. A value of $\lambda = 0.25$, therefore, seems certainly realistic and is, if anything, too small when we compare it to figures from the U.S. housing market.

Table 4 also reveals that the trading volume of the 1-default bond remains stable up to default costs of around 10 percent when other default bonds are no longer traded. The 1-default bond remains an attractive asset in this economy even for moderate default costs. This bond enables agent 1 to insure against the worst disaster state. This shock happens with very low probability, but when it occurs then the financial consequences for the tree owner are severe.

Table 4 also shows that the volatility of the tree return increases as cost of default increases, and for sufficiently high default cost the economy is the same as the baseline economy with a single risk-free bond. It appears that an economy with default costs of 20 percent and trade in the 1-default bond exhibits slightly higher return volatility than the baseline economy. This feature is due to the fact that default implies real losses in our economy which make the economic impact of the worst disaster shock even worse since default leads to a further drop in aggregate endowment.

3.3   Volatility with regulated margin requirements

As a final step in the analysis of economies with a single collateralizable tree, we consider the case of regulated collateral requirements as described in Section 2.1. We assume that there is a regulatory agency setting minimal margin requirements (just as in stock markets). We first consider margin requirements that are constant across all shocks, so $m_{a(j)}^{j}(s^t)$ does not depend on the current date-event s^t. As margin requirements become larger, we observe two opposing effects. On the one hand, the amount of leverage decreases in equilibrium which leads to less de-leveraging in disaster shocks which in turn leads to smaller price changes. On the other hand, the collateral constraint is more likely to become binding in equilibrium which increases the probability of de-leveraging episodes which in turn should lead to a higher volatility of the tree return. The solid line in Figure 4 displays the resulting tree return volatility.

Initially, volatility increases as margin requirements increase. At a margin level of about 70 percent, the volatility reaches its maximum. A further tightening of margins then decreases volatility substantially. Of course, as the margin level approaches one the economy approaches the benchmark model *(B1: No bonds)* without borrowing and so volatility becomes very small.

At a margin level of 60 percent, the implied collateral requirement uniformly exceeds the corresponding varying levels for the no-default bond under the rule of endogenous collateral requirements in our baseline economy analyzed above. Therefore, the regulated bond is default-free for all possible values of $m_{a(j)}^{j}$ in Figure 4. Interestingly, for values of the margin level between 60 and 80 percent, the regulated bond leads to higher tree return volatility than the no-default bond under the rule of endogenous collateral requirements.

As a last exercise, we examine an economy in which margins are only regulated in booms while in recessions and disasters they are left to the market. In particular, we assume that
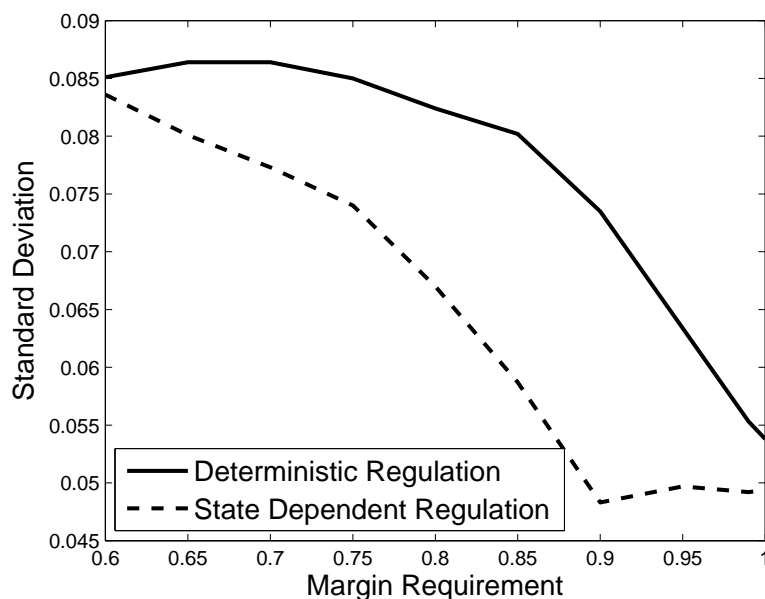
Figure 4: Volatility as a function of the margin requirement

in shocks 1 through 4 collateral requirements are endogenously determined at the level of the risk-free bond as in our baseline economy, while a regulating agency sets margin requirements in the shocks with positive growth. We assume that the margin levels are set to the same level in both shocks 5 and 6. The dashed line in Figure 4 shows the resulting tree return volatility.

It is readily apparent that limiting the regulation of margin requirements to boom times reduces the tree return volatility substantially if margin levels are sufficiently high. For example, boom-time margin levels of 80 percent lead to a return volatility of 6.5 percent as compared to values exceeding 8 percent when collateral requirements are determined endogenously or margin regulation is state-independent.

Why is state-dependent regulation so much better in reducing volatility? As with state-independent margins, agent 1 holds less leverage in good times, which leaves him with more financial wealth if a bad shock hits. In addition, collateral constraints are now looser in case of a bad shock and agent 1 may retain an even larger portion of the tree. In the extreme, if margin requirements in booms are well above 80 percent, agent 1 even increases its tree holding in case of a bad shock. This increases the relative price of the tree and thus dampens the drop in the absolute price. All in all, setting conservative margins in good times turns out to be a powerful tool to dampen the negative impact of bad shocks.

4 Two trees

Up to this point our analysis focused on an economy with a single tree representing aggregate collateralizable wealth in the economy. However, households trade in various assets and durable goods. Some of them, e.g. houses, can be used as collateral very easily and at comparatively low interest rates, others assets, e.g. stocks, can only be used as collateral for loans with high margin requirements and typically very high interest rates (see Willen and Kubler, 2006), and still others, like works of art, cannot be used as collateral at all. These observations motivate us to examine a model with two Lucas trees. For simplicity, we assume that the two trees have identical cash-flows and distinguish themselves only by the extent to which they can be used as collateral. This model feature allows for a clean analysis of the effect of collateral. We consider two different cases. First, we assume that tree 1 can be used as collateral with endogenous margin requirements, while tree 2 cannot be used as collateral. We then allow the second tree to serve as collateral, but we assume that the collateral requirements on loans backed by tree 2 are exogenously regulated. In both cases we find that the two assets' price dynamics are substantially different, despite the fact that they have identical cash-flows. Furthermore, we show that tightening the margin requirements on the regulated tree has a strong impact on the return volatility of the non-regulated tree. This effect proves to be quantitatively important. Our analysis suggests that this effect should be carefully considered in any policy discussion on the regulation of margin requirements.

4.1 Only one tree can be used as collateral

We first consider the case where the second tree cannot be used as collateral. As before in an economy with a single tree, default costs of $\lambda = 0.25$ suffice to shut down all trade in default bonds. We therefore restrict attention to an economy in which only the no-default bond is traded. We conclude the analysis in this section below with a brief discussion of an economy with costless default and argue that it produces similar quantitative results.

	Std returns	EP agg	Std returns agg	Risk-free rate	Equity-premium
Tree 1	6.64	3.69	7.04	0.38	4.50
Tree 2	8.05	6.31			

Table 5: Moments of trees' returns (only tree 1 collateralizable)

Table 5 reports moments of the two trees' returns as well as the interest rate and aggregate moments. Observe that the two trees exhibit substantially different returns despite the fact that the two trees have identical cash-flows. The tree that can be used as collateral, tree 1, now exhibits much lower return volatility and a slightly lower expected excess return than the single tree in the baseline economy in Section 3. The standard deviation of

returns of the second tree is much higher than that of tree 1. In fact, it is comparable to the corresponding value (8.14) of the single tree in the baseline economy. Turning to equity premia, the excess return of tree 2 — the tree that cannot be used as collateral — is now almost twice as large as it is for the single tree in the baseline economy and is similar to figures observed in the data.

To understand the price dynamics of the two trees, we consider the analogue of Figure 1. Figure 5 shows the time series of eight variables along 'our' sample path. The first two graphs show the (normalized) price and the first agent's holding of tree 1, respectively. The next two graphs display the corresponding values for tree 2. The fifth and sixth graph show the corresponding values for the no-default bond. The price and holding graphs reveal three features of the equilibrium. First, the price volatility of tree 1 is much lower than that of the single tree in the baseline economy. Secondly, the price volatility for tree 2 is larger than for tree 1 and its average price is much smaller. Lastly, agent 1 holds tree 1 the entire time (except for a tiny blip in disaster shock 1) but frequently sells tree 2. The second-to-last graph in the figure shows the endogenous margin requirement and the last graph depicts the *collateral premium* for tree 1. This quantity is the difference between the actual price of the tree and next period's payoff, normalized with agent 1's marginal utilities. Whenever agent 1 is unconstrained then this value is zero. However, when agent 1 becomes constrained, the collateral premium is significant.

Our observations lead us to a simple explanation of the first moments for the two tree prices. Tree 1 is more valuable to agent 1 because of its collateral value — when agent 1 is fully leveraged the value of the tree exceeds next period's discounted (with agent 1's state prices) cash-flows since it provides value for agent 1 as collateral. Since both trees have identical cash-flows, an agent can only be induced to hold tree 2 if it pays a higher average return. The specific magnitude of the difference between the two tree prices is, of course, a quantitative issue. In our calibration with a reasonable market price of risk, the effect is indeed large — the average excess return of the second tree is now comparable to that observed in U.S. stock market data.

There are several key factors that play a role for asset price volatility in the two-tree economy. For a discussion of these factors it is helpful to consider the policy and price functions in Figure 6. When faced with financial difficulties after a bad shock, agent 1 holds on to tree 1 for as long as possible, because this tree allows her to hold a short-position in the bond. (In fact, as the bond-holding function of agent 1 in Figure 6 shows, agent 2 never goes short in the bond. Therefore, the collateral value is one of the reasons why tree 1 is much more valuable to agent 1.) So, after suffering a reduction in financial wealth, agent 1 first sells tree 2. In fact, in our calibration agent 1 only sells a portion of tree 1 after she sold off the entire tree 2. In our sample path this happens only after the worst disaster shock in period 50. (Of course, the policy functions in Figure 6 show that it would happen in a more pronounced way after two or more consecutive disaster shocks but such a sequence has extremely low probability.) Whenever agent 1 sells a portion of a
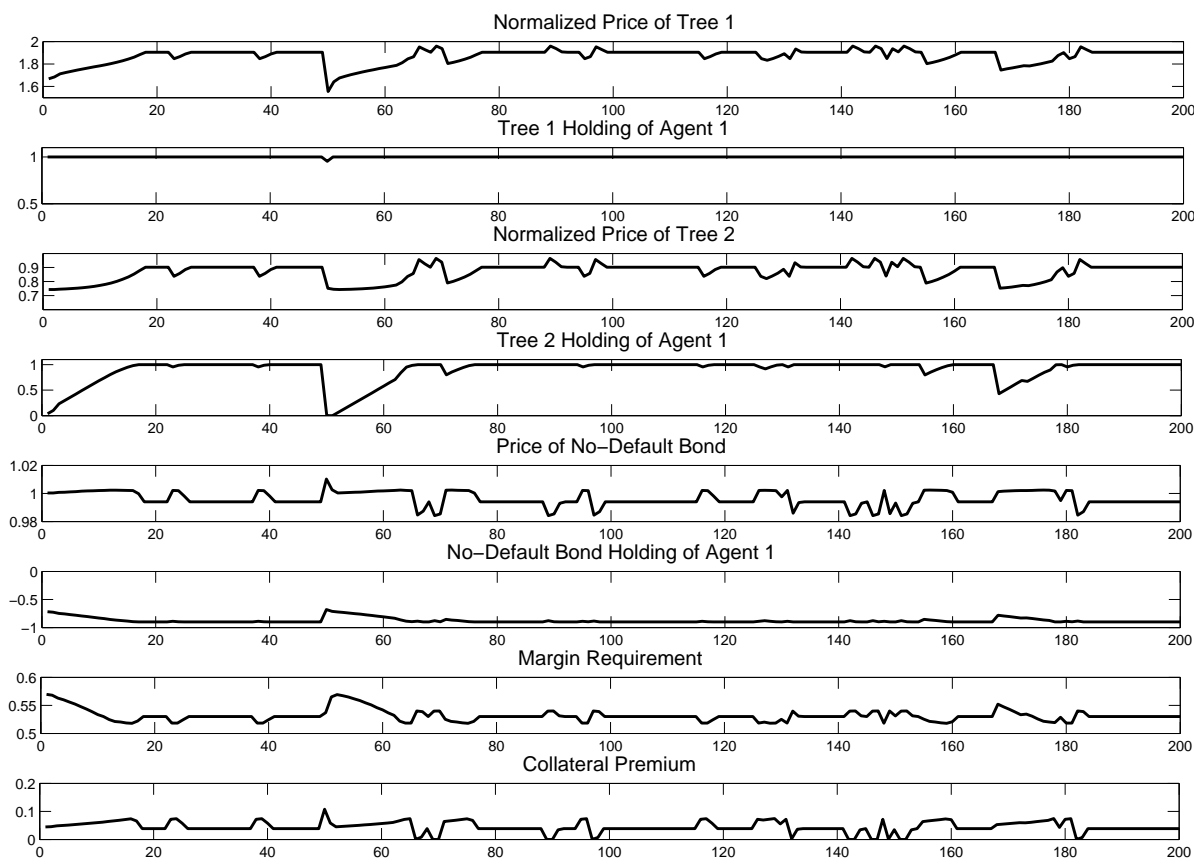
Figure 5: Snapshot from a simulation of the model with 2 trees and 5 bonds

risky tree to agent 2 its price must fall, just as in the single-tree baseline economy. And so one key factor contributing to the different volatility levels of the two trees is that tree 2 is traded much more often and in larger quantities than tree 1.

Furthermore, since tree 2 is not collateralizable, only half of the aggregate tree can be used as collateral. This constraint limits the ability of agent 1 to leverage and consequently makes her less vulnerable to negative aggregate shocks. This factor reduces the return volatility of both trees.

If agent 1 holds both trees and then becomes poorer after a bad shock, the prices of both trees fall. But since the agent first sells tree 2, the price of tree 2 falls much faster than the price of tree 1. In fact, the price drop for tree 1 is dampened by the onset of the collateral premium. This effect also contributes to the difference in the return volatilities of the two trees.

Finally, there is another key effect that was not present in the one-tree baseline economy. Now the financial accelerator plays an important role! In 'normal times' agent 1 holds both trees but is fully leveraged. In a bad shock, agent 1 must sell part of tree 2 which makes
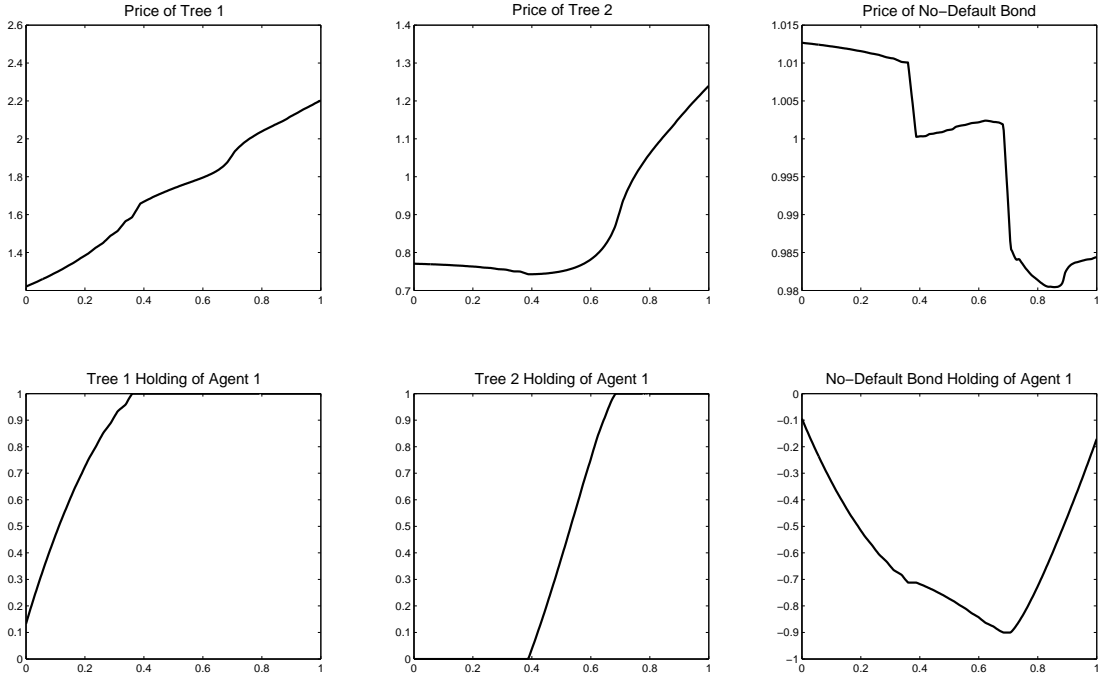
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Figure 6: Price and policy functions of the model with 2 trees and 5 bonds

him poorer in the subsequent period. This in turn increases the collateral requirement this period, leading to an increase in the margin requirement despite the fact that the interest rate decreases. This effect is clearly visible in the second-to-last graph of Figure 5. Whenever a bad shock occurs the margin requirement increases.

In sum, the fact that only 4 percent of aggregate output are collateralizable in this economy leads to a decrease in leverage and to much smaller movements in the wealth distribution than in the baseline economy. This effect reduces the return volatility of tree 1. For tree 2 such a reduction effect is strongly counteracted through two channels. First, the price of tree 2 is not stabilized by a collateral premium since this tree cannot be used as collateral. Secondly, a decrease in the holdings of tree 2 leads to an increase in the margin requirements for loans on tree 1 which in terms forces agent 1 to sell more of tree 2 (recall that initially he does not sell tree 1, since only this tree can be used as collateral).

While we do not want to push the interpretation of our results too far, it is worthwhile to note that a natural interpretation of the two trees is the aggregate stock market versus the aggregate housing market. As Willen and Kubler (2006) report, it is much more difficult to use stocks instead of a house as collateral. The data clearly shows that volatility in the stock market is much higher than in the housing market, see Fei et al. (2008). This interpretation clearly should be taken with some caution, since we do not really have a good model of the housing market — such a model would need to include transaction costs,

non-divisibilities, and certainly different cash-flow dynamics. Nevertheless it is worthwhile to point out that the equity premium for tree 2 is similar to what can be observed in the data for stock returns. Moreover, volatility of "housing returns" (tree 1) is much smaller than that of stock returns.

We complete our discussion of the economy in which the second tree cannot be used as collateral with a robustness check and consider the case of costless default, $\lambda = 0$. Just as in the economy with a single tree, the default bonds are traded if the economy experiences a disaster shock. However, trade in these bonds is typically much smaller because agent 1's financial wealth remains larger, as we discussed above. Overall, zero default costs lead to very small changes in the first and second moments. Without default costs, the standard deviation of tree 1's return drops from 6.64% to 6.56% while the standard deviation of tree 2's return drops from 8.05% to 7.98%.

4.2 One tree is regulated

Until now we have assumed that tree 2 cannot be used as collateral. This assumption is rather restrictive if not unrealistic. Stocks can be used as collateral, however, margins are regulated and large, and interest rates are much higher than mortgage rates. Therefore, we assume now that margins for tree 2 are set exogenously while collateral requirements for tree 1 are endogenous. Throughout this section, we assume default costs of $\lambda = 0.25$ which suffice to shut down all trade in default bonds.

4.2.1 State-independent regulation of tree 2

As before, we first consider margin requirements that are constant across states. The effect of an exogenous margin requirement is obvious in the limit as the requirement m_2 for tree 2 approaches one. In this case the resulting collateral requirement k_2 diverges to infinity and so the model tends to the economy of Section 4.1 in which this tree cannot be used as collateral. Figure 7 display the volatility of both trees' returns as a function of the margin m_2 set for tree 2. Observe that as m_2 tends to one, the return volatilities for the two trees approach the values from Table 5, namely 6.64% and 8.05%, respectively.

Figure 7 shows the return volatilities for values of m_2 between 0.6 and 1. The lowest value of 0.6 of the margin requirement exceeds the endogenously determined (unregulated) margin requirement of tree 1 in all states. As a result, the return volatility of tree 2 is higher that that of tree 1. If margin requirements on tree 2 are now increased, the volatility of this tree's return initially increases, while the volatility of the freely collateralizable tree 1 substantially decreases. The volatility of tree 2 is largest when its exogenous margin requirement is quite high (about 75 percent). After this peak, the volatility of tree 2 decreases until the boundary value of one has been reached. At this point tree 2 can no longer be used as collateral. The quantitatively most interesting case is a regulated margin requirement of 75 percent. At this point, the volatility of tree 2 is above 8.6 percent while
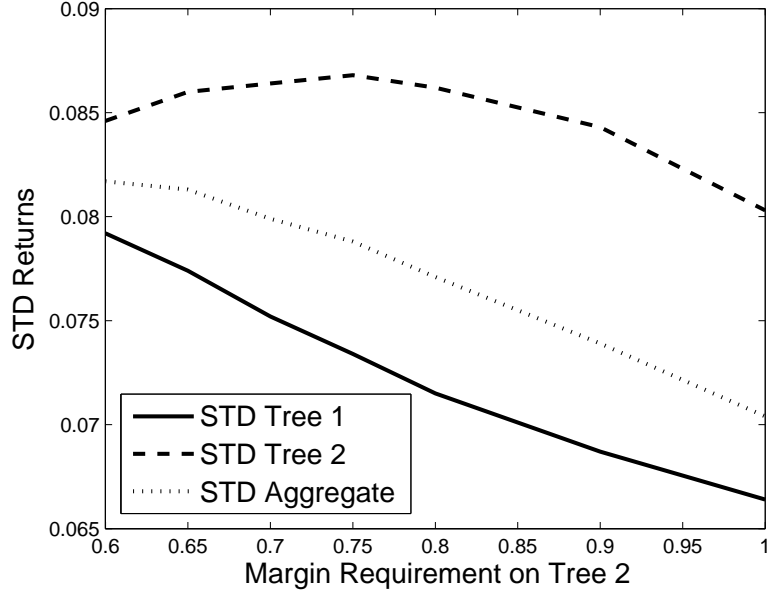
Figure 7: Volatility of tree 1 and 2 as a function of the margin requirement on tree 2

the volatility of tree 2 is below 7.5 percent. Aggregated volatility is still high, but it is readily apparent that the regulation of tree 2 has substantial effects on its own volatility as well as on the volatility of the other, unregulated, tree 1.

For an interpretation of the observed volatility variation, note that an increase of the margin requirement m_2 of tree 2 has two immediate effects. This tree becomes less attractive as collateral and the agents' (aggregated) ability to leverage decreases. These two effects influence agent 1's portfolio decisions after a bad shock occurs. First, when agent 1 must de-leverage her position, then she first sells tree 2. In equilibrium, this effect occurs more often as m_2 increases. Initially this effect leads to an increase in the return volatility of tree 2. The second effect, a reduced ability to leverage, decreases the return volatility of tree 1. Similar to the effect we observed in the one-tree economy in Section 3, the return volatility of tree 1 decreases as agent 1's ability to leverage decreases. The reason for this effect is the increased probability with which she can hold onto the tree after a bad shock.

Observe that the two described effects counteract each other for tree 2. For small increases of m_2 above 0.6, the first effect dominates the second and the tree's return volatility increases. As m_2 increases further, the second effect eventually dominates and the return volatility of tree 2 starts to decrease. Moreover, as the margin requirement on tree 2 becomes large, price effects as a result of agent 1 de-leveraging her positions become smaller. Recall that whenever agent 1 is collateral constrained, then the price of the underlying collateralized tree reflects a collateral premium. Since agent 2 never enters leveraged positions, this price impact is never present when agent 2 holds tree 2. As a result the collateral premium affects the price volatility of tree 2. This effect is greatly diminished as m_2 be-

comes sufficiently large. Put differently, the impact of the collateral premium on the return volatility fades as m_2 gets large.

To support this interpretation of our results, it is interesting to consider the excess returns of the two trees as a function of the margin requirement m_2 on tree 2. Figure 8 shows that the relation between excess return and m_2 is monotone for tree 2. As its margin
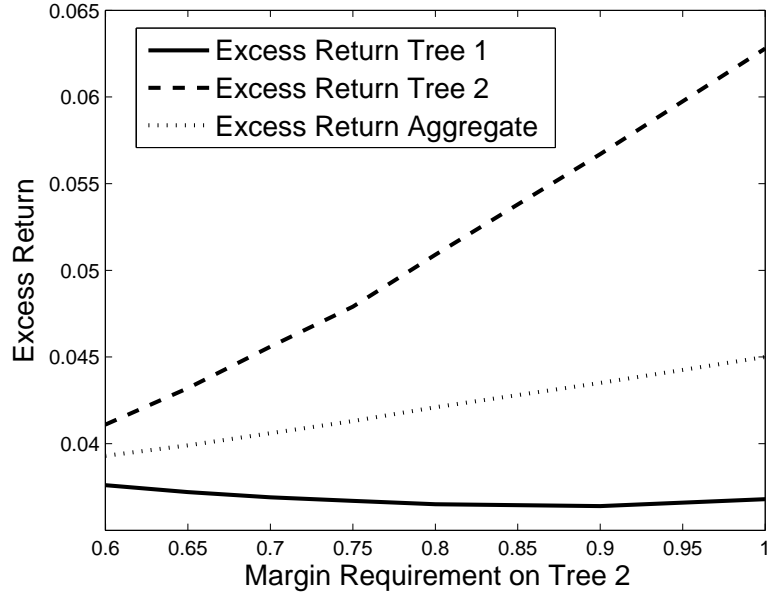

Figure 8: Excess returns

requirement increases, the collateral premium and the price of the tree decrease and the average return increases. For tree 1, average excess returns remain more or less constant. They initially decrease slightly, then increase slightly. Aggregate excess returns increase, but clearly the quantitatively most striking effect is on the returns of tree 2. Collateral constraints and regulated margins clearly have a quantitatively significant impact on asset prices in this economy.

4.2.2 State-dependent regulation of tree 2

Our results so far have shown that, for moderate margin requirements between 0.6 and 0.75, it is impossible to reduce volatility for both trees by adjusting the regulated margin of tree 2. A small change of the margin requirement always reduces volatility of one tree at the expense of the other. We now analyze whether a state-dependent regulation of the second tree can solve this dilemma.

We examine an economy in which the margins of tree 2 are only regulated for positive-growth shocks 5 and 6 while they are endogenously determined for the remaining four shocks. Figure 9 shows that the return volatilities of both trees are monotonically decreasing in the margin requirement imposed on the regulated tree 2 in good shocks. Not only does

increasing the regulated margin now reduce the volatilities of both assets, but, in fact, it does reduce aggregate market volatility much more than in the economy with state-independent regulation. For instance, an increase of state-dependent margin requirements from 0.6 to 0.7 on tree 2 decreases aggregate volatility by about 4.5% (see Figure 9), while such an increase would bring about a reduction of only 2% in the case of state-independent regulation (see Figure 7). Therefore, concerning the regulation of margin requirements, the result from
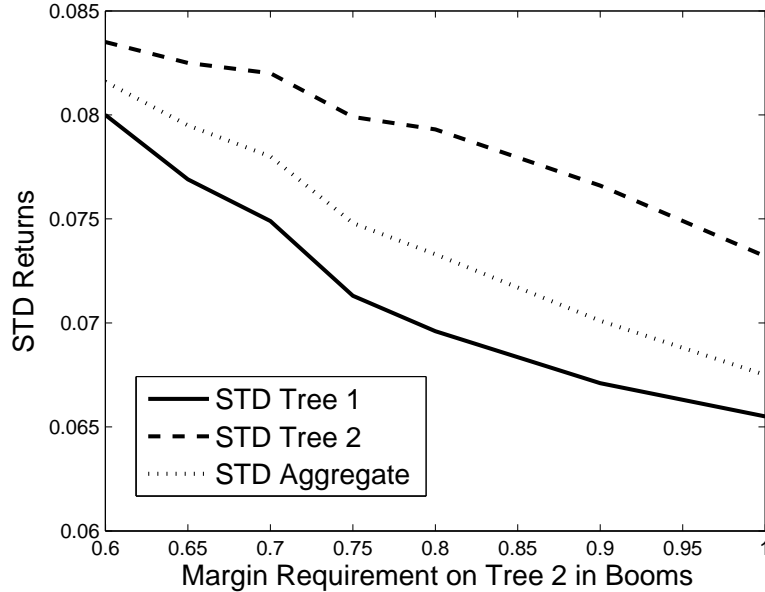

Figure 9: Volatility as a function of the margin requirement on tree 2 in booms

the single-tree economy is strongly confirmed by the analysis of the two-tree economy: regulation is much more efficient at reducing price volatility, if it is state-dependent.

5 Sensitivity analysis and extensions

As in any quantitative study, our results above hinge on the parametrization of the economy. In this section, we first discuss how our results change with other preference parameters. Then we highlight the important role of the disaster shocks for our quantitative results. Finally, we present an example which has less severe disaster shocks but nevertheless exhibits strong quantitative effects of collateral constraints.

5.1 Different preferences in the baseline model

As a robustness check for the results in our baseline model (with one tree and one bond) from Section 3, we consider different specifications for the IES, the coefficients of risk aversion, and the discount factor, β. Obviously, changes in the IES and the risk aversion coefficients

affect the risk-free rate. For these cases, we also examine specifications with an adjusted β so that the risk-free rates remain comparable. Table 6 reports asset-price moments for several different combinations of these parameters. For convenience, we repeat the results for our baseline model, $(IES, RA, \beta) = ((1.5, 1.5), (0.5, 6), (0.95, 0.95))$, and report them as the case (P1). For each model specification, we also report the standard deviation of returns for the benchmark case *B1: No bonds*.

$(IES^1, IES^2), (RA^1, RA^2), (\beta^1, \beta^2)$	Std returns	Risk-free rate	EP	Std in B1
(P1): (1.5,1.5),(0.5,6),(0.95,0.95)	8.14	1.10	3.86	5.33
(P2): (0.5,0.5),(0.5,6),(0.95,0.95)	7.20	1.75	4.18	5.33
(P3): (1.5,1.5),(0.5,6),(0.92,0.92)	7.70	4.07	3.77	5.51
(P4): (1.5,1.5),(0.5,6),(0.98,0.98)	8.57	-1.17	3.95	5.23
(P5): (1.5,1.5),(0.5,10),(0.95,0.95)	10.79	-8.58	12.55	5.34
(P6): (1.5,1.5),(0.5,10),(0.81,0.81)	8.50	1.25	13.36	6.24
(P7): (1.5,1.5),(0.5,4),(0.95,0.95)	6.58	1.59	4.22	5.34
(P8): (1.5,1.5),(0.5,4),(0.98,0.98)	6.97	1.18	1.73	5.22

Table 6: Sensitivity analysis for preferences (all reported figures in percent)

In case (P2), a model in which both agents have an IES of 0.5, the tree return volatility is considerably lower than in the baseline case (P1). However, it is still much higher than in an economy with the same preferences but without borrowing, see column B1 of (P2). We checked this result for other values of the IES below 1.5 and always observed the same phenomenon: Volatility effects are qualitatively similar but quantitatively less pronounced.[5]

Next we consider a change in the discount factor β. For the benchmark case *B1*, a higher β decreases return volatility simply because it decreases levels of returns and we report absolute volatility as opposed to the coefficient of variation. The effects in our model with one tree and one bond are quite different. As β increases from 0.95 in our baseline case (P1) to 0.98 in (P4), the return volatility increases from 8.14 to 8.47. The reason for this increase is simple. As β increases and the stock becomes more expensive, it is more difficult for agent 1 to buy a significant portion of the stock when he is in financial difficulties. This fact depresses the price of the stock when agent 1 is poor. Changes in the wealth distribution are large when agent 1 is fully leveraged and lead now to larger swings in the tree price.

In light of the intuition that we developed for the baseline case in Section 3, we expect an increase in the risk aversion of agent 2 to lead to both a higher price volatility and a higher

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[5]For low values of the IES, there is an additional unwanted effect. As one agent holds most of the wealth (that is, as the other agent becomes poor), asset prices increase because of the desire of the rich agent to save. This effect on the boundary of the state space is absent when the IES is set to 1.5 which we, therefore, do for the remainder of our analysis.

equity premium. This intuition is strongly confirmed by the comparison of (P1) and (P5). However, the increase in the second agent's risk aversion also leads to a large reduction of the interest rate to unrealistically low levels. In (P6) we recalibrate the model to obtain a positive interest rate and we find that the previously described effect of a smaller β dampens the impact of a higher risk aversion. But still, overall volatility increases substantially once the risk aversion and β are changed simultaneously: For risk aversions of 4, 6, and 10, (cases (P8), (P1) and (P6)) the return volatility is 6.97, 8.14, and 8.50 respectively.

5.2 Endowments

As we have seen repeatedly in our analysis, our model produces asset pricing moments that are comparable to observed values in the data. Clearly, this nice feature of our model depends on the magnitude of the disaster shocks. We now report results for models with less severe disaster shocks and demonstrate that the results remain qualitatively the same.

We conduct two different types of sensitivity analysis for our shock process. First, in the case (E1) we hold the magnitude of the disaster shocks constant, but reduce the overall probability of a disaster by 50 percent. Instead of setting the probabilities of shocks 1, 2, and 3 to 0.005, 0.005, and 0.024, respectively, we set them at 0.0025, 0.0025, and 0.012, respectively, and increase the probability of shock 5 accordingly. Secondly, in the case (E2) we leave the probabilities of the shocks unchanged but shift their support. In particular, we replace the growth rates in shocks 1, 2, and 3 of 0.566, 0.717, and 0.867, respectively, by the new values of 0.783, 0.8585, and 0.9335, respectively. Table 7 shows the analogue of Table 6 for these two cases.

	Std returns	Risk-free rate	Equity-premium	Std in B1
Case (E1)	5.95	3.44	2.17	4.15
Case (E2)	3.92	5.97	0.36	3.51

Table 7: Sensitivity analysis for endowments (all figures are in percent)

The table shows that a decrease in the probability of disaster has a relatively small effect on volatility while a change in the support has quite a large effect. As we explained above, the disaster states play two roles in our model. First, they lead to high excess returns of the tree, in particular whenever the risk-averse agent 2 must hold the tree. Secondly, they lead to endogenously high margin requirements. As we decrease the probability of disaster, the second effect remains unchanged. In contrast, the change in the support of the disaster shocks mitigates both effects above.

5.3 Large effects with smaller shocks

The results for the case (E2) above show that the quantitative impact of collateral constraints depends heavily on the size of the disaster shocks. However, we now demonstrate

that even with halved disaster shocks as in (E2), there are still substantial effects. For this purpose, we consider the model with two trees where tree 1 is collateralizable and tree 2 is not, and assume that agent 1's risk aversion is 10. We recalibrate the discount factor β to be 0.98, which results in a risk-free rate of 1.94. Table 8 shows that aggregate volatility with collateral constraints is now 48% higher than in the benchmark B1. This increase is of similar magnitude as in the baseline model. The high aggregate volatility is mostly driven by the volatility of tree 2, which increases by 95% compared to this benchmark.

	Std returns	EP	Std returns agg	Risk-free rate	EP agg	Std in B1
Tree 1	4.41	0.77	5.05	1.94	1.02	3.42
Tree 2	6.68	1.65				

Table 8: Moments of trees' returns (tree 1 collateralizable, tree 2 not)

6   Conclusion

In this paper we show that collateral and margin requirements play a quantitatively important role for prices of long-lived assets. This is true even for assets that cannot be used as collateral. In fact, somewhat surprisingly, we show that the presence of collateral constraints has a larger effect on the volatility of non-collateralizable assets than on the underlying collateral.

The recent financial crisis has lead researchers to suggest that central banks should regulate collateral requirements, see, for example, Ashcroft et al. (2010) or Geanakoplos (2010). We show that tightening margins uniformly over the business cycle can increase the price volatility of the underlying collateral but typically decreases price volatility of other long-lived assets in the economy that are not directly affected by the regulation. The only policy to achieve a decrease of the price volatility of all assets is to tighten margins only in boom times but leave them to market forces in recessions or crises.

Our calibration assumes the presence of disaster shocks as in Barro (2009). We provide alternative parameterizations of preferences and endowments under which our main qualitative results continue to hold.

Appendix

A   Details on computations

A.1   Time iteration algorithm

The algorithm used to solve all versions of the model is based on Brumm and Grill (2010). Equilibrium policy functions are computed by iterating on the per-period equilibrium con-

ditions, which are transformed into a system of equations. We use KNITRO to solve this system of equations for each grid point. Policy functions are approximated by piecewise linear functions. By using fractions of financial wealth as the endogenous state variables, the dimension of the state space is equal to the number of agents minus one. Hence with two agents, the model has an endogenous state space of one dimension only. This makes computations much easier than in Brumm and Grill (2010), where two and three dimensional problems are solved. In particular, in one dimension reasonable accuracy may be achieved without adapting the grid to the kinks. For the reported results we used 320 or 640 grid points depending on the complexity of the version of the model, which results in average (relative) Euler errors with order of magnitude 10^{-4}, while maximal errors are about ten times higher. If the number of grid points is increased to a few thousands, then Euler errors fall about one order of magnitude. However, the considered moments only change by about 0.1 percent. Hence, using 320 or 640 points provides a solution which is precise enough for our purposes. Compared to other models the ratio of Euler errors to the number of grid points used might seem large. However, note that due to the number of assets and inequality constraints our model is numerically much harder to handle than standard models. For example, in the version with one tree and five bonds, eleven assets are needed (as long and short positions in bonds have to the treated as separate assets) and we have to impose eleven inequality constraints per agent.

A.2   Simulations

The moments reported in the paper are averages of 50 different simulations with a length of 10.000 periods each (of which the first 100 are dropped). This is enough to let the law of large numbers do its job even for the rare disasters.

A.3   Equilibrium conditions

We state the equilibrium equations as we implemented them in Matlab for economies with a single tree and a single bond. For our computation of financial markets equilibria we normalized all variables by the aggregate endowment $\bar{e}$. To simplify the notation, we drop the dependence on the date-event s^t and, in an abuse of notation, denote the normalized parameters and variables by e_t, d_t and c_t, q_t, p_t, r_t, f_t, respectively. Similarly, we normalize both the objective function and the budget constraint of agents' utility maximization problem. The resulting maximization problem is then as follows (index h is dropped).

$$\max \quad u_t(c_t) = \left\{ (c_t)^\rho + \beta \left[E \left(u_{t+1} g_{t+1} \right)^\alpha \right]^{\frac{\rho}{\alpha}} \right\}^{\frac{1}{\rho}}$$

$$\text{s.t.} \quad 0 = c_t + \phi_t p_t + \theta_t q_t - e_t - [\phi_{t-1}]^+ \frac{r_t}{g_t} + [\phi_{t-1}]^- \frac{f_t}{g_t} - \theta_{t-1} (q_t + d_t)$$

$$0 \leq \theta_t + k_t [\phi_t]^-, \quad 0 \leq [\phi_t]^+, \quad [\phi_t]^- \leq 0,$$

The latter two inequalities are imposed because, for the computations, we treat the long and short position in the bond, $[\phi_t]^+$ and $[\phi_t]^-$, as separate assets. Note that $\phi_t = [\phi_t]^+ + [\phi_t]^-$. Let λ_t denote the Lagrange multiplier on the budget constraint. The first-order condition with respect to c_t is as follows,

$$0 = (u_t)^{1-\rho}(c_t)^{\rho-1} - \lambda_t.$$

Next we state the first-order condition with respect to c_{t+1}.

$$0 = \beta u_t^{1-\rho} \left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} \left(u_{t+1}g_{t+1}\right)^{\alpha-1} g_{t+1}(u_{t+1})^{1-\rho}(c_{t+1})^{\rho-1} - \lambda_{t+1}.$$

Below we need the ratio of the Lagrange multipliers,

$$\frac{\lambda_{t+1}}{\lambda_t} = \beta \left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} \left(u_{t+1}\right)^{\alpha-\rho}(g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}$$

Let μ_t denote the multiplier for the collateral constraint and let $\hat{\mu}_t = \frac{\mu_t}{\lambda_t}$. We divide the first-order condition with respect to θ_t,

$$0 = -\lambda_t q_t + \mu_t + E\left(\lambda_{t+1}\left(q_{t+1} + d_{t+1}\right)\right)$$

by λ_t and obtain the equation

$$0 = -q_t + \hat{\mu}_t + \beta \left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} E\left((u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}(q_{t+1} + d_{t+1})\right)$$

Similarly, the first-order conditions for $[\phi_t]^+$ and $[\phi_t]^-$ are as follows,

$$0 = -p_t + \nu_t^+ + \beta \left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} E\left((u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\frac{r_{t+1}}{g_{t+1}}\right)\right)$$

$$0 = -p_t + \hat{\mu}_t k_t - \nu_t^- + \beta \left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} E\left((u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\frac{f_{t+1}}{g_{t+1}}\right)\right),$$

where ν_t^+ and ν_t^- denote the multipliers on $0 \leq [\phi_t]^+$ and $[\phi_t]^- \leq 0$, respectively.

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