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Abstract

Recent New Keynesian models of macroeconomy view nominal cost rigidities, rather than nominal price rigidities, as the key feature that accounts for the observed persistence in output and inflation. Kryvtsov and Midrigan (2010a,b) reassess these conclusions by combining a theory based on nominal rigidities and storable goods with direct evidence on inventories for the U.S. This paper applies Kryvtsov and Midrigan's model to the case of Canada. The model predicts that if costs of production are sticky and markups do not vary much in response to, say, expansionary monetary policy, firms react by excessively accumulating inventories in anticipation of future cost increases. In contrast, in the Canadian data inventories are fairly constant over the cycle and in response to changes in monetary policy. Similarly to Kryvtsov and Midrigan, we show that markups must decline sufficiently in times of a monetary expansion in order to reduce firms' incentive to hold inventories and thus bring the model's inventory predictions in line with the data. The model consistent with salient features of the dynamics of inventories in the Canadian data implies that countercyclical markups account for a sizable (50-80%) fraction of the response of real variables to monetary shocks.

JEL classification: E31, F12
Bank classification: Business fluctuations and cycles; Transmission of monetary policy

Résumé

Dans les récents modèles macroéconomiques des nouveaux économistes keynésiens, la rigidité des coûts nominaux – plutôt que celle des prix nominaux – s'avère la source principale de la persistance observée des variations de la production et de l'inflation. Kryvtsov et Midrigan (2010a et b) testent la validité de cette conclusion en confrontant les prédictions d'une théorie fondée sur l'existence de rigidités nominales et de biens stockables avec le comportement effectif des stocks aux États-Unis. Dans la présente étude, Kryvtsov et Midrigan appliquent leur méthodologie au cas canadien. Leur modèle prévoit que si les coûts de production sont rigides et que les taux de marge varient peu en réponse, par exemple, à une politique monétaire expansionniste, la réaction des entreprises est d'accumuler des stocks excessifs en prévision de futures hausses des coûts. Or, il se trouve que, selon les données canadiennes, les stocks demeurent relativement constants au cours du cycle et sont peu sensibles aux changements apportés à la politique monétaire. Comme ils l'avaient fait pour les États-Unis, les auteurs montrent que les taux de marge doivent baisser suffisamment en période d'expansion monétaire pour réduire l'incitation des entreprises à stocker, et ainsi accorder les prévisions du modèle en matière de stocks avec les données. D'après le modèle qui reproduit les traits saillants de la dynamique des stocks au Canada, le caractère contracyclique des taux de marge explique une part considérable (de 50 à 80 %) de la réaction des variables réelles aux chocs monétaires.

Classification JEL : E31, F12
Classification de la Banque : Cycles et fluctuations économiques; Transmission de la politique monétaire

1. Introduction

In their recent papers Kryvtsov and Midrigan (2010a,b) revisit the debate in the business cycles and monetary policy literature that asks: How does the cost of production respond to monetary policy shocks? They note that the predictions of New Keynesian sticky price models, widely used for business cycle and policy analysis, are critically determined by the assumptions researchers make about the behavior of costs. If the real marginal cost of production responds strongly to monetary policy shocks, these models predict that such shocks have small and short-lived real effects, as in the work of Chari, Kehoe and McGrattan (2000). In contrast, if the real marginal cost responds slowly to monetary policy shocks, as in the work of Woodford (2002), Christiano, Eichenbaum and Evans (2005) and Dotsey and King (2006), such shocks have much larger and more persistent real effects.

In a standard New Keynesian model prices may be also slow to adjust even if production costs are volatile, for example, when price changes require fixed adjustment "menu" cost. It then follows that markups and cost are inversely related: more more volatile cost imply markups that are more countercyclical. Then an alternative way to ask the question in Kryvtsov and Midrigan is: How do markups respond to monetary policy shocks? Are the real effects of monetary policy shocks mostly accounted for by nominal cost rigidities or rather, by countercyclical variation in markups? Following Kryvtsov and Midrigan, who did their analysis using the U.S. data, we apply their methodology using the evidence for Canada.[1]

We answer the above question by studying data on inventories in Canada through the lens of a New Keynesian model in which we embed a motive for inventory accumulation. The focus on inventories is dictated by a tight relationship between prices, costs and inventories in theory, as argued by Bils and Kahn (2000). If goods are storable, firm prices are determined by the marginal valuation of inventories. In turn, firms produce to the point at which the marginal valuation of inventories is equal to the marginal cost. We exploit these predictions of the theory to show that countercyclical markups account for a sizable fraction of the real effects of monetary policy shocks in versions of the model that replicate the behavior of inventories in the Canadian data. Hence, as Kryvtsov and Midrigan do for the U.S., we find that markups are strongly countercyclical in

[1] Most of the analysis in this paper, especially pertaining to theoretical model, heavily draws on the discussion in Kryvtsov and Midrigan (2010a).

Canada.[2]

We begin our analysis by reviewing several well-known facts about inventories[3]. In the data, inventories are procyclical, but much less volatile then sales. The aggregate Canadian stock of inventories increases by about 0.39% for every 1% increase in sales during a business cycle expansion. We reach a similar conclusion when conditioning fluctuations on identified measures of monetary policy shocks. In response to an expansionary monetary policy shock, the stock of inventories increases by about 0.75% for every 1% increase in sales. These elasticities are even smaller for Retail, 0.13% and 0.15% respectively. Hence, the aggregate stock of inventories is somewhat sticky (relative to sales) and the aggregate inventory-sales ratio is countercyclical.

We then employ the model from Kryvtsov and Midrigan (2010a), in which nominal prices and wages change infrequently and firms hold inventories. In the model inventories arise due to a precautionary stockout-avoidance motive. Stockouts are especially costly for firms that have higher markups since the profit lost by failing to make a sale is greater. Similarly, a higher return to holding inventories (conversely, a lower carrying cost) makes it optimal to increase the stock of inventories available for sale.

We use the model economy to study how the response of inventories to monetary policy shocks depends on the assumptions we make about the nature of costs. A key prediction of the model is that the stock of inventories firms hold is very sensitive to changes in production costs and somewhat less sensitive to changes in markups. This feature is an outcome of the fact that in the model, as in the data, the cost of carrying inventories is fairly low. The low cost of carrying inventories makes it easy for firms to substitute intertemporally by producing and storing goods when production costs are relatively low and drawing down the stock of inventories when production costs are relatively high.

The findings can briefly summarized as follows. We first study a version of our model with nominal wage stickiness and no price rigidities, that is, in which markups are constant. We show that this model accounts extremely poorly for the dynamics of inventories in the data. This is true regardless of whether labor is the only factor of production and hence the marginal cost is

[2] For a detailed review of the related literature, see Kryvtsov and Midrigan (2010a).

[3] See Kryvtsov and Midrigan (2010a,b) and references therein. Kryvtsov and Zhang (2010a,b) incorporate inventories in the Bank of Canada forecasting model, ToTEM.

proportional to the nominal wage, or whether we introduce capital that render marginal costs more volatile than wages. In all these variations of the model inventories increase much more strongly in response to an expansionary monetary shock then they do in the data. The reason is that production costs are expected to increase after a monetary expansion as more and more unions reset their nominal wages, thus making it optimal for firms to accumulate inventories in anticipation of future cost increases.

We then introduce nominal price rigidities in addition to nominal wage stickiness. Price rigidities are important for our analysis since they imply countercyclical markups: e.g., during booms costs rise faster than prices, thus reducing the firms' incentives to hold inventories. We show that this version of the model can indeed account for the dynamics of inventories in the data, as long as production costs are sufficiently responsive to monetary shocks, due to sufficiently strong diminishing returns to labor. Production costs must be sufficiently responsive to monetary shocks in order to reduce the intertemporal substitution motive. Moreover, when costs are volatile, price rigidities generate strongly countercyclical markups and further reduce the incentive to hold inventories. Overall, we find that versions of our model that account for the dynamics of inventories in the data imply that countercyclical variation in markups accounts for 50-80% of the response of real variables to monetary policy shocks. This stands in sharp contrast to the findings of Christiano, Eichenbaum and Evans (2005) who estimate parameters values that imply that markups play essentially no role in accounting for the real effects of monetary shocks.

2. Data

In this section we review several salient facts regarding the cyclical behavior of inventories. These facts are well-known from earlier work[4]. We discuss them briefly for completeness, as they are central to our quantitative analysis below.[5]

We employ the Quarterly Survey of Financial Statistics for Enterprises with data on sales and inventories for Manufacturing, Wholesale and Retail from Q1:1988 to Q1:2008.[6] Although we

[4] See Kryvtsov and Midrigan (2010a) and references therein.

[5] Results for the unconditional moment statistics draw on those reported in Kryvtsov and Zhang (2010a).

[6] We also computed moments using the Monthly Survey of Manufacturing containing data on shipments, inventories (by stage of fabrication) for Manufacturing industries from January 1992 to April 2008. Our main results from simulations of the model calibrated to monthly data do not alter our main conclusions. We therefore used broader quarterly data.

focus on a subset of the Canadian economy, our data accounts for most of Canada's aggregate inventory stock. Manufacturing and Trade inventories (value added) comprise 85% (74%) of the total private nonfarm inventory stock (value added); the remaining industries are mining, utilities, and construction.[7]

Output is the sum of sales and the change in the end-of-period inventory stock. Inventory-to-sales ratio is defined as the ratio of the end-of-period inventory stock to sales in that period. All series are real: nominal variables for Manufacturing and Trade are deflated by GDP deflator, for Trade - by core CPI, and for Manufacturing by Industrial PPI. All data are seasonally-adjusted and HP-filtered with a smoothing parameter equal to 1600. Output, sales and inventory-sale ratios are defined in % deviations from respective HP trends. Inventory investment is defined as a fraction of output and we report it in percentage points deviations from its HP trend. Finally, to rule out breaks in time series, we restrict our analysis to inflation-targeting period in Canada by starting the data sample in 1993. Below we also use a measure of identified monetary policy shocks to report statistics conditional on monetary policy disturbances.

Panel A of Figure 1 presents the time-series of sales and the inventory-sales ratio for Manufacturing and Trade. The Figure shows that the two series are strongly negatively correlated and are almost equally volatile. Every recession is associated with a decline in sales and a similarly-sized increase in the inventory-sales ratio. Likewise, every expansion is associated with an increase in sales and a decline in the inventory-sales ratio of a similar magnitude.

Table 1 quantifies what is evident in the Figure. Panel A reports unconditional statistics for these series. We focus on the series for the entire Manufacturing and Trade sector and briefly discuss the Retail sector to gauge the robustness of these facts.

Notice in the first column of Panel A that the correlation between the inventory-sales ratio and sales for the entire Manufacturing and Trade sector is equal to -0.56. The standard deviation of the inventory-sales ratio is almost as large (1.09 times larger) as the standard deviation of sales. Consequently, the elasticity of the inventory-sales ratio with respect to sales is equal to -0.61.[8] In

[7] The surveys collect inventory and sales data based on potentially different accounting methods by firms. To our knowledge, most Canadian firms (except oil producers) use FIFO. The data from the national income and expenditure accounts are more limited but are based on an inventory valuation adjustment to revalue inventory holdings to replacement cost. For the data that is available in NIEA we will cross check the facts with those we obtained using survey data.

[8] This elasticity is defined as the product of the correlation and the ratio of the standard deviations, or equivalently,

other words, for every 1% increase in sales at business cycle frequencies, the inventory-to-sales ratio declines by about 0.61%. The stock of inventories is thus fairly constant over the cycle, increasing by only 0.39% (= -0.61+1) for every 1% increase in sales. Notice in Table 1 that production and sales are strongly correlated and that production is 1.14 times more volatile than sales. We will use this fact, in addition to the facts on the stock of inventories, in order to evaluate the model.

The other columns of Table 1 present several additional robustness checks. We note that the facts above hold if we focus separately on the Retail sector: the elasticity of inventories to sales is equal to 0.13 and production is 1.18 as volatile as sales. These facts also hold conditional on measures of monetary policy shocks. To see this run a simple VAR on (log deviations of) output, sales, inventory-sales ratios and the Bank rate and recompute the statistics conditional on identified measures of monetary shocks in Canada.[9] We report the resulting series in Panel B of Figure 1. Although monetary shocks account for a small fraction of the business cycle (the standard deviation of these series is about one third as large when conditioning on measures of monetary shocks), the main pattern is evident in this Panel as well. In particular, we again find that the inventory-sales ratio is countercyclical. As Table 1 shows, the elasticity of inventories to sales is somewhat higher for Manufacturing and Trade at 0.75 but still low for Retail, 0.15, and production is 1.06 times more volatile than sales (1.19 in Retail). Thus, in response to an expansionary monetary policy shock, both sales and inventory investment increase, but inventory investment increases much less than sales, and so the inventory-sales ratio declines[10].

3. Model

We study a monetary economy populated by a large number of infinitely lived households, a continuum of monopolistically competitive firms that produce differentiated intermediate goods, a continuum of perfectly competitive firms that produce a final good, and a government.[11] In each period t the commodities are differentiated varieties of labor services, a final labor service, money, a continuum of intermediate goods indexed by $i \in [0, 1]$, and a final good. The final good is used

as the slope coefficient in a regression of log inventory-sales ratio on log sales.

[9] Four lags are included. The results of VAR simulation are robust to the order of variables.

[10] See Jung and Yun (2005), Kryvtsov and Midrigan (2010a,b) who provide similar evidence for U.S. The evidence in this Section is also robust to the detrending method, the level of aggregation and stage-of-fabrication of inventories.

[11] The model is taken as is from Kryvtsov and Midrigan (2010a). This Section provides a concise overview of the model, for more details about the solution method, recursive formulation of the problem, firm-level decreasing returns, non-convexities, etc. see Kryvtsov and Midrigan (2010a).

for consumption and investment. In each period t, this economy experiences one of infinitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history (or state) of events up through and including period t. The probability density, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given.

In the model, we have aggregate shocks to the money supply and idiosyncratic demand shocks. We describe the idiosyncratic shocks below. In terms of the money supply shocks, we assume, throughout most of the paper, that the supply of money follows a random-walk process of the form

$$\log M(s^t) = \log M(s^{t-1}) + \log \mu(s^t), \tag{1}$$

where $\log \mu\left(s^t\right)$ is money growth, a normally distributed i.i.d. random variable with mean 0 and standard deviation σ_μ. We consider alternative specifications of monetary policy in a robustness section below.

A. Households

Households consume, trade bonds, and work. They also own the capital stock and rent it to intermediate goods producers. We assume frictions in the labor market in the form of sticky wages. In particular, we assume that households are organized in monopolistically competitive unions, indexed by j. Each union supplies a differentiated variety of labor services, $l_j\left(s^t\right)$, that aggregates into a final labor service, $l\left(s^t\right)$ according to

$$l\left(s^t\right) = \left(\int l_j\left(s^t\right)^{\frac{\vartheta-1}{\vartheta}} dj\right)^{\frac{\vartheta}{\vartheta-1}}$$

where ϑ is the elasticity of substitution across different types of labor services. Each union sets its wage $W_j\left(s^t\right)$ and therefore faces demand for its services given by

$$l_j\left(s^t\right) = \left(\frac{W_j\left(s^t\right)}{W\left(s^t\right)}\right)^{-\vartheta} l\left(s^t\right) \tag{2}$$

where $l\left(s^t\right)$ is the amount of labor hired by firms, and $W\left(s^t\right)$ is the aggregate wage rate:

$$W(s^t) = \left(\int W_j\left(s^t\right)^{1-\vartheta} dj\right)^{\frac{1}{1-\vartheta}}.$$

In this economy the markets for state-contingent money claims are complete. We represent the asset structure by having complete, state-contingent, one-period nominal bonds. Let $B_j\left(s^{t+1}\right)$ denote the consumer's holdings of such a bond purchased in period t and state s^t with payoffs contingent on a particular state s^{t+1} at date $t+1$. One unit of this bond pays one unit of money at date $t+1$ if the particular state s^{t+1} occurs and 0 otherwise. Let $Q\left(s^{t+1}|s^t\right)$ denote the price of this bond in period t and state s^t. Clearly, $Q\left(s^{t+1}|s^t\right) = \frac{Q\left(s^{t+1}\right)}{Q\left(s^t\right)}$ where $Q\left(s^t\right)$ is the date 0 price of a security that pays one unit if history s^t is realized.

The problem of union j is to choose its members' money holdings $M_j\left(s^t\right)$, consumption $c_j\left(s^t\right)$, investment $x_j\left(s^t\right)$, state-contingent bonds $B_j\left(s^{t+1}\right)$, as well as a wage $W_j\left(s^t\right)$, to maximize the household's utility:

$$\sum_{t=0}^{\infty} \int_{s^t} \beta^t \pi\left(s^t\right)\left[u\left(c_j\left(s^t\right)\right) - v\left(l_j\left(s^t\right)\right)\right] ds^t \tag{3}$$

subject to the budget constraint

$$P\left(s^t\right)\left[x_j\left(s^t\right) + \frac{\xi}{2}\left(\frac{x_j\left(s^t\right)}{k_j(s^{t-1})} - \delta\right)^2 k_j(s^{t-1})\right] + \int_{s^{t+1}} Q\left(s^{t+1}|s^t\right) B_j\left(s^{t+1}\right) ds^{t+1} + M_j(s^t)$$

$$\leqslant M_j\left(s^{t-1}\right) - P\left(s^{t-1}\right) c_j\left(s^{t-1}\right) + W\left(s^t\right) l_j\left(s^t\right) + \Pi_j\left(s^t\right) + B_j\left(s^t\right) + R\left(s^t\right) k_j\left(s^t\right),$$

a cash-in-advance constraint,

$$P\left(s^t\right) c_j\left(s^t\right) \leq M_j(s^t),$$

and subject to the demand for labor given by (2) as well as subject to the frictions on wage setting. We assume that utility is separable between consumption and leisure.

Here $P(s^t)$ is the price of the final good, $x_j\left(s^t\right) = k_j\left(s^t\right) - (1-\delta) k_j\left(s^{t-1}\right)$ is investment, $W(s^t)$ is the nominal wage, $\Pi_j(s^t)$ are firm dividends, and $R\left(s^t\right)$ is the rental rate of capital.

Investment is subject to capital adjustment costs, the size of which is governed by ξ. The budget constraint says that the household's beginning-of-period balances are equal to unspent money from the previous period, $M_j\left(s^{t-1}\right) - P\left(s^{t-1}\right) c_j\left(s^{t-1}\right)$, labor income, dividends, as well as returns from asset market activity and from rental of the capital stock to firms. The household divides these balances into money holdings, $M_j\left(s^t\right)$, finances investment spending, as well as purchases of state-contingent bonds.

We assume Calvo-type frictions on wage setting. The probability that any given union is allowed to reset its wage at date t is constant and equal to $1-\lambda_w$. A measure λ_w of the unions leave their nominal wages unchanged. We choose the initial bond holdings of unions so that each union has the same present discounted value of income. Even though unions differ in the wages they set and hence the amount of labor they supply, the presence of a complete set of securities and the separability between consumption and leisure implies that they make identical consumption and investment choices in equilibrium. Since these decision rules are well-understood, we simply note that the bond prices satisfy

$$Q\left(s^{t+1}|s^t\right) = \beta\pi\left(s^{t+1}|s^t\right) \frac{u_c\left(c\left(s^{t+1}\right)\right)}{u_c\left(c\left(s^t\right)\right)} \frac{P\left(s^t\right)}{P\left(s^{t+1}\right)}$$

where $\pi\left(s^{t+1}|s^t\right)$ is the conditional probability of s^{t+1} given s^t and we have dropped the j subscript. Similarly, the date 0 prices satisfy:

$$Q\left(s^t\right) = \beta^t \pi\left(s^t\right) \frac{u_c\left(c\left(s^t\right)\right)}{P\left(s^t\right)}.$$

B. Final good producers

The final good sector consists of a unit mass of identical and perfectly competitive firms. The final good is produced by combining the goods produced by intermediate goods firms (we refer to these goods as *varieties*) according to:

$$q\left(s^t\right) = \left(\int_0^1 v_i(s^t)^{\frac{1}{\theta}} q_i(s^t)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

where $q_i(s^t)$ is the amount of variety i purchases by a final good firm, $v_i(s^t)$ is a variety-specific shock and θ is the elasticity of substitution across varieties. For simplicity we assume that $v_i\left(s^t\right)$ is an iid log-normal random variable.

In this economy, intermediate good firms sell out of their existing stock of inventories, $z_i(s^t)$. We describe the evolution of a firm's stock of inventories below. Given the price and inventory adjustment frictions we assume, this stock of inventories will occasionally be insufficient to meet all demand and intermediate good firms will *stockout*. In such a case, we assume a rationing rule under which all final good firms are allowed to purchase an equal share of that intermediate good's stock of inventories. Since the mass of final good firms is equal to 1, $z_i\left(s^t\right)$ is both the amount of inventories the intermediate good firm has available for sale, as well as the amount of inventories that any particular final good firm can purchase.

The problem of a firm in the final good's sector is therefore:

$$\max_{q_i(s^t)} P\left(s^t\right) q\left(s^t\right) - \int_0^1 P_i\left(s^t\right) q_i\left(s^t\right) di,$$

subject to the inventory constraint

$$\text{s.t. } q_i\left(s^t\right) \leqslant z_i\left(s^t\right) \;\; \forall i$$

and the final good production technology. Cost minimization by the final good firms implies the following demand for each variety:

$$q_i\left(s^t\right) = v_i(s^t)\left(\frac{P_i\left(s^t\right) + \mu_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right)$$

where $\mu_i\left(s^t\right)$ is the multiplier on the inventory constraint. Notice here that the shocks $v_i(s^t)$, act as a demand shock for an intermediate goods firm. We will thus refer to such shocks as *demand* shocks. Perfect competition implies that the price of the final good, $P\left(s^t\right)$, is equal to

$$P\left(s^t\right) = \left[\int_0^1 v_i(s^t)\left[P_i\left(s^t\right) + \mu_i\left(s^t\right)\right]^{1-\theta} di\right]^{\frac{1}{1-\theta}}.$$

Also note that if $\mu_i(s^t) > 0$ so that the inventory constraint binds, then it satisfies:

$$P_i(s^t) + \mu_i(s^t) = \left(\frac{z_i(s^t)}{v_i(s^t) P(s^t)^\theta q(s^t)} \right)^{\frac{1}{\theta}}.$$

The left hand side of this expression is the price that a firm that stocks out would have chosen absent price adjustment frictions. Since such a firm faces an inelastic demand curve, it would like to increase its price to the point at which final good firms demand exactly all of its stock of inventories. Together with the inventory frictions we describe below, price adjustment frictions give rise to stockouts in the equilibrium of this economy since they prevent firms from increasing their prices.

C. Intermediate goods firms

The intermediate good firms are monopolistically competitive. Any given such firm sells a single variety i, rents capital from consumers, hires labor and produces the intermediate good. It then sells the good to final good firms. The critical assumption we make is that the firm makes the decision of how much to produce, $q_i(s^t)$, prior to learning the value of $v_i(s^t)$, the demand shock. This assumption introduces a precautionary motive for holding inventories, the stockout-avoidance motive.

We assume a production function

$$y_i(s^t) = \left(l_i(s^t)^\alpha k_i(s^t)^{1-\alpha} \right)^\gamma,$$

where $y_i(s^t)$ is output, $k_i(s^t)$ is the amount of capital firm i rents and $l_i(s^t)$ is the amount of labor it hires, while $\gamma \leq 1$ determines the degree of returns to scale. Letting $R(s^t)$ and $W(s^t)$ denote the rental rate of capital and the aggregate nominal wage rate, respectively, this production function implies that the minimum cost of producing $y_i(s^t)$ units of the intermediate good is given by $\Omega(s^t) y_i(s^t)^{\frac{1}{\gamma}}$, where

$$\Omega(s^t) = \chi W(s^t)^\alpha R(s^t)^{1-\alpha},$$

and χ is a constant.

Intermediate good firms face two frictions. First, they must choose how much to produce, $y_i\left(s^t\right)$, and the price to set, $P_i\left(s^t\right)$, prior to learning their demand shock, $v_i\left(s^t\right)$. Second, they change prices infrequently, in a Calvo fashion. An exogenously chosen faction $1-\lambda_p$ of firms are allowed to reset their nominal prices in any given period; the remaining λ_p of firms leave their prices unchanged.

Let $m_i\left(s^{t-1}\right)$ denote the stock of inventories firm i has at the beginning of date t. If the firm produces $y_i\left(s^t\right)$ additional units, the amount it has available for sale is equal to $z_i\left(s^t\right) = m_i\left(s^{t-1}\right)+y_i\left(s^t\right)$. Recall that, given a price $P_i\left(s^t\right)$ and stock $z_i\left(s^t\right)$, the firm's sales are equal to:

$$q_i\left(s^t\right)=\min\left(v_i\left(s^t\right)\left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta}q\left(s^t\right),\ z_i\left(s^t\right)\right) \tag{4}$$

The firm's problem is therefore to choose $P_i\left(s^t\right)$ and $z_i\left(s^t\right)\geqslant m_i\left(s^{t-1}\right)$, so as to maximize its objective given by

$$\max_{P_i(s^t),\ z_i(s^t)\ \geqslant\ m_i(s^{t-1})}\sum_{t=0}^{\infty}\int_{s^t}Q\left(s^t\right)\left[P_i\left(s^t\right)q_i\left(s^t\right)-\Omega\left(s^t\right)\left[z_i\left(s^t\right)-m_i\left(s^{t-1}\right)\right]^{\frac{1}{\gamma}}\right]ds^t$$

where recall $Q\left(s^t\right)$ is the date 0 price of one unit of currency to be delivered in state s^t and $m_i\left(s_0\right)$ is given. The constraints are the demand function in (4), the restriction that $z_i\left(s^t\right)$ and $P_i\left(s^t\right)$ are not measurable with respect to $v_i\left(s^t\right)$, as well as the constraint that $P_i\left(s^t\right)=P_i\left(s^{t-1}\right)$ in the absence of a price adjustment opportunity, as well as the law of motion for inventories :

$$m_i\left(s^t\right)=\left(1-\delta_z\right)\left(z_i\left(s^t\right)-q_i\left(s^t\right)\right)$$

where δ_z is the rate at which inventories depreciate.

D. Equilibrium

Consider now this economy's market-clearing conditions and the definition of *equilibrium*. The market-clearing conditions on labor are:

$$\left(\int l_j\left(s^t\right)^{\frac{\vartheta-1}{\vartheta}} dj\right)^{\frac{\vartheta}{\vartheta-1}} = l(s^t)$$

and

$$l\left(s^t\right) = \int_i l_i(s^t)di$$

The first expression is the production function for producing final labor services $l\left(s^t\right)$ out of the differentiated services supplied by each union. The second expression says that the total amount of the final labor service must be equal to the amount of labor hired by each intermediate goods firm.

Similarly, the market clearing conditions for the final good are:

$$q\left(s^t\right) = \left(\int_0^1 v_i(s^t)^{\frac{1}{\theta}} q_i(s^t)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$$

and

$$\int_0^1 \left(c_j\left(s^t\right) + x_j\left(s^t\right) + \frac{\xi}{2}\left(\frac{x_j\left(s^t\right)}{k_j(s^{t-1})} - \delta\right)^2 k_j(s^{t-1})\right) dj = q\left(s^t\right)$$

The first expression is the final good production function and the second says that the total consumption and investment of the different households must sum up to the total amount of the final good produced. Since all households make identical consumption and investment decisions, we can write the resource constraint for final goods as:

$$c\left(s^t\right) + x\left(s^t\right) + \frac{\xi}{2}\left(\frac{x\left(s^t\right)}{k(s^{t-1})} - \delta\right)^2 k(s^{t-1}) = q\left(s^t\right)$$

Next, the market-clearing condition on bonds is $B(s^t) = 0$ and the cash-in-advance constraint requires $P\left(s^t\right) c\left(s^t\right) = M\left(s^t\right)$. Finally, the market clearing condition for capital is

$$\int_i k_i\left(s^t\right) di = k\left(s^{t-1}\right)$$

An *equilibrium* for this economy is a collection of allocations for households $c(s^t)$, $M(s^t)$, $B(s^{t+1})$, $k\left(s^t\right)$, $x\left(s^t\right)$, $l_j\left(s^t\right)$ and $W_j(s^t)$; prices and allocations for firms $p_i(s^t)$, $q_i\left(s^t\right)$, $y_i(s^t)$, $l_i\left(s^t\right)$, $k_i\left(s^t\right)$, $z_i\left(s^t\right)$; and aggregate prices $W(s^t)$, $P(s^t)$, $R\left(s^t\right)$ and $Q(s^{t+1}|s^t)$, all of which satisfy the following conditions: (i) the consumer allocations solve the consumers' problem; (ii) the prices and allocations of firms solve their maximization problem; (iii) the market-clearing conditions hold; and (iv) the money supply process satisfies the specifications above.

E. The Workings of the Model

We next discuss the decision rules in this economy by studying a version of the model with constant returns at the firm level.[12]

To build intuition, assume away the irreversibility constraint $z_i\left(s^t\right) \geqslant m_i\left(s^{t-1}\right)$. This constraint turns out not to bind for most of the experiments we describe here, with the exception of the economy with non-convexities we describe later on. Moreover, assume that prices are flexible, $\lambda_p = 0$. Recall that $v_i\left(s^t\right)$, the demand shocks, are iid and log-normal. Let Φ denote the cdf and σ_v^2 the variance of these shocks. Then we can write the firm's expected sales as

$$R\left(P_i\left(s^t\right), z_i\left(s^t\right)\right) = \int_0^\infty \min\left(v\left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right), z_i\left(s^t\right)\right) d\Phi\left(v\right) =$$

$$= \left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right) \exp\left(\frac{\sigma_v^2}{2}\right) \Phi\left(\log v_i^*\left(s^t\right) - \sigma_v^2\right) + z_i\left(s^t\right)\left(1 - \Phi\left(\log v_i^*\left(s^t\right)\right)\right) \ , \tag{5}$$

where

$$v_i^*\left(s^t\right) = \frac{z_i\left(s^t\right)}{\left(\frac{P_i\left(s^t\right)}{P\left(s^t\right)}\right)^{-\theta} q\left(s^t\right)}$$

is the highest value of the demand shock for which the firm does not stockout. To understand expression (5), notice that the first term is the expected value of sales in those states in which the firm does not stockout, while the second term is the amount of inventories the firm has, $z_i\left(s^t\right)$, times the probability of a stockout. Clearly, $R_z = \left(1 - \Phi\left(\log v_i^*\left(s^t\right)\right)\right) > 0$: an increase in its stock of inventories allows the firm to sell in those states in which it would otherwise stockout[13].

[12] Kryvtsov and Midrigan (2010a) also discuss the case with decreasing returns as well as provide some empirical evidence using micro-level data.

[13] To derive this expression, notice that z enters (5) in three places, but two of these terms cancel out.

With constant returns and no irreversibility, the value of a firm is linear in the stock of inventories it has inherited from the previous period: an unsold unit of inventories at date t depreciates to $(1-\delta_z)$ units next period and saves the firm production costs equal to $(1-\delta_z)\, Q\left(s^{t+1}|s^t\right)\Omega\left(s^{t+1}\right)$ evaluated at date t prices. Let

$$\Omega'\left(s^t\right) = (1-\delta_z)\int_{s^{t+1}} Q\left(s^{t+1}|s^t\right)\Omega\left(s^{t+1}\right) ds^{t+1}$$

denote the expected value of these savings. The problem of the firm thus reduces to:

$$\max_{P_i(s^t),\, z_i(s^t)} \left(P_i\left(s^t\right)-\Omega'\left(s^t\right)\right) R\left(P_i\left(s^t\right), z_i\left(s^t\right)\right) - \left(\Omega\left(s^t\right)-\Omega'\left(s^t\right)\right) z_i\left(s^t\right),$$

where, recall, $\Omega(s^t)$ is the marginal cost of production.

To understand this expression, notice that the choice of prices is similar to that in the standard problem of a monopolist, except that $R\left(P_i\left(s^t\right), z_i\left(s^t\right)\right)$ is the demand function and $\Omega'\left(s^t\right)$ is the marginal valuation of the goods the firm sells. The choice of inventories, $z_i\left(s^t\right)$, is also straightforward: on one hand a higher $z_i\left(s^t\right)$ increases expected sales, but the firm expects to lose $\left(\Omega\left(s^t\right)-\Omega'\left(s^t\right)\right) z_i\left(s^t\right)$ in inventory carrying costs.

The firm's optimal price is then a markup over its shadow valuation of inventories:

$$P_i\left(s^t\right) = \frac{\varepsilon_i\left(s^t\right)}{\varepsilon_i\left(s^t\right)-1}\Omega'\left(s^t\right).$$

Here $\varepsilon_i\left(s^t\right)$ is the price elasticity of expected sales and is equal to θ (the elasticity of substitution across varieties) times the share of sales in the states in which the firm does not stockout:

$$\varepsilon_i\left(s^t\right) = \theta \times \frac{\exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v_i^*\left(s^t\right)-\sigma_v^2\right)}{\exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v_i^*\left(s^t\right)-\sigma_v^2\right) + v_i^*\left(s^t\right)\left(1-\Phi\left(\log v_i^*\left(s^t\right)\right)\right)}.$$

We next turn to the inventory accumulation decision. The choice of $z_i\left(s^t\right)$ satisfies

$$1-\Phi\left(\log v_i^*\left(s^t\right)\right) = \frac{1-r_i\left(s^t\right)}{P_i\left(s^t\right)/\Omega\left(s^t\right)-r_i\left(s^t\right)}. \tag{6}$$

The left-hand side of this expression is the probability that the firm stocks out. As in Bils and Kahn (2000), the firm chooses a higher stock of inventories (a lower stockout probability) the higher the markup $P_i\left(s^t\right)/\Omega\left(s^t\right)$, and the higher the return to holding inventories, $r_i\left(s^t\right)$, where

$$r_i\left(s^t\right) = \frac{\Omega'\left(s^t\right)}{\Omega\left(s^t\right)} = (1-\delta_z)\int_{s^{t+1}} Q\left(s^{t+1}|s^t\right)\frac{\Omega\left(s^{t+1}\right)}{\Omega\left(s^t\right)}ds^{t+1}. \tag{7}$$

Stockouts are especially costly for firms that have higher markups since the profit lost by failing to make a sale is greater. Similarly, a higher return to holding inventories (conversely, a lower carrying cost) makes it optimal to increase the stock of inventories available for sale.

One important implication of the model is that inventories are much more sensitive to changes in the return to holding inventories, rather than changes in markups. To see this, we find it useful to log-linearize (6) around the steady-state:

$$\bar{v}\hat{v}_i^*\left(s^t\right) = \frac{1}{\bar{\Phi}\left[\bar{b}-\beta\left(1-\delta_z\right)\right]}\left[\left(1-\bar{\Phi}\right)\bar{b}\left[\hat{P}_i\left(s^t\right)-\hat{\Omega}\left(s^t\right)\right]+\beta\left(1-\delta_z\right)\bar{\Phi}\hat{r}_i\left(s^t\right)\right],$$

where hats denote log-deviations from the steady state, $\bar{b}$ is the steady-state markup, $\bar{v}$ is the pre-sale steady-state inventory-sales ratio, and $1-\bar{\Phi}$ is the steady-state probability of a stockout.

If the stockout probability and markups are low, as in the data, and $\left(1-\bar{\Phi}\right)\bar{b}\approx 0$, then inventories are relatively insensitive to markups. In contrast, as long as the cost of carrying inventories (as determined by δ_z) is sufficiently small, inventories are much more sensitive to fluctuations in the return to holding inventories. Intuitively, if the cost of carrying inventories is sufficiently low, firms find it optimal to intertemporally substitute production in order to react to expected changes in the marginal cost of production and/or changes in the interest rate[14]. Hence, the dynamics of inventories is closely related to the dynamics of costs but also influenced by the behavior of markups. In the next section we exploit this key feature of the model to draw implications for the dynamics of costs in response to monetary policy shocks.

So far we have discussed the model's implications for $v_i^*\left(s^t\right)$. This object, on its own, is not useful to evaluate the model empirically as we do not directly observe it in the data. Notice however

[14] See House (2008) who makes a similar argument in the context of a model with investment.

that there is a monotonic relationship between the aggregate inventory-to-sale ratio and $v_i^*\left(s^t\right)$. In particular, integrating the distribution of demand shocks, and noting that all firms make the same $v_i^*\left(s^t\right)$ choices, it follows that the end-of-period inventory-sales ratio, which we do observe in the data, is equal to:

$$IS\left(s^t\right) = \frac{v^*\left(s^t\right)\Phi\left(\log v^*\left(s^t\right)\right) - \exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v^*\left(s^t\right) - \sigma_v^2\right)}{\exp\left(\frac{\sigma_v^2}{2}\right)\Phi\left(\log v^*\left(s^t\right) - \sigma_v^2\right) + v_t^*\left(1 - \Phi\left(\log v^*\left(s^t\right)\right)\right)}. \tag{8}$$

F. Parametrization

We next describe how we have chosen parameters to evaluate the model's quantitative implications. We set the length of the period as one quarter and therefore choose a discount factor of $\beta = .96^{1/4}$. We assume preferences of the form $u(c) - v(n) = \log c - n$. These preferences imply an infinite Frisch labor supply elasticity, and can be interpreted as the outcome of indivisibilities combined with Hansen (1985) and Rogerson (1988) –type lotteries. We focus on these preferences since they imply, in a version of the model without capital and nominal wage rigidities and with constant returns at the firm level, that the marginal cost of production increases one-for-one with the monetary shock[15]. Below we consider the implications of changing the assumptions we make about preferences.

Table 2 reports the parameter values we used in our quantitative analysis. We set the rate at which capital depreciates, δ, equal to 0.03. We set the elasticity of substitution across intermediate goods and varieties of labor, $\theta = \vartheta = 5$, implying a 25% markup, in the range of estimates in existing work. Finally, we assume a frequency of wages changes of once a year, $1 - \lambda_w = 1/4$, consistent with what is typically assumed in existing studies.

We calibrate the inventory parameters, namely, the rate at which inventories depreciate, δ_z, and the volatility of demand shocks, σ_v, to ensure that the model accounts for two facts about inventories and stockouts in the data. First, as can be seen from the decision rules (6)-(7) above, δ_z directly affects the frequency of stockouts: a higher cost of carrying inventories make it optimal for firms to stockout more often. Bils (2004) uses micro CPI data from the BLS and reports a frequency of stockouts of 5% at a monthly frequency. Hence in all of the experiments we consider

[15] This particular parametrization has been widely used in the menu cost literature. See for example Golosov and Lucas (2007).

below we choose δ_z so that the model generates a 15% quarterly frequency of stockouts. Second, σ_v, the volatility of demand shocks, directly maps into the average inventory-sales ratio in the model, as (8) shows. We thus choose this parameter so as to match an (end-of-period) inventory-sales ratio of 0.5 quarters, as in the Canadian Manufacturing and Trade sector.

For example, as Panel A. I. of Table 2 shows, in the economy with constant firm-level returns, the value of σ_v necessary to match these two facts is equal to 0.42, while the rate of depreciation is equal to $\delta_z = 4\%$. This estimate of the depreciation rate is in the range of the inventory-carrying costs measured directly in the logistics literature, see for example Richardson (1995).

4. Quantitative Investigation

We use the model to make two related points. First, versions of the model with flexible prices (that imply nearly constant markups) predict a much stronger response of inventories to a monetary policy shock than in the data. Second, models with countercyclical markups (sticky prices) can account for the dynamics of inventories in the data, but only if markups decline (real marginal costs increase) sufficiently in response to an expansionary monetary shock.

To make our first point we study a version of our economy with sticky wages and flexible prices. We then introduce nominal price rigidities and show that if the marginal cost is sufficiently responsive to monetary shocks, the model can indeed account for the dynamics of inventories in the data.

A. Economy with flexible prices

We start by studying an economy with constant returns to labor. We then allow for decreasing returns to labor by introducing capital in the production function.[16]

Constant Returns

We set $\alpha = 1$. The marginal cost of production is thus equal to nominal wages:

$$\Omega\left(s^t\right) = W\left(s^t\right).$$

[16] Kryvtsov and Midrigan (2010a) also consider a case in which decreasing returns to labor are introduced by assuming decreasing returns to scale in production.

Since we have assumed an infinite Frisch elasticity of labor supply and iid money growth, reset wages are proportional to the money supply and the aggregate wage evolve according to

$$\hat{w}\left(s^t\right) = \lambda_w \hat{w}\left(s^{t-1}\right) - \lambda_w \mu\left(s^t\right),$$

where $\hat{w}\left(s^t\right)$ is the log-deviation of $W\left(s^t\right)/M(s^t)$ from its steady state level.

Figure 2 shows the impulse responses of nominal and real variables in the model to a 1% increase in the money supply. Panel A shows that nominal wages respond gradually to the shock. Since prices are flexible, they track nominal wages closely, although prices decline somewhat relative to wages. This happens because of a decline in the optimal markup induced by inventory accumulation. Panel B shows the response of inventories and sales. We report, as in the data, the response of the real aggregate end-of-period inventory stock defined as:

$$I\left(s^t\right) = \int_0^1 \frac{m_i\left(s^t\right)}{1-\delta_z} di = \int_0^1 \left[z_i\left(s^t\right) - q_i\left(s^t\right)\right] di.$$

Similarly, real sales are computed using:

$$S\left(s^t\right) = \int_0^1 q_i\left(s^t\right) di.$$

Notice that sales rise immediately by about 0.8% and gradually decline. Moreover, the response of inventories is much greater than that of sales: inventories increase by about 2.6% on impact and gradually decline. Panel C shows that the reason inventories increase much more than sales is a sharp increase in production. Production is defined as

$$Y\left(s^t\right) = \int \left[z_i\left(s^t\right) - m_i\left(s^{t-1}\right)\right] di$$

and is, by definition, equal to sales plus inventory investment:

$$Y\left(s^t\right) = \int_0^1 \left[q_i\left(s^t\right) + \frac{m_i\left(s^t\right)}{1-\delta_z} - m_i\left(s^{t-1}\right)\right] di = S\left(s^t\right) + I\left(s^t\right) - \left(1-\delta_z\right) I\left(s^{t-1}\right)$$

Since production increases by about 2.1% in response to the monetary shock and sales by only 0.8%, the excess production contributes to the large increase in the stock of inventories.

Table 3 summarizes our findings. In Panel A we report two sets of statistics. The first set are measures of the real effects of monetary shocks which summarize the impulse response of aggregate consumption, $c\left(s^t\right)$, to a monetary shock. The first row shows that the average consumption response in the first 2 years after the shock, i.e., the area under the impulse response function in Panel D of Figure 2, is equal to 0.37%. The maximum consumption response is equal to 0.83%. Finally, the half-life of consumption, our measure of the persistence of the real effects, is equal to 2.4 quarters.

The second set of statistics we report are those that characterize the behavior of inventories, sales and production. To compute these statistics, we HP-filter these series, as in the data, with a smoothing parameter of 1600. We then contrast the model's predictions with those in the data for which we focus on the Manufacturing and Trade sector, the series conditional on money shocks.

The model does very poorly in accounting for the behavior of inventories in the data. It predicts a strongly procyclical inventory-sales ratio (the correlation with sales is 1 vs. -0.58 in the data) and that the inventory-sales ratio is much more volatile than in the data. The elasticity of the inventory-sales ratio to sales is equal to 2.20 in the model (-0.85 in the data), thus implying that the stock of inventories increases by 3.20% (0.15% in the data) for every 1% increase in sales. The model's counterfactual implications for the stock of inventories imply counterfactual implications for the behavior of inventory investment. The model predicts that production is 3.20 times more volatile than sales (1.19 in the data).

The reason the stock of inventories is very sensitive to monetary shocks in the model is the intertemporal substitution effect. The return to holding inventories is equal to

$$r_i\left(s^t\right) = (1-\delta_z)\int \frac{1}{r\left(s^t\right)}\frac{W\left(s^{t+1}\right)}{W\left(s^t\right)}\pi\left(s^{t+1}|s^t\right)ds^{t+1}$$

where $r\left(s^t\right)$ is the nominal risk-free rate. Since we have assumed that preferences are log-linear and money growth is iid, the nominal interest rate is equal to the expected growth rate of the money supply,

$$r\left(s^t\right) = \frac{1}{\beta}\int \frac{P\left(s^{t+1}\right)c\left(s^{t+1}\right)}{P\left(s^t\right)c\left(s^t\right)}\pi\left(s^{t+1}|s^t\right)ds^{t+1} = \frac{1}{\beta}\int \frac{M\left(s^{t+1}\right)}{M\left(s^t\right)}\pi\left(s^{t+1}|s^t\right)ds^{t+1}$$

and is therefore constant. As a result the return to holding inventories increases after an increase in the growth rate of the money supply, since the nominal interest rate is constant and the cost of production (here the nominal wage) is expected to increase.

We thus conclude that this version of the model generates real effects of monetary shocks for the wrong reasons, by implying a sluggish response of costs to monetary shocks and making it optimal for firms to take advantage of the lower costs by investing in inventories much more than they do in the data.

Decreasing Returns at the Aggregate Level

We next assume decreasing returns (to labor) at the aggregate level by introducing capital as a factor of production. We now have $\alpha = \frac{2}{3}$, implying a capital share of 1/3. Capital accumulation is subject to adjustment costs, the size of which is chosen so that the model implies a relative variability of investment to consumption equal to 4, as in the Canadian data. In this economy the marginal cost of production is equal across firms and given by:

$$\Omega\left(s^t\right) = \chi W\left(s^t\right)^{\alpha} R\left(s^t\right)^{1-\alpha}$$

The household's preference for smooth consumption imply that the rental rate of capital increases with an expansionary monetary shock due to the increased demand for capital. Hence, the marginal cost increases more strongly than in the economy with labor only.

Panel B of Table 3 shows that adding capital bridges the gap between the dynamics of inventories in the model and in the data: this version of the model predicts an elasticity of inventories to sales of 1.82 and a relative volatility of production to sales of 1.38. Both of these are much greater than in the data, but smaller than in the economy with constant returns.

Why are inventories more sluggish in the economy with capital than in the economy with constant returns? The reason is that capital accumulation generates more persistence in the marginal cost of production since investment in capital lowers its rental rate in future periods. Since the return to holding inventories is proportional to the expected change in costs, capital accumulation imparts sluggishness in the return to holding inventories and therefore in the inventory stock. To see this, panel C. of Table 3 also reports statistics for an economy with capital in which the stock of capital is fixed.

Panel D reports an alternative extreme experiment in which we assume away capital adjustment costs altogether. In this economy the behavior of inventories is much more in line with the data. The inventory-sales ratio is countercyclical: its correlation with sales is -0.66. Moreover, the elasticity of inventories to sales is now equal to 0.73 (0.15 in the data) and production is only 1.15 times more volatile than sales (1.19 in the data). This improved fit with regards to inventories comes however, at the cost of the model's implications for investment variability. In this version of the model investment is 17.5 times more volatile than consumption, thus substantially more volatile than in the data. Moreover, the real interest rate now increases during a monetary expansion, in contrast to the data in which real interest rates persistently decline following an expansionary monetary shock[17]. Since the real interest rate is (in addition to the expected change in the real marginal cost) one of the two components that directly affects the cost of carrying inventories, we find this version of the model without capital adjustment costs an unsatisfactory one. We thus conclude that models with constant markups and sticky wages cannot account for the response of inventories to monetary shocks in the data.

B. Economy with Sticky Prices

We next assume that prices as well as wages are sticky. Consistent with the evidence in Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008a) who report that prices change on average once about every 6-10 months, we assume a quarterly frequency of price changes of $1 - \lambda_p = \frac{3}{8}$. (Since sticky prices do not change much the inventory-accumulation decisions of firms, the inventory moments are unaffected, and so we keep all other parameter values unchanged. See Panel B of Table 2 for the parameter values and targets in this version of the model). We show that this version of the model can indeed account for the behavior of inventories, but only in the presence of sufficiently large decreasing returns to labor that make the marginal cost of production

[17] See, for example, Christiano, Eichenbaum and Evans (2005).

responsive to monetary shocks.

Constant Returns

Panel A. of Table 4 reports statistics from the economy with constant returns ($\alpha = 1$). Now that prices are sticky, the real effects of money are somewhat greater than in the economy with flexible prices: the average response of consumption is 0.44, thus about 1.19 greater than with flexible prices (0.37%). As for inventories, these are, as earlier, strongly procyclical and very volatile, with an elasticity of inventories to sales equal to 2.75, only slightly lower than in the economy with flexible prices (3.20). Thus sticky prices, on their own, do not improve much the model's ability to account for the inventory facts.

This result is driven by two features of the model. First, the optimal inventory-sales ratio is not very sensitive to variation in markups, as we have shown earlier, and much more sensitive to variation in the return to carrying inventories. Second, when wages are sticky and there are no decreasing returns, sticky prices do not greatly reduce markups: even though prices are sticky, costs are sticky as well.

To see that markups are not very countercyclical here, even though prices are sticky, we conduct the following decomposition of the real response of monetary shocks to a) nominal cost rigidities and b) markup variation. Recall that a cash-in-advance constraint holds in our model:

$$\ln(c_t) = \ln(M_t) - \ln(P_t) = \underbrace{[\ln(M_t) - \ln(\Omega_t)]}_{cost\ term} + \underbrace{[\ln(\Omega_t) - \ln(P_t)]}_{markup\ term}$$

The response of consumption is thus equal to the sum of two terms: one that captures the extent to which costs, Ω_t, decline relative to the money stock (cost term), and another that captures the extent to which prices decline relative to costs (markup term). We report, in Table 4, the average response of the second term (the markup decline) relative to the average response of consumption in order to measure the fraction of the real effects accounted for by countercyclical markups. In terms of the impulse responses of Figure 3, this ratio is equal to the area between the price and cost impulse responses relative to the area between the money supply and price responses. As the row labeled 'markup contribution' in Table 4 shows, markups account for only one-fourth of the real effects of money in this economy. Since consumption increases by about 0.44% on average in the first 2 years following the monetary shock, this implies that markups decline by an average of only about 0.11%.

Decreasing Returns at the Aggregate Level

We next introduce decreasing returns at the aggregate level, by assuming $\alpha = 2/3$. Figure 3 reports the impulse responses to a monetary shock in this economy. Panel A shows that although the nominal wage is sticky, the average marginal cost of production, $\bar{\Omega}\left(s^t\right) = \int \Omega_i\left(s^t\right) di$, increases sharply after the money shock. This decreases the incentive to invest in inventories since the expected growth in marginal cost is much lower than under constant returns. Panel B shows that now inventories gradually rise after the shock and increase much less than sales do. Moreover, since inventory investment is low, production is only slightly more volatile than sales. Overall, these impulse responses are much more in line with the data.

We report the quantitative predictions of the model in Panel B of Table 4. As in the data, the inventory-sales ratio is countercyclical (the correlation is -0.92 vs. -0.58 in the data). The elasticity of inventories to sales is only slightly greater than in the data (0.18 vs. 0.15 in the data), and production is only 1.05 times more volatile than sales (1.19 in the data).

The fit of the model improves for two reasons. First, marginal costs are more responsive to monetary shocks thereby reducing the intertemporal substitution motive. Second, now that costs are more volatile, sticky prices generate a greater decline in the markups of the firms that do not reset their prices. This drop in markups reduces the incentive to hold inventories and lower the inventory-sales ratio.

The decomposition of the real effects of money shows that in this version of the model countercyclical markups play a more important role: 43% of the average response of consumption to a monetary shock is accounted for by a decrease in markups, calculated as the ratio of the aggregate price level to the average marginal cost, $P(s^t)/\bar{\Omega}\left(s^t\right)$. This is about twice greater than in the economy with constant returns. We show below that variations of the model that do a better job at accounting for the facts on interest rates (in our model nominal interest rates are constant whereas in the data they decline following a monetary expansion), predict an even more important role for markups.

Thus, contrary to what Khan and Thomas (2007) find for *technology* shocks, for the model to account for the response of inventories to *monetary* shocks, countercyclical markups must play an important role. The difference stems from the special nature of monetary shocks. Unlike technology shocks, which shift the production possibilities frontier, monetary shocks can only affect output if either markups adjust or if nominal costs are sticky. The latter induces strong intertemporal substitution in production and investment in inventories, and is thus at odds with the data.

Panel C of Table 4 shows that the economy with capital adjustment costs predicts countercyclical inventory-sales ratio and an elasticity of inventories to sales of 0.75, thus in the neighborhood of the 0.15 elasticity in the data. Similarly, production is only 1.17 more volatile than sales (1.19 in the data). The drop in markups accounts for almost half of the increase in consumption due to a monetary shock.

5. Measuring the Response of Markups

We have shown above that variations of the model with strongly countercyclical markups do a much better job of accounting for the inventory facts than economies with no or little variation in markups. We next attempt to measure precisely the extent to which markups must decline in the aftermath of a monetary expansion in order for the model to account for the response of inventories in the data. We do so by calibrating the degree of decreasing returns necessary to account exactly for the elasticity of inventories to sales in the data. For simplicity, we focus on the version of the model with a fixed stock of capital at the aggregate level. We pin down the share of this fixed factor by matching the elasticity of inventories to sales of 0.15 in the data and then back out the contribution of markups to the total real effects of monetary shocks. We conduct this experiment using our Benchmark economy with Calvo sticky prices and wages and then consider several additional perturbations of the model to gauge the robustness of our results[18].

A. Benchmark model

Panel A. of Table 5 reports the results of this experiment for our Benchmark model with sticky prices and wages. There are two columns in this panel. The first, labeled "Constant Returns," presents results from the economy with no decreasing returns at the aggregate level (i.e., no capital). The second, labeled "Decreasing Returns," is the economy with a fixed stock of capital at the

[18] See also Kryvtsov and Midrigan (2010b) who conduct a number of additional robustness experiments in a Smets-Wouters (2007) - type economy with inventories.

aggregate level. We choose the share of the fixed factor to match exactly the 0.15 elasticity of inventories to sales in the data.

The table shows that the share of the fixed factor that matches the elasticity of inventories to sales in the data is equal to 0.46. Recall that we interpret this number as simply a measure of how important decreasing returns and other forms of adjustment costs are, and hence a measure of how volatile marginal costs are over the cycle. We do not interpret this number literally as an estimate of the share of capital in production.

With such a share of the fixed factor the model accounts well for the variability of inventory investment: production is 1.04 times more volatile than sales (1.19 times more volatile in the data). Also notice that the average response of consumption is 0.78 as large as in the economy with constant returns (0.29% vs 0.37%). Finally, our decomposition of the consumption response shows that a decline in markups accounts for more than a half (56%) of the overall increase in consumption after the monetary shock. We argue next that this number under-estimates the importance of countercyclical markups since the model fails to account for the behavior of interest rates, one of the two key components that determine the returns to holding inventories.[19]

B. Taylor Rule

Our Benchmark economy counterfactually predicts that the nominal interest rate is constant following a monetary policy expansion since the nominal interest rate is equal to the expected growth rate of the money supply which is iid. We next modify our assumptions regarding monetary policy and assume that it follows a Taylor-type interest rate rule. We follow Murchison and Rennison (2006) and assume that the monetary authority chooses its instrument so as to ensure that the nominal interest rate evolves according to:

$$r\left(s^t\right) = 0.8\ r(s^{t-1}) + (1 - 0.8)\ 2.5\ \Delta \log P\left(s^t\right) + \varepsilon_{it}$$

where $\Delta \log P\left(s^t\right)$ is inflation and ϵ_{it} is a disturbance. Notice that, as is standard in recent studies, we assume interest-rate smoothing, captured by the positive term 0.8 on the lagged nominal interest

[19] Our results do not critically depend on the assumptions we have made about the size of the inventory carrying costs, as captured by δ_z, the rate at which inventories depreciate. For example, doubling depreciation rate of stock only slightly decreases the elasticity of inventories to sales relative to the benchmark of 3.20.

rate, as well as that the nominal interest rate reacts to deviations of inflation from their steady-state level. We use the same coefficients in this interest rate rule as in Murchison and Rennison (2006) and study the response of our economy to a monetary expansion given by a negative shock ϵ_{it}. With such an interest rate rule, the nominal and real interest rates persistently decline following a monetary policy expansion, as in the data.

Notice in Panel B of Table 5 that we now require a somewhat higher share of the fixed factor (0.47) to match the elasticity of inventories to sales in the data. Intuitively the decline in interest rates makes the return to holding inventories increase even more after a monetary expansion, thereby increasing the incentive to invest in inventories. As a result we need even stronger decreasing returns at the aggregate level to undo the incentive for inventory accumulation. The greater decreasing returns assign an even more important role to countercyclical markups, since costs are now more responsive to a monetary shock. Our markup decomposition shows that 70% of the increase in consumption is accounted for by a decline in markups.

C. Higher elasticity of intertemporal substitution

We next assume $\sigma > 1$, and in particular, $\sigma = 1.5$, which is an alternative approach to ensure that the model predicts a decline in nominal interest rates after a monetary expansion, as in the data. Now the nominal interest rate declines following an increase in the growth rate of money supply since

$$r\left(s^t\right) = \frac{1}{\beta}\int \frac{P\left(s^{t+1}\right)c\left(s^{t+1}\right)^{\sigma}}{P\left(s^t\right)c\left(s^t\right)^{\sigma}}\pi\left(s^{t+1}|s^t\right)ds^{t+1} = \frac{1}{\beta}\int \mu(s^{t+1})\left(\frac{c\left(s^{t+1}\right)}{c\left(s^t\right)}\right)^{\sigma-1}\pi\left(s^{t+1}|s^t\right)ds^{t+1}$$

as consumption is highest immediately after the monetary shock and expected to mean-revert in future periods.

Panel C of Table 5 reports the predictions of the model under this parametrization. Once again we find that a greater share of the fixed factor (0.56) than in the Benchmark experiment is necessary to undo the incentive for inventory accumulation and account for the response of inventories in the data. With such a high share the marginal cost responds fairly strongly to the monetary shock and so countercyclical markups once again account for the majority of the real effects of monetary shocks. In this case 83% of the average consumption response is accounted for by the decline in markups.[20]

Overall, we conclude that our results are robust to variations in parameters governing preferences. Moreover, versions of our model that more closely match the dynamics of interest rates in the data predicts an even more important role for countercyclical markups in accounting for the real effects of monetary shocks.

D. Materials

So far we have assumed that sticky wages account for the sluggish response of costs to a monetary shock. The literature has identified a number of other mechanisms that give rise to similar outcomes, including use of materials (produced inputs) as a factor of production, as well as variable capital and labor utilization (see e.g., Dotsey and King (2006)). We show below that our conclusions are not specific to any particular source of such 'real rigidities'. In particular, we assume next that wages are flexible but rather, materials are a factor of production, alongside with labor and capital. These materials are purchased from final goods producers and, since prices are sticky, are sold at a price that does not fully react to monetary policy shocks.

Specifically, we now modify the production function of intermediate goods firms to:

$$y_i\left(s^t\right)=\left(l_i\left(s^t\right)^{\alpha}k_i\left(s^t\right)^{1-\alpha}\right)^{\gamma}n_i\left(s^t\right)^{1-\gamma}$$

where $n_i\left(s^t\right)$ is the amount of materials employed by the firm. Materials are purchased from final goods firms at a price $P\left(s^t\right)$ and so the unit cost of production is equal to:

$$\Omega\left(s^t\right)=\chi\left[W\left(s^t\right)^{\alpha}R\left(s^t\right)^{1-\alpha}\right]^{\gamma}P\left(s^t\right)^{1-\gamma}.$$

Even though wages are now flexible, the aggregate price level inherits the stickiness of the intermediate goods' prices and so reacts slowly to monetary shocks. Finally, the resource constraint for

[20] We have also considered an economy with a lower supply elasticity.It turns out however that the value of the labor supply elasticity does not matter much in our economy since unions face frictions on wage setting – if anything, a lower labor supply elasticity makes wages stickier because of a strategic complementarity in wage setting. Hence, when we lower the labor supply elasticity, we find very similar results to those above.

final goods is modified to:

$$c\left(s^{t}\right)+x\left(s^{t}\right)+\frac{\xi}{2}\left(\frac{x\left(s^{t}\right)}{k(s^{t-1})}-\delta\right)^{2}k(s^{t-1})+\int_{0}^{1}n_{i}\left(s^{t}\right)di=q\left(s^{t}\right)$$

We set the share of intermediate inputs equal to 0.60, consistent with the evidence in Basu (1995). (See also Nakamura and Steinsson (2008)).

Panel D of Table 5 shows that our earlier conclusions are unchanged under this alternative view of costs. We found, for our baseline case of log-linear preferences, that the share of the fixed factor must be equal to 0.68 in order for the model to match the variability of inventories in the data. Under such parametrization the markups once again play a much more important role and account for 65% of the real effects of monetary shocks.

6. Conclusions

We employ a model from Kryvtsov and Midrigan (2010a) that embeds a motive for inventory accumulation in a standard New Keynesian model with price and wage rigidities. The model predicts a tight relationship between inventories and the dynamics of costs and markups. Similarly to Kryvtsov and Midrigan, who applied their methodology to U.S., we evaluate the role of cost rigidities and markups in accounting for the real effects of monetary policy shocks in Canada. In the data inventories adjust slowly in response to shocks and are much less volatile than sales. Kryvtsov and Midrigan's theory interprets this fact as implying that countercyclical markups account for a sizable fraction of the real effects of monetary shocks.

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Table 1: Inventory Facts, Canada, 1993:1 - 2008:1

	A. Unconditional		B. Conditional on monetary shocks	
	Manufacturing and Trade	Retail	Manufacturing and Trade	Retail
$\rho(IS_t, S_t)$	-0.56	-0.60	-0.44	-0.58
$\sigma(IS_t) / \sigma(S_t)$	1.09	1.45	0.58	1.46
elast. IS_t w.r.t. S_t	-0.61	-0.87	-0.25	-0.85
elast. I_t w.r.t. S_t	0.39	0.13	0.75	0.15
$\rho(Y_t, S_t)$	0.94	0.91	0.95	0.99
$\sigma(Y_t) / \sigma(S_t)$	1.14	1.18	1.06	1.19

Notes: All series are real, at quarterly frequency.

The column labeled 'Unconditional' reports statistics for HP (1600)-filtered data

The column labeled "Conditional on monetary shocks" reports statistics computed using VAR model with 4 lags for HP-filtered output, sales and inventory-sales ratios for 1993:1 - 2008:1

Table 2: Parametrization

	Data	Model			
		Parameters			
		A. Flexible Prices		B. Sticky Prices	
		I. Firm-level, no capital	II. Firm-level, with capital	I. Firm-level, no capital	II. Firm-level, with capital
θ		5	5	5	5
δ		0.03	0.03	0.03	0.03
$1-\lambda_w$		1/4	1/4	1/4	1/4
$1-\lambda_p$		1	1	3/8	3/8
σ_v		0.42	0.42	0.42	0.42
δ_z		0.04	0.04	0.04	0.04
α		1	2/3	1	2/3
		Targets			
I/S ratio	0.5	0.5	0.5	0.5	0.5
Frequency stockouts	0.15	0.15	0.15	0.15	0.15

Table 3: Business Cycle Predictions of Flexible Price Economies

	Data	Model			
		No Capital	With Capital		
		A. Constant Returns	B. Adjustment costs	C. Fixed Capital	D. No adjustment costs
		Impulse response of consumption to monetary shock			
average response		0.37	0.24	0.24	0.26
maximum response		0.83	0.36	0.41	0.29
half-life, quarters		2.4	5.7	3.9	20.4
		Inventory Statistics			
$\rho(I/S_t, S_t)$	-0.58	1.00	0.90	0.93	-0.66
elast. I/S_t to S_t	-0.85	2.20	0.82	1.59	-0.27
elast. I_t to S_t	0.15	3.20	1.82	2.59	0.73
$\sigma(M_t) / \sigma(S_t)$	1.19	3.20	1.38	1.58	1.15
		Investment Statistics			
$\sigma(x_t) / \sigma(c_t)$	4	-	4	0	17.50

Note: all variables HP-filtered with smoothing parameter 1600

average output response computed for first 8 quarters after shock

Table 4: Business Cycle Predictions of Sticky Price Economies

	Data	Model			
		No Capital	With Capital		
		A. Constant Returns	B. Adjustment costs	C. Fixed Capital	D. No adjustment costs
Impulse response of consumption to monetary shock					
average response		0.44	0.35	0.33	0.45
maximum response		0.87	0.70	0.72	0.51
half-life, quarters		3.0	3.0	2.5	20.4
markup contribution		0.24	0.43	0.44	0.30
Inventory Statistics					
$\rho(I/S_t, S_t)$	-0.58	0.98	-0.92	-0.45	-0.97
elast. I/S_t to S_t	-0.85	1.75	-0.82	-0.25	-2.05
elast. I_t to S_t	0.15	2.75	0.18	0.75	-1.05
$\sigma(M_t) / \sigma(S_t)$	1.19	1.87	1.05	1.17	0.57
Investment Statistics					
$\sigma(x_t) / \sigma(c_t)$	4	-	4	0	48.94

Note: all variables HP-filtered with smoothing parameter 1600

average output response computed for first 8 quarters after shock

Table 5: Measuring the Response of Markups

	Data	Model							
		A. Benchmark		**B. Taylor rule**		**C. Higher IES**		**D. Intermediate inputs**	
		Constant Returns	Decreasing returns	Constant Returns	Decreasing returns	Constant Returns	Decreasing returns	Constant Returns	Decreasing returns
Share of fixed factor		0.00	0.46	0.00	0.47	0.00	0.56	0.00	0.68
Impulse response of consumption to monetary shock									
average response		0.44	0.29	0.61	0.36	0.39	0.24	0.44	0.26
markup contribution		0.24	0.56	0.37	0.70	0.36	0.83	0.25	0.65
Inventory Statistics									
elast. I_t to S_t	0.15	2.75	0.15	4.49	0.15	4.16	0.15	1.89	0.15
$\sigma(M_t) / \sigma(S_t)$	1.19	1.87	1.04	3.00	1.03	2.49	1.05	1.51	1.02

Note: all variables HP-filtered with smoothing parameter 1600

average output response computed for first 8 quarters after shock

Figure 1A. I/S and S dynamics, Manufacturing and Trade

unconditional HP-filtered series

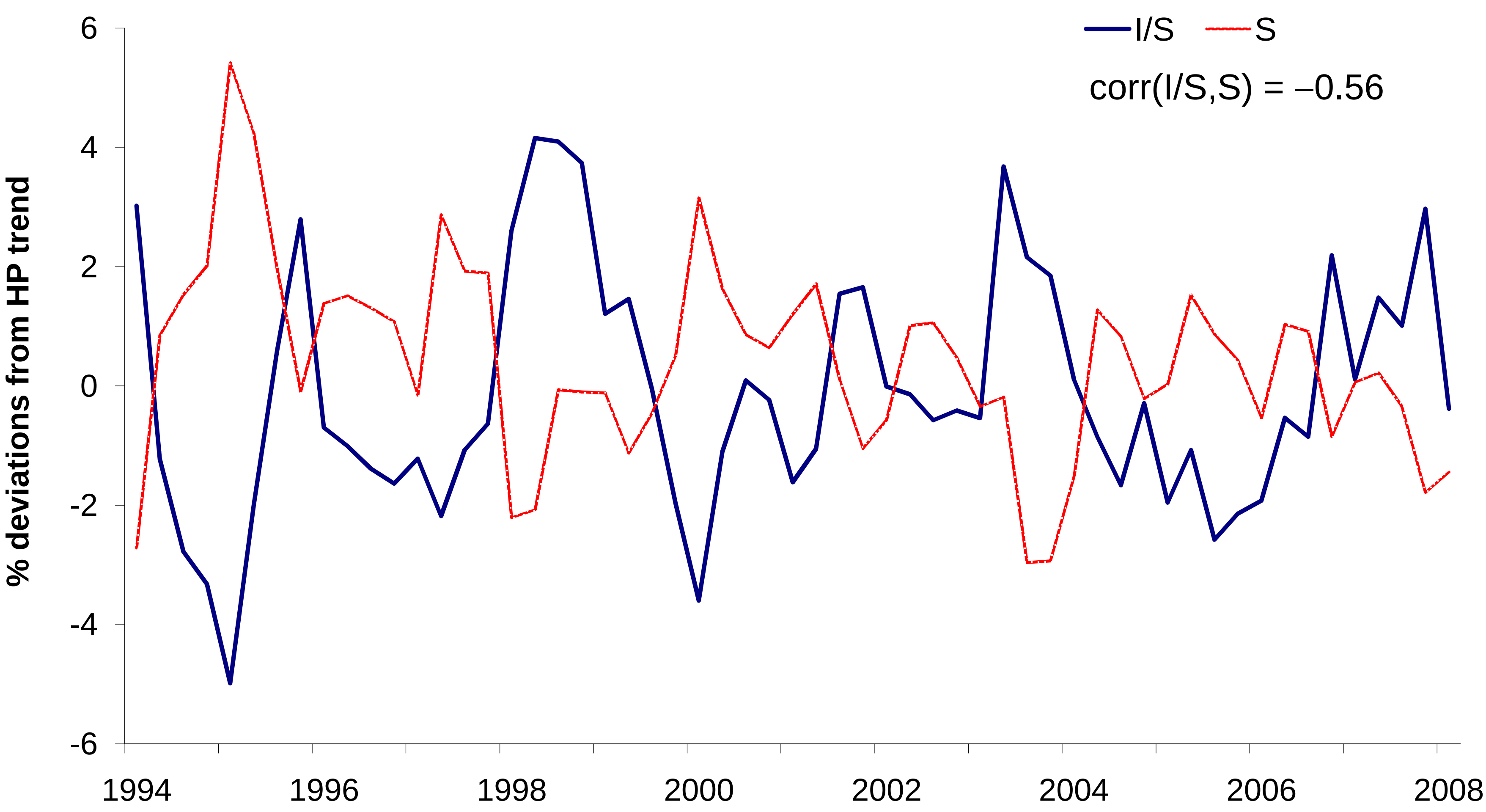

Figure 1B. I/S and S dynamics, Manufacturing and Trade

conditional on monetary policy shocks

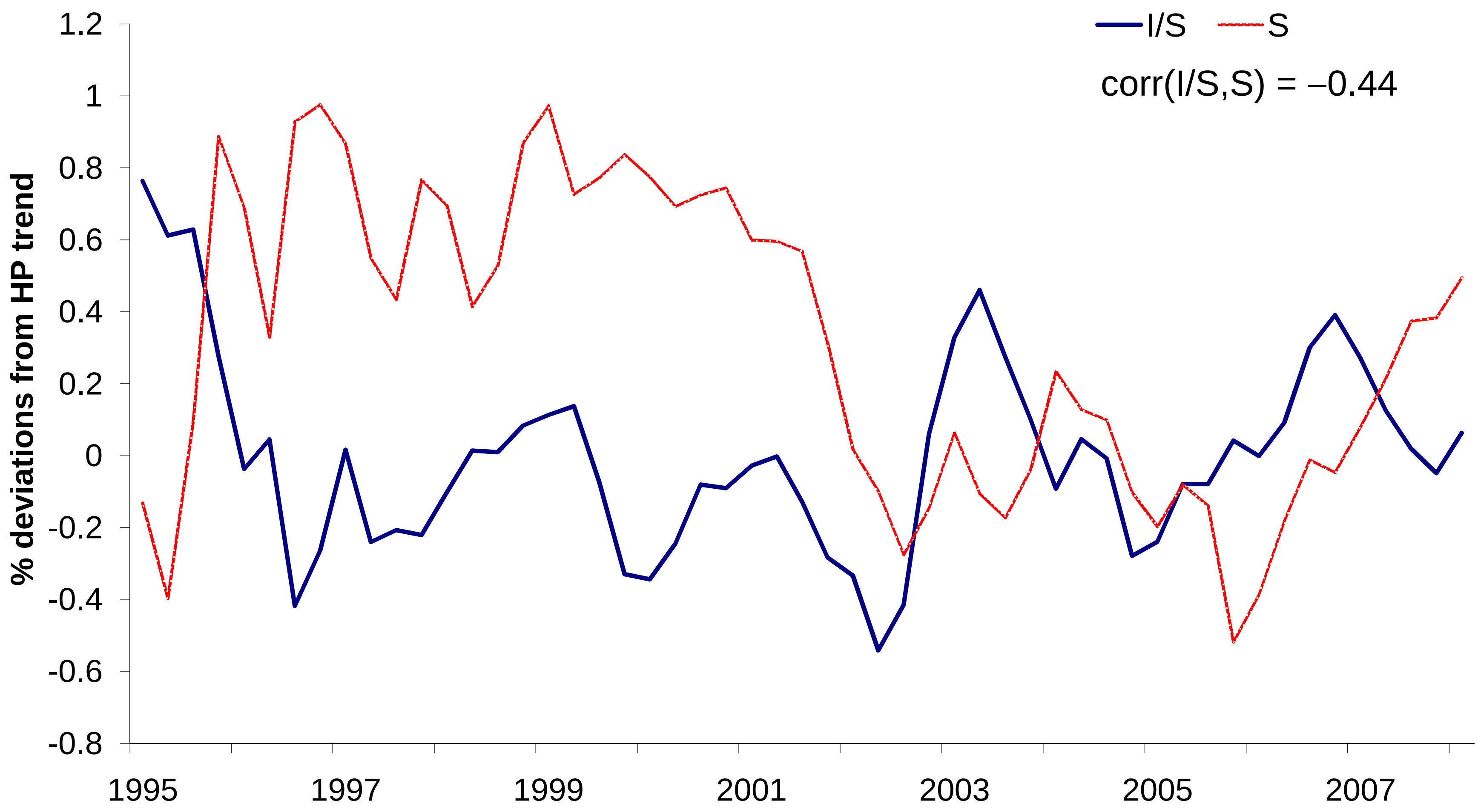

Figure 2. Impulse response to money shock. Flexible prices.

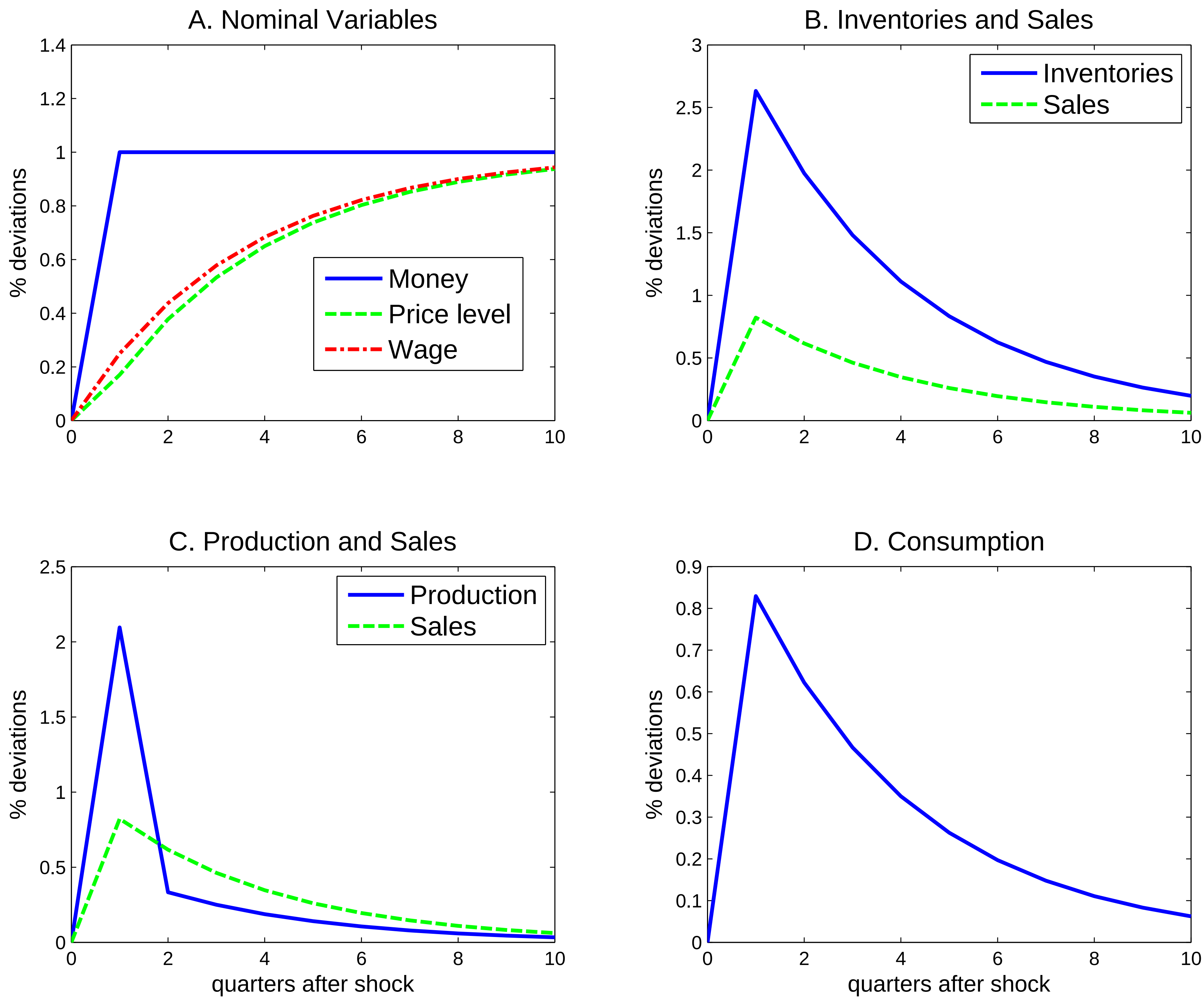

Figure 3. Impulse response to money shock. Sticky prices and decreasing returns.