Money and Costly Credit

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Acknowledgements

Abstract

I study an economy in which money and credit coexist as means of payment and the settlement of credit requires money. The model extends recent developments in microfounded monetary theory to address the choice of payment methods and the effects of inflation. Whether a buyer uses money or credit depends on the fixed cost of credit and the inflation rate. In particular, inflation not only makes money less valuable, but also makes credit more expensive because of delayed settlement. Based on quantitative analysis, the model suggests that the relationship between inflation and credit exhibits an inverse U-shape which is broadly consistent with anecdotal evidence. Compared to an economy without credit, allowing credit as a means of payment has three implications: [1] it lowers money demand at low to moderate inflation rates; [2] it improves society's welfare when the inflation rate exceeds a specific threshold; and [3] it can raise the welfare cost of inflation for some reasonable values of the credit cost parameter.

JEL classification: E41, E50
Bank classification: Credit and credit aggregates; Inflation: costs and benefits

Résumé

L'auteure étudie une économie où la monnaie et le crédit coexistent comme moyens de paiement et où le remboursement du crédit requiert de la monnaie. Son modèle tire parti, en les prolongeant, des progrès récents réalisés dans l'élaboration des fondements microéconomiques de la théorie monétaire pour analyser le choix de la méthode de paiement et les effets de l'inflation. La méthode retenue pour le règlement des achats dépend du coût fixe du crédit et du taux d'inflation. En particulier, l'inflation non seulement érode la valeur de la monnaie, mais elle renchérit le crédit du fait que celui-ci est remboursé avec un délai. Les résultats de la résolution numérique du modèle indiquent que la relation entre l'inflation et le crédit a la forme d'une bosse, ce qui cadre dans l'ensemble avec les observations. Par rapport à une économie sans crédit, l'inclusion du crédit parmi les modes de paiement a trois conséquences : 1) elle entraîne un recul de la demande de monnaie quand l'inflation est faible ou modérée; 2) elle a pour effet d'améliorer le bien-être de la société lorsque l'inflation dépasse un seuil déterminé; 3) elle peut accroître le coût de l'inflation sur le plan du bien-être pour certaines valeurs raisonnables du paramètre relatif au coût du crédit.

Classification JEL : E41, E50
Classification de la Banque : Crédit et agrégats du crédit; Inflation : coûts et avantages

1 Introduction

This paper develops a model of money and credit in order to study issues in monetary economics, in particular, the choice of payment methods and the effects of inflation. Not too long ago, consumers typically paid for their purchases using either cash or checks. In recent decades, the payment instruments available to consumers have expanded to include debit and credit cards as well as other electronic payments. In a credit transaction, credit is usually offered by a third party, and this third party is willing to postpone debt settlement to the indefinite future – at high rates of interest. A prominent difference between credit and other means of payment is that acquisition of money in advance is not necessary to make a purchase using credit. In other words, credit transactions are settled "later".[1]

Credit has become increasingly popular as a means of payment in many countries. Based on the Payment System Statistics from the BIS, the percentage of total number of transactions using cards with a credit function in 2005 was 24.8% in Canada, 23.4% in the United States, and 13% in the United Kingdom. Table 1 contains the ratio of transaction value by cards with credit function to GDP in Canada and the United States. Figure 1 shows the number of card with credit function issued from 1990 to 2008 in selected countries. These statistics suggest that more credit cards have been issued and more transactions have been paid for with credit cards.

	2000	2002	2004	2006	2008
Canada	11.68	13.26	14.04	16.1	18.29
the United States	12.54	12.66	13.6	14.58	14.95

Table 1: Ratio: Value of Transaction by Card with Credit Function
to GDP (units: $\times 10^{-5}$)

If consumers can use both money and credit to pay for their purchases, how will money and credit interact in such an environment? In particular, what is the effect of monetary policy or inflation on credit? How does the introduction of credit affect money demand, social welfare and the effects of monetary policy? These questions are interesting and important for the conduct of monetary policy in an economy in which money and credit coexist as means of payment.

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[1]Checks can be viewed as a short-term form credit. However, since the time until settlement is typically very short for checks, checks are not considered as "settled later" in this paper.
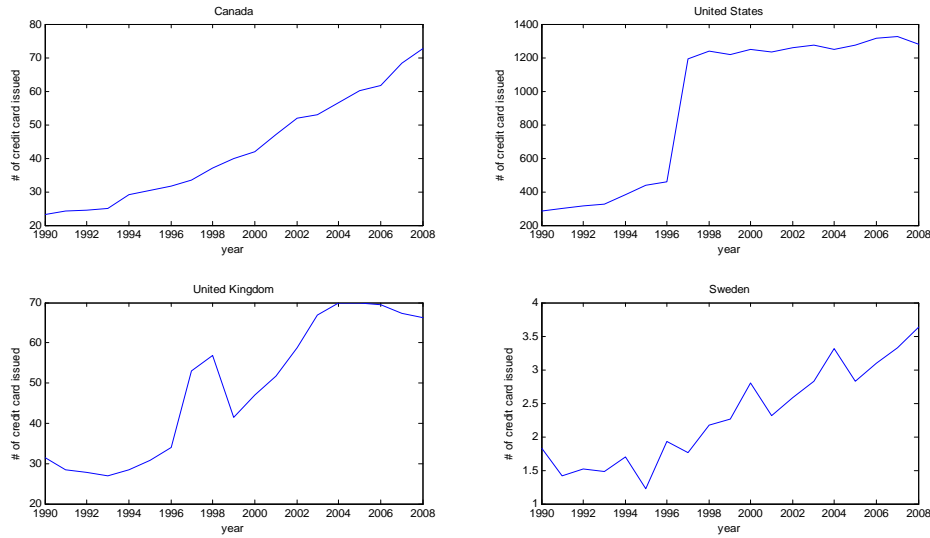
Figure 1: Number of Card Issued with Credit Function (in Millions)

Technological advances in electronic record-keeping is one of the main factors that have facilitated the use of consumer credit.[2] Specialized intermediaries are now able to offer consumer credit with limits that can vary with each person's credit history. These technological advances allow credit to substitute for money and other means of payment in transactions. For example, based on U.S. household-level data, Duca and Whitesell (1995) estimate that for every 10% increase in the probability of owning a credit card, checking balances are reduced by 9%.

However, how inflation affects the money-credit margin in practice is less clear. Empirical investigation here is hampered to some extent by data limitations and the recent improvements in credit card technology. Nevertheless, evidence from countries that suffered high inflation episodes tends to suggest that credit card usage increased after inflation was brought under control. For example, credit cards gained widespread popularity in Brazil following the successful reduction of inflation to sustainable levels, and the number of cards issued grew by 88% between 2000 and 2004. High inflation episodes also delayed the adoption and widespread use of credit cards in Turkey. Even in Australia, households' debts increased dramatically when there were lower inflation rates and thus lower cost of borrowing during the 1990s.[3] Reducing inflation seems to be consistent with

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[2]Financial deregulation is another important factor that explains the increase in credit usage.

[3]The inflation rate was on average around 8% in Australia in the 1980s and was reduced to around 3% in the 1990s.

the increasing use of credit.

Evidence based on total private credit to GDP ratio, a broader measure of credit, suggests that inflation tends to have a negative impact on credit at high rates, but not at low rates.[4] Using a sample of 97 countries, Boyd et al. (2001) conclude that inflation has a negative impact on credit. See also Boyd and Champ (2003). Later, Khan et al. (2006) also use a large cross-country sample, but they find that there is a threshold effect of inflation on credit. Inflation has a negative impact on credit when it exceeds a threshold.

In this paper, I construct a model that is able to replicate the observed relationship between inflation, money demand and credit. The model is built on Lagos and Wright (2005). In monetary theory, frictions that render money essential make credit arrangements impossible. In order for credit to exist, I assume that there exist competitive financial intermediaries that can identify agents and have access to a record-keeping technology. There are two frictions associated with credit arrangements. First, arranging credit is costly. In a bilateral trade, if a buyer wants to use credit, he must incur a fixed utility cost in order to make the seller and himself identified to a financial intermediary. When buyers have heterogeneous preferences, the fixed cost of credit will endogenously determine the fraction of buyers using credit. Second, the settlement of credit is available only at a particular time in each period, during which the financial intermediaries accept repayment of credit and settle debts. Due to the timing structure of the model, settlement is "delayed" and money becomes the only means of settlement.

These two features of the model allow some interesting interactions between money and credit. Inflation tends to increase the fraction of buyers using credit at low inflation rates, but decrease the fraction of buyers using credit at high inflation rates. That is, the relationship between inflation and credit exhibits an inverse U-shape. Compared to an economy without credit, the model has three implications: [1] the real demand for money is lower at low to moderate inflation rates; [2] social welfare is higher when the inflation rate exceeds a specific threshold; and [3] for a given inflation rate, the welfare cost of inflation can be higher for some reasonable values of the credit cost parameter.

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[4]Total private credit may be too broad in comparison to consumer credit. Given that [1] it is difficult to obtain data on consumer credit for a large sample of countries over an adequately long time span; and [2] different measures of credit tend to highly comove, as can be seen from the U.S. data, it seems useful and reasonable to review this evidence.

This paper contributes to the literature by developing a model which incorporates the following three features observed in an economy with money and credit: first, money and credit coexist as means of payment; second, the choice of using money or credit is endogenous; and third, the settlement of credit requires money. Several recent papers have attempted to construct models with both money and credit.[5] However, they are not able to capture all three features. In Berentsen et al. (2007), banks can record financial history, but they cannot record goods transaction history. In this case, credit is available through bank loans in the form of money. Therefore, money is the only means of payment. Chiu and Meh (forthcoming) extend Berentsen et al. (2007) to study how credit/financial intermediation affects allocations and welfare in an economy where ideas (or projects) are traded among investors and heterogeneous entrepreneurs. Again, credit does not serve as a means of payment. See also Camera and Ruscitti (2008) for a paper on inflation and financial intermediation. Telyukova and Wright (2008) build a model where agents can use money and credit to explain the credit card debt puzzle. Their market structure determines that agents do not use money and credit simultaneously. Sanches and Williamson (2010) adopt the notion of limited participation in the sense that only an exogenous subset of agents can use credit. The choice of using credit is not endogenous. See also Monnet and Roberds (2008).

Another contribution of the paper is that the model has a novel prediction of the relationship between inflation and credit. To generate the inverse U-shape relationship, both frictions associated with credit arrangement are important. Indeed, the two frictions imply opposite effects of inflation on credit. Costly credit has been used in the cash-in-advance model or in the OLG model.[6] With the presence of the fixed cost, inflation always decreases money demand and increases credit demand. I label the effect of inflation on credit through the fixed cost channel as the fixed cost effect. "Delayed" settlement is used in Ferraris (2006), where money and credit are complements. In fact, this idea can be traced back to Stockman (1981), where inflation reduces the capital stock if money and capital are complements. The delayed settlement effect of inflation on credit is that inflation should reduce credit. Overall, it turns out that the fixed cost effect dominates at low inflation rates

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[5] The literature on money and credit is vast and thus I do not intend to review all these papers. Rather, I will only mention a few papers that are more recent or are more directly related to this paper. For example, for recent attempts to rationalize the coexistence of inside and outside money, see Cavalcanti and Wallace (1998), Kocherlakota and Wallace (1998), Mills (2007), Sun (2007). A more recent paper by Lester et al. (2008) studies the coexistence of multiple assets which differ in their return and liquidity.

[6] For cash-in-advance models, see Lacker and Shreft (1996), Aiyagari et al. (1998), and English (1999) for examples. For the OLG framework, see Freeman and Huffman (1991) for an example.

and the delayed settlement effect dominates at high inflation rates. This prediction is consistent with the anecdotal evidence. Azariadis and Smith (1996) obtain a similar result. However, the key friction driving their result is asymmetric information associated with credit.

The rest of the paper is organized as follows. Section 2 lays out the physical environment. Section 3 solves for the equilibrium and analyzes the equilibrium when the repayment of credit can be enforced. I numerically study the model in Section 4. In Section 5, I consider monetary equilibrium when the repayment of credit cannot be enforced. Finally, Section 6 concludes. All proofs are provided in the Appendix.

2   Environment

Time is discrete and runs forever. In each period, there are three submarkets that open sequentially. The first submarket is characterized by bilateral trades and is labelled as market 1. The second submarket is characterized by a centrally located competitive spot market and is labelled as market 2. No trade occurs in the third submarket and it is labelled as market 3. The activity in the third submarket will be described in detail later. There are two permanent types of agents – buyers and sellers, each with measure 1. Buyers are those who want to consume in market 1 and sellers are those who produce in market 1. All agents are anonymous and lack commitment. Each buyer receives a preference shock ε at the beginning of each period, which determines the buyer's preference in market 1.[7] The preference shock ε is drawn from a *c.d.f.* $G(\varepsilon)$. The preference shocks are *iid* across buyers and across time. The realization of these preference shocks is public information. There are two types of goods. Goods that are produced and consumed in market 1(2) are called good 1(2). All goods are nonstorable.

In market 1, buyers and sellers are matched randomly according to a matching technology. The probability that a buyer (seller) meets a seller (buyer) is σ with $0 < \sigma \leq 1$. Given that a buyer and a seller meet, the terms of trade are determined by the buyer's take-it-or-leave-it offer. After exiting market 1, all agents enter market 2. Buyers supply labor for production and consume good 2. Sellers only consume good 2. The production technology in market 2 is assumed to be linear and 1 unit labor can be converted into 1 unit of good 2.

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[7] There are a variety of ways to model the heterogeneity in this model. For example, one can model heterogeneous sellers that have different cost functions or model heterogeneity as match specific shocks.

The preference of a buyer with a preference shock ε is $\varepsilon u(q) + v(x) - h$, where $\varepsilon u(q)$ is the buyer's utility from consuming q units of good 1. The utility function $u(q)$ satisfies $u(0) = 0$, $u'(0) = \infty$ and $u''(q) < 0 < u'(q)$. In market 2, the buyer's utility from consuming x units of good 2 is $v(x)$, where $\lim_{x \to 0} v'(x) = \infty$ and $v''(x) < 0 < v'(x)$. The buyer's disutility from working is h. The preference of a seller is $-c(q) + y$, where $c(q)$ is the seller's disutility from producing q units of good 1 with $c(0) = 0$, $c'(0) = 0$, $c'(q) > 0$ and $c''(q) \geq 0$. The seller has a linear utility in market 2, where y is the amount of consumption of good 2. All agents discount between market 3 and the next market 1. The discount rate is β.

In this economy, money is essential because agents are anonymous and lack commitment. I assume that there is a monetary authority that controls the supply of money. Let M denote the aggregate money supply at any given date. It grows at a gross rate $\gamma > 0$, i.e., $\hat{M} = \gamma M$. Here the hat denotes the variable in the next period. I will consider $\gamma > \beta$ and $\gamma \to \beta$ from above. New money is injected (or withdrawn) via a lump-sum transfer (or tax) to each buyer at the beginning of each period and the transfer is $\hat{\tau} = (\gamma - 1)M$.

Besides the monetary authority, there exist competitive financial intermediaries. These financial intermediaries possess a record-keeping technology, which allows them to identify agents and keep track of goods market transaction history. Clearly the availability of the record-keeping technology makes credit arrangements through the financial intermediaries possible in this economy. To sustain the essentiality of money, I assume two frictions associated with the record-keeping technology. The first friction is that the record-keeping technology or the financial intermediaries are not available in market 2. This restriction implies that agents may arrange credit transactions in market 1, but cannot settle their debts in market 2. As the financial intermediaries are available in market 3, buyers who have used credit in market 1 repay their debts and sellers who have extended credit get repayment in market 3. One can think of market 3 as an overnight market for settlement. Without such a restriction, agents would settle their debts in market 2. In some sense, the settlement of debts is delayed. Money becomes the only means of settlement because goods are nonstorable. The second friction associated with the record-keeping technology is that it is costly. As all agents are anonymous, the buyer in a match in market 1 can incur a fixed utility cost k to make the pair identifiable to a financial intermediary so that the seller can extend credit to the buyer.[8]

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[8]One may argue that in reality, sellers actually pay the cost of using credit. The model can be modified to have

Without incurring the fixed cost, the buyer and the seller remain anonymous and cannot make credit arrangements. Figure 2 provides a timeline of events.[9]
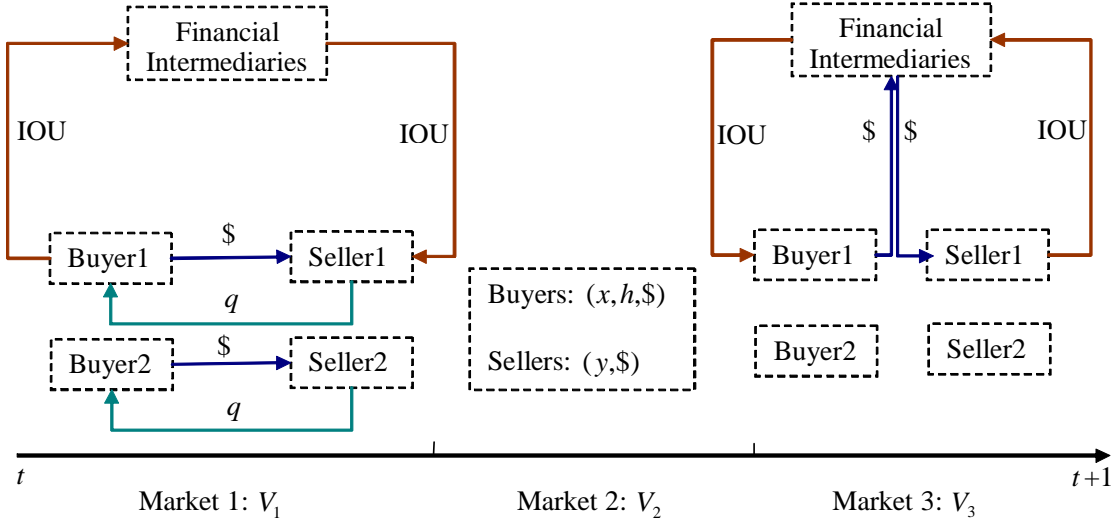

Figure 2: Timeline of Events

Now I consider a planner's problem as the benchmark allocation. Suppose that the planner weights all agents equally and is subject to the random matching technology. I restrict the attention to stationary allocations in what follows. In market 1 of each period, given a buyer's preference shock ε and the buyer meeting a seller, the planner instructs the seller to produce $q(\varepsilon)$ for the buyer. Those agents who do not find a match consume and produce nothing. In market 2 of each period, the planner assigns the consumption of good 2 x, y and the labor supply h subject to the resource constraint. Formally, the planner's problem is

$$\max_{q(\varepsilon),x,y,h} \left\{ \sigma \int [\varepsilon u(q(\varepsilon)) - c(q(\varepsilon))]dG(\varepsilon) + v(x) - h + y \right\} \tag{1}$$

$$\text{s.t. } x + y = h.$$

The optimal $(x^*, q^*(\varepsilon))$ are characterized by $v'(x) = 1$ and $\varepsilon u'(q(\varepsilon)) = c'(q(\varepsilon))$ for all ε. The optimal $q(\varepsilon)$ is increasing in ε. In fact, the optimal allocation features a slight indeterminacy. That is, given the quasi-linear preference structure, h and y are indeterminate as long as $h - y = x^*$.

the seller pay the fixed cost in a match. All the main results hold.

[9]Note that the IOU in Figure 2 refers to the record kept by the financial intermediaries. Financial intermediaries do not issue physical IOUs. I also rule out the existence of private claims on credit.

8

3 Monetary Equilibrium with Enforcement

In this section, I assume that there is enforcement in the economy. It implies that the financial intermediaries can enforce the repayment of credit, so there is no credit limit for buyers. It also implies that the monetary authority can impose lump-sum taxes, i.e., $\gamma < 1$ is feasible.

3.1 Buyers

To facilitate the analysis, I begin with buyers in market 2. Suppose that in nominal terms, a buyer carries money balance m and debt ℓ at the beginning of market 2. Let $V_2^b(m, \ell)$ and $V_3^b(z, \ell)$ be the value functions for a buyer in market 2 and 3, respectively. Notice that the buyer cannot pay off his debt in market 2 because the financial intermediaries are not available. However, the buyer can accumulate money balance. Let z denote the money balance that the buyer carries to market 3. The buyer's choice problem is

$$V_2^b(m, \ell) = \max_{x, h, z} \left\{ v(x) - h + V_3^b(z, \ell) \right\} \tag{2}$$

$$\text{s.t. } x + \phi z = \phi m + h,$$

where ϕ is the inverse of the price level (or the value of money). Substituting h from the buyer's budget constraint into (2), the unconstrained problem is

$$V_2^b(m, \ell) = \phi m + \max_{x, z} \{ v(x) - x - \phi z + V_3^b(z, \ell) \}.$$

The first order conditions for interior solutions are $v'(x) = 1$ and

$$\frac{\partial V_3^b(z, \ell)}{\partial z} = \phi. \tag{3}$$

As is standard, the choice of z does not depend on m; however, it depends on ℓ. Intuitively, if the buyer incurs more debt in market 1, he must accumulate more money in market 2 for repayment

in market 3. The envelope conditions imply

$$\frac{\partial V_2^b(m, \ell)}{\partial m} = \phi, \tag{4}$$

$$\frac{\partial V_2^b(m, \ell)}{\partial \ell} = \frac{\partial V_3^b(z, \ell)}{\partial \ell}. \tag{5}$$

Note that $V_2^b(m, \ell)$ is linear in m.

For the buyer entering market 3, the value function is

$$V_3^b(z, \ell) = \beta \int \hat{V}_1^b(z - \ell + \hat{\tau}, 0; \varepsilon) dG(\varepsilon).$$

The only activity for the buyer in market 3 is to repay his debt. Due to the quasilinear structure of the buyer's preference, I focus on one period contract without loss of generality. That is, if the buyer has any debt, he repays in the current market 3. It will become clear later that the buyer should be indifferent between repaying the debt in the current market 3 or in any future market 3. Let $\hat{m} = z - \ell + \hat{\tau}$ be the buyer's money holding at the beginning of the next period. For a buyer receiving a preference shock ε at the beginning of the next period, let $\hat{V}_1^b(\hat{m}, 0; \varepsilon)$ be the buyer's value function. As $\hat{V}_1^b(\hat{m}, 0; \varepsilon)$ depends on ε, I take the expected value for the buyer in market 3 and discount it by β. The envelope conditions yield

$$\frac{\partial V_3^b(z, \ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon), \tag{6}$$

$$\frac{\partial V_3^b(z, \ell)}{\partial \ell} = -\beta \int \frac{\partial \hat{V}_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon). \tag{7}$$

Combining (3) and (5) with (6) and (7),

$$\frac{\partial V_3^b(z, \ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon) = -\frac{\partial V_3^b(z, \ell)}{\partial \ell} = -\frac{\partial V_2^b(m, \ell)}{\partial \ell} = \phi. \tag{8}$$

From (8), $V_2^b(m, \ell)$ is linear in ℓ and $V_2^b(m, \ell) = \phi m - \phi \ell + V_2^b(0, 0)$.

After exiting market 3, each buyer realizes a preference shock ε. For a buyer with ε, the value function in market 1 is

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - k \cdot I(a) + V_2^b(m - d, a \cdot I(a))] + (1 - \sigma)V_2^b(m, 0),$$

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where (q, d, a) are the terms of trade. With probability σ, the buyer spends d units of money and uses a units of credit in nominal terms in exchange for q units of good 1 from the seller. An indicator function $I(a)$ is such that $I(a) = 1$ if $a > 0$ and $I(a) = 0$ if $a = 0$. With probability $1 - \sigma$, the buyer is not matched and carries his money to market 2.

3.2 Sellers

Let $V_2^s(m, \ell)$ and $V_3^s(z, \ell)$ be a seller's value functions in market 2 and 3, respectively. Since the seller is the creditor, ℓ should be either 0 or negative. The seller's value function in market 2 is

$$V_2^s(m, \ell) = \max_{y, z} \{y + V_3^s(z, \ell)\} \tag{9}$$

$$\text{s.t. } y + \phi z = \phi m.$$

By substituting y from the constraint into (9), the first order condition of the unconstrained problem is

$$\frac{\partial V_3^s(z, \ell)}{\partial z} \leq \phi, \text{ and } z = 0 \text{ if } \frac{\partial V_3^s(z, \ell)}{\partial z} < \phi. \tag{10}$$

The envelope conditions yield

$$\frac{\partial V_2^s(m, \ell)}{\partial m} = \phi, \tag{11}$$

$$\frac{\partial V_2^s(m, \ell)}{\partial \ell} = \frac{\partial V_3^s(z, \ell)}{\partial \ell}. \tag{12}$$

Again, $V_2^s(m, \ell)$ is linear in m.

For the seller in market 3, the value function is

$$V_3^s(z, \ell) = \beta \int \hat{V}_1^s(z - \ell, 0; \varepsilon) dG(\varepsilon).$$

If the seller has extended any credit in the previous market 1, the seller will receive repayment from the financial intermediary in market 3. I take the expected value function of the seller because the seller anticipates that a potential buyer he will meet in the next market 1 may have a preference shock ε drawn from $G(\varepsilon)$. Let $\hat{m} = z - \ell$ denote the seller's money holding at the beginning of the

next market 1. The envelope conditions are

$$\frac{\partial V_3^s(z, \ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon), \tag{13}$$

$$\frac{\partial V_3^s(z, \ell)}{\partial \ell} = -\beta \int \frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon). \tag{14}$$

Now combining (12) with (13) and (14), I obtain

$$\frac{\partial V_3^s(z, \ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon) = -\frac{\partial V_3^s(z, \ell)}{\partial \ell} = -\frac{\partial V_2^s(m, \ell)}{\partial \ell}. \tag{15}$$

For the seller who potentially meets a buyer with ε, the value function in market 1 is

$$V_1^s(m, 0; \varepsilon) = \sigma[-c(q) + V_2^s(m + d, -a)] + (1 - \sigma)V_2^s(m, 0). \tag{16}$$

If the seller meets a buyer, the seller sells q units of good 1, receives d units of money and extends credit with the nominal value a if the buyer chooses to use credit.

3.3 Equilibrium

3.3.1 Take-it-or-Leave-it Offer

Before deriving the equilibrium conditions, I solve for the terms of trade in market 1. The terms of trade in a match are determined by the buyer's take-it-or-leave-it offer.[10] There are two types of trades in market 1, depending on whether the buyer in a match uses credit or not.

Suppose that a buyer with ε uses only money. Recall that V_1^b and V_1^s are linear in m. The buyer's problem is

$$\max_{q,d}[\varepsilon u(q) - \phi d]$$

$$\text{s.t. } c(q) = \phi d \text{ and } d \leq m,$$

where m is the buyer's money holding. Let λ_1 and λ_2 be the Lagrangian multipliers associated

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[10]It will be interesting to generalize the buyer's bargaining power from 1 to less than 1. It will also be interesting to study other pricing mechanisms that have been used in the literature such as competitive pricing and price posting. In this paper, I focus only on the buyer's take-it-or-leave-it offer to get the main intuition from the model and leave those extensions for future work.

with the two constraints.

$$\mathcal{L} = \max_{q,d,\lambda_1,\lambda_2} [\varepsilon u(q) - \phi d] + \lambda_1 [\phi d - c(q)] + \lambda_2 (m - d).$$

It is straightforward that the solution is the following.

$$
\begin{cases}
\lambda_2 = 0: \ (q,d) \text{ are given by } \varepsilon u'(q) = c'(q) \text{ and } \phi d = c(q), \\[2mm]
\lambda_2 > 0: \ (q,d) \text{ are given by } d = m \text{ and } c(q) = \phi d.
\end{cases}
$$

Suppose that the buyer with ε uses credit. From (8), V_2^b is also linear in ℓ. However, it is not clear that V_2^s must be linear in ℓ at this stage. So I define the buyer's problem as

$$\max_{q,d,a} [\varepsilon u(q) - k - \phi d - \phi a]$$

$$\text{s.t. } c(q) = \phi d + V_2^s(0, -a) - V_2^s(0, 0) \text{ and } d \leq m.$$

It is obvious that the seller's money holding does not appear in the above problem. Therefore, the terms of trade with credit do not depend on the seller's money holding. In addition, recall that the terms of trade without credit do not depend on the seller's money holding. It follows from (16) that

$$\frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} = \sigma \hat{\phi} + (1 - \sigma)\hat{\phi} = \hat{\phi}. \tag{17}$$

From (15) and (17),

$$\frac{\partial V_3^s(z, \ell)}{\partial z} = -\frac{\partial V_3^s(z, \ell)}{\partial \ell} = -\frac{\partial V_2^s(m, \ell)}{\partial \ell} = \beta \hat{\phi}. \tag{18}$$

Two results follow from (18). First, V_2^s is linear in ℓ and $V_2^s(m, \ell) = \phi m - \beta \hat{\phi} \ell + V_2^s(0, 0)$. Second, s . As I only consider $\gamma > \beta$ and $\gamma \to \beta$ from above, the second result is derived from (10) and (18).

Using (18), the Lagrangian is

$$\mathcal{L} = \max_{q,d,a,\lambda_1,\lambda_2} [\varepsilon u(q) - k - \phi d - \phi a] + \lambda_1 [\phi d + \beta \hat{\phi} a - c(q)] + \lambda_2 (m - d).$$

It turns out that the inequality constraint is always binding, and thus $d = m$. The solutions for

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(q, a) are

$$\varepsilon u'(q) = \frac{\gamma}{\beta} c'(q), \tag{19}$$

$$\beta \hat{\phi} a = c(q) - \phi d. \tag{20}$$

It is interesting to note that q depends on γ. In this economy, if a buyer uses credit in market 1, he will accumulate money for debt repayment in market 2. However, for the seller who extends the credit in the match, he will not be paid in the same market 2. Instead, the seller must wait to get settled in market 3. After receiving the money, the seller carries the money to the next market 1, but he cannot spend it since he does not want to consume. Hence, the seller actually spends the money one period after the buyer accumulates the money. There is an asymmetry between the time at which the buyer accumulates the money for repayment (production) and the time at which the seller can spend the money from repayment (consumption). The buyer must compensate the seller for the loss in the value of money. From (20), the buyer essentially borrows $\frac{\beta \hat{\phi}}{\phi} a$ and repays a in nominal terms. The nominal interest rate of credit is $1 + i = \frac{\phi}{\beta \hat{\phi}} = \frac{\gamma}{\beta}$. As γ is higher, credit is more costly in nominal terms. For any given ε, q is decreasing in γ. Credit transactions are subject to inflation distortion.

As the structure of the model implies that money is the only means to settle credit, one may think that it is natural inflation affects a credit transaction's terms of trade. However, this is not necessarily true. The key feature that makes credit subject to inflation distortion is the time lag between buyer's production for repayment and seller's consumption from repayment.[11] Imagine an environment in which financial intermediaries exist in market 2 and the settlement of credit requires money. Sellers receive the repayment in the form of money and can spend it in the same market 2. It is clear that a credit transaction's terms of trade do not depend on the inflation rate in this scenario although money is imposed as the only means of settlement.

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[11] If sellers can deposit the money from repayment in a bank and earn interest as in Berentsen et al. (2007), then they would be fully compensated and the terms of trade would not be affected by inflation. However, a full general equilibrium analysis that incorporates bank deposits involves more careful consideration. For example, since financial intermediaries are not available in market 2, sellers cannot withdraw their deposits (if there is any) in market 2, which impedes the use of deposits. It is also not obvious who would borrow those deposits from the financial intermediary in the current model.

3.3.2 Money versus Credit

Having solved the terms of trade, I proceed to find the condition that determines whether a buyer uses credit or not. For a buyer with ε in market 1, if he uses only money,

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - c(q)] + \phi m + V_2^b(0, 0).$$

If the buyer uses credit,

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - \frac{\gamma}{\beta}c(q) + (\frac{\gamma}{\beta} - 1)\phi m - k] + \phi m + V_2^b(0, 0).$$

Let $T(\varepsilon)$ be the net benefit of using credit for the buyer, where

$$T(\varepsilon) = \sigma[\varepsilon u(q^c) - \frac{\gamma}{\beta}c(q^c) + (\frac{\gamma}{\beta} - 1)\phi m - k] - \sigma[\varepsilon u(q^m) - c(q^m)]. \tag{21}$$

I use q^c to denote the quantity traded with credit and q^m to denote the quantity traded without credit. For the rest of the paper, I assume that ε is uniformly distributed, $\varepsilon \sim U(0, 1]$.

Lemma 1 *For any given inflation rate γ, there exist two threshold values of ε, ε_0 and ε_1 such that*

$$\begin{cases} 0 \leq \varepsilon \leq \varepsilon_0, \text{ the buyer spends } d < m, \ a = 0 \text{ and consumes } q^* \text{ where } \varepsilon u'(q^*) = c'(q^*), \\ \\ \varepsilon_0 \leq \varepsilon \leq \varepsilon_1, \text{ the buyer spends } d = m, \ a = 0 \text{ and consumes } q \text{ where } c(q) = \phi m, \\ \\ \varepsilon_1 \leq \varepsilon \leq 1, \text{ the buyer spends } d = m, \ a > 0 \text{ and consumes } q^c \text{ where } \varepsilon u'(q^c) = \frac{\gamma}{\beta}c'(q^c). \end{cases}$$

Lemma 1 is very intuitive. If a buyer receives a very low ε, he has enough money to afford q^*, which is the optimal consumption for him. Here ε_0 is the threshold that determines whether a buyer is liquidity constrained. For a buyer who receives an intermediate ε, the money may not be enough to afford his q^*. The buyer is liquidity constrained. Using credit can relax the buyer's liquidity constraint, but this is costly. Therefore, buyers with intermediate εs find it optimal not to use credit, because the benefit from using credit is not enough to cover the fixed cost. For those buyers who have large εs, paying the fixed cost to relax their liquidity constraints becomes optimal. The threshold ε_1 determines whether a buyer uses credit.

The decision to use credit is endogenous in this environment. Buyers use credit for large purchases. This result is in accordance with the evidence on consumers' choices of payment methods. Empirically, the mean value of cash purchases is smaller than the mean value of credit purchases. In English (1999), the mean values of credit card purchases and cash purchases are \$54 and \$11, respectively. Klee (2008) documents that these respective mean values are \$30.85 and \$14.2.

3.3.3 Monetary Equilibrium

With different groups of buyers in terms of their choices of payment methods, I can now characterize the equilibrium. I define (q_0, q_1) such that

$$\varepsilon_0 u'(q_0) = c'(q_0), \tag{22}$$

$$\varepsilon_1 u'(q_1) = \frac{\gamma}{\beta} c'(q_1). \tag{23}$$

Notice that $c(q_0) = \phi m$ represents the transaction demand for money. In market 3, the expected marginal benefit of 1 unit money is

$$\beta \int \frac{\partial \hat{V}_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon) = \beta \hat{\phi} \left\{ \int_0^{\varepsilon_0} dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} [\sigma \varepsilon \frac{u'(q_0)}{c'(q_0)} + (1 - \sigma)] dG(\varepsilon) + \int_{\varepsilon_1}^1 [\sigma \frac{\gamma}{\beta} + (1 - \sigma)] dG(\varepsilon) \right\}.$$

From (8), the marginal cost of 1 unit money is ϕ. The optimal q_0 is determined by

$$\varepsilon_0 + \frac{1}{2} \frac{u'(q_0)}{c'(q_0)} (\varepsilon_1^2 - \varepsilon_0^2) + \frac{\gamma}{\beta} (1 - \varepsilon_1) = 1 + \frac{\gamma - \beta}{\beta \sigma}. \tag{24}$$

The last condition that completes the characterization of the equilibrium is derived from $T(\varepsilon) = 0$,

$$\varepsilon_1 u(q_1) - \frac{\gamma}{\beta} c(q_1) - k = \varepsilon_1 u(q_0) - \frac{\gamma}{\beta} c(q_0). \tag{25}$$

Lemma 2 *When γ is close to β or approaches ∞, $\varepsilon_1 = 1$.*

Following Lemma 2, it is possible that no buyer wants to use credit. When γ is close to β, the rate of return of money is high enough so that there is no need to use credit. As γ is higher, the terms of trade with credit become worse. When γ approaches ∞, the gain from using credit cannot cover the fixed utility cost k. In (21), $T(\varepsilon)$ is negative. Depending on the parameter values

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of (γ, k, σ), there are two types of monetary equilibrium.

Definition 1 *When repayment of credit can be enforced, a monetary equilibrium with credit is characterized by $(\varepsilon_0, \varepsilon_1, q_0, q_1)$ satisfying (22), (23), (24) and (25). A monetary equilibrium without credit is characterized by $\varepsilon_1 = 1$ and (ε_0, q_0) satisfying (22) and (24).*

Proposition 1 *For any inflation rate above the Friedman rule $(\gamma > \beta)$, monetary equilibrium exists. The optimal monetary policy is the Friedman rule $(\gamma \to \beta)$.*

Proposition 1 establishes the existence of monetary equilibrium. In general, monetary equilibrium without credit is unique. The uniqueness of monetary equilibrium with credit is less clear. However, one can show that monetary equilibrium with credit is unique for generic values of σ following the proof in Wright (2010). It is not surprising that the Friedman rule is the optimal monetary policy. If the monetary authority can run the Friedman rule, there is no cost to hold money.[12] Buyers would hold enough money to afford q^*. Credit is driven out as a means of payment.

In the model, the fixed cost k of using credit affects a buyer's choice of payment methods. A lower k can be viewed as an improvement in credit transaction technology, which is likely to promote the use of credit and lower the transaction demand for money. The effects of k are examined in Proposition 2.

Proposition 2 *(The Effects of the Fixed Cost) In a monetary equilibrium with credit, the thresholds are increasing in k, i.e., $\frac{d\varepsilon_0}{dk} > 0$ and $\frac{d\varepsilon_1}{dk} > 0$. Moreover, $\frac{dq_0}{dk} > 0$ and $\frac{dq_1}{dk} > 0$.*

If k is too big, no buyer will use credit because it is too costly. The economy would function as the one where money is the only means of payment. The other extreme case is where credit is not costly at all. When $k = 0$, $\varepsilon_0 = \varepsilon_1 = q_0 = 0$. In equilibrium, credit becomes the only means of payment in market 1 and money is driven out by credit as a means of payment in market 1. The transaction demand for money is 0. However, the total demand for money is not 0 as money is needed for settlement. Monetary equilibrium still exists, but money functions mainly as a means of settlement.

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[12] If money is subject to theft, inflation lowers the rate of return of money and hence reduces theft. As a result, the optimal monetary policy may deviate from the Friedman rule. See Sanches and Willamson (2010) for an example. Similarly, introducing counterfeiting is another way that may generate the optimal monetary policy above the Friedman rule.

Given that both money and credit are the means of payment, it seems that they substitute each other. Money and credit are also complements because the settlement of credit requires money. In this economy, the demand for real money balance is

$$\phi M = c(q_0) + \frac{\sigma}{\beta} \int_{\varepsilon_1}^{1} [c(q^c(\varepsilon)) - c(q_0)] dG(\varepsilon), \tag{26}$$

where $c(q_0)$ reflects the transaction demand for real money balance. From Proposition 2, the introduction of credit lowers q_0. It does not follow that the total real money balance must be lower as k decreases. Given that money is the only means of settlement, the second term in (26) represents the repayment demand for real money balance. It may increase as k decreases because a lower k makes ε_1 smaller and induces more buyers to use credit. Therefore, the overall effect of k on the total real money balance is ambiguous.

Another parameter of interest is the trading probability σ. A higher σ implies less trading frictions in goods market. Proposition 3 addresses the effects of σ on the economy.

Proposition 3 *(The Effects of the Trading Probability) In a monetary equilibrium with credit, the thresholds are increasing in σ, i.e., $\frac{d\varepsilon_0}{d\sigma} > 0$ and $\frac{d\varepsilon_1}{d\sigma} > 0$. Moreover, $\frac{dq_0}{d\sigma} > 0$ and $\frac{dq_1}{d\sigma} > 0$.*

Money becomes more popular as a means of payment when the trading probability increases. The search friction in goods market promotes the use of credit. Recall that a key difference between money and credit is that money has to be acquired before making a purchase. In the case of not finding a trade, the value of money depreciates when the inflation rate is above the Friedman rule. Credit allows buyers to avoid such a distortion because the money required for repayment is accumulated after making a purchase. If it is easier to find a trade, holding money is less costly so that money is more desirable. However, even if $\sigma = 1$, credit may still be useful as a means of payment depending on the inflation rate.

As the paper is motivated by the observations on inflation and credit, it is useful to study how γ affects $(\varepsilon_0, \varepsilon_1, q_0, q_1)$. One can show that $\frac{d\varepsilon_0}{d\gamma} < 0$ and $\frac{dq_0}{d\gamma} < 0$ under some conditions, e.g., $\sigma = 1$ or $\gamma < 2\beta$ in a monetary equilibrium with credit. Intuitively, inflation should have negative impacts on ε_0 and q_0 because inflation is a tax on money. The effects of inflation on ε_1 and q_1 are less clear. The two frictions associated with using credit generate two channels through which γ affects

ε_1. A higher γ lowers the rate of return of money and makes more buyers liquidity constrained. As a result, more buyers may find that the gain from relaxing the liquidity constraint by using credit can cover the fixed cost. Through the fixed cost channel, γ decreases ε_1. Indeed, this type of effect has been predicted by many other models using the cash-in-advance framework or the OLG framework. The other friction associated with credit is the delayed settlement. From (19), γ affects the marginal benefit of using credit. When γ is higher, the terms of trade using credit become worse. Therefore, buyers have less incentive to use credit. Through the delayed settlement channel, γ increases ε_1.

Having analyzed these two channels, it would be interesting to know the effect from which channel dominates. From Lemma 2, ε_1 hits the boundary 1 when either $\gamma \to \beta$ or $\gamma \to \infty$. Thus, the total effect of γ on ε_1 should be non-monotonic. In fact, it is likely that the effect displays a U-shape. I will rely on numerical results in the next section to verify these conjectures.

3.4   Welfare

In order to analyze the effect of monetary policy on aggregate welfare, I define aggregate welfare in this economy as $\mathcal{W}$ and

$$(1 - \beta)\mathcal{W} = \sigma\Psi(\varepsilon_0, \varepsilon_1, q_0) + [v(x^*) - x^*] - \sigma(\frac{1-\beta}{\beta})\int_{\varepsilon_1}^{1}[c(q^c(\varepsilon) - c(q_0)]dG(\varepsilon), \qquad (27)$$

where

$$\Psi(\varepsilon_0, \varepsilon_1, q_0) = \int_{0}^{\varepsilon_0}[\varepsilon u(q^*(\varepsilon)) - c(q^*(\varepsilon))]dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1}[\varepsilon u(q_0) - c(q_0)]dG(\varepsilon) + \int_{\varepsilon_1}^{1}[\varepsilon u(q^c(\varepsilon)) - c(q^c(\varepsilon)) - k]dG(\varepsilon).$$

Note that aggregate welfare is also buyers' aggregate welfare since sellers in this economy earn 0 surplus from trades and their aggregate welfare is 0. The first and second terms in the aggregate welfare function are standard. What's new in (27) is the third term, which is the production distortion from using credit in the following sense. After a seller extends credit in market 1, he receives payment from the financial intermediary in market 3 and must wait until the next market 2 to spend the money. As discussed earlier, the buyer who uses credit should pay the nominal interests to compensate the seller. The actual extra payment the buyer has to accumulate by working is the

real interest rate because buyers receive monetary transfers from the monetary authority in each period. This part is reflected in the third term, which can be viewed as the production distortion from using credit. Without knowing how γ affects $(\varepsilon_0, \varepsilon_1)$, it is not obvious how aggregate welfare responds to γ. Again, I leave a more general analysis in the next section.

4   Quantitative Analysis

In this section, I solve the model numerically and derive more implications from the model. For the numerical exercise, I adopt some specific functional forms for $u(q)$, $c(q)$ and $v(x)$ that have been used in the literature. Let $u(q) = \frac{1}{\rho} q^\rho$, $v(x) = B \log x$, and $c(q) = q$, where $0 < \rho < 1$. In market 1, the matching technology that I specify is the urn-ball matching function, where $\sigma = 1 - e^{-1}$. There are four parameters (β, B, ρ, k) to be determined. The period length in this model is set to be 1 year mainly to facilitate comparisons with past work on the welfare cost of inflation.

The time preference parameter β is set $\beta^{-1} = 1.04$, so the implied annual real interest rate is 0.04. For the other parameters, I follow Lucas (2000) and Lagos and Wright (2005), and fit the model's money demand to the U.S. money demand data by nonlinear least square. The data covers annual nominal interest rate and the "money demand" (or the inverse of the velocity of money) for the period $1900 - 2000$.[13] The "money demand" predicted by the model is

$$L(i) = \frac{M}{PY} = \frac{c(q_0) + \frac{\sigma}{\beta} \int_{\varepsilon_1}^1 [c(q^c(\varepsilon)) - c(q_0)] dG(\varepsilon)}{Y_c + \sigma [\int_0^{\varepsilon_0} c(q^*(\varepsilon)) dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} c(q_0) dG(\varepsilon) + \int_{\varepsilon_1}^1 c(q^c(\varepsilon)) dG(\varepsilon)]}.$$

where

$$
\begin{aligned}
Y_c &= x + \sigma \left[\int_0^{\varepsilon_0} c(q^*(\varepsilon)) dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} c(q_0) dG(\varepsilon) + \int_{\varepsilon_1}^1 c(q^c(\varepsilon)) dG(\varepsilon) \right] \\
&\quad + \frac{\sigma(1-\beta)}{\beta} \int_{\varepsilon_1}^1 [c(q^c(\varepsilon)) - c(q_0)] dG(\varepsilon).
\end{aligned}
$$

The parameters from the best fit are in Table 2.[14]

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[13] The data are originally from Craig and Rocheteau (2008).

[14] From the numerical exercise, multiple equilibria have not been found although there is no general proof of a unique monetary equilibrium with credit.

Parameters	ρ	B	k
Values	0.4732	1.4436	0.0739

Table 2: Parameter Values

The values of (ρ, B) are in the ballpark of existing studies. I use a consumption equivalence measure to evaluate the plausibility of the value of k. The utility cost $k = 0.0739$ is worth of 1% of consumption for buyers. Based on these parameter values, I numerically solve the model and show the results in Figure 3.
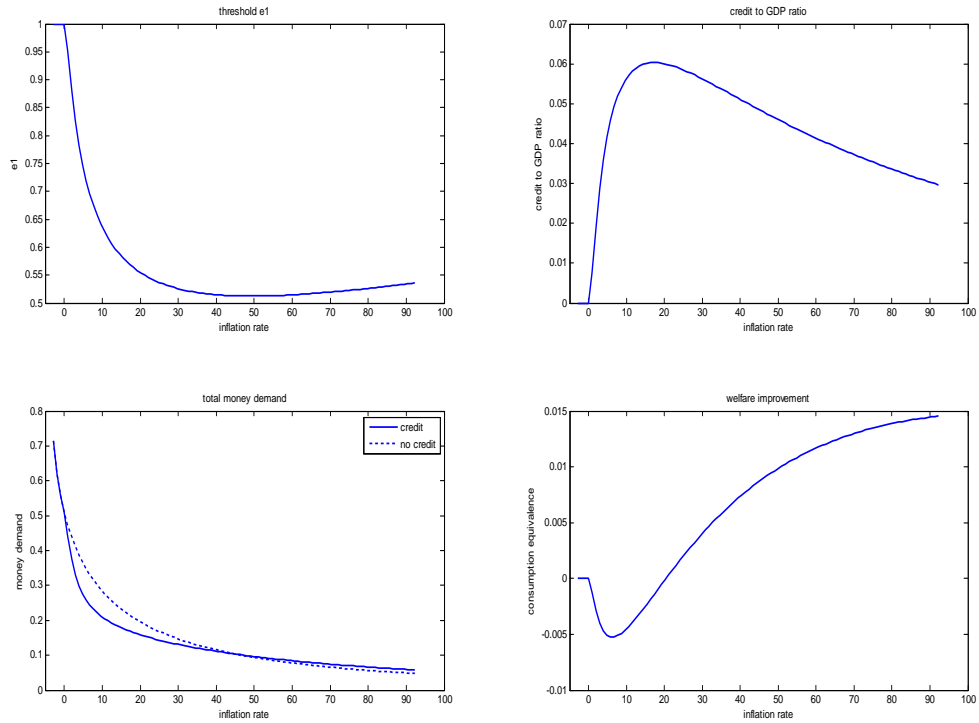

Figure 3: The Effect of Inflation – Benchmark

The upper-left and upper-right panels are the effects of inflation on the threshold ε_1 and the credit to GDP ratio, respectively.[15] The lower panels are the comparisons with a no-credit economy.

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[15]The predicted credit to GDP ratio from the model is

$$\frac{\sigma \int_{\varepsilon_1}^{1} [c(q^c(\varepsilon)) - c(q_0)] dG(\varepsilon)}{Y_c + \sigma[\int_0^{\varepsilon_0} c(q^*(\varepsilon)) dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} c(q_0) dG(\varepsilon) + \int_{\varepsilon_1}^{1} c(q^c(\varepsilon)) dG(\varepsilon)]}.$$

The lower-left panel presents the total demand for real money balances in the credit economy and the no-credit economy. The lower-right panel shows the welfare improvement of having credit based on a consumption equivalence measure. That is, the number on the vertical axis is the fraction of consumption that a buyer is willing to give up to live in a credit economy instead of a no-credit economy.

There are several interesting findings from Figure 3. It is clear that inflation induces more buyers to use credit at low inflation rates and less buyers to use credit at high inflation rates. Moreover, the credit to GDP ratio predicted by the model has an inverse U-shape against inflation. As discussed in the previous section, inflation has two effects on ε_1. The fixed cost effect implies that inflation makes more buyers use credit. This is because high inflation causes more buyers liquidity constrained so that more buyers may find using credit beneficial enough to cover the fixed cost. The delayed settlement effect on the other hand lowers incentives for buyers to use credit because of a deterioration in the terms of trade. The numerical results suggest that the fixed cost effect dominates the delayed settlement effect at low inflation rates, but the delayed settlement effect dominates the fixed cost effect at high inflation rates. High inflation makes credit involve high repayment and hence unfavorable terms. This exactly describes the consumer credit market in Brazil during the late 1980s.[16]

Compared to a no-credit economy, credit lowers demand for real money balances at low to moderate inflation rates, but slightly increases demand for real money balances at high inflation rates. One can show that the transaction demand in a credit economy is always lower than in a no-credit economy. As the repayment of credit also requires money, the repayment demand may increase as the inflation rate increases. It seems that credit and money are substitutes at low to moderate inflation rates, but are complements at high inflation rates. The first half of the result can be supported by the empirical work using U.S. data, since the inflation rates in the U.S. have been low to moderate in recent decades. See Duca and Whitesell (1995) for an example. The latter half of the result, however, has not been verified empirically.

The lower-right panel reveals that having credit does not always benefit the society in terms

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[16]Due to the long time delay in credit card charges clearing through the banking system, vendors have been documented to normally add on a 20 to 30 percent surcharge to the price of the purchased item. In this way, vendors can protect themselves from the depreciation of money during the time the vendors are waiting to be paid by the credit card companies.

of aggregate welfare. Knowing that individuals optimally choose to use money versus credit, it seems puzzling that credit can hurt the economy. From (27), credit improves welfare by relaxing the liquidity constraint for some buyers, but may hurt welfare because of the production distortion. Besides these direct effects, credit affects welfare through the general equilibrium effect. As analyzed above, credit may lower the demand for real money balances and thus the value of money, which will generate a negative externality on agents who use money. On the other hand, credit may increase the demand for real money balances and the value of money, which will generate a positive externality on agents who use money. The general equilibrium effect implies that credit may hurt welfare at low to moderate inflation rates, but improve welfare at high inflation rates. Similar results appear in Chiu and Meh (forthcoming). Overall, credit improves aggregate welfare when the inflation rate exceeds a threshold.

In terms of the effect of monetary policy, the model predicts that aggregate welfare and aggregate output are decreasing in the inflation rate. This is not surprising although the model does introduce a channel through which inflation may potentially increase output levels at low inflation rates by encouraging more buyers to use credit. However, the effect from this channel does not appear to be strong.

In Figure 3, the threshold value for credit to improve welfare is around 20% inflation rate, which is fairly high. The potential problem is that fitting (ρ, B, k) together implicitly assumes that these parameters do not change over the hundred years. However, it is hard to believe that the cost of credit transactions stays constant over time. Nevertheless, since there is no direct data that measures how k evolves over time and the focus of the paper is not to match any moment in the data, I take a simple approach to evaluate the model's predictions by varying k and fixing (ρ, B). To highlight the effect of changing k, I show in Figure 4 the credit to GDP ratio and the welfare improvement when $k = 0.01$, 0.05 and 0.1.
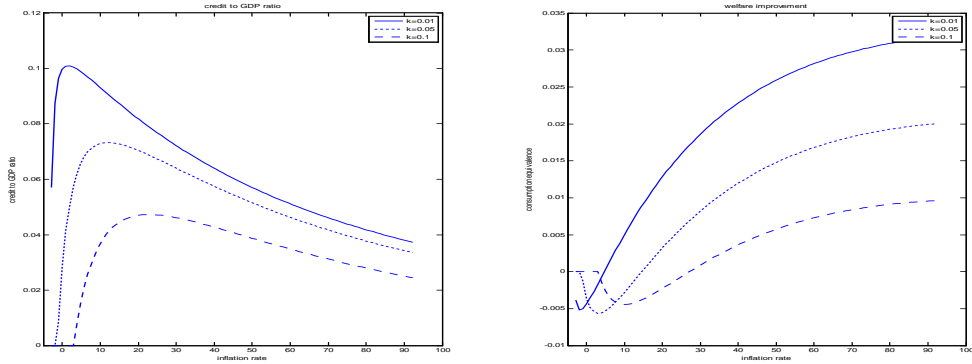
Figure 4: Comparative Statics – Varying k

According to Figure 4, a lower cost of credit promotes the use of credit. Using the average inflation rate 7.387 from $1969 - 2000$, the predicted credit to GDP ratio is 0.26% and the predicted "money demand" is 0.395 when $k = 0.1$. For $k = 0.01$, the predicted credit to GDP ratio is 10.02% and the predicted "money demand" is 0.201. The low cost credit regime is featured by more credit and less "money demand". It has been noted that there is a trend decline in "money demand" in the recent decade, which has been viewed as a shift of the money demand curve.[17] Clearly, improvements in the credit transaction technology contribute to the trend decline in "money demand".

In terms of welfare, more costly credit makes credit less beneficial to the society. One can see that the threshold for credit to be welfare-improving is higher when k is higher. In practice, if sellers receive repayment in the form of money, they may deposit the money to avoid any inflation distortion, which makes credit more beneficial. This type of argument can be built into the model by allowing a fraction of agents to settle in market 2 and the rest to settle in market 3. While this is a nice extension, the current model still serves as a benchmark for analyzing the effect of inflation on credit in a world in which credit is not entirely free of inflation distortion.

As a robustness check, I choose values of (ρ, B) by varying the sample period and evaluate the

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[17]Considering an economy without credit, I can find the values of (ρ, B) by fitting the money demand curve. Starting in the 1980s, the predicted money demand diverges from the data. If one is willing to assume a specific functional form of the trend of financial innovation and assume that each year is in a steady state, then fitting the money demand data can generate the values of (ρ, B) and the trend of financial innovation. By doing such an exercise, I found that the predicted money demand in recent years is much closer to the money demand data. A similar exercise is in Faig and Jerez (2006).

model's predictions. In these experiments, the value of ρ varies from 0.387 to 0.589, but the value of B does not change much, which is around 1.4. The model's predictions emerging from Figure 3 are very robust.[18]

To study how introducing credit affects the welfare cost of inflation, I compute the welfare cost of 10% inflation based on the parameters given in Table 2. The measure of the welfare cost follows the recent literature by using the consumption equivalence measure. The numbers reported in Table 3 is the fraction of consumption a buyer is willing to give up to have 0% inflation rather than 10% inflation. As a benchmark, I compute the welfare cost for a no-credit economy, i.e., $k = \infty$ and hence $\varepsilon_1 = 1$. The welfare cost of 10% inflation is 1.12% in the benchmark economy, which is relatively small because the take-it-or-leave-it offer by buyers avoids the holdup problem.[19]

k	benchmark	0.01	0.05	0.0739	0.1
Welfare Cost	1.12%	0.22%	0.85%	1.32%	1.40%

Table 3: Welfare Cost of 10% inflation

I then compute the welfare cost of inflation for different values of k. The introduction of credit can raise the welfare cost when credit is costly enough. For low values of k, the cost for buyers to substitute credit for money is relatively low. Therefore, inflation does not generate a large welfare loss. On the contrary, if the cost for buyers to switch from money to credit is high, inflation can result in a higher welfare loss compared to the benchmark economy. Note that if k is too big, no buyer uses credit and the economy is essentially the benchmark economy. Dotsey and Ireland (1996) and Lacker and Shreft (1996) both emphasize that credit costs are quantitatively important as a component of the welfare cost of inflation. The results in Table 3 further confirm their results.

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[18]Taking one year as the period length in the model implies that the settlement is delayed for one year, which seems to be too long. To address this issue, I can take one month or one quarter as the model's period length and compute the model's annual "money demand" by aggregating from monthly or quarterly "money demand". I can also take one quarter as the period length and use quarterly data on treasury bill rate per annum, M1 and GDP from IFS. The quarterly "money demand" data is computed by converting treasury bill rate and GDP into quarterly measure. All these methods do not change the qualitative predictions of the model including the inverse U-shape relationship between inflation and credit usage. However, the quantitative predictions depend on the specific method and the initial values assigned to (ρ, B, k).

[19]In Lagos and Wright (2005), the welfare cost of 10% inflation is 1.4% when buyers have all the bargaining power. The current result does not deviate from their estimate.

5 Monetary Equilibrium without Enforcement

So far I have assumed that financial intermediaries can enforce the repayment of credit, which implies that buyers are not credit constrained. In this section, I relax this assumption. Financial intermediaries can identify agents and keep records of goods market transactions, but they cannot enforce the repayment of credit. As in Berentsen et al. (2007) and Sanches and Williamson (2010), the punishment for default is permanent exclusion from the financial system. That is, if a buyer defaults, the buyer will never be able to use credit in any future period. Given the punishment, the amount of credit extended to a buyer should be consistent with the buyer's incentive to repay. In an environment without enforcement, the government (or the monetary authority) cannot enforce buyers to pay taxes either. It implies that $\gamma \geq 1$.

With this modification, buyers and sellers face the same choice problems as before in market 2 and 3. Only in market 1, the buyer's take-it-or-leave-it offer should be reformulated.

$$\max_{q,d,a}[\varepsilon u(q) - \phi d - \phi a]$$

$$\text{s.t. } c(q) = \phi d + \beta \hat{\phi} a, \ d \leq m \text{ and } a \leq \bar{a}.$$

Here $\bar{a}$ is the credit limit faced by the buyer. An individual buyer takes $\bar{a}$ as given. In equilibrium, $\bar{a}$ will be endogenously determined. The Lagrangian is

$$\mathcal{L} = \max_{q,d,a,\lambda_1,\lambda_2,\lambda_3} [\varepsilon u(q) - \phi d - \phi a] + \lambda_1[\phi d + \beta \hat{\phi} a - c(q)] + \lambda_2(m - d) + \lambda_3(\bar{a} - a).$$

If the credit constraint is not binding, i.e., $\lambda_3 = 0$, then $d = m$ and (q, a) are given by

$$\varepsilon u'(q) = \frac{\gamma}{\beta} c'(q),$$

$$\beta \hat{\phi} a = c(q) - \phi d.$$

If the credit constraint is binding, i.e., $\lambda_3 > 0$, I have $d = m$, $a = \bar{a}$ and q solving

$$c(q) = \phi d + \beta \hat{\phi} \bar{a}.$$

Introducing the credit limit may cause some buyers to be credit constrained. If such buyers exist, there are potentially four groups of buyers.

Lemma 3 *For any given inflation rate* γ, *there exist three thresholds of* ε, ε_0, ε_1 *and* ε_2 *such that*

$$
\begin{cases}
0 \leq \varepsilon \leq \varepsilon_0, \text{ the buyer spends } d < m, \ a = 0, \text{ and consumes } q^* \text{ where } \varepsilon u'(q^*) = c'(q^*), \\[2mm]
\varepsilon_0 \leq \varepsilon \leq \varepsilon_1, \text{ the buyer spends } d = m, \ a = 0 \text{ and consumes } q \text{ where } c(q) = \phi m, \\[2mm]
\varepsilon_1 \leq \varepsilon \leq \varepsilon_2, \text{ the buyer spends } d = m, \ a < \bar{a} \text{ and consumes } q^c \text{ where } \varepsilon u'(q^c) = \frac{\gamma}{\beta} c'(q^c), \\[2mm]
\varepsilon_2 \leq \varepsilon \leq 1, \text{ the buyer spends } d = m, \ a = \bar{a} \text{ and consumes } q \text{ where } c(q) = \phi m + \beta \hat{\phi} \bar{a}.
\end{cases}
$$

The expected marginal value of money in market 3 is

$$
\beta \int \frac{\partial \hat{V}_1(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} G(\varepsilon) = \beta \hat{\phi} \left\{ \int_0^{\varepsilon_0} dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} [\sigma \frac{\varepsilon u'(q_0)}{c'(q_0)} + (1-\sigma)] dG(\varepsilon) \right.
$$
$$
\left. + \int_{\varepsilon_1}^{\varepsilon_2} [\sigma \frac{\gamma}{\beta} + (1-\sigma)] dG(\varepsilon) + \int_{\varepsilon_2}^1 [\sigma \frac{\varepsilon \gamma}{\varepsilon_2 \beta} + (1-\sigma)] dG(\varepsilon) \right\},
$$

where q_0 is defined in (22). Combining with (8), the optimal q_0 is implicitly given by

$$
\varepsilon_0 + \frac{1}{2} \frac{u'(q_0)}{c'(q_0)} (\varepsilon_1^2 - \varepsilon_0^2) + \frac{\gamma}{\beta} (\varepsilon_2 - \varepsilon_1) + \frac{1}{2} \frac{\gamma}{\beta} (\frac{1}{\varepsilon_2} - \varepsilon_2) = 1 + \frac{\gamma - \beta}{\beta \sigma}. \tag{28}
$$

Define q_2 such that

$$
\frac{\varepsilon_2 u'(q_2)}{c'(q_2)} = \frac{\gamma}{\beta}, \tag{29}
$$

and q_2 satisfies

$$
c(q_2) = c(q_0) + \beta \hat{\phi} \bar{a}. \tag{30}
$$

Now suppose that $\varepsilon_2 < 1$. Consider a buyer with ε carrying debt a into market 2. The buyer must have spent all of his money. If he repays the debt, the buyer should work to accumulate the money for repayment. His value function is $V_2^b(0, a)$. If the buyer defaults, he does not need to work as much in market 2. Denote the payoff from default by $V_2^{bD}(0, a)$ where

$$
V_2^{bD}(0, a) = v(x^*) - x^* + \max_{z^D} \left\{ -\phi z^D + V_3^{bD}(z^D, 0) \right\}.
$$

The superscript D represents variables associated with default. The real credit limit is the value of $\beta\hat{\phi}\bar{a}$ that solves $V_2^b(0,a) = V_2^{bD}(0,a)$. After some algebra, the real credit limit is

$$
\begin{aligned}
\beta\hat{\phi}\bar{a} &= \frac{\beta^2\sigma}{\gamma(1-\beta)} \Bigg\{ \int_0^{\varepsilon_0} [\varepsilon u(q^*(\varepsilon)) - c(q^*(\varepsilon))]dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} [\varepsilon u(q_0) - c(q_0)]dG(\varepsilon) \\
&\quad + \int_{\varepsilon_1}^{\varepsilon_2} [\varepsilon u(q^c(\varepsilon)) - k - \frac{\gamma}{\beta}c(q^c(\varepsilon)) + (\frac{\gamma}{\beta}-1)c(q_0)]dG(\varepsilon) \\
&\quad + \int_{\varepsilon_2}^1 [\varepsilon u(q_2) - k - \frac{\gamma}{\beta}c(q_2) + (\frac{\gamma}{\beta}-1)c(q_0)]dG(\varepsilon) \\
&\quad - \int_0^{\varepsilon_0^D} [\varepsilon u(q^*(\varepsilon) - c(q^*(\varepsilon))]dG(\varepsilon) - \int_{\varepsilon_0^D}^1 [\varepsilon u(q_0^D) - c(q_0^D)]dG(\varepsilon) \Bigg\} \\
&\quad + \frac{(\gamma-\beta)\beta}{\gamma(1-\beta)}[c(q_0^D) - c(q_0)].
\end{aligned}
\tag{31}
$$

When the repayment of credit cannot be enforced, a monetary equilibrium with constrained credit is defined by a list of $(\varepsilon_0, \varepsilon_1, \varepsilon_2, q_0, q_1, q_2, \beta\hat{\phi}\bar{a})$ characterized by (22), (23), (25), (28), (29), (30) and (31). It is possible that $(\varepsilon_1, \varepsilon_2)$ hit the boundary 1 in equilibrium. In particular, there are two special cases: [1] $\varepsilon_2 = 1$ and [2] $\varepsilon_1 = \varepsilon_2 = 1$. In case [1], the equilibrium corresponds to the monetary equilibrium with credit in Section 3. In case [2], the equilibrium corresponds to the monetary equilibrium without credit. Notice that in these two cases, the endogenous credit limit still exists, but it is not binding.

It becomes very complicated to derive any analytical results. Hence, I use a numerical example to show how endogenizing credit limit affects equilibrium and welfare. Based on the numerical exercise in Section 4, I again set $\rho = 0.4732$ and $B = 1.4436$. As for k, I choose $k = 0.03$. The left panel in Figure 5 shows the endogenous credit limit and the maximum amount of borrowing for various inflation rates. The main finding is that buyers are credit constrained only at very low inflation rates, which is consistent with Berentsen et al. (2007). High inflation rates help to relax the credit constraint because punishment of default becomes more severe. Inflation also lowers the surplus from credit trades in this model. The overall effect of inflation on credit limit is that inflation first relaxes and then tightens the credit limit. At high inflation rates, because buyers borrow less, the credit constraint does not bind although the credit limit is lower.
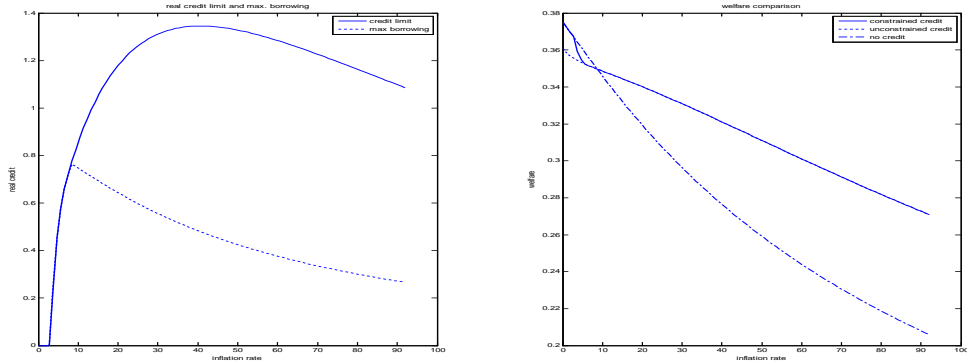
Figure 5: The Effect of Inflation – Endogenous Credit Limit

In the right panel of Figure 5, I compare welfare in an economy with endogenous credit limit with welfare in an economy with either unconstrained credit or no credit. The implication is that having the endogenous credit limit can improve social welfare at very low inflation rates. It sounds counterintuitive that imposing a constraint would make the allocation better. However, money functions well as a means of payment at low inflation rates. It is more likely that buyers would choose to default because the punishment may make them better off. If the credit limit reaches zero, the economy converts into a pure monetary economy. At very low inflation rates, I demonstrated in Section 4 that a monetary equilibrium without credit is better in terms of aggregate welfare than a monetary equilibrium with credit. Having a zero credit limit is essentially welfare-enhancing.

6 Conclusion

Both money and credit are widely used as means of payment. It is important to understand how the use of credit affects money demand and hence the effects of monetary policy. I constructed a model in which money and credit coexist as means of payment and money is the means of settlement. There are two frictions associated with using credit – a fixed utility cost and delayed settlement. In this environment, a buyer's choice of payment methods is endogenous. Credit lowers money demand at low to moderate inflation, but it slightly increases money demand at high inflation rates. The relationship between inflation and the credit to GDP ratio exhibits an inverse U-shape which is broadly consistent with existing evidence. Costly credit does not always improve social

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welfare. Depending on the fixed utility cost of credit, allowing credit as a means of payment may raise the welfare cost of inflation.

In a modified environment where enforcement is imperfect, the endogenous credit limit tends to increase at low inflation rates, but decrease at high inflation rates. However, credit constraints only bind at very low inflation rates. Interestingly, imperfect enforcement may improve social welfare because it avoids socially inefficient borrowing.

One testable implication from the model is that the relationship between inflation and the credit to GDP ratio has an inverse U-shape. There are many studies testing the long run effect of inflation on credit. Most of them use the total private credit to GDP ratio as the measure of credit. In the future, it would be of interest to use the consumer credit to GDP ratio and test its long run relationship with inflation, particularly using cross-country data.[20]

A   Appendix

A.1   Proof of Lemma 1

Proof.   As discussed in the paper, if a buyer uses credit, he must spend all his money. Define $\bar{q}$ as the solution to $u'(q) = c'(q)$. One can show that $\phi m < c(\bar{q})$ as long as $\gamma > \beta$. It implies that there exists a threshold ε_0 such that $\varepsilon_0 u'(q_0) = c'(q_0)$ where q_0 is from $c(q_0) = \phi m$. For buyers with $\varepsilon > \varepsilon_0$,

$$T'(\varepsilon) = \sigma[\varepsilon u'(q^c) - \frac{\gamma}{\beta}c'(q^c)]\frac{dq^c}{d\varepsilon} - \sigma[\varepsilon u'(q^m) - c'(q^m)]\frac{dq^m}{d\varepsilon} + \sigma[u(q^c) - u(q^m)].$$

Notice that $\varepsilon u'(q^c) - \frac{\gamma}{\beta}c'(q^c) = 0$ from (19) and $\frac{dq^m}{d\varepsilon} = 0$. Because $q^c > q^m$, it follows that $T'(\varepsilon) > 0$ for $\varepsilon > \varepsilon_0$. Define ε_1 such that $T(\varepsilon_1) = 0$. If an interior ε_1 exists, then [1] buyers with $\varepsilon_0 < \varepsilon < \varepsilon_1$ are liquidity constrained and do not use credit; and [2] buyers with $\varepsilon > \varepsilon_1$ are liquidity constrained and use credit. ■

---

[20] I tested the long run relationship between inflation and the private credit to GDP ratio following Bullard and Keating (1995). The results moderately support the inverse U-shape prediction. In addition, I used Canada and the U.S. consumer credit data to perform the same test. For these two low inflation countries, a permanent increase in inflation increases the consumer credit to GDP ratio, which is consistent with the left half of the inverse U-shape. However, due to data availability, I do not have any consumer credit data for high inflation countries. Therefore, it is hard to verify the right half of the U-shape at this stage. A detailed description of the estimation is available upon request.

A.2 Proof of Lemma 2

Proof. When γ is close to β, $\varepsilon_0 \to 1$, $T(\varepsilon) \to \sigma[\varepsilon u(q^c) - c(q^c) - k] - \sigma[\varepsilon u(q^m) - c(q^m)]$ and $q^c \to q^m$. It implies that $T(\varepsilon) < 0$ for all $\varepsilon > \varepsilon_0$, which means $\varepsilon_1 = 1$.

Recall that $T'(\varepsilon) > 0$ for $\varepsilon > \varepsilon_0$. For $\varepsilon > \varepsilon_0$, it is true that $\phi m = c(q^m)$. The maximum of $T(\varepsilon)$ is achieved at $\varepsilon = 1$ and $T(1) = \sigma[u(q^c) - \frac{\gamma}{\beta}c(q^c) - k] - \sigma[u(q^m) - \frac{\gamma}{\beta}c(q^m)]$. When γ approaches ∞, $T(1) \to -\sigma k$. It is clear that $\varepsilon_1 = 1$ when $\gamma \to \infty$. ∎

A.3 Proof of Proposition 1

Proof. Notice that (24) determines q_0, where ε_0 and ε_1 are functions of q_0. From (22), one can solve for $\varepsilon_0(q_0)$ and $\frac{d\varepsilon_0}{dq_0} > 0$. To solve for ε_1, both (23) and are used. The choice problem of q_0 can be rewritten as

$$\frac{1}{2}\frac{c'(q_0)}{u'(q_0)} + \frac{1}{2}\frac{u'(q_0)}{c'(q_0)}\varepsilon_1^2(q_0) - \frac{\gamma}{\beta}\varepsilon_1(q_0) = 1 + \frac{\gamma(1-\sigma) - \beta}{\beta\sigma}.$$

As I focus on $q_0 \in [0, q^*]$ where q^* is given by $u'(q) = c'(q)$, a solution of q_0 must exist by the Theorem of Maximum. For any $\gamma > \beta$, $q_0 > 0$ for any monetary equilibrium with or without credit. It follows that monetary equilibrium must exist for $\gamma > \beta$.

In monetary equilibrium without credit, the second hold condition is negative so that q_0 is unique and hence ε_0 is unique. In monetary equilibrium with credit, it is less clear that q_0 must be unique. Following the proof in Wright (2010), one can show that equilibrium q_0 is unique for generic values of σ. If q_0 is unique, ε_0 is also unique from (22). In addition, $\frac{d\varepsilon_1}{dq_0} > 0$ from (25). It implies that ε_1 is unique. Given that $\frac{d\varepsilon_1}{dq_1} > 0$ from (23), q_1 is unique.

When the monetary authority implements $\gamma \to \beta$, $\varepsilon_0 = \varepsilon_1 = 1$ and $q(\varepsilon) = q^*(\varepsilon)$ for all ε. The Friedman rule achieves the efficient allocation. ∎

A.4 Proof of Proposition 2

Proof. Total differentiate (24), (22), (23) and (25) with respect to k:

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2)\frac{dq_0}{dk} + [\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}]\frac{d\varepsilon_1}{dk} = 0,$$

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{[c'(q_0)]^2}\frac{dq_0}{dk} + \frac{1}{\varepsilon_0^2}\frac{d\varepsilon_0}{dk} = 0,$$

$$\frac{c'(q_1)u''(q_1) - u'(q_1)c''(q_1)}{[c'(q_1)]^2}\frac{dq_1}{dk} + \frac{\gamma}{\beta\varepsilon_1^2}\frac{d\varepsilon_1}{dk} = 0,$$

$$[u(q_1) - u(q_0)]\frac{d\varepsilon_1}{dk} - 1 - [\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]\frac{dq_0}{dk} = 0.$$

From (25), one can show that $q_1 > q_0$. Together with (22) and (23), $\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta} > 0$. It follows that $\frac{d\varepsilon_0}{dk}$, $\frac{dq_0}{dk}$, $\frac{d\varepsilon_0}{dk}$ and $\frac{d\varepsilon_1}{dk}$ are equal in sign. Moreover,

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2)\frac{dq_0}{dk} + \frac{[\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}][\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]}{[u(q_1) - u(q_0)]}\frac{dq_0}{dk} = -\frac{1}{u(q_1) - u(q_0)}. \tag{32}$$

Define

$$f(q_0; \gamma) = \varepsilon_0(q_0) + \frac{1}{2}\frac{u'(q_0)}{c'(q_0)}[\varepsilon_1^2(q_0) - \varepsilon_0^2(q_0)] + \frac{\gamma}{\beta}[1 - \varepsilon_1(q_0)] - 1 - \frac{\gamma - \beta}{\beta\sigma},$$

where $\varepsilon_0(q_0)$ is from (22) and $\varepsilon_1(q_0)$ is from (23) and (25). As the equilibrium q_0 must be local maximum, the second order condition must hold. It must be true that $f'(q_0; \gamma) < 0$, so

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2) + \frac{[\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}][\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]}{[u(q_1) - u(q_0)]} < 0. \tag{33}$$

From (32), I have $\frac{dq_0}{dk} > 0$ and hence, $\frac{d\varepsilon_0}{dk} > 0$, $\frac{d\varepsilon_1}{dk} > 0$ and $\frac{dq_1}{dk} > 0$. ∎

A.5 Proof of Proposition 3

Proof. The proof is similar to the proof of Proposition 2. Total differentiate (24), (22), (23) and (25) with respect to σ:

$$
\begin{aligned}
\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2)\frac{dq_0}{d\sigma} + [\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}]\frac{d\varepsilon_1}{d\sigma} &= -\frac{\gamma - \beta}{\beta\sigma^2}, \\
\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{[c'(q_0)]^2}\frac{dq_0}{d\sigma} + \frac{1}{\varepsilon_0^2}\frac{d\varepsilon_0}{d\sigma} &= 0, \\
\frac{c'(q_1)u''(q_1) - u'(q_1)c''(q_1)}{[c'(q_1)]^2}\frac{dq_1}{d\sigma} + \frac{\gamma}{\beta\varepsilon_1^2}\frac{d\varepsilon_1}{d\sigma} &= 0, \\
[u(q_1) - u(q_0)]\frac{d\varepsilon_1}{d\sigma} - [\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]\frac{dq_0}{d\sigma} &= 0.
\end{aligned}
$$

It is clear that $\frac{d\varepsilon_0}{d\sigma}$, $\frac{dq_0}{d\sigma}$, $\frac{d\varepsilon_1}{d\sigma}$ and $\frac{d\varepsilon_1}{d\sigma}$ have the same sign. Using (33), $\frac{dq_0}{d\sigma} > 0$. Moreover, $\frac{d\varepsilon_0}{dk} > 0$, $\frac{d\varepsilon_1}{dk} > 0$ and $\frac{dq_1}{dk} > 0$. ∎

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