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Adverse Selection, Liquidity, and Market Breakdown

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Abstract

This paper studies the interaction between adverse selection, liquidity risk and beliefs about systemic risk in determining market liquidity, asset prices and welfare. Even a small amount of adverse selection in the asset market can lead to fire-sale pricing and possibly to a market breakdown if it is accompanied by a flight-to-liquidity, a misassessment of systemic risk, or uncertainty about asset values. The ability to trade based on private information improves welfare if adverse selection does not lead to a market breakdown. Informed trading allows financial institutions to reduce idiosyncratic risks, but it exacerbates their exposure to systemic risk. Further, I show that in a market equilibrium, financial institutions overinvest into risky illiquid assets (relative to the constrained efficient allocation), which creates systemic externalities. Also, I explore possible policy responses and discuss their effectiveness.

JEL classification: G01, G11, D82
Bank classification: Financial institutions; Financial markets; Financial stability

Résumé

L'auteure étudie l'interaction entre l'antisélection, le risque de liquidité et les croyances concernant le risque systémique dans la détermination du degré de liquidité du marché, des prix des actifs et du niveau de bien-être. La présence, même faible, d'antisélection sur le marché des actifs peut entraîner l'effondrement des prix de vente, voire la défaillance du marché si elle s'accompagne d'une ruée vers la liquidité, d'une évaluation incorrecte du risque systémique ou d'une incertitude quant à la valeur des actifs. La possibilité de négocier sur la base d'informations privées accroît le bien-être lorsque l'antisélection ne mène pas à la défaillance du marché. Les transactions entre opérateurs informés permettent aux institutions financières de réduire les risques idiosyncrasiques, mais elles accentuent l'exposition de ces dernières au risque systémique. L'auteure montre également qu'en situation d'équilibre de marché (par rapport à une situation d'efficience allocative sous contraintes), les institutions financières surinvestissent dans des actifs risqués peu liquides, ce qui crée des effets externes systémiques. Enfin, l'auteure examine diverses interventions possibles de l'État et analyse leur efficacité.

Classification JEL : G01, G11, D82
Classification de la Banque : Institutions financières; Marchés financiers; Stabilité financière

1 Introduction

In the recent crisis of 2007-2009, trading in some financial markets was dramatically reduced or stopped completely. The following questions emerged: What caused market freezes? For trades that did occur, why were the assets traded at a significant discount? How were problems in a relatively small part of financial market amplified into the systemic crisis? The prevalent explanations focus on the increased uncertainty and information asymmetries about asset values. In particular, the difficulty in assessing the fundamental value of securities may lead to adverse selection problems. A flight-to-liquidity and a misassessment of systemic risk can further amplify the adverse selection problem into a severe financial crisis.

In this paper, I develop a model to analyze the interaction between adverse selection, liquidity risk and beliefs about systemic risk in determining market liquidity, asset prices and welfare. I characterize the financial institutions' portfolio choices between safe and risky assets when systemic risk is anticipated, and examine how their beliefs may contribute to market freezes.

In my model, financial institutions (investors) are ex-ante identical but ex-post different with respect to realizations of liquidity shocks and investment quality. Preference for liquidity is characterized by Diamond-Dybvig [21] type of preferences: investors need liquidity in period one or in period two, depending on whether they receive a liquidity shock in period one. In period zero, investors choose the portfolio allocation between the safe asset and the risky long-term asset with an idiosyncratic payoff. In period one, they privately observe their asset quality, and then risky assets can be traded in the market. The investors who have not experienced a liquidity shock are buyers in the asset market, while the sellers are those who have low quality assets or have received a liquidity shock.

Market liquidity is characterized by the cost (in terms of the foregone payoff) of selling a long-term asset before its maturity.[1] Two factors contribute to illiquidity in the market: a shortage of safe assets and adverse selection (characterized by the fraction of low quality assets in the market). On one hand, market liquidity depends on the amount of the safe asset

[1] This characterization of liquidity is similar to Eisfeldt [25], where liquidity is described as the cost of transferring the value of expected future payoffs from long-term assets into the current income.

held by investors that is available to buy risky assets from liquidity traders. Following the Allen and Gale ([9], [11]) "cash-in-the-market" framework, the market price is determined by the lesser of the following two amounts: expected payoff and the amount of the safe asset available from buyers per unit of assets sold. Therefore, this "cash-in-the-market" pricing may lead to market prices below fundamentals if there is not enough cash (safe assets) to absorb asset trades. On the other hand, market liquidity depends on the quality of assets traded in the market. In particular, adverse selection can cause market illiquidity if assets sold in the market are likely to be of low quality (as in Eisfeldt [25]).

The long-term investment is risky not only because of its uncertain quality but also because of the cost associated with its premature liquidation or sale. Therefore, investors are exposed to the market liquidity risk through their holding of long-term assets. Holdings of the safe asset provide partial insurance against the possibility of a liquidity shock as well as against low asset quality realizations. In addition to the value as means of storage, the safe asset has value as means for reallocating risky assets from investors who have experienced a liquidity shock to those who have not. This is similar to the concept of liquidity value for ability to transfer resources in Kiyotaki and Moore [35].[2]

As a benchmark, I examine portfolio choice when investors have private information about their investment quality but the identity of investors hit by a liquidity shock is public information. Then I analyze the situation when the investor's type (both liquidity needs and asset quality) is private information. In the latter case investors can take advantage of their private information by selling the low-payoff investments and keeping the high quality ones. This generates the lemons problem: buyers do not know whether an asset is sold because of its low quality or because the seller experienced a sudden need for liquidity.[3]

There are two possible states of the economy: normal times and a crisis. The crisis state is characterized by a larger fraction of low quality assets in the market and a higher preference for liquidity relative to normal times.[4] The aggregate uncertainty about the state

[2] This is also similar to Acharya, Shin, and Yorulmazer [3] and Diamond and Rajan [22] where one of the motives for holding liquid assets by banks is potential future acquisitions at fire-sale prices.

[3] This setting is different from models where investors have private information about aggregate (common) payoff and information can be revealed through trading.

[4] This characterization is consistent with the fact that liquidity crises tend to be associated with economic

of the economy captures systemic risk, while idiosyncratic realizations of liquidity shocks and asset payoffs represent individual exposures to the systemic risk.[5] When the economy is in the normal state, adverse selection does not significantly affect market liquidity. If the market is liquid then informed investors can gain from trading on private information at the expense of liquidity traders.[6]

In the crisis state, adverse selection leads to depressed asset prices and possibly to a breakdown of market trading. There are two types of equilibria depending on trading behavior during the crisis state: (I) with market trading when both high and low quality assets are sold, and (II) with the market breakdown. The type I equilibrium is characterized by asset price volatility across states with fire-sale pricing in the crisis state. It prevails, as a unique equilibrium, when a crisis is relatively mild (preference for liquidity and the fraction of low quality assets are relatively low). In a type II equilibrium, in the crisis state investors with high quality assets choose not to participate which causes market breakdown and liquidity hoarding. This type prevails, as a unique equilibrium, when a crisis is severe (preference for liquidity and the fraction of low quality assets are sufficiently high). In between, there is a possibility of multiple equilibria when both types coexist. In this case the equilibrium type depends on investors' initial beliefs about the average quality of assets sold in the market.

The ability to trade based on private information about asset quality increases aggregate welfare if adverse selection does not lead to the market breakdown. In normal times, informed trading is welfare beneficial since it allows financial institutions to share idiosyncratic risks through market trading.[7] However, more risk-sharing leads to more risk-taking by financial institutions which may result in significant losses during crises if market trading

downturns. (Eisfeldt [25] and Eisfeldt and Rampini [26])

[5] So investors are exposed to the systemic risk through their holdings of long-term risky assets.

[6] Usually *liquidity traders* are modeled as "noise traders" whose endowments and preference for consumption are left unspecified. Modeling liquidity traders as consumers with well-specified preferences allows one to examine the impact of informed trading on welfare.

[7] In this setting, the benefits from informed trading are *only* from risk sharing, there is no information revelation since investors have private information about idiosyncratic realizations of liquidity shocks and investment quality. All aggregate uncertainty is revealed in period one before market trading takes place.

halts.[8] Therefore, informed trading reduces idiosyncratic risks of financial institutions but it induces and exacerbates systemic risk by causing market breakdowns.

Furthermore, I show that even a small amount of adverse selection can lead to the equilibrium with no market trading during a crisis if it is accompanied by any of the following phenomena: an increase in liquidity preference during the crisis[9], underestimating the systemic risk, or uncertainty about asset values.

Increase in liquidity preference On the one hand, a higher preference for liquidity alleviates adverse selection since assets are more likely to be sold due to the seller's liquidity needs than due to their low quality. On the other hand, higher liquidity preference implies lower demand for (illiquid) risky assets. If demand is sufficiently low then the asset price is determined by cash in the market (i.e., by liquidity available in the market to absorb the asset trades) rather than by the asset's expected payoff. Hence, an increase in liquidity preference can lead to fire-sale pricing and possibly to a complete breakdown of trade.

Underestimating systemic risk Adverse selection is likely to cause a more severe crisis if systemic risk is underestimated. If the crisis is (or believed to be) a rare event, then financial institutions may not hold enough safe (liquid) assets to cushion the impact of a systemic shock when it occurs.

Uncertainty about asset values The Knightian uncertainty (ambiguity) about the fraction of low quality assets in the market can also cause market illiquidity. In this case, investors' beliefs about the extent of adverse selection are crucial: if investors believe there may be too many low quality assets in the market, then the market breaks down.

I show that the investment allocation is not constrained efficient:[10] there is overinvest-

[8] This is similar to the Hirshleifer effect when more information reduces risk sharing.

[9] The higher preference for liquidity during the crisis can viewed as precautionary liquidity hoarding due to the tightening in funding liquidity (see Brunnermeier and Pedersen [15] for dividing the concept of liquidity into two categories: funding liquidity and market liquidity).

[10] It is known since Greenwald and Stiglitz [31] that the market equilibrium is not constraint efficient if there are information imperfections. What is interesting is the source of inefficiencies, and the systemic externalities it creates. For example, Bhattacharya and Gale [12] show that even in the absence of aggregate liquidity shocks, heterogeneity in liquidity preferences leads to underinvestment into liquid assets. In my model, underinvestment into liquid (safe) assets is caused by adverse selection. If there is no adverse selection, financial institutions will hold safe assets above socially optimal level.

ment into risky assets relative to the constrained efficient investment allocation. In the market equilibrium, investors do not take into account the effect of their investment choice on market prices, thereby creating systemic externalities. Because of adverse selection, there are more assets traded in the market, in particular, more assets of low quality. To absorb this trading, more liquidity (safe assets) is needed. The social planner allocation increases the consumption of liquidity investors and investors with low quality assets which reduces ex-ante consumption volatility and improves aggregate welfare.

There are policy implications for government interventions during a crisis as well as for preemptive policy regulations. The effectiveness of policy responses during crises depends on which amplification effect contributes to a market breakdown. If it is due to an increase in liquidity preferences or to a small probability of the crisis then liquidity provision can restore the trading. However, if the no-trade outcome is caused by a large fraction of lemons or by the Knightian uncertainty about it, then it is more effective to remove these low quality assets from the market. The preemptive policy response is an ex-ante requirement of larger liquidity holdings, which prevents market breakdowns during crises, especially if the economy is in the multiple equilibria range. Also, I examine the effect of a liquidity provision during the crisis when it is financed by an ex-ante tax on holdings of risky assets. Such tax corrects the moral hazard problem associated with government interventions during crises and increases market liquidity which makes market breakdowns less likely.

This paper is organized as follows. In the next section I discuss the related literature. Section 3 describes the model environment. Section 4 characterizes the equilibrium and analyzes the role of government. Section 5 applies the model to the recent financial crisis and discusses possible policy responses and their effectiveness. Section 6 concludes the paper. All results are proved in the Appendix.

2 Related Literature

As has been demonstrated in the line of work started by Akerlof [5], asymmetric information between buyers and sellers can lead to a complete breakdown of trade. Morris and Shin [40], Bolton, Santos, and Scheinkman [13], Heider, Hoerova, and Holthausen [32], Acharya, Gale and Yorulmazer [1], Chiu and Koeppl [19], and Malherbe [39] provide explanations for market freezes and inefficient asset liquidation based on asymmetric information.[11]

My paper complements this literature on adverse selection in financial markets by accommodating market frictions such as aggregate uncertainty about liquidity preferences and asset returns in addition to asymmetric information about asset quality. Also, I explore the role of investors beliefs about asset values and the likelihood of a crisis as additional sources of market freezes. In particular, the liquidity holdings are determined endogenously, and the market price depends not only on the asset's average quality but also on the amount of liquidity available in the market. Therefore, a market breakdown can be caused by a shortage of liquid assets during the crisis, which results in depressed asset prices and causes non-participation of investors with high quality assets.[12]

Allen and Gale ([9], [10], [11]) developed a liquidity-based approach to study financial crises. When supply and demand for liquidity are inelastic in the short run, a small degree of aggregate uncertainty can have a large effect on asset prices and lead to financial instability. Allen and Carletti ([8], [7]) analyze the role of aggregate liquidity shortages in financial crises. Acharya, Shin, and Yorulmazer [3] and Diamond and Rajan [22] show that banks may hoard liquidity (above the socially optimal level) in anticipation of future gains from acquiring assets at fire-sale prices. In my paper, the market breakdown is a result of overinvestment in illiquid risky assets which causes a shortage of liquidity during crises.[13]

The importance of Knightian uncertainty in financial crises has been emphasized by Caballero [16], Caballero and Krishnamurthy [17], Krishnamurthy [38], Easley and O'Hara [24] and Uhlig [43]. In particular, Uhlig [43] develops a model of a systemic bank run with two variants: uncertainty aversion and adverse selection. He shows that only the former generates the following feature of a financial crisis: a larger share of troubled financial institutions results in a steeper asset price discount. Contrary to Uhlig [43], in my model

[11] Allen, Babus, Carletti [6] provide an extensive survey of recent papers that study the role of asymmetric information in credit markets.

[12] This unlike Malherbe [39], where agents self-insure through the ex-ante hoarding of non-productive but liquid assets, which reduces ex-post market participation and dries up market liquidity.

[13] Kahn and Wagner [34] develop a model where inefficiency in bank liquidity holdings depends on the relative costs of raising external liquidity. In particular, if liquidity supply is elastic, then there is a bias towards illiquid holdings.

it is possible that the adverse selection can lead to a larger price discount even if there is no Knightian uncertainty about asset values. In terms of policy responses to market breakdowns due to adverse selection, my paper is related to Chiu and Koeppl [19] and Philippon and Skreta [41].

My paper contributes to the literature by combining aggregate uncertainty about liquidity risk with aggregate uncertainty and asymmetric information about asset returns. My model builds on the cash-in-the-market framework developed by Allen and Gale which is well suited for studying financial crises accompanied by liquidity dry-ups. This framework captures the maturity transformation by banks (as in Diamond and Dybvig [21]) as well as the exposure of long-term assets to market liquidity risk through the "cash-in-the-market" pricing. I introduce asymmetric information in this framework, which generates an additional component of illiquidity due to the adverse selection.

3 Model

I consider a model with three dates indexed by $t = 0, 1, 2$. There is a continuum of ex-ante identical financial institutions (*investors*[14], for short) of measure one. There is only one good in the economy which can be used for consumption and investment. All investors are endowed with *one* unit of good at date $t = 0$, and nothing at the later dates. There are two states of nature $s = 1$ and $s = 2$ that are revealed at date $t = 1$. State 1 is the normal state and state 2 is the crisis state. These states are realized with ex-ante probabilities $(1 - q)$ and q, respectively. (I will also use the notation $q_1 = 1 - q$ and $q_2 = q$.) The states differ with respect to aggregate (market) productivity and the probability of a liquidity shock. There are more high-quality investments and less investors are affected by liquidity shocks in the normal state than in the crisis state.

[14] Financial institutions can also be referred to as *banks*. These are the market-based financial institutions (shadow banking) such as investment banks, money-market mutual funds, and mortgage brokers.

3.1 Preferences

Investors consume at date one or two, depending on whether they receive a liquidity shock at date one. The probability of receiving a liquidity shock in period one in state s is denoted by λ_s. (So λ_s is also the fraction of investors hit by a liquidity shock.) Investors who receive a liquidity shock have to sell or liquidate their risky long-term asset holdings and consume all their wealth in period one. They are effectively early consumers who value consumption only at date $t = 1$. The rest are the late consumers who value the consumption only at date $t = 2$. I will refer to the early consumers as *liquidity* investors, and to the late consumers as *informed* or *non-liquidity* investors.[15]

Investors have Diamond-Dybvig [21] type of preferences:

$$U(c_1, c_2)] = \lambda_s u(c_{1s}) + (1 - \lambda_s)u(c_{2s}) \tag{1}$$

where c_{ts} is the consumption at dates $t = 1, 2$ in state s. In each period, investors have logarithmic utility: $u(c_{ts}) = \log c_{ts}$.

3.2 Investment technology

Investors have access to two types of constant returns investment technologies. One is a storage technology (also called a *safe asset* or *cash*), which has zero net return: one unit of safe asset pays out one unit of consumption good in the next period. The other type of technology is a long-term risky investment project (also called a *risky asset*). The risky asset pays off in period two $\widetilde{R} \in \{R_H, R_L\}$ per unit of investment which represents an idiosyncratic (investment specific) productivity. The risky investment with payoff R_H is called a *high-quality asset* while an investment with payoff R_L is called a *low-quality asset* (*lemon*).

The quality of assets is independent across investors. Each investor i has a choice of starting his own investment project i by investing a fraction of his endowment. The investor

[15] Note both types of investors receive private information about quality of their assets. Assuming that liquidity investors are informed is without loss of generality since they cannot take advantage of this information. The structure of investment payoff and information are described in the next two subsections.

can start only one project, and each project has only one owner.[16] The idiosyncratic payoff of each investment i is an independent realization of a random variable $\widetilde{R}^i$ that takes two values: a low value R_L with probability π_s and a high value R_H with probability $(1-\pi_s)$ where $s \in \{1,2\}$ is the state. In the normal state, the fraction of low quality assets is small: $\pi_1 << 0.5$. In the crisis state, the fraction of low quality assets is larger : $\pi_2 > \pi_1$.

Remark 1 *An alternative specification*[17] *is that the payoff of each investment i consists of two components: $\widetilde{R}^i(s) = \alpha_i(s)\Re_L + (1-\alpha_i(s))\Re_H$, where $\alpha_i(s)$ represents the exposure to an asset with low payoff $\Re_L$. The individual exposure $\alpha_i(s)$ is a random variable that takes two values: a high value α_h with probability π_s and a low value α_l with probability $(1-\pi_s)$, where $s \in \{1,2\}$ is the state. Then the market (aggregate) exposure is $\alpha_m(s) = \pi_s\alpha_h + (1-\pi_s)\alpha_l$ and the market payoff is $R_m(s) = \alpha_m(s)\Re_L + (1-\alpha_m(s))\Re_H$. As before, state 1 is the normal state where the fraction of low quality assets is small. State 2 is the crisis state with more low quality assets: $\pi_2 > \pi_1$, so that $R_m(s=1) > R_m(s=2)$. To express this specification in terms of the previous one denote the payoff of low-quality investment as R_L, i.e., $R_L \equiv \alpha_h\Re^L + (1-\alpha_h)\Re^H$. Similarly, the high-quality investment payoff, denoted by R_H, is $R_H \equiv \alpha_l\Re^L + (1-\alpha_l)\Re^H$.*

The expected payoff of each individual risky project in state s is denoted by $\overline{R}_s = \pi_s R_L + (1-\pi_s) R_H$ with $R_L < 1 < R_H$. The expected payoff when the economy is in the normal state is higher than when it is in the crisis state: $\overline{R}_1 > \overline{R}_2$. The expected payoff before state is realized is denoted by $\overline{R} = (1-q)\overline{R}_1 + q\overline{R}_2$ with $\overline{R} > 1$.

The long-term asset can be liquidated prematurely at date $t=1$, in which case, one unit of the high (low) quality asset yields r_H (r_L) units of the good, and $0 \le r_L = R_L < r_H < 1$. This private liquidation technology can be interpreted as an outside funding option. Suppose there are (outside) experts who have an ability to value assets but have limited demand and their services are expensive. Alternatively, this liquidation technology can be interpreted

[16] I assume that several agents cannot coinvest into one project in order to diversify away the idiosyncratic risk. This assumption can be justified by the benifits of securitization which reflect the limitations of ex-ante project pooling.

[17] This specification is equivalent to the above (although more complicated) but it makes the model more applicable to the ABS market.

as costly restructuring of assets or terminating loans before maturity (similarly to Heider, Hoerova, and Holthausen [32]).[18] The holdings of the two-period risky asset can also be traded in the financial market at date $t = 1$. Figure 1 summarizes the payoff structure.

time	0	1	2
safe asset	1	1	1
risky asset	1	r_k	R_k

Figure 1. Payoffs, $k = L, H$

3.3 Information and Timeline

At date $t = 0$, each investor makes an investment choice between the two investment technologies, risky and safe, in proportion x and $(1-x)$, respectively. Investors choose their asset holdings to maximize their expected utility.

At date $t = 1$, liquidity shocks and the aggregate state are realized, and the financial market opens. Investors privately observe their asset payoffs and liquidity needs. The supply of risky assets comes from the investors who have experienced a liquidity shock, whereas the demand comes from those who have not.[19] Any investor can liquidate his investment project at date one, receiving r_k units of the good per unit of investment (where $k = L, H$ depending on whether the project is of high or low quality).

Note that the markets are incomplete since there are two frictions in this economy: asymmetric information about asset quality and liquidity shocks, which generates four possible types of investors in each state. The holdings of the safe asset provides partial insurance against the liquidity risk as well as the asset quality risk. The difference between returns on risky and safe assets can also be viewed as a liquidity premium.

The timeline of the model is summarized in the figure below.

[18] I assume the lemons can be liquidated at the same value as their payoff. The important assumption is that lemons have sufficiently low payoff so that there is no losses from premature liquidation. A simple case is when $R_L = r_L = 0$. It can be assumed that there are gains from restructuring, i.e. $r_L \geq R_L$ - it does not affect the results. Also, the liquidation values r_k can be state dependent, it would not qualitatively affect any results. Appendix 7.1 describes additional assumptions imposed on parameters values.

[19] Informed investors can simultaneously be sellers and buyers in the market. This assumption does not affect any results.

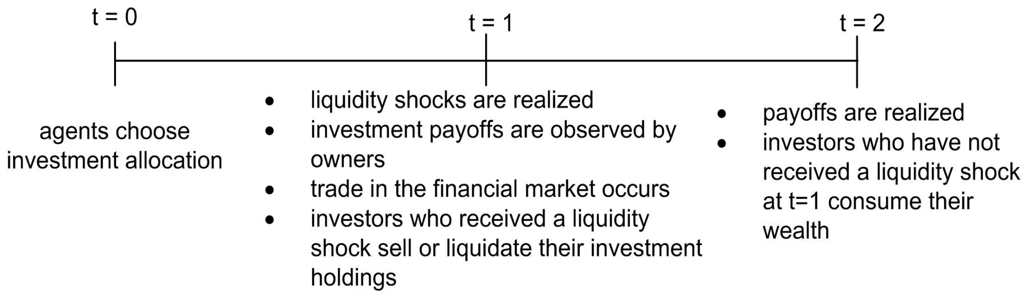

Figure 2. Timeline

I will consider two cases. In the first case, it is public information which investors have experienced a liquidity shock. The liquidity investors sell or liquidate their holdings of the risky asset in order to consume as much as possible in period one. In the second case, the identity of investors hit by a liquidity shock is private information. Therefore, after privately observing investment payoffs, non-liquidity investors can take advantage of their information by selling low quality projects in the market at date $t = 1$. Buyers can not distinguish whether an investor is selling his asset because of its low payoff or because of his liquidity needs. This generates adverse selection problem, and leads to a discount on the investments sold in the market at date $t = 1$.

4 Equilibrium

4.1 Equilibrium without Adverse Selection (benchmark model)

First, as a benchmark, I consider the case with no adverse selection where the identity of investors who have experienced a liquidity shock is public information.[20] Then all risky assets at date $t = 1$ are sold only by liquidity traders who cannot wait for the maturity of their investments at date $t = 2$.

Since realizations of asset quality are independent from realization of liquidity shocks, the expected payoff of the risky asset sold in period one is $\overline{R}_s$ in state s. All risky assets sold

[20] In this setting, absence of adverse selection refers to inability of investors to trade based on their private information about asset quality. In the equilibrium, both bad and good assets are traded in the market in period one, however, they are sold only by liquidity investors.

at $t = 1$ are aggregated in the market, therefore, the variance of an asset bought at date $t = 1$ is zero (since all investments have idiosyncratic payoffs).[21] Therefore, the expected return on risky assets bought in period one is $\overline{R}_s/p_s$, where p_s is the market price in state s. Late consumers are willing to buy risky assets at date $t = 1$ if the market price p_s is less than or equal to the expected payoff $\overline{R}_s$. The early consumers are willing to sell their projects if the market price p_s is greater than the liquidation value of their asset: r_k.[22]

The consumption of early consumers in state s is denoted by $c_{1k}(s)$ and the consumption of late consumers in state s is denoted by $c_{2k}(s)$ where $k = L, H$ refers to the quality of investment project i. The investor's maximization problem is given by

$$\max_{x\in[0,1]} \quad \sum_{s=1,2} q_s \begin{bmatrix} \lambda_s \left(\pi_s \log c_{1L}(s) + (1-\pi_s) \log c_{1H}(s)\right) \\ + (1-\lambda_s) \left(\pi_s \log c_{2L}(s) + (1-\pi_s) \log c_{2H}(s)\right) \end{bmatrix} \tag{2}$$

$$s.t. \quad \text{(i)} \quad c_{1k}(s) = \begin{cases} 1 - x + p_s x & \text{if} \quad p_s > r_k, \\ 1 - x + r_k x & \text{if} \quad p_s \leq r_k. \end{cases}$$

$$\text{(ii)} \quad c_{2k}(s) = \begin{cases} xR_k + x_{1s}\overline{R}_s & \text{if} \quad p_s > r_k, \\ xR_k + (1-x) & \text{if} \quad p_s \leq r_k. \end{cases}$$

where x_{1s} is the demand for risky assets at $t = 1$ in state s:

$$x_{1s} \begin{cases} = \frac{1-x}{p_s} & \text{if} \quad r_H < p_s < \overline{R}_s, \\ \in \left[0, \frac{1-x}{p_s}\right] & \text{if} \quad p_s = \overline{R}_s, \\ = 0 & \text{if} \quad \text{otherwise.} \end{cases} \tag{3}$$

If the market price $p_s \leq r_H$ then all liquidity investors with high quality assets choose to liquidate their assets so that only lemons (assets with low payoffs) are traded in the market. Then the expected payoff of a traded risky asset is r_L. Therefore, there is no trade since no one is willing to buy these low quality assets ($x_{1s} = 0$).

[21] Assuming the law of large numbers holds. As shown by Judd (1985), one can find a measure that makes a law of large numbers valid for a continuum of i.i.d. random variables.

[22] For simplicity, I assume that if the asset price is equal to the liquidation value, investors choose to liquidate their assets rather than to sell them. This assumption rules out equilibria with partial pooling.

Therefore, aggregate demand at $t = 1$ in state s is given by

$$D(s) \begin{cases} = & (1-\lambda_s)\frac{1-x}{p_s} & \text{if} \quad r_H < p_s < \overline{R}_s, \\ \in & \left[0, (1-\lambda_s)\frac{1-x}{p_s}\right] & \text{if} \quad p_s = \overline{R}_s, \\ = & 0 & \text{if} \quad \text{otherwise}, \end{cases} \tag{4}$$

and aggregate supply at $t = 1$ in state s is given by

$$S(s) = \begin{cases} \lambda_s x & \text{if} \quad p_s > r_H, \\ \lambda_s \pi_s x & \text{if} \quad r_L < p_s \leq r_H, \\ 0 & \text{if} \quad p_s \leq r_L. \end{cases} \tag{5}$$

Then market clearing conditions are

$$\lambda_s x p_s \leq (1-\lambda_s)(1-x). \tag{6}$$

The price in state s is equal to the lesser of the amount of cash available from buyers per unit of assets sold and the expected payoff,

$$p_s = \min\left\{\frac{(1-\lambda_s)(1-x)}{\lambda_s x}, \overline{R}_s\right\}. \tag{7}$$

This cash-in-the-market pricing captures the effect of liquidity on asset pricing. When there is sufficient liquidity in the market, the price is equal to the asset's expected payoff. However, when liquidity is scarce, the price is determined by the holdings of safe asset (*cash*) available in the market.

The aggregate uncertainty about the fraction of investors affected by liquidity shocks generates asset price volatility: the equilibrium market price is lower during the crisis than in normal times ($p_2 < p_1$).[23] The investment allocation x is smaller than the first-best investment allocation since the investment quality is not observable.[24]

4.2 Equilibrium with Adverse Selection

Now suppose the identity of liquidity investors is private information. Then, after observing investment payoff, non-liquidity investors can take advantage of their private information

[23] This result is similar to Allen and Gale [9].

[24] See Appendix 7.2 for the proof.

by selling low productive investments in the market at date $t = 1$. This generates the adverse selection problem, and therefore leads to a discount on the price of risky assets sold at $t = 1$.

An investor who buys a risky asset at date $t = 1$ does not know whether it is sold due to a liquidity shock or because of its low payoff. Buyers believe that with probability $\frac{\lambda_s}{\lambda_s+(1-\lambda_s)\pi_s}$ investment is sold due to a liquidity shock, and with probability $\frac{(1-\lambda_s)\pi_s}{\lambda_s+(1-\lambda_s)\pi_s}$ it sold because of its low quality. Hence, buyers believe that the expected payoff of risky assets sold in state s is $\widehat{R}_s$,

$$\widehat{R}_s = \frac{\lambda_s}{\lambda_s + (1-\lambda_s)\pi_s}\overline{R}_s + \frac{(1-\lambda_s)\pi_s}{\lambda_s + (1-\lambda_s)\pi_s}R_L. \tag{8}$$

The non-liquidity investors are willing to buy risky assets at $t = 1$ if the market price p_s is less than or equal to the expected payoff $\widehat{R}_s$. Therefore, the demand for risky assets at $t = 1$ is given by

$$x_{1s}\begin{cases} = \frac{1-x}{p_s} & \text{if} \quad r_H < p_s < \widehat{R}_s, \\ \in \left[0, \frac{1-x}{p_s}\right] & \text{if} \quad r_H < p_s = \widehat{R}_s, \\ = 0 & \text{if} \quad \text{otherwise.} \end{cases} \tag{9}$$

The liquidity investors are willing to sell their investment if the market price p_s is greater than the liquidation value of their assets. If the market price is less than or equal to the liquidation value of high quality assets ($p_s \leq r_H$) then only low quality assets are traded in the market with the expected payoff of r_L. Since no one is willing to buy these low quality assets, there is no trade. Similarly, if the fraction of low quality assets π_s is sufficiently large so that the expected payoff $\widehat{R}_s \leq r_H$, then there is no market trading as well.

If there is trading in state s, the market clearing conditions are

$$\forall s = 1, 2 : (\lambda_s + (1-\lambda_s)\,\pi_s)\,xp_s \leq (1-\lambda_s)\,(1-x)\,. \tag{10}$$

Note, ability to trade based on private information increases the supply of risky assets.

The market price in state s can be expressed as the lesser of the amount of cash per unit of assets sold and the expected payoff $\widehat{R}_s$,

$$p_s = \min\left\{\frac{(1-\lambda_s)\,(1-x)}{(\lambda_s + (1-\lambda_s)\,\pi_s)\,x}, \widehat{R}_s\right\}. \tag{11}$$

Market liquidity is characterized by the cost of selling long-term assets before maturity,

$$C(s) = \frac{\widehat{R}_s - p_s}{\widehat{R}_s} \tag{12}$$

A lower cost implies higher market liquidity. Therefore, there is a trade-off between asset payoff and liquidity: risky assets have larger expected payoff but there is a cost associate with premature liquidation or sale of the asset. This cost is increasing in the amount of adverse selection in the market.

Denote aggregate liquidity holdings in state s by $L(s)$,

$$L(s) = (1 - \lambda_s)(1 - x). \tag{13}$$

Even though the safe asset has lower expected return, it has additional value for its ability to reallocate risky assets from liquidity investors to non-liquidity ones. This value of liquidity is characterized by the payoff on risky asset bought in period one: $\widehat{R}_s/p_s \geq 1$.

I distinguish two types of equilibria: type I with market trading in both states, and type II with a market breakdown in the crisis state.[25] Type I is a pooling equilibrium where both high and low quality assets are sold in each state. Type II is a separating equilibrium where in the crisis liquidity investors choose to liquidate their high quality assets rather than to sell them, which leads to the no-trade outcome.

Proposition 1 *If the crisis is mild (λ_2 and π_2 are relatively small) then there is a unique type I equilibrium with market price volatility across states: $p_1 > p_2$. If the crisis is severe (λ_2 and π_2 are sufficiently large) then there is a unique type II equilibrium with no trade in the crisis state. For the intermediate range of parameters λ_2 and π_2, there is a possibility of multiple equilibria: one of each type. In the case of multiple equilibria, in the type I equilibrium market liquidity and liquidity holdings are larger, and the expected utility is higher than in the type II equilibrium.*

Adverse selection leads to the increased price volatility across states because a larger share of lemons in the market during the crisis. Therefore, market liquidity is larger in the

[25] Equilibria with partial pooling are ruled out by assuming that if the asset price is equal to the liquidation value, investors choose to liquidate their assets rather than to sell them. This assumption is for simplicity only, and does not qualitatively affect the results.

normal state than in the crisis state. Also, the payoff on the risky asset bought in period one is larger in the crisis state relative to the normal state: $\widehat{R}_2/p_2 > \widehat{R}_1/p_1$. This reflects the fire-sale phenomenon when the value of liquidity is high during crises.

However, the scarcity of liquidity holdings in the market could lead to a market breakdown, in which case the role of the safe asset is reduced to the storage technology. If there are too many low-quality assets in the market $\left(\pi_2 \geq \frac{\lambda_s(R_H - r_H)}{\lambda_s R_H + (1-\lambda_s)r_H - r_L}\right)$ so that the expected payoff $\widehat{R}_2$ falls below the liquidation value r_H, then the market breaks down. As a result, the value of the (liquid) safe asset is lower in the type II equilibrium than in the type I equilibrium.

A high preference for liquidity (λ_2) can also lead to a breakdown of trade. On the one hand, higher preference for liquidity alleviates the adverse selection and increases the expected payoff since assets are more likely to be sold due to seller' liquidity needs than due to their low quality. On the other hand, higher liquidity preference implies lower demand for risky assets. If demand is sufficiently low then the asset price is determined by liquidity available in the market to absorb the asset trades (rather than by the expected payoff). Therefore, an increase in liquidity preference during the crisis may amplify the adverse selection problem by pushing the asset prices further down, possibly to the extent of causing a market breakdown. This is consistent with the asset fire-sales when depressed prices reflect the difficulty of finding buyers during the crisis.

For some range of parameters, two types of equilibria coexist. These are sunspot equilibria when the equilibrium type is determined by investors' self-fulfilling beliefs. In particular, if investors believe there is no trade during the crisis than they hold less of the safe asset. Then if the crisis state is realized, there is not enough liquidity to absorb the informed trading, so the market does indeed break down. Note that the market breakdown is caused by aggregate overinvestment into the risky long-term asset. Furthermore, an equilibrium with market breakdown is (ex-ante[26]) inefficient since it achieves a lower expected utility relative to the equilibrium with market trading during the crisis.

[26] Note, ex-post Pareto efficiency is violated for investors with high quality assets in the normal state.

4.3 Welfare Implications

In the setting where trading based on private information is not possible, the market provides insurance only against liquidity risk. So, there are possible welfare gains from allowing investors to benefit from private information on their asset quality. I show that informed trading is welfare beneficial if it does not cause a market breakdown.

Proposition 2 *The ability to trade based on private information increases expected utility if there is market trading during the crisis and may decrease expected utility if there is no trade during the crisis and the probability of a crisis is sufficiently large. The market liquidity in each state and aggregate liquidity holdings are smaller in the equilibrium with adverse selection than in the equilibrium without adverse selection.*

Market trading in the interim period ($t = 1$) allows investors with low quality assets to benefit from their private information at the expense of liquidity traders. The ability to trade based on private information provides partial ex-ante insurance against a low asset quality realization, which is especially relevant in the crisis state. As a result, it leads to consumption smoothening across different types of investors, and therefore improves ex-ante welfare.

However, if there is no trade during a crisis then investors are left with their low quality assets. So, the breakdown of trade prevents risk sharing. Moreover, some of the high quality assets are liquidated before maturity contributing to a welfare loss. Therefore, the market breakdown increases consumption volatility and leads to lower aggregate welfare.

Because it provides partial insurance, informed trading makes a risky investment ex-ante more attractive, which is reflected in lower aggregate liquidity holdings. Also, the supply of risky assets in period $t = 1$ (in particular, the supply of low quality assets) is larger. As a result, market prices are lower relative to the equilibrium without adverse selection. Therefore, adverse selection leads to a less liquid market, i.e., the cost of selling a risky asset before maturity is higher.

4.4 Numerical Example

Adverse Selection To illustrate the impact of adverse selection, consider the following numerical example. The asset return parameters are $R_H = 1.2$, $r_H = 0.5$, $R_L = r_L = 0.3$, the fraction of low quality investments in the normal state is $\pi_1 = 0.05$, the probability of a liquidity shock in the normal state and the crisis state, respectively, are $\lambda_1 = 0.2$ and $\lambda_2 = 0.3$, and the probability of the crisis is $q = 0.1$. Figure 3a depicts the equilibrium values of investment (x), prices (p_s) and expected utility (EU) as a function of the fraction of low quality assets in the crisis (π_2). The solid and dashed lines depict equilibrium values with and without adverse selection.

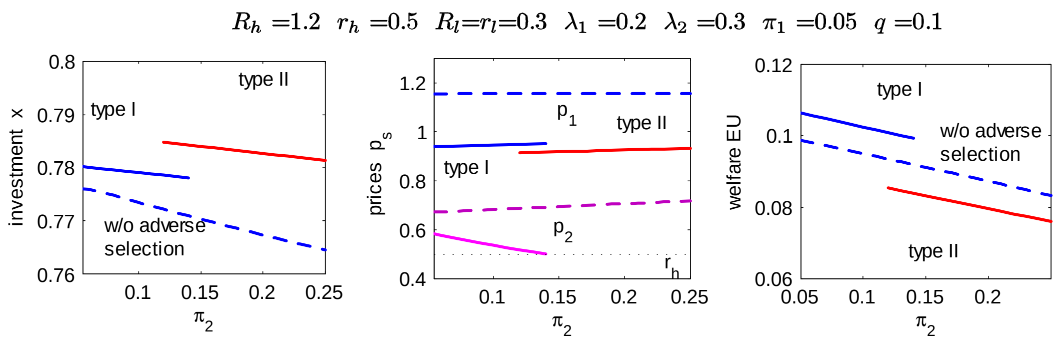

Figure 3a. Equilibrium values of investment, prices and welfare as a function of π_2.

As the fraction of low quality assets increases, the economy moves from the equilibrium with trading to the equilibrium with no trade in the crisis state. If the fraction of lemons is relatively small (less than 12%) then there is a unique equilibrium with market trading in both states. If the fraction of lemons is sufficiently large (more than 14.2%) then there is a unique equilibrium with no trade during the crisis. In between, the two types of equilibria coexist.

If market trading breaks down, the safe asset has lower value and, as a result, the holdings of risky assets is larger in a type II equilibrium relative to a type I equilibrium and to an equilibrium without adverse selection. Adverse selection increases asset price volatility, and results in an increase in welfare if there is market trading, otherwise, it leads to a welfare loss.

Figure 3b depicts aggregate liquidity holdings in each state ($L(s)$), market liquidity as the cost of foregone payoff when assets are sold before maturity ($C(s)$), and the return on asset bought in the market (R_s/p_s). The cost of selling assets before maturity and the return on assets bought in period one are higher in the crisis state than in the normal state, reflecting the lack of liquidity in the market during the crisis.

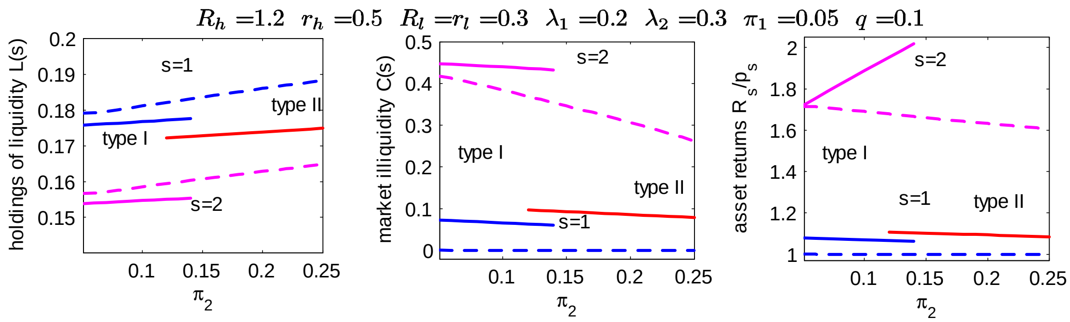

Figure 3b. Equilibrium values of market liquidity and asset returns as a function of π_2.

Liquidity preference Now consider the effect of change in preferences for liquidity during the crisis (λ_2). As before, asset returns are $R_H = 1.2$, $r_H = 0.5, R_L = r_L = 0.3$, the fraction of low quality investments in the normal state and in the crisis state, respectively, are $\pi_1 = 0.05$ and $\pi_2 = 0.25$, the probability of the crisis is $q = 0.1$. The probability of a liquidity shock in the normal state is $\lambda_1 = 0.2$. The figure below illustrates the effect on equilibrium values of an increase in liquidity preferences in the crisis state (λ_2) from 0.2 to 0.4.

For $\lambda_2 \leq 0.31$, there is a unique equilibrium with market trading in both states; for $\lambda_2 > 0.32$ there is a unique equilibrium with no trade during the crisis; otherwise, there are multiple equilibria. The higher preference for liquidity magnifies the effect of adverse selection on asset prices and market liquidity. The difference in the payoff of assets bought in period one across two states ($R_2/p_2 - R_1/p_1$) is increasing in the preference for liquidity

λ_2 which is consistent with the fire-sale pricing.

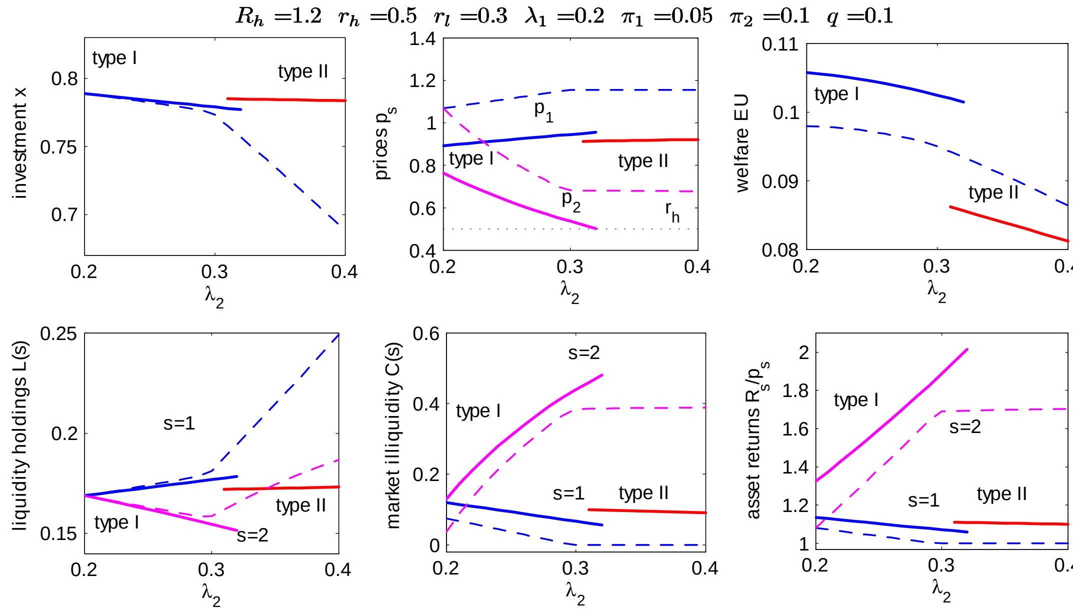

Figure 5. Equilibrium values as a function of preference for liquidity during the crisis λ_2.

Figure 6 illustrates how the equilibrium type depends on the interaction between liquidity preference (λ_2) and the fraction of low quality assets (π_2). The figure depicts the possible equilibria regions for different values of π_2 and λ_2. Each point in the (π_2, λ_2) plane corresponds to a particular type of equilibria: type I or type II, except for the region with multiple equilibria when both type I and II occur together.

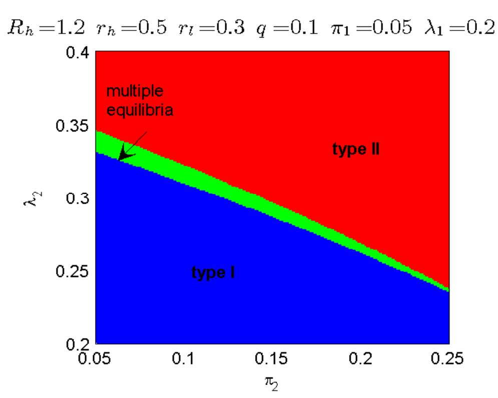

Figure 6. Equilibrium types for different values of π_2 and λ_2.

As can be seen from the figure, even a small amount of adverse selection (small π_2) can lead to the no-trade outcome if the preference for liquidity is sufficiently high (large λ_2).

4.5 Properties of Equilibrium

In this subsection, I examine how changes in the probability (q) of the crisis state and beliefs about it affect the equilibrium types and values.

4.5.1 Probability of the crisis state

Corollary 1. *If the probability of the crisis state q is smaller then (i) investment allocation is larger ; (ii) market prices are lower ; and (iii) expected utility is higher. If the economy is in the type I equilibrium (with market trading in both states) then an increase in q may lead to the type II equilibrium (with market breakdown in the crisis state).*

The lower probability of the crisis state q implies that an asset is less likely to become a lemon, which makes it ex-ante more profitable. Therefore, a lower q leads to a higher level of investment x. More investment at date $t = 0$ implies larger supply and lower demand for risky assets at date $t = 1$. As a result, market prices are lower in both equilibrium types. The fact that the market price is increasing in the crisis probability makes it is possible to move from one equilibrium type to the other.

Consider the numerical example discussed before. The asset return parameters are $R_H = 1.2, r_H = 0.5, R_L = r_L = 0.3$, the fraction of low quality assets in the normal state is $\pi_1 = 0.05$ and in the crisis state is $\pi_2 = 0.15$, and the probability of a liquidity shock in the normal state is $\lambda_1 = 0.2$, and in the crisis state is $\lambda_2 = 0.3$. Figure 7 depicts the equilibrium values as a function of the probability of the crisis state q.

As the probability of the crisis increases, the economy moves from the unique equilibrium with market breakdown to the multiple equilibria (for $q > 11.8\%$), and then to the unique equilibrium with market trading (for $q > 20.6\%$). So, if the crisis is a rare event then there is no trade during the crisis.

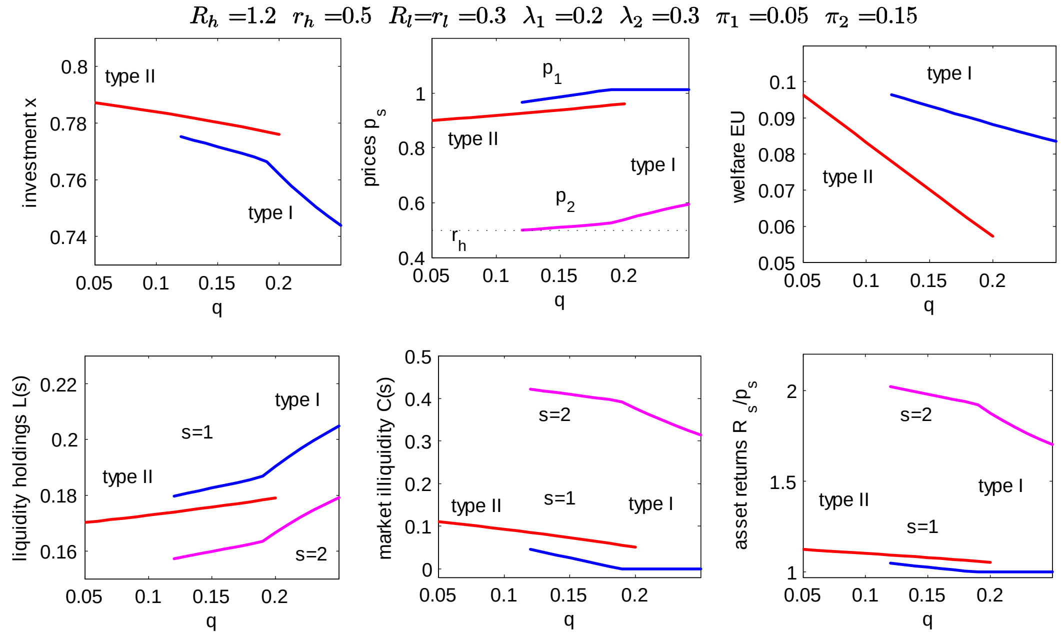

Figure 7. Equilibrium values as a function of probability of the crisis state q.

Assume that the probability of the crisis q depends on the previously realized state, and compare equilibria sequentially.[27] The transition matrix is given by $\begin{bmatrix} 1-q_{12} & q_{12} \\ 1-q_{22} & q_{22} \end{bmatrix}$ where $q_{jk} = \Pr(s = s_k | s = s_j), k, j \in \{1, 2\}$ is the conditional probability of transition from state j to state k, and $q_{22} > q_{12}$. So that it is more likely that the economy continues to stay in the crisis state once it is realized.

Let us look again at the numerical example. Suppose $q_{12} = 0.05$ and $q_{22} = 0.25$. If the economy is in the normal state then it is in the type II equilibrium with no trade. Once the crisis occurs, probability of the crisis next period changes and investment allocations are adjusted (liquidity holdings are increased), and the economy moves to the type I equilibrium. So, the market trading is resumed next period even if the crisis persists.

Next I examine how equilibrium types depend on the interaction between liquidity pref-

[27] This assumption creates generic dynamics where each time period $T = 1, 2, ...$ consists of three subperiods: $t = 0, 1, 2$.

erence (λ_2), probability of the crisis (q), and the fraction of lemons (π_2). Figure 8 illustrates the possible equilibria regions for different values of q and λ_2. Again, I consider two examples with the same values of $R_H = 1.2$, $r_H = 0.5$, $R_L = r_L = 0.3$ and different values of π_2: $\pi_2 = 0.1$ and $\pi_2 = 0.2$.

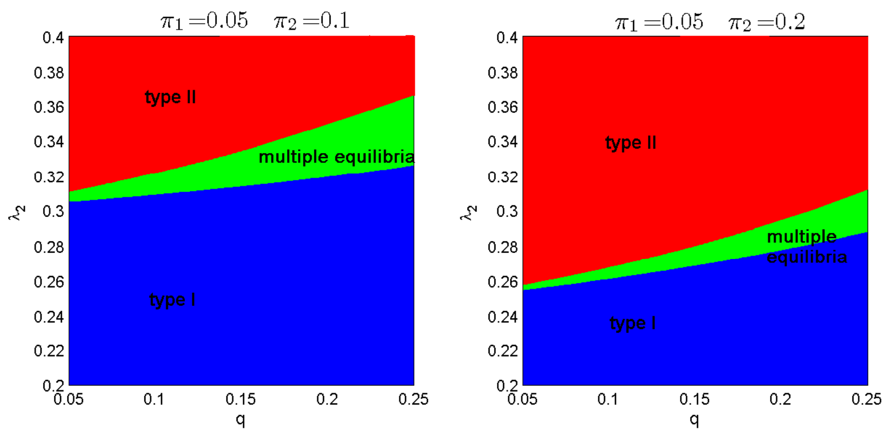

Figure 8. Equilibrium types for different values of q and λ_2.

As illustrated in Figure 8, even a small amount of adverse selection (small π_2) can lead to the market breakdown if the crisis is a rare event (small q) and preference for liquidity is high (large λ_2). The threshold value of the crisis probability when the economy moves from trade to no-trade equilibrium is increasing in λ_2. So, if the crisis is accompanied by flight to liquidity (large increase in λ_2), then market breakdowns are more persistent.

4.5.2 Role of beliefs about the crisis

Next I analyze the role of beliefs about a crisis probability. Suppose the (true) probability of a crisis is q_o but investors believe that the probability is q which could be less or greater than q_o. Let us look again at the numerical example. Suppose the probability of a crisis is $q_o = 10\%$. Figure 9 depicts the equilibrium values of investment and expected utility as a function of q. If a crisis is considered to be a rare event ($q < 3.2\%$), then the economy is in the unique equilibrium with no trade during the crisis. If investors believe $q > 6.6\%$, then the economy is in the unique equilibrium with market trading in both states. For $q \in [3.2\%, 6.6\%]$, there are multiple equilibria.

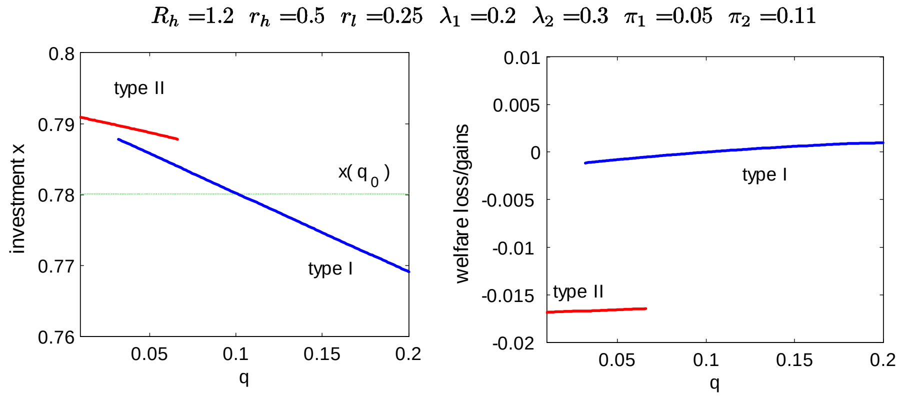

Figure 9. Equilibrium values as a function of beliefs q.

Underestimating the crisis probability is more costly in terms of welfare than overestimating it because the former may result in a market breakdown. Moreover, overestimating the probability of a crisis may actually be welfare beneficial since the market equilibrium is not efficient: investors overinvest into risky assets at date $t = 0$ relative to the second-best investment allocation.[28] Thus, pessimistic beliefs about the crisis probability lead to larger liquidity holdings, and may therefore improve welfare.

4.6 Equilibrium with Adverse Selection and Knightian Uncertainty

Suppose the crisis is accompanied by an unanticipated shock in period one. The shock is an "unforeseen contingency", an event that investors are not aware of so they do not plan for it.[29] As a result of this shock, investors face Knightian uncertainty (ambiguity) about the fraction of low quality assets in the crisis state. Investors do not know the actual probability $\widehat{\pi}_2$ of an asset being a lemon, instead they believe it belongs to some interval: $\widehat{\pi}_2 \in [\underline{\pi}_2, \overline{\pi}_2]$ such that $\pi_1 \leq \underline{\pi}_2 \leq \overline{\pi}_2$. Investors are assumed to have Gilboa-Schmeidler [28] maxmin utility: $U(c) = \min_{\widehat{\pi}_2 \in [\underline{\pi}_2, \overline{\pi}_2]} E[\log(c)]$.

[28] See section 4.7.1 for the Social Planner solution.

[29] *Unforseen contingencies* are defined as "possibilities that the agent does not think about or recognize as possibilities at the time he makes a decisoon" (Lipman, The New Plagrave Dictionary of Economics 2008). In modeling unanticipated uncertainty about the asset value, I am following Easley and O'Hara (2008) and Uhlig (2009).

This assumption does not change the investment decision made at date $t = 0$ since there is no ambiguity at date $t = 0$. The investment allocation x is determined by the initial beliefs π_2 (before the unanticipated shock is realized) so that $x = x(\pi_2)$. Assume $\pi_2 < \overline{\pi}_2$.

Non-liquidity investors decide whether to buy assets at date $t = 1$ based on the worst among possible priors: $\overline{\pi}_2$. Therefore, investors are willing to buy risky assets at $t = 1$ during the crisis if the market price p_2,

$$p_2 = \frac{(1-\lambda_2)}{(\lambda_2 + (1-\lambda)\,\widehat{\pi}_2)} \frac{(1-x(\pi_2))}{x(\pi_2)}, \tag{14}$$

is less than the (worst) expected payoff $\widehat{R}(\overline{\pi}_2)$,

$$\widehat{R}(\overline{\pi}_2) = \frac{\lambda_2(1-\overline{\pi}_2)}{\lambda_2 + (1-\lambda_2)\overline{\pi}_2} R_H + \frac{\overline{\pi}_2}{\lambda_2 + (1-\lambda_2)\overline{\pi}_2} R_L. \tag{15}$$

Consider the case when $p_2 > r_H$, which also implies that $p_1 > r_H$. So, if there is no ambiguity about $\widehat{\pi}_2$ (i.e., $\underline{\pi}_2 = \widehat{\pi}_2 = \overline{\pi}_2$) then there is market trading in both states. However, with ambiguity about $\widehat{\pi}_2$, there is a breakdown of trade when $\overline{\pi}_2$ is sufficiently large so that $\widehat{R}_2(\overline{\pi}_2) \leq r_H$, i.e.,

$$\overline{\pi}_2 \geq \frac{\lambda_2\,(R_H - r_H)}{\lambda_2 R_H + (1-\lambda_2) r_H - R_L}. \tag{16}$$

Therefore, the ambiguity (Knightian uncertainty) about the fraction of low quality assets can amplify the effect of adverse selection and cause a market breakdown.

4.7 Government

4.7.1 Social Planner

In this section, I analyze this model from the social planner perspective, and compare it with a market equilibrium.

First-best allocation Under full information (when it is known who receives a liquidity shock and the asset quality is observable) the optimal investment allocation is $x = \left(1 - \sum_{s=1,2} q_s \lambda_s\right)$, consumption allocations of liquidity investors are $c_1(s) = \frac{1}{\lambda_s} \sum_{s=1,2} q_s \lambda_s$, and non-liquidity investors receive $c_2(s) = \overline{R}_s \left(1 - \sum_{s=1,2} q_s \lambda_s\right) / (1-\lambda_s)$.

Second-best allocation With asymmetric information about the quality of assets and the identity of liquidity investors, the first-best allocation is not incentive compatible because investors with low quality assets have the incentive to pretend to be liquidity traders.

The incentive-compatible utility maximization problem is given by

$$\max_x \sum_{s=1,2} \sum_{k=L,H} q_s \left(\lambda_s \pi_{sk} \log c_{1k}(s) + (1-\lambda_s)\, \pi_{sk} \log c_{2k}\,(s)\right) \tag{17}$$

$$\begin{aligned} s.t. \quad (i) \quad & \lambda_s c_1(s) \leq 1 - x, \\ (ii) \quad & (1-\lambda_s) \sum_{k=L,H} \pi_{sk} c_{2k}\,(s) = x\,(1-\pi_s)\, R_h + 1 - x - \lambda_s \sum_{k=L,H} \pi_{sk} c_{1k}(s), \\ (iii) \quad & c_1(s) \leq c_{2k}\,(s)\,.\ \forall k, s \end{aligned}$$

for all $s = 1, 2$ and $k = L, H$.[30]

Since the quality of assets is not observable, all liquidity investors consume the same amount: $c_{1k}(s) \equiv c_1(s)$ for each k, s.[31] The constraints (i) and (ii) are resource constraints for period one and two, respectively. The constraints (iii) are incentive compatibility constraints. In equilibrium, constraints (v) are binding for investors with low quality assets: $c_1(s) = c_{2L}\,(s)$ in each state s.

Proposition 3 *The optimal holdings of the safe asset in the incentive-compatible social planner's solution are larger than in the market equilibrium. The social planner achieves higher aggregate welfare relative to the market equilibrium.*

In the market equilibrium, investors do not take into account the effect of their investment choice on prices. This creates an externality which distorts the efficient investment allocation. In particular, this externality leads to overinvestment in risky assets that contributes to the market breakdown. Due to the adverse selection, there are more assets traded in the market at date $t = 1$, in particular, more assets of low quality. To absorb this trading, more market liquidity is need.

[30] $\pi_{ks} \equiv \begin{cases} \pi_s & if \quad k = L \\ 1-\pi_s & if \quad k = H \end{cases} \quad \forall s = 1, 2$

[31] The social planner can differentiate liquidity investors with bad and good assets by offering a contract with a lower price and a lower quantity/probability. However, such contracts are not optimal since it results in the liquidation of some high quality assets before maturity, and therefore, leads to a loss in welfare.

The effect of prices on expected utility depends on the investors' type: liquidity investors and investors with low quality assets benefit from higher prices, while non-liquidity investors with high quality assets benefit from lower prices. Overall, the price effect evaluated at the market equilibrium is positive[32]. Therefore, aggregate welfare can be improved by increasing holdings of the safe asset which leads to higher asset prices. The social planner problem is equivalent to the investor maximization problem when the price effect is taken into account. As a result, a larger allocation of liquidity by social planner smooths ex-ante consumption and increases aggregate welfare. The social planner reduces the adverse selection problem but does not completely eliminate it.

4.7.2 Policy Implications

Ex-ante liquidity requirements The social planner solution suggests the following policy implication: requiring ex-ante larger liquidity holdings would alleviate the adverse selection problem and prevent the market breakdown during crises, especially if the economy is in the multiple equilibria range. The government can require agents to hold the safe asset at date $t = 0$ so that the second-best allocation is implemented.

Liquidity provision during a crisis

Tax-financed liquidity provision Alternatively, the government can intervene ex-post when the economy is in the crisis state. If the market breakdown is due to the high liquidity preference or the low crisis probability, then liquidity provision into the market can restore trading. Consider the situation when the economy is in the no-trade equilibrium and the government decides to intervene. Suppose the price needs to be increased by Δ to restore trading, then government should inject Λ amount of liquidity such that $\Lambda = \Delta\left(\lambda_2 + (1-\lambda_2)\pi_2\right)x$. Alternatively, the government can buy γ amount of assets such that $\gamma = \Delta\left(\lambda_2 + (1-\lambda_2)\pi_2\right)x / \left(\frac{(1-\lambda_2)(1-x)}{(\lambda_2+(1-\lambda_2)\pi_2)x} + \Delta\right)$. This policy is effective if the expected payoff of assets sold in the market is above the liquidation value of the high quality asset: $\widehat{R}_2 > r_H$.

[32] See Appendix 7.5 for the proof.

It should be noted that there is a moral hazard problem associated with government interventions during crises. If market participants anticipate government interventions then the optimal holdings of risky assets are larger. Therefore, a larger intervention is required. The moral hazard problem can be corrected if the liquidity provision at date $t = 1$ is financed by a tax τ per unit of investment, which is imposed at date $t = 0$. The tax τx should be equal to the amount of liquidity Λ that is required to restore market price to the level of $\overline{p}_2$,

$$\tau x = (\lambda_2 + (1 - \lambda_2)\,\pi_2)\, x\overline{p}_2 - (1 - \lambda_2)\,(1 - x)\,. \tag{18}$$

Imposing such tax increases liquidity holdings at $t = 0$ and prevents market breakdowns at $t = 1$, leading to a higher expected utility.[33]

Numerical example To illustrate the effect of the government policy, consider the numerical example. The asset return parameters are $R_H = 1.2$, $r_H = 0.5, R_L = r_L = 0.3$, the fraction of low quality investments in the normal state is $\pi_1 = 0.05$, the probabilities of a liquidity shock in the normal state and in the crisis state, respectively, are $\lambda_1 = 0.2$ and $\lambda_2 = 0.3$, the probability of the crisis is $q = 0.1$. Figure 11a depicts the values of investment x, prices p_s and expected utility as a function of π_2, for the market equilibrium (types I and II), the equilibrium with government intervention (G), and the social planner solution (second-best).

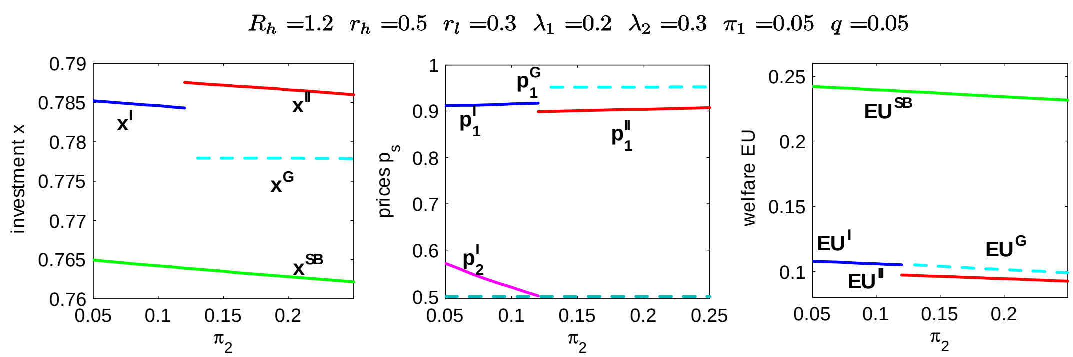

Figure 11a. Equilibrium values of investment,prices and welfare as a function of π_2.

Imposing a tax at date $t = 0$ to finance liquidity provision at date $t = 1$ leads to the larger investor's holdings of liquidity at $t = 0$: $(1 - x^G) > (1 - x^{II})$. As a result, the market prices

[33]See Appendix 7.5 for the detailed analysis.

are higher, and the market breakdown is avoided. Also, it leads to a higher expected utility: $EU^G > EU^{II}$.

Figure 11b depicts the aggregate holdings of liquidity ($L(s)$), the cost of foregone payoff when risky assets are sold before maturity ($C(s)$), and the return on assets bought on the secondary market (R_s/p_s).

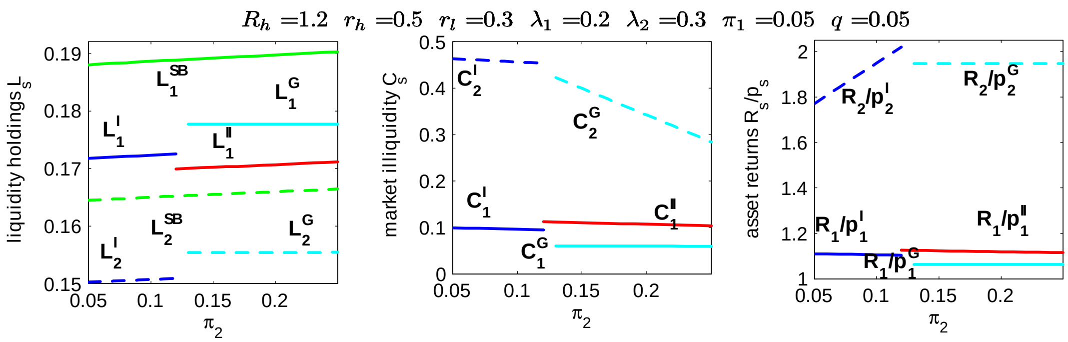

Figure 11b. Equilibrium values of market liquidity and asset returns as a function of π_2.

The liquidity available in the market at $t = 1$ for purchasing risky assets with tax-financed government interventions $\left(L_s^G\right)$ is larger than liquidity holdings in the market equilibrium $\left(L_s^{II}\right)$, but smaller than the second-best allocation $\left(L_s^{SB}\right)$. Also, the government intervention reduces the cost of selling assets before maturity: $C_s^G < C_s^{II}$, and decreases the return on assets bought at $t = 1$: $R_s/p_s^G < R_s/p_s^{II}$. So the tax-financed liquidity provision during crises also leads to a larger market liquidity in normal times. It reduces the adverse selection problem and improves welfare, although not as much as the social planner's solution.

Liquidity injection (recapitalization) Another type of intervention is injecting liquidity directly into financial institutions. In this case, the government can offer liquidity Λ_i to a financial institution i in exchange for a fraction of its future consumption $\Delta_i c_{tk}$ depending on the i's type. As demonstrated earlier, an increase in liquidity holdings leads to higher asset prices and higher expected utility. Therefore, financial institutions, especially those in need of liquidity or those with lemons, would be willing to participate in this exchange.[34]

[34] For some parameter values, non-liquidity investors with high quality assets may choose not to participate. Then the welfare impact of this policy is lower.

Asset purchases If the no-trade outcome is a result of the large fraction of lemons in the market or Knightian uncertainty about it then it is more effective for the government to purchase these assets. The liquidity injection is not useful since it does not affect the expected value of assets, and therefore leads to further liquidity hoarding. Removing such assets from the market reduces the adverse selection and uncertainty problems. In particular, the fraction δ of low quality assets needs to be removed from the market in order to restore trading $\left(\text{where } \delta = 1 - \frac{\lambda_2(R_H - r_H)}{\lambda_2 R_H + (1-\lambda_2) r_H - R_L} / \pi_2\right)$. Note that if the market breakdown is caused by a loss of confidence due to the ambiguity about the asset values then government interventions can restore market confidence without generating the moral hazard.

5 Model Implications and the Financial Crisis

5.1 Financial Crisis of 2007-2009

In the recent crisis of 2007-2009, financial institutions held a significant amount of asset backed securities (ABS). These securities had skewed payoffs: they had high expected return prior to the crisis but incurred substantial losses during the crisis. In particular, the haircut on ABS increased from 3-5% in August 2007 to 40-50% in August 2008 (Gorton and Metrick [30]). Furthermore, the demand for ABS collapsed from over $500 billion in 2007 to $20 billion in 2009 (see Figure 12 taken from Adrian, Ashcraft, and Pozsar (2010)).

Financial institutions were exposed to systemic risk through securities holdings which had skewed payoffs: they produced high returns in normal times but incurred substantial losses during the crisis. Before the crisis, many of these created securities were rated AAA, which implied a minimal risk of default. In particular, these assets were considered very liquid: if needed, these securities could be sold at a fair market price. During the crisis, the value of securities became more sensitive to private information. When in February 2007 subprime mortgage defaults increased, triggering the liquidity crisis, a large fraction of these securities were downgraded.[35] The impact of declining housing prices on securities

[35] For example, 27 of the 30 tranches of asset-backed CDOs underwritten by Merrill Lynch in 2007 were downgraded from AAA ratings to "junk" (Coval, Jurek and Stafford [20]).

depended on the exact composition of assets and mortgages that backed them. Due to the complexity of structured financial products and heterogeneity of the underlying asset pool, owners had an informational advantage in estimating how much those securities were worth.[36]

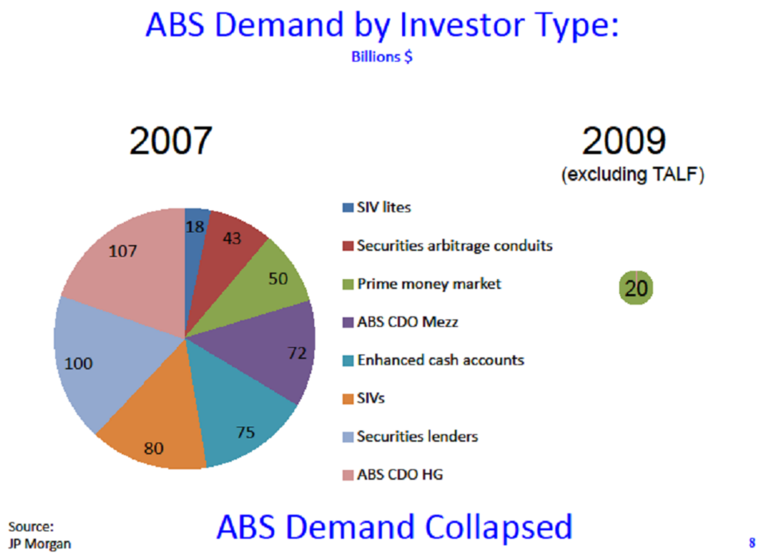

Figure 12. Demand for ABS in 2007 and 2009.

The asymmetric information about the assets' value leads to the lemons problem: a buyer does did not know whether the seller is selling the security because of a sudden need for liquidity, or because the seller is trying to unload the toxic assets. The adverse selection issue can cause market freezes reflecting buyers' beliefs that most securities offered for sale are of low quality.[37]

Also, as market condition worsened, investors' value for liquidity had increased. Finan-

[36] This problem was especially pronounced with the junior equity tranches (a.k.a. "toxic waste"). These tranches were hard to value since they were traded infrequently and were usually held by the issuing bank. Moreover, these securities received overly optimistic ratings from the credit rating agencies. (Brunnermeier [14])

[37] For example, Krishnamurthy [37] identifies adverse selection as one of the diagnoses of the recent crisis: market participants may fear that if they transact they will be left with a "lemon". Also, Drucker and Mayer [23] find that underwriters of prime MBS appeared to exploit access to better information when trading in the secondary market. Elul [27] also finds evidence of adverse selection in the prime mortgage market.

cial institutions were exposed to the market liquidity risk through the maturity mismatch of their balance sheets: they financed their long-term asset holdings with shorter maturity instruments. Diamond and Rajan [22] and Brunnermeier [14] identify maturity mismatch as an important factor contributing to the fragility of financial system. Because of the losses on their assets, some banks became undercapitalized; however, their attempts to recapitalize pushed the market price further down.[38]

Furthermore, market participants underestimated the systemic risk, in particular, they failed to see the correlation of risks induced by securitization (Coval, Jurek and Stafford [20]). Overly optimistic ratings from the credit rating agencies further contributed to mis-assessment of systemic risk ([14]). Increasing defaults on subprime mortgages and the lack of historical evidence caused an increase in market uncertainty about the impact of economic shocks on the value of financial securities. According to Gorton [29], the size and location of expected losses were not fully known because of the complexity and opaqueness of securitization. As the safest AAA subprime tranches experienced losses, investors started to question the valuation of models of all securitized products. This resulted in a dramatic increase in uncertainty and investors' panic.

5.2 Model Implications

The model captures the following important features of the financial crisis:

- adverse selection generated by asymmetric information about assets quality,
- increase in preference for liquidity which causes asset sales for exogenous reasons (unrelated to asset returns),
- considering a crisis as a low probability event,
- uncertainty about assets' value due to the unexpected shock.

The model demonstrates how adverse selection can lead to liquidity hoarding, increased asset price volatility, lower trading volume and possibly to complete breakdown of trade

[38] Brunnermeier and Pedersen [15] refer to this phenomena as a "loss spiral" and a "margin spiral". Indeed, Adrian and Shin [4] documented evidence of these phenomena for investments banks.

during a crisis. Investors are exposed to systemic risk through their holdings of risky assets. Although adverse selection is generated by idiosyncratic asymmetric information, the extent of adverse selection depends on the aggregate state. In normal times, when the fraction of lemons is small, adverse selection does not have a significant effect on the market. However, if the fraction of lemons is large or potential buyers believe it may be large, then adverse selection can lead to a market breakdown. Furthermore, even a small amount of adverse selection can be amplified to a full scale crisis with market freezes and liquidity hoarding[39] if it is accompanied by a flight-to-liquidity, a misassessment of systemic risk, or by uncertainty about asset values.

In my model, I show that the ability to trade based on private information is welfare improving if adverse selection does not lead to a market breakdown during the crisis. In normal times, it is welfare beneficial, but during the crisis it may lead to significant losses if market trading halts. Therefore, informed trading reduces the idiosyncratic risks of financial institutions but exacerbates the systemic risk.

This result is consistent with arguments in Holmstrom [33] and Stiglitz [42] that the problem in the recent crisis was not the lack of transparency as such but the sensitivity of securities to systemic risk. In particular, it suggests that the increase in transparency is not necessarily beneficial unless it reaches the level of full (symmetric) information. (Holmstrom [33]).

The results can be applied to *a cross-country analysis of financial crises*. The model prediction offers an explanation for the following observation: while capital flows into emerging countries are often speculative and volatile, capital flows into the US are mostly nonspeculative and driven by a search for safe assets (Caballero and Krishnamurthy [18]).[40] The countries with a history of rare financial crises tend to have less aggregate liquidity holdings relative to (illiquid) long-term investment. In these countries, if the crisis does occur then it is more severe and more likely to be accompanied by market freezes. On the other hand, countries that are more prone to financial crises have more aggregate liquidity holdings

[39] Indeed, the size of subprime market were small compared to the total ABS market. In 2007, subprime issuance about 30% of the total non-agency MBS issuance. (*BIS Quarterly Review, December 2007*)

[40] Furthermore, Acharya and Schnabl [2] show that global banking flows, not just global imbalances, determined the geography of the financial crisis.

which alleviates the crisis.[41] In the model (Figure 7), an economy where crises are accompanied by a market breakdown but do not occur often has a higher expected utility than an economy with more frequent crises but without market breakdowns. In the former case, government interventions can restore market trading and further increase welfare.

Also, the model can be applied to the *credit markets* by changing the initial assumption: investors borrow ω units of good at $t = 0$ (instead of receiving it as an endowment) and have to repay it at date $t = 1$ with probability λ (else they repay it at $t = 2$). Then in period one investors who have not received a liquidity shock are creditors, and liquidity investors are borrowers. The risky asset is used as a collateral in the credit market. In this setting, the cost $C(s) = \frac{\widehat{R}_s - p_s}{\widehat{R}_s}$ corresponds to the haircut on asset expected value $\widehat{R}_s$. This cost is larger in the crisis than in the normal state and it is increasing with amount of adverse selection in the market.

5.3 Policy Responses

In the market equilibrium, the investment allocation is not constrained efficient since financial markets are subject to the following two frictions: asymmetric information about investment quality and liquidity risk. Financial institutions do not take into account the effect of their investment choices on the market prices. As a result, they overinvest into risky illiquid assets (relative to the constraint efficient allocation), which creates systemic externalities.[42]

The inefficiency of a market equilibrium provides a rationale for government interventions to alleviate the crisis and ex-ante regulation targeted to prevent market freezes.[43] The appropriate policy response during the crisis depends on which amplification mechanism(s)

[41] This is consistent with empirical evidence that bank liquidity is countercyclical (Acharya, Shin, and Yorulmazer [3])

[42] For example, Caballero and Krishnamurthy [18] argue that the aggregate shortage of safe assets was one of the key factors contributing to the financial crisis.

[43] Indeed, during the crises, central banks in advanced economies intervened on an unprecedented scale. Some central banks used unconventional measures such as providing liquidity to banks on extraordinary terms and at longer maturities, and intervening in selected credit markets to support secondary market liquidity. As a result, central banks' balance sheets expanded significantly. For example, the balance sheet of the Federal Reserve exceeded 15% of GDP in 2009 relative to 6% of GDP in 2007 and 2008 (IMF, IFS).

cause the breakdown of trade. If it is due to a flight-to-liquidity or an underestimating of systemic risk, then liquidity provision through open market operations or direct injection of liquidity into financial institutions can restore asset trading. However, if the no-trade outcome is a result of a large fraction of lemons in the market or uncertainty about it, then the liquidity provision is not efficient and leads to further liquidity hoarding.[44] In this case, it is more effective to purchase the most illiquid assets. Removing such assets from the market reduces adverse selection and uncertainty problems.[45]

There is a moral hazard problem associated with government interventions during crises: if market participants anticipate a government intervention then the optimal holdings of risky assets are larger. For example, Kocherlakota [36] argues that during crises government bailouts are inevitable, and these bailouts (debt guarantees) lead to the inefficient allocation of capital towards risky investments. He proposes to use taxes to address the resulting risk externalities. Financing government liquidity injection by imposing an ex-ante tax on financial institutions corrects the moral hazard problem and increases market liquidity. Another preemptive policy response is an ex-ante requirement of larger liquidity (safe asset) holdings, which corrects the systemic externalities and prevents market breakdowns during crises.

6 Conclusion

I analyze the effect of adverse selection in the asset market. Asymmetric information about asset returns generates the lemons problem when buyers do not know whether the asset is sold because of its low quality or because the seller's sudden need for liquidity. Market trading based on asymmetric information allows financial institutions to reduce idiosyncratic

[44] As noted by Bernanke (2008), traditional liquidity provision was inadequate for addressing the strains in short-term funding markets. For example, despite massive liquidity injections by Federal Reserve, many over-the-counter markets continued to experience liquidity problems (BIS (2008)).

[45] This is consistent with arguments about the effectiveness of the Troubled Asset Relief Program (TARP). TARP was originally established to buy "troubled assets" from financial institutions in order to restore their financial solvency. However, as has been extensively noted, there were various implementation issues associated with it. Ultimately, the funds ($700 billion) were used for direct capital injections into financial institutions and for other purposes.

risks, but it exacerbates their exposure to systemic risk. In normal times, adverse selection does not significantly affect market liquidity. However, when the economy is in a crisis, adverse selection may lead to market freezes and liquidity hoarding.

Further, I examine the following amplification mechanisms: an increase in liquidity preferences, underestimating the likelihood of a crisis, and ambiguity about the fraction of low quality assets. Any of these phenomena can amplify the effect of adverse selection, leading to increased asset price volatility, fire-sale pricing and possibly to a breakdown of trade during crises. The government can mitigate adverse selection problems and increase aggregate welfare by requiring larger holdings of safe liquid assets. The choice and effectiveness of policy responses during a crisis depends on which amplification mechanisms cause market freezes.

References

[1] V. Acharya, D. Gale, and T. Yorulmazer. Rollover risk and market freezes. *NBER Working Paper* (2010).

[2] V. Acharya and P. Schnabl. Do Global Banks Spread Global Imbalances? The Case of Asset-Backed Commercial Paper during the Financial Crisis of 2007-09. *IMF Economic Review* (2010).

[3] V. Acharya, H. Shin, and T. Yorulmazer. Crisis Resolution and Bank Liquidity. *Review of Financial Studies* (2010).

[4] T. Adrian and H. Shin. Money, Liquidity, and Monetary Policy. *American Economic Review* **99**(2), 600–605 (2009).

[5] G. Akerlof. The market for" lemons": Quality uncertainty and the market mechanism. *The quarterly journal of economics* **84**(3), 488–500 (1970).

[6] F. Allen, A. Babus, and E. Carletti. Financial Crises: Theory and Evidence. (2009).

[7] F. Allen and E. Carletti. Mark-to-market accounting and liquidity pricing. *Journal of Accounting and Economics* **45**(2-3), 358–378 (2008).

[8] F. Allen and E. Carletti. The role of liquidity in financial crises. *Working Paper* (2008).

[9] F. Allen and D. Gale. Limited Market Participation and Volatility of Asset Prices. *The American Economic Review* **84**(4), 933–955 (1994).

[10] F. Allen and D. Gale. Optimal financial crises. *Journal of Finance* pp. 1245–1284 (1998).

[11] F. Allen and D. Gale. Financial intermediaries and markets. *Econometrica* pp. 1023–1061 (2004).

[12] S. Bhattacharya, D. Gale, U. of Pennsylvania. Center for Analytic Research in Economics, and the Social Sciences. Preference shocks, liquidity and central bank policy. (1986).

[13] P. Bolton, J. Santos, and J. Scheinkman. Outside and Inside Liquidity. *NBER Working paper* (2009).

[14] M. Brunnermeier. Deciphering the 2007-08 liquidity and credit crunch. *Journal of Economic Perspectives* (2008).

[15] M. Brunnermeier and L. Pedersen. Market Liquidity and Funding Liquidity. *NBER Working Paper* (2007).

[16] R. Caballero. Sudden Financial Arrest. *Working paper* (2009).

[17] R. Caballero and A. Krishnamurthy. Collective Risk Management in a Flight to Quality Episode. *Journal of Finance* **63**(5), 2195–2230 (2008).

[18] R. Caballero and A. Krishnamurthy. Global Imbalances and Financial Fragility. *The American economic review* **99**(2), 584–588 (2009).

[19] J. Chiu and T. Koeppl. Market Freeze and Recovery: Trading Dynamics under Optimal Intervention by a Market-Maker-of-Last-Resort. (2010).

[20] J. Coval, J. Jurek, and E. Stafford. The Economics of Structured Finance. *Journal of Economic Perspectives* **23**(1), 3–25 (2009).

[21] D. Diamond and P. Dybvig. Bank Runs, Deposit Insurance, and Liquidity. *The Journal of Political Economy* **91**(3), 401–419 (1983).

[22] D. Diamond and R. Rajan. The credit crisis: Conjectures about causes and remedies. *American Economic Review* **99**(2), 606–610 (2009).

[23] S. Drucker and C. Mayer. Inside Information and Market Making in Secondary Mortgage Markets. *Working paper* (2008).

[24] D. Easley and M. OHara. Liquidity and Valuation in an Uncertain World. *Working Paper, Cornell University* (2008).

[25] A. Eisfeldt. Endogenous liquidity in asset markets. *Journal of Finance* **59**(1), 1–30 (2004).

[26] A. Eisfeldt and A. Rampini. Capital reallocation and liquidity. *Journal of Monetary Economics* **53**(3), 369–399 (2006).

[27] R. Elul. Securitization and Mortgage Default: Reputation vs. Adverse Selection. *Working Papers* (2009).

[28] I. Gilboa and D. Schmeidler. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics* **18**(2), 141–153 (1989).

[29] G. Gorton. The panic of 2007. *NBER Working Paper* (2008).

[30] G. Gorton and A. Metrick. Securitized Banking and the Run on Repo. *NBER Working Paper* (2009).

[31] B. Greenwald and J. Stiglitz. Financial market imperfections and business cycles. *The Quarterly Journal of Economics* **108**(1), 77–114 (1993).

[32] F. Heider, M. Hoerova, and C. Holthausen. Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk. *ECB Working Paper* (2009).

[33] B. Holmstrom. Discussion of "The Panic of 2007" by Gary Gorton. In "Mantaining Stability in a Changing Financial System, Proceedings of the 2008 Jackson Hole Conference, Federal Reserve of Kansas City" (2008).

[34] C. Kahn and W. Wagner. Sources of Liquidity and Liquidity Shortages. (2010).

[35] N. Kiyotaki and J. Moore. Liquidity, business cycles, and monetary policy. (2008).

[36] N. Kocherlakota. Taxing risk and the optimal regulation of financial institutions. *Economic Policy Paper* (2010).

[37] A. Krishnamurthy. The financial meltdown: Data and diagnoses. *Northwestern working paper* (2008).

[38] A. Krishnamurthy. Amplification Mechanisms in Liquidity Crises. *NBER Working Paper* (2009).

[39] F. Malherbe. Self-fulfilling liquidity dry-ups. *Research series* (2010).

[40] S. Morris and H. Shin. Contagious Adverse Selection. *Working Paper* (2009).

[41] T. Philippon and V. Skreta. Optimal Interventions in Markets with Adverse Selection. *NBER Working Paper* (2010).

[42] J. Stiglitz. The Financial Crisis of 2007-2008 and its Macroeconomic Consequences. (2008).

[43] H. Uhlig. A model of a Systemic Bank Run. *NBER working paper* (2009).

7 Appendix

7.1 Assumptions:

parameters: $q, \pi_1, \pi_2, \lambda_1, \lambda_2, R_h, r_h, r_l$

$$
\begin{aligned}
(i) \quad &: \quad \sum_{k=h.l} (1-\pi_k)\left(1-\overline{\lambda}\right) R_k + \pi_k \overline{\lambda} r_k > 1 \\
(ii) \quad &: \quad r_h < \frac{\lambda_2 + \left(\pi_2 \frac{\overline{R}_2}{R_l + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}} + (1-\pi_2)\frac{\overline{R}_2}{R_H + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}}\right)}{\lambda_2 + \left(\pi_2 \frac{R_l}{R_l + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}} + (1-\pi_2)\frac{R_H}{R_H + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}}\right)} \\
(iii) \quad &: \quad \lambda_2 > \frac{1}{1 + \overline{R}_2 \frac{(1-\lambda_1)}{\lambda_1}\left[\frac{\lambda_1 + \left(\pi_1 \frac{R_l}{R_l + \overline{R}_1 \frac{\lambda_1}{1-\lambda_1}} + (1-\pi_1)\frac{R_H}{R_H + \overline{R}_1 \frac{\lambda_1}{1-\lambda_1}}\right)}{\lambda_1 + \left(\pi_1 \frac{\overline{R}_1}{R_l + \overline{R}_s \frac{\lambda_1}{1-\lambda_1}} + (1-\pi_1)\frac{\overline{R}_1}{R_H + \overline{R}_1 \frac{\lambda_1}{1-\lambda_1}}\right)}\right] + 1}
\end{aligned}
$$

(i) $\Rightarrow$ there is always positive holding of risky asset

(ii) $\Rightarrow$ there is always trade in the crisis state without adverse selection

(iii) $\Rightarrow$in the crisis state price is always determined by market clearing conditions, hence, $p_2 < \overline{R}_2$

7.2 Equilibrium without Adverse Selection

The investor's maximization problem is given by

$$
\max\left\{\sum_{s=1,2}\left[\lambda_s \log(1-x+p_s x) + (1-\lambda_s)\left(\pi_s \log\left(xR_l + (1-x)\overline{R}_s/p_s\right) + (1-\pi_s)\log\left(xR_h + (1-x)\overline{R}_s/p_s\right)\right)\right]\right\}
$$

Market clearing conditions imply

$$
p_s = \min\left\{\frac{(1-\lambda_s)}{\lambda_s}\frac{(1-x)}{x}; \overline{R}_s\right\}
$$

case 1: $p_s \leq \overline{R}_s$

$$
\begin{aligned}
x &= \sum_{s=1,2} q_s (1-\lambda_s)\left(\lambda_s + \pi_s \frac{R_l}{\left(R_l + \overline{R}_s \frac{\lambda}{1-\lambda}\right)} + (1-\pi_s)\frac{R_h}{\left(R_h + \overline{R}_s \frac{\lambda}{1-\lambda}\right)}\right) \\
p_s &= \frac{(1-\lambda_s)}{\lambda_s} \frac{\sum_{s=1,2} q_s \lambda_s \left(\lambda_s + \pi_s \frac{\overline{R}_s}{\left(R_l + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)} + (1-\pi_s)\frac{\overline{R}_s}{\left(R_h + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)}\right)}{\sum_{s=1,2} q_s (1-\lambda_s)\left(\lambda_s + \pi_s \frac{R_l}{\left(R_l + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)} + (1-\pi_s)\frac{R_h}{\left(R_h + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)}\right)}
\end{aligned}
$$

case 2: $p_1 = \overline{R}_1, p_2 \leq \overline{R}_2$

x^* is a solution to the following equation:

$$\sum_{s=1,2} q_s \left[\begin{array}{c} \lambda_1 \frac{\overline{R}_1 - 1}{\overline{R}_1 x + (1-x)} + (1-\lambda_1)\pi_1 \frac{R_l - 1}{xR_l + (1-x)} + (1-\lambda_1)(1-\pi_1)\frac{R_h - 1}{xR_h + (1-x)} + \\ (1-\lambda_2)\frac{1}{(1-x)x}\left(\left(\lambda_2 + \pi_2 \frac{R_l}{\left(R_l + \overline{R}_2 \frac{\lambda_2}{1-\lambda_2}\right)} + (1-\pi_s)\frac{R_h}{\left(R_h + \overline{R}_s \frac{\lambda_2}{1-\lambda_2}\right)}\right) - x\right) \end{array} \right] = 0$$

$$\begin{aligned} p_1 &= \overline{R}_1 \\ p_2 &= \frac{(1-\lambda_2)}{\lambda_2}\frac{(1-x^*)}{x^*} \end{aligned}$$

It can be verified that $x^* \leq \frac{1}{\frac{\lambda}{1-\lambda}\overline{R}_1 + 1}$ so that market clearing condition in state $s = 1$ is satisfied.

First-Best investment and consumption allocations:

$$investment \quad : \quad x^o = 1 - \sum_{s=1,2} q_s \lambda_s = \left(1 - \overline{\lambda}\right)$$

$$consumption \quad : \quad \begin{array}{l} c_1(s) = \frac{\overline{\lambda}}{\lambda_s} \\ c_2(s) = \frac{\left(1-\overline{\lambda}\right)}{(1-\lambda_s)}\overline{R}_s \end{array}$$

The market equilibrium investment allocation is less than the first-best: $x^* < x^o$,

$$\begin{aligned} x^* &\leq \sum_{s=1,2} q_s (1-\lambda_s)\left(\lambda_s + \pi_s \frac{R_l}{R_l + \frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s} + (1-\pi_s)\frac{R_h}{R_h + \frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s}\right) < \\ &< \sum_{s=1,2} q_s (1-\lambda_s) \end{aligned}$$

7.3 Equilibrium with Adverse Selection

7.3.1 Proof of Proposition 1

Proof. **Type I equilibrium**

Let us start with a type I equilibrium with market trading in both states. The investors' maximization problem is given by

$$\begin{aligned} \max_x \quad & \lambda_s \log(1 - x + p_s x) + (1-\lambda_s)\sum_{s=1,2} q_s \left(\pi_s \log\left(xp_s + (1-x)\widehat{R}_s/p_s\right) + (1-\pi_s)\log\left(xR_H + (1-x)\widehat{R}_s/p_s\right)\right) \\ s.t \quad & (i)\ \ 0 \leq x \leq 1 \\ & (ii)\ p_s > r_h \ \ \forall s \end{aligned}$$

Therefore, an investment allocation x and market prices p_s are determined by the following equations:

$$\begin{aligned} F(x) &\equiv \sum_{s=1,2} q_s F_s(x, p_s) = 0 \\ p_s &= \max\left(\frac{(1-\lambda_s)}{(\lambda_s + (1-\lambda_s)\pi_s)}\frac{(1-x)}{x}, \widehat{R}_s\right) \quad \forall s \end{aligned}$$

where

$$F_s(x,p_s) \equiv \lambda_s \frac{p_s-1}{1-x+p_s x} + (1-\lambda_s)\left(\pi \frac{p_s-\widehat{R}_s/p_s}{xp_s+(1-x)\widehat{R}_s/p_s} + (1-\pi)\frac{R_H-\widehat{R}_s/p_s}{xR_H+(1-x)\widehat{R}_s/p_s}\right)$$

If prices are determined by cash-in-the-market, then by substituting prices p_s, we get

$$F_s(x) = \left(\lambda_s \frac{1}{\left(\frac{1}{(1-\lambda_s)}+\pi_s\right)} + (1-\lambda_s)\pi_s \frac{(1-x)}{(1-x)+\widehat{R}_s\left(\frac{\lambda_s}{(1-\lambda_s)}+\pi_s\right)^2 x} + (1-\lambda_s)(1-\pi_s)\frac{R_H}{R_H+\widehat{R}_1\left(\frac{\lambda_s}{(1-\lambda_s)}+\pi_s\right)} - x\right)$$

This is a monotonically decreasing function of x. At $x=0$, F is greater than 0 and at $x=1$, F is less than zero. Therefore, by Intermediate Function Theorem, there exist a unique x^* such that at $F(x)=0$ The x^* can be derived as a root to a cubic equation:$a_1x^3+a_2x^2+a_3x+a_4=0$, where $a_1=-d_1d_2$, $a_2=d_1d_2d_3-((1-\lambda_1)q_1\pi_1+1)d_2-((1-\lambda_2)q_2\pi_2+1)d_1$, $a_3=(d_1+d_2)d_3-1+(1-\lambda_1)q_1\pi_1(d_2-1)+(1-\lambda_2)q_2\pi_2(d_1-1)$, and $a_4=d_3+(1-\lambda_1)q_1\pi_1+(1-\lambda_2)q_2\pi_2$, where

$$\begin{aligned}
d_1 &= \left(\widehat{R}_1\left(\frac{\lambda_1}{(1-\lambda_1)}+\pi_1\right)^2-1\right),\\
d_2 &= \left(\widehat{R}_2\left(\frac{\lambda_2}{(1-\lambda_2)}+\pi_2\right)^2-1\right),\\
d_3 &= \sum_{s=1,2} q_s\left(\lambda_s\frac{1}{\left(\frac{1}{(1-\lambda_s)}+\pi_s\right)}+(1-\lambda_s)\frac{(1-\pi_s)R_H}{R_H+\widehat{R}_s\left(\frac{\lambda_s}{(1-\lambda_s)}+\pi_s\right)}\right).
\end{aligned}$$

Denote the solution as x^*. Then the prices are given by

$$p_s^* = \max\left(\frac{(1-\lambda_s)}{(\lambda_s+(1-\lambda_s)\pi_s)}\frac{(1-x^*)}{x^*},\widehat{R}_s\right)$$

If $r_H < \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)}\frac{(1-x^*)}{x^*} \le \widehat{R}_s$ then (x^*,p_1^*,p_2^*) are equilibrium investment and prices. If $\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_1)}\frac{(1-x^*)}{x^*} > \widehat{R}_1$ then $p_1^*=\widehat{R}_1$, and (x^*, p_2^*) are determined by

$$\begin{aligned}
(i) &: \quad q_1F_1(x^*,\widehat{R}_1)+q_2F_2(x^*,p_2^*)=0\\
(ii) &: \quad p_2^*=\max\left(\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{(1-x^*)}{x^*},\widehat{R}_2\right)
\end{aligned}$$

If $\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_2)}\frac{(1-x^*)}{x^*} \le \widehat{R}_2$ then (x^*,p_1^*,p_2^*) is an equilibrium. It can verified that $x^* \le \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_1)\widehat{R}_1+(1-\lambda)}$ so that market clearing condition in state $s=1$ is satisfied. If $\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_2)}\frac{(1-x^*)}{x^*} > \widehat{R}_2$ then $p_2^*=\widehat{R}_2$ and by assumption 3, $p_1^*=\widehat{R}_1$. Hence, equilibrium investment x^* is a solution to $\sum_{s=1,2} q_sF_s(x^*,\widehat{R}_s)=0$.

If $p_2^* \le r_h$ then in the crisis state liquidity traders with high quality investment choose to liquidate their investment rather than selling it at $t=1$. Therefore, the expected return $\widehat{R}_2=R_l$, so there no demand for risky assets. Hence, (x^*,p_1^*,p_2^*) cannot be an equilibrium investment and prices if $p_2^* \le r_h$.

If λ_2 and π_2 are sufficiently large such that $p_2^* \le r_h$ then the type I does no longer exist. $F_s(x)$ is decreasing in λ_s and π_s. Also, $F_s(x)$ is decreasing in x. Hence, x is decreasing in λ_s and π_s. If p_2^* is determined by cash-in-the-market-pricing then the effect of an increase in λ_2 or π_2 on the price in state $s = 2$ is determined by

$$\begin{aligned}
\frac{\partial p_2}{\partial \lambda_2} &= -\frac{\frac{1}{(1-\lambda_2)^2}}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)^2}\frac{(1-x)}{x} - \frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)}\frac{1}{x^2}\frac{\partial x}{\partial \lambda_2} \\
\frac{\partial p_2}{\partial \pi_2} &= -\frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)^2}\frac{(1-x)}{x} - \frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)}\frac{1}{x^2}\frac{\partial x}{\partial \lambda_2}
\end{aligned}$$

Therefore, increase in λ_2 and/or $\pi_2$can lead to the decrease in p_2^*, potentially resulting in $p_2^* \leq r_H$. If $p_2^* = \widehat{R}_2$ then it is again decreasing in π_2 but increasing in λ_2, however, if λ_2 increases sufficiently then cash-in-the-market-pricing binds and p_2^* becomes decreasing function of λ_2.

The consumption allocation of early and late consumers in a type I equilibrium are given by

$$\begin{aligned}
c_1(s) &= (1-x^*)\frac{1+(1-\lambda_s)\pi_s}{\lambda_s+(1-\lambda_s)\pi_s} \\
c_{2L}(s) &= (1-x^*)\frac{(1-\lambda_s)}{(\lambda_s+(1-\lambda_s)\pi_s)} + x^*\left(\frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_s R_l\right) \\
c_{2H}(s) &= x^*\left(R_h + \frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_s R_l\right)
\end{aligned}$$

Type II equilibrium

Now consider a type II equilibrium with no trading in the crisis state. The investors maximization problem becomes

$$\max_x \left\{ \begin{array}{l} \lambda \log(1-x+p_1x) + (1-\lambda)(1-q)\left(\pi_1 \log\left(xp_1 + (1-x)\widehat{R}_1/p_1\right) + (1-\pi_s)\log\left(xR_H + (1-x)\widehat{R}_1/p_1\right)\right) + \\ +\lambda \log(1-x+r_kx) + (1-\lambda) + q\left(\pi_2 \log(xr + (1-x)) + (1-\pi_2)\log(xR_H + (1-x))\right) \end{array} \right\}$$

$$\begin{aligned}
s.t \quad & (i) \;\; 0 \leq x \leq 1 \\
& (ii) \;\; p_1 > r_h
\end{aligned}$$

Therefore, an investment allocation x and market prices p_s are determined by the following equations:

$$\begin{aligned}
G(x) &\equiv q_1 F_1(x, p_1) + q_2 G_2(x) = 0 \\
p_1 &= \max\left(\frac{(1-\lambda_1)}{(\lambda_1 + (1-\lambda_1)\pi_1)}\frac{(1-x)}{x}, \widehat{R}_1\right)
\end{aligned}$$

where

$$G_2(x) = \left(\pi_2 \frac{R_l - 1}{xR_l + (1-x)} + \lambda_2(1-\pi_2)\frac{r_H - 1}{xr_H + (1-x)} + (1-\lambda_2)(1-\pi_2)\frac{R_H - 1}{xR_H + (1-x)}\right)$$

If price p_1 is determined by cash-in-the-market, then $G(x)$ is a decreasing function in x, and it is positive at $x = 0$ and negative at $x = 1$. Therefore, a solution $x^{**} : G(x^{**}) = 0$ exists and it is unique.

If $\frac{(1-\lambda_1)}{(\lambda_1+(1-\lambda_1)\pi_1)}\frac{\left(1-x^{**}\right)}{x^{**}} \leq \widehat{R}_1$, then (x^{**}, p_1^{**}) is an equilibrium. If $\frac{(1-\lambda_1)}{(\lambda_1+(1-\lambda_1)\pi_1)}\frac{\left(1-x^{**}\right)}{x^{**}} > \widehat{R}_1$, then $p_1^{**} = \widehat{R}_1$ and $x^{**} : q_1F_1(x^{**}, \widehat{R}_1) + q_2G_2(x^{**}) = 0$. It can be shown that $x^{**} < \frac{1}{\left(\frac{\lambda_1}{(1-\lambda_1)}+\pi_1\right)+1} < \frac{1}{\left(r_h\left(\frac{\lambda_1}{(1-\lambda_1)}+\pi_1\right)+1\right)}$. Therefore, $p_1^{**} > r_h$. Hence, there is always market trading in the normal state. Also, it can be verified that $\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{\left(1-x^{**}\right)}{x^{**}} < r_h$, i.e., there is indeed no market trading during the crisis.

Define hypothetical price $p_2^{**}(\lambda_2, \pi_2) = \frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{\left(1-x^{**}(\lambda_2,\pi_2)\right)}{x^{**}(\lambda_2,\pi_2)}$ (this is an implied price in the crisis state, it is hypothetical since there no market trading). This hypothetical price p_2^{**} is decreasing in λ_2 and π_2. *Therefore, if λ_2 and π_2 are sufficiently small such that $p_2^{**} > r_h$ then the type II does not exist.*

The consumption allocation of early and late consumers in a type II equilibrium are given by

$$c_{1k}(s) = \begin{cases} (1-x^{**})\frac{1+(1-\lambda_s)\pi_s}{\lambda_s+(1-\lambda_s)\pi_s} & if \quad p_s > r_k \\ 1-x^{**}+r_kx^{**} & if \quad p_s \leq r_k \end{cases}$$
$$c_{2H}(s) = \begin{cases} x^{**}\left(R_h + \frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_sR_l\right) & if \quad p_s > r_k \\ x^{**}R_H + (1-x^{**}) & if \quad p_s \leq r_k \end{cases}$$
$$c_{2L}(s) = \begin{cases} (1-x^{**})\frac{(1-\lambda_s)}{(\lambda_s+(1-\lambda_s)\pi_s)} + x^{**}\left(\frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_sR_l\right) & if \quad p_s > r_k \\ x^{**}R_l + (1-x^{**}) & if \quad p_s \leq r_k \end{cases}$$

Multiple Equilibria

The equilibria of type I and II coexist for λ_2 *and* π_2 such that $p_2^{*}(\lambda_2, \pi_2) > r_h \geq p_2^{**}(\lambda_2, \pi_2)$. Consider a type I equilibrium investment allocation x^*. It can be shown that $G(x^*) \geq 0$, which implies $x^{**} > x^*$. Hence, $p_2^{**}(\lambda_2, \pi_2) < p_2^{*}(\lambda_2, \pi_2)$. Therefore, there is a possibility that $p_2^{*}(\lambda_2, \pi_2) > r_h \geq p_2^{**}(\lambda_2, \pi_2)$.

The expected utility is higher when there is a market trading in both states. Consider investment allocation in type II equilibrium x^{**}. Since $p_2 > r_h > R_l$ then consumption for all $k = L, H,$ and $t = 1, 2$: $c_{tk}^{I}(x^{**}) = c_{tk}^{II}(x^{**})$ for $s = 1$ and $c_{tk}^{I}(x^{**}) > c_{tk}^{II}(x^{**})$ for $s = 2$. Hence, $V^{II}(x^{**}) < V^{I}(x^{**}) \leq V^{I}(x^*)$. Therefore, type I equilibrium is ex-ante Pareto dominant. However, type I equilibrium is not ex-post Pareto dominant since investor with high quality asset have higher expected utility in the normal state in type II relative to type I equilibrium: $c_{2H}^{I}(s=1) < c_{2H}^{II}(s=2)$. ■

7.3.2 Proof of Proposition 2

Proof. Denote investment allocation in an equilibrium without adverse selection by x', and investment allocations in type and I and II equilibrium with adverse selection by x^* and x^{**},respectively. Similarly, denote expected utility in state s for an equilibrium without adverse selection by $V_s(x')$, and for type and I and II equilibrium with adverse selection by $V_s^{I}(x^*)$and $V_s^{II}(x^*)$, respectively.

It can be shown that for $\forall s=1,2: F_s(x', p_s(x'))>0$. Therefore, we have $x^{**}>x^*>x'$ since $G_2(x^*)>F_2(x^*)$, and functions F_s and G_2 are decreasing in x.

Consider the difference in expected welfare V^I-V in state s,

$$\begin{aligned} V_s^I(x') - V_s(x') &= \left(\lambda_s \log\left(\frac{c_1^I(s)}{c_1(s)}\right) + (1-\lambda)\pi_s \log\left(\frac{c_{2L}^I(s)}{c_{2L}(s)}\right) + (1-\lambda)(1-\pi_s)\log\left(\frac{c_{2H}^I(s)}{c_{2H}(s)}\right)\right) \geq \\ &\geq \log\left(\lambda_s \frac{c_1^I(s)}{c_1(s)} + (1-\lambda)\pi_s \frac{c_{2L}^I(s)}{c_{2L}(s)} + (1-\lambda)(1-\pi_s)\frac{c_{2H}^I(s)}{c_{2H}(s)}\right) \geq 0 \end{aligned}$$

Therefore, $V^I(x^*) \equiv \sum_{s=1,2} q_s V_s^I(x^*) \geq \sum_{s=1,2} q_s V_s^I(x') > \sum_{s=1,2} q_s V_s(x') \equiv V(x')$, i.e., ability to trade based on private information increases the expected utility if there is market trading in both states.

Next, consider the difference in expected welfare $V^{II}-V$ in state $s=2$. Given the investment allocation, all types of investors consume less in the crisis state in a type II (no-trade) equilibrium than in an equilibrium without adverse selection: $c_{t,k}^{II}(x)<c_{t,k}(x)$. Therefore, $V_2\left(x'\right) - V_2(x^{**}) \geq V_2(x^{**}) - V_2^{II}(x^{**}) > 0$. (Let $x'' = \arg\max_x V_2(x)$, then $x'' \leq x' < x^{**}$ since $\frac{\partial V_2(x)}{\partial x}|_{x=x'} < 0$. Hence, $V_2(x^{**}) < V_2\left(x'\right)$). Also, we have $V_1^{II}(x^{**}) - V_1(x^{**}) \geq V_1^{II}(x^{**}) - V_1(x\prime) > 0$.

Therefore, informed trading leads to the welfare gains in the normal state: $V_1^{II}(x^{**}) > V_1(x\prime)$ and welfare loss in the crisis state: $V_2^{II}(x^{**}) < V_2\left(x'\right)$. The ex-ante effect depends on the probability of the crisis state. Define, $\Delta V \equiv V^{II}(x^{**}) - V(x') = \sum_{s=1,2} q_s\left(V_s^{II}(x^{**}) - V_s(x')\right)$. For $q=0$, $V^{II}(x^{**}) > V(x\prime)$ and for $q=1$, $V^{II}(x^{**}) < V(x\prime)$. Therefore, $\exists \widetilde{q} \in (0,1): \forall q<\widetilde{q}$, $V^{II}(x^{**}) > V(x\prime)$ and $\forall q > \widetilde{q}$, $V^{II}(x^{**}) > V(x\prime)$ since ΔV is linear in q. ■

7.4 Comparative Statics

7.4.1 Proof of Corollary 1

Proof. First consider an equilibrium with trade in both states. The equilibrium investment allocation is determined from the following equation: $\sum_{s=1,2} q_s F_s(x, p_s) = 0$ ($F_s(x,p_s)$ is defined in the proof of Proposition 1). $F_s(x,p_s)$ is decreasing in π_s, therefore, $\sum_{s=1,2} q_s F_s(x,p_s)$ is decreasing in q. Also, $\sum_{s=1,2} q_s F_s(x,p_s)$ is decreasing in x. Hence, the solution x^* is decreasing in q. If the prices are determined by cash-in-the-market constraint, then the prices p_s^* are increasing in q. Also, the expected utility V^I decreases as q becomes larger.

Now consider an equilibrium with the market breakdown in the crisis state. If we compute $x\prime$ such that $G_2(x')=0$ and x'' such that $F_1(x'', p_1(x''))=0$ then $x''>x^{**}>x'$. The equilibrium x^{**} is determined by $G(x,p_1) = (1-q)F_1(x,p_1) + qG_2(x,p_2) = 0$. Since G is decreasing in x then the optimal x^{**} is decreasing in q. Therefore, p_1 is increasing in q since it negatively depends on x. Since $V_2^{II}(x^{**}) < V_1^{II}(x^{**})$, as q becomes larger the expected utility V^{II} decreases. The market breaks down when the price in the crisis state falls below the liquidation value r_H. Consider again the hypothetical price $p_2^{**} = \max\left(\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{(1-x^{**}(q))}{x^{**}(q)}, \widehat{R}_2\right)$ defined in the proof of Proposition 1 . The increase in q may increase p_2^{**} sufficiently to restore the trading.

Consider some q such that $p_2^{**} = r_H - \varepsilon$ with $\varepsilon > 0$., so there is no trading in state 2. Therefore, $F_2(x^{**},p_2) > G_2(x^{**}) = 0$. If q increases sufficiently so that x^{**} goes down by more than $\frac{\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)\varepsilon}{\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)(r+\varepsilon)\right)\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)r\right)}$ then the trading in the crisis state restores. ■

7.5 Government

7.5.1 Proof of Proposition 3

Proof. The social planner maximization problem can be written as following,

$$\max_x\{\sum_{s=1,2} q_s\left(\lambda_s \log c_1(s) + (1-\lambda_s)\left(\pi_s \log c_{2L}(s) + (1-\pi_s)\log c_{2H}(s)\right)\right)\}$$

$$\begin{aligned}
s.t.\quad & (i) && (\lambda + (1-\lambda)\pi_s)\,\Delta c_1(s) = (1-\lambda)(1-x) \\
& (ii) && (1-\lambda)\,\Delta c_2(s) = (\lambda + (1-\lambda)\pi_s)\,x\widehat{R} \\
& (iii) && c_1(s) = \Delta c_1(s) + (1-x) \\
& (iv) && c_{2L}(s) = \Delta c_1(s) + \Delta c_2(s) \\
& (v) && c_{2H}(s) = xR_H + \Delta c_2(s) \\
& (vi) && c_1 \geq xr_H + (1-x) \\
& (vii) && c_{2H} \geq xR_H + (1-x) \\
& (viii) && c_{2L} \geq c_1 \\
& (ix) && \Delta c_t(s) \geq 0
\end{aligned}$$

where $\Delta c_1(s)$ is a transfer of cash holdings to liquidity investors in exchange of their risky asset holdings at date $t = 1$ and $\Delta c_2(s)$ is a transfer of risky asset holdings in exchange for cash holding to non-liquidity traders. The social planner problem is set up so that it is comparable with a market equilibrium, i.e., the planner does not have any additional advantages over market. For example, a partial pooling equilibrium is explicitly ruled out. Note, even though it may be feasible for the planner to differentiate liquidity investors with bad and good assets by offering a contract with a lower price and a lower quantity or probability, it is not optimal because of a loss in welfare due to the premature liquidation of some high quality assets.

The maximization problem can be reduced to the following,

$$\max_{x} \quad \left\{ \sum_{s=1,2} q_s \left[\lambda \log(1-x)\left(\tfrac{1+(1-\lambda)\pi_s}{\lambda+(1-\lambda)\pi_s}\right) + (1-\lambda) \left(\begin{array}{c} \pi_s \log\left((1-x)\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)} + x\frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\widehat{R}_s\right) \\ +(1-\pi_s)\log x\left(R_h + \frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\widehat{R}_s\right) \end{array} \right) \right] \right\}$$

$$s.t. \quad \left(\max_s \left(\tfrac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\widehat{R}_s\right)+1\right)^{-1} \leq x \leq \left(\tfrac{(\lambda+(1-\lambda)\pi_1)}{(1-\lambda)} r_h + 1\right)^{-1}$$

The optimal investment x is a solution to the following equation $H(x) \equiv \sum_{s=1,2} q_s H_s(x) = 0$, where

$$H_s(x) \equiv \left(-\lambda \frac{1}{(1-x)} + (1-\lambda)\pi_s \frac{\left(\frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\right)^2 \widehat{R}_s - 1}{\left((1-x) + x\left(\frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\right)^2 \widehat{R}_s\right)} + (1-\lambda)(1-\pi_s)\frac{1}{x} \right)$$

This is a monotonically decreasing function of x. At $x=0$, H is greater than 0 and at $x=1$, H is less than zero. Therefore, by Intermediate Function Theorem, there exist a unique x^o such that at $H(x^o)=0$ The x^o can be derived as a root to a cubic equation.

Furthermore at $x = 1-\overline{\lambda}$, we have $H(x) < 0$ which implies that $x^o < 1-\overline{\lambda}$, i.e., the investment allocation in the incentive compatible equilibrium is smaller than the first-best investment allocation. Denote the optimal expected utility by V^o. To compare welfare achieved by social planner with a market equilibrium. If prices are determined by cash-in-the-market constraints then the expected utility in a market equilibrium with trade in both states is given by

$$V(x) = \left\{ \sum_{s=1,2} q_s \left[\lambda \log(1-x)\left(\frac{1+(1-\lambda)\pi_s}{\lambda+(1-\lambda)\pi_s}\right) + (1-\lambda) \left(\begin{array}{c} \pi_s \log\left((1-x)\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)} + x\frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\widehat{R}_s\right) \\ +(1-\pi_s)\log x\left(R_h + \frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)}\widehat{R}_s\right) \end{array} \right) \right] \right\}$$

Therefore, $V^I(x^*) \leq V(x^*) \leq V^o$ since $x^o = \arg\max V(x)$, i.e., the social planner always achieves a higher welfare level. Furthermore, comparing $F_s(x)$ and $H(x)$, it can be shown that $\sum_{s=1,2} q_s F_s(x^o) \geq 0$. It implies that $x^* \geq x^o$ where $\sum_{s=1,2} q_s F_s(x^*) = 0$, i.e., the investment allocation in a market equilibrium is larger than the social planner investment allocation.

Note, social planner achieves larger consumption allocation in both states $c_{tk}(x^o) > c^I_{tk}(x^{**})$ for all types of investors, except the ones with high quality asset: $c_{2H}(x^o) < c^I_{2H}(x^{**})$. Therefore, the social solution is not ex-post Pareto dominant. ■

Price effect From market clearing, we have $\frac{\partial p_s}{\partial x} < 0$. The effect of the market price on expected utility is given by

$$\frac{\partial EU}{\partial p_s} = \left(\lambda_s \frac{x}{1-x+p_s x} + (1-\lambda_s)\left(\pi_s \frac{x-(1-x)\widehat{R}_s/p_s^2}{xp_s + (1-x)\widehat{R}_s/p_s} - (1-\pi_s)\frac{\widehat{R}_s/p_s^2}{xR_H + (1-x)\widehat{R}_s/p_s} \right) \right)$$

At market equilibrium investment allocation, we have $\frac{\partial EU}{\partial p_s}|_{x=x^*} > 0$. Hence, $\frac{\partial EU}{\partial p_s}|_{x=x^*} \frac{\partial p_s}{\partial x}|_{x=x^*} < 0$, i.e., by decreasing investment allocation, we can increase expected utility.

7.5.2 Government Intervention

Suppose the economy is in no-trade equilibrium such that $\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)} \frac{(1-x)}{x} < r_h$. Then the government intervene by providing liquidity to the market in order to restore trading:

$$
\begin{aligned}
p_2^G \quad & = \frac{(1-\lambda_2)(1-x) \ +\Lambda}{(\lambda_2+(1-\lambda_2)\pi_2) \ x} = r_h \ +\varepsilon \\
& \Rightarrow \Lambda = (r_h+\varepsilon)(\lambda_2+(1-\lambda_2)\pi_2) \ x - (1-\lambda_2)(1-x)
\end{aligned}
$$

Total amount of liquidity intervention: $\Lambda = (r_h+\varepsilon)(\lambda_2+(1-\lambda_2)\pi_2)x - (1-\lambda_2)(1-x)$. Tax (per unit of investment) imposed on investors at date $t=0$, to finance liquidity provision at $t=0$:

$$
\tau = p_2^G(\lambda_2+(1-\lambda_2)\pi_2) - (1-\lambda_2)\frac{(1-x)}{x}
$$

Then investors maximization problem becomes:

$$
\max_x \quad \sum_{s=1,2} q_s \left[\lambda_s \log\left(1-x+p_s^G(1-\tau)x\right) + (1-\lambda_s)\left(\begin{array}{c} \pi_s \log\left(x(1-\tau)p_s^G + (1-x)\widehat{R}_s/p_s^G\right) \\ +(1-\pi_s)\log\left(x(1-\tau)R_H + (1-x)\widehat{R}_s/p_s^G\right)\end{array}\right)\right]
$$

$$
s.t \quad \tau = \begin{cases} p_2^G(\lambda_2+(1-\lambda_2)\pi_2) - (1-\lambda_2)\frac{(1-x)}{x} & if \quad \frac{(1-\lambda_2)}{p_2^G(\lambda_2+(1-\lambda_2)\pi_2)+(1-\lambda_2)} \le x \le 1 \\ 0 & if \quad x < \frac{(1-\lambda_2)}{p_2^G(\lambda_2+(1-\lambda_2)\pi_2)+(1-\lambda_2)} \end{cases}
$$

where $p_1^G = \max\left(\frac{(1-\lambda_1)(1-x)}{(\lambda_1+(1-\lambda_1)\pi_1)x}, \widehat{R}_1\right)$ and $p_2^G = r_h+\varepsilon$.