Adverse Selection, Liquidity, and Market Breakdown

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Abstract

This paper develops a model that illustrates how even a small amount of adverse selection in the asset market can lead to the market breakdown during the crisis. Asymmetric information about asset returns generates the "lemons" problem when buyers do not know whether an asset is sold because of its low quality or because the seller experienced a sudden need for liquidity. The adverse selection can lead to an equilibrium with no trade, reflecting the buyers' belief that most assets offered for sale are of low quality. However, the ability to trade based on private information maybe welfare improving if adverse selection does not cause the market breakdown. I analyze the role of market liquidity, uncertainty about the assets value, and beliefs about the probability of a crisis in amplifying the effect of adverse selection and leading to the increased asset price volatility, fire-sale pricing, and possibly to the breakdown of trade during the crisis. Furthermore, I discuss the policy implications and its efficiency depending on which amplification mechanism causes the no-trade outcome.

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1 Introduction

In the recent crisis of 2007-2009, the market for securities backed by subprime mortgages was the first to suffer a sudden dry up in liquidity. Among possible explanations for market freezes are increased uncertainty and information asymmetries about the value of assets. In particular, the difficulty in assessing the fundamental value of securities may lead to the adverse selection problems and loss of liquidity. Flight-to-liquidity that accompany the initial shock can further amplify the adverse selection problem into severe financial crisis.

In this paper, I develop a model that illustrates how even a small amount of adverse selection in the asset market can lead to the liquidity hoarding and market breakdown during the crisis. I analyze the role of market liquidity, beliefs of the crisis, and uncertainty about assets value, in amplifying the effect of adverse selection.

In my model, agents have the Diamond-Dybvig[1] type of preferences: they consume in period one or in period two, depending on whether they receive a liquidity shock in period one. In period zero, investors choose how much to invest into risky long-term assets which have idiosyncratic payoffs. In period one, liquidity shocks are realized, investment quality is privately observed, and subsequently, risky investments are traded in the market. The investors who have not experienced a liquidity shock are the buyers in the financial market, while the sellers are those who have low quality assets or have received a liquidity shock.

There are two sources of illiquidity in the market: shortage of liquid assets and adverse selection (characterized by a fraction of low quality assets in the market). On one hand, market liquidity, defined as the demand for risky investments in the interim period, depends on the amount of the safe asset held by the investors which is available to buy risky assets from liquidity traders. Similarly to the Allen and Gale [5] "cash-in-the-market" framework, the greater is the average holdings of the safe asset in investors' portfolios, the greater is the market ability to absorb liquidity trading without large price changes. However, if the preference for liquidity is high, the "cash-in-the-market" pricing[2] may lead to the market

[1] Diamond and Dybvig (1983)

[2] The equilibrium price of the risky asset is equal to the lesser of two amounts: the discounted value of future dividends and the amount of cash available from buyers divided by the number of shares sold. (Allen and Gale [7])

prices below fundamentals. In addition, adverse selection can also cause market illiquidity if assets sold in the market are likely to be of low quality (similarly to Eisfeldt [18]). Therefore, market liquidity can also be characterized by the cost (in terms of foregone payoff) of selling long-term asset before maturity.[3]

I begin by examining the portfolio choice when investors have private information about their investment payoff and it is public information which investors have received a liquidity shock. Then I analyze the situation when the identity of investors hit by a liquidity shock is private information. In the latter case, investors can take advantage of their private information by selling the low-payoff investments and keeping the ones with high payoffs. This generates the lemons problem: buyers do not know whether an asset is sold because of its low quality or because the seller experienced a sudden need for liquidity.[4]

When economy is in a normal state with a small fraction of low quality assets and relatively low preference for liquidity, adverse selection does not significantly affect market liquidity. If the market is liquid then informed investors can gain from trading on private information at the expense of liquidity traders.

The crisis state is characterized by a larger fraction of low quality assets in the market and by higher preference for liquidity[5] relative to normal times.[6] Therefore, during the crisis adverse selection further depress the asset prices exacerbating price volatility, and possibly leading to the market breakdown. There are two types of equilibria: (I) with market trading during the crisis when both high and low quality assets are sold in the market, and (II) with market breakdown during the crisis. The type I equilibrium is characterized by asset price volatility and trading volume volatility across states. This type prevails when crisis is relatively mild. The type II equilibrium occurs when investors with high quality choose

[3] This characterization of liquidity is similar to Eisfeldt [18], where liquidity is described as the cost of transferring the value of expected future payoffs from long-term assets into current income.

[4] This setting is different from models where investors have private information about aggregate (common) payoff and information can be revealed through trading.

[5] The higher preference for liquidity during the crisis can viewed as precautionary liquidity hoarding due to the tighter *funding liquidity*. (See Brunnermeier and Pedersen [11] for dividing the concept of liquidity into two categories: funding liquidity and market liquidity.)

[6] This result is consistent with the fact that liquidity crises tend to be associated with economic downturns. (*Eisfeldt (2004) and Eisfeldt and Rampini (2003)*)

not to participate which causes market breakdown and liquidity hoarding. This happens when the crisis is sufficiently severe. In between, there is a possibility of multiple equilibria when both types coexist. In this case, the equilibrium type depends on the investors' beliefs about quality of assets sold in the market.

The private information about asset quality may be welfare beneficial if adverse selection does not lead to the market breakdown. The ability to trade based on private information smoothens the ex-ante consumption and consequently may lead to an increase in welfare.

Furthermore, I show that even a small amount of adverse selection can lead to an equilibrium with no market trading during the crisis if it is accompanied by any of the following amplification mechanisms: increase in the liquidity preference during the crisis, beliefs about the likelihood of a crisis, or uncertainty about assets returns. On one hand, higher preference for liquidity alleviates the adverse selection since assets are more likely to be sold due to seller' liquidity needs than due to their low quality. On the other hand, higher liquidity preference implies lower demand for risky assets and therefore leads to lower prices. I show that if a crisis is accompanied by the flight to liquidity, the effect of adverse selection can be amplified, leading to the fire-sale pricing (when assets are priced significantly below their expected payoffs) and possibly to a complete breakdown of trade.

The increase in adverse selection or liquidity preferences during the crisis is more likely to lead to the market breakdown if the probability of a crisis is smaller. Furthermore, underestimating the likelihood of a crisis can also aggravate the effect of adverse selection, leading to increased asset price volatility or market breakdown during the crisis. If a crisis is considered to be a rare event, then investors hold more of illiquid risky asset. So when a crisis state is realized, there are not enough holdings of liquid (safe) asset to absorb asset sales.

A Knightian uncertainty (ambiguity) about the fraction of low quality assets in the market can also cause market illiquidity. In this case, the investors beliefs about the extent of adverse selection is crucial: if investors beliefs there may be too many low quality assets in the market, then market breaks down.

These amplification mechanisms lead to different policy implications. If the market breakdown is due to an increase in the liquidity preference or underestimating the probability of the crisis then injecting liquidity into the market can restore the trading. However, if the no-trade outcome is caused by a large fraction of lemons or by the Knightian uncertainty about it, then it is more effective to remove these low quality assets from the market. The requirement of larger liquidity holdings prevents the market breakdown during the crisis, especially the economy is in the multiple equilibria range.

I show that investment allocation is not efficient, the central planner (government) can reduce the adverse selection problem by increasing holdings of liquid asset. Since adverse selection leads to the larger supply of low quality assets, more market liquidity is needed to absorb these trades. The central planner allocation reduces consumption volatility by improving consumption of liquidity investors and investors with low quality assets. As a result, it achieves a higher welfare than any of the market equilibria. The welfare improvement is more significant relative to an equilibrium with the market breakdown.

This paper is organized as follows. In the next section I discuss the related literature. Section 3 describes the model environment, and Section 4 characterizes the equilibrium. Section 5 applies model to the recent financial crisis and discuses the policy implications. Section 6 concludes the paper. All results are proved in the Appendix.

2 Related Literature

As has been demonstrated in line of work started by Akerlof [2], asymmetric information between buyers and sellers can lead to a complete breakdown of trade. Morris and Shin [26] show that adverse selection may lead to the failure of trade in a coordination game among differently informed traders. Eisfeldt [18] shows that higher investment productivity leads to the increased liquidity in a model where long-term risky assets are illiquid due to the adverse selection. Bolton, Santos, and Scheinkman [9] analyze the efficiency of trading equilibria in the presence of asymmetric information about asset values. The delay in trading increases the adverse selection problem and may inefficiently accelerate asset liquidation. Heider, Hoerova, and Holthausen [22] study the interbank market in the presence of counterparty risk. They show that private information about the risk of banks' assets and heterogeneous liquidity needs can result in a market breakdown and liquidity hoarding.

Malherbe [25] analyzes how adverse selection may lead to self-fulfilling liquidity dry-ups. When agents expect the market to be illiquid, they self-insure through the ex-ante hoarding of non-productive but liquid assets, that reduces ex-post market participation and dries up market liquidity. In my model, the market breakdown is actually caused by the shortage of liquid assets during the crisis, which results in depressed asset prices and causes non-participation of investors with high quality assets.

The importance of Knightian uncertainty has been emphasized by Easley and O'Hara [17], Caballero [12], Caballero and Krishnamurthy [13], Krishnamurthy [24], and Uhlig [27].

Uhlig [27] develops a model of a systemic bank run. He considers two variants, uncertainty aversion and adverse selection, and illustrates that the former generates the following feature of financial crisis: a larger share of troubled financial institutions results in a steeper asset price discount. However, in my model it is possible that the adverse selection can lead to a larger price discount even if there is no uncertainty about assets value.

Caballero [12] argues that complexity and Knightian uncertainty are key multipliers that can greatly increase the impact of an initial shock. Krishnamurthy [24] examines two amplification mechanisms that operate during liquidity crises. The first mechanism involves asset prices and balance sheets: a negative shock to agents' balance sheets causes them to liquidate assets, lowering prices, further deteriorating balance sheets and amplifying the shock. The second mechanism involves investors' Knightian uncertainty: shocks to financial innovations increase agents' uncertainty about their investments, causing them to disengage from risk and seek liquid investments, which amplifies the crisis. Caballero and Simsek [14] developed a model of fire sales due to the endogenously increased complexity of financial network during crises. Easley and O'Hara [17] show that uncertainty about the true value of an asset can lead to a no-trade equilibrium when investors have incomplete preferences over portfolios.

Allen and Gale ([5], [6], [7], [8]) developed a liquidity-based approach to study financial crises. When supply and demand for liquidity are inelastic in the short run, a small degree of aggregate uncertainty can have a large effect on asset prices and lead to financial instability. Allen and Carletti [4], [3] analyze the role of liquidity in financial crises.

My paper contributes to the literature by combining aggregate uncertainty about liquidity risk with aggregate uncertainty and asymmetric information about asset returns. The cash-in-the-market framework developed by Allen and Gale is well suited for studying financial crisis accompanied by liquidity dry-ups. Introducing asymmetric information in this framework generates an additional component of illiquidity due to the adverse selection. I analyze the interaction between adverse selection and liquidity preferences in determining the market liquidity, asset prices and welfare. Furthermore, I explore the role of investors beliefs about assets value and about the likelihood of a crisis as an additional source of market breakdown.

3 Model

I consider a model with three dates indexed by $t = 0, 1, 2$. There is a continuum of ex-ante identical financial institutions (*investors*, for short) with an aggregate Lebesgue measure of unity. There is only one good in the economy that can be used for consumption and investment. All investors are endowed with one unit of good at date $t = 0$, and nothing at the later dates.

3.1 Preferences

Investors consume at date one or two, depending on whether they receive a liquidity shock at date one. The probability of receiving a liquidity shock in period one is denoted by λ. So λ is also a fraction of investors hit by a liquidity shock. Investors who receive a liquidity shock have to liquidate their risky long-term asset holdings and consume all their wealth in period one. So they are effectively early consumers who value consumption only at date $t = 1$. The rest are the late consumers who value the consumption only at date $t = 2$. I will refer to the early consumers as *liquidity traders*, and to the late consumers as *informed* investors.[7]

Investors have Diamond-Dybvig type of preferences:

$$U(c_1, c_2) = \lambda u(c_1) + (1 - \lambda)u(c_2) \tag{1}$$

[7] Note both types of investors receive private information about quality of their assets, however, liquidity traders cannot take advantage of this information. The structure of investment payoff and information are described in the next two subsections.

where c_t is the consumption at dates $t = 1, 2$. In each period, investors have logarithmic utility: $u(c_t) = \log c_t$.

3.2 Investment technology

Investors have access to two types of constant returns investment technologies. One is a storage technology (also called the *safe asset* or cash), which has zero net return: one unit of safe asset pays out one unit of safe asset in the next period. Another type of technology is a long-term risky investment project (also called a *risky asset*). The risky assets pay off $\widetilde{R} \in \{R_H, R_L\}$ per unit of investment at date two that represents an idiosyncratic (investment specific) productivity. The risky investment with payoff R_H is a high-quality asset while an investment with payoff R_L is a low-quality asset (*lemon*).

There are two states of nature $s = 1$ and $s = 2$ that are revealed at date $t = 1$. The state 1 is a *normal* state and the state 2 is a *crisis* state. These states are realized with ex-ante probabilities $(1 - q)$ and q. I will also use the notation $q_1 = 1 - q$ and $q_2 = q$. The states differ with respect to aggregate (market) productivity and probability of a liquidity shock. There are more high-quality investments and less investors are affected by liquidity shocks in the normal state $s = 1$ than in the crisis state $s = 2$.

The quality of assets are independent across investors. Each investor i has a choice of starting his own investment project i by investing a fraction of his endowment. The investor can start only one project, and each project has only one owner.[8] The idiosyncratic payoff of each investment i is an independent realization of a random variable $\widetilde{R}^i$ that takes two values: a low value R_L with probability π_s and a high value R_H with probability $(1 - \pi_s)$ where $s \in \{1, 2\}$. In the normal state, the fraction of low quality assets is small: $\pi_s > \pi_1$. In the crisis state, the fraction of low quality assets is larger: $\pi_s = \pi_2 > \pi_1$.

Alternative specification[9] is that the payoff of each investment i consists of two components: $\widetilde{R}^i(s) = \alpha_i(s)\Re^L + (1 - \alpha_i(s))\,\Re^H$. The fraction $\alpha_i(s)$ represents the invest-

[8] I assume that several agents cannot coinvest into one project in order to diversify away the idiosyncratic risk. This assumption can be justified by the benifits of securitization which reflect the limitations of ex-ante projects pooling.

[9] This specification is equivalent to the above but it makes the model more applicable to the MBS market.

ment's exposure to an asset with a low payoff $\Re^L$. The individual exposure $\alpha_i(s)$ is a random variable that takes two values: a high value α_h with probability π_s and a low value α_l with probability $(1-\pi_s)$ where $s \in \{1,2\}$. So that the market exposure is given by $\alpha_m(s) = \pi_s\alpha_h + (1-\pi_s)\,\alpha_l$ and the market (aggregate) payoff is $R^m(s) = \alpha_m(s)\Re^L + (1-\alpha_m(s))\,\Re^H$. As before, the state 1 is a normal state where the fraction of low quality assets is small: $\pi = \pi_1$. The state 2 is a crisis state with more low quality assets: $\pi = \pi_2 > \pi_1$, so that $R^m(1) > R^m(2)$. Denote the payoff of low-quality investment as R_L, i.e., $R_L = \alpha_h\Re^L + (1-\alpha_h)\,\Re^H$. Similarly, the high-quality investment payoff is denoted by R_H such that $R_H = \alpha_l\Re^L + (1-\alpha_l)\,\Re^H$.

The expected payoff of each individual risky project in state s is denoted by $\overline{R}_s = \pi_s R_L + (1-\pi_s)\,R_H$ with $R_L < 1 < R_H$. The expected payoff when an economy is in a normal state is higher than when it is in a crisis state: $\overline{R}_1 > \overline{R}_2$. The expected payoff before states are realized is denoted by $\overline{R} = (1-q)\,\overline{R}_1 + q\overline{R}_2$ with $\overline{R} > 1$.

The long-term asset can be liquidated prematurely at date $t = 1$, in this case, one unit of the risky asset R_k yields r_k units of the good, where $k = L, H$ and $0 \leq r_L \leq R_L < r_H < 1$.[10] The holdings of the two-period risky asset can be traded in financial market at date $t = 1$. Figure 1 summarizes the payoff structure.

time	0	1	2
safe asset	1	1	1
risky asset	1	r_k	R_k

Figure 1. Payoff structure.

3.3 Information

At date $t = 0$, investors make investment choices between the two technologies, safe and risky, in proportion x and $(1-x)$ respectively. They choose their asset holdings to maximize their expected utility.

At date $t = 1$, the liquidity shocks and the aggregate state are realized, and the financial market opens. Investors privately observe their asset payoffs. The supply of the risky assets comes from the investors who have experienced a liquidity shock. The demand for risky

[10] Appendix 7.1 describes additional assumptions imposed on parameters values.

assets comes from investors who have not received a liquidity shock. Any investor can liquidate his investment project at date one, receiving r_k units of the good per unit of investment.

The timeline of the model is summarized in the figure below.

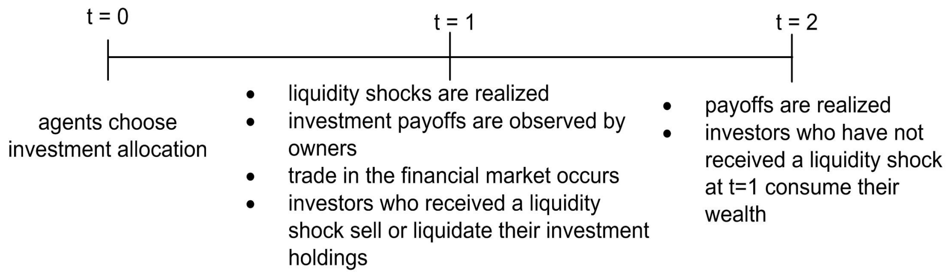

Figure 2. Timeline

Note the markets are incomplete since there are two frictions in this economy: liquidity shock and asymmetric information about asset quality, which generates four possible types of investors in each state. Investors are ex-ante identical but ex-post differ in terms of realization of liquidity shocks and quality of their investments. The holding of safe asset provides partial insurance against the possibility of liquidity shock and low quality assets.

I will consider two cases. In the first case, it is public information which investors have experienced a liquidity shock. If an investor gets a liquidity shock, he sells or liquidates his holdings of the risky asset in order to consume as much as possible in period one. In the second case, identity of investors hit by a liquidity shock is private information. Therefore, after observing investment payoffs, agents can take advantage of this private information by selling low quality projects in the market at date $t = 1$. In this case, buyers are not able to distinguish whether an investor is selling his asset holdings because of its low payoff or because of the liquidity needs. This generates adverse selection problem, and leads to a discount on the investments sold in the market at date $t = 1$.

4 Equilibrium

4.1 Equilibrium without Adverse Selection

First, I consider the case where identity of investors hit by a liquidity shock is public information. Therefore, there is no adverse selection. All risky assets at $t = 1$ are sold by liquidity traders who cannot wait for the maturity of their investments at date $t = 2$.

Since all the investments have idiosyncratic productivity, the expected payoff of the risky asset sold in period one is $\overline{R}_s$ in state s. All risky assets sold at $t = 1$ are aggregated in the market, hence, the variance of an asset bought at date $t = 1$ is zero. Therefore, the return on risky asset bought in period one is $\overline{R}_s/p_s$, where p_s is the market price in state s. The late consumers are willing to buy risky asset at date $t = 1$ if the market price p_s is less than or equal to the expected payoff $\overline{R}_s$. The earlier consumers are willing to sell their projects if the market price p_s is greater than the liquidation value r_k.[11]

At date $t = 0$, investors choose the investment allocations between the risky and safe technologies, in proportion x and $(1-x)$ respectively, in order to maximize their expected utility. The consumption of early consumers in state s is denoted by $c_{1k}(s)$ and the consumption of late consumers in state s is denoted by $c_{2k}(s)$ where $k = L, H$ refers to payoff of an investment project i.

$$\max_{c_{tk}(s)} \quad \sum_{s=1,2} q_s \begin{bmatrix} \lambda_s \log\left(\pi_s \log c_{1L}(s) + (1-\pi_s)\log c_{1H}(s)\right) + \\ + (1-\lambda_s)\left(\pi_s \log c_{2L}(s) + (1-\pi_s)\log c_{2H}(s)\right) \end{bmatrix} \tag{2}$$

$$s.t. \quad (i) \quad c_{1k}(s) = \begin{cases} 1 - x + p_s x & if \quad p_s > r_k \\ 1 - x + r_k x & if \quad p_s \leq r_k \end{cases}$$

$$(ii) \quad c_{2k}(s) = \begin{cases} xR_k + y_s \overline{R}_s & if \quad p_s > r_k \\ xR_k + (1-x) & if \quad p_s \leq r_k \end{cases}$$

The late consumers are willing to buy risky assets at $t = 1$ if the market price p_s is less than or equal to the expected payoff $\overline{R}_s$. Therefore, the demand for risky asset at $t = 1$ in

[11] For simplicity, I assume that if the asset price is equal to the liquidation value, investors choose to liquidate their assets rather than to sell.

state s is given by

$$y(s) = \begin{cases} \frac{1-x}{p_s} & if \quad p_s < \overline{R}_s \\ \left[0, \frac{1-x}{p_s}\right] & if \quad p_s = \overline{R}_s \\ 0 & if \quad p_s > \overline{R}_s \end{cases} \tag{3}$$

Therefore, the aggregate demand at $t = 1$ in state s is given by

$$D(s) = \begin{cases} (1-\lambda_s)\frac{1-x}{p_s} & if \quad p_s < \overline{R}_s \\ \left[0, (1-\lambda_s)\frac{1-x}{p_s}\right] & if \quad p_s = \overline{R}_s \\ 0 & if \quad p_s > \overline{R}_s \end{cases} \tag{4}$$

The early consumers are willing to sell their investments if the market price p_s is greater than the liquidation value r_k. Therefore, the aggregate supply at $t = 1$ in state s is given by

$$S(s) = \begin{cases} \lambda x & if \quad p_s > r_H \\ \lambda \pi_s x & if \quad r_L < p_s \leq r_H \\ 0 & if \quad p_s \leq r_L \end{cases} \tag{5}$$

Market clearing conditions are given by

$$\lambda_s x p_s = (1-\lambda_s)(1-x) \tag{6}$$

The price in state s is equal to the lesser of two amounts: the expected payoff and the amount of cash available from buyers divided by the amount of assets sold. :

$$p_s = \min\left\{\frac{(1-\lambda_s)(1-x)}{\lambda_s x}, \overline{R}_s\right\} \tag{7}$$

This cash-in-the-market pricing captures the effect of liquidity on asset pricing. When there is sufficient liquidity in the market, the price is equal to the asset's expected payoff. However, when liquidity is scarce, the price is determined by the holdings of safe asset (cash) available in the market.

The aggregate uncertainty about liquidity shock generates asset price volatility: the equilibrium market price is lower during the crisis than in normal times, $p_2 < p_1$.[12] The investment allocation x is smaller than the first-best investment allocation since the investment quality is not observable.[13]

[12] This result is similar to Allen and Gale [5].

[13] see Appendix 7.2

4.2 Equilibrium with Adverse Selection

Now suppose identity of investors who have received a liquidity shock is private information. Therefore, after observing investment payoff, agents can take advantage of this private information by selling low productive investments in the market at date $t = 1$. This generates the adverse selection problem and therefore, leads to the discount on the price of risky assets sold at $t = 1$. Adverse selection cause the risky assets sold at date $t = 1$ to be less liquid since the fraction of low quality assets in the market increases. Investors always can choose to liquidate their asset holdings if the market price is too low.

An investor who buys a risky asset at date $t = 1$, does not know whether it is sold due to a liquidity shock or because of its low payoff. The buyers believe that with probability λ_s investment is sold due to a liquidity shock, and with probability $(1-\lambda_s)(1-\pi_s)$ it sold because of its low quality. Hence, buyers believe that the payoff of risky assets sold in state s is $\widehat{R}_s$ such that

$$\widehat{R}_s = \frac{\lambda_s}{\lambda_s + (1-\lambda_s)\pi_s}\overline{R}_s + \frac{(1-\lambda_s)\pi_s}{\lambda_s + (1-\lambda_s)\pi_s}R_L \tag{8}$$

The late consumers are willing to buy risky asset at $t = 1$ if the market price p_s is less than or equal to the expected payoff $\widehat{R}_s$. Therefore, the demand for risky asset at $t = 1$ is given by

$$y(s) = \begin{cases} \frac{1-x}{p_s} & if \quad p_s < \widehat{R}_s \\ \left[0, \frac{1-x}{p_s}\right] & if \quad p_s = \widehat{R}_s \\ 0 & if \quad p_s > \widehat{R}_s \end{cases} \tag{9}$$

The earlier consumers are willing to sell their investment if the market price p_s is greater than the liquidation value r_k.

The market clearing conditions are given by

$$(\lambda_s + (1-\lambda_s)\pi_s)\, x p_s = (1-\lambda_s)(1-x) \tag{10}$$

Note, the supply of risky assets is larger because of the adverse selection.

Therefore, the market price in state s can be expressed as the lesser of the two: expected payoff $\widehat{R}_s$ and the amount of cash per unit of assets sold.

$$p_s = \min\left\{\frac{(1-\lambda_s)(1-x)}{(\lambda_s + (1-\lambda_s)\pi_s)\, x}, \widehat{R}_s\right\} \tag{11}$$

If the market price p_s is such that $r_L < p_s \leq r_H$ then all asset with high payoffs will be liquidated so that only lemons (assets with low payoffs) are traded in the market. Therefore, the expected payoff of a risky asset is r_L. In this case, there is no trading as no one would be willing to buy these low quality assets. If the fraction of low quality assets π_2 is sufficiently large so that the expected payoff is less than or equal to the liquidation value: $\widehat{R}_2 \leq r_H$, then there is no trading as well.

Proposition 1 *If the crisis is mild (λ_2 and π_2 are relatively small) then there is a unique type I equilibrium with market trading in both states and the market price in a crisis state p_2 being lower than the market price in a normal state p_1. If the crisis is severe (λ_2 and π_2 are sufficiently large) then there is a unique type II equilibrium with market trading in normal state $s = 1$ and no trade in a crisis state $s = 2$. For intermediate parameters range, there is a possibility of multiple equilibria when two types coexist. In case of multiple equilibria, the expected utility is higher and holdings of safe asset are larger in a type I than in a type II equilibrium.*

Type I is a pooling equilibrium where both high and low quality assets are sold. Type II is a separating equilibrium where in a crisis state investors choose to liquidate high quality assets rather than to sell them, which leads to a no-trade outcome.

Consider an equilibrium of type I with market trading in both states. Because of the adverse selection, assets offered for sale at $t = 1$ have lower expected return. Also, the supply of risky asset in period $t = 1$, in particular the supply of low quality assets, is larger. As a result, the market prices are lower relative to the equilibrium without adverse selection. Furthermore, adverse selection leads to the increased price volatility across states due to the larger share of lemons in the market during the crisis. Also, the payoff on risky asset bought at $t = 1$ is larger in the crisis relative to a normal state: $\widehat{R}_2/p_2 > \widehat{R}_1/p_1$. This reflects the fire-sale phenomena when the value of liquidity is high during the crisis.

Type II equilibrium prevails when the fraction of lemons in the market and/or preference for liquidity are sufficiently large such that the price of risky asset falls below liquidation value. Then investors with high quality assets chose not to participate in the market, and as a result, there is no trade since only low quality assets are available in the market.

In a crisis state, the price of risky asset is determined either by the expected payoff $\widehat{R}_2$ or by aggregate holding of safe asset per unit of risky asset sold in the market. If there too many lemons $\left(\pi_2 \geq \frac{\lambda_s(R_H - r_H)}{\lambda_s R_H + (1-\lambda_s) r_H - R_L}\right)$ so that the expected payoff $\widehat{R}_2$ is below the liquidation value r_H, then there is no trading during the crisis state. As a result, the value of liquid safe asset is lower in a type II equilibrium than in a type I.

The higher probability of receiving a liquidity shock implies less lemons are sold in the market since assets are more likely to be sold due to seller' liquidity needs than due to their low quality. This reduces the adverse selection problem and, therefore, increases the expected payoff on asset sold before maturity. However, higher liquidity preference also implies larger supply and smaller demand for risky assets.

If the market price is determined by "cash-in-the-market" pricing then higher preference for liquidity leads to lower prices. Therefore, the increase in preference for liquidity in a crisis state results in the further price decrease relative to a normal state. Hence, a lack of liquidity during the crisis may amplify the adverse selection problem pushing the asset prices further down, possibly to the extent of market breakdown. This is consistent with the asset fire-sales when depressed prices reflect the difficulty of finding buyers during the crisis.

For some parameters range two types of equilibria coexist. The equilibrium type is determined by investors' initial beliefs. If investors believe there is no trading during the crisis than they hold less of the safe asset. When crisis state is realized, there are not enough liquidity to absorb the informed trading, so market indeed breaks down. In this case, the market breakdown is caused by aggregate overinvestment into the risky long-term asset.

Furthermore, an equilibrium with market breakdown is (ex-ante[14]) inefficient since it achieves a lower expected utility relative to equilibrium with market trading during the crisis.

4.2.1 Market Liquidity

Market liquidity can be defined as an aggregate holding of safe asset available in the market:

$$L(s) = (1 - \lambda_s)(1 - x)$$

[14] Note, ex-post Pareto efficiency is violated for investors with high quality asset in a normal state.

Also, market liquidity can be characterized by the cost (in terms of foregone payoff) of selling long-term asset before maturity[15]. A lower cost implies higher liquidity.

$$C(s) = \frac{\widehat{R}_s - p_s}{\widehat{R}_s}$$

Therefore, there is a trade-off between asset payoff and liquidity: risky assets have larger expected payoff but there is a cost associate with premature liquidation or sale of the asset. This cost is increasing with amount of adverse selection in the market.

Even though the safe asset has lower expected return, it has additional value for ability to reallocate risky assets from liquidity traders to investors who didn't receive a liquidity shock. This value of liquidity is reflected in the payoff on risky asset bought in period one: $\widehat{R}_s/p_s > 1$. The payoff is larger in the crisis relative to a normal state: $\widehat{R}_2/p_2 > \widehat{R}_1/p_1$ reflecting the higher value of liquidity during the crisis. However, the scarcity of liquidity holdings in the market could lead to the market breakdown, in which case the role of safe asset reduces to the storage technology.

The market liquidity, measured both as aggregate holdings of safe asset and as the cost of premature liquidation of risky asset, is larger in a normal state than in a crisis state[16]. Also, in each state market liquidity is larger when there is no adverse selection.

4.3 Welfare

The informed trading is beneficial if does not cause market breakdown during the crisis.

Proposition 2 *The ability to trade based on private information increases expected utility if there is market trading and may decrease expected utility if there no-trade during the crisis and probability of a crisis is sufficiently large. The investors hold less of safe asset in an equilibrium with adverse selection than in an equilibrium without adverse selection.*

The market trading in the interim period allows investors with low quality assets benefit from the private information at the expense of liquidity traders. The ability to trade based on private information provides some ex-ante insurance against a low asset quality realization, especially in the crisis state. As a result, adverse selection makes risky investment

[15] This characterization of liquidity is similar to Eisfeldt [18], where liquidity is described as the cost of transferring the value of expected future payoffs from long-term assets into current income.

[16] This is consistent with emperical evedinces that market liquidity is procyclical.

ex-ante more attractive, which is reflected in the larger optimal investment allocation. Also, it leads to the consumption smoothening across different types of investors, and therefore, improves the welfare.

However, if there is no trade during the crisis, then investors are left with their low quality assets. So, the market breakdown prevents risk sharing. Moreover, some of the high quality asses are liquidated before maturity. This increases consumption volatility and leads to a lower welfare relative to an equilibrium where informed trading is not possible.

Furthermore, larger investment allocation implies smaller holdings of safe asset in an equilibrium with adverse selection. As a result, the market is less liquid and the cost selling risky asset before maturity is higher than in the absence of adverse selection.

In the setting where trading based on private information is not possible, the market provides insurance only against liquidity risk. So, there are possible welfare gains from allowing investors to benefit from private information on their asset quality.

4.4 Example

Adverse Selection To illustrate the impact of adverse selection, consider the following numerical example. The asset return parameters are given $R_H = 1.2, r_H = 0.5, r_L = 0.3$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$, probability of a liquidity shock in a normal state and a crisis state, respectively: $\lambda_1 = 0.2$ and $\lambda_2 = 0.3$, the probability of a crisis: $q = 0.1$. Figure 3a depicts the equilibrium values of investment, prices and expected utility as a function of low quality assets in the crisis. The solid lines depict values of equilibria with adverse selection, and dashed lines represent values of equilibria without adverse selection.

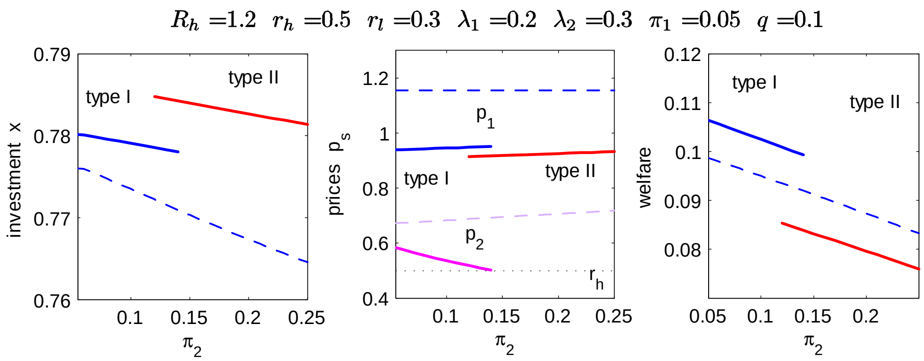

Figure 3a. Equilibrium values of investment,prices and welfare as a function of π_2.

As the fraction of low quality assets increases, the economy moves from an equilibrium with trading to an equilibrium with no-trade in the crisis state. If the fraction of lemons is relatively small (*less than 12%*) then there is a unique equilibrium with market trading in both states. If the fraction of lemons is sufficiently large (*more than 14.2%*) then there is a unique equilibrium with no-trade during the crisis. In between, the two types of equilibria coexist.

The holdings of risky asset is larger in type II equilibrium since the value of safe asset is lower. The asset price volatility increases with adverse selection. The private information about asset quality results in an increase in welfare if there is market trading. However, if adverse selection causes the market breakdown, there is a welfare loss.

Figure 3b depicts market liquidity as an aggregate holdings of safe asset, the cost of foregone payoff when asset is sold before maturity, and the return on asset bought on the secondary market. The liquidity available in the market at $t = 1$ for purchasing risky assets is larger in a normal state than in a crisis state. Also, in each state market liquidity is larger when there is no adverse selection. The cost of selling asset before maturity is higher in the crisis state implying the lower market liquidity. The asset return is higher during the crisis reflecting the lack of liquidity in the market.

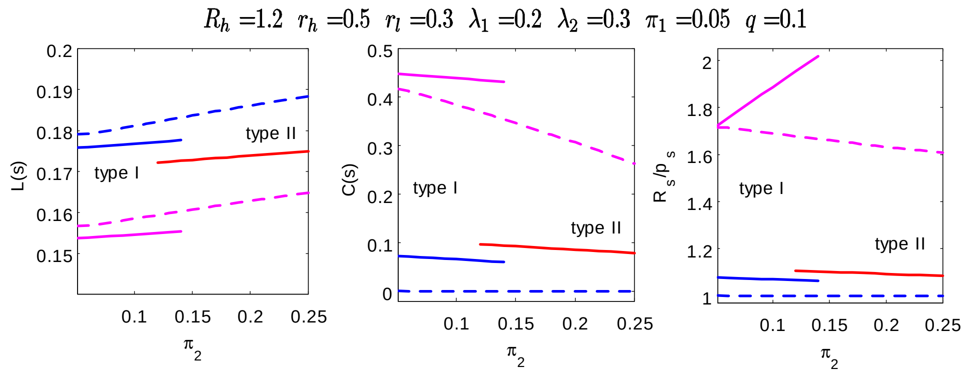

Figure 3b. Equilibrium values of market liquidity and asset returns as a function of π_2.

The adverse selection leads to a low market liquidity, low trading volume, and high return on assets bought during the crisis. If an economy is in a no-trade equilibrium, especially in a multiple equilibria range, then providing a liquidity to the market may restore trading.

Liquidity preference Now consider the effect of higher preference for liquidity during the crisis. The increase in liquidity preference λ_2 in a crisis state may lead to shift to a type II equilibrium with market trading in a normal state and no trade in a crisis state.

As before, asset returns are given by $R_H = 1.2$, $r_H = 0.5, R_L = r_l = 0.3$, the fraction of low quality investments in a normal state and in a crisis state, respectively: $\pi_1 = 0.05$ and $\pi_2 = 0.25$, the probability of a crisis: $q = 0.1$. The probability of a liquidity shock in a normal state is $\lambda_1 = 0.2$. The figure below illustrates the effect of an increase in the liquidity preference in a crisis state λ_2 from 0.2 to 0.4 on the equilibrium values. For $\lambda_2 \leq 0.3$, there is market trading in both states; for $\lambda_2 > 0.32$ there is no trade during the crisis, otherwise there are multiple equilibria.

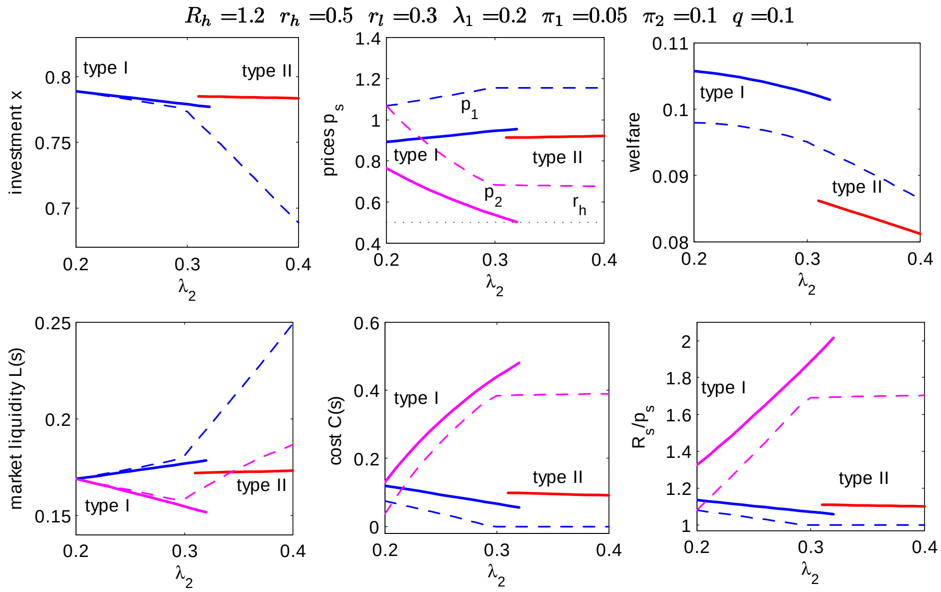

Figure 5. Equilibrium values as a function of λ_2.

The flight to liquidity during the crisis magnifies the effect of adverse selection on asset prices and market liquidity. The higher preference for liquidity ex-ante results in a lower market prices and a higher cost of selling investment before maturity. As a result, the payoff of assets bought in the market during the crises is higher, which is consistent the fire-sale pricing.

If a crisis is accompanied by the flight to liquidity, the effect of adverse selection can be amplified, leading to the fire-sale pricing (assets are priced significantly below their expected payoffs) and possibly to a complete breakdown of trade.

Next figure illustrates how equilibrium types depends on the interaction between liquidity preference (λ_2) and fraction of low quality assets (π_2). Figure 8 depicts the possible equilibria regions for different values of π_2 and λ_2. Each point in the (π_2, λ_2) plane corresponds to a particular type of equilibria: type I or type II, except for the region with multiple equilibria when type I and II occur simultaneously.

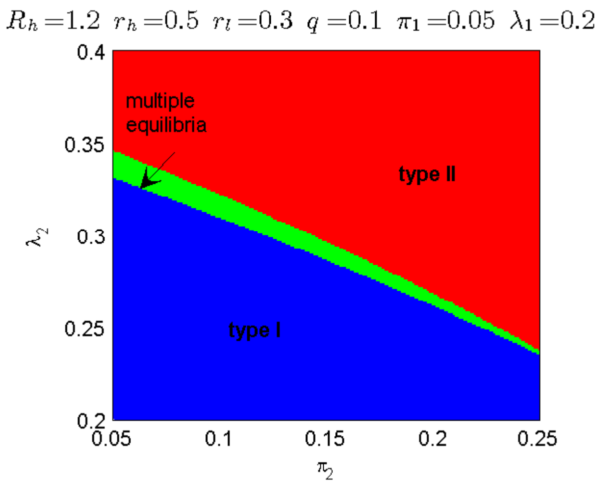

Figure 6. Equilibrium types for different values of π_2 and λ_2.

As can be seen from the figure, even small amount of adverse selection (small π_2) can lead to the no-trade outcome if the preference for liquidity is sufficiently high (large λ_2).

If the market breakdown is caused by high liquidity preference, (i.e., the expected payoff $\widehat{R}$ is higher than the liquidation value r_H) then the trading can potentially be restored by liquidity (safe asset) provision to the market.

4.5 Properties of Equilibrium

4.5.1 Probability of a crisis state

The probability of a crisis state q reflects the investors' beliefs about the likelihood of a crisis. In this section, I examine how changes in q affect the equilibrium values.

Corollary 1. *If investors believe a crisis state is more likely to occur (q is larger) then (i) liquidity holdings are larger; (ii) market prices are higher; (iii) expected utility is lower. If the economy is in a type II equilibrium with market trading in a normal state and no trade in a crisis state then an increase in q may lead to a type I equilibrium with trading in both states.*

The higher probability of a crisis state q implies that an asset is more likely to become a lemon, which makes it ex-ante less profitable. Therefore, an increase in q leads to a lower level of investment. The smaller investment at date $t = 0$ implies less supply and more demand for risky assets at date $t = 1$. As a result, market prices are higher, both in a type I and a type II equilibria.

The fact that the market price is increasing in the probability of a crisis makes it is possible to move from one equilibrium type to another. Suppose an economy is in a type II equilibrium with no market in a crisis state, and the probability of a crisis q increases. Then it is possible that the price in a crisis state will increase sufficiently to switch to a type I equilibrium with market trading in both states. (If an economy is initially in a type I equilibrium then the equilibrium type does not change if q is increased. If an economy is in a type II equilibrium and the probability q is decreased then the equilibrium type does not change either.)

Consider again the numerical example. The asset return parameters are given $R_H = 1.2$, $r_H = 0.5, r_L = 0.3$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$ and in a crisis state: $\pi_2 = 0.15$, and probability of a liquidity shock in a normal state: $\lambda_1 = 0.2$, and in a crisis state: $\lambda_2 = 0.3$. Figure 7 depicts the equilibrium values of investment, prices, welfare, and market liquidity as a function of probability of a crisis state q.

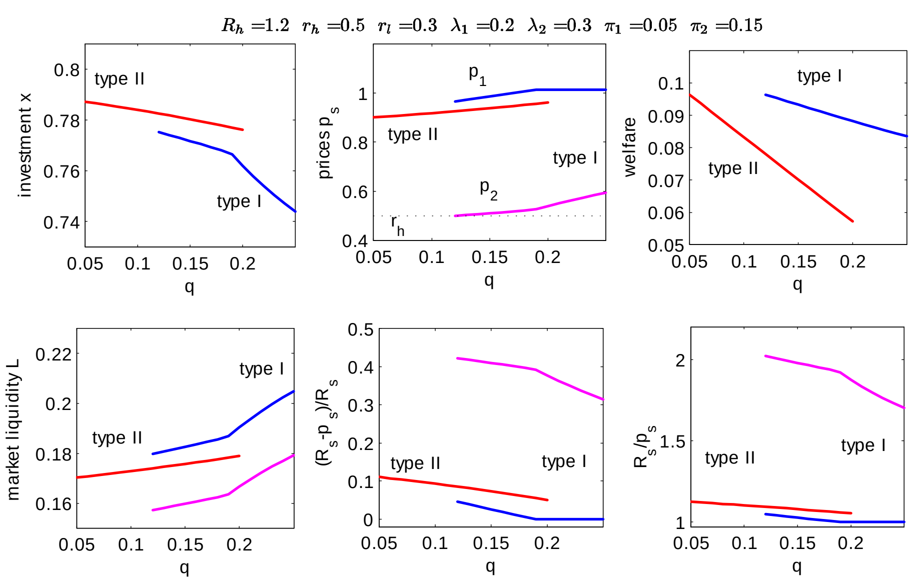

Figure 7. Equilibrium values as a function of q.

As the probability of a crisis increases, the economy moves from a unique equilibrium with market trading to multiple equilibria (for $q > 11..8\%$), and then to a unique equilibrium with no-trade in the crisis state (for $q > 20.6\%$). So, if a crisis is considered to be a rare event (probability is small) then there is no market trading during the crisis.

Let us compare equilibria sequentially. Suppose the probability of a crisis q depends on the previously realized state. So that conditional probability of transition from a normal state to a crisis state is smaller than the conditional probability of remaining in a crisis state. The transition matrix is given by $\begin{bmatrix} 1-q_{12} & q_{12} \\ 1-q_{22} & q_{22} \end{bmatrix}$ where $q_{22} > q_{12}$ and $q_{jk} = \Pr(s = s_k | s = s_j), k, j \in \{1, 2\}$, so it is more likely that an economy continues to stay in a crisis state if it is realized. Let us look again at the numerical example. Suppose $q_{11} = 0.05$ and $q_{22} = 0.25$. If an economy is in a normal state then it is in a type II equilibrium with no trading during the crisis. Once an economy is in a crisis state, beliefs are revised and investment allocations are adjusted, and an economy moves to a type I equilibrium. So, the market trading is resumed next period even if the crisis persists.

Next I examine how equilibrium types depends on the interaction between liquidity preference (λ_2), probability of a crisis (q), and fraction of lemons (π_2). Figure 8 illustrates the possible equilibria regions for different values of q and λ_2. Again, I consider two examples with the same values of $R_H = 1.2$, $r_H = 0.5, r_L = 0.3$ and different values of π_2: $\pi_2 = 0.15$ and $\pi_2 = 0.25$.

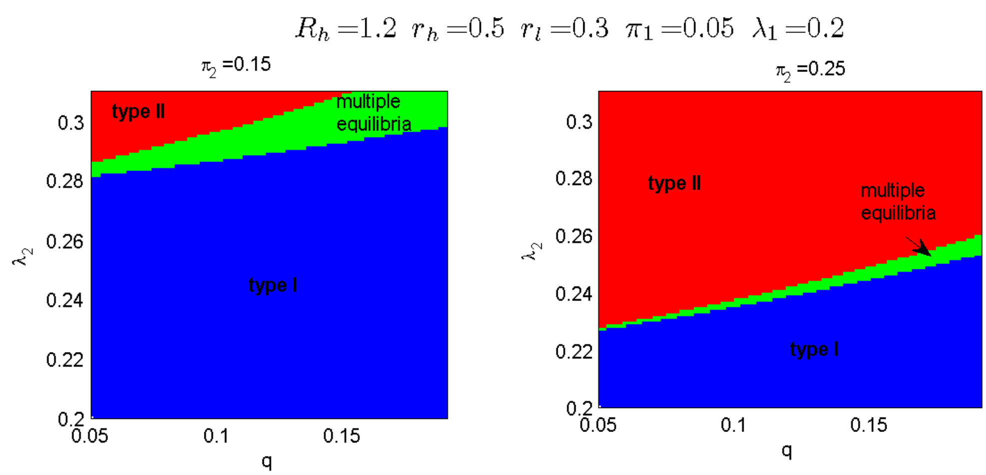

Figure 8. Equilibrium types for different values of q and λ_2.

As can be seen from the figure, even small amount of adverse selection can lead to the market breakdown if a crisis is considered to be a rare event (small q) and preference for liquidity is high (large λ_2). If crisis is likely to occur then there is trading even if there are many low quality assets in the market. The threshold value of the crisis probability when economy switches from trade to no-trade equilibrium is increasing in λ_2. So, if a crisis is accompanied by significant flight to liquidity, then no trade outcome can be more persistent.

4.5.2 Role of beliefs about the crisis

In this section, I analyze the role of beliefs about the likelihood of a crisis. Suppose the (true) probability of a crisis is q_o, however, investors believe that the probability is q which can be less or greater than q_o. Let us look again at the numerical example considered before. Suppose the probability of a crisis is $q_o = 0.1$. Figures 9 depicts the equilibrium values of investment and expected utility as a function of beliefs about probability of a crisis state $q \in (0, 0.2)$. If a crisis is considered to be unlikely ($q < 3.2\%$), then there is market breakdown during the crisis. If probability of the crisis is above 6.6%, then economy is in a unique equilibrium with market trading in both states. In between $q \in [0.032, 0.066]$, there are multiple equilibria of both types.

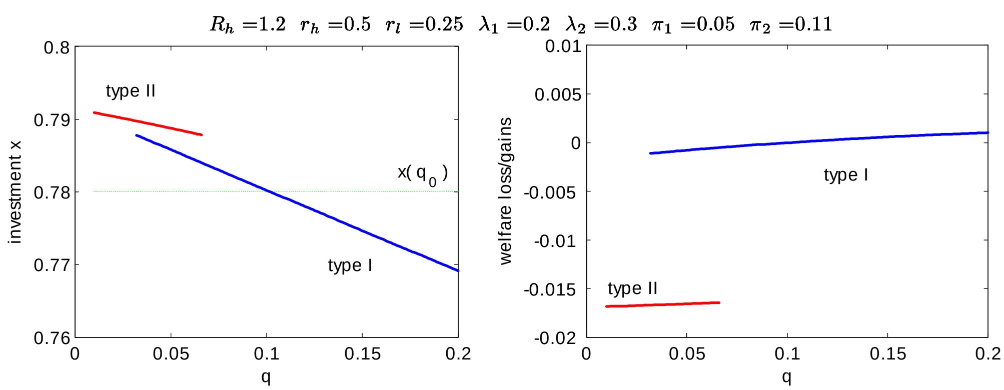

Figure 9. Equilibrium values of investment,prices and expected utility as a function of beliefs q.

Underestimating the probability of the crisis may result in a no-trade outcome. Overestimating the probability of the crisis may actually be welfare beneficial since competitive equilibrium is not efficient.[17] Investors overinvest into risky asset at date $t = 0$ relative to the second-best investment allocation. Therefore, the pessimistic beliefs about likelihood of the crisis leads to the larger holdings of safe asset, which increases market liquidity and improves the welfare.

Therefore, expectations of the crisis can affect the equilibrium type. Underestimating the probability of a crisis is more costly in term of welfare than overestimating as it may result in the market breakdown during the crisis.

4.6 Equilibrium with Adverse Selection and Knightian Uncertainty

Now consider the case when a crisis state is accompanied by an unanticipated shock in period one. The shock can be viewed as an "unforeseen contingency", an event that investors are not aware about so they do not plan for it.[18] As a result of this shock, investors face Knightian uncertainty (ambiguity) about the fraction of low quality assets in a crisis state, i.e., $\widehat{\pi}_2 \in [\underline{\pi}_2, \overline{\pi}_2]$ where $\pi_1 \leq \underline{\pi} < \overline{\pi}$. Investors do not know the actual probability of an asset being a lemon, instead they believe the probability $\widehat{\pi}_2$ belongs to the set: $[\underline{\pi}, \overline{\pi}]$. Investors are assumed to have Gilboa-Schmeidler maxmin utility: $U(c) = \min_{\widehat{\pi}_2} E_{\widehat{\pi}_2}[\log(c)]$. This assumption does not change the investment decision made at date $t = 0$ since there are no ambiguity at date $t = 0$. The investment allocation x depends on the initial beliefs π_2 (before the unanticipated shock is realized).

The investment decisions of liquidity traders are unaffected by this uncertainty about $\widehat{\pi}_2$. The late consumers make decision about buying assets at date $t = 1$ based on the worst among possible priors: $\overline{\pi}$. Therefore, investors are willing to buy risky asset at $t = 1$ during the crisis if the market price p_2 is less than the (worst) expected payoff $\widehat{R}(\overline{\pi}_2)$ which is given by

$$\widehat{R}(\overline{\pi}_2) = \frac{\lambda_2(1-\overline{\pi}_2)}{\lambda_2 + (1-\lambda_2)\overline{\pi}_2} R_H + \frac{\overline{\pi}_2}{\lambda_2 + (1-\lambda_2)\overline{\pi}_2} r_L \tag{12}$$

[17] See section 4.4. for the Central Planner solution.

[18] *Unforseen contingencies* are defined as "possibilities that the agent does not think about or recognize as possibilities at the time he makes a decisison" (Lipman, The New Plagrave Dictionary of Economics 2008). In modeling unanticipated uncertainty about the asset value, I am following Easley and O'Hara (2008) and Uhlig (2009).

Suppose $\widehat{\pi}_2$ is the actual (true) fraction of low quality assets such that $\pi_2 \leq \widehat{\pi}_2 < \overline{\pi}_2$. Therefore, the price p_2 is given by

$$p_2 = \frac{(1-\lambda_2)}{(\lambda_2 + (1-\lambda)\,\widehat{\pi}_2)} \frac{(1-x(\pi_2))}{x(\pi_2)} \tag{13}$$

Consider the case when $p_2 > r_H$. This implies that $p_1 > r_H$. So, if there are no ambiguity about $\widehat{\pi}_2$ then there is market trading in each state. However, in the presence of ambiguity about $\widehat{\pi}_2$ there is no trade equilibrium if $\overline{\pi}_2$ is sufficiently large so that $\widehat{R}_2(\overline{\pi}_2) \leq r_H$, i.e.,

$$\overline{\pi}_2 \geq \frac{\lambda_2\,(R_H - r_H)}{\lambda_2 R_H + (1-\lambda_2) r_H - r_L} \tag{14}$$

Again consider the numerical example: asset returns are given by $R_H = 1.3$, $r_H = 0.5, r_L = 0.3$, $\pi_1 = 0.03$, $q = 0.1$, and $\lambda = 0.2$. The figure below illustrates the effect of an increase in the fraction of lemons in a crisis state π_2 from 0.05 to 0.5 on the equilibrium values of investment and prices. If the fraction of lemons during a crisis exceeds 37% then the market breaks down.

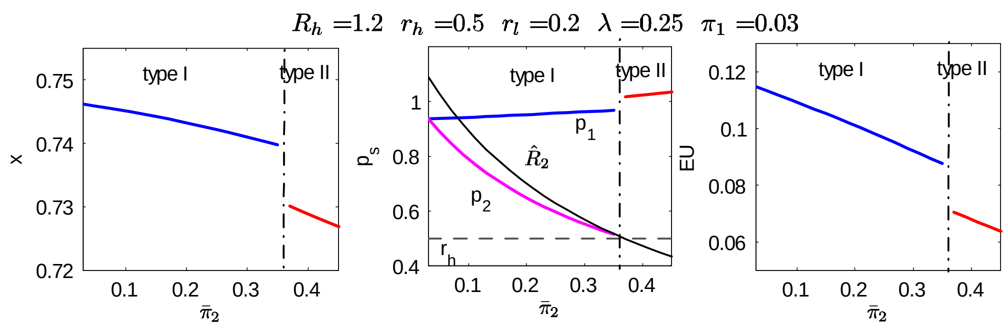

Figure 10. Equilibrium values of investment,prices and expected utility as a function of beliefs $\overline{\pi}_2$.

Therefore, the uncertainty about fraction of low quality assets can amplify the affect of adverse selection and result in a breakdown of trade. If the market breakdown of trade is caused by large fraction of low quality assets then liquidity provision is not helpful since it does not affect the expected payoff, and therefore, results in further hoarding of liquidity. In this case, it is more effective to liquidate some of low quality assets. This reduces adverse selection, and therefore can restore the market trading.

4.7 Government

In this section, I analyze this model from the central planner perspective, and compare it with the market equilibria.

First-best allocation Under full information (when it is known who receives a liquidity shock and the quality of asset is observable) the optimal investment allocation is $x = \left(1 - \sum_{s=1,2} q_s \lambda_s\right)$, consumption allocation of liquidity investors $c_1(s) = \frac{1}{\lambda_s} \sum_{s=1,2} q_s \lambda_s$, and late consumers receive $c_2(s) = \overline{R}_s \left(1 - \sum_{s=1,2} q_s \lambda_s\right) / (1 - \lambda_s)$.

Second-best allocation With asymmetric information about the quality of assets and identity of liquidity traders, the first-best allocation is not incentive compatible because investors with low quality assets have an incentive to pretend to be liquidity traders to get one unit of good per unit of low quality asset instead of liquidating it for r_L units of good since $r_L < 1$.

Therefore, the incentive-compatible maximization problem becomes:

$$\max_x \sum_{s=1,2} \sum_{k=L,H} q_s \left(\lambda \pi_{sk} \log c_{1k}(s) + (1-\lambda) \pi_{sk} \log c_{2k}(s)\right)^{19} \tag{15}$$

$$\begin{aligned}
s.t. \quad & (i) && \lambda c_1(s) \leq 1 - x \\
& (ii) && (1-\lambda) \sum_{k=L,H} \pi_{sk} c_{2k}(s) = x(1-\pi_s) R_h + 1 - x - \lambda \sum_{k=L,H} \pi_{sk} c_{1k}(s) \\
& (iii) && c_1(s) \geq x r_H + 1 - x \\
& (iv) && c_{2L}(s) \geq x r_L + 1 - x \\
& (v) && c_{2H}(s) \geq x R_H + 1 - x \\
& (vi) && c_1(s) \leq c_{2k}(s) \ \ \forall k, s
\end{aligned}$$

Since the quality of assets is not observable, all liquidity investors consume the same amount: $c_{1k}(s) \equiv c_1(s)$ for each k, s. The constraints (i) and (ii) are resource constraints for period one and two, respectively. The constraints $(iii), (iv)$ and (v) are participation constraints for each type. The constraints (vi) are incentive compatibility constraints. In equilibrium, these constraints bind for investors with low quality assets: $c_1(s) = c_{2L}(s)$ in each state s.

Proposition 3. *The optimal holdings of safe asset in the incentive-compatible central planner solution are larger than the first-best allocation and than in the market equilibrium. The central planner achieves higher welfare relative to the market equilibrium.*

The central planner can reduce the adverse selection problem but cannot completely eliminate it. Due to the adverse selection, there are more assets traded in the market at date $t = 1$, in particular, more assets of low quality. To absorb this trading, more market liquidity is required. In the market equilibrium, investors do not take into account the effect of their investment choice on prices. The effect of prices on expected utility depends on the investors' type: liquidity investors and investors with low quality assets benefit from higher prices, while investors with high quality assets benefit from asset low prices. Overall, effect evaluated at the market equilibrium is positive. This means that the ex-ante welfare can be improved by increasing holdings of safe asset which results in higher prices. So, the central planner problem is equivalent to the investor maximization problem when price effect is taken into account, which leads to a larger fraction of endowment allocated to the safe asset at date $t = 0$. This larger liquidity allocation smooths the ex-ante consumption by improving consumption of liquidity investors and investors with lemons. As a result, it achieves higher welfare.

The central planner solution suggests another policy implication: requiring ex-ante a larger holdings of safe asset (liquidity) would alleviate the adverse selection problem and prevent market breakdown during the crisis.

4.7.1 Policy Implications

Liquidity requirement at t=0 The central planner solution suggests the following policy implication: requiring ex-ante a larger holdings of safe asset (liquidity) would alleviate the adverse selection problem and prevent the market breakdown during crises, especially if the economy is in the multiple equilibria range. The government can require to hold liquidity $(1 - x)$ at date $t = 0$ such that the second-best allocation is implemented.

Market Intervention at t=1

Alternatively, government can intervene ex-post when the economy is in the crisis state. If the market breakdown is due to the higher liquidity preference or to underestimating the likelihood of a crisis, then liquidity provision into the market can restore the trading. Consider the situation when government decides to intervene if an economy is no-trade equilibrium during the crisis. Suppose the price has to be increased by Δ to restore trading, then government should inject Λ amount of liquidity such that $\Lambda = \Delta\left(\lambda_2 + (1-\lambda_2)\,\pi_2\right)x$. Alternatively, the government can buy γ amount of assets such that $\gamma = \Delta\left(\lambda_2 + (1-\lambda_2)\,\pi_2\right)x / \left(\frac{(1-\lambda_2)(1-x)}{(\lambda_2+(1-\lambda_2)\pi_2)x} + \Delta\right)$. This policy is effective if the expected payoff of assets sold in the market is above liquidation value: $\widehat{R}_2 > r_H$.

If the no-trade outcome is a result of large fraction of lemons in the market or Knightian uncertainty about it then it is more effective to purchase these assets. In this case, the liquidity injection is not useful since it does not affect the expected value of assets, and therefore, leads to the further liquidity hoarding. If $\pi_2 : \widehat{R}_2\left(\pi_2\right) > r_H$ then fraction δ of bad assets needs to be removed from the market such that $\delta = 1 - \frac{\lambda_2(R_H - r_H)}{\lambda_2 R_H + (1-\lambda_2) r_H - R_L} / \pi_2$. Removing such assets from the market reduces adverse selection and uncertainty problems.[20]

It should be noted that there is a moral hazard problem associated with government interventions during crises. If market participants anticipate government interventions then the optimal holdings of risky assets is larger. This implies that the amount of liquidity that needs to be provided to the market or amount of assets that needs to purchased would also be larger. The moral hazard problem can be corrected if liquidity injection at date $t = 1$ is financed by a tax τ per unit of investment imposed at date $t = 0$. Therefore, the tax τx should be equal liquidity provision Λ that required in order to restore market price $\overline{p}_2$.

$$\tau x = \left(\lambda_2 + (1-\lambda_2)\,\pi_2\right) x \overline{p}_2 - (1-\lambda_2)\,(1-x)$$

Imposing such tax increases liquidity holdings at $t = 0$ and prevents market breakdowns at $t = 1$, which results in a higher expected utility.[21]

To illustrate the effect of government policy consider again the numerical example. The asset return parameters are given $R_H = 1.2$, $r_H = 0.5, R_L = r_L = 0.3$, the fraction of low quality investments in a normal state: $\pi_1 = 0.05$, probability of a liquidity shock in a normal state and a crisis state, respectively: $\lambda_1 = 0.2$ and $\lambda_2 = 0.3$, the probability

[20] This is consistent with arguments about effectiveness of the TARP proposal. However, as has been extensively noted, there are various implementation issues associated with it.

[21] See Appendix

of a crisis: $q = 0.1$. Figure 11a depicts the equilibrium values of investment, prices and expected utility as a function of low quality assets in the crisis. The solid lines depict values of market equilibria with adverse selection, and dashed lines represent values of equilibria with government intervention.

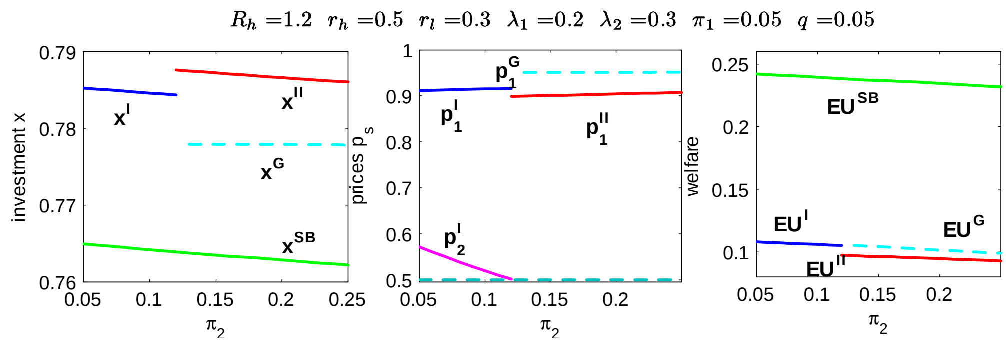

Figure 11a. Equilibrium values of investment,prices and welfare as a function of π_2.

Imposing the tax at date $t = 0$ to finance liquidity provision at $t = 1$ leads to the larger investor's holdings of liquidity at $t = 0$. As a result, the market prices are higher, and market breakdown is avoided. Also, it is leads to a higher expected utility.

Figure 11b depicts market liquidity as an aggregate holdings of safe asset, the cost of foregone payoff when asset is sold before maturity, and the return on asset bought on the secondary market. The liquidity available in the market at $t = 1$ for purchasing risky assets with tax-financed government interventions L_s^G is larger than liquidity holdings in type I and II market equilibria: L_s^I and L_s^{II}, but smaller than the second-best allocation L_s^{SB}. Also, government intervention reduces the cost of selling asset before maturity: $C_s^G < C_s^{II}$ and asset return bought at date $t = 1 : R_s/p_s^G < R_s/p_s^{II}$.

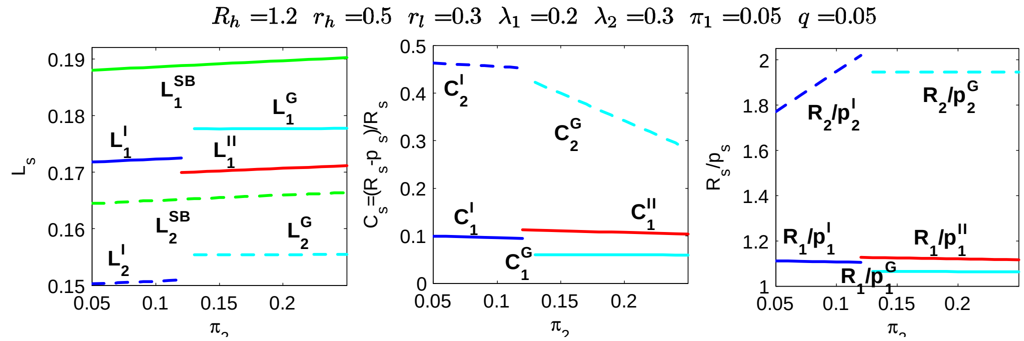

Figure 11b. Equilibrium values of market liquidity and asset returns as a function of π_2.

Therefore, the tax-financed liquidity provision during crises also leads to a larger market liquidty in normal times. It reduces adverse selection problem and improves welfare relative to the market equilibria, although not as much as the central planner solution.

5 Model Implications and Financial Crisis

Financial institutions held significant amount of mortgage backed securities (MBS).[22] Before the crisis, many of those MBS were rated AAA, which implied a minimal risk of default. These assets were considered liquid: if a financial institution needed cash, it could sell these securities at a fair market price. When in February 2007 subprime mortgage defaults had increased, triggering the liquidity crisis, a large fraction of these securities have been downgraded.[23] The impact of declining housing prices on securities depended on the exact composition of assets and mortgages that backed them. Due to the complexity of structured financial products and heterogeneity of the underlying asset pool, owners had an informational advantage in estimating how much those securities are worth.[24]

[22] These securities have skewed payoffs: they offer high expected return in most states of nature but suffer substantial losses in extremely bad states. When an economy is in a normal state with strong fundamentals, the asymmetric information does not significantly affect the asset value. However, when an economy is subject to a negative shock, the value of securities becomes more sensitive to private information and the adverse selection may influence the trading decisions. (Morris and Shin [26])

[23] For example, 27 of the 30 tranches of asset-backed CDOs underwritten by Merrill Lynch in 2007 were downgraded from AAA ratings to “junk” (Coval, Jurek and Stafford [15]).

[24] The junior equity tranches (also referred to as "toxic waste") were usually held by the issuing bank; they were traded infrequently and were therefore hard to value. Also, these structured finance products received

The asymmetric information about the assets value leads to the Akerlof (1970) lemons problem: a buyer does not know whether the seller is selling the security because of a sudden need for liquidity, or because the seller is trying to unload the toxic assets. This adverse selection issue can generate the market illiquidity reflecting buyers' beliefs that most securities offered for sale are of low quality.[25] Krishnamurthy [23] identifies this issue as one of diagnoses of the current crisis: market participants may fear that if they transact they will be left with a "lemon". Drucker and Mayer [16] find that underwriters of prime MBS appeared to exploit access to better information when trading in the secondary market. Elul [19] also finds evidence of adverse selection in the prime mortgage market.

Moreover, the extent of asymmetric information was not fully known. Gorton [20] argues that there was a loss of information about size and location of expected losses due to the complexity and opaqueness of securitization.

Furthermore, as market condition worsened, investors' value for liquidity had increased which was reflected in the high spreads of MBS relative to Treasury bills (Krishnamurthy [23]). The deleveraging that accompanies the initial shock can further aggravate the adverse selection problem.[26] Because of the losses on their MBS, some banks became undercapitalized; however, their attempts to recapitalize pushed their market price further down. This reflects the investors' fear that any bank that issues new equity or debt may be overvalued, leading to the liquidity crunch.[27] As market liquidity falls, it becomes difficult to find trading partners which leads to the fire-sale pricing.[28]

The demand for ABS collapsed from over $500 billion in 2007 to $20 billion in 2009 as

overly optimistic ratings from the credit rating agencies. One of the reason the underlying securities default risks were underestimated is that the statistical models were based on the historically low mortgage default and delinquency rates.(Brunnermeier [10])

[25] For example, the repo market in 2007-2009, as described by Gorton and Metrick [21].

[26] "The large haircuts on some securities could be seen as a response by leveraged entitites to the potential drying up of trading possibilities in the asset-backed securities (ABS) market. The equity market, in contrast, is populated mainly with non-leveraged entities such as mutual funds, pension funds, insurance companies and households, and hence is less vulnerable to the drying up of trading partners." Morris and Shin [26]

[27] Brunnermeier and Pedersen [11] refer to this phenomena as a "loss spiral" and a "margin spiral". Adrian and Shin [1] documented evidence of these phenomena for investments banks.

[28] The haircut on ABSs increased from 3-5% in August 2007 to 50-60% in August 2008. The haircut on equities increased from 15% to 20% for the same period (Gorton and Metrick [21]).

illustrated in Figure 11 which is taken from Adrian, Ashcraft, and Pozsar (2010).

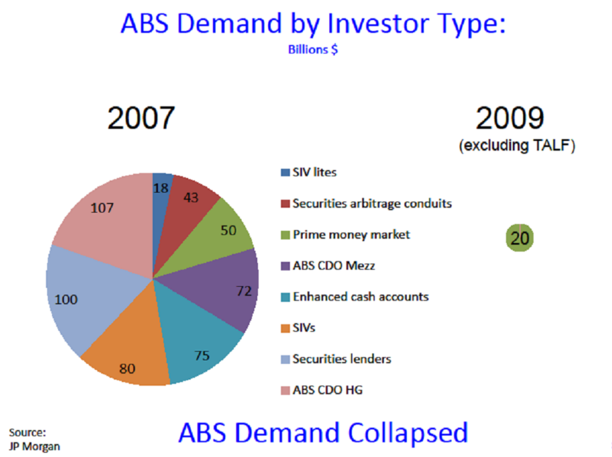

Figure 12 shows the prices of the ABX index which includes 2007 securities with AAA ratings. The index opened in 2007 at a price below par (of 100) and was trading below 30 in the summer of 2009.

Figure 12. 2007-2 vintage of the ABX index for the AAA tranche.

My model provides the framework which captures the important ingredients of the crisis:

- adverse selection generated by the asymmetric information about asset quality
- increase in preference for liquidity which causes asset sales for exogenous reasons (unrelated to asset returns) in order to raise the liquidity
- considering the crisis as a low probability event
- uncertainty about assets value due to the unexpected shock

My model demonstrates how adverse selection can lead to the liquidity hoarding, lower asset pricing, lower trading volume and possibly to the complete market breakdown during the crisis. Although adverse selection is generated by idiosyncratic asymmetric information, the extent of adverse selection depends on the aggregate state. It can be viewed as individual exposure to the systemic risk. The focus is on the size of the adverse selection problem (rather than its location) as the cause of market illiquidity. In normal times, when the fraction of lemons is small, adverse selection does not have significant effect on the market. However, if the fraction of lemons is large or potential buyers believe it maybe large, then adverse selection can lead to the market breakdown. Furthermore, even a small amount of adverse selection can be amplified to a full scale crisis with market freezes and liquidity hoarding if it is accompanied by high preference for liquidity, low probability of the crisis, and uncertainty about assets value.[29]

Also, the model can be applied to the credit markets. Changing the initial assumption: investors borrow one unit of good at $t = 0$ (instead of receiving it as an endowment) and have to repay it at date $t = 1$ with probability λ. Then in period one investors who have not received a liquidity shock are creditors, and liquidity investors are borrowers. The risky asset is used as a collateral in the credit market. In this setting, the cost $C(s) = \frac{\widehat{R}_s - p_s}{\widehat{R}_s}$ corresponds to the haircut on asset expected value $\widehat{R}_s$. This cost is is larger in the crisis relative to a normal state and it is increasing with amount of adverse selection in the market.

Furthermore, the results can be applied to the cross-countries analysis of financial crises. The countries with a history of rare financial crises tend have less aggregate liquidity holdings relative to (illiquid) long-term investment. In these countries if a crisis does occur then it is more severe and more likely to be accompanied by market freezes. On the other hand, countries that are more prone to financial crises, have more aggregate liquidity holdings which smoothen a crisis when it occurs.

Policy Responses The policy responses during crises depend on which mechanism causes the market breakdown. If it is due to the higher liquidity preference or to underestimating the likelihood of a crisis, then liquidity provision into the market can restore

[29] The size of subprime market were small compared to the total ABS market. In 2007, subprime issuance about 30% of the total non-agency MBS issuance. (*BIS Quarterly Review, December 2007*)

the trading. However, if the no-trade outcome is a result of large fraction of lemons in the market or Knightian uncertainty about it then it is more effective to purchase these assets. Removing such assets from the market reduces adverse selection and uncertainty problems. In this case, the liquidity injection is not useful since it does not affect the expected value of assets, and therefore, leads to the further liquidity hoarding. There is a moral hazard problem associated with government interventions during the crisis: if market participants anticipate the government market intervention then the optimal holdings of risky assets is larger. One way to avoid this problem is to finance liquidity injection by ex-ante tax per unit of investment Imposing such tax increases market liquidity and prevents market breakdowns during crises. The preemptive policy response is ex-ante requirement of larger liquidity holdings which prevents the market breakdown during crises, especially if the economy is in the multiple equilibria range.

6 Conclusion

I analyze the effect of adverse selection in the asset market. The asymmetric information about asset returns generates the lemons problem when buyers do not know whether the asset is sold because of its low quality or because the seller's sudden need for liquidity. This adverse selection can lead to market breakdown reflecting the buyers' belief that most assets that are offered for sale are of low quality.

Further, I examine the following amplification mechanisms: increase in the liquidity preference during the crisis, underestimating the likelihood of a crisis, and uncertainty about the fraction of low quality assets. Any of these phenomena can amplify the effect of adverse selection leading to the increased asset price volatility, fire-sale pricing and possibly to the breakdown of trade during the crisis.

The ability to trade based on private information may be welfare improving if adverse selection does not lead to the market breakdown. The central planner can reduce the adverse selection problem by requiring larger liquidity holdings, which prevents market breakdown during the crisis and increases the aggregate welfare.

The policy implication depends on which mechanism causes the market breakdown. If it is due to the higher liquidity preference or to underestimating the likelihood of a crisis, then liquidity provision into the market can restore the trading. However, if the no-trade outcome is a result of large fraction of lemons in the market or Knightian uncertainty about it then it is more effective to purchase these assets. Removing such assets from the market reduces adverse selection and uncertainty problems. In this case, the liquidity injection is not useful since it does not affect the expected value of assets, and therefore, leads to the further liquidity hoarding. The requirement of larger liquidity holdings prevents the market breakdown during the crisis, especially if the economy is in the multiple equilibria range.

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7 Appendix

7.1 Assumptions:

parameters: $q, \pi_1, \pi_2, \lambda_1, \lambda_2, R_h, r_h, r_l$

$$
\begin{aligned}
(i) \quad &: \quad \overline{\pi} r_l + (1-\overline{\pi})\left(1-\overline{\lambda}\right) R_h + (1-\overline{\pi})\overline{\lambda} r_h > 1 \\
(ii) \quad &: \quad r_h < \frac{\lambda_2 + \left(\pi_2 \frac{\overline{R}_2}{r_l + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}} + (1-\pi_2)\frac{\overline{R}_2}{R_H + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}}\right)}{\lambda_2 + \left(\pi_2 \frac{r_l}{r_l + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}} + (1-\pi_2)\frac{R_H}{R_H + \overline{R}_2 \frac{\lambda_2}{(1-\lambda_2)}}\right)} \\
(iii) \quad &: \quad \lambda_2 > \frac{1}{1 + \overline{R}_2 \frac{(1-\lambda_1)}{\lambda_1}\left[\frac{\lambda_1 + \left(\pi_1 \frac{r_l}{r_l + \overline{R}_1 \frac{\lambda_1}{1-\lambda_1}} + (1-\pi_1)\frac{R_H}{R_H + \overline{R}_1 \frac{\lambda_1}{1-\lambda_1}}\right)}{\lambda_1 + \left(\pi_1 \frac{\overline{R}_1}{r_l + \overline{R}_s \frac{\lambda_1}{1-\lambda_1}} + (1-\pi_1)\frac{\overline{R}_1}{R_H + \overline{R}_1 \frac{\lambda_1}{1-\lambda_1}}\right)}\right] + 1}
\end{aligned}
$$

(i) $\Rightarrow$ there is always positive holding of risky asset

(ii) $\Rightarrow$ there is always trade in a crisis state without adverse selection

(iii) $\Rightarrow$in a crisis state price is always determined by market clearing conditions, hence, $p_2 < \overline{R}_2$

7.2 Equilibrium without Adverse Selection

The investor's maximization problem is given by

$$
\max\left\{\sum_{s=1,2}\left[\lambda_s \log(1-x+p_s x) + (1-\lambda_s)\left(\pi_s \log\left(xR_l + (1-x)\overline{R}_s/p_s\right) + (1-\pi_s)\log\left(xR_h + (1-x)\overline{R}_s/p_s\right)\right)\right]\right\}
$$

Market clearing conditions imply

$$
p_s = \min\left\{\frac{(1-\lambda_s)}{\lambda_s}\frac{(1-x)}{x}; \overline{R}_s\right\}
$$

case 1: $p_s \leq \overline{R}_s$

$$
\begin{aligned}
x &= \sum_{s=1,2} q_s (1-\lambda_s)\left(\lambda_s + \pi_s \frac{r_l}{\left(r_l + \overline{R}_s \frac{\lambda}{1-\lambda}\right)} + (1-\pi_s)\frac{R_h}{\left(R_h + \overline{R}_s \frac{\lambda}{1-\lambda}\right)}\right) \\
p_s &= \frac{(1-\lambda_s)}{\lambda_s}\frac{\sum_{s=1,2} q_s \lambda_s \left(\lambda_s + \pi_s \frac{\overline{R}_s}{\left(r_l + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)} + (1-\pi_s)\frac{\overline{R}_s}{\left(R_h + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)}\right)}{\sum_{s=1,2} q_s (1-\lambda_s)\left(\lambda_s + \pi_s \frac{r_l}{\left(r_l + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)} + (1-\pi_s)\frac{R_h}{\left(R_h + \overline{R}_s \frac{\lambda_s}{1-\lambda_s}\right)}\right)}
\end{aligned}
$$

case 2: $p_1 = \overline{R}_1, p_2 \leq \overline{R}_2$

x^* is a solution to the following equation:

$$\sum_{s=1,2} q_s \left[\begin{array}{c} \lambda_1 \frac{\overline{R}_1 - 1}{\overline{R}_1 x + (1-x)} + (1-\lambda_1)\,\pi_1 \frac{r_l - 1}{x r_l + (1-x)} + (1-\lambda_1)(1-\pi_1) \frac{R_h - 1}{x R_h + (1-x)} + \\ (1-\lambda_2) \frac{1}{(1-x)x} \left(\left(\lambda_2 + \pi_2 \frac{r_l}{\left(r_l + \overline{R}_2 \frac{\lambda_2}{1-\lambda_2}\right)} + (1-\pi_s) \frac{R_h}{\left(R_h + \overline{R}_s \frac{\lambda_2}{1-\lambda_2}\right)} \right) - x \right) \end{array} \right] = 0$$

$$\begin{aligned} p_1 &= \overline{R}_1 \\ p_2 &= \frac{(1-\lambda_2)}{\lambda_2} \frac{(1-x^*)}{x^*} \end{aligned}$$

It can be verified that $x^* \leq \frac{1}{\frac{\lambda}{1-\lambda}\overline{R}_1 + 1}$ so that market clearing condition in state $s = 1$ is satisfied.

First-Best investment and consumption allocations:

$$\begin{aligned} &investment &:& \quad x^o = 1 - \sum_{s=1,2} q_s \lambda_s = \left(1 - \overline{\lambda}\right) \\ &consumption &:& \quad \begin{array}{l} c_1(s) = \frac{\overline{\lambda}}{\lambda_s} \\ c_2(s) = \frac{\left(1-\overline{\lambda}\right)}{(1-\lambda_s)} \overline{R}_s \end{array} \end{aligned}$$

The market equilibrium investment allocation is less than the first-best: $x^* < x^o$,

$$\begin{aligned} x^* &\leq \sum_{s=1,2} q_s (1-\lambda_s) \left(\lambda_s + \pi_s \frac{r_l}{r_l + \frac{\lambda_s}{(1-\lambda_s)} \overline{R}_s} + (1-\pi_s) \frac{R_h}{R_h + \frac{\lambda_s}{(1-\lambda_s)} \overline{R}_s} \right) < \\ &< \sum_{s=1,2} q_s (1-\lambda_s) \end{aligned}$$

7.3 Equilibrium with Adverse Selection

7.3.1 Proof of Proposition 1

Proof. **Type I equilibrium**

Let us start with a type I equilibrium with market trading in both states. The investors' maximization problem is given by

$$\begin{aligned} \max_x \quad & \lambda_s \log(1 - x + p_s x) + (1-\lambda_s) \sum_{s=1,2} q_s \left(\pi_s \log\left(x p_s + (1-x)\widehat{R}_s / p_s \right) + (1-\pi_s) \log\left(x R_H + (1-x)\widehat{R}_s / p_s \right) \right) \\ s.t \quad & (i) \;\; 0 \leq x \leq 1 \\ & (ii) \; p_s > r_h \;\; \forall s \end{aligned}$$

Therefore, an investment allocation x and market prices p_s are determined by the following equations:

$$
\begin{aligned}
F(x) &\equiv \sum_{s=1,2} q_s F_s(x,p_s) = 0 \\
p_s &= \max\left(\frac{(1-\lambda_s)}{(\lambda_s + (1-\lambda_s)\pi_s)}\frac{(1-x)}{x}, \widehat{R}_s\right) \quad \forall s
\end{aligned}
$$

where

$$
F_s(x,p_s) \equiv \lambda_s \frac{p_s - 1}{1 - x + p_s x} + (1-\lambda_s)\left(\pi \frac{p_s - \widehat{R}_s/p_s}{xp_s + (1-x)\widehat{R}_s/p_s} + (1-\pi)\frac{R_H - \widehat{R}_s/p_s}{xR_H + (1-x)\widehat{R}_s/p_s}\right)
$$

If prices are determined by cash-in-the-market, then by substituting prices p_s, we get

$$
F_s(x) = \left(\lambda_s \frac{1}{\left(\frac{1}{(1-\lambda_s)} + \pi_s\right)} + (1-\lambda_s)\pi_s \frac{(1-x)}{(1-x) + \widehat{R}_s\left(\frac{\lambda_s}{(1-\lambda_s)} + \pi_s\right)^2 x} + (1-\lambda_s)(1-\pi_s)\frac{R_H}{R_H + \widehat{R}_1\left(\frac{\lambda_s}{(1-\lambda_s)} + \pi_s\right)} - x\right)
$$

This is a monotonically decreasing function of x. At $x = 0$, F is greater than 0 and at $x = 1$, F is less than zero. Therefore, by Intermediate Function Theorem, there exist a unique x^* such that at $F(x) = 0$ The x^* can be derived as a root to a cubic equation:$a_1x^3 + a_2x^2 + a_3x + a_4 = 0$, where

$$
\begin{aligned}
a_1 &= -d_1 d_2 \\
a_2 &= d_1 d_2 d_3 - ((1-\lambda_1) q_1 \pi_1 + 1) d_2 - ((1-\lambda_2) q_2 \pi_2 + 1) d_1 \\
a_3 &= ((d_1 + d_2) d_3 - 1) + ((1-\lambda_1) q_1 \pi_1 (d_2 - 1) + (1-\lambda_2) q_2 \pi_2 (d_1 - 1)) \\
a_4 &= d_3 + (1-\lambda_1) q_1 \pi_1 + (1-\lambda_2) q_2 \pi_2
\end{aligned}
$$

$$
\begin{aligned}
d_1 &= \left(\widehat{R}_1\left(\frac{\lambda_1}{(1-\lambda_1)} + \pi_1\right)^2 - 1\right) \\
d_2 &= \left(\widehat{R}_2\left(\frac{\lambda_2}{(1-\lambda_2)} + \pi_2\right)^2 - 1\right) \\
d_3 &= \sum_{s=1,2} q_s \left(\lambda_s \frac{1}{\left(\frac{1}{(1-\lambda_s)} + \pi_s\right)} + (1-\lambda_s)\frac{(1-\pi_s) R_H}{R_H + \widehat{R}_s\left(\frac{\lambda_s}{(1-\lambda_s)} + \pi_s\right)}\right)
\end{aligned}
$$

Denote the solution as x^*, then the prices are given by

$$
p_s^* = \max\left(\frac{(1-\lambda_s)}{(\lambda_s + (1-\lambda_s)\pi_s)}\frac{(1-x^*)}{x^*}, \widehat{R}_s\right)
$$

If $r_H < \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)}\frac{(1-x^*)}{x^*} \leq \widehat{R}_s$ then (x^*, p_1^*, p_2^*) are equilibrium investment and prices. If $\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_1)}\frac{(1-x^*)}{x^*} > \widehat{R}_1$ then $p_1^* = \widehat{R}_1$, and (x^*, p_2^*) are determined by

$$
\begin{aligned}
(i) &: \quad q_1 F_1(x^*, \widehat{R}_1) + q_2 F_2(x^*, p_2^*) = 0 \\
(ii) &: \quad p_2^* = \max\left(\frac{(1-\lambda_2)}{(\lambda_2 + (1-\lambda_2)\pi_2)}\frac{(1-x^*)}{x^*}, \widehat{R}_2\right)
\end{aligned}
$$

If $\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_2)}\frac{\left(1-x^*\right)}{x^*} \leq \widehat{R}_2$ then (x^*, p_1^*, p_2^*) is an equilibrium. It can verified that $x^* \leq \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_1)\widehat{R}_1+(1-\lambda)}$ so that market clearing condition in state $s=1$ is satisfied. If $\frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_2)}\frac{\left(1-x^*\right)}{x^*} > \widehat{R}_2$ then $p_2^* = \widehat{R}_2$ and by assumption 3, $p_1^* = \widehat{R}_1$. Hence, equilibrium investment x^* is a solution to $\sum_{s=1,2} q_s F_s(x^*, \widehat{R}_s) = 0$.

If $p_2^* \leq r_h$ then in the crisis state liquidity traders with high quality investment choose to liquidate their investment rather than selling it at $t=1$. Therefore, the expected return $\widehat{R}_2 = r_l$, so there no demand for risky assets. Hence, (x^*, p_1^*, p_2^*) cannot be an equilibrium investment and prices if $p_2^* \leq r_h$.

If λ_2 and π_2 are sufficiently large such that $p_2^* \leq r_h$ then the type I does no longer exist. $F_s(x)$ is decreasing in λ_s and π_s. Also, $F_s(x)$ is decreasing in x. Hence, x is decreasing in λ_s and π_s. If p_2^* is determined by cash-in-the-market-pricing then the effect of an increase in λ_2 or π_2 on the price in state $s=2$ is determined by

$$
\begin{aligned}
\frac{\partial p_2}{\partial \lambda_2} &= -\frac{\frac{1}{(1-\lambda_2)^2}}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)^2}\frac{(1-x)}{x} - \frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)}\frac{1}{x^2}\frac{\partial x}{\partial \lambda_2} \\
\frac{\partial p_2}{\partial \pi_2} &= -\frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)^2}\frac{(1-x)}{x} - \frac{1}{\left(\frac{\lambda_2}{1-\lambda_2}+\pi_2\right)}\frac{1}{x^2}\frac{\partial x}{\partial \lambda_2}
\end{aligned}
$$

Therefore, increase in λ_2 and/or $\pi_2$can lead to the decrease in p_2^*, potentially resulting in $p_2^* \leq r_H$. If $p_2^* = \widehat{R}_2$ then it is again decreasing in π_2 but increasing in λ_2, however, if λ_2 increases sufficiently then cash-in-the-market-pricing binds and p_2^* becomes decreasing function of λ_2.

The consumption allocation of early and late consumers in a type I equilibrium are given by

$$
\begin{aligned}
c_1(s) &= (1-x^*)\frac{1+(1-\lambda_s)\pi_s}{\lambda_s+(1-\lambda_s)\pi_s} \\
c_{2L}(s) &= (1-x^*)\frac{(1-\lambda_s)}{(\lambda_s+(1-\lambda_s)\pi_s)} + x^*\left(\frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_s r_l\right) \\
c_{2H}(s) &= x^*\left(R_h + \frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_s r_l\right)
\end{aligned}
$$

Type II equilibrium

Now consider a type II equilibrium with no trading in a crisis state. The investors maximization problem becomes

$$
\begin{aligned}
\max_x \quad & \left\{ \begin{array}{l} \lambda \log(1-x+p_1x) + (1-\lambda)(1-q)\left(\pi_1 \log\left(xp_1 + (1-x)\widehat{R}_1/p_1\right) + (1-\pi_s)\log\left(xR_H + (1-x)\widehat{R}_1/p_1\right)\right) + \\ +\lambda \log(1-x+r_kx) + (1-\lambda) + q\left(\pi_2 \log(xr + (1-x)) + (1-\pi_2)\log(xR_H + (1-x))\right)\end{array} \right\} \\
s.t \quad & (i)\ \ 0 \leq x \leq 1 \\
& (ii)\ p_1 > r_h
\end{aligned}
$$

Therefore, an investment allocation x and market prices p_s are determined by the following equations:

$$
\begin{aligned}
G(x) &\equiv q_1F_1(x, p_1) + q_2G_2(x) = 0 \\
p_1 &= \max\left(\frac{(1-\lambda_1)}{(\lambda_1+(1-\lambda_1)\pi_1)}\frac{(1-x)}{x}, \widehat{R}_1\right)
\end{aligned}
$$

where

$$G_2(x) = \left(\pi_2 \frac{r_L - 1}{xr_L + (1-x)} + \lambda_2(1-\pi_2)\frac{r_H - 1}{xr_H + (1-x)} + (1-\lambda_2)(1-\pi_2)\frac{R_H - 1}{xR_H + (1-x)}\right)$$

If price p_1 is determined by cash-in-the-market, then $G(x)$ is a decreasing function in x, and it is positive at $x = 0$ and negative at $x = 1$. Therefore, a solution $x^{**} : G(x^{**}) = 0$ exists and it is unique.

If $\frac{(1-\lambda_1)}{(\lambda_1+(1-\lambda_1)\pi_1)}\frac{\left(1-x^{**}\right)}{x^{**}} \leq \widehat{R}_1$, then (x^{**}, p_1^{**}) is an equilibrium. If $\frac{(1-\lambda_1)}{(\lambda_1+(1-\lambda_1)\pi_1)}\frac{\left(1-x^{**}\right)}{x^{**}} > \widehat{R}_1$, then $p_1^{**} = \widehat{R}_1$ and $x^{**} : q_1 F_1(x^{**}, \widehat{R}_1) + q_2 G_2(x^{**}) = 0$. It can be shown that $x^{**} < \frac{1}{\left(\frac{\lambda_1}{(1-\lambda_1)}+\pi_1\right)+1} < \frac{1}{\left(r_h\left(\frac{\lambda_1}{(1-\lambda_1)}+\pi_1\right)+1\right)}$. Therefore, $p_1^{**} > r_h$. Hence, there is always market trading in a normal state. Also, it can be verified that $\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{\left(1-x^{**}\right)}{x^{**}} < r_h$, i.e., there is indeed no market trading during the crisis.

Define hypothetical price $p_2^{**}(\lambda_2, \pi_2) = \frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{\left(1-x^{**}(\lambda_2,\pi_2)\right)}{x^{**}(\lambda_2,\pi_2)}$ (this is an implied price in the crisis state, it is hypothetical since there no market trading). This hypothetical price p_2^{**} is decreasing in λ_2 and π_2. *Therefore, if λ_2 and π_2 are sufficiently small such that $p_2^{**} > r_h$ then the type II does not exist.*

The consumption allocation of early and late consumers in a type II equilibrium are given by

$$c_{1k}(s) = \begin{cases} (1-x^{**})\frac{1+(1-\lambda_s)\pi_s}{\lambda_s+(1-\lambda_s)\pi_s} & if \quad p_s > r_k \\ 1 - x^{**} + r_k x^{**} & if \quad p_s \leq r_k \end{cases}$$

$$c_{2H}(s) = \begin{cases} x^{**}\left(R_h + \frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_s r_l\right) & if \quad p_s > r_k \\ x^{**}R_H + (1-x^{**}) & if \quad p_s \leq r_k \end{cases}$$

$$c_{2L}(s) = \begin{cases} (1-x^{**})\frac{(1-\lambda_s)}{(\lambda_s+(1-\lambda_s)\pi_s)} + x^{**}\left(\frac{\lambda_s}{(1-\lambda_s)}\overline{R}_s + \pi_s r_l\right) & if \quad p_s > r_k \\ x^{**}r_L + (1-x^{**}) & if \quad p_s \leq r_k \end{cases}$$

Multiple Equilibria

The equilibria of type I and II coexist for λ_2 *and* π_2 such that $p_2^{*}(\lambda_2, \pi_2) > r_h \geq p_2^{**}(\lambda_2, \pi_2)$. Consider a type I equilibrium investment allocation x^*. It can be shown that $G(x^*) \geq 0$, which implies $x^{**} > x^*$. Hence, $p_2^{**}(\lambda_2, \pi_2) < p_2^{*}(\lambda_2, \pi_2)$. Therefore, there is a possibility that $p_2^{*}(\lambda_2, \pi_2) > r_h \geq p_2^{**}(\lambda_2, \pi_2)$.

The expected utility is higher when there is a market trading in both states. Consider investment allocation in type II equilibrium x^{**}. Since $p_2 > r_h > r_l$ then consumption for all $k = L, H$, and $t = 1, 2:$ $c_{tk}^{I}(x^{**}) = c_{tk}^{II}(x^{**})$ for $s = 1$ and $c_{tk}^{I}(x^{**}) > c_{tk}^{II}(x^{**})$ for $s = 2$. Hence, $V^{II}(x^{**}) < V^{I}(x^{**}) \leq V^{I}(x^*)$. Therefore, type I equilibrium is ex-ante Pareto dominant. However, type I equilibrium is not ex-post Pareto dominant since investor with high quality asset have higher expected utility in the normal state in type II relative to type I equilibrium: $c_{2H}^{I}(s=1) < c_{2H}^{II}(s=2)$. ■

7.3.2 Proof of Proposition 2

Proof. Denote investment allocation in an equilibrium without adverse selection by x', and investment allocations in type and I and II equilibrium with adverse selection by x^* and x^{**},respectively. Similarly, denote expected utility in state s for an equilibrium without adverse selection by $V_s(x')$, and for type and I and II equilibrium with adverse selection by $V_s^I(x^*)$and $V_s^{II}(x^*)$, respectively.

It can be shown that for $\forall s = 1, 2 : F_s(x', p_s(x')) > 0$. Therefore, we have $x^{**} > x^* > x'$ since $G_2(x^*) > F_2(x^*)$, and functions F_s and G_2 are decreasing in x.

Consider the difference in expected welfare $V^I - V$ in state s,

$$\begin{aligned} V_s^I(x') - V_s(x') &= \left(\lambda_s \log\left(\frac{c_1^I(s)}{c_1(s)}\right) + (1-\lambda)\pi_s \log\left(\frac{c_{2L}^I(s)}{c_{2L}(s)}\right) + (1-\lambda)(1-\pi_s)\log\left(\frac{c_{2H}^I(s)}{c_{2H}(s)}\right)\right) \geq \\ &\geq \log\left(\lambda_s \frac{c_1^I(s)}{c_1(s)} + (1-\lambda)\pi_s \frac{c_{2L}^I(s)}{c_{2L}(s)} + (1-\lambda)(1-\pi_s)\frac{c_{2H}^I(s)}{c_{2H}(s)}\right) \geq 0 \end{aligned}$$

Therefore, $V^I(x^*) \equiv \sum_{s=1,2} q_s V_s^I(x^*) \geq \sum_{s=1,2} q_s V_s^I(x') > \sum_{s=1,2} q_s V_s(x') \equiv V(x')$, i.e., ability to trade based on private information increases the expected utility if there is market trading in both states.

Next, consider the difference in expected welfare $V^{II} - V$ in state $s = 2$. Given the investment allocation, all types of investors consume less in the crisis state in a type II (no-trade) equilibrium than in an equilibrium without adverse selection: $c_{t,k}^{II}(x) < c_{t,k}(x)$. Therefore, $V_2\left(x'\right) - V_2(x^{**}) \geq V_2(x^{**}) - V_2^{II}(x^{**}) > 0$. (Let $x'' = \arg\max_x V_2(x)$, then $x'' \leq x' < x^{**}$ since $\frac{\partial V_2(x)}{\partial x}|_{x=x'} < 0$. Hence, $V_2(x^{**}) < V_2\left(x'\right)$). Also, we have $V_1^{II}(x^{**}) - V_1(x^{**}) \geq V_1^{II}(x^{**}) - V_1(x\prime) > 0$.

Therefore, informed trading leads to the welfare gains in a normal state: $V_1^{II}(x^{**}) > V_1(x\prime)$ and welfare loss in a crisis state: $V_2^{II}(x^{**}) < V_2\left(x'\right)$. The ex-ante effect depends on the probability of a crisis state. Define, $\Delta V \equiv V^{II}(x^{**}) - V(x') = \sum_{s=1,2} q_s\left(V_s^{II}(x^{**}) - V_s(x')\right)$. For $q = 0$, $V^{II}(x^{**}) > V(x\prime)$ and for $q = 1$, $V^{II}(x^{**}) < V(x\prime)$. Therefore, $\exists \widetilde{q} \in (0, 1) : \forall q < \widetilde{q}$, $V^{II}(x^{**}) > V(x\prime)$ and $\forall q > \widetilde{q}$, $V^{II}(x^{**}) > V(x\prime)$ since ΔV is linear in q. ■

7.4 Comparative Statics

7.4.1 Proof of Corollary 1

Proof. First consider an equilibrium with trade in both states. The equilibrium investment allocation is determined from the following equation: $\sum_{s=1,2} q_s F_s(x, p_s) = 0$ ($F_s(x, p_s)$ is defined in the proof of Proposition 1). $F_s(x, p_s)$ is decreasing in π_s, therefore, $\sum_{s=1,2} q_s F_s(x, p_s)$ is decreasing in q. Also, $\sum_{s=1,2} q_s F_s(x, p_s)$ is decreasing in x. Hence, the solution x^* is decreasing in q. If the prices are determined by cash-in-the-market constraint, then the prices p_s^* are increasing in q. Also, the expected utility V^I decreases as q becomes larger.

Now consider an equilibrium with market breakdown in a crisis state. If we compute $x\prime$ such that $G_2(x') = 0$ and x'' such that $F_1(x'', p_1(x'')) = 0$ then $x'' > x^{**} > x'$. The equilibrium x^{**} is determined by $G(x, p_1) = (1-q)F_1(x, p_1) + qG_2(x, p_2) = 0$. Since G is decreasing in x then the optimal x^{**} is decreasing in q. Therefore, p_1 is increasing in q since it negatively depends on x. Since $V_2^{II}(x^{**}) < V_1^{II}(x^{**})$, as q becomes larger the expected utility V^{II} decreases. The market breaks down when the price in a crisis state falls below the liquidation value r_H. Consider again the hypothetical price $p_2^{**} = \max\left(\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)}\frac{\left(1-x^{**}(q)\right)}{x^{**}(q)}, \widehat{R}_2\right)$ defined in the proof of Proposition 1 . The increase in q may increase p_2^{**} sufficiently to restore the trading.

Consider some q such that $p_2^{**} = r_H - \varepsilon$ with $\varepsilon > 0$., so there is no trading in state 2. Therefore, $F_2(x^{**}, p_2) > G_2(x^{**}) = 0$. If q increases sufficiently so that x^{**} goes down by more than $\frac{\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)\varepsilon}{\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)(r+\varepsilon)\right)\left(1+\left(\frac{\lambda}{(1-\lambda)}+\pi_2\right)r\right)}$ then the trading in a crisis state restores. ■

7.5 Government

7.5.1 Proof of Proposition 3

Proof. The central planner maximization problem can be written as following,

$$\max_x \{\sum_{s=1,2} q_s \left(\lambda \log(1-x) + (1-\lambda)\left(\pi_s \log c_{2L} + (1-\pi_s)\log c_{2H}\right)\right)\}$$

$$\begin{aligned}
s.t.\quad & (i) && (\lambda + (1-\lambda)\pi_s)\,\Delta c_1(s) = (1-\lambda)(1-x) \\
& (ii) && (1-\lambda)\,\Delta c_2(s) = (\lambda + (1-\lambda)\pi_s)\, x\widehat{R} \\
& (iii) && c_1(s) = \Delta c_1(s) + (1-x) \\
& (iv) && c_{2L}(s) = \Delta c_1(s) + \Delta c_2(s) \\
& (v) && c_{2H}(s) = xR_H + \Delta c_2(s) \\
& (vi) && c_1 \geq xr_H + (1-x) \\
& (vii) && c_{2H} \geq xR_H + (1-x) \\
& (viii) && c_{2L} \geq c_1
\end{aligned}$$

where $\Delta c_1(s)$ is a transfer of cash holdings to liquidity investors in exchange of their risky asset holdings at date $t = 1$ and $\Delta c_2(s)$ is a transfer of risky asset holdings in exchange for cash holding to non-liquidity traders.

The maximization problem can be reduced to the following,

$$\max_{x} \quad \left\{ \sum_{s=1,2} q_s \left[\lambda \log(1-x) \left(\tfrac{1+(1-\lambda)\pi_s}{\lambda+(1-\lambda)\pi_s} \right) + (1-\lambda) \left(\begin{array}{c} \pi_s \log\left((1-x) \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)} + x \frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)} \widehat{R}_s \right) \\ + (1-\pi_s) \log x \left(R_h + \frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)} \widehat{R}_s \right) \end{array} \right) \right] \right\}$$

$$s.t. \quad 0 \leq x \leq 1$$

The optimal investment x is a solution to the following equation $H(x) \equiv \sum_{s=1,2} q_s H_s(x) = 0$, where

$$H_s(x) \equiv \left(-\lambda \frac{1}{(1-x)} + (1-\lambda)\pi_s \frac{\left(\frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)} \right)^2 \widehat{R}_s - 1}{\left((1-x) + x \left(\frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)} \right)^2 \widehat{R}_s \right)} + (1-\lambda)(1-\pi_s) \frac{1}{x} \right)$$

This is a monotonically decreasing function of x. At $x = 0$, H is greater than 0 and at $x = 1$, H is less than zero. Therefore, by Intermediate Function Theorem, there exist a unique x^o such that at $H(x^o) = 0$ The x^o can be derived as a root to a cubic equation.

Furthermore at $x = 1-\overline{\lambda}$, we have $H(x) < 0$ which implies that $x^o < 1-\overline{\lambda}$, i.e., the investment allocation in the incentive compatible equilibrium is smaller than the first-best investment allocation. Denote the optimal expected utility by V^o. To compare welfare achieved by central planner with a market equilibrium. If prices are determined by cash-in-the-market constraints then the expected utility in a market equilibrium with trade in both states is given by

$$V(x) = \left\{ \sum_{s=1,2} q_s \left[\lambda \log(1-x) \left(\frac{1+(1-\lambda)\pi_s}{\lambda+(1-\lambda)\pi_s} \right) + (1-\lambda) \left(\begin{array}{c} \pi_s \log\left((1-x) \frac{(1-\lambda)}{(\lambda+(1-\lambda)\pi_s)} + x \frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)} \widehat{R}_s \right) \\ + (1-\pi_s) \log x \left(R_h + \frac{(\lambda+(1-\lambda)\pi_s)}{(1-\lambda)} \widehat{R}_s \right) \end{array} \right) \right] \right\}$$

Therefore, $V^I(x^*) \leq V(x^*) \leq V^o$ since $x^o = \arg\max V(x)$, i.e., the central planner always achieves a higher welfare level. Furthermore, comparing $F_s(x)$ and $H(x)$, it can be shown that $\sum_{s=1,2} q_s F_s(x^o) \geq 0$. It implies that $x^* \geq x^o$ where $\sum_{s=1,2} q_s F_s(x^*) = 0$, i.e., the investment allocation in a market equilibrium is larger than the central planner investment allocation.

Note, central planner achieves larger consumption allocation in both states $c_{tk}(x^o) > c^I_{tk}(x^{**})$ for all types of investors, except the ones with high quality asset: $c_{2H}(x^o) < c^I_{2H}(x^{**})$. Therefore, the central solution is not ex-post Pareto dominant. ■

Price effect From market clearing, we have $\frac{\partial p_s}{\partial x} < 0$. The effect of the market price on expected utility is given by

$$\frac{\partial EU}{\partial p_s} = \left(\lambda_s \frac{x}{1-x+p_s x} + (1-\lambda_s) \left(\pi_s \frac{x-(1-x)\widehat{R}_s/p_s^2}{xp_s + (1-x)\widehat{R}_s/p_s} - (1-\pi_s) \frac{\widehat{R}_s/p_s^2}{xR_H + (1-x)\widehat{R}_s/p_s} \right) \right)$$

At market equilibrium investment allocation, we have $\frac{\partial EU}{\partial p_s}|_{x=x^*} > 0$. Hence, $\frac{\partial EU}{\partial p_s}|_{x=x^*} \frac{\partial p_s}{\partial x}|_{x=x^*} < 0$, i.e., by decreasing investment allocation, we can increase expected utility.

Government Intervention Suppose the economy is in no-trade equilibrium such that $\frac{(1-\lambda_2)}{(\lambda_2+(1-\lambda_2)\pi_2)} \frac{(1-x)}{x} < r_h$. Then the government intervene by providing liquidity to the market in order to restore trading:

$$\begin{aligned} p_2^G &= \frac{(1-\lambda_2)(1-x)\ +\Lambda}{(\lambda_2+(1-\lambda_2)\pi_2)\ x} = r_h\ +\varepsilon \\ &\Rightarrow \quad \Lambda = (r_h+\varepsilon)(\lambda_2+(1-\lambda_2)\pi_2)\ x-(1-\lambda_2)(1-x) \end{aligned}$$

Total amount of liquidity intervention: $\Lambda = (r_h+\varepsilon)(\lambda_2+(1-\lambda_2)\pi_2)x - (1-\lambda_2)(1-x)$. Tax (per unit of investment) imposed on investors at date t=0, to finance liquidity provision at $t = 0$: $\tau = (r_h+\varepsilon)(\lambda_2+(1-\lambda_2)\pi_2) - (1-\lambda_2)\frac{(1-x)}{x}$

Then investors maximization problem becomes:

$$\begin{aligned} \max_x \quad & \lambda_s \log\left(1-x+p_s^G(1-\tau)x\right) + (1-\lambda_s)\sum_{s=1,2} q_s\left(\pi_s \log\left(x(1-\tau)p_s^G + (1-x)\widehat{R}_s/p_s^G\right) + (1-\pi_s)\log\left(x(1-\tau)R_H + (1-x)\widehat{R}_s\right.\right. \\ s.t \quad & (i)\ \ 0 \le x \le 1 \end{aligned}$$

Proof. where $p_1^G = \max\left(\frac{(1-\lambda_1)(1-x)}{(\lambda_1+(1-\lambda_1)\pi_1)x}, \widehat{R}_1\right)$ and $p_2^G = r_h+\varepsilon$. ■