Which Bank is the "Central" Bank? An Application of Markov Theory to the Canadian Large Value Transfer System.*

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Abstract

In a modern financial system the importance of a given institution depends on both its individual characteristics as well as the nature of its relationships with other financial institutions. In this paper we examine the network defined by the credit controls in the Canadian Large Value Transfer System (LVTS). We provide a ranking with respect to the predicted liquidity holdings. We define these liquidity holdings as functions of the network structure defined by the credit controls in LVTS. An institution is deemed most important if, based on our network analysis, it is predicted to hold the most liquidity. In addition we provide a unique measure of how fast an institution is in terms of processing its payments.

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"Why do I rob banks? Because that's where the money is." – Willie Sutton

PRELIMINARY

1 Introduction

Recently, economists have argued that the importance of banks within the financial system cannot be determined in isolation. In addition to its individual

*The views expressed here are those of the authors and not necessarily those of the Bank of Canada, the Federal Reserve Bank of New York or the Federal Reserve System.

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characteristics, the position of a bank within the banking network matters.[1] In this paper we examine the payments network defined by credit controls in the Canadian Large Value Transfer System (LVTS). We provide a ranking with respect to predicted liquidity holdings, which we derive from the network structure. A bank is deemed most important if, based on our network analysis, it is predicted to hold the most liquidity.

We focus on the Tranche 2 component of the LVTS.[2] In this component, participants set bilateral credit limits (BCLs) with each other that determine, via these limits and an associated multilateral constraint, the maximum amount of money any one participant can transfer to any other without offsetting funds. Because banks start off the day with zero outside balances, these credit limits define the initial liquidity holdings of banks.[3] However, as payments are made and received throughout the day the initial liquidity holdings are shuffled around in ways that need not conform to the initial allocation. Banks with high credit limits may not be major holders of liquidity throughout the day if they make payments more quickly than they receive them. Likewise, banks that delay in making payments may tie up large amounts of liquidity even though they have a low initial allocation. Hence, knowledge of the initial distribution alone does not tell us how liquidity is allocated throughout the day, nor does it provide us with the desired ranking.

[1]Allen and Gale (2000) analyze the role network structure plays in contagion of bank failures caused by preference shocks to depositors in a Diamond-Dybvig type model and find more complete networks are more resilient. Bech and Garratt (2007) explore how the network topology of the underlying payment flow among banks affects the resiliency of the interbank payment system.

[2]See Arjani and McVanel (2006) for an overview of the Canadian LVTS.

[3]This is not the case in all payment systems. In Fedwire opening balances are with the exception of discount window borrowing and a few accounting entries equal to yesterday's closing balance. In CHIPS each participant has a pre-established opening position requirement, which, once funded via Fedwire funds transfer to the CHIPS account, is used to settle payment orders throughout the day. The amount of the initial prefunding for each participant is calculated weekly by CHIPS based on the size and number of transactions by the participant. A participant cannot send or receive CHIPS payment orders until it transfers its opening position requirement to the CHIPS account.

In order to predict the allocation of liquidity in the LTVS we apply a well known result from Markov chain theory, known as the Perron-Frobenius theorem. This theorem outlines conditions under which the transition probability matrix of a Markov chain has a stationary distribution.

In the present application, we define a transition probability matrix for the LVTS using the normalized BCL vectors for each bank. This approach is based on the premise that money flows out of a bank in the proportions given by the BCLs the bank has with the other banks. We also allow the possibility that banks will hold on to money. This is captured by a positive probability that money stays put. Assuming money flows through the banking system in a manner dictated by our proposed transition probability matrix, the values of its stationary vector represent the fraction of time a dollar spends at each location in the network. The bank with the highest value in the stationary vector is predicted to hold the most liquidity and is thus the most "central" bank.

An attractive feature of our application of Markov chain theory is that it allows us to estimate an important, yet unobservable characteristic of banks, namely, their relative waiting times for using funds. The Bank of Canada observes when payments are processed by banks, but does not know when the underlying payment requests arrive at the banks. We are able to estimate these wait times using a Bayesian framework. We find that processing speed plays a significant factor in explaining the liquidity holdings and causes the ranking of banks in terms of predicting liquidity holdings to be different from initial distribution of liquidity.

Once we have estimates for the wait times we are able to see how well the daily stationary distributions match the daily observed distributions of liquidity. We find that they match closely. This validates our approach and suggests that Markov analysis could be a useful tool for examining the impact of changes in credit policies (for example a change in the system wide percentage) by the central bank on the distribution of liquidity in the LVTS and for examining the effects of changes in the credit policies of individual banks.[4]

Our approach has much in common with Google's PageRank procedure, which was developed as a way of ranking web pages for use in a search engine by Sergey Brin and Larry Page.[5] In the Google PageRank system, the ranking of a web page is given by the weighted sum of the rankings of every other web page, where the weights on a given page are small if that page points to a lot of places. The vector of weights associated with any one page sum to one (by construction), and hence the matrix of weights is a transition probability matrix that governs the flow of information through the world wide web. Google's PageRank ranking is the stationary vector of this matrix (after some modifications which are necessary for convergence).

The potential usefulness of Markov theory for examining money flows was proposed by Borgatti (2005). He suggests that the money exchange process (between individuals) can be modelled as a random walk through a network, where money moves from one person to any other person with equal probability. Under Borgatti's scenario, the underlying transition probability matrix is symmetric. Hence, as he points out, "the limiting probabilities for the nodes are proportional to degree." The transition probability matrix defined by the BCLs and the patience parameters of banks is not symmetric and hence, this proportionality does not hold in our application.

Others have looked at network topologies of banking systems defined by observed payment flows. Boss, Elsinger, Summer, and Thurner (2004) used Austrian data on liabilities and Soramäki, Bech, Arnold, Glass, and Beyeler

[4]Progress along these lines will require a model of how banks choose credit limits. We are working on such a model.

[5]The PageRank method has also been adapted by the founders of Eigenfactor.org to rank journals. See Bergstrom (2007)

(2006) used U.S. data on payment flows and volumes to characterize the topology of interbank networks. These works show that payment flow networks share structural features (degree distributions, clustering etc.) that are common to other real world networks and, in the latter case, discuss how certain events (9/11) impact this topology. In terms of methodology our work is completely different from these works. We prespecify a network based on fixed parameters of the payment system and use this network to predict flows. The other papers provide a characterization of actual flows in terms of a network.

2 Data

The data set used in the study consists of all tranche 2 transactions in the LVTS from October 1st 2005 to October 1st 2006. This data set consists of 272 days in which the LVTS was running.

The participants in the sample consist of members of the LVTS and the Bank of Canada. For the purposes of this study we exclude the Bank of Canada since it does not send any significant payments in tranche 2.[6]

2.1 Credit Controls

The analysis uses data on daily cyclical bilateral credit limits set by the fourteen banks over the sample period. Sample statistics for the daily cyclical limits are presented in Table 1. BCLs granted by banks vary by a large amount (at least an order of magnitude). The BCLs are fairly symmetric since the min through the 50th percentile of absolute differences of the BCLs between pairs of banks are zero and even the 75 percentile of the cyclical is only 16 million compared to the average cyclical BCL of 699 million. While it is not evident from table 1, BCLs vary across pairs of banks by a large amount (at least an order of

[6]We discuss implications of this in section 3.

	BCLs	abs diff
min	0.00	0.00
25 percentile	50.00	0.00
median	200.00	0.00
mean	417.33	59.47
75 percentile	698.64	16.33
max	2464.68	1201.10
std. dev.	495.81	182.50

Table 1: Daily cyclical limits in millions of dollars

magnitude) in some instances.

3 Initial Versus Average Liquidity Holdings

Let W_t denote the array of Tranche 2 debt caps (or BCLs) in place at time t, where element w_{ijt} denotes the BCL bank j has granted to bank i on date t. The initial distribution of liquidity is determined by the bilateral debt caps that are in place when the day begins. By taking the row sum of the matrix W_t, we obtain the sum of bilateral credit limits granted to bank i. However, a bank's initial payments cannot exceed this amount times the system wide percentage, which is currently 24%. Using the notation from Arjani and McVanel (2006), let

$$T2NDC_{it} = .24 * \sum_j w_{ijt}, \tag{1}$$

denote the tranche 2 multilateral debit cap of bank i on date t. Since we are summing over the BCLs that each bank $j \neq i$ has granted to bank i, this is the conventional measure of the status (a lá Katz) of bank i as determined by the "opinions" of all the other banks. The BCL bank j grants to i defines i's ability to send payments to j. Hence, in terms of the weighted, directed network associated with W_t, w_{ijt} is the weight on the directed link from i to j. Hence, $T2NDC_{it}/.24$ is also the (weighted) outdegree centrality of bank i on date t.

The multilateral debt caps specified in (1) represent the amount of liquidity available to each bank for making payments at the start of the day. Thus, the initial distribution of liquidity on date t is $d_t = (d_{1t}, ..., d_{nt})$, where

$$d_{it} = \frac{T2NDC_{it}}{\sum_{j=1}^{n} T2NDC_{jt}},\ i = 1, ..., n.$$

During the day, however, the liquidity holdings of bank i changes to reflect payments made and received. The average amount of liquidity that bank i holds on date t, denoted Y_{it}, is the time weighted sum of their balance in tranche 2 and the maximum cyclical T2NDC on date t. To compute this we divide the day into T (not necessarily equal) time intervals, where T is the number of transactions that occurred that day. Then

$$Y_{it} = \sum_{j=1}^{T} b_{itj}\delta_{j,j+1} + T2NDC_{it} \tag{2}$$

where $\delta_{j,j+1}$ is the length of time between transaction j and $j+1$ and b_{itj} is i's aggregate balance of incoming and outgoing payments on date t following transaction j.

In a closed system the aggregate payment balances at any point must sum to zero across all participants. Therefore the total potential liquidity in the system is the sum of the T2NDCs. In practice this is not quite true since the Bank of Canada is also a participant in the LVTS and acts as a drain of liquidity in tranche 2. Specifically, the Bank of Canada receives payments on behalf of various other systems (e.g. Continuous Linked Settlement (CLS) Bank pay-ins). Therefore, in practice the summation of net payments across participants sums to a negative number; since the Bank of Canada primarily uses tranche 1 for outgoing payments. To account for this drain, we use as our definition of liquidity in the system at any one time the summation, across all banks, of (2).

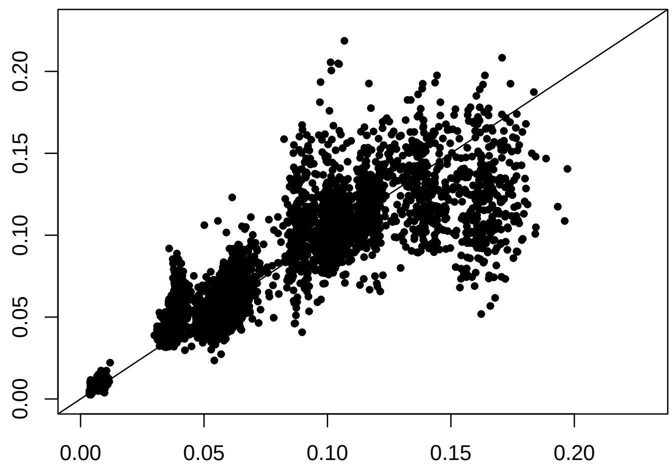

Figure 1: Initial versus average liquidity holdings.

Thus, the average share of total liquidity that i has on date t is equal to

$$y_{it} = \frac{Y_{it}}{\sum_{i=1}^{14} Y_{it}}. \tag{3}$$

The vector $y_t = (y_{1t}, .., y_{nt})$ is our measure of the observed date t distribution of money holdings for the n banks.

A comparison of the initial distribution of liquidity, d_t, to the observed daily liquidity holdings, y_t, over the 272 days of the sample period is shown in Figure 1. Each point in the figure represents a matching initial and observed value (the former is measured on the horizontal axis and the latter is measured on the vertical axis) for a given bank on a given day. Hence, there are $272 \times 14 = 3808$ points on the graph. If the two distributions matched exactly all the points would lie on the 45 degree line.

We will present a formal (statistical) comparison of the two distributions in a future revision. For now, we point out that the worst match between the two distributions occurs for the points on the far right. This vertical clustering below the 45 degree line reflects the fact that for some banks the value in the initial distribution is almost always greater than the observed liquidity holdings over the day. This occurs because, as we shall see in section 5, these banks, in particular bank 11, are speedy payment processors.

4 Markov Analysis

We begin with the weighted adjacency matrix W defined from the BCLs in Section 3 and normalize the components so that the rows sum to one. That is, we define the stochastic matrix $W^N = [w_{ij}^N]$, where

$$w_{ij}^N = \frac{w_{ij}}{\sum_j w_{ij}}. \tag{4}$$

Row i of W^N is a probability distribution over the destinations of a dollar that leaves bank i that is defined using the vector of BCLs granted to bank i from all the other banks. Conditional on the fact that a dollar leaves bank i, its movement is described by the matrix W^N. However, we need to make an important modification to address the fact that banks sometimes delay in processing payment requests.

Delay is accounted for by (i) specifying delay probabilities θ_i for each bank i and (ii) re-scaling the off-diagonal elements of W^N to make these the appropriate conditional probabilities. Specifically, we create a new stochastic matrix $B = [b_{ij}]$, where

$$b_{ii} = \theta_i,\ i = 1, ..., n,\ \text{and } b_{ij} = (1 - \theta_i) w_{ij}^N \text{ for } i \neq j. \tag{5}$$

The delay parameters θ_i can be interpreted as the probability that bank i sends a payment to itself. These are allowed to differ across banks.

By the Perron-Frobenius theorem (see, for example, Seneta (1981, chapter 1) we know that the power method applied to the matrix B converges to a unique, positive stationary vector from any starting point so long as B is stochastic, irreducible and aperiodic. These conditions are met by construction and because of the high degree of connectedness of banks in the LVTS.[7] Given a vector of delay parameters $\theta = (\theta_1, ..., \theta_n)$, the desired stationary vector, which we denote by $x(\theta)$, is the leading (left) eigenvector of B:

$$x^T(\theta) = x^T(\theta)B.$$

Where do the θ_i's come from? Unfortunately data is available on when payment requests are processed, but not on when they were first received by the bank. Hence, we do not have data on the delay tendencies of each bank. Consequently we estimate the delay parameters using our assumption that on average the distribution of liquidity in the system throughout the day achieves the stationary distribution that corresponds to the transition probability matrix B.

5 Estimation of the Delay Parameters

Let t denote day t in the sample period. Then, for each day of the sample we can compute:

$$x_t^T(\theta) = x_t^T(\theta)B_t. \tag{6}$$

We want to choose the vector θ so that over the sample period the eigenvectors defined by (6) are as close as possible to the observed distributions of liquidity.

[7] In the case of Google, many pages exist which do not link to other pages and hence the transition probability matrix constructed from the world wide web using links is only substochastic. Moreover, this hyperlink matrix, as it is called in Langville and Meyer (2006), is neither irreducible nor aperiodic. Hence, modifications of the initial hyperlink matrix are required to derive the Google rankings.

5.1 Bayesian Estimation Procedure

Our model of the observable distribution of liquidity is

$$y_{it} = x_{it}(\theta) + \epsilon_{it}, \tag{7}$$

where θ is the vector of unknown diagonal parameters of B, y_{it} is the observed amount of liquidity being held by bank i on date t, $x_{it}(\theta)$ is the stationary amount of liquidity held by i on date t according to (6), and ϵ_{it} is the forecast error, which has a mean zero symmetric distribution.

In this preliminary exploration we are interested in explaining mean levels of liquidity as opposed to the forecast errors. Therefore, for the moment we assume a simple distribution of errors that is independent across observations.[8] The process of finding the unobservable θs can be done either via a GMM estimation or via a Bayesian framework; the latter is described below.

The family of distributions used for the forecast error is the normal family with precision τ.[9] In this case the likelihood for an observation is

$$L(y_{it}|\theta, B_t, \tau) = N(y_{it}|x_{it}(\theta), \tau).$$

Assuming independence of the errors, a likelihood for the whole sample is

$$L(\{y_{it}\}_{t=1}^{T}|\theta, \{B_t\}_{t=1}^{T}, \tau) = \prod_{t=1}^{T}\prod_{i=1}^{n} L(y_{it}|\theta, B_t, \tau).$$

We assume a flat uniform prior on the θs and a diffuse Gamma prior on the precision with a shape parameter of 1/2 and a scale parameter of 2. The former distribution embodies our lack of information about the θs and the

[8] A plausible next step would be to include the correlations between the errors on a given data t induced by the fact that $y_{it][}$s have to sum to one. Given the difficulty in estimating the mean parameters estimating these covariance parameters will be left for a later exercise.

[9] The precision is just the inverse of the variance.

latter distribution embodies our lack of information of the error term, and also exploits the conjugacy of the normal-gamma likelihood.

The MCMC algorithm used to calculate the above model is a Metropolis-in-Gibbs. The first block is a draw of τ (conditional on the current realization of the θs) from its posterior distribution of Gamma with the scale parameter of $1/2+nT$ where nT is the total number observations, and a shape parameter of $1 + \text{SSE}$ where SSE is the sum of squared errors (i.e the sum of squared differences between the cash distribution and the stationary distribution). The second block is a random walk Metropolis-Hastings step to draw a realization of the θs conditional on the current realization of τ. The proposal density is a multivariate normal distribution with mean of the current θs and a covariance matrix tuned so that the acceptance probability is approximately 25%-30%. The drawing procedure consists of simultaneously drawing the mean of the θs, which is denoted $\bar{\theta}$, and then drawing deviations of this mean, which are denoted $\theta_{\epsilon,i}$. An individual θ is then defined as

$$\theta_i = \bar{\theta} + \theta_{\epsilon,i},$$

$i = 1, ..., n$. This allows good movement along the likelihood surface as described by Gelman and Hill (2007).

6 Empirical Results

The algorithm was started at θ_i equal to 0.5 for all banks except for bank eleven which was set at (roughly) 0.3. After this, the MCMC algorithm was run for 530,000 iterations and a posterior sample was collected.[10] The first 30,000 iterations were discarded as a burn-in phase. Total computing time was roughly 84 hours.

[10]The identification problems discussed below necessitate the large amount of iterations.

Bank	θ_i	Lower 95% HPD	Upper 95% HPD
1	0.3126	0.2396	0.4538
2	0.2285	0.0178	0.4632
3	0.3305	0.2580	0.4682
4	0.3251	0.2344	0.4677
5	0.4220	0.0357	0.7454
6	0.3815	0.0809	0.6019
7	0.1992	0.0921	0.3671
8	0.3348	0.2611	0.4721
9	0.4131	0.3400	0.5359
10	0.4154	0.3504	0.5369
11	0.0778	0.0021	0.2649
12	0.3591	0.2867	0.4923
13	0.4158	0.3222	0.5438
14	0.4962	0.4287	0.6015

Table 2: Posterior Averages

The posterior sample averages and the 95% HPDs are presented in Table 2. Precise estimates of θ have a fairly large amount of uncertainty to them. This is due to an identification problem in how the θs are defined. This comes from the fact that if all θs are identical (say zero) then the stationary distribution that comes from this set of θs will be the same as that from any other identical vector of θs. This holds for the case when all θs are identical and not equal to one. Another issue is that the surface of the likelihood is very flat in certain directions (e.g. the direction of the unit vector) and falls off rapidly in other directions. Because of this the sampler can only move slowly around the surface of the likelihood.[11]

The most striking feature is the degree of heterogeneity among the estimates. Looking at the ratio of estimates for banks 14 and 11, for instance, we see that bank 14 is over 6 times more likely to delay in making a payment than bank 11. We do not, at this point, attempt to explain the variation in the θs. However, we do note that there does seem to exist a negative relationship

[11]This is a problem of the likelihood not the method. In a classical exercise, like GMM, the optimizer would get stuck at non-optimal points since as the optimizer gets close to (for example) the unit vector it will stop moving (or slow down in its movements) due to the flatness.

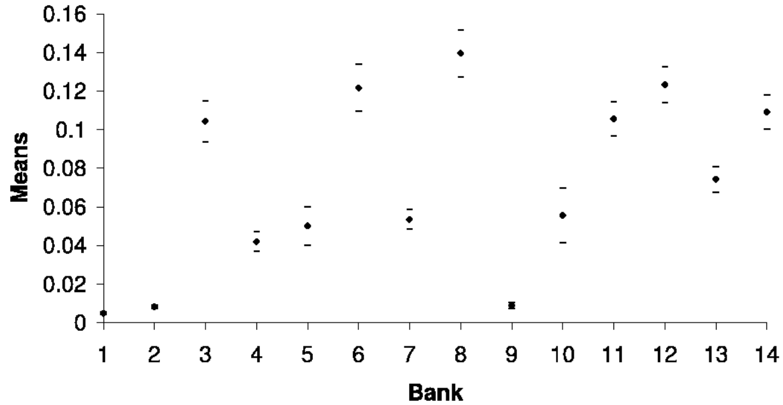

Figure 2: Average stationary distribution of liquidity for each bank along with bar-indicators at 2 standard deviations above and below each mean value.

between delay tendencies and initial liquidity holdings. Classical ordinary least squares regression of the average initial distribution on the θ_is provides estimates of .4056 for the intercept (standard deviation equals 0.0472) and -0.9669 for the slope (standard deviation equals 0.5442). This suggests that banks with higher liquidity holdings delay less, however this is not quite significant at the 10% confidence level (The p-value of the slope of the trend line is .1009).

Figure 2 shows a plot of the average stationary distribution of liquidity for each bank along with bar-indicators at 2 standard deviations above and below each mean value (As a benchmark, note that in case of the normal distribution these bars would include about 95% of the observations used to compute the mean). In terms of ranking frequencies, bank 1 has the highest predicted liquidity on 260 of 272 days and in contrast the similarly sized bank 11 is the highest on 5 days.

Insight into the differences between bank 1 and 11 can be seen by looking at Table 2. Bank 11 has a delay parameter of only .0778 compared to .3126 for bank 1. Hence, despite its relatively lower level of initial liquidity bank 1 is over 4 times more likely to hold onto liquidity sent to it than bank 11, and hence bank 1 holds more liquidity over the course of the day. The difference between average liquidity according to the initial and stationary distributions

Bank	Difference
1	0.0025
2	0.0012
3	-0.0018
4	-0.0002
5	-0.0006
6	-0.0008
7	0.0086
8	-0.0023
9	-0.0103
10	-0.0140
11	0.0428
12	-0.0053
13	-0.0059
14	-0.0140

Table 3: Difference between average initial and average stationary distributions.

for all banks are shown in Table 3.

6.1 Comparison of the stationary distribution to the observed distribution of liquidity.

Figure 3 shows the daily stationary distributions (using the posterior means for the θ vector) and the daily observed liquidity distributions over the 272 days of the sample period.[12] Each point in the figure represents a matching stationary distribution value and observed value (the former is measured on the horizontal axis and the latter is measured on the vertical axis) for a given bank on a given day. Hence, as in Figure 1, there are 3808 points on the graph and if the two distributions matched exactly all the points would lie on the 45 degree line. Different colors represent different banks.

Compared to Figure 1, which involves the initial distribution of liquidity, there is improved clustering around the 45 degree line. In particular, the cluster of points associated with the fastest processor, bank 11, (magenta) is centered

[12] An animated presentation of the data is available at http://www.econ.ucsb.edu/∼garratt/daily.m1v.

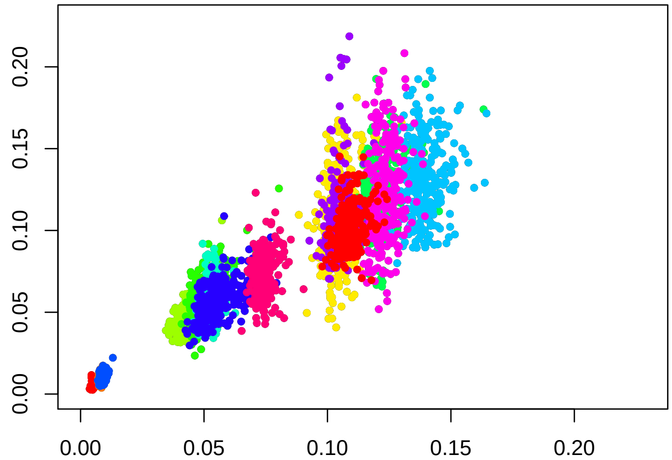

Figure 3: Actual liquidity and the stationary distribution at the posterior average values of θ.

closely on the 45 degree line. In figure 1 bank 11 was one of the several banks which contributed to the vertical clustering below the forty-five degree line. This was due to the fact that in figure 1 the speed with which bank 11 (among others) processes payments was not taken into account. Again, formal statistical analysis will follow.

7 Conclusion

In this paper we have developed an empirical measure of which banks in the Canadian LVTS payment system are likely to be holding the most liquidity at any given time. This measure is based on the implicit network structure defined by the BCLs that LVTS members grant each other.

Our measure of predicted liquidity is based on the idea that credit limits are a good indicator of likely liquidity flows. This idea is born out by comparing predicted liquidity with the realized average liquidity. One crucial parameter that we estimate is an unobserved processing speed parameter. We then show that when processing speed is taken into account our measure of predicted liquidity is a good predictor of average daily liquidity.

While we estimate a constant unobserved processing speed parameter it is probable that the processing speed varies by day.[13] In future work we plan to estimate a daily processing speed per bank and compare this to our constant parameter.

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[13] We thank Thor Koeppl for pointing this out.

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