Comments: Modeling Default Correlation

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Outline of Presentation

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The Importance of the topic

The modeling of default dependence is critical both for risk management and for pricing credit structures.

In risk management it is well known that a small change in default correlation can have a major impact on the value-at-risk.

For pricing different types of credit structures, such as collateralized debt obligations, it is well known that a small change in default correlation can have a major impact on the prices of different tranches.

The Importance of the topic

The critical issue is the modeling of default dependence. For n obligors, we want to specify the joint distribution of the stopping times. This is a non trivial issue.

For example, a low interest rate environment generates a search for yield enhancement. This generates demand for the securitization of high yield assets, such as subprime mortgages, auto loans, and credit cards. A decline of lending standards, failure to use up-date data, declining house prices generates a increase in delinquencies. This eventually lends to closing of asset markets closing, tighter credit conditions, home builders defaulting, banks failing etc. How do you model this type of outcome?

If a competitor fails, is that good news or bad?

Not withstanding the importance of the modeling default dependence, there is little to guide us to an appropriate methodology.

Contributions

This paper examines the properties of three measure of correlation

1. Base correlation, assuming a normal copula;
2. The correlation of an implied year average intensity function
3. Equity return correlation.

They find that all three measures are time varying and there is significant correlation between the measures.

Questions

After reading this paper I had the following questions.

1. What is the relationship between the three measures. The authors never directly address this issue.
2. Why is there time variation? Again, the authors never explain why time variation exists.
3. Why are the different measures correlated? The authors provide empirical evidence that they are related, but never explain why.

Modeling Default Dependence

Two approaches have been employed: structural models and starting with Merton (1974) and Black and Cox (1976) and reduced form starting with Jarrow and Turnbull (1992, 1995) and Lando (1994).

The Merton model provides the foundation for the modeling of default dependence in CreditMetrics and in the pricing of structural products, as will be explained.

Lando (1994) introduced modeling the intensity function as a Cox process. One of the properties of Cox processes is that if you condition on the path of the covariates, then you have conditional independence.

Modeling Default Dependence: reduced form

Das, Duffie, Kapadia and Saita (2007) test a joint hypothesis. Given a well specified intensity intensity function, then by conditioning on the path of the covariates, defaults are independence.

They reject the joint hypothesis and argue that the issue is the assumption of a Cox process.

However, Fan Yu (2005) argues that a well specified intensity function can generate the observed levels of default dependence observed in the data.

The authors of this paper never look at this approach, citing the Das et al paper as justification.

I would suggest that they revisit this approach, though this might take the paper in other directions.

Normal Copula

The normal copula has become the work horse in credit the area.

For n random variables and with cumulative marginal distribution functions

$$F(T_j) = P(\Gamma_j \le T_j)$$

j=1,..n, the normal copula is defined as.,

$$P[\Gamma_1 \le T_1, ..., \Gamma_n \le T_n) = N_n[N^{-1}(F_1(T_1)), ..., N_\upsilon^{-1}(F_n(T_n)), \Sigma]$$

The risk management model in CreditMetrics and Li (2000) assumed a normal copula. The marginal distributions are inferred by using credit ratings and the logic behind the Merton (1974) model.

For pricing credit structures, the marginal distributions are inferred from credit default swap prices.

Normal Copula

The critical question is the specification of the covariance matrix.

By appealing to Merton (1974), the correlation matrix is estimated using equity returns.

For base correlation, the non-diagonal elements of the correlation matrix are assumed to a constant denoted by ρ.

A separate ρ is estimated for each tranche.

The copula model is very much a static, mechanical type of approach.

The only inputs that change are the marginal distributions.

The base correlation is the only degree of freedom to describe default dependence.

Five Year Average Intensity

Use credit default prices for an obligor to infer the term structure of intensity functions. Calculate the five year survival probability

$$S(t, t+5) \equiv \exp(-\bar{\lambda}\, 5)$$

and the average intensity.

Question: Given you have 1,3, 5 and 7 year CDS prices, did you look at the dynamics of different parts of the terms structure for a given obligor? and across obligors? Do part of the term structure of survival probabilities have different properties?

More Questions

One of the difficulties I had with this paper is knowing how to interpret the results.

For example, suppose we used a simple barrier model with a jump to default. A closed form solution exists for the survival probability.

The unknowns are

the volatility of equity

the barrier

the intensity of the jump to default.

How does one calibrate these parameters?

We could use the term structure of CDS prices for a particular obligor each day and test whether the calibrated parameters are stationary?

More Questions

What do you do about a portfolio of names?

We know that a normal copula is unable to price the different tranches, so it comes as no surprise that base correlation varies with the tranches and over time. The question is why. The authors should provide evidence to answer this question.

If you are going to assume a complex structure for the dynamics of volatility, then your simple pricing models for individual obligors disappears.

Summary

Default dependence is a very important topic, both for risk management and pricing.

It is not clear how to model default dependence. Hence, it is important that we learn more about the properties of different measures.

This paper is an interesting contribution to this important area.