Mo	tivation

Bond Premium in a DSGE Model

EZ Preferences

Long-Run Risks

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Conclusions o

# The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

Glenn D. Rudebusch Eric T. Swanson

Economic Research Federal Reserve Bank of San Francisco

Conference on Fixed Income Markets Bank of Canada September 13, 2008

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2 The Bond Premium in the Standard New Keynesian Model

- 3 Epstein-Zin Preferences
- 4 Long-Run Risks
- 5 Conclusions

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The Bor	nd Premium Puzzle	e		

The equity premium puzzle: excess returns on stocks are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Mehra and Prescott, 1985).

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The bond premium puzzle: excess returns on long-term bonds are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Backus, Gregory, and Zin, 1989).

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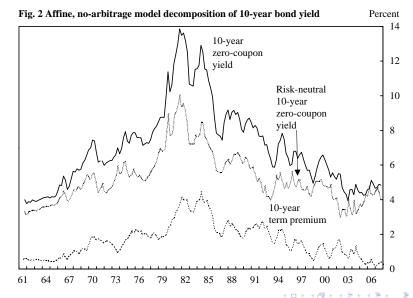
The bond premium puzzle: excess returns on long-term bonds are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Backus, Gregory, and Zin, 1989).

Note:

• Since Backus, Gregory, and Zin (1989), DSGE models with nominal rigidities have advanced considerably

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks 0000000	Conclusions o
Why Stu	udy the Term Prem	ium?		

The term premium is important:

• DSGE models increasingly used for policy analysis; total failure to explain term premium may signal flaws in the model

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 many empirical questions about term premium require a structural DSGE model to provide reliable answers

Motivation 00●00	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ೦೦೦೦೦೦೦	Conclusions o
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- many empirical questions about term premium require a structural DSGE model to provide reliable answers

The equity premium has received more attention in the literature, but the term premium:

- provides an additional perspective on the model
- tests nominal rigidities in the model
- only requires modeling short-term interest rate process, not dividends

applies to a larger volume of U.S. securities

Motivation Bo	ond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions
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- Wachter (2005)
  - can resolve bond premium puzzle using Campbell-Cochrane preferences in endowment economy

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We examine to what extent the Piazzesi-Schneider results generalize to the DSGE case

Motivation 0000●	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions o
Related	Strands of the Lit	erature		

The Bond Premium in a DSGE Model:

 Backus-Gregory-Zin (1989), Donaldson-Johnson-Mehra (1990), Den Haan (1995), Rudebusch-Swanson (2008)

Epstein-Zin Preferences and the Bond Premium in an Endowment Economy:

 Piazzesi-Schneider (2006), Colacito-Croce (2007), Backus-Routledge-Zin (2007), Gallmeyer-Hollifield-Palomino-Zin (2007), Bansal-Shaliastovich (2008)

Epstein-Zin Preferences in a DSGE Model:

 Tallarini (2000), Croce (2007), Levin-Lopez-Salido-Nelson-Yun (2008)

Epstein-Zin Preferences and the Bond Premium in a DSGE Model:

• van Binsbergen-Fernandez-Villaverde-Koijen-Rubio-Ramirez (2008)

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# The Term Premium in a Standard DSGE Model



## The Bond Premium in the Standard New Keynesian Model

- Define Standard New Keynesian DSGE Model
- Review Asset Pricing
- Solve the Model
- Results with the Standard Model

Motivation 00000	Bond Premium in a DSGE Model ●ooooooo	EZ Preferences	Long-Run Risks	Conclusions o
New Ke	ynesian Model (Ve	ery Standa	lrd)	

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \right)$$

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standard model:  $h_t \equiv bC_{t-1}$ 

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standard model:  $h_t \equiv bC_{t-1}$ 

Stochastic discount factor:

$$m_{t+1} = \frac{\beta (C_{t+1} - bC_t)^{-\gamma}}{(C_t - bC_{t-1})^{-\gamma}} \frac{P_t}{P_{t+1}}$$

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Parameters:  $\beta$  = .99, b = .66,  $\gamma$  = 2,  $\chi$  = 1.5

Motivation	Bond Premium in a DSGE Model o●oooooo	EZ Preferences	Long-Run Risks	Conclusions o
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#### New Keynesian Model (Very Standard)

Continuum of differentiated firms:

- face Dixit-Stiglitz demand with elasticity  $\frac{1+\theta}{\theta}$ , markup  $\theta$
- set prices in Calvo contracts with avg. duration 4 quarters
- identical production functions  $y_t = A_t \bar{k}^{1-\alpha} I_t^{\alpha}$
- have firm-specific capital stocks
- face aggregate technology  $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

Parameters  $\theta = .2$ ,  $\rho_A = .9$ ,  $\sigma_A^2 = .01^2$ 

Perfectly competitive goods aggregation sector

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# New Keynesian Model (Very Standard)

Government:

- imposes lump-sum taxes G<sub>t</sub> on households
- destroys the resources it collects

• 
$$\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \overline{G} + \varepsilon_t^G$$

Parameters  $\bar{G} = .17 \bar{Y}$ ,  $\rho_G = .9$ ,  $\sigma_G^2 = .004^2$ 

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# New Keynesian Model (Very Standard)

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$$\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon_t^G$$

Parameters  $\bar{G} = .17 \bar{Y}$ ,  $\rho_G = .9$ ,  $\sigma_G^2 = .004^2$ 

Monetary Authority:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [1/\beta + \pi_t + g_y(y_t - \bar{y}) + g_\pi(\bar{\pi}_t - \pi^*)] + \varepsilon_t^i$$

Parameters  $\rho_i = .73$ ,  $g_y = .53$ ,  $g_{\pi} = .93$ ,  $\pi^* = 0$ ,  $\sigma_i^2 = .004^2$ 

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Asset P	ricing			

Asset pricing:

 $p_t = d_t + E_t[m_{t+1}p_{t+1}]$ 



Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks 0000000	Conclusions o
Asset F	Pricing			

Asset pricing:

$$p_t = d_t + E_t[m_{t+1}p_{t+1}]$$

Zero-coupon bond pricing:

$$p_t^{(n)} = E_t[m_{t+1}p_{t+1}^{(n-1)}]$$
$$i_t^{(n)} = -\frac{1}{n}\log p_t^{(n)}$$

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Notation: let  $i_t \equiv i_t^{(1)}$ 

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## The Term Premium in the Standard NK Model

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# The Term Premium in the Standard NK Model

In DSGE framework, convenient to work with a default-free consol,

Motivation	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions
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In DSGE framework, convenient to work with a default-free *consol*, a perpetuity that pays \$1,  $\delta_c$ ,  $\delta_c^2$ ,  $\delta_c^3$ , ... (nominal)

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Price of the consol:

$$\widetilde{p}_t^{(n)} = 1 + \delta_c \, E_t m_{t+1} \widetilde{p}_{t+1}^{(n)}$$

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Risk-neutral consol price:

$$\widehat{p}_t^{(n)} = 1 + \delta_c \, e^{-i_t} E_t \widehat{p}_{t+1}^{(n)}$$

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$$\widehat{oldsymbol{
ho}}_t^{(n)} = oldsymbol{1} + \delta_c \, oldsymbol{e}^{-i_t} oldsymbol{E}_t \widehat{oldsymbol{
ho}}_{t+1}^{(n)}$$

Term premium:

$$\psi_t^{(n)} \equiv \log\left(\frac{\delta_c \widetilde{p}_t^{(n)}}{\widetilde{p}_t^{(n)} - 1}\right) - \log\left(\frac{\delta_c \widehat{p}_t^{(n)}}{\widehat{p}_t^{(n)} - 1}\right)$$

Motivation	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions o
Solving	the Model			

The standard NK model above has a relatively large numer of state variables:  $C_{t-1}$ ,  $A_{t-1}$ ,  $G_{t-1}$ ,  $i_{t-1}$ ,  $\Delta_{t-1}$ ,  $\bar{\pi}_{t-1}$ ,  $\varepsilon_t^A$ ,  $\varepsilon_t^G$ ,  $\varepsilon_t^i$ 

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Motivation	Bond Premium in a DSGE Model ○○○○○●○○	EZ Preferences	Long-Run Risks 0000000	Conclusions o
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We solve the model by approximation around the nonstochastic steady state (perturbation methods)

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Solving	Solving the Model					

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We solve the model by approximation around the nonstochastic steady state (perturbation methods)

- In a first-order approximation, term premium is zero
- In a second-order approximation, term premium is a constant (sum of variances)
- So we compute a *third*-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes *n*th order approximations

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions o
Results				

In the standard NK model:

- mean term premium: 1.4 bp
- unconditional standard deviation of term premium: 0.1 bp

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Results				

In the standard NK model:

- mean term premium: 1.4 bp
- unconditional standard deviation of term premium: 0.1 bp

Intuition:

• shocks in macro models have standard deviations pprox .01

- 2nd-order terms in macro models  $\sim (.01)^2$
- 3rd-order terms  $\sim (.01)^3$

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Results				

In the standard NK model:

- mean term premium: 1.4 bp
- unconditional standard deviation of term premium: 0.1 bp

Intuition:

- shocks in macro models have standard deviations  $\approx$  .01
- 2nd-order terms in macro models  $\sim (.01)^2$
- 3rd-order terms  $\sim (.01)^3$

To make these higher-order terms important,

need "high curvature" modifications from finance literature

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• or shocks with standard deviations  $\gg .01$ 

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## Additional Robustness Checks

This basic finding is extremely robust:

- Campbell-Cochrane habits:  $\bar{\psi}^{(10)} =$  2.4 bp, sd( $\psi^{(10)}$ ) = 0.1 bp
- "best fit" parameters:
- Iarger models (CEE):
- models with investment
- internal habits
- markup shocks
- nominal wage rigidities
- real wage rigidities
- time-varying  $\pi_t^*$  (long-run risk)

 $\bar{\psi}^{(10)} = 10.6$  bp, sd( $\psi^{(10)}$ ) = 1.3 bp

$$ar{\psi}^{(10)} =$$
 1.0 bp, sd $(\psi^{(10)}) =$  0.1 bp

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- internal habits
- markup shocks
- nominal wage rigidities
- real wage rigidities
- time-varying  $\pi_t^*$  (long-run risk)

Basic problem: even if agents in these habit-based models are very risk averse, in a DSGE setting they are able to offset the risk that they hate (high-frequency variation in *C*)

 $\bar{\psi}^{(10)} = 10.6 \text{ bp, sd}(\psi^{(10)}) = 1.3 \text{ bp}$ 

$$ar{\psi}^{(10)} =$$
 1.0 bp, sd( $\psi^{(10)}) =$  0.1 bp

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences ●000	Long-Run Risks	Conclusions o
Epstein	-Zin Preferences			

Modify the standard NK model to incorporate Epstein-Zin preferences.

The model then has three key ingredients:

- Intrinsic nominal rigidities
  - makes bond pricing interesting
- 2 Epstein-Zin preferences
  - makes households risk averse
- Long-run risk (productivity or inflation)
  - introduces a risk households cannot offset

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makes bonds risky

Motivation	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions
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Epsteir	-Zin Preferences			

 $V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1}$ 

Motivation	Bond Premium in a DSGE Model	EZ Preferences o●oo	Long-Run Risks 0000000	Conclusions o
Epstein	-Zin Preferences			

$$V_t \equiv u(c_t, I_t) + \beta E_t V_{t+1}$$

Epstein-Zin preferences:

$$V_t \equiv u(c_t, l_t) + \beta \left( E_t V_{t+1}^{1-\alpha} \right)^{1/(1-\alpha)}$$

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Note:

• need to impose  $u \ge 0$ 

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Note:

- need to impose  $u \ge 0$
- or  $u \le 0$  and  $V_t \equiv u(c_t, l_t) \beta (E_t(-V_{t+1})^{1-\alpha})^{1/(1-\alpha)}$

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks 0000000	Conclusions o
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• or 
$$u \leq 0$$
 and  $V_t \equiv u(c_t, l_t) - \beta \left( E_t (-V_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}$ 

We'll use standard NK utility kernel:

$$u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi},$$

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Motivation	Bond Premium in a DSGE Model	EZ Preferences ○○●○	Long-Run Risks 0000000	Conclusions o
Epstein	-Zin Preferences			

Household optimality conditions with EZ preferences:

$$\mu_{t} u_{1}|_{(c_{t}, h)} = P_{t} \lambda_{t}$$

$$-\mu_{t} u_{2}|_{(c_{t}, h)} = w_{t} \lambda_{t}$$

$$\lambda_{t} = \beta E_{t} \lambda_{t+1} (1 + r_{t+1})$$

$$\mu_{t} = \mu_{t-1} (E_{t-1} V_{t}^{1-\alpha})^{\alpha/(1-\alpha)} V_{t}^{-\alpha}, \quad \mu_{0} = 1$$

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences oo●o	Long-Run Risks	Conclusions o
Epstein	-Zin Preferences			

Household optimality conditions with EZ preferences:

$$\mu_{t} u_{1}|_{(c_{t}, l_{t})} = P_{t} \lambda_{t}$$

$$-\mu_{t} u_{2}|_{(c_{t}, l_{t})} = w_{t} \lambda_{t}$$

$$\lambda_{t} = \beta E_{t} \lambda_{t+1} (1 + r_{t+1})$$

$$\mu_{t} = \mu_{t-1} (E_{t-1} V_{t}^{1-\alpha})^{\alpha/(1-\alpha)} V_{t}^{-\alpha}, \quad \mu_{0} = 1$$

Stochastic discount factor:

$$m_{t,t+1} \equiv \frac{\beta u_1 \big|_{(c_{t+1}, l_{t+1})}}{u_1 \big|_{(c_t, l_t)}} \left( \frac{V_{t+1}}{\left( E_t V_{t+1}^{1-\alpha} \right)^{1/(1-\alpha)}} \right)^{\alpha} \frac{P_t}{P_{t+1}}$$

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences ○○○●	Long-Run Risks	Conclusions o
Results				

### Table 2: Empirical and Model-Based Unconditional Moments

Variable	U.S. Data	EU Preferences	"best fit" EZ Preferences
sd[ <i>C</i> ]	1.19	1.42	2.53
sd[ <i>L</i> ]	1.71	2.56	2.21
sd[w <sup>r</sup> ]	0.82	2.08	1.52
$sd[\pi]$	2.52	2.25	2.71
sd[ <i>i</i> ]	2.71	1.90	2.27
sd[ <i>i</i> <sup>(10)</sup> ]	2.41	0.54	1.03
mean[ $\psi^{(10)}$ ]	1.06	.010	1.05
$sd[\psi^{(10)}]$	0.54	.000	.184
mean[ <i>i</i> <sup>(10)</sup> – <i>i</i> ]	1.43	047	0.99
sd[ <i>i</i> <sup>(10)</sup> - <i>i</i> ]	1.33	1.43	1.33
mean[ $x^{(10)}$ ]	1.76	.015	1.04
sd[x <sup>(10)</sup> ]	23.43	6.56	9.02
memo: quasi-CRRA		2	75

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions o
Long-R	un Risks			

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### 4 Long-Run Risks

- Long-Run Inflation Risk
- Long-Run Real Risk

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ●oooooo	Conclusions o
Long-R	un Inflation Risk			

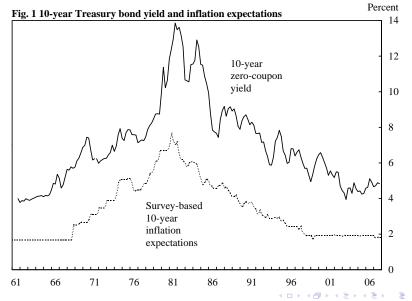
Introduce long-run inflation risk to make long-term bonds more risky:

- same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk
- long-term inflation expectations more observable than long-term consumption growth
- other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary

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## Long-Run Inflation Risk



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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks oo●oooo	Conclusions o
Long-F	Run Inflation Risk			

Suppose:

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + \varepsilon_t^{\pi^*}$$

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks oo●oooo	Conclusions o
Long-R	un Inflation Risk			

Suppose:

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + \varepsilon_t^{\pi^*}$$

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Then:

- inflation is volatile, but not risky
- in fact, long-term bonds act like insurance: when π<sup>\*</sup> ↑, then C ↑ and p<sup>(10)</sup> ↓
- result: term premium is *negative*

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ०००●०००	Conclusions o
Long-R	un Inflation Risk			

Consider instead:

$$\pi_t^* = \rho_{\pi}^* \pi_{t-1}^* + (1 - \rho_{\pi}^*) \theta_{\pi^*} (\overline{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}$$



Motivation	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ०००●०००	Conclusions o
Long-R	Iun Inflation Risk			

Consider instead:

$$\pi_t^* = \rho_{\pi}^* \pi_{t-1}^* + (1 - \rho_{\pi}^*) \theta_{\pi^*} (\overline{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}$$

- $\theta_{\pi^*}$  describes pass-through from current  $\pi$  to long-term  $\pi^*$
- Gürkaynak, Sack, and Swanson (2005) found evidence for  $\theta_{\pi^*} > 0$  in U.S. bond response to macro data releases
- makes long-term bonds act less like insurance: when technology/supply shock, then π ↑, C ↓, and p<sup>(10)</sup> ↓ supply shocks become very costly
- The term premium is *positive*, closely associated with θ<sub>π\*</sub>

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ○○○○●○○	Conclusions o
Results				

#### Table 4: Model-Based Moments with Long-Run Inflation Risk

Variable	U.S. Data	EU Preferences & LR Risk	EZ Prefs & LR Risk
sd[C]	1.19	1.92	1.86
sd[ <i>L</i> ]	1.71	3.33	1.73
sd[w <sup>r</sup> ]	0.82	2.55	1.45
$sd[\pi]$	2.52	5.00	3.22
sd[ <i>i</i> ]	2.71	4.74	2.99
sd[ <i>i</i> <sup>(10)</sup> ]	2.41	3.32	1.94
mean[ $\psi^{(10)}$ ]	1.06	.002	.748
$sd[\psi^{(10)}]$	0.54	.001	.431
mean[ <i>i</i> <sup>(10)</sup> – <i>i</i> ]	1.43	062	.668
sd[ <i>i</i> <sup>(10)</sup> − <i>i</i> ]	1.33	1.60	1.11
mean[x <sup>(10)</sup> ]	1.76	.003	.737
$sd[x^{(10)}]$	23.43	16.96	11.83
memo: quasi-CRRA		2	65

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ○○○○●○	Conclusions o
Long-R	un Productivity Ris	sk		

Following Bansal and Yaron (2004), introduce long-run real risk to make the economy more risky:

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Assume productivity follows:

$$\log A_t^* = \rho_{A^*} \log A_{t-1}^* + \varepsilon_t^{A^*}$$
$$\log A_t = \log A_t^* + \varepsilon_t^A$$
where  $\rho_{A^*} = .98$ ,  $\sigma_{A^*} = .002$ , and  $\sigma_A = .005$ .

- makes the economy much riskier to agents
- increases volatility of stochastic discount factor

Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks ○○○○○●	Conclusions o
Results				

#### Table 3: Model-Based Moments with Long-Run Productivity Risk

Variable	U.S. Data	EU Preferences & LR Risk	EZ Prefs & LR Risk
sd[C]	1.19	0.92	2.95
sd[ <i>L</i> ]	1.71	1.03	1.32
sd[w <sup>r</sup> ]	0.82	1.43	1.90
$sd[\pi]$	2.52	1.12	3.14
sd[ <i>i</i> ]	2.71	1.17	2.88
sd[ <i>i</i> <sup>(10)</sup> ]	2.41	0.65	1.84
mean[ $\psi^{(10)}$ ]	1.06	.005	.872
$sd[\psi^{(10)}]$	0.54	.000	.183
mean[ <i>i</i> <sup>(10)</sup> – <i>i</i> ]	1.43	018	.758
sd[ <i>i</i> <sup>(10)</sup> - <i>i</i> ]	1.33	0.64	1.15
mean[x <sup>(10)</sup> ]	1.76	.005	.859
sd[x <sup>(10)</sup> ]	23.43	4.39	11.59
memo: quasi-CRRA		2	35

Motivation	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions •	
Conclu	Conclusions				

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The term premium in standard NK DSGE models is very small, even more stable

Motivation	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions
Conclus	sions			

- The term premium in standard NK DSGE models is very small, even more stable
- Habit-based preferences can solve bond premium puzzle in endowment economy, but fail in NK DSGE framework: although agents are risk-averse, they can offset that risk

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions •			
Conclusions							

- The term premium in standard NK DSGE models is very small, even more stable
- Habit-based preferences can solve bond premium puzzle in endowment economy, but fail in NK DSGE framework: although agents are risk-averse, they can offset that risk
- Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework:

agents are risk-averse and cannot offset long-run real or nominal risks

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Motivation 00000	Bond Premium in a DSGE Model	EZ Preferences	Long-Run Risks	Conclusions			
Conclusions							

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- Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework: agents are risk-averse and cannot offset long-run real or

nominal risks

Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments

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