Models of foreign exchange settlement and informational efficiency in liquidity risk management

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Abstract: Large, international banking groups have increasingly sought to centralise their cross-currency liquidity management: liquidity shortages in one currency are financed using liquidity surpluses in another currency. We investigate how the degree of coordination in settlement of the associated foreign exchange transactions affects two aspects of banks' liquidity management: the likelihood that a liquidity-short bank does not meet its obligations but declares technical default, and the likelihood that other banks suffer losses if the liquidity-short bank is also hit by a solvency shock. Our main assumption is that intra-group lending takes place under symmetric information, while interbank market loans are extended under asymmetric information. Better coordinated settlement increases the exposure of the intra-group lender relative to the external lender and leads to more informed lending. We find that better coordination increases the incidence of technical default but reduces the likelihood that solvency shocks are transmitted.

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1 Introduction

In 2006, a report by The Joint Forum[1] found that financial groups have adopted a wide range of approaches to liquidity risk management in response to greater internationalisation, a defining characteristic of which is the degree of centralisation. Under local liquidity management, each subsidiary of a financial group maintains a separate pool of liquidity in its local currency and funds its obligations domestically in each market. Under global liquidity management, financial groups also fund liquidity shortfalls (or recycle liquidity surpluses) via intra-group, cross-currency and / or cross-border transfers of liquidity or collateral: there is a global flow of liquidity within the group.

In practice, there are many barriers to this global flow of liquidity. These include: transaction fees; time zone frictions; restrictions on intra-group lending; constraints to accessing intraday credit at foreign central banks; and a lack of connectivity and harmonisation between the various national and international payment and settlement systems. We focus on the design of the settlement infrastructure for the cross-currency transfer of liquidity. Would financial stability benefit from a greater degree of centralisation of liquidity management? And if so, how can the design of the settlement infrastructure facilitate it? These are current topics of discussions at the Bank for International Settlement's Committee for Payment and Settlement Systems (CPSS), and the New York Fed's Payments Risk Committee.

A key design feature is the degree of coordination of the settlements of the two currencies which are exchanged in a foreign exchange (FX) transaction. In July 2007, the CPSS found that an important share of FX transactions continues to settle 'in ways that still generate significant potential risks across the global financial system.' One-third of the \$3.8 trillion that settle every day in FX markets were still subject to such risks. Shocks to an institution could propagate through the financial system: FX exposures to single counterparties typically exceed 5% of an institution's total capital in about one-fourth of the institutions on an average day, and in every second institution on a peak day.[2] According to CPSS, one of the reasons for this risk is the absence of complete coordination of settlement for FX transactions which settle on the day on which they are traded. Such complete coordination would be offered by a simultaneous ('Payment-versus-Payment', or PvP) settlement of both currencies.[3] This paper shows that while there are benefits to increased coordination for same-day settlement of foreign exchange transactions, there may also be costs for financial stability.

[1] The Joint Forum (2006)

[2] CPSS (2007, Table 2). The CPSS's definition of total capital is, approximately, equal to the sum of tier 1 capital (equity and retained earnings), tier 2 capital (supplementary capital) and tier 3 capital (short-term subordinated debt) as defined by the Basel Committee on Banking Supervision.

[3] CLS Bank offers a corresponding service for overnight settlement.

Our starting point is the assumption that only external but not internal (within-group) credit relationships suffer from asymmetric information between the borrower and the lender. When liquidity is managed locally, a liquidity-short bank has to resort to external finance. In contrast, when liquidity is managed globally, such a bank can additionally access foreign-denominated liquidity at a foreign subsidiary by exchanging it into the home currency. We show that this FX transaction involves a mixture of external and internal finance. The higher the degree of coordination in FX settlement, the larger the share of internal finance, and the higher the informational efficiency of the credit relationship. Informational efficiency matters in particular in situations of crises, in which liquidity is needed on the same day, and in which the lender has little time to evaluate the borrower's solvency. Indeed, The Joint Forum report found that most financial groups expect to rely upon intra-group, cross-border and cross-currency transfers in stress situations.

We show that the transition from local to global liquidity management, and better coordination of settlement of FX transactions, have two consequences on financial stability. First, the *transmission* of solvency shocks from one institution to another would be less likely because banks with high solvency risks would not be able to refinance themselves at all in response to liquidity outflows, neither domestically nor via FX transactions. But this implies that these banks would have to delay the payment of their obligations beyond their due-date: they would be in 'technical default'. Hence the second consequence: Technical default might become more likely. These results continue to hold when we endogenise banks' ex-ante liquidity holdings.

The following section reviews related literature. Sections 3-5 present the model's setup and guide the reader through the derivation of the main results. Section 6 shows how these results extend to a comparison of local with global liquidity management. A discussion of the model's main assumptions can be found in section 7. Proofs are presented in the appendix.

2 Related literature

Net redemptions from the banking system are an exception in economies with well-developed financial markets: if liquidity leaves one bank, it generally flows via a payment system directly into the accounts of another bank. If there was no market failure in the domestic interbank market, no bank would ever experience a liquidity crisis, because it could always re-borrow the liquidity it lost. Global liquidity management would not have any advantages.

The key assumption in this paper is that there is a market failure in the domestic interbank market that prevents liquidity from being lent out by a liquidity-rich bank to a liquidity-poor bank.

This market failure is due to asymmetric information and was described as a screening problem by Stiglitz and Weiss (1981). Stiglitz and Weiss argued that borrower/lender relationships are characterised by asymmetric information. Credit rationing occurs in equilibrium because of two effects: First, a bank attracts only riskier borrowers when it increases its interest rate (adverse selection). Second, the borrower might be inclined to increase the risk of his project if the bank cannot perfectly monitor his choice (moral hazard). Gorton and Huang (2004), Mallick (2004) and Skeie (2004) suggest other reasons for market failures; but these are more likely to hold for longer-term exposures. In our model, exposures do not last more than 24 hours; hence our choice to make adverse selection and not moral hazard the cornerstone of the market imperfection in our model. Screening problems appear to be very important in reality: banks generally refuse to grant intraday credit to counterparties which do not have excellent credit status (instead of simply charging them a higher interest rate); and our market intelligence suggests that even high-quality credit institutions do not rely on uncollateralised borrowing in their contingency plans.

There are several other strands of models that dealt with related questions. Manning and Willison (2005) analyse the benefits of the cross-border use of collateral in payment systems. In contrast, we focus on a (more extreme) situation in which banks have no collateralisable assets: thus, banks have to resort to unsecured borrowing and/or an exchange of foreign currency to transfer their foreign liquidity holdings into domestic currency. (See section 7 for a discussion.) The literature on (the limits of) internal capital markets discusses the degree to which internal borrowing takes place under symmetric information and identifies a variety of factors that inhibit the information flow within a multinational company (e.g., Stein (1997), Scharfstein and Stein (2000)). We abstract from these frictions to keep the analysis tractable. Kahn and Roberds (2001) describe settlement banks' incentives in CLS Bank, which started to offered PvP settlement for some foreign exchange transactions (but not those requiring same-day settlement) in 2002. They argue that PvP increases the certainty that the counterparty will settle its part of the FX transaction. This improves banks' incentives to have good liquidity management procedures, but may also reduce their incentive to monitor counterparties. The authors do not discuss the relative merits of global and local liquidity management, nor do they show the trade-off between the likelihoods of transmission of shocks and of technical default. Freixas and Holthausen (2004) study the cross-country integration of interbank markets. They argue that cross-country interbank market integration may not be perfect when banks have less knowledge about the solvency of foreign banks. Over the short time-frame we have in mind, we abstract from the possibility of raising liquidity abroad. Ratnovski (2007) investigates banks' liquidity holdings under assumptions not unlike our own. In his model, a bank cannot access liquidity held elsewhere within the same group. However, he allows a bank to invest in transparency, which facilitates the communication of its solvency status when they needs to raise liquidity. We abstract from investment in communication; if there are serious doubts about a bank's solvency, this bank's statements are unlikely to be believed, and in the short time horizon we focus on a bank is unlikely to be able to prove its solvency. Fujiki (2006) shows that PvP settlement, together with free daylight overdrafts from the central bank, is one possibility to yield efficiency gains in an extension of Freeman's (1996) island model. Finally, there is a distinct strand of models investigating banks' reserve management: Tapking (2006) and Ewerhart et al (2004) show that interbank interest rates do not necessarily have to increase when liquidity becomes scarce. Ho and Saunders (1985) derive optimal interbank lending to meet reserve requirements.

3 Setup

This section describes the main assumptions (in particular regarding the distribution of information) and the timing of the game. Figure 1 contains a stylised game tree with the global bank's most important decisions. Variables are also listed on page 38 for ease of reference. There are three days and three players: One global bank with two subsidiaries, G_E and G_W, and two local banks D_E and D_W, one in each country (East or West). Banks are owned by their depositors (equity is zero) and maximise undiscounted end-of-day-two payoffs. Local banks are, by assumption, liquidity-rich and 'safe' in that their illiquid assets are not subject to real shocks.

Day zero. The global bank invests its deposits into a risky, illiquid and a risk-free liquid asset. The risk-free asset pays off one for each unit invested, ie, it has a return of zero. With probability p_i, one unit of the risky asset has a payoff of $(1+\rho)/p_i$. With probability $(1-p_i)$, it becomes worthless. Thus, one unit of the risky asset is expected to pay off $(1+\rho)$; after investment outlays, the expected profit is simply ρ per unit invested. (Notice that the payoff assumptions imply that ρ does not depend on p_i: this simplifies the following analysis.) Both p_E and p_W are distributed as follows: Either $p_E = 1$, in which case p_W is distributed uniformly in $[0,1]$; or $p_W = 1$, in which case p_E is distributed uniformly in $[0,1]$. Both events have probability $1/2$. Their realisations remain the global bank's private information. Notice that p_E and p_W can be interpreted as the likelihood that a subsidiary's loan portfolio will not default within the next 24 hours. These probabilities should generally be very close to one. Thus, it appears justifiable to exclude the case that both portfolios have a non-zero likelihood of default within the next 24 hours. (See section 7 for a discussion.)

After having learnt the risk of its illiquid assets, the global bank decides how to invest the deposit base of 1 in each country into a risk-free liquid asset (shares L_E and L_W) and a risky illiquid asset (shares $1 - L_E$ and $1 - L_W$ of its deposit base). The exchange rate between currencies is 1:1.

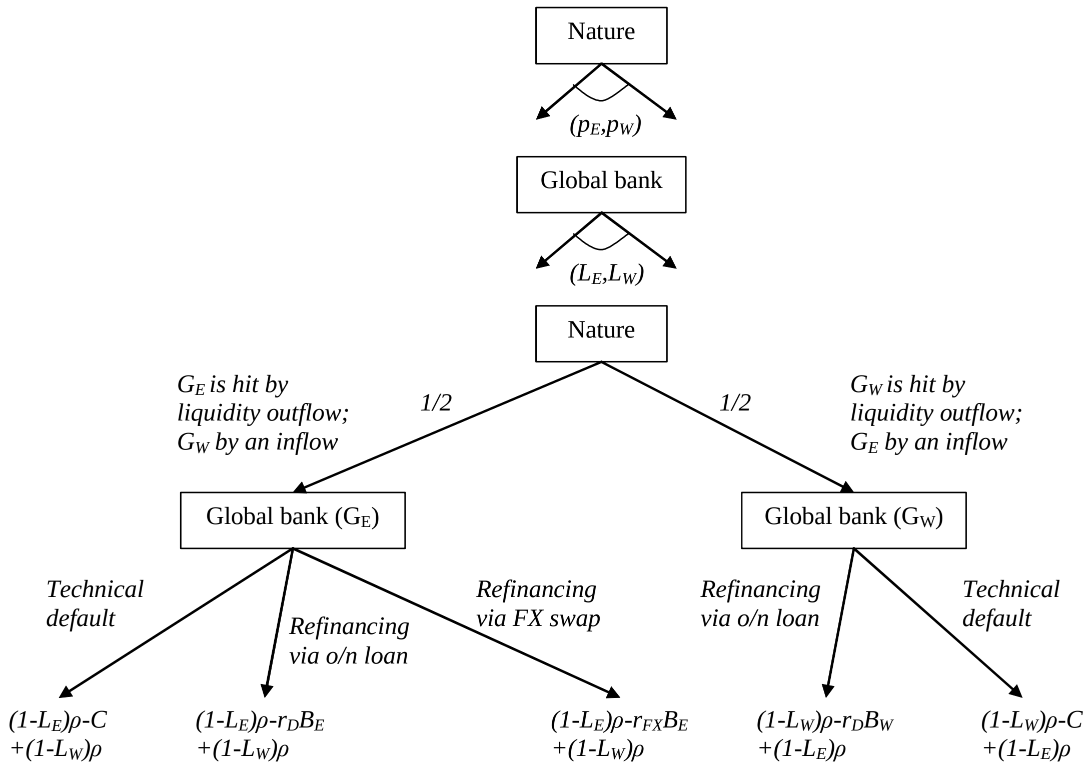

Figure 1: The global bank's main decisions under global liquidity management.

Day one. Banks are hit by liquidity shocks of size λ. These liquidity shocks are independent of p_i and sum to zero in two ways: First, if a subsidiary in country i experiences a liquidity outflow, the domestic bank in the same country experiences a liquidity inflow of the same size. This is a quite natural assumption, given that without central bank intervention, the total amount of central bank money in an economy remains constant. Second, if a subsidiary in country i experiences a liquidity outflow, the subsidiary in the other country j experiences an inflow of the same size. This assumption is made to maximise the benefits of global liquidity management. It also means that one subsidiary always has sufficient liquidity available to lend out to the other subsidiary, even when it decides ex ante to hold no liquidity. Thus, the liquidity shock has the following two realisations, where each has probability 1/2 of occurring:

	Realisation 1	Realisation 2
G_E	λ	$-\lambda$
G_W	$-\lambda$	λ
D_E	$-\lambda$	λ
D_W	λ	$-\lambda$

In response to the liquidity shock, the global bank has several options. It may have sufficient liquidity to absorb the shock. If, in contrast, there is a shortfall, it may raise B_E east dollars (or B_W west dollars, respectively). For its eastern subsidiary, G_E, it can choose between refinancing via a domestic interbank loan, transferring liquidity between subsidiaries via an FX swap (both with D_E as counterparty), and a declaration that it will have to delay payment until day 2, also referred to as 'technical default'. We assume that the per-dollar cost of the domestic interbank loan, r_D, and the per-dollar cost of the FX swap, r_{FX}, are such that all parties with an exposure to G_E expect to break even. Technical default entails a fixed cost of C, independent of the amount of the payments that did not settle on day 1: A proxy for the associated reputational damage. (We abstract from any contagious effect technical default may have on other banks.)

For its western subsidiary, G_W, the options are more restricted. This is because of date conventions used in the settlement of foreign exchange transactions explained in more detail in section 5. The result is that G_W only has two options: Technical default and a domestic interbank loan with D_W as counterparty.

Day two. During the subsequent 24 hours, each subsidiary's illiquid asset may be hit by the aforementioned real shock. The payoffs in figure 1 show expected payoffs over the possible realisations of the real shock. If the shock hits a subsidiary, it is forced to default on its obligations, such that its creditor (if there is one) loses the entire principal amount of its loan. (Notice that this implies that the illiquid asset cannot be used to collateralise the liquidity-short bank's loan.) This is referred to as 'transmission of losses' from the debtor to the creditor.

On day 2, if the real shock did not hit G_i's illiquid asset (probability $p_i \in [0,1]$), it pays off sufficient liquid assets such that obligations incurred on day 1 can be fulfilled. The assumption that the asset's payoff is 'sufficient' is made so as to abstract from repercussions that refinancing decisions on day 1 may have for the availability of liquidity on day 2.[4] Outstanding interbank loans are paid back and FX swaps reversed. Each bank distributes its assets among its (local) depositors. (The need to enter an FX swap rather than just a simple FX transaction is motivated by the latter

[4]This is clearly a strong simplifation but we would have faced this problem in all games with a finite number of periods.

assumption).

The following sections first derive the optimal liquidity holdings and the likelihood of technical default and transmission of losses for the base case of symmetric information. The paper's main results refer to the case of asymmetric information. Section 4's results for local liquidity management are also informative for global liquidity management: as we will argue in section 5, the decisions of the Western subsidiary are identical in both cases.

4 Local liquidity management

When liquidity is managed locally, each subsidiary of the global bank seeks recourse to the domestic interbank market if it experiences a liquidity outflow and does not want to declare technical default. In market i, it borrows B_i at rate r_{Di}. Its expected payoffs are independent of the realisation of p_i. This follows from our assumptions that first, the risky asset's return is independent of p_i; second, that the real shock is distributed independently of the liquidity shock; and third, that its potential lenders only know the prior distribution of p_i, such that r_{Di} does not depend on the realisation of p_i. The global bank's expected payoff can be written as

$$\begin{aligned}\pi_G &= \pi_{GE} + \pi_{GW} \\ &= (1 - L_E)\,\rho - \frac{1}{2} p_E \min\{C, r_{DE} B_E\} \\ &\quad + (1 - L_W)\,\rho - \frac{1}{2} p_W \min\{C, r_{DW} B_W\}\end{aligned}$$

where the second line is the expected profit from the eastern, and the third line the expected profit from the western subsidiary. $(1 - L_i)\,\rho$ is the expected payoff from the illiquid asset, given that a share $(1 - L_i)$ of G_i's balance sheet is invested in it. $\frac{1}{2} p_i r_{Di} B_i$ is the expected borrowing cost if G_i decides to raise funds when short of liquidity (rather than declare technical default). $r_{Di} B_i$, the interest payment on the loan over B_i, is only made if G_i is still solvent on day 2. This happens with probability p_i. Finally, G_i only suffers a liquidity outflow on day 1 with probability 1/2. If, in contrast, G_i experiences a liquidity inflow on day 1, it cannot profitably lend out its excess liquidity on the interbank market because by assumption, the domestic interbank market is already liquidity rich (i.e., D_i does not need to borrow).[5] Correspondingly, $\frac{1}{2} p_i C$ is the expected cost of declaring technical default. (The associated costs C only matter if G_i expects to survive until day 2.)

[5] Including a demand for liquidity from a domestic bank would increase the benefit the global bank derives from holding liquidity, but not change the dependence between liquidity holding and solvency risk, which is in the focus of this paper.

Two assumptions enable us to solve each subsidiary's problem independently: Most importantly, each subsidiary can go bankrupt independently; there are no spillovers from this bankruptcy to the surviving subsidiary above and beyond a loss resulting from direct credit exposure. (See section 7 for a discussion.) Second, because liquidity management is local in this section, such intra-group credit exposures never arise. The following results refer to the eastern subsidiary's decision problem; the western subsidiary's are identical.

Because borrowing is costly, G_E never borrows more than necessary to cover its liquidity shortfall: $B_E \leq \lambda - L_E$ in all equilibria. Because the cost of technical default is independent of the amount outstanding at the end of day 1, $B_E \in \{0, \lambda - L_E\}$ in equilibrium: G_E either borrows sufficiently to avoid technical default, or nothing at all. If $B_E = \lambda - L_E$, its expected profit is

$$\pi_{GE}(L_E, p_E) = \begin{cases} (1 - L_E)\rho - \frac{1}{2} p_E r_{DE} (\lambda - L_E) & \text{if} \quad \lambda - C/r_{DE} \leq L_E \\ (1 - L_E)\rho - \frac{1}{2} p_E C & \text{if} \quad \lambda - C/r_{DE} > L_E \end{cases}$$

The following lemma shows that the profit-maximising ex-ante liquidity holdings must lie in a corner.

Lemma 1 $L_E \in \{0, \lambda\}$

Proof. $\pi_{GE}(L_E, p_E)$ is declining in the ex-ante liquidity holding L_E with slope ρ for small $L_E \leq \lambda - C/r_D$. For larger L_E, the slope increases to $\rho + \frac{1}{2} p_E r_{DE}$. Thus, $\pi_{GE}(L_E, p_E)$ is convex in L_E. ■

We also refer to the choice $L_E = \lambda$ as 'hoarding liquidity'. Whether hoarding liquidity is profitable depends, among other factors, on the interest rate r_{DE} which the domestic lender D_E charges. By assumption, liquidity is provided at an interest rate at which D_E expects to break even. Break-even is achieved if

$$r_{DE} = \frac{1 - E[p_E | p_E \in P_{DE}]}{E[p_E | p_E \in P_{DE}]}$$

where P_{DE} is the set of risk types that opt for the interbank loan in equilibrium.

There are three types of equilibria: Those in which $P_D = \varnothing$ (either because both the opportunity costs of holding liquidity and the cost of technical default are low, or because the lender would expect to face bad risks if, off equilibrium, he was approached for an interbank loan); those in which $P_D = [0, 1]$ (because the opportunity costs of holding liquidity are low, and the cost of technical default high); and those in which some, but not all risks take out the overnight loan. Lemma 2 provides details.

Lemma 2 *G_E's equilibrium strategy fulfils one of the following:*

(D1): $P_D = \varnothing$. Then

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$

and bad risks declare technical default if hit by a liquidity outflow. This equilibrium exists for all λ, C, and ρ.

(D2): $P_D = [0, 2(1-\rho)]$. Then

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2(1-\rho) \\ \lambda & \text{if} \quad p_E \geq 2(1-\rho) \end{cases}$$

and bad risks take out an overnight loan if hit by a liquidity outflow. This equilibrium exists if and only if $\lambda \leq C$ and $\rho > 1/2$.

(D3): $P_D = [0, 1]$. Then $L_E = 0$ for all risks and all risks take out an overnight loan if hit by a liquidity outflow. This equilibrium exists if and only if $\lambda \leq C$ and $\rho > 1/2$.

Notice that in both D1 and D2, a bank's liquidity shortfall is correctly interpreted by the potential lender as a reflection of high solvency risk, even though the solvency shock and the liquidity shock are independent. In case D1, this leads the market to deny credit to the liquidity-short bank. We abstract from equilibrium D2 because it is unstable, in a sense made more precise in the appendix, and hence is unlikely to be observed in reality.

The following lemma collects results regarding the ex-ante likelihood of transmission of losses and technical default under symmetric information for the 'stable' equilibria D1 and D3. Transmission of losses (from G_E to its domestic creditor D_E) occurs in circumstances in which G_E's illiquid assets fail (probability $(1-p_E)$) given that G_E opted for refinancing after having been hit by a liquidity outflow (probability 1/2). Technical default occurs in circumstances in which G_E did not opt for refinancing $\Pr(p_E < 2\lambda\rho/C)$ after having been hit by a liquidity outflow (probability 1/2).

Lemma 3 *Likelihood of technical default and transmission of losses under local liquidity management.*

1. *In equilibrium (D1), the likelihood of transmission of losses is zero. The likelihood of technical default is*

$$\frac{1}{2}\Pr(p_E \leq 2\lambda\rho/C) = \frac{\lambda\rho}{C}$$

2. *In equilibrium (D3), the likelihood of technical default is zero. The likelihood of transmission of losses is*

$$\frac{1}{2}E[1-p_E|p_E \in [0,1]]\Pr(p_E \in [0,1]) = \frac{1}{2}\int_0^1 (1-p_E)\,dp_E = \frac{1}{4}$$

Unsurprisingly, the likelihood of technical default is increasing in ρ (because this increases the opportunity costs of holding liquidity ex ante) and λ (because this makes refinancing more costly), and decreasing in C (because for lower C, technical default is associated with a lower penalty).

5 Globally centralised liquidity management

We assume that each subsidiary holds liquidity in its own currency but not in other currencies. A single unit within the global bank decides on how much liquidity each subsidiary holds. We impose that intra-group loans of liquidity are charged at an interest rate at which the lender just expects to break even; a more explicit modelling of the bargaining between lender and borrower is left for future research. We abstract from regulatory barriers to the transfer of liquidity across jurisdictions and focus instead on the influence the model of settlement of foreign exchange transactions has on the informational efficiency of the FX market, and thereby on the likelihoods of technical default and transmission of losses. Importantly, we assume that each subsidiary can default independently, and that there are no obligations by the surviving part of the global bank to support an insolvent subsidiary. See section 7 for a discussion.

In reality, an important constraint to foreign exchange trades arises from calendar day conventions. When two banks trade dollars for euros, both the sale and the purchase (the transfer of dollars and the transfer of euros) have to settle on the same calendar day; otherwise the bank that receives its payment only on the next calendar day effectively grants the other bank an overnight loan which is repaid in another currency. This implies that a US subsidiary of a global bank that finds itself short of liquidity will not be able to sell euros against dollars without incurring charges for an overnight loan if the euro payment only settles on the following calendar day. We capture this asymmetry by assuming that only the eastern subsidiary G_E, but not the western subsidiary G_W, has the option to raise liquidity via an FX transaction.

Because the liquidity shocks are by assumption perfectly negatively correlated across subsidiaries of the global bank, G_W always has sufficient liquidity available when G_E experiences a liquidity outflow. Thus, there is no need for G_W to hold ex ante additional liquidity to support G_E in case G_E experiences a liquidity outflow. Equally, G_E's liquidity holdings are independent of G_W's needs because G_W does not have access to FX swaps. This implies that the optimal liquidity holdings can be derived independently for both subsidiaries and that G_W's optimal liquidity holdings are the same under global and local liquidity management.

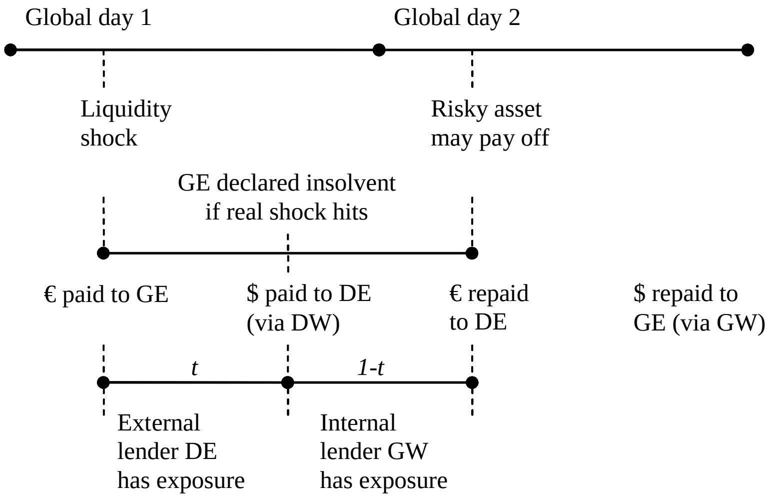

Figure 2: Time-line for the settlement of an swap.

The remainder of this section deals with G_E's optimal liquidity holdings. We analyse G_E's choice between declaring technical default, raising liquidity via a domestic interbank loan from D_E, and raising liquidity via an FX swap when G_E is hit by a liquidity shortage on day 1. Figure 2 illustrates which of G_E's counterparties has exposure to G_E when G_E enters an FX swap. To fix ideas, it identifies the Eastern currency with euros, and the Western currency with dollars.

In an FX swap, D_E has an exposure to G_E during the period between the settlement of the euro and the dollar legs (a fraction $t \in [0, 1)$ of 24 hours). If G_E is declared insolvent, D_E may lose the full face value of the swap. (This form of principal risk is also referred to as FX settlement risk, or Herstatt risk.) The duration of this exposure can be lengthy: For example, if a domestic European bank pays euros at 9am in the European day, it may only receive the dollar via its correspondent at 5pm in the US day, some 14 hours later. In our model, once the dollars have been paid to D_E, the risk is transferred to G_W, which granted G_E an overnight loan. If t is small, we speak of (relatively) coordinated settlement; if t is large, of (relatively) uncoordinated settlement of the two currency transactions in the first (spot) leg of the FX swap. The case $t = 0$ corresponds to payment-versus-payment settlement. For $t \to 1$, the FX swap approaches a domestic overnight loan.

We solve the model backwards. Section 5.1 derives the lenders' and the borrower's participation constraints; section 5.2 provides the results for the global bank's optimal refinancing choice, and 5.3 derives the link between the degree of coordination in settlement and the likelihood of technical default and transmission of losses.

5.1 Participation constraints: Determination of the cost of refinancing

This section derives the lenders' and the borrower's participation constraints. Consider first the internal lender:

Lemma 4 *G_W charges the borrower a per-dollar fee of*

$$r_G = (1-t)\,\frac{1-p_E}{p_E}$$

for the intra-group loan.

Notice that this per-dollar fee is the full-information interest rate appropriate to the borrower's risk, $(1-p_E)/p_E$, times a factor for the duration of G_W's exposure to G_E's default risk, $(1-t)$. Of course, the shorter the duration of the exposure, and the lower G_E's risk of default, the lower the fee. The proof is in the appendix.

The eastern domestic bank, D_E, now offers two products: an interbank loan at interest rate r_D, and an FX swap at a fee s_D. For the interbank loan, the participation constraint is, as before,

$$r_D \geq \frac{1-E\left[p|p\in P_I\right]}{E\left[p|p\in P_I\right]}$$

where all $p_E \in P_I$ opt for the interbank loan. Lemma 5 provides the corresponding per-dollar fee for the FX swap:

Lemma 5 *As compensation for Herstatt risk, D_E charges the borrower a fee of*

$$s_D = \frac{t\left(1-E\left[p|p\in P_{FX}\right]\right)}{1-t\left(1-E\left[p|p\in P_{FX}\right]\right)}$$

for each unit borrowed, where all $p_E \in P_{FX}$ opt for the FX swap.

This fee is, of course, decreasing in the time t that D_E has exposure to G_E. It is also decreasing in D_E's expectation of the borrower's risk. The proof is in the appendix.

The total per-dollar cost of the FX swap is the sum of both lenders' fees:

$$r_{FX}\left(p_E,t,P_{FX}\right) = (1-t)\,\frac{1-p_E}{p_E} + t\left(\frac{1-E\left[p|p\in P_{FX}\right]}{1-t\left(1-E\left[p|p\in P_{FX}\right]\right)}\right)$$

Lemma 6 contains comparative static properties:

Lemma 6 *For a given $P_{FX} \neq \varnothing$, the following holds:*

1. *r_{FX} is strictly declining in p_E:*

$$\frac{\partial r_{FX}}{\partial p_E} = \frac{\partial\left((1-t)\frac{1-p_E}{p_E}+t\left(\frac{1-x}{1-t(1-x)}\right)\right)}{\partial p_E} = -\frac{1-t}{p_E^2}<0$$

2. *G_E's expected payment for the FX swap, $p_E r_{FX}(p_E,t)\,B_E$, is strictly monotone in p_E: There is a unique $t'(P_{FX})\in(0,1]$ such that*

$$\frac{\partial\left(p_E r_{FX}\right)}{\partial p_E}\begin{cases} >0 & \text{if} \quad t>t' \\ =0 & \text{if} \quad t=t' \\ <0 & \text{if} \quad t<t' \end{cases}$$

That r_{FX} decreases in p_E should be intuitive: A part of the FX transaction is financed by an informed lender, who charges bad risks a higher fee. The strict monotonicity of $p_E r_{FX}(p_E,t)$ is important for the structure of the equilibria. A decline in the borrower's risk has two effects: First, he is less likely to fail, implying that r_{FX} falls, such that his expected costs of taking out the FX swap falls. (Recall that only survivors pay.) But for the same reason, he is more likely to have to pay back the loan, increasing the expected cost of the FX swap. If t is small, both legs of the FX swap are settled within a short time interval. This implies that the informed lender, G_W, carries most of the exposure. Then r_{FX} declines rapidly in response to better risks, dominating the multiplication by p_E. This case will, in particular, apply for PvP settlement. If, in contrast, t is large, some time passes until the second currency transaction of the spot leg is settled, such that the external, uninformed lender carries most of the exposure. The total cost of the FX swap then does not react much to changes in the borrower's risk, such that $p_E r_{FX}(p_E,t)$ is increasing in p_E.

5.2 The liquidity-short subsidiary's refinancing decision

This section starts out with a presentation of the formal results. The following subsections provide an intuition.

5.2.1 Equilibrium refinancing decisions - formal results.

Proposition 1 characterises the forms G_E's equilibrium strategy under global liquidity management. (There may be additional mixed equilibria for specific parameter constellations, e.g., for $t=t'$; these are ignored here.)

There are three main classes of equilibria. The first two are not able to inform us about global liquidity management. In the first, no loan is made in equilibrium, and the external lender would be very pessimistic about a potential borrower's risk should he be approached by one. In the second, all refinancing is via the interbank loan and no FX swap is offered in equilibrium because the external lender would be very pessimistic about a potential counterparty's risk should he be approached for an FX swap. This type of equilibrium does not appear to be very informative about actual behaviour because it assumes that D_E is happy to take over the exposure to G_E overnight, but not for a fraction of that time.

Of more interest is the third class of equilibria. Here, some refinancing is carried out via FX swaps. Proposition 1 shows that if some refinancing is done via an FX swap ($P_{FX} \neq \varnothing$), there is no recourse to overnight loans ($P_I = \varnothing$). Put differently, the model predicts that once the tools for global liquidity management are in place, a liquidity manager will whenever possible refinance himself using internal funds rather than rely on borrowing externally. If the expected return, ρ, on the illiquid, risky asset is sufficiently high, the liquidity manager will opt to hold no liquid assets ex ante (equilibria of type G3 in proposition 1 below). Here, good risks will refinance themselves, whereas bad risks will opt for technical default if hit by a liquidity outflow. If, in contrast, the expected return is low, some risks will opt for hoarding liquidity to avoid borrowing altogether. Which risks do indeed hold $L_E = \lambda$ depends on whether the expected cost of refinancing via an FX swap is increasing or decreasing in p_E. If $p_E r_{FX}(p_E)$ is strictly increasing in p_E (equilibria of type G4), then the best risks face the highest expected cost of refinancing (and of technical default), so they opt for hoarding liquidity. If $p_E r_{FX}(p_E)$ is strictly decreasing in p_E (equilibria of type G5), then the best risks expect to face a relatively low cost of refinancing via an FX swap, while the worst risks expect to face a relatively low cost of technical default, such that only the intermediate risks may have an interest in hoarding liquidity.

Notice that in all equilibria in which there is global refinancing, the worst risks will prefer to declare technical default. This is because any FX swap involves at least some portion of informed lending ($t < 1$), and because the expected costs, $p_E C$, of the reputational penalty, is very small for bad risks.

The following two definitions will be used:

Definition 1 *Indifferent risk types.*

$$\begin{aligned} p'' &: \quad p'' \in [0,1] \;\; \textit{and} \; r_{FX}(p'', t, P_{FX}) = C/\lambda \\ p''' &: \quad p''' \in [0,1] \;\; \textit{and} \; \rho = \frac{1}{2} p''' r_{FX}(p''', t, P_{FX}) \end{aligned}$$

Type p'' is indifferent between refinancing via an FX swap and technical default, given that he has experienced a liquidity outflow of size λ. All types $p_E > p''$ strictly prefer to refinance. Type p'''

is indifferent between hoarding liquidity ex ante (and incurring an opportunity cost of ρ), and relying on refinancing via an FX swap should he experience a liquidity outflow. Notice that, depending on the equilibrium, P_{FX} may depend on p'' and/or p''', so neither uniqueness nor existence of p'' and p''' is guaranteed.

Proposition 1 *G_E's (Bayesian Nash) equilibrium strategy fulfils one of the following:*

- *(G1) No refinancing: $P_{FX} = P_I = \varnothing$ and*

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$

- *(G2) Only local refinancing: $P_I \neq \varnothing$ and $P_{FX} = \varnothing$. Here, the equilibria have the same form as under local liquidity management.*

- *Global refinancing: $P_{FX} \neq \varnothing$. Then $P_I = \varnothing$ and p'' exists and*

  - *Large opportunity cost of holding liquidity:*

    - *G3: If $\rho > pr_{FX}(p)/2$ for all $p > p''$, p'' is unique. $L_E = 0$ for all types and $P_{FX} = [p'', 1]$.*

  - *Small opportunity costs of holding liquidity:*

    - *(G4): If $p_E r_{FX}(p_E)$ is strictly increasing in p_E, then for all $p''' > p''$, $P_{FX} = [p'', p''']$ and*

    $$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq p''' \\ \lambda & \text{if} \quad p_E > p''' \end{cases}$$

    G5: If $p_E r_{FX}(p_E)$ is strictly decreasing in p_E, then for all $p''' > p''$, $P_{FX} = [p''', 1]$ and

    $$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\rho\lambda/C \text{ or } p_E \geq p''' \\ \lambda & \text{if} \quad 2\rho\lambda/C < p_E < p''' \end{cases}$$

The formal proof can be found in the appendix. As in the case of local liquidity management, equilibria of the type G4 are unstable in the best-reply dynamic sense. In contrast, equilibria of types G3 and G5 do not suffer from this instability.[6]

Corollary 1 contains specific results for the very policy-relevant case of payment-versus-payment settlement ($t = 0$). Here, the uninformed lender has no exposure, and information is symmetric.

[6] To see this, consider equilibria of the form G3 or G5. Here, $P_{FX} = [p', 1]$ for $p' \in \{p'', p'''\}$. A slight exogenous increase in p' would improve D_E's expectation and make the FX swap attractive for types below $p' + \varepsilon$, such that p' would return to equilibrium in a best-reply dynamic.

Corollary 1 *(PvP settlement) If* $t = 0$*,*

$$\begin{aligned} p'' &= \lambda / (\lambda + C) \\ p''' &= 1 - 2\rho \\ p_E r_{FX}(p_E) &= 1 - p_E \end{aligned}$$

and there are two equilibria:

- *G3 If* $\rho > \left(1 - \frac{\lambda}{C+\lambda}\right)/2$*, then* $P_{FX} = [p'', 1]$ *and* $L_E = 0$
- *G5 If* $\rho < \left(1 - \frac{\lambda}{C+\lambda}\right)/2$*, then* $P_{FX} = [p''', 1]$ *and*

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\rho\lambda/C \ \text{or} \ p_E \geq p''' \\ \lambda & \text{if} \quad 2\rho\lambda/C < p_E < p''' \end{cases}$$

Notice that for $t = 0$, $p_E r_{FX}(p_E)$ is strictly decreasing in p_E, so G4 does not exist. G1 and G2 also do not exist; in the game under incomplete information (i.e., for $t > 0$), they owe their existence to specific off-equilibrium beliefs.

The following two subsections provide some intuition for the result. The first illustrates why a global liquidity manager generally prefers to refinance internally. The second shows how the equilibria of type G3 and G5 are derived.

5.2.2 The availability of an FX swap crowds out refinancing via domestic overnight loans.

To see why internal refinancing is preferred, consider figure 3. It is drawn for a special case, in which C is high (such that technical default is not an option for any type); ρ is high (such that G_E would opt to hold no liquidity ex ante independently of its risk), and in which settlement is PvP for the FX swap (such that G_E faces a fully informed counterparty when it enters an FX swap, and an uninformed counterparty when it refinances itself domestically). The x-axis shows the likelihood p_E that G_E's illiquid assets pay off on day 2. Thus, G_E's default risk is increasing from the right to the left. The y-axis shows the interest rates at which G_W is willing to grant the intra-group loan (the downwards sloping line), and the interest rate at which D_E is willing to grant the overnight loan (the horizontal line).

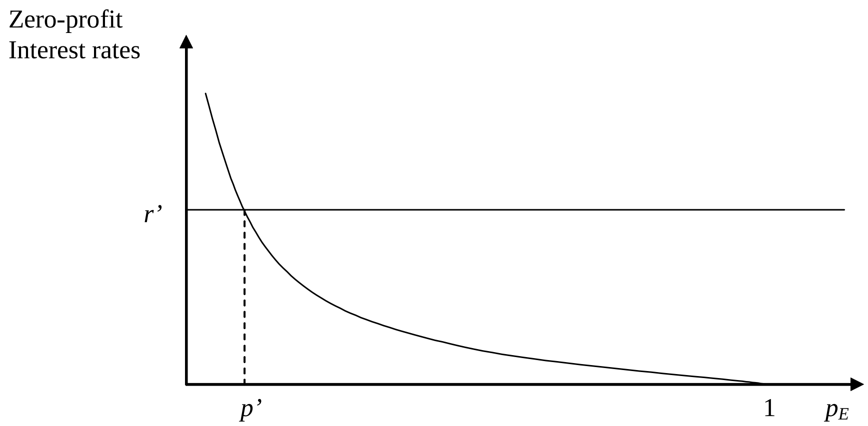

Figure 3: Crowding out of domestic interbank loans

The external lender, D_E, is uninformed and must offer the same contract to all G_E types. By contrast, the informed lender, G_W, knows the borrower's type and offers better risks (higher p_E) a lower interest rate.[7] The proof that all risk types prefer G_W's offer now proceeds by contradiction. Suppose instead that there was a type $p_E = p' > 0$ which found itself confronted with the same offer r' by the external and the internal lender. By construction, both the informed and the uninformed lender expect to break even by offering $r_D = r_G(p')$. Then all riskier types $p_E < p'$ would strictly prefer to take the uninformed lender's offer. All less risky types, in contrast, would strictly prefer the informed lender's offer. But this contradicts the assumption that both lenders expect to break even on their contracts. Thus, there is no $p' > 0$ at which both curves intersect. In any equilibrium, all types $p_E > 0$ choose the FX swap.

Notice that this argument does not preclude equilibrium G2 in which there is only local refinancing. It simply shows that there cannot be an equilibrium in which G_E's decision on whether to refinance via an interbank loan or via an FX swap depends on its solvency risk.

[7] Notice that this line is constructed assuming that G_W offers contracts such that he expects to break even on *each* contract.

5.2.3 Derivation of proposition 1 - intuition

This section provides some graphical intuition for the derivation of equilibria of type G5. Figure 4 shows how the model is solved backwards in this case. It takes the result that the availability of an FX swap crowds out refinancing via domestic overnight loans as given.

The first panel shows the (second-stage) decision between refinancing via an FX swap and technical default. Only good risks ($p_E \geq p''$) prefer to take out the FX swap. The second panel shows the (first-stage) decision about how much liquidity to hold ex ante. The opportunity cost of holding liquidity is constant in p_E. (This is where the assumption enters that the expected return on the risky asset is independent of p_E.) Refinancing via an FX swap is preferred by the best risks ($p_E \geq p'''$). The worst risks ($p_E \leq 2\lambda\rho/C$) opt for technical default, while intermediate risks prefer to hoard sufficient liquidity to avoid any borrowing. The third panel summarises the results. The arrows indicate reactions to a decline in the degree of coordination in FX settlement (an increase in t). Notice that the equilibrium only exists for sufficiently small $t < t'$; only for these t is the expected cost of refinancing via an FX swap a declining function of p_E. They ultimately determine the reaction of the likelihood of technical default and of transmission of losses. Their direction is proven in the following section, also for equilibria of type G3.

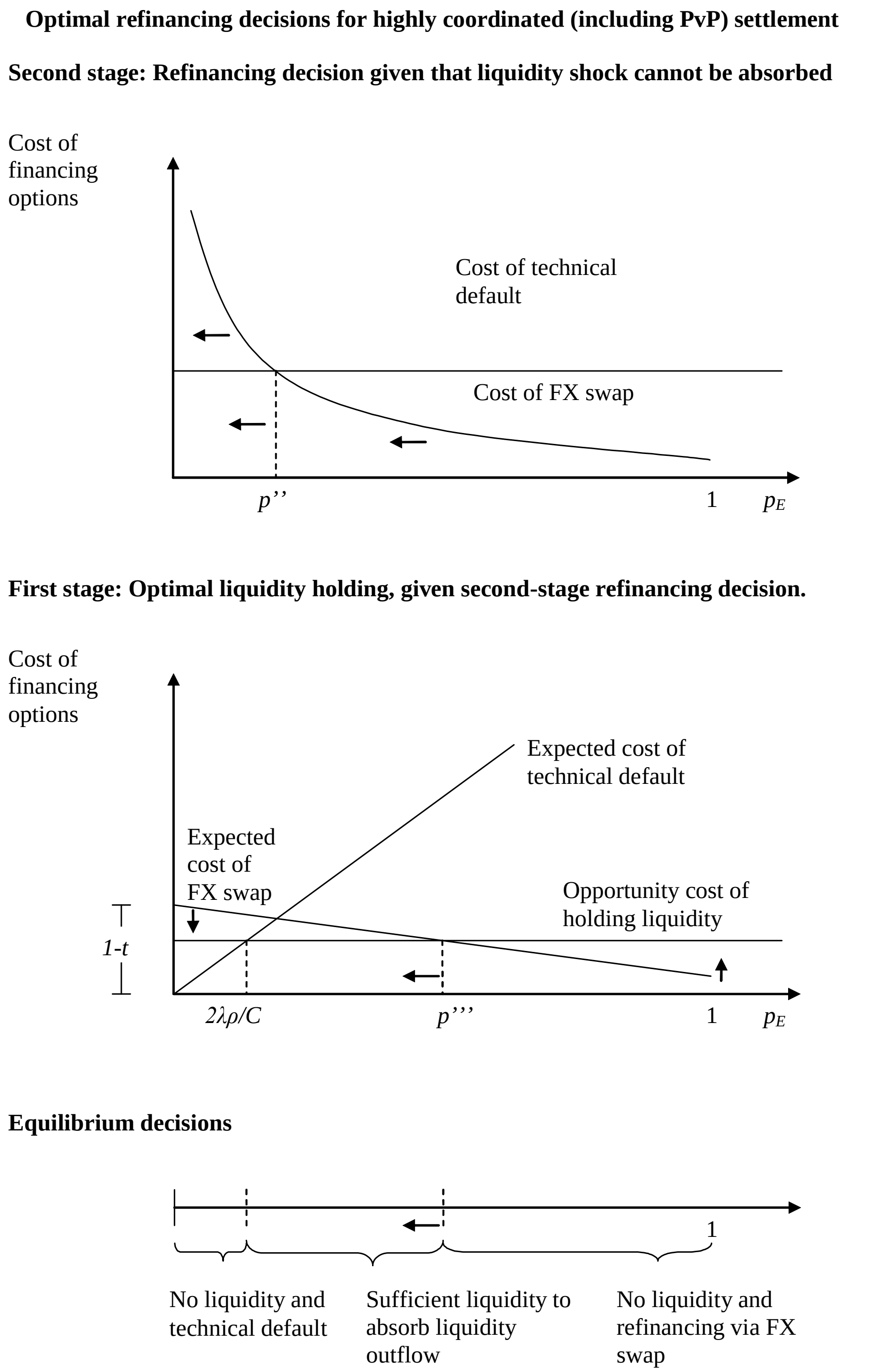

Figure 4: Construction of equilibrium G5. Arrows indicate comparative static reactions to an increase in t (decline in coordination).

5.3 Likelihood of technical default and of transmission of losses

As argued above, it seems reasonable to focus in the following on equilibria of types G3 and G5. Lemma 7 describes how the degree of coordination of settlement of FX transactions influences the equilibrium proportion of risks which take out an FX swap. This proportion determines the likelihoods of technical default and transmission of losses (propositions 2 and 3).

Lemma 7 *In all stable equilibria in which there is refinancing via an FX swap, the total derivatives of p'' and p''' with respect to t are*

$$\begin{aligned}\frac{dp''}{dt} &= -\frac{\partial r_{FX}(p'',t)}{\partial t} / \frac{\partial r_{FX}(p,t)}{\partial p}_{|p=p''} < 0 \\ \frac{dp'''}{dt} &= -\frac{\partial r_{FX}(p''',t)}{\partial t} / \frac{\partial pr_{FX}(p,t)}{\partial p}_{|p=p'''} < 0\end{aligned}$$

The more time elapses between the settlement of the first (euro) and the second (dollar) transactions of the spot leg of the FX swap, the larger the share of exposure born by the uninformed lender, the lower the fee the worst risk in P_{FX} is charged for the FX swap and hence the more attractive it is to take out the FX swap. As a result, P_{FX} contains worse risks in equilibrium. Essentially, highly uncoordinated settlement allows relatively bad risks to hide among the good risks when taking out an FX swap. Put differently, by introducing PvP settlement, the informed lender carries all the risk, so he increases the charge for bad risks, which in response opt for other alternatives (technical default in G3; hoarding liquidity in G5).

Ex ante, the increase in the likelihood that G_E takes out an FX swap also increases the likelihood of transmission of losses: G_E's insolvency only matters for other banks when G_E owes them money. In the equilibria in which the best risks take out the FX swaps (cases G3 and G5), losses are transmitted if G_E is hit by a liquidity outflow on day 1 (probability 1/2) and, given this outflow, refinances itself (ex ante with probability $\Pr(p_E \in [p',1])$ for $p' \in \{p'',p'''\}$) and, given this refinancing decision, its risky assets are hit by the real shock; that is, with probability $\frac{1}{2}E[1-p_E|p_E \in [p',1]]\Pr(p_E \in [p',1])$. Proposition 2 states the result formally:

Proposition 2 *In all stable equilibria in which there is refinancing via an FX swap, losses are more likely to be transmitted the less coordinated the FX settlement. Formally, for $p' \in \{p'',p'''\}$,*

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\frac{1}{2}E[1-p_E|p_E \in [p',1]]\Pr(p_E \in [p',1])\right) \\ =\ &\frac{\partial}{\partial t}\left(\frac{1}{4}(1-p')^2\right) = \frac{1}{2}(1-p')\left(-\frac{\partial p'}{\partial t}\right) > 0\end{aligned}$$

The distribution between domestic and global transmission of losses is also of interest. If G_E's default occurs while D_E has exposure, losses are contained domestically. If, in contrast, G_E's default occurs while G_W has exposure, losses are transmitted only within the same banking group, but across countries. Clearly, the likelihood of domestic transmission of losses is increasing in t. In contrast, the likelihood of global transmission of losses may be declining in t, given that G_W has, on average, exposure to worse risks, but only for a shorter duration.

If G_E opts against taking out an FX swap, he may choose to declare technical default instead. In equilibria of the form G3, G_E declares technical default whenever he does not take out the FX swap. Consequently, the ex-ante likelihood of technical default is falling in t. In equilibria of the form G5, there may be an intermediate interval of risks ($p_E \in [2\rho\lambda/C, p''']$) which opt for hoarding liquidity. If this interval is non-empty, the likelihood of technical default is constant in t: only risks worse than $2\rho\lambda/C$ opt for technical default, and none of these parameters depend on t. If, in contrast, the interval is empty, all risks worse than p''' opt for technical default, and p''' is declining in t. So, again, the ex-ante likelihood of technical default is falling in t. The following proposition summarises the results:

Proposition 3 *In all stable equilibria in which there is refinancing via an FX swap, the likelihood of technical default falls the less coordinated the settlement of the two currency transactions in the spot leg of the FX swap.*

The following pictures illustrate these comparative static properties for a special case in which $C/\lambda = 2$ and relatively high opportunity costs of liquidity holdings ($\rho = 20\%$). For some t, both equilibria G3 and G5 exist. The reader might want to compare the charts with figure 4, which is drawn for a specific value of t. Shifting the respective lines in figure 4 in the directions given by the arrows, that is, increasing t, yields the results in figures 5.4-5.6.

Equilibrium G3 | Equilibrium G5

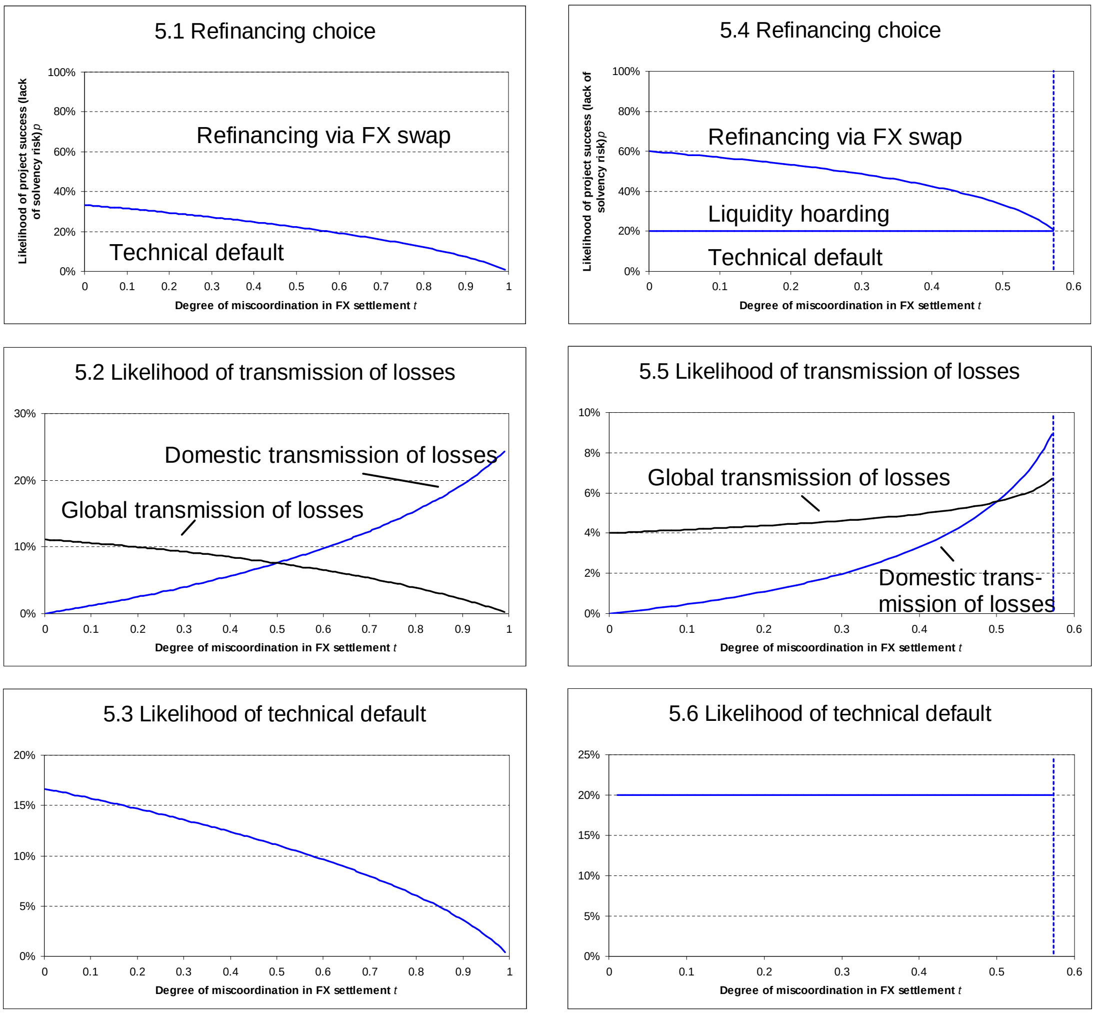

Figure 5.1 in the top left panel illustrates that as the degree of miscoordination increases, riskier borrowers are able to take out the FX swap because the uninformed lender determines a larger part of the FX swap. For $t \to 1$, the FX swap becomes formally equivalent to a domestic overnight loan, and equilibrium G3 approaches equilibrium D3 (see lemma 8). As a consequence of the decreased coordination, the likelihood of transmission of shocks rises. Figure 5.2 splits this up into domestic and global transmission of losses. Also, as riskier borrowers find the FX swap more attractive, the likelihood of technical default decreases (figure 5.3).

Figure 5.4 illustrates that because the expected cost of refinancing via an FX swap is decreasing in t in equilibrium G5, more refinancing takes place as t rises. Both the opportunity cost of hoarding liquidity and the expected costs of technical default are constant in t, which is reflected in the fact that the line that separates the regions in which the subsidiary opts for liquidity hoarding, and the one in which it prefers technical default, is horizontal. In contrast to the opportunity cost of hoarding liquidity, the expected costs of technical default are decreasing in p_E. Thus, the worst risks opt for technical default, whereas intermediate risks hoard liquidity. Figures 5.5 and 5.6 show the resulting consequences on the transmission of losses and the likelihood of technical default.

It is interesting to see how the likelihood of technical default and transmission of losses react to a change in the size of the liquidity shock, and the cost of technical default. For the interesting equilibria G3 and G5, proposition 4 shows that the likelihood of technical default is decreasing the higher the cost of technical default, and the smaller the liquidity shock. This is intuitive: The smaller the liquidity shock, the lower the refinancing costs via an FX swap, so the more likely a liquidity-short bank is to either hoard liquidity or to refinance when hit by a liquidity outflow. But a greater likelihood of refinancing also implies that transmission of losses has become more likely. Equally intuitive, both likelihoods of transmission of losses and technical default rise in the opportunity cost of hoarding liquidity. The proof is in the appendix.

Proposition 4 *In all stable equilibria in which there is refinancing via an FX swap, the likelihood of technical default falls and the likelihood of transmission of losses rises in C/λ. Both rise in ρ.*

6 Comparison between global and local liquidity management

When global liquidity management is introduced, equilibrium D1, in which some types refinance domestically, does not survive. This is because some of the lending involved in an FX swap is done by an informed lender, who can offer better terms than an uninformed lender for each level of the borrower's risk (see section 5.2). Lemma 8 states that equilibrium D3, in which all risks refinance domestically, survives as the limit case when the coordination of FX settlement is close to zero:

Lemma 8 *For $t \to 1$, equilibrium G3 converges to D3.*

The statement is made more precise and proven in the appendix, but the reader can gather the intuition from figure 5.1, which shows that for $t \to 1$, all risks refinance via an FX swap, which is virtually identical to a domestic overnight loan (because the domestic lender holds the exposure for virtually an entire day). Thus, the results in propositions 2 and 3 are equally applicable for a comparison between local and global liquidity management: Technical default becomes more likely, and transmission of losses less likely when we move from local to global liquidity management. A considerable caveat to these statements is, of course, that there are multiple equilibria under global as well under local liquidity management; it is not certain whether banks would indeed move from equilibrium D3 to G3 once they switch to global liquidity management.

7 Discussion

This section discusses the implications of some of the key assumptions made in the model.

Absence of reputational contagion. We assume that the Eastern subsidiary's default only impacts the Western subsidiary to the extent that the Western subsidiary loses the principal amount of the intragroup loan. However, in practice, the Western subsidiary would probably suffer some reputational damage as well.[8] If, in response, the Western subsidiary required a higher compensation for the intragroup loan, the cost of an FX swap would increase, and domestic interbank lending might not be crowded out. However, one might also argue that the Western subsidiary would be willing to lend at a lower rate if the Eastern subsidiary otherwise suffered refinancing problems on the following day.

If the presence of reputational contagion did not change the price of the FX swap, our results would presumably still hold with respect to actual losses incurred by the depositors: Reputational contagion is by definition independent of the existence of actual exposures. Should depositors decide to run the surviving subsidiary, fewer assets could be distributed among them if the surviving subsidiary had to write off a loan to the failed subsidiary. Also, reputational contagion would presumably be independent of whether the bank manages its liquidity locally or globally. The comparison between local and global management would then still hold.

It would probably be interesting to study risks of transmission of losses and technical default in a model in which both G_E and G_W are not subsidiaries, but branches, which could not fail separately. This is left for future research.[9]

Abstraction from refinancing via cross-border collateral movements. Another option

[8] Notice that for simplicity, we also abstract from any contagion as a consequence of technical default. This would have required more detailed modelling of the payment flows on each day and is left for future research.

[9] Relatedly, notice that we restricted attention to the case that only one subsidiary's loan portfolio was risky at a time. This appeared to be justifiable given our focus on risk of imminent failure (ie, within the next 24 hours). If we allowed both portfolios to default and G_W defaults before it takes over the exposure to G_E in the FX swap, but after G_E received Eastern dollars from D_E, G_E would not be able to raise Western dollars in time to pay D_E. Then D_E would effectively grant G_E an overnight loan and require additional payments. This would raise the costs of the FX swap for good risks, and reduce it for bad risks of G_E. In expectation, more risks might opt to take out the FX swap, but we do not think that our results would change qualitatively.

that banks might have available when managing their liquidity globally is the movement of collateral across borders. Continuing the example in which the Eastern subsidiary suffers a liquidity outflow, the Western subsidiary could sell (or repo) West-$ denominated collateral in the domestic market. The Eastern subsidiary could access this liquidity via an FX swap, exactly as discussed above. The situation would change if the Eastern subsidiary could use West-$ denominated collateral to raise liquidity from the Eastern subsidiary. In this case, the Western subsidiary could lend the Eastern subsidiary the West-$ denominated collateral; this would enable the Eastern subsidiary to raise liquidity under symmetric information. From a modelling perspective, this corresponds exactly to the situation of PvP settlement of FX swaps. Thus, widening collateral requirements, and simplifying the process for cross-border transfer of collateral, could be another policy option to improve the informational efficiency in global liquidity management.

Opportunity costs of liquidity-rich banks and their bargaining power. We assume that both the internal lender and the external lender are willing to grant a loan at an interest rate at which they just break even. It might be desirable to endogenise the bargaining between the liquidity-short bank and its potential lenders. At the moment, we effectively assume that the liquidity-short bank has all the bargaining power. Consider for the moment the opposite case in which the liquidity-short bank has no bargaining power, and the lender(s) make take-it-or-leave-it offers. Proposition 1 would then not hold any more. To see this, consider for simplicity the case of PvP settlement. The informed lender would charge an interest rate high enough to ensure that the liquidity-short bank would not declare technical default, i.e. such that

$$\rho(1 - L_E) - p_E r_G B_E \geq \rho(1 - L_E) - p_E C$$

holds as an equality, leading to $r_G = C/B_E$. Then the cost of an FX swap is constant in p_E, and identical to what the outside lender would charge for a domestic overnight loan: r_{DE} would be set such that

$$\rho(1 - L_E) - E[p_E | p_E \in P_I] r_D B_E \geq \rho(1 - L_E) - E[p_E | p_E \in P_I] C$$

holds as an equality, leading to $r_{DE} = C/B_E$. In practice, the borrower is likely to have some bargaining power, such that better risks pay less for an FX swap than worse risks, and proposition 1 would presumably go through as in the extreme case we consider.

The **specification of the outside option**, that is, of declaring technical default, also has important consequences for the results we derived. First, we assumed that a bank maximises its payoff at the end of day 2; the cost of technical default incurred on day 1 does not matter for a bank that is declared bankrupt on day 2. In equilibrium, a bank whose illiquid assets do not pay off has no assets left (it either had zero liquid assets when the liquidity shock hit, or borrowed only just enough to make all necessary payments). There is no additional penalty depositors could suffer from technical default. This assumption implies that if funds are insufficient to absorb the liquidity shock, the liquidity-short bank either declares technical default or takes out the overnight loan independently of its risk, because the *relative* size of the expected borrowing cost $p_E r_D B_E$ and the expected cost of technical default $p_E C$ is independent of p_E. Second, the assumption that the costs of failing to raise sufficient funds are independent of the amount that a bank falls short appears reasonable when the alternative is a declaration of technical default. However, it would be less convincing if one interpreted C as the cost of fire-selling assets; an option that is usually open to banks. Increasing the banks' action space by allowing them to fire-sell assets appears to be an interesting extension which is left for future research.

8 Conclusion and future research

We show that the transition from local to global liquidity management, and better coordination of settlement of FX transactions, have two consequences on financial stability. First, the transmission of solvency shocks from one institution to another would be less likely because banks with high solvency risks would not be able to refinance themselves at all in response to liquidity outflows, neither domestically nor via FX transactions. But this implies that these banks would have to delay the payment of their obligations beyond their due-date. Hence the second consequence: Technical default might become more likely. These results continue to hold when we endogenise banks' ex-ante liquidity holdings.

The paper could be extended in a number of ways. Most importantly, the second-round effects of technical default and insolvency could be modelled in more detail. In the current version of the model, only the bank that declares technical default suffers a loss. In practice, other banks might have expected to receive liquidity from this bank and now find themselves short of liquidity as well. However, given that the total amount of (central bank) liquidity is constant in each system, we would need a slightly more complex scenario for the liquidity shocks. Similarly, there might be second-round effects for the solvency shock as well: The equity of the defaulting bank's creditor is impaired. It might then be more likely to default on its own liabilities and could find it difficult to fund itself, if necessary, on the following day.

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10 Appendix

10.1 Proofs

10.1.1 Local liquidity management

Proof of lemma 2 First recall that the (second-stage) decision over borrowing vs technical default is independent of p_E. Thus, abstracting from the special case that all risks are indifferent between these options, either all $p_E : L_E(p_E) = 0$ declare technical default (such that $P_D = \varnothing$), or all $p_E : L_E(p_E) = 0$ take out an interbank loan (such that $P_D = \{p_E \in [0,1] : L_E(p_E) = 0\}$).

1. Suppose $P_D = \varnothing$. This equilibrium always exists given the off-equilibrium belief $\Pr(p_E = 0 | P_D = \varnothing) = 1$. Then $L_E = 0$ is preferred over $L_E = \lambda$ if $(1-0)\rho - \frac{1}{2}p_E C \geq (1-\lambda)\rho$, such that

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$

2. Suppose instead $P_D \neq \varnothing$. Then all $p_E \in P_D$ prefer holding $L_E = 0$ over holding $L_E = \lambda$ if

$$(1-0)\rho - \frac{1}{2}p_E r_D(\lambda - 0) \geq (1-\lambda)\rho \tag{1}$$

equivalently, $p_E \leq 2\rho/r_D$. This is an upper bound to p_E, implying that P_D must have the form $[0, p']$ where $p' \leq 1$. However, such an equilibrium is unstable in the following sense: Consider the sequences $\{r_D(k), p'(k), P_D(k)\}_{k=1}^{\infty}$ where $P_D(k) = [0, p'(k)]$,

$$r_D(k) = \frac{1 - E[p_E | p_E \in P_D(k)]}{E[p_E | p_E \in P_D(k)]} = \frac{1 - \frac{1}{2}(1 - p'(k))}{\frac{1}{2}(1 - p'(k))} = \frac{2 - p'(k)}{p'(k)}$$

and $p'(k+1) = 2\rho/p'(k)$, starting at $p'(0) = p'$. Then

$$p'(k+1) - p'(k) = \frac{2\rho p'}{2 - p'} - p' = p'(k)\frac{p'(k) - 2(1-\rho)}{2 - p'(k)}$$

This difference is strictly positive for all k if $p'(k) > 2(1-\rho)$, and strictly negative for all k if $p'(k) < 2(1-\rho)$. But given $P_D(0) = [0, p'(0)]$, $p' \in \{0, 2(1-\rho)\}$ solve 1 as an equality (and $P_D \neq \varnothing$ only if $p' = 2(1-\rho)$), so $p'(0) = 2(1-\rho)$. Hence, if the initial shock to p' was positive ($\varepsilon > 0$), the sequence $\{p'(k)\}_{k=1}^{\infty}$will at some point increase above 1, whereas it will fall below zero if $\varepsilon < 0$.

3. Finally, consider an equilibrium in which $P_D = [0,1]$ and there is no p' which is indifferent between $L_E = 0$ and $L_E = \lambda$. Then $r_D = 1$. This equilibrium exists under two conditions: first, in the second stage of the game, that borrowing is preferred over technical default given $L_E = 0$:
$$(1-0)\,\rho - \frac{1}{2}p_E\,(1)\,(\lambda - 0) \geq (1-0)\,\rho - \frac{1}{2}p_E C$$
equivalently, $\lambda \leq C$, and second, in the first stage of the game, that $L_E = 0$ is strictly preferred over $L_E = \lambda$ for all $p_E \in [0,1]$, i.e., that
$$(1-0)\,\rho - \frac{1}{2}p_E\,(1)\,(\lambda - 0) > (1-\lambda)\,\rho$$
equivalently, $2\rho > p_E$ holds for all types, so also for $p_E = 1$, so this equilibrium exists if $\rho > 1/2$.

10.1.2 Global liquidity management

Proof of lemma 4 This is very straightforward. If G_W does not grant the intragroup loan, his payoff is $\pi_{GW} = (1 - L_W)\,\rho$. If he grants the intragroup loan, and G_E defaults, his payoff is (notice that we assume that the occurrence of the real shock is uniformly distributed during the day)

$$\begin{aligned}
\pi_{GW} &= \Pr\left(G_E \text{ defaults while } D_E \text{ has exposure}\right)(1-L_W)\,\rho \\
&\quad + \Pr\left(G_E \text{ defaults while } G_W \text{ has exposure}\right)\left((1-L_W)\,\rho - B_E\right) \\
&= t\,(1-L_W)\,\rho + (1-t)\left((1-L_W)\,\rho - (1-t)\,B_E\right) \\
&= (1-L_W)\,\rho - (1-t)\,B_E
\end{aligned}$$

If G_E does not default, the payoff is $\pi_{GW} = (1-L_W)\,\rho + r_G B_E$. Then G_W is willing to grant the intragroup loan if

$$(1-p_E)\left((1-L_W)\,\rho - (1-t)\,B_E\right) + p_E\left((1-L_W)\,\rho + r_G B_E\right) \geq (1-L_W)\,\rho$$

equivalently, if $-(1-p_E)\,(1-t) + p_E r_G \geq 0$. G_W's break-even per-unit fee $r_G = (1-t)\,(1-p_E)\,/p_E$ solves this equation as an equality.

Proof of lemma 5 Assuming that all $p_E \in P_{FX}$ choose the swap, D_E's payoffs are $\pi_{DE} = s_D B_E$ if G_E does not default and, if G_E defaults,

$$\begin{aligned}
\pi_{DE} &= \Pr\left(G_E \text{ defaults while } D_E \text{ has exposure}\right)(-B_E) \\
&\quad + \Pr\left(G_E \text{ defaults while } G_W \text{ has exposure}\right)(s_D B_E) \\
&= t\,(-B_E) + (1-t)\,(s_D B_E)
\end{aligned}$$

D_E is willing to offer the FX swap if his expected payoff from doing is (at least) equal to zero, i.e., if

$$E\left[(1-p_E)\left(t\left(-B_E\right)+(1-t)\,s_D B_E\right)+p_E s_D B_E | p_E\in P_{FX}\right]\geq 0$$

D_E's break-even per-unit fee $s_D\left(t\right)=t\left(1-E\left[p|p\in P_{FX}\right]\right)/\left(1-t\left(1-E\left[p|p\in P_{FX}\right]\right)\right)$ solves this equation as an equality.

Proof of lemma 6 The change of the total expected cost of the FX swap with respect to p_E is, for a given P_E, independent of p_E:

$$\frac{\partial p_E r_{FX}}{\partial p_E}=-\left(1-t\right)+t\left(\frac{1-E\left[p|p\in P_{FX}\right]}{1-t\left(1-E\left[p|p\in P_{FX}\right]\right)}\right)$$

This implies that the expected costs of an FX swap are monotone in p_E for all $p_E\in P_{FX}$. This derivative is positive if

$$t^2-\frac{3-2E\left[p|p\in P_{FX}\right]}{1-E\left[p|p\in P_{FX}\right]}t+\frac{1}{1-E\left[p|p\in P_{FX}\right]}\leq 0$$

The solutions to the corresponding quadratic equation are

$$t_{1,2}=\frac{3-2E\left[p|p\in P_{FX}\right]}{2\left(1-E\left[p|p\in P_{FX}\right]\right)}\pm\frac{\sqrt{4\left(1-E\left[p|p\in P_{FX}\right]\right)^2+1}}{2\left(1-E\left[p|p\in P_{FX}\right]\right)}$$

It is straighforward to show that the larger root is always larger than 1. Thus,

$$\frac{\partial\left(p_E r_{FX}\right)}{\partial p_E}\begin{cases}>0 & \text{if} \quad t\geq t_2\\ <0 & \text{if} \quad t<t_2\end{cases}$$

One can show that the smaller root t_2 is strictly increasing in $E\left[p|p\in P_{FX}\right]$. Also, $t_2\in[0.38,1]$ because

$$\begin{aligned}\lim_{E[p|p\in P_{FX}]\to 0} t_2 &= \frac{3-2\,(0)}{2\,(1-0)}-\frac{\sqrt{4\,(1-0)^2+1}}{2\,(1-0)}=\frac{3}{2}-\frac{1}{2}\sqrt{5}=0.38\\ \lim_{E[p|p\in P_{FX}]\to 1} t_2 &= \lim_{x\to 1}\left(\frac{3-2x}{2\,(1-x)}-\frac{\sqrt{4\,(1-x)^2+1}}{2\,(1-x)}\right)=1\end{aligned}$$

Proof of proposition 1 As with local liquidity management, G_E's expected payoffs

$$\pi_{GE}\left(L_E,p_E\right)=\begin{cases}\left(1-L_E\right)\rho & \text{if} \quad L_E\geq\lambda\\ \left(1-L_E\right)\rho-\frac{1}{2}p_E\min\left\{r_D,r_{FX}\right\}B_E & \text{if} \quad L_E<\lambda \text{ and } \min\left\{r_D,r_{FX}\right\}B_E\leq C\\ \left(1-L_E\right)\rho-\frac{1}{2}p_E C & \text{if} \quad L_E<\lambda \text{ and } \min\left\{r_D,r_{FX}\right\}B_E\geq C\end{cases}$$

are declining in B_E because borrowing is costly, $B_E \leq \lambda - L_E$ in all equilibria. Because the cost of technical default is independent of the amount outstanding at the end of day 1, $B_E \in \{0, \lambda - L_E\}$ in equilibrium: G_E either borrows sufficiently to avoid technical default, or nothing at all. If $B_E = \lambda - L_E$, then

$$\begin{aligned}\pi_{GE}(L_E, p_E) &= \begin{cases} (1 - L_E)\rho - \frac{1}{2}p_E r_D(\lambda - L_E) & \text{if} \quad \min\{r_D, r_{FX}\}(\lambda - L_E) \leq C \\ (1 - L_E)\rho - \frac{1}{2}p_E C & \text{if} \quad \min\{r_D, r_{FX}\}(\lambda - L_E) \geq C \end{cases} \\ &= \begin{cases} (1 - L_E)\rho - \frac{1}{2}p_E \min\{r_D, r_{FX}\}(\lambda - L_E) & \text{if} \quad \lambda - C/\min\{r_D, r_{FX}\} \leq L_E \\ (1 - L_E)\rho - \frac{1}{2}p_E C & \text{if} \quad \lambda - C/\min\{r_D, r_{FX}\} > L_E \end{cases}\end{aligned}$$

Let P_I be the set of types that opt for an interbank loan, and P_{FX} the set of types that opt for an FX swap. The following paragraphs establish the existence conditions for the equilibria. The equilibria are grouped by whether or not some risks choose to refinance themselves via an FX swap or an interbank loan: i.e., whether in a certain class of equilibria, P_{FX} and P_I are non-empty. For each of the possible cases, conditions for the existence of such equilibria are derived.

1. Suppose $P_{FX} = P_I = \varnothing$. As in the case of local liquidity management, this equilibrium always exists if $t > 0$ given G_E's off-equilibrium belief that he faces the worst risk if G_E asks for an interbank loan or an FX swap. Then $L_E = 0$ is preferred over $L_E = \lambda$ under the same conditions as in local liquidity management, and

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \\ \lambda & \text{if} \quad p_E \geq 2\lambda\rho/C \end{cases}$$

2. Suppose $P_{FX} \neq \varnothing$.

   First notice that in all equilibria in which there is global refinancing, the worst risks will prefer to declare technical default. This is because any FX swap involves at least some portion of informed lending ($t < 1$), and because the expected costs, $p_E C$, of the reputational penalty, is very small for bad risks. Thus, $P_{FX} \neq \varnothing$ requires that p'' exists.

   Proof that $P_I = \varnothing$. Suppose in contrast that in equilibrium, $P_I \neq \varnothing$. For this to be an equilibrium, we need that $r_D \leq r_{FX}(p'_E)$ for any $p'_E \in P_I$. Now r_{FX} is strictly decreasing in p_E, so all $p_E < p'_E$ strictly prefer the external loan to the FX swap. Thus, for any $p'_E \in P_I$ and $p''_E \in P_{FX}$, $p''_E > p'_E$ (in words: If $P_I \neq \varnothing$, in equilibrium the better risks go for the FX swap). Because all $p'_E \in P_I$ take out the interbank loan in equilibrium, all $p_E < \max(P_I)$ also prefer refinancing via an interbank loan over technical default. (Recall that the choice between an interbank loan and technical default is independent of p_E.) Because all p'_E decided to hold no liquidity ex ante, all $p_E < \max(P_I)$ prefer to hold no liquidity ex ante because their expected borrowing costs, $p_E r_D$, are even smaller than the best risk's in P_I, while the opportunity costs of holding liquidity are constant for all risks. Thus, all $p_E < \max(P_I)$ take out the interbank loan in equilibrium. (Notice that this uses the assumption that the expected return from the illiquid asset is independent of p_E.) Thus, if $P_I \neq \varnothing$, then $P_I = [0, p']$ for some $p' \in (0, 1]$.

Now consider $p_E = p'$. p' is indifferent between an interbank loan and the cheaper of his other two options: an FX swap and technical default. But $P_I = [0, p']$ implies that first, $r_D > (1-p')/p'$, and second, that $s_D < (1-p')/p'$ because only risks better than p' choose the FX swap. This implies that for all t, $r_{FX}(p') = (1-t)(1-p')/p' + ts_D < r_D$. But then p' strictly prefers the FX swap over the interbank loan, a contradiction. Thus, we cannot have $P_I \neq \varnothing$ if $P_{FX} \neq \varnothing$.

Having established that $P_{FX} \neq \varnothing$ implies $P_I = \varnothing$, we can rewrite G_E's expected profit as

$$\pi_{GE}(L_E, p_E) = \begin{cases} (1-L_E)\rho - \frac{1}{2}p_E r_{FX}(\lambda - L_E) & \text{if} \quad \lambda - C/r_{FX} \leq L_E \\ (1-L_E)\rho - \frac{1}{2}p_E C & \text{if} \quad \lambda - C/r_{FX} > L_E \end{cases}$$

For a given $P_{FX} \neq \varnothing$, r_{FX} does not depend on L_E. Then $\pi_{GE}(L_E, p_E)$ is declining in L_E with slope ρ for small $L_E \leq \lambda - C/\min\{r_D, r_{FX}\}$. For larger L_E, the slope increases to $\rho + \frac{1}{2}p_E$. Thus, in all equilibria in which $P_{FX} \neq \varnothing$, $\pi_{GE}(L_E, p_E)$ is convex in L_E and the L_E that maximises π_{GE} must be a corner solution, i.e., in $\{0, \lambda\}$. Clearly, if $L_E = \lambda$, only $B_E = 0$ is optimal. If $L_E = 0$, both $B_E \in \{0, \lambda\}$ can be optimal depending on whether the costs of raising liquidity exceed the costs of technical default.

Notice that the calculations of the following paragraphs are illustrated in figure 4 for the case that $p_E r_{FX}(p_E)$ is strictly decreasing in p_E.

(a) Second stage. Consider the refinancing decision for those types which do not have enough liquidity to absorb the liquidity shock. Risk p_E prefers refinancing over technical default if

$$(1-0)\rho - \frac{1}{2}p_E r_{FX}(p_E)(\lambda - 0) \geq (1-0)\rho - \frac{1}{2}p_E C$$

equivalently, $r_{FX}(p_E) \leq C/\lambda$. Let $P'' = \{p \in [0,1] : r_{FX}(p) = C/\lambda\}$.

(b) First stage. Given the second-round decision, $L_E = \lambda$ is preferred if

$$(1-\lambda)\rho \geq \max\left\{(1-0)\rho - \frac{1}{2}p_E r_{FX}(p_E)(\lambda - 0), (1-0)\rho - \frac{1}{2}p_E C\right\}$$

equivalently, $2\lambda\rho \leq p_E \min\{\lambda r_{FX}(p_E), C\}$. Let $P''' = \{p \in [0,1] : 2\lambda\rho = p\max\{\lambda r_{FX}(p), C\}\}$

Notice that we have not shown that P'' and P''' contain only one point. This is because r_{FX} depends on P_{FX}, which, in turn, depends on P'' and P'''. However, we can say something about the structure of P'' and P''' in all equilibria in which $P_{FX} \neq \varnothing$: For any $P_{FX} \neq \varnothing$, r_{FX} is strictly declining in p_E. Thus

$$P'' = \{p'' : p'' \in [0,1], r_{FX}(p'') = C/\lambda\}$$

Also, for any $P_{FX} \neq \varnothing$, $pr_{FX}(p)$ is strictly monotone in p. Thus

$$P''' = \{p''' : p'' \in [0,1], 2\lambda\rho = p''' \min\{\lambda r_{FX}(p'''), C\}\}$$

The following paragraphs simply characterise the equilibria depending on whether P'' and P''' are empty. This is done first for the case that $p_E r_{FX}(p_E)$ is strictly increasing in p_E, and then for the case that $p_E r_{FX}(p_E)$ is strictly decreasing in p_E.

(a) Suppose $p_E r_{FX}(p_E)$ is strictly increasing in p_E. Then the expected costs of refinancing, and the expected costs of declaring technical default are highest for the best risks. Thus, in equilibrium, at most the best risks prefer to hold $L_E = \lambda$.

  i. $P'' = \varnothing$. This may occur for two reasons. First, $r_{FX}(p) > C/\lambda$ for all p. Then technical default would always be preferred over refinancing via an FX swap. But then $P_{FX} = \varnothing$. Second, $r_{FX}(p) < C/\lambda$ for all p, implying that refinancing via an FX swap would always be preferred over technical default. If $P''' \neq \varnothing$, then all p''' fulfil $2\rho = p''' r_{FX}(p''')$ and for each p''' there is an equilibrium in which

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq p''' \\ \lambda & \text{if} \quad p_E > p''' \end{cases}$$

  and $P_{FX} = [0, p''']$. If $P''' = \varnothing$ because for all $p \in [0,1]$, $2\rho > pr_{FX}(p)$, then $L_E = 0$ for all p and $P_{FX} = [0,1]$. If $P''' = \varnothing$ because for all $p \in [0,1]$, $2\rho < pr_{FX}(p)$, there is no equilibrium in which $P_{FX} \neq \varnothing$.

  ii. $P'' \neq \varnothing$.

    A. Suppose first that $P''' = \varnothing$. This may occur for two reasons. First, for all $p \in [0,1]$, $2\lambda\rho > p \min\{\lambda r_{FX}(p), C\}$. In this case, the expected costs of refinancing or technical default are smaller than the opportunity cost for holding liquidity, and $L_E = 0$ for all types. For each $p'' \in P''$, there is an equilibrium in which $L_E = 0$ for all types. If a liquidity outflow occurs, only good risks will refinance: $P'' \neq \varnothing$ implies that $p_E C$ and $p_E r_{FX}(p_E)$ intersect at least once; also, both are linear in p_E, and for bad risks, technical default is expected to be less expensive than refinancing because

$$\lim_{p_E \to 0} p_E C = 0 < \lim_{p_E \to 0} p_E r_{FX}(p_E) = 1 - t$$

Thus, in these equilibria, $P_{FX} = [p'', 1]$. Second, if $P''' = \varnothing$ because for all $p \in [0,1]$, $2\lambda\rho < p \min\{\lambda r_{FX}(p), C\}$, then there is no equilibrium in which $P_{FX} \neq \varnothing$.

B. Now suppose that $P''' \neq \varnothing$. Take any $p'' \in P''$, $p''' \in P'''$. Assume first that $p''' < p''$. But then $P_{FX} = \varnothing$: $P'' \neq \varnothing$ implies that $p_E C$ and $p_E r_{FX}(p_E)$ intersect at least once; hence, $\lim_{p_E \to 0} p_E C < \lim_{p_E \to 0} p_E r_{FX}(p_E)$ implies that $p_E C < p_E r_{FX}(p_E)$ for all $p_E < p''$. But by definition of p''' and increasingness of $p_E r_{FX}(p_E)$, all risks $p_E > p'''$ prefer to hoard liquidity over refinancing. Assume instead that $p''' \geq p''$. Then p''' is determined by indifference between $L_E = 0$ cum refinancing and $L_E = \lambda$; that is, $p''' : 2\rho < p''' r_{FX}(p''')$. Then the equilibrium takes the form

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq p''' \\ \lambda & \text{if} \quad p_E > p''' \end{cases}$$

and $P_{FX} = [p'', p''']$. All risks worse than p'' declare technical default if hit by a liquidity outflow.

(b) Suppose $p_E r_{FX}(p_E)$ is strictly decreasing in p_E. Then the expected costs of refinancing are decreasing in p_E, and the expected costs of declaring technical default are increasing in p_E. Thus, if $P_{FX} \neq \varnothing$, then $P_{FX} = \left[p^{IV}, 1\right]$ for some $p^{IV} \in [0, 1)$. Then p^{IV} is determined either by indifference between $L_E = 0$ cum refinancing and $L_E = \lambda$, or by indifference between $L_E = 0$ cum refinancing and $L_E = 0$ cum technical default.

i. $P'' = \varnothing$. As above, we need $r_{FX}(p) < C/\lambda$ for all p for $P_{FX} \neq \varnothing$. Then p^{IV} is determined by indifference between $L_E = 0$ cum refinancing and $L_E = \lambda$. $L_E = \lambda$ is preferred if $2\rho < p_E r_{FX}(p_E)$, i.e., if p_E is sufficiently small (recall that here we assume that $p_E r_{FX}(p_E)$ is strictly decreasing in p_E). Thus, $p^{IV} = p'''$ and there is an equilibrium such that

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \geq p''' \\ \lambda & \text{if} \quad p_E < p''' \end{cases}$$

and $P_{FX} = [p''', 1]$.

ii. $P'' \neq \varnothing$. p_E prefers $L_E = 0$ if $2\lambda\rho \geq p_E \min\{\lambda r_{FX}(p_E), C\}$. Strict decreasingness of $p_E r_{FX}(p_E)$ and strict increasingness of $p_E C$ in p_E imply that $p_E \min\{\lambda r_{FX}(p_E), C\}$ is maximal at $p''C$. Then there are two cases:

A. First, $2\lambda\rho > p_E \min\{\lambda r_{FX}(p_E), C\}$ for all p_E, equivalently, $2\lambda\rho > p''C$ (or again equivalently, $2\rho > p'' r_{FX}(p'')$). Then p^{IV} is determined by indifference between $L_E = 0$ cum refinancing and $L_E = 0$ cum technical default. $L_E = 0$ for all p_E. Consequently, $p^{IV} = p''$ and $P_{FX} = [p'', 1]$ and all $p_E \leq p''$ declare technical default.

B. Alternatively, there are p_E such that $2\lambda\rho = p_E \min\{\lambda r_{FX}(p_E), C\}$. Then p^{IV} is determined by indifference between $L_E = 0$ cum refinancing and $L_E = \lambda$. Consequently, $p^{IV} = p'''$. In this case, sufficiently low p_E prefer $L_E = 0$ cum technical default over $L_E = \lambda$ if, as before, $p_E \leq 2\lambda\rho/C$. ($p'' < p^{IV}$ implies $p''' > 2\lambda\rho/C$.) Thus, there is an equilibrium such that

$$L_E = \begin{cases} 0 & \text{if} \quad p_E \leq 2\lambda\rho/C \text{ or } p_E \geq p''' \\ \lambda & \text{if} \quad 2\lambda\rho/C < p_E < p''' \end{cases}$$

and $P_{FX} = [p''', 1]$.

3. Finally, suppose $P_{FX} = \varnothing$ and $P_I \neq \varnothing$. This equilibrium only exists for non-PvP settlement and is supported by the (not very convincing) off-equilibrium belief that if G_E asks off equilibrium for an FX swap, D_E refuses, believing that it faces the worst risk. Given these beliefs, the FX swap is essentially unavailable for G_E, so the situation is exactly the same as under local liquidity management. The off-equilibrium belief is classified as unconvincing because it implies that D_E is ready to accept an overnight exposure to G_E, but not the shorter exposure it would carry if it granted an FX swap.

Proof of lemma 7 p'' is defined by $r_{FX}(p'') = C/\lambda$. Taking total derivatives yields

$$\left(\frac{\partial r_{FX}}{\partial p} - \frac{\partial (C/\lambda)}{\partial p}\right) dp + \left(\frac{\partial r_{FX}}{\partial t} - \frac{\partial (C/\lambda)}{\partial t}\right) dt = \frac{\partial r_{FX}}{\partial p} dp + \frac{\partial r_{FX}}{\partial t} dt = 0$$

such that

$$\frac{dp''}{dt} = -\frac{\partial r_{FX}}{\partial t} \Big/ \frac{\partial r_{FX}}{\partial p}$$

p''' is defined by $p''' r_{FX}(p''') = 2\rho$. Taking total derivatives yields

$$\begin{aligned} & \left(\frac{\partial (p r_{FX})}{\partial p} - \frac{\partial (2\rho)}{\partial p}\right) dp + \left(\frac{\partial (p r_{FX})}{\partial t} - \frac{\partial (2\rho)}{\partial t}\right) dt \\ = \; & \frac{\partial (p r_{FX})}{\partial p} dp + p \frac{\partial r_{FX}}{\partial t} dt = 0 \end{aligned}$$

such that

$$\frac{dp'''}{dt} = -\frac{\partial r_{FX}}{\partial t} / \frac{\partial p r_{FX}}{\partial p}$$

The remainder of the proof computes the signs of the derivatives. In the equilibria under consideration, $P_{FX} = [p, 1]$ for $p \in \{p'', p'''\}$. Then $E[p|p \in P_{FX}] = (p+1)/2$. Then

$$r_{FX}(p,t) = (1-t)\frac{1-p}{p} + t\left(\frac{1-(p+1)/2}{1-t(1-(p+1)/2)}\right)$$

- Regarding the derivative with respect to p,

$$\frac{\partial r_{FX}}{\partial p} = \frac{\partial\left((1-t)\frac{1-p}{p} + t\left(\frac{1-(p+1)/2}{1-t(1-(p+1)/2)}\right)\right)}{\partial p} = -(2-t)\frac{(p-1)^2 t^2 + \left(p^2+2p-3\right)t + 2}{p^2(-t+pt+2)^2} < 0$$

  if $(p-1)^2 t^2 + \left(p^2+2p-3\right)t + 2 > 0$, equivalently,

$$t^2 + \frac{\left(p^2+2p-3\right)}{(p-1)^2}t + \frac{2}{(p-1)^2} > 0$$

  This holds always true because the roots of the corresponding quadratic equation

$$t_{1,2} = -\frac{\left(p^2+2p-3\right)}{2(p-1)^2} \pm \sqrt{\left(\frac{\left(p^2+2p-3\right)}{2(p-1)^2}\right)^2 - \frac{2}{(p-1)^2}}$$

  are both negative.

- Regarding the derivative with respect to t,

$$\frac{\partial r_{FX}}{\partial t} = \frac{\partial\left((1-t)\frac{1-p}{p} + t\left(\frac{1-(p+1)/2}{1-t(1-(p+1)/2)}\right)\right)}{\partial t} = -(1-p)\frac{(-t+pt+2)^2 - 2p}{p(-t+pt+2)^2} < 0$$

  It is straightforward to show that the right-hand side is negative as intuition suggests: A decrease in the share of informed lending reduces the costs for the marginal lender. Thus,

$$\frac{dp''}{dt} = -\frac{\partial r_{FX}}{\partial t} / \frac{\partial r_{FX}}{\partial p} < 0$$

- Regarding dp'''/dt, in equilibria of type (C5), we have $\partial p r_{FX}/\partial p < 0$, so

$$\frac{dp'''}{dt} = -\frac{\partial r_{FX}}{\partial t} / \frac{\partial p r_{FX}}{\partial p} < 0$$

Proof of proposition 4 p'' is defined by $r_{FX}(p'') = C/\lambda$. Taking total derivatives yields

$$\left(\frac{\partial r_{FX}}{\partial p} - \frac{\partial(C/\lambda)}{\partial p}\right)dp + \left(\frac{\partial r_{FX}}{\partial(C/\lambda)} - \frac{\partial(C/\lambda)}{\partial(C/\lambda)}\right)d(C/\lambda) = \frac{\partial r_{FX}}{\partial p}dp - d(C/\lambda) = 0$$

such that

$$\frac{dp''}{d(C/\lambda)} = 1/\frac{\partial r_{FX}}{\partial p} < 0$$

p''' is defined by $2\rho = p''' r_{FX}(p''')$. Neither side of this equation depends on C/λ. (Also notice that there is no indirect dependence on C/λ via p'': in equilibria of type (C5), $P_{FX} = [p''', 1]$. C/λ only determines the parameter range of ρ for which the equilibrium (C5) exists (cf. figure 4).

From proposition 2, the likelihood of transmission of losses is equal to $\frac{1}{4}(1-p')^2$ or $p' \in \{p'', p'''\}$. Thus, the likelihood of transmission of losses is declining in p', and decreasing in (C/λ). From proposition 1, the likelihood of technical default is $\Pr(p_E < p'')$ in equilibria of type (C3), and $\Pr(p_E < 2\rho/(C/\lambda))$ in equilibria of type (C5). Both p'' and $2\rho/(C/\lambda)$ are decreasing in C/λ, implying that the likelihood of technical default is decreasing in the cost of technical default, and increasing in the size of the liquidity shock.

The comparative statics with respect to ρ are straightforward: ρ only affects the existence of equilibrium G3 when $\rho = \frac{1}{2} p'' r_{FX}(p'')$, but not p''; hence, if G3 exists, there is no effect of a marginal change in ρ on the likelihood of technical default and transmission of losses. For equilibria in G5, an increase in ρ makes hoarding liquidity more expensive. This increases the likelihood of technical default. Regarding the likelihood of transmission of losses, recall that $p''' : p''' \in [0,1]$ and $\rho = \frac{1}{2} p''' r_{FX}(p''', t, P_{FX})$ and, in G5, $P_{FX} = [p''', 1]$. Taking total derivatives yields

$$\left(\frac{\partial p r_{FX}}{\partial p} - \frac{\partial \rho}{\partial p}\right) dp + \left(\frac{\partial p r_{FX}}{\partial \rho} - \frac{\partial \rho}{\partial \rho}\right) d\rho = \frac{\partial p r_{FX}}{\partial p} dp - d\rho = 0$$

such that

$$\frac{dp'''}{d\rho} = 1 / \frac{\partial p r_{FX}}{\partial p} < 0$$

10.1.3 Proof of lemma 8

G3 converges to D3 in the following sense: For $t \to 1$, G3 exists if and only if D3 exists, and liquidity holding and refinancing conditions are identical in the sense that all types that refinance in G3 also refinance in D3.

Using $P_{FX} = [p'', 1]$ and $p'' : r_{FX}(p'') = C/\lambda$, one can solve explicitly for p'':

$$p'' = -\frac{\left(2C + 2\lambda - 5t\lambda + 2t^2\lambda - Ct - \sqrt{-(t-2)\left(4C\lambda + 2\lambda^2 - C^2 t - t\lambda^2 + 2C^2 - 6Ct\lambda\right)}\right)}{2t\left(C + \lambda(2-t)\right)}$$

Taking limits yields

$$\lim_{t \to 1} p'' = \frac{\lambda - C + \sqrt{(C-\lambda)^2}}{2(C+\lambda)} = \begin{cases} 0 & \text{if} \quad C > \lambda \\ (\lambda - C)/(\lambda + C) & \text{if} \quad C \leq \lambda \end{cases}$$

D3 exists if and only if $\rho > p r_{FX}(p)/2$ for all $p > p''$. Given $p'' < 1$, we have $t' < 1$ (cf. the proof of lemma 6), so $\partial p_E r_{FX}(p_E)/\partial p_E < 0$. Thus, D3 exists if and only if $\rho > r_{FX}(1)/2$. Taking limits wrt t yields

$$\begin{aligned}
\rho &> \frac{1}{2}\lim_{t\to 1}\left((1-t)\frac{1-1}{1}+t\left(\frac{1-E\left[p|p\in P_{FX}\right]}{1-t\left(1-E\left[p|p\in P_{FX}\right]\right)}\right)\right) \\
&= \frac{1}{2}\left(\frac{1-\frac{1}{2}\left(1+\max\left\{0,\frac{\lambda-C}{C+\lambda}\right\}\right)}{1-\left(1-\frac{1}{2}\left(1+\max\left\{0,\frac{\lambda-C}{C+\lambda}\right\}\right)\right)}\right)=\frac{1}{2}\frac{1-\max\left\{0,\frac{\lambda-C}{C+\lambda}\right\}}{1+\max\left\{0,\frac{\lambda-C}{C+\lambda}\right\}}
\end{aligned}$$

Now $C > \lambda$ implies that this condition is equal to $\rho > 1/2$, which is the condition under which D3 exists. Also, for $C > \lambda$, $\lim_{t\to 1} p'' = 0$, such that $P_{FX} = [0,1] = P_D$ and $L_E = 0$ in both equilibria. That is, for $t \to 1$, the global bank's subsidiaries do not hold liquidity in either equilibrium, and if hit by a liquidity outflow, they both refinance independently of their risk using an overnight loan (or, in G3, an FX swap that is indistinguishable from an overnight loan).

10.2 List of symbols

- D_E and D_W for the domestic banks in countries E and W; G_E and G_W for the subsidiaries of the global bank.
- λ is the size of the liquidity shock
- p_E and p_W are the likelihoods that the subsidiaries' illiquid assets pay off R/p_E (R/p_W)
- ρ is the expected net payoff of each subsidiary's illiquid assets (i.e., expected gross return minus 1)
- C is the cost of technical default
- L_E and L_W: Liquidity holdings of the global bank's subsidiaries. Total balance sheet size is 1 in period zero.
- B_E and B_W is the amount the liquidity-short branch borrows.
- r_{Di} is the interest rate the domestic bank D_i charges for its loan
- s_D is the per-dollar fee the domestic bank requires as compensation for Herstatt risk in an FX swap
- r_G is the intragroup per-dollar fee the liquidity-rich subsidiary charges the liquidity-poor subsidiary as part of the FX swap