The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks*

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Abstract

The term premium on nominal long-term bonds in the standard dynamic stochastic general equilibrium (DSGE) model used in macroeconomics is far too small and stable relative to empirical measures obtained from the data—an example of the "bond premium puzzle." However, in models of endowment economies, researchers have been able to generate reasonable term premiums by assuming that investors face long-run economic risks and have recursive Epstein-Zin preferences. We show that introducing these two elements into a canonical DSGE model can also produce a large and variable term premium without compromising the model's ability to fit key macroeconomic variables.

*The views expressed in this paper are those of the authors and do not necessarily reflect the views of other individuals within the Federal Reserve System.

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1 Introduction

The term premium on long-term nominal bonds compensates investors for inflation and consumption risks over the lifetime of the bond. A large finance literature finds that these risk premiums are substantial and vary significantly over time (e.g., Campbell and Shiller, 1991, and Cochrane and Piazzesi, 2005); however, the economic forces that can justify such large and variable term premiums are less clear. Piazzesi and Schneider (2006) provide some economic insight into the source of a large positive mean term premium in a consumption-based asset pricing model of an endowment economy. Their analysis relies on two crucial features: first, the structural assumption that investors have Epstein-Zin recursive utility preferences,[1] and second, an estimated reduced-form process for the joint determination of consumption and inflation. With these two elements, they show that investors require a premium for holding nominal bonds because a positive inflation surprise lowers a bond's value and is associated with lower future consumption growth. In such a situation, bondholders' wealth decreases just as their marginal utility rises, so they require a premium to offset this risk. Using a similar structure—characterized by both Epstein-Zin preferences and reduced-form consumption and inflation empirics—Bansal and Shaliastovich (2008) also obtain significant time variation in the term premium.

An important shortcoming of such analyses is that they rely on reduced-form empirical correlations between consumption growth and inflation that have no direct structural foundation and may not be stable over time. For example, if the relative importance of technology and demand shocks shifts over time, the reduced-form correlations may change. Therefore, it is important to investigate the bond pricing implications of Epstein-Zin preferences in a structural economic model of preferences and technology. The canonical structural model connecting consumption and inflation is the dynamic stochastic general equilibrium (DSGE) model in which households and firms solve explicit optimization problems and form rational expectations in the face of fundamental shocks to productivity and other factors. In

[1] Early on, Kreps and Porteus (1978) established the theoretical framework for such recursive preferences, which were further developed by Epstein and Zin (1989) and Weil (1989).

this paper, we explore whether the above results with an exogenous reduced-form empirical process for consumption and inflation can be obtained in a structural model.

Our analysis also examines whether the earlier results in an endowment economy with Epstein-Zin investors can be generalized to a production economy. There is some reason to be skeptical in this regard. Although Wachter (2006) obtained a significant mean term premium in an endowment economy using long-memory habit preferences (à la Campbell and Cochrane, 1999), Rudebusch and Swanson (2008) showed that such long-memory habits generated only a negligible term premium in a DSGE model. In particular, because households in a production economy can endogenously trade off labor and consumption, they are much better insulated from consumption risk than households in an endowment economy, who must consume whatever endowment they receive.[2] In a production economy, when households are hit by a negative shock, they can compensate by increasing their labor supply and working more hours, which provides partial insurance against shocks to consumption. Households in an endowment economy do not have this opportunity, so the consumption cost of shocks is correspondingly greater, and risky assets thus carry a larger risk premium. Therefore, it is important to explore whether the endowment economy results with Epstein-Zin preferences hold in a production economy.

In this paper, we use an augmented DSGE model to illuminate the economic forces behind movements in long-term nominal bond premiums by trying to match both macroeconomic moments (e.g., the standard deviations of consumption and inflation) and bond pricing moments (e.g., the means and volatilities of the yield curve slope and bond excess holding period returns). The underlying form of our model follows the standard structure of DSGE models (e.g., Christiano, Eichenbaum, and Evans, 2005, and Smets and Wouters, 2003) and, notably, contains an important role for nominal rigidities in order to endogenously describe the behavior of inflation and other nominal quantities. However, to produce a significant term premium, we make two key additions to the model. First, we assume that households in the model have Epstein-Zin preferences, so risk aversion can be modeled independently from

[2] Jermann (1998), Lettau and Uhlig (2000), and Boldrin, Christiano, and Fisher (2001) also stress this difference between endowment and production economies in accounting for the equity premium.

the intertemporal elasticity of substitution. Such a separation allows the model to match risk premiums even in the face of the intertemporal substitution possibilities associated with variable labor supply.[3] Second, our model includes long-run economic risks. Bansal and Yaron (2004) have stressed that uncertainty about the economy's long-run growth prospects can play an important role in generating sizable equity risk premiums, and persistent real shocks to technology will also play a role in our model. However, because we are pricing a nominal asset, we also consider long-run nominal risks as the central bank's long-run inflation objective is allowed to vary over time with the recent history of inflation.[4]

Together, these two key ingredients—Epstein-Zin preferences and long-run economic risk—allow our model to replicate the level and variability of the term premium without compromising its ability to fit macroeconomic variables. Intuitively, our model is identical to first order to the standard macroeconomic DSGE representations because the first-order approximation to Epstein-Zin preferences is the same as the first-order approximation to standard expected utility preferences. Furthermore, the macroeconomic moments of the model are not very sensitive to the additional second and higher-order terms introduced by Epstein-Zin preferences, while risk premiums are unaffected by first-order terms and completely determined by those second- and higher-order terms. Therefore, by varying the Epstein-Zin risk aversion parameter while holding the other parameters of the model constant, we are able to fit the asset pricing facts without compromising the model's ability to fit the macroeconomic data.

Our analysis has implications for both the finance and macroeconomic literatures. For finance, our analysis can illuminate the earlier reduced-form results with an economic structural interpretation. For macroeconomics, our results suggest a path to transform the standard DSGE model into a complete description of the economy. As a theoretical matter, asset prices and the macroeconomy are inextricably linked; indeed, as emphasized by Cochrane

[3] Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2008) also price bonds in a DSGE model with Epstein-Zin preferences, although their model treats inflation as an exogenous stochastic process and thus suffers from some of the same drawbacks as Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2007).

[4] Gürkaynak, Sack, and Swanson (2005) showed that a small degree of inflation "pass-through" of this form helps account for the "excess sensitivity" of U.S. long-term bond yields to macroeconomic news.

(2007), asset markets are the mechanism in the model by which consumption and investment are allocated across time and states of nature. Therefore, the usual macroeconomic modeling strategy of ignoring asset prices is untenable, as any complete DSGE model must match the long-term nominal interest rate and other asset prices as well as consumption and inflation.

The remainder of the paper proceeds as follows. Section 2 lays out a stylized canonical DSGE model with Epstein-Zin preferences. Section 3 presents results for this model and shows how it is able to match the term premium without impairing the model's ability to fit macroeconomic variables. Section 4 introduces a model with enhanced long-run economic risks, which improves the model's overall fit to the data. Section 5 concludes. A technical appendix provides additional details of how to incorporate and solve Epstein-Zin preferences in an otherwise standard DSGE model.

2 A DSGE Model with Epstein-Zin Preferences

In this section, we describe a standard DSGE model that is modified to include Epstein-Zin preferences. We also price nominal bonds in this model and present a variety of measures of the term premium and bond risk.

2.1 Epstein-Zin Preferences

It is standard practice in macroeconomics to assume that a representative household chooses state-contingent plans for consumption, c, and labor, l, so as to maximize an expected utility functional:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{1}$$

subject to an asset accumulation equation, where $\beta \in (0,1)$ is the household's discount factor and the period utility kernel $u(c_t, l_t)$ is twice-differentiable, concave, increasing in c, and decreasing in l. The maximand in equation (1) can be expressed in first-order recursive form as:

$$V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1}, \tag{2}$$

where the household's state-contingent plans at time t are chosen so as to maximize V_t.

In this paper, we follow the finance literature and generalize (2) to an Epstein-Zin specification:

$$V_t \equiv u(c_t, l_t) + \beta \left(E_t V_{t+1}^{1-\alpha}\right)^{1/(1-\alpha)}, \tag{3}$$

where the parameter α can take on any real value.[5] If $u \geq 0$ everywhere, then the proof of Theorem 3.1 in Epstein and Zin (1989) shows that there exists a solution V to (3) with $V \geq 0$. If $u \leq 0$ everywhere, then it is natural to let $V \leq 0$ and reformulate the recursion as:

$$V_t \equiv u(c_t, l_t) - \beta \left[E_t(-V_{t+1})^{1-\alpha}\right]^{1/(1-\alpha)}. \tag{4}$$

The proof in Epstein and Zin (1989) also demonstrates the existence of a solution V to (4) with $V \leq 0$ in this case.[6] When $\alpha = 0$, both (3) and (4) reduce to the standard case of expected utility (2). When $u \geq 0$ everywhere, higher (lower) values of α correspond to greater (lesser) degrees of risk aversion. When $u \leq 0$ everywhere, the opposite is true: higher (lower) values of α correspond to lesser (greater) degrees of risk aversion.

Note that, traditionally, Epstein-Zin preferences over consumption streams have been written as:

$$\widetilde{V}_t \equiv \left[c_t^\rho + \beta \left(E_t \widetilde{V}_{t+1}^{\widetilde{\alpha}}\right)^{\rho/\widetilde{\alpha}}\right]^{1/\rho}, \tag{5}$$

but by setting $V_t = \widetilde{V}_t^\rho$ and $\alpha = 1 - \widetilde{\alpha}/\rho$, this can be seen to correspond to (3). Moreover, the form (3) has the advantage that it allows us to consider standard DSGE utility kernels involving both labor and inelastic intertemporal substitution ($\rho < 0$), which the form (5) cannot easily handle.

The key advantage of using Epstein-Zin utility (3) is that it breaks the equivalence between the inverse of the intertemporal elasticity of substitution and the coefficient of relative risk aversion that has long been noted in the literature regarding expected utility (2)—see, e.g., Mehra and Prescott (1985) and Hall (1988). In (3), the intertemporal elasticity

[5] The case $\alpha = 1$ corresponds to $V_t = u(c_t, l_t) + \beta \exp(E_t \log V_{t+1})$ for the case $u \geq 0$, and $V_t = u(c_t, l_t) - \beta \exp[E_t \log(-V_{t+1})]$ for $u \leq 0$.

[6] We exclude the case where u is sometimes positive and sometimes negative, although for local approximations around a deterministic steady state with infinitesimal uncertainty, this case does not present any particular difficulties.

of substitution over deterministic consumption paths is exactly the same as in (2), but now the household's risk aversion to uncertain lotteries over V_{t+1} can be amplified by the additional parameter α, a feature which is crucial for allowing us to fit both the asset pricing and macroeconomic facts below.[7]

We now turn to the utility kernel u, and we adopt the usual DSGE specification:

$$u(c_t, l_t) \equiv \frac{{c_t}^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}, \tag{6}$$

which allows for tractable modeling of nominal wage and price rigidities—an essential ingredient of models in this literature. If $\gamma > 1$, then (6) is nonpositive everywhere and V is defined by (4). If $\gamma \leq 1$, then there are two main approaches to ensure that the utility kernel u is everywhere positive. The first is to add a constant:

$$u(c_t, l_t) \equiv \frac{{c_t}^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} + \frac{\chi_0 \bar{l}^{1+\chi}}{1+\chi}, \tag{7}$$

where $\bar{l}$ denotes the household's time endowment. Note, however, that additive shifts of the utility kernel, as in (7), are nonneutral and affect the household's attitude towards risk, except for the special case of expected utility, $\alpha = 0$. (This will become apparent when we derive the household's stochastic pricing kernel, below.) The second approach is to use (6) but impose that there is some subsistence level $\underline{c} \geq 0$ for consumption below which households cannot go. By setting $\underline{c}$ high enough, we can ensure that u is positive over the range of admissible values for c and l. Of these two approaches, we will generally opt for the latter, which does a better job of explaining the term premium below (although preliminary results suggest that in the larger-scale Christiano, Eichenbaum, and Evans (2005) model both approaches work about equally well).

2.2 The Household's Optimization Problem

We now turn to the representative household's optimization problem under Epstein-Zin preferences. We assume that households are representative and choose state-contingent

[7] Indeed, the linearization or log-linearization of (3) is exactly the same as that of (2), which turns out to be very useful for matching the model to macroeconomic variables, since models with (2) are already known to be able to fit macroeconomic quantities reasonably well. We will return to this point in Section 3, below.

consumption and labor plans so as to maximize (3) subject to an intertemporal flow budget constraint, specified below. We will solve the household's optimization problem as a Lagrange problem with the states of nature explicitly specified. To that end, let $s^0 \in S_0$ denote the initial state of the economy at time 0, let $s_t \in S$ denote the realizations of the shocks that hit the economy in period t, and let $s^t \equiv \{s^{t-1}, s_t\} \in S_0 \times S^t$ denote the initial state and history of all shocks up through time t. We define s^t_{t-1} to be the projection of the history s^t onto its first t components; that is, s^t_{t-1} is the history s^t as it would have been viewed at time $t-1$, before time-t shocks have been realized.

Households have access to an asset whose price is given by p_{t,s^t} in each period t and state of the world s^t. In each period t, households choose the quantity of consumption c_{t,s^t}, labor l_{t,s^t}, and asset holdings a_{t,s^t} that will carry through to the next period, subject to a constraint that the household's asset holdings a_{t,s^t} are always greater than some lower bound $\underline{a} \ll 0$, which does not bind in equilibrium but rules out Ponzi schemes. Households are price takers in consumption, asset, and labor markets, and face a price per unit of consumption of P_{t,s^t}, and nominal wage rate w_{t,s^t}. Households also own an aliquot share of firms and receive a per-period lump-sum transfer from firms in the amount d_{t,s^t}. The household's flow budget constraint is thus:

$$p_{t,s^t} a_{t,s^t} + P_{t,s^t} c_{t,s^t} = w_{t,s^t} l_{t,s^t} + d_{t,s^t} + p_{t,s^t} a_{t-1,s^t_{t-1}}. \tag{8}$$

The household's optimization problem is to choose a sequence of vector-valued functions, $[c_t(s^t), l_t(s^t), a_t(s^t)] \colon S_0 \times S^t \to [\underline{c}, \infty] \times [0, \bar{l}] \times [\underline{a}, \infty]$ so as to maximize (3) subject to the sequence of budget constraints (8). For clarity in what follows, we assume that s^0 and s_t can take on only a finite number of possible values (i.e., S_0 and S have finite support), and we let $\pi_{s^\tau | s^t}$, $\tau \geq t \geq 0$, denote the probability of realizing state s^τ at time τ conditional on being in state s^t at time t.

The household's optimization problem can be formulated as a Lagrangean, where the household chooses state-contingent plans for consumption, labor, and asset holdings, $(c_{t,s^t}, l_{t,s^t}, a_{t,s^t})$, that maximize V_0 subject to the infinite sequence of state-contingent constraints (3) and (8), that is, maximize:

$$\mathcal{L} \equiv V_{0,s^0} - \sum_{t=0}^{\infty}\sum_{s^t}\mu_{t,s^t}\left\{V_{t,s^t} - u(c_{t,s^t}, l_{t,s^t}) - \beta\left(\sum_{s^{t+1}}\pi_{s^{t+1}|s^t}V_{t+1,s^{t+1}}^{1-\alpha}\right)^{1/(1-\alpha)}\right\} -$$
$$\sum_{t=0}^{\infty}\sum_{s^t}\lambda_{t,s^t}\{p_{t,s^t}a_{t,s^t} + P_{t,s^t}c_{t,s^t} - w_{t,s^t}l_{t,s^t} - d_{t,s^t} - p_{t,s^t}a_{t-1,s^t_{t-1}}\}. \tag{9}$$

The household's first-order conditions for (9) are then:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial c_{t,s^t}} &: \quad \mu_{t,s^t}u_1|_{(c_{t,s^t},l_{t,s^t})} = P_{t,s^t}\lambda_{t,s^t},\\
\frac{\partial\mathcal{L}}{\partial l_{t,s^t}} &: \quad -\mu_{t,s^t}u_2|_{(c_{t,s^t},l_{t,s^t})} = w_{t,s^t}\lambda_{t,s^t},\\
\frac{\partial\mathcal{L}}{\partial a_{t,s^t}} &: \quad \lambda_{t,s^t}p_{t,s^t} = \sum_{s^{t+1}\supseteq s^t}\lambda_{t+1,s^{t+1}}p_{t+1,s^{t+1}},\\
\frac{\partial\mathcal{L}}{\partial V_{t,s^t}} &: \quad \mu_{t,s^t} = \beta\pi_{s^t|s^t_{t-1}}\mu_{t-1,s^t_{t-1}}\left(\sum_{\tilde{s}^t\supseteq s^t_{t-1}}\pi_{\tilde{s}^t|s^t_{t-1}}V_{t,\tilde{s}^t}^{1-\alpha}\right)^{\alpha/(1-\alpha)}V_{t,s^t}^{-\alpha}; \qquad \mu_{0,s^0} = 1
\end{aligned}$$

Letting $(1 + r_{t+1,s^{t+1}}) \equiv p_{t+1,s^{t+1}}/p_{t,s^t}$, the gross rate of return on the asset, making substitutions, and defining the stationary Lagrange multipliers $\widetilde{\lambda}_{t,s^t} \equiv \beta^{-t}\pi^{-1}_{s^t|s^0}\lambda_{t,s^t}$ and $\widetilde{\mu}_{t,s^t} \equiv \beta^{-t}\pi^{-1}_{s^t|s^0}\mu_{t,s^t}$, these become:

$$\frac{\partial\mathcal{L}}{\partial c_{t,s^t}} : \quad \widetilde{\mu}_{t,s^t}u_1|_{(c_{t,s^t},l_{t,s^t})} = P_{t,s^t}\widetilde{\lambda}_{t,s^t} \tag{10}$$
$$\frac{\partial\mathcal{L}}{\partial l_{t,s^t}} : \quad -\widetilde{\mu}_{t,s^t}u_2|_{(c_{t,s^t},l_{t,s^t})} = w_{t,s^t}\widetilde{\lambda}_{t,s^t} \tag{11}$$
$$\frac{\partial\mathcal{L}}{\partial a_{t,s^t}} : \quad \widetilde{\lambda}_{t,s^t} = \beta E_{t,s^t}\widetilde{\lambda}_{t+1,s^{t+1}}(1 + r_{t+1,s^{t+1}}) \tag{12}$$
$$\frac{\partial\mathcal{L}}{\partial V_{t,s^t}} : \quad \widetilde{\mu}_{t,s^t} = \widetilde{\mu}_{t-1,s^t_{t-1}}(E_{t-1,s^t_{t-1}}V_{t,\tilde{s}^t}^{1-\alpha})^{\alpha/(1-\alpha)}V_{t,s^t}^{-\alpha}; \qquad \widetilde{\mu}_{0,s^0} = 1 \tag{13}$$

These first-order conditions are very similar to the expected utility case except for the introduction of the additional Lagrange multipliers $\widetilde{\mu}_{t,s^t}$, which translate utils at time t into utils at time 0, allowing for the "twisting" of the value function by α that takes place at each time $1, 2, \ldots, t$. Note that in the expected utility case, $\widetilde{\mu}_{t,s^t} = 1$ for every t and s^t, and equations (10) through (13) reduce to the standard optimality conditions. Substituting out for

$\widetilde{\lambda}_{t,s^t}$ and $\widetilde{\mu}_{t,s^t}$ in (10) through (13), we get the household's intratemporal and intertemporal (Euler) optimality conditions:

$$\begin{aligned} \frac{-u_2|_{(c_{t,s^t},l_{t,s^t})}}{u_1|_{(c_{t,s^t},l_{t,s^t})}} &= \frac{w_{t,s^t}}{P_{t,s^t}} \\ u_1|_{(c_{t,s^t},l_{t,s^t})} &= \beta E_{t,s^t}(E_{t,s^t} V_{t+1,s^{t+1}}^{1-\alpha})^{\alpha/(1-\alpha)} V_{t+1,s^{t+1}}^{-\alpha}\, u_1|_{(c_{t+1,s^{t+1}},l_{t+1,s^{t+1}})}(1+r_{t+1,s^{t+1}})\, P_{t,s^t}/P_{t+1,s^{t+1}} \end{aligned}$$

Finally, let $p_{t,s^t}^{s^\tau}$, $t \leq \tau$, denote the price at time t in state s^t of a state-contingent bond that pays one dollar at time τ in state s^τ and 0 otherwise. If we insert this state-contingent security into the household's optimization problem, we see that, for $t < \tau$:

$$p_{t,s^t}^{s^\tau} = \beta E_{t,s^t}(E_{t,s^t} V_{t+1,s^{t+1}}^{1-\alpha})^{\alpha/(1-\alpha)} V_{t+1,s^{t+1}}^{-\alpha} \frac{u_1|_{(c_{t+1,s^{t+1}},l_{t+1,s^{t+1}})}}{u_1|_{(c_{t,s^t},l_{t,s^t})}} \frac{P_{t,s^t}}{P_{t+1,s^{t+1}}}\, p_{t+1,s^{t+1}}^{s^\tau}. \tag{14}$$

That is, the household's (nominal) stochastic discount factor at time t in state s^t for stochastic payoffs at time $t+1$ is given by:

$$m_{t,s^t,t+1,s^{t+1}} \equiv \left(\frac{V_{t+1,s^{t+1}}}{(E_{t,s^t} V_{t+1,s^{t+1}}^{1-\alpha})^{1/(1-\alpha)}} \right)^{\alpha} \frac{\beta u_1|_{(c_{t+1,s^{t+1}},l_{t+1,s^{t+1}})}}{u_1|_{(c_{t,s^t},l_{t,s^t})}} \frac{P_{t,s^t}}{P_{t+1,s^{t+1}}} \tag{15}$$

Despite the twisting of the value function by α, the price $p_{t,s^t}^{s^\tau}$ nevertheless satisfies the standard relationship:

$$\begin{aligned} p_{t,s^t}^{s^\tau} &= E_{t,s^t}\, m_{t,s^t,t+1,s^{t+1}}\, m_{t+1,s^{t+1},t+2,s^{t+2}}\, p_{t+2,s^{t+2}}^{s^\tau} \\ &= E_{t,s^t}\; m_{t,s^t,t+1,s^{t+1}}\, m_{t+1,s^{t+1},t+2,s^{t+2}} \cdots m_{\tau-1,s^{\tau-1},\tau,s^\tau} \end{aligned}$$

and the asset pricing equation (14) is linear in the future state-contingent payoffs, so that we can price any compound security by summing over the prices of its individual constituent state-contingent payoffs.

2.3 The Firm's Optimization Problem

To model nominal rigidities, we assume that the economy contains a continuum of monopolistically competitive intermediate goods firms indexed by $f \in [0, 1]$ that set prices according to Calvo contracts and hire labor from households in a competitive labor market. Firms have identical Cobb-Douglas production functions:

$$y_t(f) = A_t \overline{k}^{(1-\eta)} l_t(f)^\eta, \tag{16}$$

where $\overline{k}$ is a fixed, firm-specific capital stock and A_t denotes an aggregate technology shock that affects all firms.[8] We have suppressed the explicit state-dependence of the variables in this equation and in the remainder of the paper to ease the notational burden. The technology shock A_t follows an exogenous AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A, \tag{17}$$

where ε_t^A denotes an *i.i.d.* aggregate technology shock with mean zero and variance σ_A^2.

Firms set prices according to Calvo contracts that expire with probability $1 - \xi$ each period. When the Calvo contract expires, the firm is free to reset its price as it chooses, and we denote the price that the firm f sets in period t by $p_t(f)$. There is no indexation, so the price $p_t(f)$ that the firm sets is fixed over the life of the contract. In each period $\tau \geq t$ that the contract remains in effect, the firm must supply whatever output is demanded at the contract price $p_t(f)$, hiring labor $l_\tau(f)$ from households at the market wage w_τ.

Firms are collectively owned by households and distribute profits and losses back to households each period. When a firm's price contract expires, the firm chooses the new contract price $p_t(f)$ to maximize the value to shareholders of the firm's cash flows over the lifetime of the contract (equivalently, the firm chooses a state-contingent plan for prices that maximizes the value of the firm to shareholders). That is, the firm maximizes:

$$E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j} \left[p_t(f) y_{t+j}(f) - w_{t+j} l_{t+j}(f) \right], \tag{18}$$

where $m_{t,t+j}$ is the representative household's stochastic discount factor from period t to $t+j$.

[8] Woodford (2003) Altig, Christiano, Eichenbaum, and Linde (2004), and others have emphasized the importance of firm-specific fixed factors for generating a level of inflation persistence that is consistent with the data. Firm-specific capital stocks also help to match the term premium as well as the persistence of inflation.

The output of each intermediate firm f is purchased by a perfectly competitive final goods sector that aggregates the continuum of intermediate goods into a single final good using a CES production technology:

$$Y_t = \left[\int_0^1 y_t(f)^{1/(1+\theta)} df \right]^{1+\theta}. \tag{19}$$

Each intermediate firm f thus faces a downward-sloping demand curve for its product:

$$y_t(f) = \left(\frac{p_t(f)}{P_t} \right)^{-(1+\theta)/\theta} Y_t, \tag{20}$$

where P_t is the CES aggregate price per unit of the final good:

$$P_t \equiv \left[\int_0^1 p_t(f)^{-1/\theta} df \right]^{-\theta}. \tag{21}$$

Differentiating (18) with respect to $p_t(f)$ yields the standard optimality condition for the firm's price:

$$p_t(f) = \frac{(1+\theta) E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j} mc_{t+j}(f) y_{t+j}(f)}{E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j} y_{t+j}(f)}. \tag{22}$$

where $mc_t(f)$ denotes the marginal cost for firm f at time t:

$$mc_t(f) \equiv \frac{w_t l_t(f)}{\eta y_t(f)}. \tag{23}$$

2.4 Aggregate Resource Constraints and the Government

To aggregate up from firm-level variables to aggregate quantities, it is useful to define cross-sectional price dispersion, Δ_t:

$$\Delta_t^{1/\eta} \equiv (1-\xi) \sum_{j=0}^{\infty} \xi^j p_{t-j}(f)^{-(1+\theta)/\theta\eta}, \tag{24}$$

where the occurrence of the parameter η in the exponent is dues to the firm-specificity of capital. We define L_t, the aggregate quantity of labor demanded by firms, by:

$$L_t \equiv \int_0^1 l_t(f) df. \tag{25}$$

Then L_t satisfies:

$$Y_t = \Delta_t^{-1} A_t \overline{K}^{1-\eta} L_t^{\eta}, \tag{26}$$

where $\overline{K} = \overline{k}$ is the capital stock. Equilibrium in the labor market requires that $L_t = l_t$, labor demand equals the aggregate labor supplied by the representative households.

In order to study the effects of fiscal shocks, we assume that there is a government sector in the model that levies lump-sum taxes G_t on households and destroys the resources it collects. Government consumption follows an exogenous AR(1) process:

$$\log G_t = \rho_G \log G_{t-1} + \varepsilon_t^G, \tag{27}$$

where ε_t^G denotes an *i.i.d.* government consumption shock with mean zero and variance σ_G^2.

Although agents cannot invest in physical capital in this version of the model, we do assume that an amount $\delta\overline{K}$ of output each period is devoted to maintaining the fixed capital stock. Thus, the aggregate resource constraint implies that

$$Y_t = C_t + \delta\overline{K} + G_t, \tag{28}$$

where $C_t = c_t$, the consumption of the representative household.

Finally, there is a monetary authority in the economy which sets the one-period nominal interest rate i_t according to a Taylor-type policy rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[1/\beta + \overline{\pi}_t + g_y(Y_t - \overline{Y})/\overline{Y} + g_\pi(\overline{\pi}_t - \pi^*)\right] + \varepsilon_t^i, \tag{29}$$

where $1/\beta$ is the steady-state real interest rate in the model, $\overline{Y}$ denotes the steady-state level of output, π^* denotes the steady-state rate of inflation, ε_t^i denotes an *i.i.d.* stochastic monetary policy shock with mean zero and variance σ_i^2, and ρ_i, g_y, and g_π are parameters.[9] The variable $\overline{\pi}_t$ denotes a geometric moving average of inflation:

$$\overline{\pi}_t = \theta_\pi \overline{\pi}_{t-1} + (1-\theta_\pi)\pi_t, \tag{30}$$

[9] In equation (29) (and equation (29) only), we express i_t, π_t, and $1/\beta$ in annualized terms, so that the coefficients g_π and g_y correspond directly to the estimates in the empirical literature. We also follow the literature by assuming an "inertial" policy rule with *i.i.d.* policy shocks, although there are a variety of reasons to be dissatisfied with the assumption of AR(1) processes for all stochastic disturbances except the one asociated with short-term interest rates. Indeed, Rudebusch (2002, 2006) and Carrillo, Fève, and Matheron (2007) provide strong evidence that an alternative policy specification with serially correlated shocks and little gradual adjustment is more consistent with the dynamic behavior of nominal interest rates.

where current-period inflation $\pi_t \equiv \log(P_t/P_{t-1})$ and we set $\theta_\pi = 0.7$ so that the geometric average in (30) has an effective duration of about four quarters, which is typical in estimates of the Taylor Rule. The advantage of using (30) rather than the four-quarter average inflation rate is that (30) only requires keeping track of one lagged variable ($\overline{\pi}_{t-1}$) and hence one extra state variable in the model, while a four-quarter moving average would require keeping track of three (π_{t-1}, π_{t-2}, and π_{t-3}). All of our results below are very similar whether we use (30) or a more traditional four-quarter average inflation rate in the policy rule (29).

2.5 Long-term Bonds and the Term Premium

The price of any asset in the model economy must satisfy the standard stochastic discounting relationship in which the household's stochastic discount factor is used to value the state-contingent payoffs of the asset in period $t+1$. For example, the price of a default-free n-period zero-coupon bond that pays one dollar at maturity satisfies:

$$p_t^{(n)} = \mathrm{E}_t[m_{t+1} p_{t+1}^{(n-1)}], \tag{31}$$

where $m_{t+1} \equiv m_{t,t+1}$, $p_t^{(n)}$ denotes the price of the bond at time t, and $p_t^{(0)} \equiv 1$, *i.e.*, the time-t price of one dollar delivered at time t is one dollar. The continuously-compounded yield to maturity on the n-period zero-coupon bond is defined to be:

$$i_t^{(n)} \equiv -\frac{1}{n} \log p_t^{(n)}. \tag{32}$$

In the U.S. data, the benchmark long-term bond is the ten-year Treasury note. Thus, we wish to model the term premium on a bond with a duration of about ten years. Computationally, it is inconvenient to work with a zero-coupon bond that has more than a few periods to maturity; instead, it is much easier to work with an infinitely lived consol-style bond that has a time-invariant or time-symmetric structure. Thus, we assume that households in the model can buy and sell a long-term default-free nominal consol which pays a geometrically declining coupon in every period in perpetuity. The nominal consol's price per one dollar of coupon in period t, which we denote by $\widetilde{p}_t^{(n)}$, then satisfies:

$$\widetilde{p}_t^{(n)} = 1 + \delta_c \mathrm{E}_t m_{t+1} \widetilde{p}_{t+1}^{(n)}, \tag{33}$$

where δ_c is the rate of decay of the coupon on the consol. By choosing an appropriate value for δ_c, we can thus model prices of a bond of any desired Macaulay duration or maturity n, such as the 10-year maturity that serves as our zero-coupon benchmark in the data.[10] Finally, the continuously-compounded yield to maturity on the consol, $\widetilde{\imath}_t^{(n)}$, is given by:

$$\widetilde{\imath}_t^{(n)} \equiv \log \left(\frac{\delta_c \widetilde{p}_t^{(n)}}{\widetilde{p}_t^{(n)} - 1} \right). \tag{34}$$

Note that even though the nominal bond in our model is default-free, it is still risky in the sense that its price can covary with the household's marginal utility of consumption. For example, when inflation is expected to be higher in the future, then the price of the bond generally falls, because households discount its future nominal coupons more heavily. If times of high inflation are correlated with times of low output (as is the case for technology shocks in the model), then households regard the nominal bond as being very risky, because it loses value at exactly those times when the household values consumption the most. Alternatively, if inflation is not very correlated with output and consumption, then the bond is correspondingly less risky. In the former case, we would expect the bond to carry a substantial risk premium (its price would be lower than the risk-neutral price), while in the latter case we would expect the risk premium to be smaller.

In the literature, the risk premium or *term premium* on a long-term bond is typically expressed as the difference between the yield on the bond and the unobserved risk-neutral yield for that same bond. To define the term premium in our model, then, we first define the risk-neutral price of the consol, $\widehat{p}_t^{(n)}$:

$$\widehat{p}_t^{(n)} \equiv \mathrm{E}_t \sum_{j=0}^{\infty} e^{-i_{t,t+j}} \delta_c^j, \tag{35}$$

where $i_{t,t+j} \equiv \sum_{n=0}^{j} i_n$. Equation (35) is the expected present discounted value of the coupons of the consol, where the discounting is performed using the risk-free rate rather than the

[10] As δ_c approaches 0, the consol behaves more like cash—a zero-period zero-coupon bond. As δ_c approaches 1, the consol approaches a traditional consol with a fixed (nondepreciating) nominal coupon, which, under our baseline parameter values below, has a duration of about 25 years. By setting $\delta_c > 1$, the duration of the consol can be made even longer.

household's stochastic discount factor. Equivalently, equation (35) can be expressed in first-order recursive form as:

$$\widehat{p}_t^{(n)} = 1 + \delta_c e^{-i_t} \mathrm{E}_t \widehat{p}_{t+1}^{(n)}, \tag{36}$$

which directly parallels equation (33). The implied term premium on the consol is then given by:

$$\psi_t^{(n)} \equiv \log\left(\frac{\delta_c \widetilde{p}_t^{(n)}}{\widetilde{p}_t^{(n)} - 1}\right) - \log\left(\frac{\delta_c \widehat{p}_t^{(n)}}{\widehat{p}_t^{(n)} - 1}\right), \tag{37}$$

which is the difference between the observed yield to maturity on the consol and the risk-neutral yield to maturity.

For a given set of structural parameters of the model, we will choose δ_c so that the bond has a Macaulay duration of $n = 40$ quarters, and we will multiply equation (37) by 400 in order to report the term premium in units of annualized percentage points rather than logs.

The term premium in equation (37) can also be expressed more directly in terms of the stochastic discount factor, which can be useful for gaining intuition about how the term premium is related to the various economic shocks driving our DSGE model above.

First, use (33) and (36) to write the difference between the consol price and the risk-neutral consol price as:

$$\begin{aligned}
\widetilde{p}_t^{(n)} - \widehat{p}_t^{(n)} &= \delta_c (E_t m_{t+1} \widetilde{p}_{t+1}^{(n)} - E_t m_{t+1} E_t \widehat{p}_{t+1}^{(n)}), \\
&= \delta_c \left[Cov_t(m_{t+1}, \widetilde{p}_{t+1}^{(n)}) + E_t m_{t+1} E_t (\widetilde{p}_{t+1}^{(n)} - \widehat{p}_{t+1}^{(n)}) \right], \\
&= \delta_c \left[Cov_t(m_{t+1}, \widetilde{p}_{t+1}^{(n)}) + e^{-i_t} E_t (\widetilde{p}_{t+1}^{(n)} - \widehat{p}_{t+1}^{(n)}) \right], \\
&= \sum_{j=0}^{\infty} e^{-i_{t,t+j}} \delta_c^{t+j+1} Cov_t(m_{t+j+1}, \widetilde{p}_{t+j+1}^{(n)}),
\end{aligned} \tag{38}$$

where the last equality in (38) follows from forward recursion. Equation (38) makes it clear that, even though the bond price depends only on the one-period-ahead covariance between the stochastic discount factor and next period's bond price, the term premium depends on this covariance over the entire lifetime of the bond. (An exactly analogous expression holds for the case of a zero-coupon bond.)

Of course, the term premium is usually written as the difference between the yield on the long-term bond and the risk-neutral yield on that bond. From (37),

$$\begin{aligned} \psi_t^{(n)} &= \log\left(1 - 1/\widehat{p}_t^{(n)}\right) - \log\left(1 - 1/\widetilde{p}_t^{(n)}\right), \\ &\approx -1/\widehat{p}_t^{(n)} + 1/\widetilde{p}_t^{(n)}, \\ &= \frac{-1}{\widetilde{p}_t^{(n)}\widehat{p}_t^{(n)}}(\widetilde{p}_t^{(n)} - \widehat{p}_t^{(n)}). \end{aligned} \tag{39}$$

For all of the parameterizations we consider below, the approximation on the second line of (39) is good because the 40-quarter bond price is about 40. The final line of (39) can also be well approximated by replacing the actual bond prices in the denominator with their steady-state values:

$$\psi_t^{(n)} \approx -(\widetilde{p}_t^{(n)} - \widehat{p}_t^{(n)})/\overline{p}_t^{(n)2}. \tag{40}$$

Finally, combining equations (38) and (40) gives a closed-form expression for the term premium in terms of the future covariance of the stochastic pricing kernel with the price of the bond:

$$\psi_t^{(n)} \approx \frac{-1}{\overline{p}_t^{(n)2}} \sum_{j=0}^{\infty} e^{-i_{t,t+j}} \delta_c^{t+j+1} Cov_t(m_{t+j+1}, \widetilde{p}_{t+j+1}^{(n)}). \tag{41}$$

2.6 Alternative Measures of Long-term Bond Risk

Although the term premium is the cleanest conceptual measure of the riskiness of long-term bonds, it is not directly observed in the data and must be inferred using term structure models or other methods. Accordingly, the literature has also focused on two other empirical measures that are closely related to the term premium but are more easily observed: the slope of the yield curve and the excess return to holding the long-term bond for one period relative to the one-period short rate.

The slope of the yield curve is simply the difference between the yield to maturity on the long-term bond and the one-period risk-free rate, i_t. The slope is an imperfect measure of the riskiness of the long-term bond because it can vary in response to shocks even if all investors in the model are risk-neutral. However, on average, the slope of the yield curve equals the term premium, and the volatility of the slope provides us with a noisy measure of the volatility of the term premium.

A second measure of the riskiness of long-term bonds is the excess one-period holding return—that is, the return to holding the bond for one period less the one-period risk-free rate. For the case of an n-period zero-coupon bond, this excess return is given by:

$$x_t^{(n)} \equiv \frac{p_t^{(n-1)}}{p_{t-1}^{(n)}} - e^{i_{t-1}}. \tag{42}$$

The first term on the right-hand side of (42) is the gross return to holding the bond and the second term is the gross one-period risk-free return. For the case of the consol in our model, the excess holding period return is a bit more complicated, since the consol pays a coupon in period $t-1$ and then depreciates in value by the factor δ_c, so the excess holding period return is given by:

$$\widetilde{x}_t^{(n)} \equiv \frac{\delta_c \widetilde{p}_t^{(n)} + e^{i_{t-1}}}{\widetilde{p}_{t-1}^{(n)}} - e^{i_{t-1}}. \tag{43}$$

Again, the first term on the right-hand side of (43) is the gross return to holding the consol and includes the one-dollar coupon in period $t-1$ that can be invested in the one-period security. As with the yield curve slope, the excess returns in (42) and (43) are imperfect measures of the term premium because they would vary in response to shocks even if investors were risk-neutral. However, the mean and standard deviation of the excess holding period return provide popular measures of the average term premium and the volatility of the term premium.

2.7 Model Solution Method

A technical issue in solving the model above arises from its relatively large number of state variables: A_{t-1}, G_{t-1}, i_{t-1}, Δ_{t-1}, $\overline{\pi}_{t-1}$, and the three shocks, ε_t^A, ε_t^G, and ε_t^i, make a total of eight.[11] Because of this high dimensionality, discretization and projection methods are computationally infeasible, so we solve the model using the standard macroeconomic technique

[11] The number of state variables can be reduced a bit by noting that G_t and A_t are sufficient to incorporate all of the information from G_{t-1}, A_{t-1}, ε_t^G, and ε_t^A, but the basic point remains valid, namely, that the number of state variables in the model is large from a computational point of view.

of approximation around the nonstochastic steady state—so-called perturbation methods. However, a first-order approximation of the model (i.e., a linearization or log-linearization) eliminates the term premium entirely, because equations (33) and (36) are identical to first order. A second-order approximation to the solution of the model produces a term premium that is nonzero but constant (a weighted sum of the variances σ_A^2, σ_G^2, and σ_i^2). Since our interest in this paper is not just in the level of the term premium but also in its volatility and variation over time, we compute a third-order approximate solution to the model around the nonstochastic steady state using the algorithm of Swanson, Anderson, and Levin (2006). For the baseline model above with eight state variables, a third-order accurate solution can be computed in just a few minutes on a standard laptop computer, and for the more complicated specifications we consider below with long-run risks, a third-order solution can be computed in twenty or thirty minutes. Additional details of this solution method are provided in Swanson, Anderson, and Levin (2006) and Rudebusch, Sack and Swanson (2007).

Once we have computed an approximate solution to the model, we compare the model and the data using a standard set of macroeconomic and financial moments, such as the standard deviations of consumption, labor, and other variables, and the means and standard deviations of the term premium and the alternative measures of long-term bond risk described above. One method of computing these moments is by simulation, but this method is slow and, for a nonlinear model, the simulations can sometimes diverge to infinity. We thus compute these moments in closed form, using perturbation methods. In particular, we compute the unconditional standard deviations and unconditional means of the variables of the model to second order.[12] For the term premium, the unconditional standard deviation is zero to second order, so we compute the unconditional standard deviation or the term premium to third order.[13] This method yields results that are extremely close to those that arise from

[12] To compute the standard deviations of the variables to second order, we compute a fourth-order accurate solution to the unconditional covariance matrix of the variables and then take the square root along the diagonal. Becuase $E[XY]$ involves the product of two variables, we only need a third-order accurate solution for X and Y in order to compute their product to fourth order (this is easiest to see by normalizing their constant terms to zero).

[13] The first-order approximation to the term premium is zero, as discussed above, so a third-order accurate

simulation while at the same time being quicker and more numerically robust.

3 Comparing the Epstein-Zin DSGE Model to the Data

We now investigate whether the model developed in the previous section, which is a canonical DSGE model augemented with Epstein-Zin preferences, is consistent with basic features of the data. We first describe the baseline model parameters and see whether this model can match important macroeconomic and finance moments. We then investigate the best possible fit of the model to the data.

3.1 Model Parameterization

The baseline parameter values that we use for our simple New Keynesian model are reported in Table 1 and are fairly standard in the literature (see, e.g., Levin, Onatski, Williams, and Williams, 2005). We set the household's discount factor, β, to .99 per quarter, implying a steady-state real interest rate of 4.02 percent per year. We set households' utility curvature with respect to consumption, γ, to .66, implying an intertemporal elasticity of substitution in consumption of 1.5, which is somewhat higher than estimates in the micro literature (e.g., Vising-Jorgenson (2002)), but identical to the value used by Bansal and Yaron (2004), who argue that existing estimates in the micro literature are downward-biased due to heteroskedasticity in the consumption process.[14] Households' utility curvature with respect to labor, χ, is set to 1.5, implying a Frisch elasticity of 2/3, which is in line with estimates from the microeconomics literature (e.g., Pistaferri, 2003). We discuss the parameter α and its relationship to the coefficient of relative risk aversion below.

We set firms' output elasticity with respect to labor, η, to .7, firms' steady-state markup, θ, to .2 (implying a price-elasticity of demand of 6), and the Calvo frequency of price adjustment, ξ, to .75 (implying an average price contract duration of four quarters), all of which

solution to the term premium is sufficient to compute the standard deviation of the term premium to third order.

[14] In Bansal and Yaron's (2004) example, the assumed intertermporal elasticity of substitution is 1.5, but the micro-style regression estimate, assuming constant consumption volatility, would be only 0.6.

are stadard in the literature. We set the steady-state capital-output ratio in the model to 2.5 (where output is annualized), and the capital depreciation rate to 2 percent per quarter (which implies a steady-state investment-output ratio of 20 percent). Government purchases are assumed to comprise 17 percent of output in steady state. The shock persistences ρ_A and ρ_G are set to .9, as is common, and the shock variances σ_A^2 and σ_G^2 are set to $.01^2$ and $.004^2$, respectively, consistent with typical estimates in the literature. The monetary policy rule coefficients are taken from Rudebusch (2002) and are also typical of those in the literature. Finally, the parameter χ_0 is chosen to normalize the steady-state quantity of labor to unity and the parameter δ_c is chosen to set the Macaulay duration of the consol in the model to ten years, as discussed above.

Table 1
Baseline Parameter Values for the Simple New Keynesian Model

β	.99	ρ_i	.73	$\overline{K}/(4\overline{Y})$	2.5
γ	.66	g_π	.53	$\delta\overline{K}/\overline{Y}$	.2
χ	1.5	g_y	.93	$\overline{G}/\overline{Y}$	.17
α	43	ρ_A	.9		
η	.7	ρ_G	.9		
θ	.2	σ_A^2	$.01^2$		
ξ	.75	σ_G^2	$.004^2$		
memo:					
quasi-CRRA	15				
χ_0	4.74				
δ_c	.9848				

3.2 The Coefficient of Relative Risk Aversion

In a model in which the household's optimization problem is homothetic (e.g., a model with fixed labor, $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma)$, and shocks that enter multiplicatively with respect to wealth), which is standard in the endowment economy literature using Epstein-Zin preferences, the household's value function V_t is equal to a constant (function of parameters) times

$W_t^{1-\gamma}$, where W_t denotes beginning-of-period household wealth. In that case, it is common practice in the literature to refer to $1-\widetilde{\alpha}$, or $1-(1-\alpha)(1-\gamma)$ the way we have written it in (3), as the household's coefficient of relative risk aversion with respect to gambles over wealth (since the expectation in (3) is over $E_t W_{t+1}^{(1-\alpha)(1-\gamma)}$).

In contrast, the value function for the household's optimization problem in our model is much more complex than in an endowment economy and is not separable in the level of household wealth (the utility kernel is not homothetic due to the presence of labor and the various shocks that do not enter multiplicatively with respect to wealth). Moreover, it is difficult to define risk aversion when there is more than one good or more than one state variable, as discussed by Kihlstrom and Mirman (1971). For these reasons, there is no standard or even unambiguous quantitative measure of risk aversion in our model.[15]

In order to compare our model and results to the endowment economy literature, we thus report the *quasi-CRRA* for our model, $1-(1-\alpha)(1-\gamma)$. The interpretation of this coefficient is that, *if* labor in our model were held fixed, and *if* utility were homothetic, and *if* all the shocks in the model were multiplicative with respect to wealth, *then* the CRRA in the model would be the quasi-CRRA that we report. We have experimented with alternative definitions of the CRRA for our model, and none of these has been entirely satisfactory, so at present this is the best quantitative measure of risk aversion in the model that we can offer, although we continue to search for a better measure.

In the baseline parameterization of our model given in Table 1, the Epstein-Zin coefficient α is set to 43, and γ is 0.66, which implies a quasi-CRRA of 15. This value is only slightly higher than the ones used by Bansal and Yaron (2004) and Bansal and Shaliastovich (2008) in their analysis of the equity, term, and foreign exchange premiums in an endowment economy setting.

[15] We do know from Epstein and Zin (1989) that, for $u(c_t, l_t) \geq 0$ everywhere, higher values of α correspond to greater risk aversion. The issue here is that we have no easy way to *quantify* the degree of risk aversion in our model in a way that one could compare to the empirical literature.

3.3 Model Results

For the model with baseline parameter values, various model-implied moments are reported in Table 2, along with the corresponding empirical moments for quarterly U.S. data from 1960 to 2007. For the empirical moments, consumption, C, is real personal consumption expenditures from the U.S. national income and product accounts, labor, L, is total hours of production workers from the Bureau of Labor Statistics, and the real wage, w^r, is total wages and salaries of production workers from the BLS divided by total production worker hours and deflated by the GDP price index. Standard deviations were computed for logarithmic deviations of each series from a Hodrick-Prescott trend and reported in percentage points. Standard deviations for inflation, interest rates, and the term premium were computed for the raw series rather than for deviations from trend. Inflation, π, is the annualized rate of change in the quarterly GDP price index from the Bureau of Economic Analysis. The short-term nominal interest rate, i, is the end-of-month federal funds rate from the Federal Reserve Board, in annualized percentage points. The short-term real interest rate, r, is the short-term nominal interest rate less realized quarterly inflation rate at an annual rate. The ten-year zero-coupon bond yield, $i^{(10)}$, is the end-of-month ten-year zero-coupon bond yield taken from Gurkaynak, Sack, and Wright (2008). The term premium on the ten-year zero-coupon bond, $\psi^{(10)}$, is the term premium computed by Kim and Wright (2005), in annualized percentage points.[16] The yield curve slope and one-period excess holding return are calculated from the data above and are reported in annualized percentage points.

[16] Kim and Wright (2005) use an arbitrage-free, three-latent-factor affine model of the term structure to compute the term premium. Alternative measures of the term premium using a wide variety of methods produce qualitatively similar results in terms of the overall magnitude and variability—see Rudebusch, Sack, and Swanson (2007) for a detailed discussion and comparison of several methods.

Table 2
Empirical and Model-Based Unconditional Moments

Variable	U.S. Data, 1961-2007	Model with Expected Utility Preferences	Model with Epstein-Zin Preferences	Model with EZ Preferences (best fit)
sd$[C]$	1.19	1.42	2.33	2.53
sd$[L]$	1.71	2.56	2.42	2.21
sd$[w^r]$	0.82	2.08	3.00	1.52
sd$[\pi]$	2.52	2.25	2.50	2.71
sd$[i]$	2.71	1.90	1.73	2.27
sd$[r]$	2.30	1.89	1.94	1.62
sd$[i^{(10)}]$	2.41	0.54	0.48	1.03
mean$[\psi^{(10)}]$	1.06	.010	.104	1.05
sd$[\psi^{(10)}]$	0.54	.000	.007	.184
mean$[i^{(10)} - i]$	1.43	-.047	.058	0.99
sd$[i^{(10)} - i]$	1.33	1.43	1.29	1.33
mean$[x^{(10)}]$	1.76	.015	.141	1.04
sd$[x^{(10)}]$	23.43	6.56	5.92	9.02
memo:				
quasi-CRRA		2	15	75
IES		0.5	1.5	1.3
χ		1.5	1.5	0.4
ρ_A		0.9	0.9	0.95
σ_A		.01	.01	.007

All variables are quarterly values expressed in percent. Inflation and interest rates, the term premium (ψ), and excess holding period returns (x) are expressed at an annual rate.

The second column of Table 2 reports results for the version of our stylized model with expected utility preferences, $\alpha = 0$. The model does a reasonable job of matching the U.S. data for the macroeconomic variables, the short-term nominal interest rate, and the yield to maturity on the long-term bond. However, the term premium implied by the expected utility version of the model is both too small in magnitude and has the wrong sign—the model implies a term premium of 1 basis point—and is far too stable, with an unconditional standard deviation less than one-tenth of one basis point. This basic finding of a term premium that is too small and far too stable is extremely robust with respect to wide variation of the parameters over plausible values (see Rudebusch and Swanson, 2008, for additional discussion and sensitivity analysis).

The third column of Table 2 reports results from the version of the model with Epstein-Zin preferences and a quasi-CRRA of 15 ($\alpha = 43$). The model fits all of the macroeconomic variables essentially as well as an expected utility version of the model with the same IES ($\gamma = .66$), which is a straightforward implication of two features of the model: First, the linearization or log-linearization of Epstein-Zin preferences (3) is exactly the same as that of standard expected utility preferences (2), so to first order, these two utility specifications are the same, and second, the shocks that we consider here and which are standard in macroeconomics have standard deviations of only about 1 percent or less, so a linear approximation to the model is typically very accurate. Only for models with enormous curvature (e.g., $\gamma \gg 1$ or $\chi \gg 1$), or for much larger shocks, would we expect second- or higher-order terms of the model to matter very much.

For asset prices, however, the implications of the Epstein-Zin and expected utility preferences are very different.[17] With Epstein-Zin preferences, the mean term premium is an order of magnitude larger than with expected utility preferences, and the mean yield curve slope and excess holding period return show similar marked increases. There is, however, little improvement in matching the empirical volatilities of these series, and even the mean term premium remains significantly smaller than its empirical counterpart. This last deficiency

[17] Here, second- and higher-order terms are the whole story, since to first order the model is certainty equivalent and hence there are no first-order risk premium terms.

can be remedied by boosting the risk aversion of the model, as we show in Figure 1.

The solid line in Figure 1 plots the relationship between the mean term premium ($\psi^{(10)}$) and the quasi-CRRA. As the quasi-CRRA increases, holding all the other parameters of the model fixed at their baseline values, the mean term premium rises steadily, so that a quasi-CRRA of 75 produces a mean term premium of 60 basis points, which is within the range of the empirical estimate. Finally, as shown in the third column of Table 2, a quasi-CRRA of 75 (with other parameters at their baseline values) produces fairly reasonable other bond yield moments without distorting the macroeconomic moments. That is, even for very extreme values of α, and hence very high levels of risk aversion in the model, the dynamics of the macroeconomic variables implied by the model are largely unchanged, a finding that has also been noted by Tallarini (2000) and Backus, Routledge, and Zin (2007). The fact that the models are first-order equivalent seems to dominate, for practical purposes, the additional curvature that is introduced by the parameter α. This is a very useful feature of the model, for our purposes, because it allows us to vary the parameter α to match asset prices, without jeopardizing the ability of the model to fit the behavior of macroecoomic aggregates.

Finally, the last column of Table 2 reports results from the "best-fit" parameterization of the model with Epstein-Zin preferences, where we have searched over a wide range of parameter values to find the parameterization that provides the closest joint fit to both the macroeconomic and financial moments in the data. The computational time required to solve the model for each set of parameter values is about 20 minutes, so it is generally infeasible to estimate the model using maximum likelihood or Bayesian estimation procedures. Instead, we perform a grid search over the five parameters listed in Table 2 that are among the most uncertain and of the greatest importance for the term premium—namely, α, γ, χ, ρ_A, and σ_A—and report the set of parameter values that best fits the macroeconomic and financial moments in Table 2.[18] We define the "best fit" to be the set of parameters that matches the equally-weighted sum of squared deviations from the moments in the first column of Table 2 as closely as possible (with one exception: we divide the standard deviation of the

[18] Some details of the grid search here.

excess holding period return $x^{(10)}$ by 10 in order to give it roughly as much weight as the other moments in the column).[19] With the resulting best-fitting parameter values (reported at the bottom of Table 2), the mean term premium is about 105 basis points and the unconditional standard deviation of the term premium is 18.4 basis points, a much better fit than the baseline model. To achieve this better fit, the estimation procedure picks a high value for the quasi-CRRA of 75, and a high technology shock persistence, $\rho_A = .95$. With these extreme parameter values, holding the technology shock standard deviation fixed at its baseline value would result in macroeconomic moments that are too volatile relative to the data, so the estimation chooses a lower standard deviation, $\sigma_A = .007$. The value of $\chi = 0.4$ also helps to damp down the macroeconomic volatility of the real wage and hence firms' marginal cost and inflation.

Impulse responses for the best-fit Epstein-Zin DSGE model, which are shown in Figure 2, provide further insight into the sources of movements in bond yields. The first column of Figure 2 provides the response of consumption, inflation, the bond price, and the term premium to a positive one-standard-deviation shock to technology. The second and third columns provide responses for similarly-sized shocks to government spending and monetary policy, respectively. These impulse responses demonstrate that the reduced-form correlations between consumption, inflation, and the bond price depend on the underlying type of structural shock. Recall that Piazzesi and Schneider (2006) suggested that the term premium stems from the fact that a surprise increase in inflation lowered the value of a nominal bond and was also followed by lower consumption going forward. For our structural model, these two correlations are exhibited in the first column of Figure 2 following a technology shock, as inflation falls and the long-term bond price and consumption both rise. However, these relationships take on the opposite sign for the government spending and monetary policy shocks, where a fall in inflation is associated with decreases in the bond price and in consumption. Thus, the sign of the reduced-form correlation depends on the distribution of the underlying shocks that are hitting the economy, and the sign of the correlations estimated by Piazzesi

[19] Minimizing the equal-weighted distance to these six moments provides us with a consistent estimator of our parameters, though it is not efficient.

and Schneider suggest that technology-type shocks predominated over their sample. This observation is consistent with the quantitative magnitudes exhibited in Figure 2. The bond price movements are one or two orders of magnitude larger for the technology shocks than for the other two shocks. Indeed, even if those other two shocks are eliminated, the technology shocks on their own can do essentially as good a job in matching all of the moments in Table 2 as the full model.

Although the results in Table 2 do a fairly good job of matching the macroeconomic and finance moments, that performance comes at the cost of assuming a very high degree of risk aversion. This is consistent with some earlier work, such as Piazzesi and Schneider (2006), who assume a CRRA of 59. Still, it is not clear that such severe risk aversion is consistent with the microeconomic evidence, so in the next section, we consider the addition of more persistent economic risk in order to reduce the degree of risk aversion needed to match the data.

4 Long-Run Risk

The results in Table 2 demonstrate that Epstein-Zin preferences are capable of matching both the basic macroeconomic and financial facts in a DSGE framework. This finding contrasts sharply to preference specifications based on habit, which Rudebusch and Swanson (2008) found failed in the DSGE setting despite their successes in endowment economy studies such as Campbell and Cochrane (1999) and Wachter (2006). However, the fit in the last column of Table 2 still comes at the cost of a very high quasi-CRRA of 75, which implies a level of risk aversion that is generally at odds with microeconomic surveys and experiments. In this section, we examine to what extent a long-run risk in the model (such as a long-run productivity risk or a long-run inflation risk) can help the model to fit the data with a smaller value for the quasi-CRRA.

4.1 Long-Run Productivity Risk

Since Bansal and Yaron (2004), the ability of a relatively small but highly persistent long-run consumption growth risk to account for a variety of risk premium puzzles in an endowment economy framework has been widely recognized. In our DSGE framework, it is natural to model long-run consumption risk as a long-run risk to productivity; that is, analogous to Bansal and Yaron, we now assume that the level of aggregate technology A has a small but highly persistent component A^* as well as an i.i.d. component:

$$\log A_t^* = \rho_{A^*} \log A_{t-1}^* + \varepsilon_t^{A^*}, \tag{44}$$

$$\log A_t = \log A_t^* + \varepsilon_t^A, \tag{45}$$

where the shocks $\varepsilon_t^{A^*}$ and ε_t^A are uncorrelated. We then replace equation (17) of our DSGE model above with (44)–(45). Choosing baseline parameter values for (44)–(45) is not completely straightforward, however—Bansal and Yaron's parameter values are for an exogenous consumption process, while consumption in our DSGE model is not an exogenous process but instead is an endogenous function of technology and other structural shocks. As a baseline, we set $\rho_{A^*} = .98$, similar to Bansal-Yaron's value of .979 for consumption growth. Following Bansal and Yaron, we choose values for σ_{A^*} and σ_A to match the unconditional volatility of consumption in our baseline model without long-run risk and also to set the proportion of one-step-ahead consumption growth volatility that is attributable to the long-run shock to about 5 percent, similar to Bansal and Yaron's value of 4.4 percent. This results in baseline values of $\sigma_{A^*} = .002$ and $\sigma_A = .005$.

Table 3 reports the results of incorporating this long-run productivity risk into our DSGE model above. The first column reports results for the expected utility version of the model with long-run risk and the second column reports results for our baseline Epstein-Zin parameterization of the model with long-run risk. As in Table 2, the last column of Table 3 reports results for the best fit set of parameter values from a grid search over the parameters α, γ, χ, σ_{A^*}, and σ_A.

Table 3
Model-Based Unconditional Moments with Long-Run Productivity Risk

Variable	Model with Expected Utility Preferences and Long-run Risk	Model with Epstein-Zin Preferences and Long-run Risk	Model with EZ Preferences and Long-run Risk (best fit)
sd$[C]$	.92	1.56	2.95
sd$[L]$	1.03	.93	1.32
sd$[w^r]$	1.43	1.56	1.90
sd$[\pi]$	1.12	1.64	3.14
sd$[i]$	1.17	1.52	2.88
sd$[r]$	.66	.66	1.35
sd$[i^{(10)}]$	.65	.90	1.84
mean$[\psi^{(10)}]$	.005	.084	.872
sd$[\psi^{(10)}]$	.000	.016	.183
mean$[i^{(10)} - i]$	$-.018$	.048	.758
sd$[i^{(10)} - i]$	.64	.72	1.15
mean$[x^{(10)}]$	.005	.083	.859
sd$[x^{(10)}]$	4.39	6.05	11.59
memo:			
quasi-CRRA	2	15	35
IES	0.5	1.5	1.5
χ	1.5	1.5	0.1
ρ_{A^*}	.98	.98	.98
σ_{A^*}	.002	.002	.004
σ_A	.005	.005	.001

All variables are quarterly values expressed in percent. Inflation and interest rates, the term premium (ψ), and excess holding period returns (x) are expressed at an annual rate.

In the case of expected utility (the first column), the presence of long-run productivity risk has little effect on the term premium or on other measures of bond market risk simply because households are hardly at all risk-averse. The Epstein-Zin parameterization in the middle column shows more of an effect. Relative to the baseline model without long-run risk in Table 2, the term premium is substantially more variable even though the macroeconomic variables are less variable. Finally, the best fit column of Table 3 provides notable success with long-run productivity risk. Here, a quasi-CRRA of only 35 provides the best fit to the data with a term premium about as large and variable as the best-fitting model without long-run risk.

4.2 Long-Run Inflation Risk

Since Bansal and Yaron (2004), the finance literature has stressed the importance of long-run risk in consumption growth. In contrast, there has been little attention devoted to long-run nominal risks in the economy, specifically, time-variation in the economy's long-run inflation rate. Such risk would appear to be very relevant for pricing nominal bonds. Therefore, we consider the case where the monetary authority's target rate of inflation, π_t^*, varies over time. Certainly, financial market perceptions of the long-run inflation rate in the U.S. appear to have varied considerably in recent decades. As discussed by Kozicki and Tinsley (2001), survey data on long-run inflation expectations show considerable variation over the past 50 years. Such variation is consistent with the macro-finance arbitrage-free model estimates in Rudebusch and Wu (2007, 2008) and with the evidence on the "excess sensitivity" of long-term bond yields to macroeconomic announcements found by Gürkaynak, Sack, and Swanson (2005).

From the point of view of modeling the term premium, long-run inflation risk has a number of advantages over long-run productivity or consumption risk. First, estimates of the low-frequency component of productivity or consumption are extremely imprecise, so it is very difficult to empirically test the direct predictions of a Bansal-Yaron long-run productivity or consumption risk model with observable macroeconomic variables. In contrast, survey data and other estimates on long-run inflation expectations are readily available and show considerable variation. Second, the idea that long-term nominal bonds are risky because of uncertainty about future monetary policy and long-run inflation is intuitively appealing. Third, estimates of the term premium in the finance literature are low in the 1960s, high in the late 1970s and early 1980s, and then low again in the 1990s and 2000s, which suggests that inflation and inflation variability are highly correlated with the term premium, at least over these longer, decadal samples. Modeling the linkage between long-run inflation risk and the term premium thus seems to be a promising avenue for understanding and modeling long-term bond yields.

Following the empirical evidence in Gürkaynak et al., we assume that π_t^* loads to some extent on the recent history of inflation:

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + (1-\rho_{\pi^*})\vartheta_{\pi^*}(\overline{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}. \tag{46}$$

There are two main advantages to using specification (46) rather than a simple random walk or AR(1) specification with $\vartheta_{\pi^*} = 0$. First, (46) allows long-term inflation expectations to respond to current news about inflation and economic activity in a manner that is consistent with the bond market responses documented by Gürkaynak et al. Thus, $\vartheta_{\pi^*} > 0$ seems to be consistent with the data (Gürkaynak et al. find that a value of $\vartheta_{\pi^*} = .02$ is roughly consistent with the bond market data).

Second, if $\vartheta_{\pi^*} = 0$, then even though π_t^* varies over time, it does not do so systematically with output or consumption. As a result, long-term bonds are not particularly risky, in the sense that their returns are not very correlated with the household's stochastic discount factor. Long-term bonds even have some elements of insurance in this case, because a negative shock to $\varepsilon_t^{\pi^*}$ leads the monetary authority to raise interest rates and depress output at precisely the same time that it causes long-term bond yields to fall and bond prices to rise; as a result, long-term bonds act like insurance for this type of shock and carry a negative risk premium. By contrast, if $\vartheta_{\pi^*} > 0$, then a negative technology shock today raises inflation and long-term inflation expectations and depresses bond prices at exactly the same time that it depresses output, which makes holding long-term bonds quite risky. Thus, to help the model generate a term premium that is positive on average, we will set $\vartheta_{\pi^*} > 0$.

To focus on the effects of the long-run nominal risk, we abstract away from Bansal and Yaron's long-run productivity risk in this section and consider only the effects of including equation (46) in our DSGE model. As discussed above, we set the baseline value of $\vartheta_{\pi^*} = .02$, consistent with the high-frequency bond market evidence in Gürkaynak et al. (2005). We set the baseline values for ρ_{π^*} and σ_{π^*} equal to .995 and 5bp, respectively, consistent with the Bayesian DSGE model estimates in Levin et al. (2005).

In Figure 1, we can see that the effects of the long-run nominal risk are indeed substantial. As the quasi-CRRA is varied along the horizontal axis, holding the other parameters of the model fixed at their baseline values, the term premium is always the highest for the version of the model with long-run inflation risk.

This observation is further reinforced in Table 4, which reports all of the basic macroeconomic and financial moments that result from introducing the long-run inflation risk into our DSGE model. The first column presents results for the model with expected utility preferences and long-run inflation risk, the second column present results for our baseline parameterization of the Epstein-Zin version of the model with long-run inflation risk, and the last column presents results for the best fit parameterization of the Epstein-Zin version of the model with long-run inflation risk, where we search over values for α, γ, χ, ρ_A, σ_A, ρ_{π^*}, ϑ_{π^*}, and σ_{π^*}.

With expected utility preferences, the presence of long-run inflation risk has little effect on the term premium or other measures of bond market risk—intuitively, even though the quantity of nominal bond risk is greater, households simply aren't risk averse enough for that greater quantity to have a substantial effect. Introducing long-run nominal risk into the model with Epstein-Zin preferences, however, virtually doubles the size of the term premium in the second column, relative to Table 2, and has a tremendous effect on the variability of the term premium and other measures of bond market risk. The variability of the macro variables in the second column is too high, however, due to the additional volatility introduced by the presence of long-run inflation risk.

Table 4
Model-Based Unconditional Moments with Long-Run Inflation Risk

Variable	Model with Expected Utility Preferences and Long-run Inflation Risk	Model with Epstein-Zin Preferences and Long-run Inflation Risk	Model with EZ Preferences and Long-run Inflation Risk (best fit)
sd$[C]$	1.92	3.42	1.86
sd$[L]$	3.33	3.45	1.73
sd$[w^r]$	2.55	4.45	1.45
sd$[\pi]$	5.00	6.65	3.22
sd$[i]$	4.74	5.97	2.99
sd$[r]$	2.61	3.10	1.48
sd$[i^{(10)}]$	3.32	4.39	1.94
mean$[\psi^{(10)}]$	.002	.170	.748
sd$[\psi^{(10)}]$	.001	.270	.431
mean$[i^{(10)} - i]$	$-.062$	.171	.668
sd$[i^{(10)} - i]$	1.60	1.49	1.11
mean$[x^{(10)}]$	.003	.169	.737
sd$[x^{(10)}]$	16.96	21.58	11.83
memo:			
quasi-CRRA	2	15	65
IES	0.5	1.5	1.2
χ	1.5	1.5	0.1
ρ_A	0.9	0.9	0.95
σ_A	.01	.01	.005
ρ_{π^*}	.995	.995	.99
ϑ_{π^*}	.02	.02	.02
σ_{π^*}	5bp	5bp	1bp

All variables are quarterly values expressed in percent. Inflation and interest rates, the term premium (ψ), and excess holding period returns (x) are expressed at an annual rate.

The excessive macroeconomic volatility in the second column of Table 4 can be fixed once we consider varying the parameters of the model more freely. The final column of Table 4 reports results for the best-fitting set of parameter values, which involves slightly less long-run inflation risk than in our baseline specification and a lower value for σ_A, both of which help to reduce the macroeconomic volatility of the model. The estimation also results in a much higher value for the quasi-CRRA, which increases the level and variability of the term premium and other financial moments without greatly distorting the macro moments implied by the model. The low estimated value for χ helps to keep the variability of real wages, marginal cost, and inflation low, just as in the model without long-run inflation risk in Table 2.

5 Conclusions

In stark contrast to our earlier work with habits (Rudebusch and Swanson, 2008), here we have found that introducing Epstein-Zin preferences into a DSGE model is a very successful strategy for matching both financial and macroeconomic moments. We are able to obtain a large and volatile term premium in a structural model of a production economy, thus generalizing the earlier endowment economy results in finance. Of course, it will be important to examine the robustness of our results by incorporating Epstein-Zin preferences into larger, more empirical DSGE models. A related next step would go beyond just matching sample moments and perform econometric estimation and inference of DSGE models with Epstein-Zin preferences, as in Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2008), but extended to include intrinsic nominal rigidities and endogenous inflation. Examining to what extent a DSGE model can jointly explain the risk premiums on equity, real bonds, nominal bonds, and perhaps even exchange rates would be very interesting. Finally, the relationship between the variability or uncertainty surrounding the central bank's inflation objective and the size and variability of the term premium warrants further study, in our view. In short, there appear to be many fruitful avenues for future research in this area.

6 Appendix: Equations of the Model

The following equations show exactly how we incorporate Epstein-Zin preferences into our otherwise standard DSGE model in first-order recursive form, and how bond prices and the term premium are computed in the model. The Mathematica-style syntax of these equations is consistent with the perturbation AIM algorithm of Swanson et al. (2006), which we use to solve this system to 3rd order around the nonstochastic steady state.

```
(* Value function and Euler equation *)
V == C[t]^(1-gamma) /(1-gamma) - chi0 *L[t]^(1+chi) /(1+chi) + beta *Vkp[t],
C[t]^-gamma == beta *(Exp[Int[t]]/pi[t+1]) *C[t+1]^-gamma *(V[t+1]/Vkp[t])^-alpha,

(* The following two equations define the E-Z-W-K-P certainty equivalent term
 Vkp = (E_t V[t+1]^(1-alpha))^(1/(1-alpha)). It takes two equations to do this because
 perturbationAIM sets the expected value of all equations equal to zero, E_t F(variables) = 0.
 Thus, the first equation below defines Valphaexp[t] == E_t V[t+1]^(1-alpha). The second
 equation then takes the (1-alpha)th root of this expectation.
 Note: the literature often refers to the coefficient alpha as the CRRA, but that terminology is only
 justifiable when the model has only one state variable (wealth) and the model is homothetic. The
 present model does not satisfy either of these conditions. Nevertheless, alpha is *a* measure of
 risk aversion, as shown by Epstein and Zin.
 Finally, the scaling and unscaling of Valphaexp[t] by the constant VAIMSS improves the numerical
 behavior of model; without it, the steady-state value of Valphaexp can be minuscule (e.g., 10^-50),
 which requires Mathematica to use astronomical levels of precision in order to solve. *)
Valphaexp[t] == (V[t+1]/VAIMSS)^(1-alpha),
Vkp[t] == VAIMSS *Valphaexp[t]^(1/(1-alpha)),

(* Price-setting equations *)
zn[t] == (1+theta) *MC[t] *Y[t] + xi *beta *(C[t+1]/C[t])^-gamma *(V[t+1]/Vkp[t])^-alpha
        *pi[t+1]^((1+theta)/theta/eta) *zn[t+1],
zd[t] == Y[t] + xi *beta *(C[t+1]/C[t])^-gamma *(V[t+1]/Vkp[t])^-alpha *pi[t+1]^(1/theta)
        *zd[t+1],
p0[t]^(1+(1+theta)/theta *(1-eta)/eta) == zn[t] /zd[t],
pi[t]^(-1/theta) == (1-xi) *(p0[t]*pi[t])^(-1/theta) + xi,

(* Marginal cost and real wage *)
MC[t] == wreal[t] /eta *Y[t]^((1-eta)/eta) /A[t]^(1/eta) /KBar^((1-eta)/eta),
chi0 *L[t]^chi /C[t]^-gamma == wreal[t], (* no adj costs *)
```

```
(* Output equations *)
Y[t] == A[t] *KBar^(1-eta) *L[t]^eta /Disp[t],
Disp[t]^(1/eta) == (1-xi) *p0[t]^(-(1+theta)/theta/eta)
        + xi *pi[t]^((1+theta)/theta/eta) *Disp[t-1]^(1/eta),
C[t] == Y[t] - G[t] - IBar, (* aggregate resource constraint, no adj costs *)

(* Monetary Policy Rule *)
piavg[t] == rhoinflavg *piavg[t-1] + (1-rhoinflavg) *pi[t],
4*Int[t] == (1-taylrho) * ( 4*Log[1/beta] + 4*Log[piavg[t]]
        + taylpi * (4*Log[piavg[t]] - pistar[t]) + tayly * (Y[t]-YBar)/YBar )
        + taylrho * 4*Int[t-1] + eps[Int][t], (* multiply Int, infl by 4 to put at annual rate *)

(* Exogenous Shocks *)
Log[A[t]/ABar] == rhoa * Log[A[t-1]/ABar] + eps[A][t],
Log[G[t]/GBar] == rhog * Log[G[t-1]/GBar] + eps[G][t],
pistar[t] == (1-rhopistar) *piBar + rhopistar *pistar[t-1] + gssload *(4*Log[piavg[t]] - pistar[t])
        + eps[pistar][t],

(* Term premium and other auxiliary finance equations *)
Intr[t] == Log[Exp[Int[t-1]]/pi[t]], (* ex post real short rate *)
pricebond[t] == 1 + consoldelta *beta *(C[t+1]/C[t])^-gamma *(V[t+1]/Vkp[t])^-alpha /pi[t+1]
        *pricebond[t+1],
pricebondrn[t] == 1 + consoldelta *pricebondrn[t+1] /Exp[Int[t]],
ytm[t] == Log[consoldelta*pricebond[t]/(pricebond[t]-1)] *400, (* yield in annualized pct *)
ytmrn[t] == Log[consoldelta*pricebondrn[t]/(pricebondrn[t]-1)] *400,
termprem[t] == 100 * (ytm[t] - ytmrn[t]), (* term prem in annualized basis points *)
ehpr[t] == ( (consoldelta *pricebond[t] + Exp[Int[t-1]]) /pricebond[t-1] - Exp[Int[t-1]]) *400,
slope[t] == ytm[t] - Int[t]*400
```

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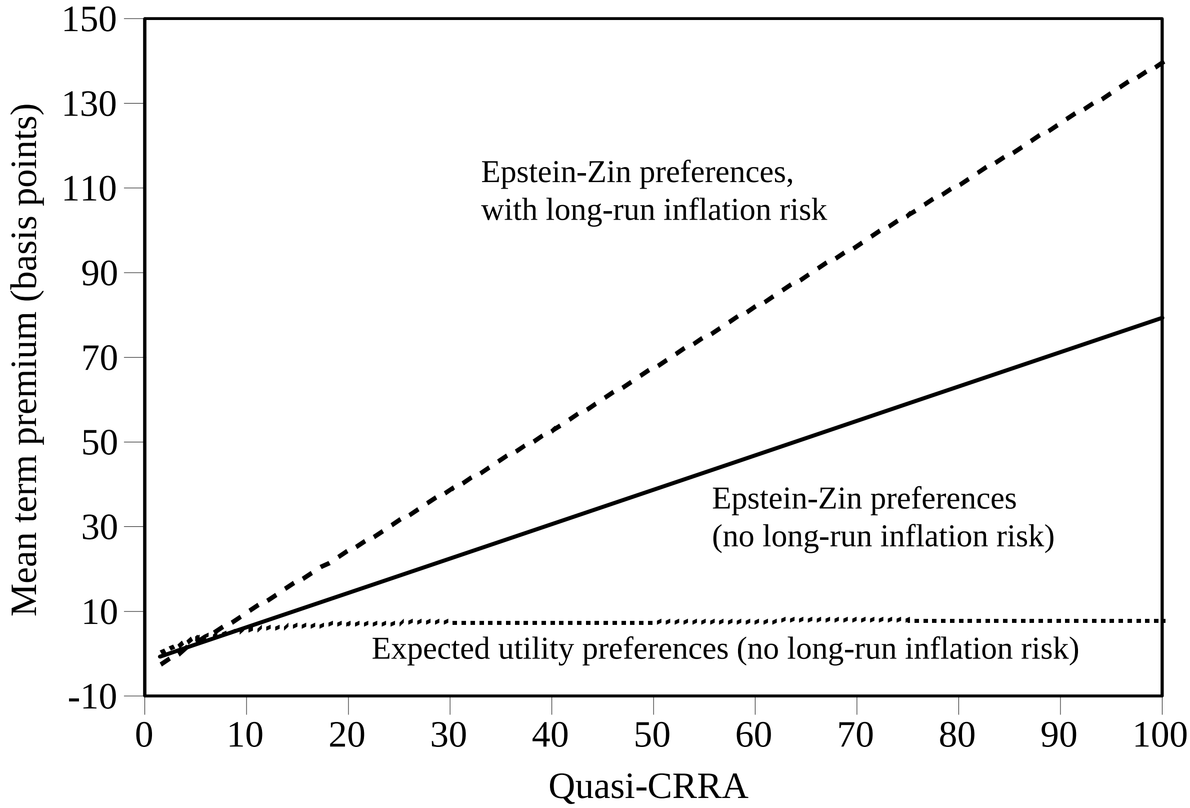

Figure 1. Mean Term premium in DSGE models with varying amounts of risk aversion. The solid and dashed lines show the mean 10-year term premium in a DSGE model without and with long-run inflation risk, respectively. The dotted line shows the mean term premium in the version of the model with expected utility preferences.

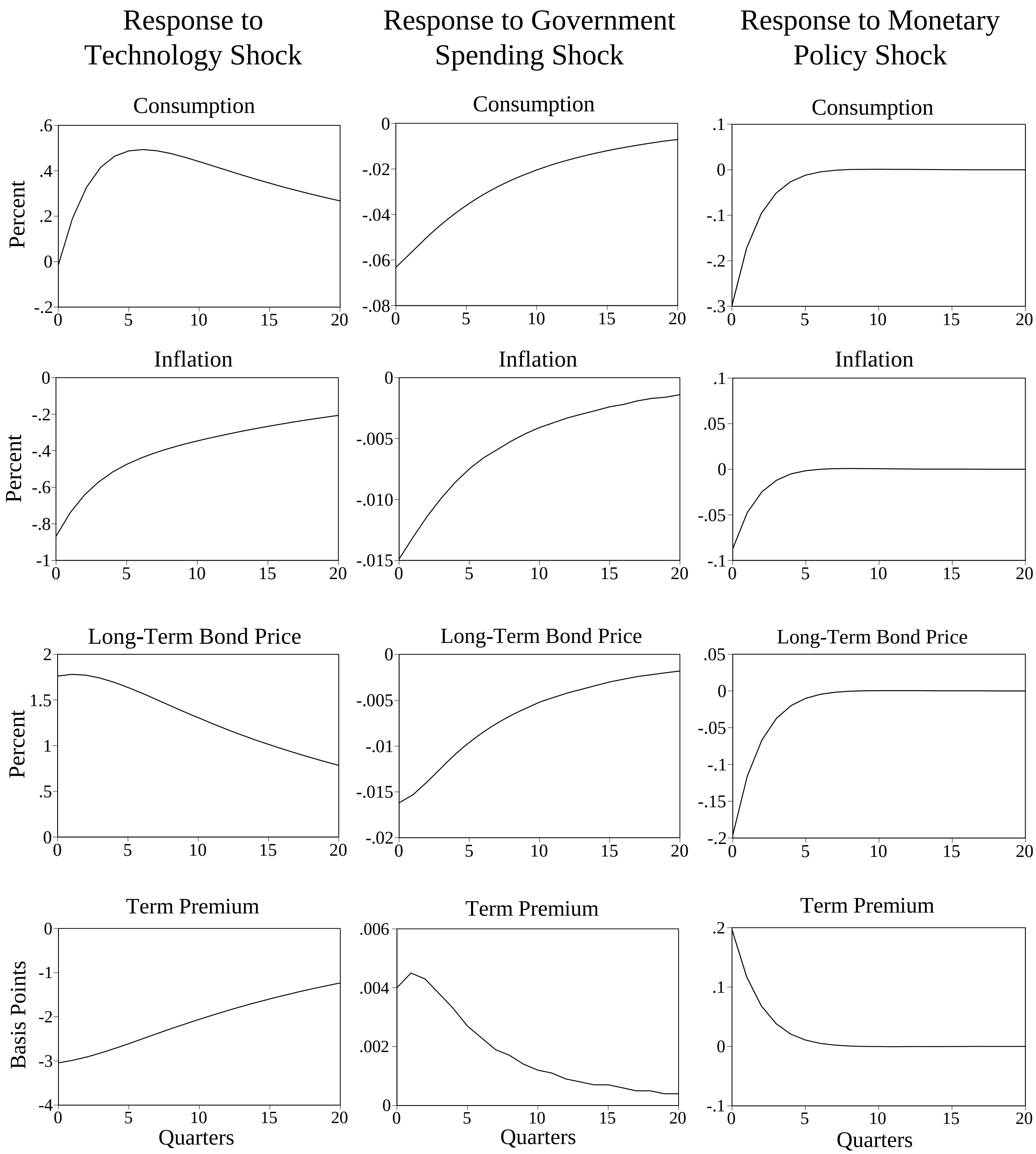

Figure 2. Impulse responses to structural shocks.
Impulse responses of consumption, inflation, long-term bond prices, and term premiums to positive one standard deviation shocks to technology, government spending, and monetary policy.