Discussion of No-Arbitrage Near-Cointegrated VAR(*p*) Term Structure Models, Term Premia and GDP Growth by C. Jardet, A. Monfort and F. Pegoraro

R. Mark Reesor

Department of Applied Mathematics
The University of Western Ontario

Conference on Fixed Income Markets
Bank of Canada
September 2008

General Comments

- VAR(p) and CVAR(p) models.
- Model averaging to get NCVAR(3) model.
- No-Arbitrage NCVAR(3) Affine Term Structure Model.
- Term premia decomposition.
- New Information Response Function—quite useful for investigating the effects of shocks, particular of filtered variables.

VAR and CVAR Models

- The VAR(p) model is

$$X_t = \nu + \sum_{j=1}^{p} \Phi_j X_{t-j} + \varepsilon_t$$

- ε_t are assumed to be iid $\mathcal{N}(0, \Omega)$.
- CVAR(p) model is a constrained (nested) version of this model with cointegrating relationship given by the spread $S_t = R_t - r_t$.
- Number of lags p=3 is selected using several criterion.
- Statistical tests validate cointegration relation.
- Validation of assumptions on errors?

Toy Example — Misspecified Error Distribution

- Examine the effect of a misspecified error distribution on model forecasts.
- Consider the AR(1) model

$$X_t = FX_{t-1} + \epsilon_t.$$

- Compute a prediction interval assuming that

$$\epsilon_t \sim \mathcal{N}(0, Q)$$

- Suppose that true error distribution is a location-scale exponential with cumulative distribution function

$$F_\lambda(x) = \begin{cases} 0 & \text{for } x \leq -\lambda \\ 1 - \mathrm{e}^{-\left(\frac{x+\lambda}{\lambda}\right)} & \text{for } x > -\lambda \end{cases},$$

Note that the mean is zero and with $\lambda^2 = Q$ this has the same variance as the normal distribution above.

Toy Example — Misspecified Error Distribution

- Under the normality assumption

$$X_{t+1}|\mathcal{F}_t \sim \mathcal{N}\left(FX_t, \lambda^2\right)$$

- A 95% prediction interval for $X_{t+1}|\mathcal{F}_t$ is

$$[FX_t - 1.96\lambda, FX_t + 1.96\lambda] = [a, b].$$

- If the errors are from the L-S exponential distribution

$$\mathbb{P}\left([X_{t+1} < a] \bigcup [X_{t+1} > b] \middle| \mathcal{F}_t\right) = 0.0518.$$

- Missed forecasts always lie above the interval.

Model Averaging to get NCVAR(3)

- NCVAR(3) parameters are given by

$$\theta_{nc}(\lambda) = \lambda\theta_{var} + (1-\lambda)\theta_{cvar}$$

  for some $\lambda \in [0,1]$.
- Selection criteria for λ is to minimise

$$\sum_{t=1}^{T}\left[\tilde{B}_t^*(h) - \hat{B}_t^*(h)\right]^2$$

  - $\tilde{B}_t^*(h)$ is the observed realisation of $\exp(-r_t - \cdots - r_{t+h-1})$.
  - $\hat{B}_t^*(h)$ is the NCVAR(3) forecast.
    - an h-step ahead forecast.
    - model is a vector autoregression.

Bruce Hansen's Work on Model Averaging

- Authors cite three of Hansen's papers
  - "Notes and Comments, Least Squares Model Averaging", *Econometrica*, Volume 75 No. 4, 2007
  - "Least Squares Forecast Averaging", *J Econometrics*, to appear 2008
  - "Averaging Estimators for Autoregressions with a Near Unit Root", *J Econometrics*, to appear 2008
- Third paper gives theory and evidence for optimal weights in the case of
  - Univariate autoregressions; and
  - 1-step ahead forecasts.
- Future Work
  - Vector Autoregressions
  - k-step ahead forecasts.
- NCVAR(3) model presented here is strong empirical evidence in favour of Mallows Model Averaging.

Optimal Weights

- Table 3 (out-of-sample forecasts of $B_t^*(h)$), compute λ^* corresponding to each residual maturity h.
- Question Is there a single choice of λ across all h for which the NCVAR performs well?
- Table 4 (out-of-sample forecasts of the state variables) NCVAR model produces excellent forecasts for (r_t, R_t, g_t) at all horizons using a single value for $\lambda = 0.2624$.

Term Structure Model — In-Sample Fit

- Pricing Error

$$PE_t = \frac{\sum_h \left|\tilde{R}_t(h) - R_t(h)\right|}{H}$$

  - $\tilde{R}_t(h)$ is observed yield
  - $R_t(h)$ is the model-implied yield
  - H is the number of maturities used to estimate the risk sensitivity parameters.
- This measure ignores any possible maturity effects on the fits of the different models.
- Table 5 gives summary stats of PE across models — unclear if there is any significant difference in model fits.

Term Structure Model — Out-of-Sample Forecasts

- NCVAR(3) model is clearly best, particularly for long forecast horizons.
- Table 6 gives forecast results for maturities up to 5 years, but no forecasting results for longer maturities?