On Informationally Efficient Markets and Efficient Market Outcomes in Monetary Economies*

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Abstract

I examine a model where monetary exchange is necessary. Inside money constitutes a claim against the stochastic dividend flow of a durable asset. The long-horizon expected return on this asset remains constant over time. An exogenous and high-frequency flow of new information (news), however, induces volatility in short-horizon expected returns. An informationally efficient asset market implies that the value of inside money will fluctuate accordingly.

I demonstrate that an informationally efficient asset market is not necessarily consistent with an efficient allocation of resource when the asset in question is a monetary instrument. The efficacy of equity as a high-velocity payment instrument is hindered by unanticipated movements in its short-term value that occasionally leave consumers "cash-constrained." When this is so, I show that the nondisclosure of high-frequency news items can improve *ex ante* welfare. If nondisclosure is not possible, then the introduction of a fiat currency may improve *ex ante* welfare.

1 Introduction

The underlying premise of the efficient-market hypothesis (EMH) is that unfettered financial markets are "informationally efficient" in the sense that new and relevant information is capitalized rapidly in the prices of traded securities. While economists have long debated the empirical relevance of EMH, relatively less attention has been devoted to the question of whether informationally efficient markets necessarily imply or promote efficient market outcomes.

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It is probably fair to say that most economists believe that the tenets of EMH go hand-in-hand with efficient market outcomes. In fact, it seems hard to argue why asset prices should not–in the interest of allocative efficiency–reflect all current and relevant information at each moment of time. Indeed, many regulatory measures–like mark-to-market accounting rules–appear clearly motivated by the idea that (informationally efficient) market prices are the best measure of value, that these values should be updated frequently, and that they should made as transparent as possible on balance sheet statements.

The sentiment expressed above is quite possibly a correct one; at least, most of the time. The purpose of this paper is to argue that it is not likely to be correct in all circumstances. In particular, while it is probably a good idea to facilitate the flow of information for most traded securities, such a policy is not necessarily desirable for that special class of assets that are used widely in making payments. The most obvious example of such an asset are those objects chosen by society to serve as money. But other examples may include securities that commonly serve as collateral in the so-called "shadow-banking" system.

To develop my argument, I consider an economy endowed with a single durable asset that delivers a stochastic dividend over time. There is an exogenous high-frequency flow of new information (news) that, if made available to market participants, would lead them to revise their short-term forecasts of the dividend flow. The question I ask is whether the nondisclosure of this news might in some circumstances enhance economic efficiency.

Naturally, this question is interesting only if one departs from the assumptions that would allow an Arrow-Debreu securities market to coordinate all economic activity. The existence of money (an asset that circulates as a means of payment) constitutes evidence that would support such a departure. For this reason, I focus on monetary economies.[1]

The agents of my model economy experience idiosyncratic shocks that determine, at various points in time, whether they have a desire to consume or an ability to produce a perishable good or service. To motivate the need for money, I assume that all agents, apart from that agent or agency in control of the durable asset, are anonymous and lack commitment. The implication of this is that private credit is infeasible, so that payment for goods and services must be made up-front with a tangible asset. A natural candidate for this tangible asset are claims against the economy's durable asset. In this way, equity shares can serve as the economy's payment instrument.

Despite these limitations on trading arrangements, I demonstrate that a competitive monetary equilibrium may be efficient. When this is so, the price of equity (money) fluctuates over time in response to news; but this in no way hinders the ability of agents realize efficient trades, even though they are restricted to using equity as a payment instrument. It follows that informationally efficient

[1] I define money to be an object that circulates widely as a medium of exchange. I do not restrict this object to be fiat money.

markets are consistent with efficient outcomes.

There are, however, circumstances in which a competitive monetary equilibrium is inefficient. This situation occurs when a subset of agents are debt-constrained (anonymous agents that lack commitment are unable borrow or short equity). In this case, the arrival of bad news temporarily depresses the purchasing power of equity shares; and an individual's existing shareholdings are insufficient to finance all desired consumption.

When this is so, the agents in my model would strictly prefer (*ex ante*) that high-frequency news items be suppressed, if it is possible to do so. Nondisclosure has the effect of mitigating (in my environment, eliminating) asset price fluctuations; and this turns out to be important for assets used in payments since it reduces the incidence of binding debt constraints. It follows that informationally efficient markets are not necessarily consistent with efficient market outcomes. This result calls into question the logic of mark-to-market accounting regulations imposed on that sector of the economy responsible for managing the economy's money supply (the banking sector).

In the second part of the paper, I assume that a nondisclosure policy is infeasible and examine the potential role of fiat money. I find that inside and outside money can coexist even though the former is dominated in expected return. Moreover, whenever fiat money is valued, it's presence is welfare-improving. Indeed, a Friedman rule policy, if it is feasible, implements the first-best allocation.

2 The Environment

The basic structure draws on Lagos and Wright (2005). There is a continuum of *ex ante* identical agents, indexed by $i \in [0,1]$. Agents live forever, with time denoted $t = 0, 1, 2, ..., \infty$. Each time period t is divided into two subperiods, labeled *day* and *night*.

Nonstorable output is produced in the day and the night. Consumption in the day is denoted $x_t(i) \in \mathbb{R}$; where $x_t(i) < 0$ is to be interpreted as production. Assume that utility is linear in day consumption/production.

At the beginning of the night, agents realize an idiosyncratic shock that determines their type: a *consumer* or *producer*. Consumption at night is denoted $c_t(i) \in \mathbb{R}_+$ and generates (for a consumer) the utility flow $u(c_t(i)) \in \mathbb{R}$; where $u'' < 0 < u'$ and $u'(0) = \infty$. Production at night is denoted $y_t(i) \in \mathbb{R}_+$ and generates (for a producer) the utility flow $-h(y_t(i)) \in \mathbb{R}$; where $h' > 0$ and $h'' \geq 0$.

Types are *i.i.d.* across agents and across time. For simplicity, assume that each type is equally likely; so that the preferences of an agent i at the beginning of time are represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t(i) + 0.5u(c_t(i)) - 0.5h(y_t(i)) \right] \tag{1}$$

where $0 < \beta < 1$.

There is a durable asset that generates an exogenous and stochastic output flow $0 \leq z_t < \infty$ at the beginning of each day. This aggregate shock follows a Markov process, $\Pr\left[z_{t+1} \leq z^+ \mid \eta_t = \eta\right] = F(z^+ \mid \eta)$; where F is a cumulative distribution function, conditional on information η_t (news) received at the beginning of the night. Assume that news is either bad or good; $\eta_t \in \{b, g\}$ and that $\pi \equiv \Pr[\eta_t = b]$. Define

$$z(\eta) \equiv \int z^+ dF(z^+ \mid \eta) \tag{2}$$

where $0 \leq z(b) \leq z(g) < \infty$. That is, $z(\eta)$ is a "short-term" conditional forecast made at night over the dividend payment that is to be realized the next day. In contrast, the "long-term" forecast (for horizons extending from one day to the next and beyond) is invariant to news; i.e.,

$$z^e \equiv \pi z(b) + (1 - \pi) z(g) \tag{3}$$

As all output is nonstorable, there are two resource constraints

$$z_t \geq \int x_t(i) di \tag{4}$$

$$\int y_t(i) di \geq \int c_t(i) di \tag{5}$$

2.1 The First-Best Allocation

The first-best allocation maximizes (1) for an *ex ante* representative agent, subject to the resource constraints (4) and (5); and assuming that expectations are consistent with (2). The first-best allocation may assign $x_t(i) = z_t$; so that each agent receives (in expectation) z^e units of output in the day.[2]

Symmetry implies $c_t(i) = c_t$ and $y_t(i) = y_t$. An equal population of types at night implies $c_t = y_t$; by virtue of the resource constraint (5) holding with equality. Optimality requires $y_t = y^*$; with $0 < y^* < \infty$ satisfying

$$u'(y^*) = h'(y^*) \tag{6}$$

The first-best allocation delivers *ex ante* utility

$$W^* = (1 - \beta)^{-1} \left[z^e + 0.5u(y^*) - 0.5h(y^*) \right]$$

[2] Note that owing to the quasilinear property of preferences, the presence of risk (whether aggregate or idiosyncratic) has no effect on *ex ante* welfare. The first-best allocation here is also consistent with any lottery over $\{x_t(i)\}$ that generates expected utility z^e for the agent.

2.2 Anonymity and Limited Commitment

Agents are anonymous and lack commitment. Together, these restrictions preclude the use of private debt. They also imply that a tangible medium of exchange (money) is necessary to facilitate intertemporal trade; see Kocherlakota (1998). Following the literature, I assume that a planner (or the government, working on behalf of society) can issue durable, divisible, and noncounterfeitable bearer notes that might serve as the economy's medium of exchange.

I also assume that the planner maintains control of the economy's asset; i.e., he is free to distribute dividends as he sees fit. The existence of this asset implies that the planner may issue money in the form of equity claims (inside money); that is, as opposed to the fiat instrument (outside money) that would be necessary in a world without capital. I will consider fiat money in a later section, but for now I assume that it takes the form of equity.

In what follows, I restrict attention to linear mechanisms. In particular, trade occurs in a sequence of anonymous competitive spot markets, involving a *quid pro quo* exchange of money and output.

3 An Economy with Inside Money

Inside money takes the form of equity shares. I normalize the number of shares to unity and assume that each agent is initially endowed with a single share. Apart from this initial period, I anticipate that the equilibrium distribution of money at the beginning of each day will fall on a two-point set $\{s_c, s_p\}$; where $s_j \geq 0$ and j denotes the agent's type in the previous night (consumer or producer). Let (ϕ_1, ϕ_2) denote the price of money (shares) measured in units of output; in the day and night, respectively. In what follows, ϕ_1 denotes the ex-dividend price.

3.1 Decision Making in the Day

Let $s \geq 0$ denote money that is carried forward into the night. Then the day budget constraint is given by

$$x = (z + \phi_1)\, s_j - \phi_1 s \tag{7}$$

Let $D(s_j, z)$ denote the value of entering the day with shares s_j and with realized dividend income z. Let $N(s, \eta)$ denote the *ex ante* (before type is known) value of entering the night-market with share-holdings s when the news is η. The value functions D and N must satisfy the following Bellman equation

$$D(s_j, z) \equiv \max_{s \geq 0} \left\{ (z + \phi_1)\, s_j - \phi_1 s + E_\eta \left[N(s, \eta) \right] \right\} \tag{8}$$

where here, I have substituted in the budget constraint (7).

Assume that the value function N is increasing and at least weakly concave in s; i.e., $N_{11} \leq 0 < N_1$. In fact, these are properties that will hold in equilibrium. If $N_{11} < 0$, then each agent leaves the day-market with identical share-holdings s characterized by

$$\phi_1 = E_\eta \left[N_1(s, \eta) \right] \tag{9}$$

If $N_{11} = 0$, then desired individual share-holdings are indeterminate; at least, beyond some strictly positive lower bound. Even in this case, however, condition (9) will continue to hold in any equilibrium.[3]

By the envelope theorem, $D_1(s_j, z) = z + \phi_1$; so that $D_1(s_j^+, z^+) = z^+ + \phi_1^+$. Given that the stochastic dividend flow is an i.i.d. process from one day to the next, and given quasi-linearity, I conjecture that $\phi_1 = \phi_1^+$. That is, the ex-dividend price of equity in the day will remain constant from one day to the next. In this case,

$$\int D_1(s_j^+, z^+) dF(z^+ \mid \eta) = z(\eta) + \phi_1 \tag{10}$$

3.2 Decision Making at Night

Let $C(s, \eta)$ denote the value of being a consumer at night, with money s and when news is η. Using $c \equiv \phi_2(s - s_c^+)$, the choice problem may be stated as

$$C(s, \eta) \equiv \max_{s_c^+ \geq 0} \left\{ u(\phi_2(s - s_c^+)) + \beta \int D(s_c^+, z^+) dF(z^+ \mid \eta) \right\} \tag{11}$$

The consumer's debt constraint $s_c^+ \geq 0$ plays an important role in what follows.[4] Utilizing (10), desired consumption is characterized by

$$\begin{array}{ll} \phi_2(\eta) u'(c(\eta)) = \beta \left[z(\eta) + \phi_1 \right] & \text{if} \quad \phi_2(\eta) s > c(\eta) \\ c(\eta) = \phi_2(\eta) s & \text{otherwise} \end{array} \tag{12}$$

Let $P(s, \eta)$ denote the value of being a producer at night, with money s and when news is η. Using $y \equiv \phi_2(s_p^+ - s)$, the choice problem may be stated as

$$P(s, \eta) \equiv \max_{s_p^+ \geq 0} \left\{ -h(\phi_2(s_p^+ - s)) + \beta \int D(s_p^+, z^+) dF(z^+ \mid \eta) \right\} \tag{13}$$

Note that as a producer has no desire to consume, his debt constraint is necessarily slack. Utilizing (10), desired production is characterized by

$$\phi_2(\eta) h'(y(\eta)) = \beta \left[z(\eta) + \phi_1 \right] \tag{14}$$

[3] If it did not hold, then the demand for shares would either be zero or infinity.

[4] That is, consumers may wish to short equity, but are prevented from borrowing because they are anonymous and lack commitment.

3.3 Market Clearing

The market-clearing conditions are given by

$$s = 1 \text{ and } c(\eta) = y(\eta) \tag{15}$$

which will, of course, imply $0.5s_c^+(\eta) + 0.5s_p^+(\eta) = 1$.

3.4 General Equilibrium

The object of interest here is the equilibrium allocation at night $y(\eta)$, together with the corresponding price system ϕ_1 and $\phi_2(\eta)$.

To begin, consider (9). Note that $N_1(s,\eta) \equiv 0.5C_1(s,\eta) + 0.5P_1(s,\eta)$. Applying the envelope theorem to (11) and (13), $N_1(s,\eta) \equiv 0.5\phi_2(\eta)u'(y(\eta)) + 0.5\phi_2(\eta)h'(y(\eta))$. Condition (9) may therefore be expressed as

$$\phi_1 = 0.5\pi\phi_2(b)\left[u'(y(b)) + h'(y(b))\right] + 0.5(1-\pi)\phi_2(g)\left[u'(y(g)) + h'(y(g))\right] \tag{16}$$

Next, note that condition (14) implies the asset-price function

$$\phi_2(\eta) = \beta\left[\frac{z(\eta) + \phi_1}{h'(y(\eta))}\right] \tag{17}$$

Finally, note that (12) and (14), together with market-clearing, imply

$$\begin{array}{ll} y(\eta) = y^* & \text{if} \quad \phi_2(\eta) > y^* \\ \phi_2(\eta) = y(\eta) < y^* & \text{otherwise} \end{array} \tag{18}$$

Conditions (16), (17) and (18) constitute the key restrictions that characterize the general equilibrium allocation and price-system for the inside-money economy.

4 Properties of the Inside-Money Economy

4.1 A No-News Economy

I begin with a useful benchmark that I call a *no-news economy*; i.e., assume that $z(\eta) = z^e$ for $\eta \in \{b, g\}$. It follows that $\phi_2(\eta) = \phi_2$ and $y(\eta) = y$.

Now, conjecture that the debt-constraint remains slack. Then (18) implies that $y = y^*$ and (17) implies $\phi_2 = \beta\left[z^e + \phi_1\right]/h'(y^*)$. This pricing function, together with $y(\eta) = y^*$ and (16) delivers

$$\phi_1 = \left(\frac{\beta}{1-\beta}\right) z^e; \tag{19}$$

which appears to be the standard asset-pricing formula that one would expect for risk-neutral agents.

I need to confirm that the conjecture I made with respect to (18) holds in equilibrium; i.e., that $\phi_2 > y^*$. Using ϕ_1 and ϕ_2 as derived above, this condition can be expressed as

$$\left(\frac{\beta}{1-\beta}\right) z^e > h'(y^*)y^*$$

Whether this condition holds or not depends on parameters. For example, it can clearly be made to hold as either $z^e \to \infty$ or $\beta \to 1$. On the other hand, it will fail as either $z^e \to 0$ or $\beta \to 0$.

Lemma 1 *For a given* $\beta \in (0, 1)$, *there exists an expected asset income* $0 < z_0 < \infty$ *satisfying*

$$\left(\frac{\beta}{1-\beta}\right) z_0 = h'(y^*)y^* \tag{20}$$

So my conjecture is confirmed for a parameterization such that $z^e \geq z_0$.[5] Nevertheless, it is instructive to see what happens when $0 < z^e < z_0$. In this case, the debt constraint binds tightly; so that (18) implies $\phi_2 = y < y^*$.

Before proceeding to characterized the debt-constrained level of y, it will be useful to define the following object

$$A(y) \equiv 0.5 \left[\frac{u'(y) + h'(y)}{h'(y)}\right]$$

Lemma 2 $A(y^*) = 1$ *and* $A'(y) < 0$.

Now, express condition (16) as $\phi_1 = \phi_2 h'(y)A(y)$. Note that this implies $\beta [z^e + \phi_1] = \beta [z^e + \phi_2 h'(y)A(y)]$. Using condition (17), this implies $\phi_2 h'(y) = \beta [z^e + \phi_2 h'(y)A(y)]$. As $\phi_2 = y$ when the debt-constraint binds, the latter expression can be written as

$$yh'(y) [1 - \beta A(y)] = \beta z^e$$

With $y < y^*$ so determined, the equilibrium price of equity in the day is $\phi_1 = yh'(y)A(y)$; or

$$\phi_1 = \left[\frac{\beta A(y)}{1 - \beta A(y)}\right] z^e \tag{21}$$

In comparing the asset price functions (19) and (21), it appears that equity is "over-valued" in the debt-constrained equilibrium relative to its "fundamental" value. That is, people would like to borrow (or short equity) at night, but

[5] I allow for an equality here, as this implies that the debt-constraint only binds weakly.

cannot. In terms of the expected rate of return on equity (from one day to the next)

$$1 < \left[\frac{z^e + \phi_1}{\phi_1}\right] < \frac{1}{\beta}$$

That is, the effect of the binding debt constraint is to lower the expected rate of return on equity (reflecting the usual precautionary saving motive).

4.2 A News Economy

By a *news economy*, I mean $0 \leq z(b) < z^e < z(g)$.

If the debt constraint never binds, then by (18), the competitive equilibrium implements the efficient allocation $y(\eta) = y^*$. As a consequence, the equilibrium asset price in the day is given by (19). Condition (17) then delivers an expression for the price of equity at night

$$\phi_2(\eta) = \beta\left[\frac{z(\eta) + \phi_1}{h'(y^*)}\right]$$

That is, the equilibrium share price at night responds to news in the way one would expect; i.e., $\phi_2(b) < \phi_2(g)$.

Thus, it is conceivable that equity might serve as an efficient payments instrument. While the price of this monetary instrument fluctuates randomly at night in response to new information, this price volatility in no way inhibits *ex ante* efficiency. At least, this is true as long as share price movements do not leave consumers debt-constrained in some states of the world; a possibility that I now wish to consider.

Lemma 3 *If $z^e = z_0$ and $z(b) < z(g)$, then the consumer debt constraint will bind tightly in the bad news state and remain slack in the good news state.*

I omit a formal proof of Lemma 3, but provide a brief sketch of the logic underpinning the result. Note that by the definition of z_0 in (20), the debt-constraint binds just weakly when $z(b) = z(g) = z_0$. Lemma 3 maintains $z^e = z_0$ and introduces a mean-preserving spread $z(b) < z_0 < z(g)$. Since the debt-constraint binds weakly at z_0, it will bind tightly at $z(b)$. Likewise, the debt-constraint will remain slack at $z(g)$.

Lemma 3 and condition (18) imply that $\phi_2(b) = y(b) < y(g) = y^*$. Appealing to (16) and (17), the equilibrium $(\phi_1, y(b))$ is characterized by

$$\begin{aligned} \phi_1 &= \pi\beta[z(b) + \phi_1]A(y(b)) + (1-\pi)\beta[z(g) + \phi_1] \\ h'(y(b))y(b) &= \beta\left[z(b) + \phi_1\right] \end{aligned}$$

Solving for the ex-dividend price of equity in the day

$$\phi_1 = \beta \left[\frac{\pi z(b) A(y(b)) + (1-\pi) z(g)}{1 - \beta(\pi A(y(b)) + 1 - \pi)} \right] \tag{22}$$

Note that (22) reduces to (19) when $y(b) = y^*$. Hence, as long as $y(b) < y^*$, equity commands a premium relative to its "fundamental" value.

As for the equilibrium price of equity at night, refer to condition (17)

$$\phi_2(b) = \frac{\beta \left[z(b) + \phi_1 \right]}{h'(y(b))} \text{ and } \phi_2(g) = \frac{\beta \left[z(g) + \phi_1 \right]}{h'(y^*)}$$

It is curious to note that $\phi_2(b) > \phi_2(g)$ appears possible here. If this is so, then the debt constraint would bind in the good news state and remain slack in the bad news state; a possibility ruled out by Lemma 3. Hence, $\phi_2(b) < \phi_2(g)$.

In this economy, consumers find themselves debt-constrained at night with probability $0 < \pi < 1$.Clearly, the competitive equilibrium does not implement the first-best allocation (is allocatively inefficient) in this case. Consider the following proposition.

Proposition 1 *Assume that $z^e = z_0$ and $z(b) < z(g)$. Then the competitive equilibrium for the inside-money economy with news is informationally efficient and allocatively inefficient. Moreover, the competitive equilibrium for the inside-money economy where news is suppressed is informationally inefficient and allocatively efficient.*

Proposition 1 asserts that a non-disclosure policy by the operator of capital over some forms of information may be socially desirable (assuming that such a policy is even feasible).[6] Note that under the conditions stated in Proposition 1, the equilibrium with nondisclosure corresponds to that of the *no-news economy* studied earlier. The implication is that in suppressing the news flow, the first-best allocation is rendered implementable.

It is true that by withholding information, asset prices are no longer informationally efficient at night. In fact, the price of equity at night is rendered entirely insensitive to news (it is a constant). Not allowing asset prices to fully reflect all relevant information is probably not a practice that one would want to encourage in general. The analysis here, however, suggests that an exception might be made for the set of assets that play a prominent role in the economy's payments system.

In the environment studied above, the public revelation of news prior to night-market trading would not destroy risk-sharing if agents were not anonymous and/or had the power to commit to their promises. But as Kocherlakota (1998) has emphasized, it is precisely the limitations along these dimensions

[6] Of course, the condition $z^e = z_0$ in Proposition 1 is sufficient, but not necessary for this result.

that make monetary exchange necessary. When this is so, agents may find themselves cash-constrained by a temporary decline in the value of their money (a price decline that bears little, if any, relation to the fundamental long-run value of their monetary asset). Welfare is enhanced here by suppressing the high-frequency information flow that generates excess volatility in the value of money.

5 An Economy with Inside and Outside Money

If a nondisclosure policy is infeasible, then the competitive equilibrium for the inside-money economy may be inefficient. In this case, one solution is a government policy that applies a distortionary and news-contingent subsidy (tax) on dividend income, financed by a lump-sum tax (transfer). But if state-contingent or lump-sum taxes are unavailable policy instruments, such a policy is not feasible.

An alternative strategy–the one I pursue here–is to introduce a fiat instrument (outside money). The supply of fiat money is denoted M and is assumed to grow at a constant rate, so that $M^{+} = \mu M$. New money $(\mu - 1)M$ is injected as a lump-sum transfer in each day. To begin, I allow for lump-sum taxation $(\mu < 1)$, but I also consider the case where all trade is restricted to be voluntary $(\mu \geq 1)$.

In the day, agents now enter with money balances a_j and shares s_j. They enter the night with money balances m and shares s. Let (v_1, v_2) denote the value of fiat money in the day and night, respectively.

5.1 Decision Making in the Day

In the day, the choice problem is given by

$$D(s_j, a_j, z) \equiv \max_{s,m\geq 0} \{(z + \phi_1)s_j - \phi_1 s + v_1(a_j - m) + (\mu - 1)v_1 M + E_\eta [N(s, m, \eta)]\}$$

The demand for inside and outside money carried into the night is characterized by

$$\phi_1 = E_\eta [N_1(s, m, \eta)] \quad (23)$$

$$v_1 = E_\eta [N_2(s, m, \eta)] \quad (24)$$

Moreover, using the envelope theorem, note that

$$E\left[D_1^{+} \mid \eta\right] = z(\eta) + \phi_1 \quad (25)$$

$$D_2^{+} = v_1^{+} \quad (26)$$

5.2 Decision Making at Night

I anticipate that if both monies are to be willingly held, they must each earn the same expected return from night to the next day; i.e.

$$\left[\frac{z(\eta)+\phi_1}{\phi_2(\eta)}\right] = \left[\frac{v_1^+}{v_2(\eta)}\right] \equiv R(\eta) \tag{27}$$

Using $c \equiv \phi_2(s - s_c^+) + v_2(m - a_c^+)$, the choice problem for a consumer is given by

$$C(s,m,\eta) \equiv \max_{s_c^+ \geq 0, a_c^+ \geq 0} \left\{ u(\phi_2(s - s_c^+) + v_2(m - a_c^+)) + \beta \int D(s_c^+, a_c^+, z^+) dF(z^+ \mid \eta) \right\}$$

Of course, given (27), only the sum $s_c^+ + a_c^+$ is determined at the individual level. At an interior, desired consumption is characterized by

$$u'(c(\eta)) = \beta R(\eta) \tag{28}$$

If the consumer is debt-constrained, then $s_c^+ = a_c^+ = 0$ and

$$c(\eta) = \phi_2(\eta)s + v_2(\eta)m \tag{29}$$

Using $y \equiv \phi_2(s_c^+ - s) + v_2(a_c^+ - m)$, the choice problem for a producer is given by

$$P(s,m,\eta) \equiv \max_{s_p^+, a_p^+} \left\{ -h(\phi_2(s_c^+ - s) + v_2(a_c^+ - m)) + \beta \int D(s_p^+, a_p^+, z^+) dF(z^+ \mid \eta) \right\}$$

Desired production is characterized by

$$h'(y(\eta)) = \beta R(\eta) \tag{30}$$

5.3 Market Clearing

The market-clearing conditions are given by

$$s = 1; \; m = M \text{ and } c(\eta) = y(\eta) \tag{31}$$

Together, these imply $0.5s_c^+(\eta) + 0.5s_p^+(\eta) = 1$ and $0.5a_c^+(\eta) + 0.5a_p^+(\eta) = M$.

5.4 General Equilibrium

The object of interest here is the equilibrium allocation at night $y(\eta)$, together with the corresponding price system $(\phi_1, \phi_2(\eta))$ and $(v_1, v_2(\eta))$.

To begin, apply the envelope theorem to the consumer and producer value functions above and use this information to express (23) and (24) as

$$\phi_1 = 0.5\pi\phi_2(b)\left[u'(y(b)) + h'(y(b))\right] + 0.5(1-\pi)\phi_2(g)\left[u'(y(g)) + h'(y(g))\right] \quad (32)$$

$$v_1 = 0.5\pi v_2(b)\left[u'(y(b)) + h'(y(b))\right] + 0.5(1-\pi)v_2(g)\left[u'(y(g)) + h'(y(g))\right] \quad (33)$$

Note that (32) corresponds exactly to (16) and that (33) is simply the analog to (32) for the case of fiat money.

Next, conditions (30) and (27) imply

$$\left[\frac{z(\eta)+\phi_1}{\phi_2(\eta)}\right] = \left[\frac{v_1^+}{v_2(\eta)}\right] = \left(\frac{1}{\beta}\right) h'(y(\eta)) \quad (34)$$

Moreover, note that (28), (29) and (30), together with market-clearing, imply

$$\begin{array}{ll} y(\eta) = y^* & \text{if} \quad \phi_2(\eta) + v_2(\eta)M > y^* \\ \phi_2(\eta) + v_2(\eta)M = y(\eta) < y^* & \text{otherwise} \end{array} \quad (35)$$

Finally, as I restrict attention to stationary allocations, market-clearing will imply

$$\left(\frac{v_1^+}{v_1}\right) = \left(\frac{1}{\mu}\right) \quad (36)$$

Conditions (32), (33), (34), (35) and (36) constitute the key restrictions that characterize the general equilibrium allocation and price-system for the "dual money" economy.

5.5 Properties of the Dual Money Economy

I consider the case for which $z^e = z_0$; so that absent fiat money, the consumer debt constraint binds tightly only when the news is bad. Now let me introduce fiat money into this economy. I conjecture that there is a sufficiently low inflation rate $\mu > \beta$ such that outside money coexists with inside money. Moreover, I conjecture that for $\mu > \beta$, consumers will remain debt-constrained when the news is bad and will remain unconstrained when the news is good. Hence, by condition (35)

$$\begin{array}{rcl} y(g) & = & y^* \\ y(b) & = & \phi_2(b) + v_2(b)M < y^* \end{array}$$

Now, consider condition (33). Applying Lemma 1, this may be expressed as

$$v_1 = \pi v_2(b)h'(y(b))A(y(b)) + (1-\pi)v_2(g)h'(y^*)$$

Condition (34) implies that $v_2(\eta)h'(y(\eta)) = \beta v_1^+$. Together with condition (36), the expression above reduces to

$$1 = \left(\frac{\beta}{\mu}\right)[\pi A(y(b)) + 1 - \pi] \tag{37}$$

Now this is rather interesting. Condition (37) implies that in a dual-money equilibrium, the equilibrium level of output in the bad-news state can be determined entirely by the monetary policy parameter μ.

Proposition 2 *Assume that $z^e = z_0$ and $z(b) < z(g)$. Then in an economy where fiat money coexists with inside money, $y(b)$ is monotonically decreasing in μ with $y(b) \nearrow y^*$ as $\mu \searrow \beta$.*

The proposition above suggests that deflating at the Friedman rule is consistent with first-best implementation. It also suggests that fiat money will be valued in this economy for any inflation rate that implies a level of $y(b)$ that is at least as high as what would prevail in an economy without fiat money.

With $y(b)$ determined by (37), let me now turn to equilibrium prices. Using Lemma 1, condition (32) may be expressed as

$$\phi_1 = \pi\phi_2(b)h'(y(b))A(y(b)) + (1-\pi)\phi_2(g)h'(y^*)$$

Condition (34) implies that $\phi_2(\eta)h'(y(\eta)) = \beta[z(\eta) + \phi_1]$. Combining this with the expression above yields the equilibrium ex-dividend price of equity in the day

$$\phi_1 = \beta\left[\frac{\pi z(b)A(y(b)) + (1-\pi)z(g)}{1 - \beta(\pi A(y(b)) + 1 - \pi)}\right] \tag{38}$$

This asset-price equation is, in fact, identical to what transpires in the inside-money economy; see (22). The only difference is that the price of equity ϕ_1 is determined here, in part, by inflation policy μ via condition (37). That is, an increase in inflation tightens the debt-constraint; which results in a greater premium on inside money.

Lemma 5 *ϕ_1 is increasing in μ.*

With ϕ_1 determined by (38), condition (34) delivers

$$\phi_2(b) = \frac{\beta[z(b) + \phi_1]}{h'(y(b))} \text{ and } \phi_2(g) = \frac{\beta[z(g) + \phi_1]}{h'(y^*)}$$

Lemma 6 *$\phi_2(b)$ is increasing in μ.*

With $y(b)$, ϕ_1 and $\phi_2(b)$ now determined, condition (35) can then be used to solve for $v_2(b)$; i.e.,

$$v_2(b)M = y(b) - \phi_2(b) \tag{39}$$

The analog to (39) for the inside-money economy is condition (18); where, implicitly, $v_2(b) = 0$. To this point, I have only conjectured that fiat money coexists with inside money. A necessary condition for the validity of this conjecture is $v_2(b) > 0$.

Proposition 2 implies that $v_2(b) > 0$ for μ sufficiently close to β. However, this proposition, together with Lemma 6 and condition (39) imply

Proposition 3 *Assume that $z^e = z_0$ and $z(b) < z(g)$. Then $v_2(b) > 0$ (fiat money has value) for μ sufficiently close to β. Moreover, there exists an upper bound on inflation $\beta < \overline{\mu}$ such that $v_2(b) = 0$ for any $\mu \geq \overline{\mu}$.*

The proof of the first part of this proposition follows as a corollary to Proposition 2. To prove the second part, note that $y(b)$ is monotonically decreasing in μ and that $\phi_2(b)$ is monotonically increasing in μ; see Lemma 6. Hence, the right-hand-side of (39) will eventually equal zero at some inflation rate. Note that there is nothing to guarantee that $\overline{\mu} > 1$. If $\overline{\mu} \in (\beta, 1)$, then fiat money will only be valued under a deflationary policy. If lump-sum taxation is infeasible, fiat money would not be valued in this case. On the other hand, if $\overline{\mu} > 1$, then fiat money can be valued (and be welfare-improving) even in the absence of lump-sum taxation (a constrained efficient policy would, in this environment, require setting $\mu = 1$).

I turn next to examining the equilibrium rates of return on inside and outside money. The full-period (day-to-day) expected return on equity is $(z_0 + \phi_1)/\phi_1$ and the full-period expected return on fiat money is $1/\mu$. Define the full-period equity premium as the difference between these two objects.

Proposition 4 *Assume that $z^e = z_0$ and $z(b) < z(g)$. Then for any $\mu \in (\beta, \overline{\mu})$, the full-period equity premium satisfies*

$$\left[\frac{z_0 + \phi_1}{\phi_1} - \frac{1}{\mu}\right] = \pi \left[\frac{1}{\mu} - \frac{z(b) + \phi_1}{\phi_1}\right] [A(y(b)) - 1] > 0 \tag{40}$$

The proof to this proposition is relegated to the appendix. Proposition 4 asserts that in the range of expected inflation rates for which fiat money is valued, fiat money is dominated in expected return by inside money (equity). It is not quite right to think of this equity premium as being the product of risky equity *vis-à-vis* safe fiat. For one thing, I could make the return to fiat as risky as equity by introducing stochastic lump-sum transfers of fiat. As long as the expected inflation rate remains unchanged, nothing important changes in the equilibrium allocation (this is a consequence of the quasi-linear preference structure).

On the other hand, equity is subject to "news risk." But to the extent that fiat must compete with equity as a means of payment at night, the value of both assets will fluctuate in response to news. Nevertheless, the equity premium is in some way related to news risk since it is news that results in an occasionally binding debt constraint; and as condition (40) makes clear, the equity premium requires $y(b) < y^*$ or $A(y(b)) > 1$. Finally, it follows as a corollary to Proposition 2 that the equity premium approaches zero as $\mu \searrow \beta$.

Next, consider the realized rate of return on fiat money from the day to the night

$$\frac{v_2(b)}{v_1} = \frac{\beta}{h'(y(b))}\frac{1}{\mu} \text{ and } \frac{v_2(g)}{v_1} = \frac{\beta}{h'(y^*)}\frac{1}{\mu} \tag{41}$$

Note that these two rates of return are the same if h is linear or if $\mu = \beta$. The realized rate of return on inside money from the day to the night is

$$\frac{\phi_2(b)}{\phi_1} = \frac{\beta}{h'(y(b))}\left[\frac{z(b)+\phi_1}{\phi_1}\right] \text{ and } \frac{\phi_2(g)}{\phi_1} = \frac{\beta}{h'(y^*)}\left[\frac{z(g)+\phi_1}{\phi_1}\right] \tag{42}$$

In the proof to Proposition 4 (see condition (A2)), I show that

$$\left[\frac{z(g)+\phi_1}{\phi_1}\right] > \frac{1}{\mu} > \left[\frac{z(b)+\phi_1}{\phi_1}\right]$$

Conditions (41), (42) and (A2) imply the following result.

Proposition 5 *Assume that $z^e = z_0$ and $z(b) < z(g)$. Then for any $\mu \in (\beta, \overline{\mu})$,*

$$\frac{\phi_2(b)}{\phi_1} < \frac{v_2(b)}{v_1} \leq \frac{v_2(g)}{v_1} < \frac{\phi_2(g)}{\phi_1}$$

Thus, in any equilibrium where fiat money coexists with inside money and the debt-constraint is only occasionally binding, fiat money offers a relatively stable "short term" (from day to night) rate of return. In fact, the short-term return on fiat is completely insensitive to news if either h is linear or if $\mu = \beta$. In contrast, the short-term return on equity continues to vary with news under either of these two conditions.

To summarize, away from the Friedman rule, equity dominates money in expected return from one day to the next. This rate of return dominance vanishes at the Friedman rule. Intuitively, equity is relatively cheap away from the Friedman rule because its value at night is more variable than money; and this variability matters when consumers are potentially debt-constrained. At the Friedman rule, consumers are not debt-constrained; so the extra volatility associated with the value of equity at night is inconsequential (eliminating its discount relative to money).

It is of some interest to point out that in a dual money economy operating at the Friedman rule, the use of equity as a means of payment at night is superfluous. That is, even if equity and fiat could be used to pay for goods at night, there is an equilibrium in which agents choose to make payments only with fiat. In contrast, there is no equilibrium in which agents might choose to make payments at night only in the form of equity.

6 Conclusion

In an economy where some trades involve anonymous agents that lack commitment, a tangible monetary instrument is needed to realize gains that would otherwise go unexploited. In principle, this monetary instrument could take the form of an asset-backed security representing a claim against a productive capital asset. One characteristic of productive capital is that information about its future return typically arrives at a higher frequency than the return itself (e.g., dividends may occur quarterly, while news may arrive daily). When asset markets are informationally efficient, this high-frequency news is embedded immediately into the market price of the security. This poses a potential problem for the use of securities as a means of financing high-frequency payments. On any given day, a consumer holding equity as a means of payment may find the value of his current holdings insufficient to finance a planned expenditure.

The implication of this is that an informationally-efficient asset market is not necessarily desirable for assets that are destined to be used in high-frequency transactions. In fact, I demonstrate above how an informationally-inefficient asset market (via a nondisclosure policy) may be necessary to realize an efficient market outcome. This latter result is in a way reminiscent of Hirschleifer (1971, pg. 568). The ideas developed here may be of some use in interpreting the nondisclosure practices used by banks (issuers of high-velocity payment instruments) in the past. For example, Gorton (2009) notes that during U.S. National Banking Era (1863-1913) panics, private clearinghouses consisting of bank coalitions would temporarily suspend public disclosure of individual bank balance sheets. A similar motive may underlie recent calls to suspend "mark-to-market" accounting practices in the banking sector.

Of course, nondisclosure of particular types of information may not be practical in reality. When this is so, a natural private sector response is the creation of low-risk "informationally insensitive" debt that might serve as money; a theme pursued by Gorton and Pennacchi (1990). In fact, banks do go appear to go to great lengths in creating low-risk debt for this purpose.[7] Such an activity would appear to extend to the so-called "shadow-banking" sector; which oversaw the creation of "low-risk" securities (e.g., AAA tranches of asset-backed securities) used extensively as collateral in the repo market; see Gorton (2009).

It is unlikely, however, that any private security can be made completely risk-free. Through its power to tax, the state can lay claim over a far larger and more diversified portfolio of assets than any private agency. To the extent that such a concentration of power can be trusted, there may be a role for government money/debt to serve as the economy's primary medium of exchange.

[7] Gorton and Pennacchi (1990) develop a model that it relies on the presence of asymmetric information between "informed" and "uninformed" traders. In their environment, one solution to this problem is for a firm to split the cash flow of their asset portfolio between risky equity and risk-free debt. The debt instrument here is "informationally insensitive" in that its value is independent of any news received by informed traders. In this manner, uninformed agents can be induced to acquire and use debt for transaction purposes.

In contrast to the standard rationale for government money as a record-keeping device (Kocherlakota, 1998), the role highlighted here is in the ability of the government to create and manage an "informationally insensitive" security that might better serve as a high-velocity payment instrument.

7 References

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8 Appendix

Proof to Proposition 4

Consider conditions (32) and (33) of the text and write these as

$$\begin{aligned}
\phi_1 &= 0.5\pi\phi_2(b)h'(y(b))A(y(b)) + 0.5(1-\pi)\phi_2(g)h'(y^*) \\
v_1 &= 0.5\pi v_2(b)h'(y(b))A(y(b)) + 0.5(1-\pi)v_2(g)h'(y^*)
\end{aligned}$$

using the fact that $A(y^*) = 1$; see Lemma 2. Now use condition (34) to express these equations as

$$\begin{aligned}
2\phi_1 &= \beta\left\{\pi\left[z(b)+\phi_1\right]A(y(b)) + (1-\pi)\left[z(g)+\phi_1\right]\right\} \\
2v_1 &= \beta\left\{\pi v_1^+ A(y(b)) + (1-\pi)v_1^+\right\}
\end{aligned}$$

Divide through by ϕ_1 and v_1, respectively.

$$
\begin{aligned}
2 &= \beta\left\{\pi\left[\frac{z(b)+\phi_1}{\phi_1}\right]A(y(b))+(1-\pi)\left[\frac{z(g)+\phi_1}{\phi_1}\right]\right\} \\
2 &= \beta\left\{\pi\frac{1}{\mu}A(y(b))+(1-\pi)\frac{1}{\mu}\right\}
\end{aligned}
$$

This implies

$$
\pi\left[\frac{z(b)+\phi_1}{\phi_1}\right]A(y(b))+(1-\pi)\left[\frac{z(g)+\phi_1}{\phi_1}\right]=\pi\frac{1}{\mu}A(y(b))+(1-\pi)\frac{1}{\mu}
$$

Rearranging this latter equation,

$$
(1-\pi)\left[\frac{z(g)+\phi_1}{\phi_1}-\frac{1}{\mu}\right]=\pi A(y(b))\left[\frac{1}{\mu}-\frac{z(b)+\phi_1}{\phi_1}\right] \tag{A1}
$$

This implies

$$
\frac{z(g)+\phi_1}{\phi_1}>\frac{1}{\mu}>\frac{z(b)+\phi_1}{\phi_1} \tag{A2}
$$

where the latter inequality is used in (40).

Next, add the stated component to each side of (A1),

$$
(1-\pi)\left[\frac{z(g)+\phi_1}{\phi_1}-\frac{1}{\mu}\right]+\pi\left[\frac{z(b)+\phi_1}{\phi_1}-\frac{1}{\mu}\right]=\pi A(y(b))\left[\frac{1}{\mu}-\frac{z(b)+\phi_1}{\phi_1}\right]+\pi\left[\frac{z(b)+\phi_1}{\phi_1}-\frac{1}{\mu}\right]
$$

or

$$
\left[\frac{z_0+\phi_1}{\phi_1}-\frac{1}{\mu}\right]=\pi\frac{1}{\mu}\left[A(y(b))-1\right]-\pi\left[\frac{z(b)+\phi_1}{\phi_1}\right]\left[A(y(b))-1\right]
$$

With some manipulation, one arrives at equation (40),

$$
\left[\frac{z_0+\phi_1}{\phi_1}-\frac{1}{\mu}\right]=\pi\left[\frac{1}{\mu}-\frac{z(b)+\phi_1}{\phi_1}\right]\left[A(y(b))-1\right]
$$