# International Integration, Common Exposure and Systemic Risk in the Banking Sector: An Empirical Investigation\*

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Abstract

This paper analyzes the impact of ongoing financial and economic integration on banks'

common exposure to shocks and on banking sector systemic risk. For that, we study

the evolution of correlations between large international banks' asset-to-debt ratios over

1993-2006 and compute a systemic risk index for the same period. We find that banks'

common exposure to shocks has significantly decreased until 2000 and rapidly increased

afterwards. Systemic risk follows a totally different pattern. No trend emerges and,

instead, we observe two peaks: one in 1998 (LTCM and Russian crisis) and one in 2002-

2003 (stock market downturn). These findings suggest that, contrary to a widespread

belief, higher correlations between banks do not necessarily induce higher systemic risk.

We then provide evidence that systemic risk is mainly driven by banks' individual risk-

taking rather than by their common exposure to risks.

Keywords: Systemic risk, Co-movements, Banking sector, International integration.

JEL-Classifications: F36, G15, G21.

## 1 Introduction

Worldwide, the banking sector has gone through profound transformations over the last decades. Technical progress in financial engineering and in communications technologies as well as global deregulation policies have significantly modified the international financial landscape. In Europe, the launch of the Euro has contributed additionally to accelerate these changes.<sup>1</sup> An obvious outcome of these developments is an acceleration of international financial markets integration. At the firm level, financial institutions — and large banks in particular — now profit from a much easier access to a wider range of geographical markets and financial instruments. At the same time, some studies find enhanced synchronization in real activity—i.e., deeper economic integration—worldwide.<sup>2</sup>

The structural changes in the environment in which financial institutions operate have raised concern about an increase in systemic risk due to banks becoming more and more exposed to the same risk factors, i.e., due to an increase in the banks' common exposure to shocks. Yet, from a theoretical point of view, it is neither clear what overall impact on banks' common exposure to shocks and on systemic risk should be expected nor how banks' common exposure to shocks is related to systemic risk in the banking sector. Enhanced synchronization in business cycles worldwide reduces the set of diversifiable risks available to banks. This increases banks' individual risk as they become more vulnerable to shocks. In addition, the probability that the banks are exposed to the same risk factors—or to risk factors that are highly correlated with each other—increases. In other words: the banks' common exposure to shocks, i.e., the risk at the system level, increases. As both the risk at the system level and banks' individual risk rise due to deeper economic integration worldwide, the latter leads to higher systemic risk in the banking sector.

<sup>&</sup>lt;sup>1</sup>See Baele *et al.* (2004).

<sup>&</sup>lt;sup>2</sup>See, e.g., De Nicolò and Tieman (2006).

In contrast to higher correlation of business cycles, access to a wider range of markets and products widens the available set of diversifiable risks for individual banks. This has two possible consequences for the banks and the banking system as a whole, respectively. First, banks may use these new investment and financing opportunities to diversify their portfolio. If all banks choose to diversify, they are all exposed to the same risk factors. The banking sector as a whole becomes more homogeneous, the banks' common exposure to shocks increases.

Diversification, however, gives rise to higher monitoring costs. Therefore—instead of diversification—banks may, second, also choose to specialize in a certain field that appears promising. In doing so, banks are exposed to different risk factors. The banking system as a whole becomes more heterogeneous, the banks' common exposure to shocks decreases. Thus, depending on whether the banks in a banking system use their access to a wider range of market and products to rather diversify or to specialize, their common exposure to shocks may increase or decrease.

The overall impact of access to a wider range of markets and products on systemic risk is similarly indeterminate. If banks choose to use the additional investment and financing opportunities to diversify, their individual risk and therefore—ceteris paribus—the systemic risk increase. At the same time, however, diversification leads to an increase in the banks' common exposure to shocks. The banks in the system are all exposed to the same risk factors, which is harmful in terms of systemic risk. Hence, while diversification leads to a decrease in each bank's individual risk, it increases the risk at the system level. The analoguous holds for the case when banks choose to specialize: Their individual risk increases while the risk at the system level decreases. As in the case with the banks' common exposure to shocks, the overall impact on systemic risk is difficult to determine a priori. This ambiguity remains even in the case where it is known if banks have rather chosen to diversify—i.e., if their common exposure to shocks has increased—or to specialize—i.e., if their common exposure to shocks

has decreased. It is not obvious how banks' common exposure to shocks is related to systemic risk. To summarize: From a theoretical point of view, it remains unclear (i) whether the observed structural changes in the banking sector's environment have increased or decreased banks' common exposure to shocks, (ii) whether they have increased or decreased systemic risk and (iii) how common exposure and systemic risk are interrelated.

In this paper, we try to clear up these ambiguities with empirical data by studying a panel of large international banks over a period running from 1993 through to 2006. Concretely, we intend to provide an answer to the following questions: (1) what was the impact of international—both economic and financiel—integration on banks' common exposure to shocks between 1993 and 2006? (2) What was the impact of international integration on systemic risk in the international banking sector between 1993 and 2006? (3) Is there an empirical link between common exposure to shocks and systemic risk during the period under consideration? As already pointed out above, the first two questions are not identical. Monnin (2004) also shows with a simple numerical example that a higher common exposure to shocks does not necessarily imply higher systemic risk. Indeed, an increase in common exposure can be compensated by a decrease in banks' total exposure, making the overall systemic risk to decrease. This situation occurs, for example, when two banks switch from independent risky strategies (no common exposure, very high individual risk) to a common safe strategy (complete common exposure, but to a very low risk).

To estimate the impact of international integration on common exposure, we analyse the evolution of the co-movements – i.e. correlation – between banks' asset-to-debt ratios (AD ratios henceforth). As shown in Section 2.1, shocks to AD ratios sum up both shocks to banks' assets and liabilities as well as their impact on each other. AD ratio movements can thus be considered as a good summary of movements in banks' overall condition. A high (low) correlation between AD ratios suggests that both

banks are similarly (differently) affected by shocks and thus that they have a high (low) common exposure to shocks.

To estimate the impact of international integration on systemic risk, we compute a systemic risk index based on Lehar (2005) and study its evolution during the period 1993-2006. Lehar's index measures the probability of observing a systemic crisis—defined as a given number of simultaneous bank defaults—in the banking sector at a given point in time. We finally try to see if there is a link between the evolution of banks' common exposure (i.e. correlation between AD ratios) and systemic risk.

Several studies have analyzed co-movements in banks' characteristics. DeNicolo and Kwast (2002) find a significant rise in stock return correlations between large US banking institutions during the 1990s. In a similar study for the European Union, Schröder and Schüler (2003) show that the correlations between 13 national bank stock indices rose significantly in the last years. Brasili and Vulpes (2005) draw a similar conclusion when studying the correlations between European banks' distance-to-defaults. Hawkesby, Marsh and Stevens (2005) analyze the correlations between equity returns (and between credit default swaps premia) for a sample of European and US large and complex financial institutions (LCFI). They find a relatively high degree of comovements in asset prices of LCFIs compared to a control group of non-financials. However, their results also show that a considerable degree of heterogeneity remains between different sub-groups of the sample: There exists, e.g., a divide between European and US banking institutions. The general conclusion from these studies is that co-movement between banks has increased in the last decade, which suggests that banks are becoming increasingly similar with time and face more and more the same risks. Most of these studies conjectured that the observed increase in co-movements leads to higher systemic risk.

Our paper differs from the previous studies in two ways: firstly, we use a new method based on Ledoit, Santa-Clara and Wolf (2003) to estimate the joint dynamic of the AD ratios as a whole (i.e. for all banks at the same time), whereas previous studies concentrated on the dynamic between pairs of banks. The resulting time-varying covariance matrix can then be used directly in the computation of the systemic risk index as well as for computing the evolution of correlation between banks. Secondly, and this is our main contribution, we study in detail the link between common exposure and systemic risk. We assess if a higher common exposure to shocks (i.e. higher correlation) is associated with higher or lower systemic risk or if it does not play any role for systemic risk. As mentioned before, other studies often claim that higher correlation yields to higher systemic risk, without formally checking this assumption.

The main results of our analysis are the followings. Firstly, we find that the correlation between banks' AD ratios decreases in the first part of the sample period, and increases after 2000. This suggests that banks have rather specialized—and thus reduced their common exposure to shocks—than diversified their portfolios in response to changes in the banking sector environment until 2000. After this date, however, the banks seem to become increasingly similar and their common exposure rises. This finding holds for the whole sample as well as for different sub-groups (namely North America and European Union). However, the degree of common exposure differs between these groups. Correlations between North-American banks tend to be higher than between their European counterparts. Co-movements between US and European banks are far less pronounced than within each regional sub-group, suggesting that these two groups are (at least partially) exposed to different shocks.

Secondly, in contrast to the correlation analysis, we do not find any significant trend in the systemic risk index. The latter is rather characterized by two peaks at the end of 1998 and at the end 2002 and the beginning of 2003. These two periods correspond to two well-known episodes of high level of stress on the banking sector: the LTCM and the Russian crisis at the end of 1998 and a persistent downturn on stock market in 2002-2003. Taking a closer look on different sub-groups, we find that

the high level of systemic risk in 2002-2003 is mainly attributable to European banks suffering additionally from bad economic conditions in the European economy. In the US sample, the crisis period 2002-2003 is less pronounced but an additional (however less explicit) crisis period is detected around 1994-1995.

Thirdly, our results point out that correlation between banks is not a reliable measure of systemic risk. The link between correlation and our systemic risk index is weak and its direction can change depending on the period considered. On the contrary, the distance-to-default, which is a combination of AD ratio's volatility and level, turns out to be a very reliable factor explaining the systemic risk index. In other words, systemic risk seems to be the consequence of each banks' individual risk taking (i.e., its distance-to-default) rather than of the banks' common exposure to shocks (correlation). This finding warns against viewing systemic risk as a pure correlation phenomenon and highlights the danger of high and volatile leverage at the individual bank level. Note, however, that once the effects of the distance-to-default are taken into account, we find that correlation is positively associated with systemic risk. In other words, for a given level of individual risk, a higher common exposure implies a higher systemic risk.

The paper is structured as follows: Section 2 explains the methodology used to estimate the correlation dynamics and the systemic risk index. The data used is described in Section 3. Section 4 studies the correlation dynamics between large international banks' AD ratios. Section 5 presents the estimated systemic risk index and compares its evolution with the correlation dynamics. Section 6 gives our conclusions and recommendations for banking sector supervisors.

## 2 Methodology

As mentioned in the previous section, we need two ingredients to answer the questions at the centre of this paper: the evolution of the correlations between the banks' AD ratios and the one of the systemic risk index. To get them, we proceed in four steps:

(1) we make some assumptions about bank asset and debt dynamics, (2) we use these assumptions to recover the AD ratios from observable equity and debt data using Merton's method,<sup>3</sup> (3) we estimate the joint dynamic of the AD ratios, including the dynamic of their covariances, using a multivariate GARCH model and (4) we use the estimated dynamic to compute the systemic risk index. The next four sections describe these steps in detail. In addition, the next section shows why shocks to AD ratios sum up both shocks to bank assets and debts as well as their effects on each others.

## 2.1 Asset and debt dynamic

The AD ratio is defined as the ratio between the asset market value of a bank and its debt face value. Unfortunately, asset market values are not directly observable, but, following Merton (1974), they can be estimated by modelling the bank's equity as a call option on the bank's assets. However, to recover the AD ratios from observed equity prices with Merton's technique, we have to assume that asset market values and debt face values follow a multivariate Itô process such as

$$\begin{bmatrix} d\mathbf{A}_t \\ d\mathbf{D}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_A \\ \boldsymbol{\mu}_D \end{bmatrix} dt + \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\Upsilon}_{AA} & \boldsymbol{\Upsilon}_{AD} \\ \boldsymbol{\Upsilon}_{DA} & \boldsymbol{\Upsilon}_{DD} \end{bmatrix} \begin{bmatrix} d\mathbf{w}_A \\ d\mathbf{w}_D \end{bmatrix}$$
(1)

We have that  $d\mathbf{A}_t$  and  $d\mathbf{D}_t$  are  $(N \times 1)$  vectors containing the instantaneous change in assets  $A_t^i$  and debts  $D_t^i$  of all banks, respectively. N is the number of banks.  $\mathbf{A}_t$  and  $\mathbf{D}_t$  are  $(N \times N)$  matrices containing the assets  $A_t^i$  and the debts  $D_t^i$  in their respective diagonals, all other elements being zeros.  $\boldsymbol{\mu}_A$  and  $\boldsymbol{\mu}_D$  are  $(N \times 1)$  vectors regrouping the constant instantaneous growth rate of banks' assets and debts, respectively.  $d\mathbf{w}_A$ and  $d\mathbf{w}_D$   $(N \times 1)$  are vectors of independent Wiener processes. They represent the individual shocks to assets and debts of each bank at time t. Finally  $\boldsymbol{\Upsilon}_{AA}$ ,  $\boldsymbol{\Upsilon}_{AB}$ ,  $\boldsymbol{\Upsilon}_{BA}$ 

<sup>&</sup>lt;sup>3</sup>The AD ratios are not directly observable because the asset market value is not directly observable.

and  $\Upsilon_{BB}$  are  $(N \times N)$  matrices which regroup the instantaneous responses of assets and debts to the different shocks. For example,  $v_{AD}^{ij}$  (the ij-th element of  $\Upsilon_{AD}$ ) is the instantaneous response of the bank i asset value to a shock in bank j debt.

The diagonals of the matrices  $\Upsilon_{AA}$  and  $\Upsilon_{DD}$  are the direct responses of the bank's assets and debt to their own shocks. The diagonal of the matrix  $\Upsilon_{AD}$  ( $\Upsilon_{DA}$ ) is the direct response of a bank's assets (debt) to a shock affecting its own debt (assets). All other elements are the indirect responses of a bank's assets and debt to shocks affecting other banks' assets and debt. They represent the contagion effects between banks through interlinkages. Note that we do not assume any symmetrical response between banks or between asset and debts. For example, the correlation between assets and debt can be different according to whether the shock affects the assets or the debt. Similarly, the response of bank i to a shock to bank j can be different from a bank j's response to bank i's shock.

Without loss of generality (see Appendix A), we can rewrite equation (1) in the reduced form

$$d\mathbf{z}_t = \boldsymbol{\mu}dt + \boldsymbol{\Upsilon}d\mathbf{w} \tag{2}$$

where

$$\Upsilon\Upsilon'=\Omega$$

 $\mathbf{z}_t$  is the  $(n \times 1)$  vector regrouping the log AD ratios  $z_t^i = \ln{(A_t^i/D_t^i)}$ ,  $\boldsymbol{\mu}$  is a  $(n \times 1)$  vector of instantaneous drifts in log AD ratios and  $\boldsymbol{\Omega}$  is the  $(n \times n)$  variance-covariance matrix between instantaneous changes in log AD ratios. As shown in Appendix A, the matrix  $\boldsymbol{\Upsilon}$  sums up all interactions between banks' assets and debts.

#### 2.2 Recovering AD ratios from equity prices

Merton (1974) first suggested to model the bank's equity as a call option on the bank's assets to compute the bank's default probability estimated by market participants. This method can also be used to recover the bank's AD ratio from the equity price. Merton's method is based on the fact that if, at debt's maturity time t + T, the value of the bank's assets is smaller than its debt  $(A_{t+T}^i < D_{t+T}^i)$ , then it is not rational for the shareholders to exercise the option, i.e., they will make the bank default. If the bank defaults, the value of the equity is then zero. Thus Merton's model states that the value of bank i's equity at time t + T is:

$$E_{t+T}^{i} = \max\left(A_{t+T}^{i} - D_{t+T}^{i}, 0\right) \tag{3}$$

where  $E_t^i$  is the bank's stock price.

An equity with such payoffs is similar to an exchange option<sup>4</sup>. If both the assets and the debt are log normally distributed, as stated in Equation (1), its initial value can easily be computed (Margrabe 1978) and is equal to:

$$E_t^i = A_t^i \Phi\left(d_1\right) - D_t^i \Phi\left(d_2\right) \tag{4}$$

with

$$d_1 = \frac{z_t^i + \left(\sigma_{z_t^i}^2/2\right)T}{\sigma_{z_t^i}\sqrt{T}}$$

$$d_2 = d_1 - \sigma_{z_t^i} \sqrt{T}$$

where  $\Phi\left(\cdot\right)$  is the cumulative normal distribution and  $\sigma_{z_{t}^{i}}^{2}$  is the conditional variance of

<sup>&</sup>lt;sup>4</sup>Exchange options are sometimes also referred to as options to exchange one asset for another.

the log AD ratio. Dividing both sides of Equation (4) by  $D_t^i$  yields

$$X_t^i = Z_t^i \Phi\left(d_1\right) - \Phi\left(d_2\right) \tag{5}$$

where  $X_t^i$  is the equity-to-debt ratio of bank i at time t and  $Z_t^i$  is its AD ratio at time t. Using Itô's lemma, we have that

$$\sigma_{x_t^i} X_t^i = \sigma_{z_t^i} Z_t^i \Phi\left(d_1\right) \tag{6}$$

where  $\sigma_{x_t^i}^2$  is the conditional variance of the log equity-to-debt ratio.

Bank i's debt  $D_t^i$  and equity price  $E_t^i$  are directly observable. With them, we can form the equity-to-debt ratio  $X_t^i$  and compute its conditional variance  $\sigma_{x_t^i}^2$ . With  $X_t^i$  and  $\sigma_{x_t^i}$  known, we are left with two unknown variables in Equation (5) and (6): the AD ratio  $Z_t^i$  and its conditional variance  $\sigma_{z_t^i}$ . The AD ratio  $Z_t^i$  can be recovered by simultaneously solving Equation (5) and (6) using a numerical iterative process.<sup>5,6</sup>

Note that this method gives the correct AD ratio only if the market participants correctly interpret the information they have about the banks. In particular, they should understand correctly the inter-dependence between banks and integrate it in their valuation. Note also that this method is not the only one that can be used to recover the AD ratio.<sup>7</sup> However, Hovakimian and Kane (2000) show that the differences in the AD ratio valuations given by these different methods are small.

<sup>&</sup>lt;sup>5</sup>Note that only the "marginal" variance of the equity-to-debt ratios is necessary to recover the AD ratios. In particular, this technique does not require to know the correlations between the banks' equities (see Zhou (2001) for another illustration with a bivariate model). Therefore, this technique can be applied separately to each bank.

<sup>&</sup>lt;sup>6</sup>Other techniques are available to recover the market asset value. Duan (1994) estimates it by maximizing a likelihood function. Vassalou and Xing (2004) use an iterative process that does not require the second equation.

<sup>&</sup>lt;sup>7</sup>See for example, Bichsel and Blum (2004) for a model without stochastic debt, Ronn and Verma (1986) for a model with forbearance or Saunders and Wilson (1995) for a model with an infinite-maturity put.

#### 2.3 Estimation of the AD ratio joint dynamics

The next step is to estimate the dynamic process in Equation (2) with empirical data. We choose to model the AD ratio dynamic with a multivariate GARCH model (M-GARCH). The main advantage of this kind of model is that it allows for *time-varying* variances and covariances. This is necessary in our context since we are interested in the evolution of the correlation through time.

The equivalent of Equation (2) in discrete time is:

$$\Delta \mathbf{z}_{t} = \boldsymbol{\mu}_{z} + \mathbf{u}_{t}$$

$$\mathbf{u}_{t} \sim N(0, \mathbf{H}_{t})$$

$$(7)$$

where  $\Delta \mathbf{z}_t$  is a  $(N \times 1)$  vector containing the changes in log AD ratios  $\Delta z_t^i$ ,  $\boldsymbol{\mu}_z$  is a  $(N \times 1)$  vector of constants,  $\mathbf{u}_t$  is a  $(N \times 1)$  vector of white noise residuals and  $\mathbf{H}_t$  denotes the  $(N \times N)$  conditional variance-covariance matrix of the residuals. We use the diagonal *Vech* model (Bollerslev, Engle and Wooldridge 1988) specification for the dynamic of  $\mathbf{H}_t$ . In this model, the conditional covariance  $h_t^{ij}$  between bank i and bank j depends only on the past covariance and the past residuals:

$$h_t^{ij} = \omega_{ij} + \alpha_{ij} u_{t-1}^i u_{t-1}^j + \beta_{ij} h_{t-1}^{ij}$$
(8)

The model has the property that when two banks experience trouble at the same time, their correlation increases and they will tend to move together in the future. This property is important as it captures the empirical finding that the correlation between banks seems to increase during bad periods. Yet, correlation will also increase when both banks are hit by a positive shock; i.e., the covariances behave symetrically.

In matrix form, Equation (8) has the following representation

$$\mathbf{H}_{t} = \mathbf{C} + \mathbf{A} \otimes \mathbf{u}_{t-1} \mathbf{u}'_{t-1} + \mathbf{B} \otimes \mathbf{H}_{t-1} \tag{9}$$

where the coefficient matrices  $\mathbf{C}$ ,  $\mathbf{A}$  and  $\mathbf{B}$  are  $(N \times N)$  matrices regrouping the parameters  $\omega_{ij}$ ,  $\alpha_{ij}$  and  $\beta_{ij}$ , respectively. The symbol  $\otimes$  denotes the Hadamar product of two matrices.<sup>8</sup> The covariance matrix  $\mathbf{H}_t$  has some distinctive characteristics to be respected by the estimation method. First,  $\mathbf{H}_t$  is symmetric implying that  $\mathbf{C}$ ,  $\mathbf{A}$  and  $\mathbf{B}$  must also be symmetric. Secondly,  $\mathbf{H}_t$  is a positive semidefinite matrix and thus, any estimation of it must also be positive semidefinite.

A natural way to estimate  $\mathbf{H}_t$  and the coefficient matrices  $\mathbf{C}$ ,  $\mathbf{A}$  and  $\mathbf{B}$  seems to use maximum likelihood estimates as it is usually done for univariate GARCH models. Unfortunately, this is not feasible because (i) the parameters are too numerous and so intricately linked that existing optimization algorithms do usually not converge, and (ii) maximum likelihood estimation does not necessarily give positive semidefinite covariance matrices. To cope with this second problem, econometricians usually impose additional conditions on the model coefficients to ensure that the matrix  $\mathbf{H}_t$  is positive semidefinite. In addition to the fact that such restrictions might not make sense from an economic point of view, Kroner and Ng (1998) have shown that M-GARCH results are very sensitive to different specifications.

We choose to follow a different approach to estimate the coefficient matrices **C**, **A** and **B**: the Flexible M-GARCH method developed by Ledoit, Santa-Clara and Wolf (2003). This procedure has the advantage to solve both problems previously mentioned without imposing a priori restrictions on the coefficients. It is based on a decentralized estimation of the coefficients. Ledoit et al. propose to estimate the coefficient matrices **C**, **A** and **B** in two steps. In the first step, each coefficient of the

<sup>&</sup>lt;sup>8</sup>The Hadamar product is the elementwise product of two matrices:  $\mathbf{U} \otimes \mathbf{V} = (u_{ij}v_{ij})$ .

<sup>&</sup>lt;sup>9</sup>See Ding and Engle (2001) for a recent comparison of the restrictions used by different models.

matrix is independently estimated with a univariate or bivariate GARCH model. Thus, the estimation of a large matrix is reduced to several univariate and bivariate problems for which conventional univariate and bivariate GARCH estimation techniques are easy to apply. As indicated above, the resulting estimated coefficient matrices  $\hat{\mathbf{C}}$ ,  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{B}}$  will not necessarily ensure the variance-covariance matrix to be positive semidefinite. Thus, in a second step, Ledoit *et al.* apply a result from Ding and Engle (2001) stating that positive semidefinite coefficient matrices are a sufficient condition to yield (almost surely) a positive semidefinite variance-covariance matrix. The second part of the procedure is thus to find the positive semidefinite matrices  $\tilde{\mathbf{C}}$ ,  $\tilde{\mathbf{A}}$  and  $\tilde{\mathbf{B}}$  that are the closest to the  $\hat{\mathbf{C}}$ ,  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{B}}$  initial matrices.<sup>10</sup> The coefficient matrices  $\tilde{\mathbf{C}}$ ,  $\tilde{\mathbf{A}}$  and  $\tilde{\mathbf{B}}$  given by the Flexible M-GARCH estimation are then used to compute the conditional variance-covariance dynamic with Equation (8) and, in the next step, to compute the systemic risk index.

## 2.4 Construction of the systemic risk index

Our systemic risk index follows Lehar (2005). The index is an estimation of the *probability of a systemic banking crisis* at time t. In this paper, a systemic crisis is defined as follows:

**Definition 1** A systemic crisis occurs when a percentage  $\theta$  of the banking system becomes insolvent within the next k periods.

Definition 1 requires another definition, which specifies under which conditions a bank defaults.

**Definition 2** The bank i defaults if the market value  $A_t^i$  of its assets falls below the face value  $D_t^i$  of its debt within the next k periods (i.e.  $A_{t+j}^i < D_{t+j}^i$  for at least one  $j \in [0, k]$  or equivalently  $Z_{t+j}^i < 1$  for at least one  $j \in [0, k]$ ).

 $<sup>^{10}</sup>$ The positive semidefinite matrice  $\mathbf{X}$ , which is the closest from the initial matrice  $\mathbf{Y}$ , can be found by using a simple algorithm developed by Sharapov (1997).

Given Definitions 1 and 2, the systemic risk index can be expressed as:

$$I_{t}(\theta) = \Pr\left[\sum_{i=1}^{N} \theta_{t}^{i} b_{t}^{i} > \theta\right]$$

$$(10)$$

where N is the number of banks,  $\theta_t^i$  is the weight<sup>11</sup> of bank i in the banking system at time t and

$$b_t^i = \begin{cases} 1 & \text{if } A_{t+j}^i < D_{t+j}^i \text{ for at least one } j \in [0, k] \\ 0 & \text{otherwise} \end{cases}$$

 $b_t^i$  is a dummy variable that takes the value 1 if bank i goes bankrupt in the next k periods. There are various ways to determine a bank's weight  $\theta_t^i$  in the banking sector (e.g. equal weight, proportion of a bank's assets in the total banking sector's assets, proportion of a bank's interbank deposits in the total system, etc.). We chose to give an equal weight to each bank.

Unfortunately, it is not possible to compute analytically the probability of a systemic crisis. To estimate it, we proceed with a Monte-Carlo simulation based on the estimated AD ratio joint dynamics described in section 2.3. An alternative would have been to use approximation techniques such as the one proposed by Carmona and Durrleman (2004). The Monte-Carlo simulation, yet, has the advantage of being easier and faster to implement and more flexible for further developments (e.g. modification of the dynamic or explicit modelling of contagion effects). A description of the simulation algorithm to estimate the systemic risk index is given in Appendix B. We set k = 12 (one year) in this paper.

The individual weights are normalized such that  $\sum_{i=1}^{N} \theta_i = 1$ .

## 3 Data

The data needed to compute the correlation between AD ratios and to construct the systemic risk index consist of individual banks' balance sheet data (debt) and market information (equity prices). Data on debt is taken from Bloomberg while equity prices stem from Datastream. As data on debt is not available on a monthly basis, quarterly and—for some banks—yearly data have been transformed into monthly data by linear interpolation.

We constructed two different datasets. The first dataset comprises monthly data on 27 large international banks from November 1992 until June 2006 (long sample).<sup>12</sup> The second dataset (short sample) comprises data on a total of 39 large international banks—including the 27 institutions already represented in the first dataset—from June 1997 until June 2006.<sup>13</sup>

## 4 Common exposure to shocks

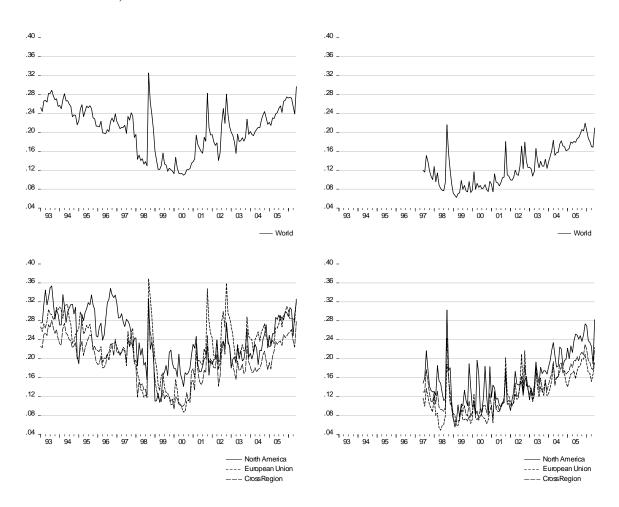
To get an idea of how banks' common exposure to shocks has evolved in time, we try to identify a potential common trend in AD ratio correlations between pairs of banks. A high AD ratio correlation indicates that two banks are both equally affected by a shock, i.e., that they have a high common exposure to shocks. Thus, if we observe a common upward trend in all correlations, we can conclude that, in the aggregate, banks' common exposure has increased and that banks have become more similar.

We can get a first idea on the trend in correlation by observing the evolution of the average correlation (cf Figure 1). In both samples, the average correlation decreases until about 2000 and then increases regularly. This pattern is also observed in the average correlation between pairs of North American banks, pairs of European Union

<sup>&</sup>lt;sup>12</sup>The long sample consists of banks from: Germany (3), France (1), Italy (2), Netherlands (2), Spain (2), Sweden (2), Switzerland (1), UK (3), USA (2), Canada (5), Australia (4).

<sup>&</sup>lt;sup>13</sup>The short sample consists of banks from: Belgium (3), Germany (3), France (2), Italy (3), Netherlands (2), Spain (2), Sweden (2), Switzerland (1), UK (5), USA (7), Canada (5), Australia (4).

FIGURE 1: CORRELATION BETWEEN AD RATIOS (LEFT: LONG SAMPLE, RIGHT: SHORT SAMPLE)



banks and between pairs of banks from each sub-group. This is a first indication that, in the aggregate, banks' common exposure to shocks has decreased until about 2000 and then increased.

To study more precisely this hypothesis, we estimate the common trend to all correlations in a panel data analysis. We then test for a break in the slope of the common trend. Concretely, we estimated the following system of equations

$$y_t^{ij} = \gamma + \delta t + u^{ij} + \varepsilon_t^{ij} \tag{11}$$

Table 1: Test for a break in the common trend and estimated slopes of the trend

Sample	Region	$\sup_{\text{stat}} F$ -	<i>p</i> -value	Break date	$\gamma$	$\delta$ before break	$\delta$ after break
1993-2006	World	1604.05	<1%	2000.04	0.5918	-0.0034	0.0033
	European Union	491.46	<1%	1999.12	0.6742	-0.0043	0.0040
	North America	100.82	<1%	2000.06	0.6845	-0.0031	0.0026
	Cross	362.65	<1%	2000.01	0.5593	-0.0032	0.0028
1997-2006	World	644.99	<1%	1999.10	0.2488	-0.0026	0.0027
	European Union	114.28	<1%	1999.08	0.2308	-0.0025	0.0027
	North America	199.37	<1%	2001.11	0.3084	-0.0011	0.0049
	Cross	276.64	<1%	1999.12	0.2698	-0.0028	0.0030

All coefficients are signficant at a 1% confidence level. A supF-statistics greater than 16.64 indicates that the null hypothesis of no break is rejected at a 1% confidence level.

where  $y_t^{ij}$  is a logit transformation<sup>14</sup> of the correlation between bank i and j at time t,  $\gamma$  is the average (logit) correlation,  $\delta$  is the slope of the common trend,  $u^{ij}$  is a fixed effect particular to each pair and  $\varepsilon_t^{ij}$  is a independant heteroskedastic error term, which is normally distributed with variance  $\sigma_{ij}^2$ . We then test the hypothesis of a break at time  $t_0$  in the slope  $\delta$  of the trend. We used the test developed by Bai and Perron (1998 and 2003) which simultaneously estimates the most probable break date and then tests if the break is statistically significant. We do the estimation and the test for both sample and for the entire set of correlations, for correlations between North American banks, for correlations between EU banks and for correlations between North American and EU banks. The results are presented in Table 1.

The estimated break dates lie between August 1999 and June 2000 (with an exception of a break date of November 2001 for the North American banks in the short sample). It indicates that a change in banks' common exposure occurred around the beginning of this century. Before this date, the trends are negative, which implies that the common exposure to shocks had a tendency to decrease. This suggests that, during this period, banks have rather chosen to specialize than to diversify their porto-

<sup>14</sup>More precisely,  $y_t^{ij} = \ln\left(q_t^{ij}/\left(1-q_t^{ij}\right)\right)$  where  $q_t^{ij} = \left(cor\left(i,j\right)+1\right)/2$ . This transformation insure that  $y_t^{ij}$  is distributed over  $]-\infty; +\infty[$  whereas the correlation is bounded between -1 and 1.

folios. After 2000, the trends reverse and common exposure to shocks increases, hinting at increasing similarities or interdependencies between banks. An increase in banks' co-movements since 1999 is also documented by Brasili and Vulpes (2005).

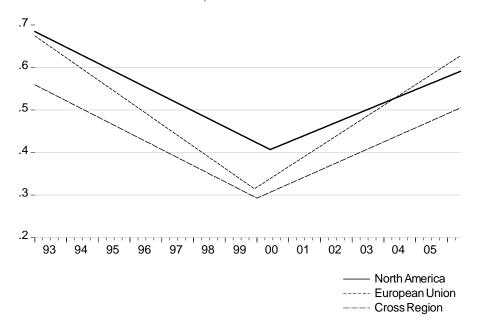
This trend reverse is observed in both regions and between these regions. Figure 2 displays the estimated trend for correlation between North American banks, between EU banks and between banks of each region. Except for the end of the sample, the correlation between North American banks is higher than between EU banks. North American banks seem to be more commonly exposed to shocks, or more homogeneous, than EU banks. The correlation between EU and North American banks is the lowest, indicating that banks from different regions are less commonly exposed to shocks or more heterogeneous. This result is in line with Hawkesby, Marsh and Stevens (2005), who find a high degree of heterogeneity between both sub-groups and a higher correlation between US banks. Note finally that the slope of the EU banks is steeper in both phases. EU banks have specialized more strongly during the pre-2000 period and then have become more similar than North American banks, to the point that the correlation between EU banks seems higher now than between US banks.

# 5 Evolution of systemic risk

Do the changes observed in the banking industry in the past years have any impact on systemic risk? In particular, does the increase in common exposure to shocks observed since 2000 generate a higher systemic risk? To answer these questions, we constructed a set of indices of systemic risk. These indices are based on Lehar (2005). They reflect the probability to observe a systemic crisis in the banking sector (cf. Section 2.4). We computed a global systemic risk index for all the banks in our sample and two regional sub-indices, one for North American banks and one for EU banks. For all three regions

The trend for the short sample are not presented here but can be obtained by the authors. The conclusions are similar.

FIGURE 2: ESTIMATED COMMON TRENDS IN AD RATIOS CORRELATION (LONG SAMPLE, IN LOGIT TRANFORMED UNITS)



(World, North America, EU), we computed two indices: one for which a default of 10% of the banking sector triggers a crisis, one for which 20% of the banking sector must default to trigger a crisis. We computed these 6 indices for both samples (cf. Section 3), which makes a total of 12 systemic risk indices. These indices are presented in Figure 3.

The global index points out two periods of high systemic risk: at the end of 1998 and at end 2002 until beginning 2003. These two episodes correspond to the LTCM and Russian crisis in 1998 and to the stock market downturn in 2002-2003. The systemic risk during the rest of the sample is less acute. The 1998 peak is observed in both the US and the EU sub-indexes. The EU banks seem to have been more affected than the North American banks in 2002-2003, probably because they were also facing bad economic conditions at that time. In the US banking system, a period of higher systemic risk is also observed in 1994-1995, which translates by a slightly higher systemic risk in the global index.

FIGURE 3: SYSTEMIC RISK INDEX (LEFT: LONG SAMPLE, RIGHT: SHORT SAMPLE)

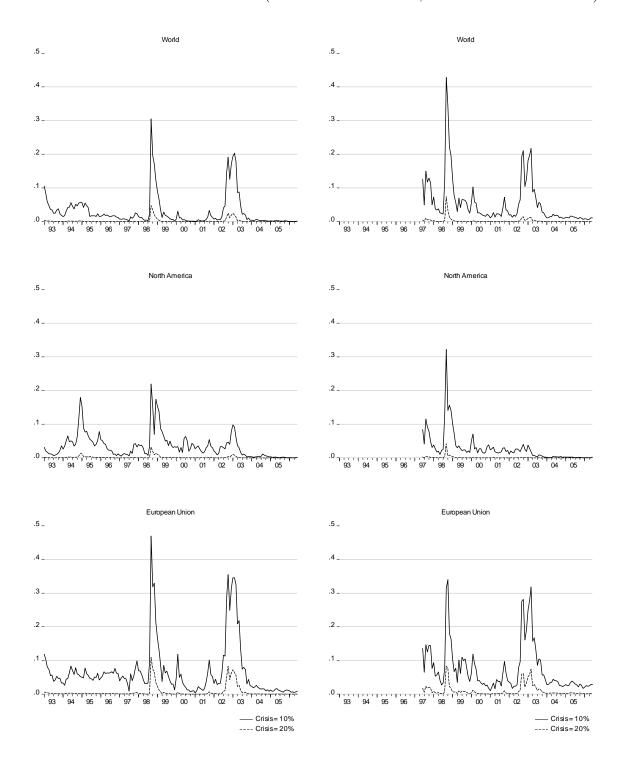


Table 2: Trend in the systemic risk index

Sample	Index	Trend before 2000	Trend after 2000
1993-2006	World 10%	-0.0196**	0.0109
	World $20\%$	-0.0233**	0.0034
	North America $10\%$	0.0075	-0.0110
	North America $20\%$	0.0018	-0.0079
	EU $10\%$	0.0031	0.0019
	EU $20\%$	0.0022	0.0142
1997-2006	World 10%	-0.0151	-0.0076
	World $20\%$	-0.0435	0.0012
	North America $10\%$	-0.0223	-0.0343**
	North America $20\%$	-0.0397	-0.0118**
	EU $10\%$	-0.0120	-0.0030
	EU $20\%$	-0.0299	-0.0004

<sup>\* (\*\*)</sup> indicates that the coefficient is significant at a 5% (1%) confidence level. The percentage associated to each index corresponds to the proportion of the banking system that need to default to trigger a crisis (e.g. World 10% corresponds to the systemic risk index for which a 10% of the banking system has to default to trigger a crisis).

#### 5.1 Trends in the systemic risk index

A quick look at Figure 3 suggests that the path of the systemic risk index is very different from the evolution of banks' common exposure to shocks presented in Figure 1. The latter has a distinct V-shape, whereas the former is characterized by two peaks of higher systemic risk for the banking sector. This visual impression is confirmed when we try to fit a trend with a break to the systemic risk index at the beginning of 2000 (which corresponds to the break date observed in the correlation trend). The results of this regression are presented in Table 2. Since the index is a probability bounded between 0 and 1, we estimate the coefficients with a logit regression using weighted least squares as suggested by Greene (2000).<sup>16</sup>

Most indices do not display any significant trend. Only the global and the North American systemic risk indices seem to have significantly decreased before 2000 in the long sample and after 2000 in the short sample, respectively. Furthermore, no significant break date is detected by the Bai and Perron test for any of the indices. This result contrasts with the unambiguous trends and breaks observed in the AD ratio

<sup>&</sup>lt;sup>16</sup>All regressions in this section are made using this method.

correlation: While a clear V-shaped trend appears in the dynamic of banks' common exposure to shocks, no apparent trend is detected in the systemic risk index' pattern. Moreover, the slope of the significant trend observed after 2000 for the North American systemic risk index does not correspond to the sign that one would a priori expect (i.e. the systemic risk decreases whereas the common exposure to shocks increases).

Many other studies record similar results as ours for banks' common exposure to shocks or banks' co-movements. Most of them conclude, without explicitly checking it, that an increase in co-movements induces a higher systemic risk. However, given our results for the systemic risk index, the existence of the link between co-movements or common exposure to shocks and systemic risk is ambiguous. The next section studies this question in more detail.

#### 5.2 Are common exposure and systemic risk related?

The results of the previous section raise questions about the existence of a link between banks' common exposure to shock (i.e. AD ratio correlations) and systemic risk in the banking sector. Do common exposures really play a role for systemic risk? How can we interpret a change in common exposure in terms of systemic risk? From the construction of the systemic risk index (see Section 2.4), it is obvious that three elements determine its value: 1) the correlation structure between banks' AD ratios, 2) the volatilities of the AD ratios and 3) the level of the AD ratios. While the first component captures the systemic characteristics of a banking sector, the last two components are bank-specific. Combined in the distance-to-default<sup>17</sup>, they describe bank's individual risk taking. The systemic risk index is a function of these systemic and bank-specific dimensions. Unfortunately, we do not know the exact form of this function. We can guess though that it is likely to be non linear.

To get an idea about each factor's influence on the systemic risk index, we compute

<sup>&</sup>lt;sup>17</sup>The distance-to-default is equal to the level of the AD ratio divided by its volatility.

the rank correlation between the systemic risk index and (i) the banks' AD ratio correlations and (ii) the banks' distance-to-default. The rank correlation statistics are preferred to the traditional (linear) correlation (Pearson coefficient) because they measure the link between two variables independently of the form taken by the function linking them. We use both the Spearman rank-order correlation coefficient and the Kendall measure of correlation to compute the rank correlation. We start by computing the rank correlation between the average correlation (or average distance-to-default) and the systemic risk index. Note, however, that it is difficult to adequately reflect the complete correlation structure (or distance-to-default structure) in one single measure such as the average. In particular, it is possible that the systemic risk index might be mainly influenced by extreme values of correlations or distance-to-defaults (i.e., by banks that are extremely commonly exposed or extremely close to default). To check for that, we also use the 75% (25%) and the 90% (10%) percentiles of the correlations (distance-to-defaults). The evolution of the average and centiles of correlations and distance-to-defaults are displayed in Figure 4. The rank correlation between the systemix risk index and these different measures are presented in Table 3.<sup>19</sup>

The results for the rank correlation show that the link between systemic risk and banks' common exposure (i.e., AD ratio correlation) is ambiguous. A positive relationship is identified in the long sample, while the same relationship appears to be negative in the short sample. On the opposite, the link between systemic risk and distance-to-default is always negative. Moreover, the rank correlation between systemic risk and distance-to-default is always stronger than the one between systemic risk and common exposure. We thus draw the following main conclusion: Low distance-to-default is a much stronger and much more reliable sign of systemic risk than high correlation. The effect of banks' common exposure to shocks on systemic risk is weaker and can even

<sup>&</sup>lt;sup>18</sup>Spearman rank-order correlation coefficient measures the *linear* correlation between the *ranks* of each observation. Kendall's  $\tau$  is even more nonparametric since it uses the relative ordering of the data, without assuming any linear relation at any point of its computation.

<sup>&</sup>lt;sup>19</sup>We present the results for the World index only. The results for the regional systemic risk indices can be obtained by the authors. The conclusions do not differ from those presented here.

FIGURE 4: EVOLUTION OF THE AD CORRELATIONS AND THE DISTANCE-TO-DEFAULTS (LEFT: LONG SAMPLE, RIGHT: SHORT SAMPLE)

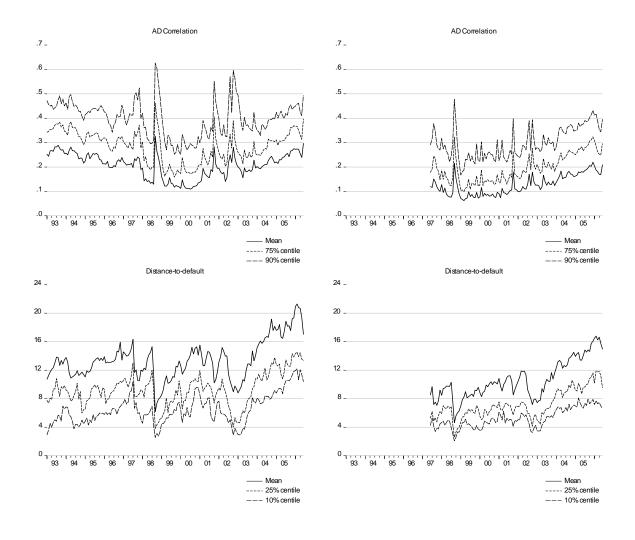


Table 3: Rank correlation between the systemic risk index and different factors

Sample	Index	Factor	Correlation		Distance-to-default		
			Spearman coefficient	Kendall's tau	Spearman coefficient	Kendall's tau	
1993-2006	World 10%	Average	0.2369	0.1620	-0.8587	-0.6833	
		75% percentile	0.2874	0.1949	-0.8549	-0.6752	
		90% percentile	0.4291	0.2880	-0.9046	-0.7429	
	World $20\%$	Average	0.3041	0.2254	-0.7880	-0.6379	
		75% percentile	0.3310	0.2464	-0.7553	-0.6093	
		90% percentile	0.4731	0.3518	-0.8008	-0.6470	
1997-2006	World 10%	Average	-0.3510	-0.2339	-0.9348	-0.7919	
		75% percentile	-0.3424	-0.2319	-0.9375	-0.7976	
		90% percentile	-0.3041	-0.2159	-0.8984	-0.7256	
	World $20\%$	Average	-0.0618	-0.0472	-0.7630	-0.5939	
		75% percentile	-0.0579	-0.0429	-0.7730	-0.6097	
		90% percentile	-0.0470	-0.0401	-0.6837	-0.5238	

The fourth (fifth) column gives the Spearman (Kendall) coefficient between different systemic risk indices and the mean, 75% centile and 90% centile of the correlation between banks' AD ratio. The sixth (seventh) column gives the Spearman (Kendall) coefficient between different systemic risk indices and the mean, 25% centile and 10% centile of the banks' distance to default.

change direction depending on the period.

The strong link between distance-to-default and systemic risk is illustrated by Figure 5, in which a logit transformation of the systemic risk index is plotted against AD ratio correlation (left) and distance-to-default (right), respectively. Clearly, the dispersion with the distance-to-default is smaller than with correlation. Interestingly, with this transformation, the link between the systemic risk and the distance-to-default seems to be relatively linear.

Table 4 presents the results of a linear regression of the (logit of the) systemic risk index on the AD ratio correlation and on the distance-to-default, respectively. The coefficient on the distance-to-default is significantly negative in all specifications. The degree of correspondence (coefficient of partial correlation  $R^2$ ) between the index and the estimated regression is very high (mostly over 80%). The results from the regression with correlation are less convincing: We have a very low coefficients of partial correlation (with the exception of the 90% centile in the long sample) and in the short sample, most coefficients are not significant.

Table 4: Regression of the systemic risk index on correlation and distance-to-default

		World 10%		World 20%	
Sample	Regressor	Coefficient	$\mathbb{R}^2$	Coefficient	$R^2$
1993-2006	Mean correlation	7.6496*	0.0396	17.4344**	0.1416
	75% centile of correlations	10.4503**	0.1271	20.1928**	0.3220
	90% centile of correlations	15.8976**	0.5599	25.3561**	0.7423
	Mean distance-to-default	-0.5073**	0.7761	-0.8087**	0.8164
	25% centile of distance-to-defaults	-0.6618**	0.8601	-1.1114**	0.8683
	10% centile of distance-to-defaults	-0.9528**	0.8545	-1.4776**	0.7810
1997-2006	Mean correlation	-4.1073	0.0234	-0.4085	0.0001
	75% centile of correlations	-3.2653	0.0306	-0.3175	0.0001
	90% centile of correlations	-3.6337*	0.0473	0.0664	0.0000
	Mean distance-to-default	-0.5672**	0.8735	-0.9852**	0.8830
	25% centile of distance-to-defaults	-0.7476**	0.8908	-1.2656**	0.8935
	10% centile of distance-to-defaults	-1.0445**	0.7599	-1.9254**	0.7650

<sup>\* (\*\*)</sup> indicates that the coefficient is significant at a 5% (1%) level.

Not surprisingly, these regression results coincide with those obtained from the rank correlation analysis. The common exposure (i.e., AD ratio correlation) is a poor predictor of the systemic risk index and the direction of its relation changes depending on the period. The distance-to-default, by contrast, explains well the systemic risk index. This difference in precision is illustrated by Figure 6. In each panel, the actual systemic risk index is compared with a proxy given by a linear function of one of the two factors (i.e., common exposure and distance-to-default, respectively). The match between the systemic risk index and its proxy based on the 10% percentile of distance-to-default is striking (right and lower panel of Figure 6). Taking distance-to-default as a proxy for measuring systemic risk seems to give a fairly good and easily obtainable estimation of it.

However, while the distance-to-default seems to be the main factor driving the systemic risk, the common exposure might account for the portion of the systemic risk index that is not explained by the distance-to-default. To check that, we compute the rank correlation between the common exposure and the residuals of a regression of the systemic risk index on the distance-to-default. The idea is to check if a positive

FIGURE 5: SYSTEMIC RISK INDEX VS. AD CORRELATIONS (LEFT) OR DISTANCE-TO-DEFAULT (RIGHT) (LONG SAMPLE)

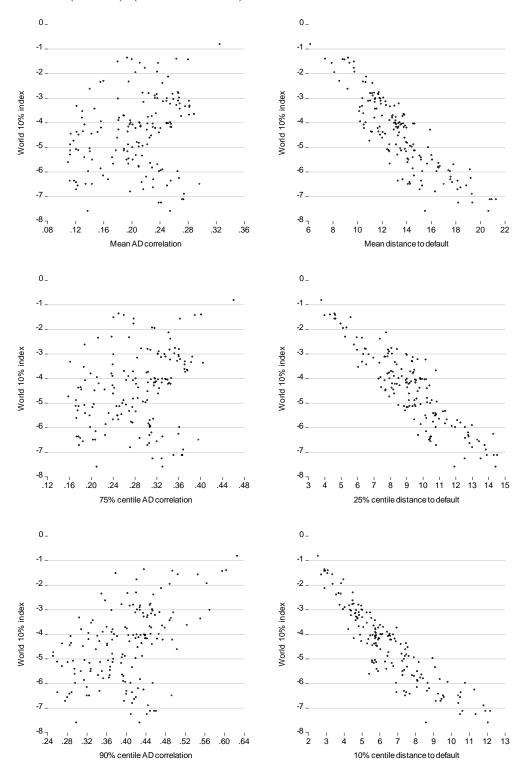
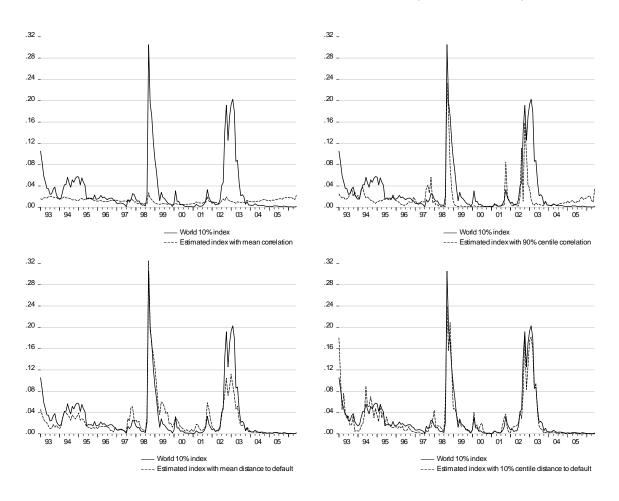


FIGURE 6: ESTIMATED SYSTEMIC RISK INDEX (LONG SAMPLE)



residual (i.e., an "excess" of systemic risk given what is estimated by distance-to-default) is associated with a high or a low common exposure. The rank correlations are presented in Table 5.

Table 5: Rank correlation with residuals

Sample	Residuals from equation using:	Factor	Spearman coefficient	Kendall's tau
Long	Mean distance-to-default	Mean correlation	0.6468	0.4638
		75% perc. correlation	0.5845	0.4142
		90% perc. correlation	0.5158	0.3677
	10% perc. distance-to-default	Mean correlation	0.3944	0.2752
		75% perc. correlation	0.3476	0.2366
		90% perc. correlation	0.3079	0.2079
Short	Mean distance-to-default	Mean correlation	0.6509	0.4540
		75% perc. correlation	0.6185	0.4166
		90% perc. correlation	0.5300	0.3683
	10% perc. distance-to-default	Mean correlation	0.4914	0.3428
		75% perc. correlation	0.4630	0.3211
		90% perc. correlation	0.3757	0.2565

The results show that residuals are positively correlated with common exposure. For example, the Spearman rank correlation between the mean AD ratio correlation and the systemic risk left unexplained by the mean distance-to-default (residuals of the regression of the systemic risk on the mean distance-to-default) is 0.65. This degree of correlation is significantly higher than between common exposures and the systemic risk index (cf. Table 3), suggesting that the effect of common exposures on the unexplained part of systemic risk is greater than on the systemic risk itself. We also see that the rank correlations are all positive. This means that, once the distance-to-default is taken into account, a higher common exposure always induces a higher systemic risk.

To conclude, we find that the systemic risk dynamic does not match the dynamic observed for banks' common exposure. This indicates that common exposure is probably not the main factor explaining systemic risk. Indeed, further analysis reveals that the banks' distance-to-defaults, which describes banks' individual risk taking, is the main driving force. However, we find that common exposures explain relatively

well the part of systemic risk left "unexplained" by the distance-to-default. We also find that, once the distance to default is taken into account, higher common exposure induces higher systemic risk.

Note that we have also tried to disentangle the effect of the distance-to-default between the AD ratios level, which represents the reserves that the banks can use to absorb shocks, and its volatility, which measures the risk of their investments. We found that both elements are of equal importance to explain the evolution of the systemic risk. The volatility plays a significant role in explaining the observed peaks whereas the AD ratios level is more relevant in other time.

## 6 Conclusion

This paper answers two questions. The first one is: how has banks' common exposure to shocks evolved in the last decade in response to the changes observed in international banking sector's environment? To answer this question, we estimate the correlations between large international banks' asset-to-debt (AD) ratios over 1993-2006 with the Flexible M-GARCH approach developed by Ledoit, Santa-Clara and Wolf (2003). We find a decreasing trend until 2000 followed by an increasing trend. This suggests that during the nineties, banks (or at least some of them) have taken advantage of the new technologies and markets available to them to pursue their own business strategy and to differentiate from the others, thus reducing their common exposure. Since 2000, however, the banks' common exposure to shocks increased rapidly, which could indicate that they adopt increasingly similar strategies or markets. This finding is also robust for different sub-groups of the sample.

The paper's second question concerns the impact of these trends on systemic risk in the banking sector. From a theoretical point of view, ongoing financial market integration and rising cross-border activities may have both favourable and adverse effects on the stability of the banking system. To explore this question empirically, we construct a systemic risk index based on Lehar (2005) where systemic risk is defined as the probability of a joint failure of a critical number of banks. In contrast to the correlation analysis, no clear trend emerges. Instead, we observe two peaks in the end of 1998 (LTCM and Russian crisis) and in 2002-2003 (stock market downturn), the latter mainly hurting European banks.

The different patterns observed for banks' common exposure and for the systemic risk contrasts with the widespread view that systemic risk increases with banks' comovement. Our further results confirm that correlation between AD ratios is not a reliable measure for systemic risk. Instead, we find that the distance-to-default is the main driver of the systemic risk index. Once this distance-to-default is taken into account, however, correlation is positively associated with systemic risk.

These findings have two direct consequences for supervisory authorities: first, they show that systemic risk cannot be viewed as a pure correlation phenomenon. Instead, they highlight the danger of high and volatile leverage. According to our results, the main driver of systemic risk is the size of the risks taken by each bank individually (reflected by their distance-to-default) rather by their common exposure to shocks (i.e. AD ratio correlation). Thus, supervisors concerned by systemic stability should first concentrate on making sure that banks are not taking disproportionate risks before trying to reduce inter-linkage or to enforce diversification in the banking sector. Second, from the monitoring point of view, co-movements between banks seem to be a spurious measure of systemic risk. Taken individually, it gives, in the best case, a weak signal about systemic risk or, in the worst case, a signal of wrong direction. To be useful and unambiguous about the evolution of systemic risk, co-movement must be interpreted in combination with distance-to-default.

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## A Reduced form asset un debt dynamic

Developping each equation of the system

$$\begin{bmatrix} d\mathbf{A}_t \\ d\mathbf{D}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_A \\ \boldsymbol{\mu}_D \end{bmatrix} dt + \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\Upsilon}_{AA} & \boldsymbol{\Upsilon}_{AD} \\ \boldsymbol{\Upsilon}_{DA} & \boldsymbol{\Upsilon}_{DD} \end{bmatrix} \begin{bmatrix} d\mathbf{w}_A \\ d\mathbf{w}_D \end{bmatrix}$$

yields

$$dA_{t}^{i} = A_{t}^{i}\mu_{A}^{i}dt + A_{t}^{i}\left(\sum_{j=1}^{n}\upsilon_{AA}^{ij}dw_{A}^{j} + \sum_{j=1}^{n}\upsilon_{AD}^{ij}dw_{D}^{j}\right) = A_{t}^{i}\mu_{A}^{i}dt + A_{t}^{i}\sigma_{A^{i}}dw_{A^{i}}$$

$$dD_{t}^{i} = D_{t}^{i}\mu_{D}^{i}dt + D_{t}^{i}\left(\sum_{j=1}^{n}\upsilon_{DA}^{ij}dw_{A}^{j} + \sum_{j=1}^{n}\upsilon_{DD}^{ij}dw_{D}^{j}\right) = D_{t}^{i}\mu_{D}^{i}dt + D_{t}^{i}\sigma_{D^{i}}dw_{D^{i}}$$

which are both Itô processes where  $\sigma_{A^i} = \left(\sum_{j=1}^n \left(v_{AA}^{ij}\right)^2 + \sum_{j=1}^n \left(v_{AD}^{ij}\right)^2\right)^{1/2}$  and  $\sigma_{D^i} = \left(\sum_{j=1}^n \left(v_{DA}^{ij}\right)^2 + \sum_{j=1}^n \left(v_{DD}^{ij}\right)^2\right)^{1/2}$ . Using Itô's formula, we get that

$$\begin{split} d\ln A_t^i &= \left(\mu_A^i - \frac{1}{2}\sigma_{A^i}^2\right)dt + \sigma_{A^i}dw_{A^i} \\ d\ln D_t^i &= \left(\mu_D^i - \frac{1}{2}\sigma_{D^i}^2\right)dt + \sigma_{D^i}dw_{D^i} \end{split}$$

To get the dynamic of  $dz_t^i$ , note that  $z_t^i = \ln A_t^i - \ln D_t^i$ , which implies that  $dz_t^i = d \ln A_t^i - d \ln D_t^i$ . Using the two previous equations in this expression gives

$$dz_t^i = \mu_z^i dt + \sigma_{z^i} dw_{z^i}$$

where

$$\mu_{z}^{i} = \mu_{A}^{i} - \mu_{D}^{i} - \frac{1}{2} \left( \sigma_{A^{i}}^{2} - \sigma_{D^{i}}^{2} \right)$$

$$\sigma_{z^{i}} = \left( \sum_{j=1}^{n} \left( v_{AA}^{ij} \right)^{2} + \sum_{j=1}^{n} \left( v_{AD}^{ij} \right)^{2} + \sum_{j=1}^{n} \left( v_{DA}^{ij} \right)^{2} + \sum_{j=1}^{n} \left( v_{DD}^{ij} \right)^{2} \right)^{1/2}$$

We can see that the dynamic of the AD ratio sums up the instantaneous growth rate of the bank's asset and debts as well as all interactions with shocks to its own and other banks' assets and debts.

Regrouping this equation for all the banks in one single system yields

$$d\mathbf{z}_t = \boldsymbol{\mu}dt + \boldsymbol{\Upsilon}d\mathbf{w}$$

## B Simulation algorithm

To compute the systemic crisis index  $I_t(\theta)$ , we start the simulation at time t with the vector of AD ratio  $\mathbf{z}_t$  and the estimated covariance matrix  $\mathbf{H}_t$ . The simulation uses the following algorithm:

- 1. Generate a vector  $\mathbf{e}_t$  containing N independent standard white noises.  $\mathbf{e}_t$  simulates the independent shocks to the AD ratios.
- 2. Generate the vector  $\mathbf{u}_t$  from the vector  $\mathbf{e}_t$  with a Choleski decomposition of the variance-covariance matrix  $\mathbf{H}_t$ .  $\mathbf{u}_t$  is the total effect of the different individual shocks  $\mathbf{e}_t$  on each AD ratio.
- 3. Generate the new vector  $\mathbf{z}_{t+1} = \mathbf{z}_t + \Delta \mathbf{z}_{t+1}$  with Equation (7).
- 4. For each bank, check if  $z_{t+1}^i < 0$  (insolvency condition). If bank i becomes insolvent, set  $b_t^i = 1$ .

- 5. Check if there is a systemic crisis, i.e. if  $\sum_{i=1}^{N} \theta_t^i b_t^i > \alpha$ . Stop the algorithm if there is a systemic crisis.
- 6. If there is no systemic crisis, compute the new variance-covariance matrix  $\mathbf{H}_{t+1}$  with Equation (9), the vector  $\mathbf{u}_t$  generated in Step 2 and the estimated coefficient matrices  $\tilde{\mathbf{C}}$ ,  $\tilde{\mathbf{A}}$  and  $\tilde{\mathbf{B}}$ .
- 7. Repeat Steps 1 to 6 k times.

For each period, repeat this algorithm M times. The probability of a systemic crisis is then estimated by the number of times that the algorithm has stopped over M.