Comments Welcome

Simple Monetary Policy Rules in Canadian Macroeconomic Models: A Comparison of the Participating Models

by

Denise Côté, John Kuszczak, Jean-Paul Lam, Ying Liu and Pierre St-Amant[1]

Bank of Canada, Ottawa, Ontario, Canada K1A 0G9

October 2002

Abstract

In this paper, we examine and compare a wide range of private and public sector models of the Canadian economy with respect to their paradigm, structure and dynamics properties. The twelve models can be classified under two economic paradigms. The first one is the "conventional" paradigm (or the Phillips-curve paradigm) and the second one is the "money matters" paradigm. Under the conventional paradigm, inflation is determined by price adjustments in reaction to inflation expectations and by factor disequilibrium in various markets (e.g., the labour market, the goods market). Under the "money matters" paradigm, inflation is mostly determined by movements in monetary aggregates. While most models fall under the "conventional" paradigm, there are nevertheless important differences within this paradigm. These differences are represented, in particular, by alternative channels through which monetary policy affects the economy (short-term interest rates or the yield curve), by differences in the inflation process (linear/non-linear Phillips curve), by alternative expectations processes (backward- or forward-looking expectations), and by the sensitivity of output and inflation to changes in interest rates. We have also examined the dynamic properties of the various models when they use a simple monetary authority reaction function, such as the one proposed by John Taylor (1993). The eight standard shocks considered in our study reveal indeed significant differences in the dynamic properties of the participating models. Finally, our comparison of the models' reaction functions with those of a VAR comprising very little economic structure suggests that some models do a better job than others in reflecting the typical response of the Canadian economy to certain shocks.

---

1. We would like to thank the following private sector firms and other organizations who have participated in the Bank of Canada project on Taylor Rules including a one-day workshop held on 25 October 2001: Conference Board of Canada, DRI-WEFA, IMF, Department of Finance of Canada, OECD and PEAP (University of Toronto). We wish to thank also Jim day and Samuel Lee for their excellent technical help, Jamie Armour, Ramdane Djoudad, Scott Hendry, Kevin Moran, Steven Murchison and Bing-Sun Wong for their valuable input and for conducting part of the simulations. Finally, we thank Steve Ambler, Don Coletti, Paul Darby, Michael Devereux, Peter Dungan, Chuck Freedman, Alain Guay, Ben Hunt, Douglas Laxton, David Laidler, Andrew Levin, Dale Orr, Benoît Robidoux, Lucie Samson, Jack Selody and all the participants of the "Taylor Rules Workshop" for their comments and suggestions. The views expressed in this paper are not those of the Bank of Canada but solely those of the authors. All errors and omissions are our own.

FR-01-012.

1. Introduction

The monetary authorities must contend with several sources of uncertainty when conducting monetary policy. The uncertainties they face include the structure of the economy, the mechanisms through which monetary policy affects the economy, the nature of the shocks affecting that economy and the channels through which these shocks are transmitted. Consequently, advice regarding monetary policy should be based not solely on one characterization of the economy, but rather on several alternative viewpoints.

When conducting monetary policy, one means of accounting for uncertainty and of mitigating its impact, is to incorporate projections from a variety of different models into the decision-making process. Another approach, proposed by Levin, Wieland and Williams (1999) and Taylor (1999) in the context of models of the U.S. economy, consists of using a "simple monetary policy rule", which yield good results in several models. Such a rule would make it possible to accommodate uncertainty in economic models. The goal of our research is to determine whether we can identify such a rule in the context of models of the Canadian economy.[2]

Our research is different from the previous literature on simple monetary policy rules in several ways. First, we use a very large number of models to evaluate simple monetary policy rules. Second, the models used are very diverse and are all used either for forecasting key variables of the Canadian economy and/or for policy analysis. By considering a large variety of models, we are able to address some of the criticisms, notably by Hetzel (2000) and Svensson (2001), that the models used in the past to evaluate simple monetary policy rules were too similar in structure and did not really constitute a test of robustness of the rules. Third, careful attention has been paid to how the various models fit the data. Sims (2001) argues that existing studies which use models to evaluate policy rules have not paid enough attention to how these models fit the data. Finally, a robustness check is also performed on the results of our evaluation of the monetary policy rules on the basis of a ranking of the participating models.

2. In the fall of 2000, we set out to organize a research framework aim at determining whether we can identify a simple monetary policy rule in the context of models of the Canadian economy. In the autumn of 2001, the results of our research effort were the subject of a Bank of Canada one-day workshop on Taylor Rules (see the Bank of Canada website at http://www.bankofcanada.ca/workshop2001/).

The goal of this report is to set forth the similarities and differences in the twelve models of the Canadian economy with respect to their paradigm, structure and dynamics properties. This will hopefully provide a better understanding of the models in order to facilitate a subsequent evaluation of different simple monetary policy rules, which is subject of another report entitled "The Performance and Robustness of Simple Monetary Policy Rules in Models of the Canadian Economy".

To further understand the structure and properties of the twelve models of the Canadian economy, i.e., the way the various models respond to different macroeconomic shocks, we perform several deterministic simulations. Because output and inflation dynamics depend in part on the specification of monetary policy, to compare and evaluate the different models we specify a common policy reaction function. The original Taylor rule is thus imposed as the baseline reaction function in each model. Eight deterministic shocks (seven temporary and one permanent) are then simulated in the twelve models. The seven temporary shocks which are simulated in these models are as follows: a domestic demand shock, an external shock, a shock to commodity prices, a price shock, a wage growth shock, a shock to short-term interest rates, and a shock to the exchange rate. Finally, the deterministic permanent shock is a shock to long-term interest rates.

The models used in this paper are very diverse and are all used either for forecasting and/or for policy analysis. Hence, to find out how and why some or all of the major models differ from each other is an interesting research avenue in itself. Moreover, recent studies on structure and properties of models of the Canadian economy as well as cross-model comparisons are practically non-existent. To our knowlegde, the last time that such a study was conducted was in July 1982, when the Bank of Canada and the Department of Finance held a one-day seminar on the structure and properties of nine major Canadian econometric models.[3]

A number of institutions have since built new models of the Canadian economy or made major improvements to their models. The passage of time have provided opportunities for improvements reflecting in part new issues for macroeconomic policy and new research on economic theory. Also access to new data or more extensive data sets and advances in computer hardware and software and in new algorithms for solving large nonlinear systems of equations

---

3. See B. O'Reilly, G. Paulin and P. Smith (1983).

have allowed the creation of more complex models. Our study provides therefore an opportunity to describe the current state of the Canadian macroeconomic models and is a source document for research studies in macroeconomic modelling in Canada.

The discussion presented in this report, regarding the structure and dynamics of the models of the Canadian economy as well as the analysis of their key properties, is based on official publications, discussions that we had directly with the participants, the answers of a questionnaire on model properties (see Table 1) and on the model responses provided by the participants. It is important to keep in mind, however, that the models involved in this project and described here may still go through ongoing modification. Hence, their current versions may differ from those used in this exercise.

Our study considers twelve private and public sector models of the Canadian economy. Five of them are maintained by private sector organizations. The models are:

i.) CEFM: Canadian Economic and Fiscal Model, Department of Finance Canada;

ii.) DRI: Data Resources of Canada;[4]

iii.) FOCUS: Policy and Economic Analysis Program (PEAP), Institute for Policy Analysis, University of Toronto;

iv.) FOCUS-CE: the version incorporating forward-looking expectations;

v.) INTERLINK: Organisation for Economic Co-operation and Development;

vi.) MTFM: The Conference Board of Canada’s Medium-Term Forecasting Model;

vii.) WEFA: Wharton Economic Forecasting Associates;

viii.) LPM: Limited Participation Model, Monetary and Financial Analysis Department, Bank of Canada;

ix.) M1-VECM: Vector-error-correction model, based on the M1 aggregate, Monetary and Financial Analysis Department, Bank of Canada;

x.) MULTIMOD: International Monetary Fund;

xi.) NAOMI: North American Open-Economy Macroeconometric Integrated Model, Department of Finance Canada;

xii.) QPM: Quarterly Projection Model, Research Department, Bank of Canada.

---

4. Data Resources of Canada and Wharton Economic and Forecasting Associates have recently merged.

Our examination and comparison of the twelve participating models reveal important differences among them. The models can be classified under two economic paradigms. The first one is the “conventional” paradigm and the second one is the “money matters” paradigm. While most models fall under the “conventional” paradigm, there are nevertheless important differences within this paradigm. These differences are represented, in particular, by alternative channels through which monetary policy affects the economy (short-term interest rates or the yield curve), by differences in the inflation process (linear/non-linear Phillips curve), by alternative expectations processes (backward- or forward-looking expectations), and by the sensitivity of output and inflation to changes in interest rates.

We have also examined the dynamic properties of the various models when they use a simple monetary authority reaction function, such as the one proposed by John Taylor (1993). The eight standard shocks considered in our study reveal indeed significant differences in the dynamic properties of the participating models. Finally, our comparison of the models’ reaction functions with those of a VAR comprising very little economic structure suggests that some models do a better job than others in reflecting the typical response of the Canadian economy to certain shocks.

The remainder of the paper is organized as follows. In Section 2, we examine in more details the twelve models emphasizing the structure and dynamics of each. In Section 3, we discuss the properties of the various models when they are subjected to deterministic macroeconomic shocks. In Section 4, we compare some of the models’ response functions with those of a vector-autoregressive model. We conclude in Section 5.

2. The Structure and Dynamics of the Models

CEFM

The CEFM[5] model incorporates four sectors: firms, households, the government, and the external sector. These sectors are described by a system of 113 estimated equations, using 61 economic and 52 fiscal variables. Firms maximize profits and use labour, capital, and natural resources to produce goods with a Cobb-Douglas production technology. Demand for capital, in

5. See Robidoux, B. and B.-S. Wong, 1998. Also see DeSerres, A., B. Robidoux, and B.-S. Wong. 1998.

turn, determines investment in machinery and equipment, and non-residential construction. Consumers maximize utility over an infinite planning horizon subject to an intertemporal budget constraint. Consequently, consumption, which is disaggregated into purchases of consumer goods and residential investment, depends on aggregate wealth. As in the MULTIMOD and QPM models, some consumers are unable to borrow on the basis of future incomes because of liquidity constraints. The external sector consists of raw materials, finished products, and services that are both exported and imported. The exchange rate is anchored by purchasing-power parity in the long run, while deviations from this value are determined by short-term uncovered interest rate parity. Compared with the other participating models, the CEFM model has a very elaborated government sector, since one of the principal goals of the Department of Finance is to predict the federal government's revenue and spending prospects with considerable details.

Inflation, in terms of wage increases, is modelled in CEFM using an augmented Phillips curve with backward-looking inflation expectations. A growth trend in total factor productivity and an unemployment gap are added to these elements. The natural unemployment rate, in turn, is determined by an index of the generosity of the Employment Insurance Program and a variable approximating the level of unionization. The central bank's instrument for monetary policy is the 90-day commercial paper rate. Various reaction functions model the monetary authority's behaviour. Other interest rates are determined through the term structure of interest rates and also exert an influence on the economy's real variables, such as consumption and investment.

DRI

DRI is a large scale econometric model of about 700 equations with a large number of variables determined exogenously. The model embodies several sectors which follows closely the National Income and Expenditure Accounts accounting framework. The supply side of the DRI model is based on an explicit production function yielding potential output. The difference between actual and potential output is the primary channel through which demand and supply imbalances influences the adjustment of prices. The exchange rate is determined by the short-term uncovered interest rate parity condition and also by movements in real commodity prices.

In DRI, the wage rate is determined in accordance with an extended Phillips curve specification as a function of backward-looking inflation expectations, productivity and the gap between the actual and full-employment unemployment rates. Prices at the producer level, are determined by industrial capacity utilization, the gap between the actual and full-employment unemployment rates, U.S. wholesale prices and the exchange rate. The financial sector contains several interest rates and the money supply. The 90-day commercial paper rate is the instrument of monetary policy intervention. Longer-term interest rates are determined by the expectations hypothesis and affect real variables such as consumption, housing and investment.

FOCUS

FOCUS[6] is a quarterly macroeconomic model of the Canadian economy maintained by the Policy and Economic Analysis Program (PEAP) of the University of Toronto. The model consists of a system of some 300 equations and identities. Like most other models considered here, FOCUS belongs to the neo-classical synthesis. This model's equations correspond to the IS-LM curve analytical framework and incorporate a Phillips curve with inflation expectations. Owing to price/wage rigidity, there is a certain trade-off between price and quantity adjustment in the short term. In the long term, however, the model retains a neo-classical character, such that production depends on only real factors and not on variations in aggregate demand. Monetary policy is thus neutral in the long run. Given the large number of equations, the components of aggregate demand are modelled with considerable detail. Aggregate supply is formulated as a Cobb-Douglas production function with decreasing returns to scale. The model's LM curve defines equilibrium on the money market. Other financial markets are not modelled (Walras's Law). The foreign sector is described with the BP curve, while short-term uncovered interest rate parity determines the exchange rate.

In the FOCUS model, the wage adjustment process is similar to that in CEFM. Wage changes are modelled using an augmented Phillips curve with backward-looking inflation expectations, to which productivity growth and the unemployment gap are added. Global inflation is thus determined as a markup added to the rate of wage growth. The consistent-expectations version of this model (the "CE" version), incorporates forward-looking inflation expectations. The instrument of monetary policy intervention is the 90-day commercial paper rate.

6. See Dungan, P. 1997.

INTERLINK

INTERLINK is the OECD's semi-annual model of the global economy.[7] It follows the tradition of many other macroeconomic models of the neo-classical synthesis, combining short-term "keynesian" features with long-term neo-classical properties. In particular, the presence of real and nominal rigidities in the wage and price setting behaviour generally imply that a protracted period of adjustment occurs before output and employment returns to potential following a shock. The Canadian model, with its 26 equations and 280 identities, has aggregate supply determined by a constant-returns-to-scale Cobb-Douglas production function with capital and labour as production factors with the exogenous trend growth rate of technical progress. Aggregate demand is divided into 12 components: private and public consumption, investment in residential and non-residential construction, public sector investment, investment in stocks, and exports and imports of manufactured and non-manufactured goods and services. The exchange rate is modelled by short-term uncovered interest rate parity.

In INTERLINK, the key price is the business sector GDP deflator, determined in an error-correction framework. In the long-run, prices are determined as a constant mark-up over marginal costs, as calculated from the Cobb-Douglas production function. However, in the short run, prices are sensitive to demand pressure and may therefore deviate from trend unit costs. Demand pressures also enter through a capacity utilisation term. There is also a short-run effect from import prices of non-manufactures, representing cost pressures from commodity prices. Wages (including non-wage compensation) come from a reduced-form bargaining model, and so depend on prices, the unemployment rate relative to NAIRU, trend labour productivity, and the wedge between consumer and producer prices. Implicitly, the NAIRU is a function of the growth rate of trend labour productivity. Most other prices feed off (are linked to) the business sector GDP deflator. For example, the consumption deflator depends on the business sector GDP deflator and import prices. Expectations are not specified anywhere in the model except in the backward-looking exchange rate equation. Monetary policy operates through the 90-day commercial paper rate. Long-term rates have a slightly larger impact than short rates. Long-term interest rates affect business investment while short-term interest rates affect consumption. Money growth has no

7. See Richardson, P. 1988.

effect in the model and monetary policy does not have a permanent effect on either the level or the growth rate of real GDP.

MTFM

The Conference Board of Canada's MTFM is a large, quarterly, input-output model comprising about 350 equations. It is estimated on a sample period beginning in 1961. In this model, aggregate demand consists of 70 components. It should be mentioned that aggregate supply is not explicitly modelled, although market conditions can be extrapolated from capacity utilization rates by industrial sector. The exchange rate is explained by both uncovered short-term interest rate parity and by commodity price variations.

Global inflation is modelled in the Conference Board's MTFM using a bottom-up approach based on more than 100 different prices whose weights are drawn from input-output tables. Inflationary pressures are established at three main stages of the goods production process (raw materials, intermediate, and finished). Thus, there are three sets of prices to determine: the price of raw materials, the price of industrial or transformed products, and the price of finished products or final demand. At each production stage, price is determined as a markup added to the costs of inputs (the marginal cost of labour, the cost of capital, materials, and changes to import prices). The markup, in turn, is influenced by market conditions, i.e., net aggregate demand as approximated by the utilization rate of production capacity or by the gap between aggregate output and its trend. It should be noted that in MTFM, inflation expectations are backward-looking and, in general, inflation is more sensitive to changes in supply than to changes in aggregate demand. Finally, the central bank uses the treasury bill rate as its instrument of monetary policy. Other rates (e.g., bonds, mortgages) are transmitted over the term structure of interest rates and also play a role, though short-term rates have the greatest impact on aggregate demand. Money is neutral in the long term.

WEFA

In many respects, the WEFA model resembles the others. It models four groups of economic agents: consumers, firms, budgetary authorities, and monetary authorities. It disaggregates global demand into several components. There are about 10 equations for consumption and three for investment. A permanent income variable partially explains consumers' behaviour. As in the FOCUS, MTFM, and M1-VECM models, the concept of potential output is determined exogenously. As for the financial variables, real interest rates and the exchange rate are linked in the usual manner to short-term uncovered interest rate parity. Note that this is true of nearly all the models presented in this study.

The WEFA model explains global inflation using a "bottom-up" approach starting from several different prices. Inflationary pressures in the economy arise at various stages of the goods-producing process. Wage increases do not affect prices directly, but rather indirectly over variables for labour income. It should be noted that, in the WEFA model, inflation expectations are of the backward-looking type. The central bank's instrument of monetary policy is the three-month treasury bill rate. Other interest rates (e.g., bonds, mortgages) enter over the term structure of interest rates, but do not play a role in the determination of real variables. As in the case of the other models, money is neutral in the long term.

LPM

The Limited Participation Model (LPM)[8] is a calibrated general-equilibrium model. LPM decomposes aggregate demand into consumption and investment. Both of these components derive from equations of the forward-looking type. In particular, households choose between three classes of consumption goods, two of which are domestic products (tradable and non-tradable), while a third is produced abroad. Monetary policy actions affect the real economy through frictions generated by agents' portfolio decisions. More precisely, rigidities in adjusting money balances are the main source of the short-run non-neutrality of monetary policy. This is unlike the other participating models, in which the short-term impact of monetary policy on the economy's real variables works over some form of price or wage rigidity. In LPM, prices are perfectly flexible in the short run, implying that the aggregate supply function is vertical. However, since firms incur debt to finance wages, variations in the interest rate alter supply conditions, causing the aggregate supply curve to shift.

8. See Hendry, Ho, and Moran, 2001.

M1-VECM

The vector-error-correction model (M1-VECM)[9] is based on the paradigm that money is at the heart of the monetary policy transmission mechanism. This model comprises four key equations in which variations in money, production, prices, and interest rates depend on the lagged values of these same variables, on a series of exogenous variables, and on the “money gap,” i.e., the difference between money supply and demand. The VECM model assigns an active role to money in the sense that changes in the supply of, relative to the estimated long-term demand for, money cause variations in production and in prices in the short term, but only prices in the long term. While the external sector is not explicitly modelled, the exchange rate is determined by uncovered interest rate parity in the short run and anchored in the relative condition of purchasing-power parity in the long run.

M1-VECM assigns an active role to money, in the sense that variations in the relative supply of, and estimated long-term demand for, money (i.e., the money gap) affect production and prices jointly. In this model, inflation is measured by core CPI. The money gap in the previous period and lagged variables for the rate of monetary expansion increase inflation appreciably. The lagged output gap, previous variations in the interest rate, in inflation, in the exchange rate, and in U.S. interest rates are also accounted for in this model. The monetary policy instrument is the overnight fund rate, and the actions of the monetary authorities are transmitted over the slope of the yield curve (overnight rate minus the rate on bonds with maturities exceeding 10 years).

MULTIMOD

MULTIMOD is the IMF’s annual model of the global economy.[10] It includes individual models for each of the seven largest industrialized countries (including Canada), one model for the remaining 14 industrialized countries, one model for developing countries, and a final model for countries in transition. Like QPM, MULTIMOD consists of a set of dynamic relationships that trace the path leading from the starting conditions to the implicit steady-state, or long-term equilibrium, solution. In MULTIMOD,[11] consumer behaviour is modelled on the Blanchard-

9. See Adam, C. and S. Hendry. 2000.

10. The model is estimated for the 1970–2000 period.

11. See Laxton, D. et al. 1998.

Weil-Buiter paradigm,[12] which assumes that economic agents plan within a finite time horizon. Consumers' lifespans are unknown, and they must plan their consumption and savings in light of this uncertainty. This paradigm is extended with the addition of remuneration profiles that vary across age groups and imply different marginal propensities to consume over the life cycle. Moreover, consumers are confronted, in part, with liquidity constraints that restrict their ability to borrow on the basis of future income. Thus, in this model, aggregate consumption is obtained by summing consumption depending on permanent income with that depending on disposable income.

MULTIMOD models investment with Tobin's q, which specifies that the desired level of investment may exceed the steady-state level to the extent that the expected marginal productivity of capital is greater than its replacement cost. The specification of the foreign sector is relatively conventional. Imports are determined by their relative prices and by a measure of domestic activity calibrated on the basis of input-output tables.[13] Exports are modelled to be compatible with other countries' imports. In the short term, exchange rates and interest rates are linked by uncovered interest rate parity adjusted for risk premiums. As in QPM, real domestic interest rates are connected to exogenous foreign values adjusted for risk premiums, while the steady-state real exchange rate is determined endogenously to generate the trade balance flows necessary for overall equilibrium in the economy's stock of assets.

The MULTIMOD models fundamental inflation (defined as core CPI inflation) similarly to QPM—i.e., using a non-linear Phillips curve with inflation expectations that are both backward- and forward-looking—except that the disequilibrium factor is approximated by the unemployment gap.[14] Global inflation, that is, the overall rise in the CPI, includes changes in the prices of imports of manufactured goods, the rate of growth of the price of oil, previous variations in global inflation, and the rate of increase in the core CPI. Expected inflation, in turn, is a linear combination of previous values of the growth rate of the CPI, the core CPI, and model-consistent

---

12. Buiter, W.H. 1988.

13. MULTIMOD is a macroeconomic model of the world economy. The share of domestic demand supplied by foreign production is established on the basis of input-output tables specific to each country.

14. In MULTIMOD, the backward-looking and forward-looking elements are weighted 0.75 and 0.25, respectively.

values for those two measures. In MULTIMOD, monetary authorities act on the nominal short-term interest rate, i.e., the three-month treasury bill rate, to achieve their inflation-control target.

NAOMI

The North American Open-Economy Macroeconometric Integrated Model[15] (NAOMI) includes six behavioural equations and 18 identities. For the sample period, 1972Q1–2000Q1, the equation system is estimated simultaneously using the full-information maximum-likelihood (FIML) procedure. The model's endogenous variables include output growth, inflation, the real exchange rate, the yield curve, and long-term interest rates. Variables exogenous to the model include potential output, U.S. variables, commodity prices, and the budget balance for the entire public sector. Like M1-VECM, NAOMI defines aggregate demand in terms of a single equation (IS curve). In particular, production growth is modelled on increases in potential output, production growth in the United States, and changes in the yield curve. Variations in the real exchange rate, relative non-energy commodity prices, and the ratio of the budget balance of the entire public sector to nominal potential GDP are also incorporated. While the foreign sector is not explicitly modelled, the exchange rate is determined in the long run by the relative purchasing-power parity condition and plays a leading role in the adjustment of the economy following external and domestic shocks.

NAOMI explains inflation by price-level adjustments in response to backward-looking inflation expectations, by the level and variation of the output gap, by changes to relative commodity prices, and, finally, by movements in the real exchange rate. Variations in the output gap are introduced to capture the predictive information it contains concerning the future level of potential production. As in the QPM and M1-VECM models, the actions of the monetary authorities are transmitted over the slope of the yield curve. Monetary policy affects the real economy in the short run because of nominal rigidities, while price flexibility ensures its neutrality in the long run.

15. See Murchison, Stephen, 2001.

QPM

The Quarterly Projection Model[16] (QPM) is used by the Bank of Canada both for policy analysis and for generating economic projections. QPM can be considered a system that consists of two calibrated models. The first, the steady-state or long-term equilibrium model (SSQPM), is based on the Blanchard-Weil paradigm with overlapping generations.[17] This model is used to study the determinants of long-term equilibrium in the economy and the permanent effects of economic shocks or policy changes. The second model, QPM, consists of a set of dynamic relationships that trace the paths leading from the starting conditions to the implicit steady-state solution, or long-term equilibrium.

QPM is designed to explain the behaviour of households, firms, foreigners, government (all levels of the public sector), and the central bank. These agents' optimization decisions interact to determine the final levels of four key stocks: household financial wealth, capital, public debt, and net foreign assets. These stocks, in turn, are key determinants of related flows, such as consumption expenditure, savings, investment spending, government outlays and revenues, and the external balance. In this model, the exchange rate plays a key role in the monetary policy transmission mechanism by promoting equilibrium between aggregate demand and supply. In the short term, the exchange rate and interest rates are linked by uncovered interest rate parity. In steady state, real domestic interest rates depend on their exogenous external analogues adjusted for risk premiums, and the real exchange rate adjusts endogenously to generate the trade balance flows required for global equilibrium in the economy's stock of assets.

QPM describes inflation (in terms of core CPI inflation) by a non-linear Phillips curve with inflation expectations that are both backward- and forward-looking.[18] This non-linearity endows it with the property that price adjustments are larger under conditions of excess demand on goods markets than under excess supply. The instrument of monetary policy intervention is the 90-day commercial paper rate, while actions of the monetary authorities are propagated over the slope of the yield curve.

16. See Black, R, D. Laxton, D. Rose, and R. Tetlow. 1994. See also D. Coletti, B. Hunt, D. Rose, and R. Tetlow. 1996.
17. Blanchard, O.J. 1985. See also Weil, P. 1989.
18. In QPM, the backward-looking and forward-looking elements are weighted 0.7 and 0.3, respectively.

3. Deterministic Shocks with the Original Taylor Rule

In this section, we examine the dynamic properties that the various models display when they use a simple reaction function by monetary authorities, such as the one proposed by John Taylor (1993). The Taylor rule is a behavioural rule that monetary authorities apply when inflation diverges from the inflation-control target and output from its potential level. In another study, we examine a variety of alternative simple monetary policy rules.[19] It is possible that the parameters of various models are not invariant to changes in monetary policy rules (Lucas's critique). Nonetheless, in our study we assume invariance of the parameters.

The original Taylor reaction function for a given inflation-control target, π_t^T, is defined by equation (1):[20]

$$r_t = r_t^* + 0.5(\pi_t - \pi_t^T) + 0.5(y_t - y_t^p) \tag{1}$$

where r_t is the real interest rate on 90-day commercial paper, r_t^* is the equilibrium real interest rate on 90-day commercial paper, $\pi_t - \pi_t^T$ the inflation gap, and $y_t - y_t^p$ the output gap.

The immediate means, or instrument, whereby monetary authorities act on the economy is the nominal interest rate, i_t, which is determined by the Fisher equation. The original Taylor rule can thus be expressed in nominal terms using the following equation:

$$i_t = i_t^* + 1.5(\pi_t - \pi_t^T) + 0.5(y_t - y_t^p) \tag{2}$$

where i_t is the nominal interest rate on 90-day commercial paper, i_t^* = the equilibrium nominal interest rate on 90-day commercial paper.

Equations (1) and (2) represent the monetary authorities' reaction function, as proposed by Taylor.

---

19. For more information and details, see "The Performance and Robustness of Simple Monetary Policy Rules in Models of the Canadian Economy," by Côté, Kuszczak, Lam, Liu and Saint-Amant.

20. Note that the value of 0.5 for the coefficients was inferred by Taylor from the properties of large-scale models of the U.S. economy.

In the context of our study, we seek to understand and compare the properties of the various models. Because output and inflation dynamics depend in part on the specification of monetary policy, we specify a common policy reaction function. The original Taylor rule is thus imposed as the baseline reaction function in each model. Within each of these models, the original Taylor rule implies that monetary authorities choose a nominal interest rate that includes a combination of the real interest rate and anticipated inflation. This allows them to attain their target given the structure and dynamics of their model.

For the purpose of understanding the structure and properties of the twelve models of the Canadian economy, i.e., the way the various models respond to different macroeconomic shocks, we submit the models examined to eight deterministic shocks: seven temporary and one permanent. We then run dynamic simulations to examine how equilibrium is reestablished when the behaviour of monetary authorities is described by the original Taylor rule. The seven temporary shocks we introduce into the dynamic simulations are as follows: a demand shock, an external shock, a shock to commodity prices, a price shock, a wage-change shock, a shock to short-term interest rates, and a shock to the exchange rate. Finally, the deterministic permanent shock is to long-term interest rates. These deterministic shocks are described in Table 2 and analyzed in detail in the next section. Several of them require some explanation. The price shock, for example, is interpreted as a temporary change to firms' profit margins. The temporary shock to short-term interest rates is interpreted as a modification of the inflation-control target, while the permanent shock to long-term interest rates represents a permanent change in the term premium. Finally, the transitory shock to the exchange rate is interpreted as a temporary loss of confidence by investors in the Canadian economy.

We asked each participant to generate a new control solution for their model when the monetary authorities' behavioural rule is approximated by the Taylor rule. The shocks were run on the models initiating from this new control solution. This allowed us to evaluate the impact of the shocks in isolation.

The shocks are introduced into the dynamic simulations when the economy is in steady-state equilibrium. It should be kept in mind that the temporary shocks we are examining here have no impact on the long-run equilibrium of the models. These dynamic simulations thus allow us to see how equilibrium is reestablished subsequent to temporary shocks and to use the observed reactions to better understand the behaviour and the dynamic structure of the various models, particularly how the original Taylor behavioural rule determines policies under these conditions.

3.1 Analysis of the Response Functions[21]

i.) A temporary domestic demand shock

We first introduce a temporary shock to aggregate demand into the models. Over four quarters, the level of consumption and investment increases by 1.00, 0.75, 0.50, and 0.25 per cent, respectively. Figures 1.1a to 1.1d illustrate the response functions of real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate in the 11 models for 24 quarters following the beginning of the dynamic simulation.

As expected, the demand shock causes inflation to rise in the short term. In CEFM and MTFM, however, the price increases are particularly small. To the extent that inflation rises above the target trajectory and production remains greater than potential output, monetary authorities raise the short-term interest rate—by about 25 basis points during the first quarter. The CEFM and WEFA models show negligible interest rate hikes during the first four quarters, while these increases are particularly high in QPM, DRI, INTERLINK, and NAOMI during that same period. The greater interest rate increases in QPM, DRI, INTERLINK, and NAOMI are consistent with the particularly strong responses of inflation (and the persistence of the output response in DRI) in those models. The non-linearity of the Phillips curve in QPM as well as the large currency depreciation may largely explain the strong rise in inflation in that model. As to NAOMI and INTERLINK, the cause may be the steeper slope of the Phillips curve in those models.

Finally, we observe that in certain models the impact of the temporary demand shock on production extends beyond the fourth quarter, either by creating secondary cycles (NAOMI and QPM) or by generating quasi-permanent effects (WEFA and DRI). Owing to the interest rate increase and the appreciation of the currency, production and inflation eventually return to their reference value between the twelfth and sixteenth quarters, on average, except in the DRI and WEFA models for production and, in the WEFA model only, for inflation.

21. Twelve models are examined. The dynamic properties of LPM are omitted here because this model does not handle the shocks considered in this study.

ii.) A temporary external shock

The second shock we introduce into the models is a temporary external shock. Real U.S. GDP is increased by 1 per cent, 0.75 per cent, 0.50 per cent, and 0.25 per cent, respectively, during the first four quarters. Note that this temporary shock also incorporates the endogenous response of some U.S. macroeconomic variables, such as inflation and short-term interest rates, as well as the endogenous response of commodity prices.[22] Figures 1.2a to 1.2d show the response functions of real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate in the 11 models for 24 quarters following the beginning of the dynamic simulation.

Like the domestic demand shock, this positive external shock stimulates the Canadian economy, though to a lesser extent. We notice that the temporary external shock's impact on domestic production dissipates soon after the fourth quarter. This shock is particularly persistent in the DRI and INTERLINK models, and generates secondary cycles in the other models. The temporary increase in foreign demand exerts upward pressure on domestic inflation. Monetary authorities react by increasing the short-term interest rate by about 10 to 20 basis points over the first year, except in the QPM, NAOMI, and INTERLINK models, where the increases are much more pronounced. As a result of a large increase in inflation, the interest rate increase is particularly steep and persistent in QPM. The magnitude of the direct and indirect impact on prices of the pronounced depreciation and the characteristics of the QPM's Phillips curve contribute to the persistence of inflation in that model when the original Taylor rule is used. In the other models, inflation returns to the target trajectory around the twelfth quarter. Note finally that the strength of the CPI inflation response in the case of WEFA is somewhat surprising, considering that this model shows little response in the case of real Canadian GDP.

iii.) A temporary shock to commodity prices

The third shock we introduce into the 11 models is a temporary shock to commodity prices over eight quarters. During the first quarter, commodity prices rise 4 per cent, during the second 3.5 per cent, during the third 3 per cent, … , and during the eighth 0.5 per cent, returning

22. As a result of a temporary increase in U.S. GDP, the U.S. vector-autoregressive model predicts an increase in U.S. output up to 11 quarters and a rise in inflation and U.S. interest rates during the first 8 quarters. The endogenous variables gradually revert to their steady-state values afterwards. See Appendix 1 for more information on the U.S. VAR used to generate the endogeneous responses.

to their initial value by the ninth quarter. Note that this shock incorporates the endogenous response of several U.S. macroeconomic variables, such as production, inflation, and short-term interest rates.[23]

The short-term impact on real Canadian GDP is expected to be positive, since this shock implies an improvement in Canada's terms of trade that boosts the value of exports. The ensuing growth in wealth should, in turn, stimulate consumption.[24] This shock may also have a positive impact on inflation: directly entering into the CPI and indirectly affecting prices because of the expansion of economic activity, and possibly contributing to inflation expectations. The impact on inflation should only be temporary, however, since monetary authorities are committed to bringing it back onto the target trajectory.

Figures 1.3a to 1.3d present the response functions for real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate to a temporary increase in commodity prices in the 11 models for 24 quarters following the beginning of the dynamic simulation. This shock generates a moderate increase in real GDP through eight quarters in most of the models, while MTFM and INTERLINK show a decline. This shock drives a temporary increase in inflation in all but the FOCUS model. We also notice that in general this shock has a greater impact on prices than on quantities.

In the short term, monetary authorities respond by raising the interest rate, then gradually lowering it as the inflation and output gaps close. The interest rate increases vary substantially from one model to the next reflecting differences in inflation and output gaps among models. Overall, the production and inflation gaps close between the twelfth and sixteenth quarters, except in the INTERLINK and WEFA models for output. In fact, the response of GDP is particularly persistent in these models. Moreover, in contrast to the other models, QPM predicts a persistent depreciation of the Canadian currency.

---

23. Subsequent to a temporary increase in commodity prices, the U.S. vector-autoregressive model predicts a fall in U.S. output for the first 16 quarters and a rise in inflation and U.S. interest rates during the first 10 quarters. The endogenous variables gradually revert to their steady-state values afterwards. See Appendix 1 for more information on the U.S. VAR used to generate these endogeneous responses.

24. Although Canada is a net exporter of energy and non-energy commodities, the increase in energy prices has a negative impact on the Canadian economy. The higher input prices facing certain industrial sectors more than cancel out the positive effect on the energy sector. A rise in the prices of non-energy commodities is considered beneficial to the Canadian economy.

iv.) A temporary shock to price levels

The price-level shock is a temporary increase in the CPI, excluding food, energy, and indirect taxes. The price index increases by 1 per cent, 0.75 per cent, 0.50 per cent, and 0.25 per cent, respectively, during the first four quarters, returning to its original level in the fifth quarter. Figures 1.4a to 1.4d present the response functions for real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate in the 11 models for 24 quarters following the beginning of the dynamic simulation.

This shock can be interpreted as a temporary increase in firms' profit margins. Although this shock is temporary, it may have repercussions on inflation expectations, driving inflation away from its target trajectory. Monetary authorities must therefore act to counter the shock's impact on inflation expectations and to attenuate secondary upward pressures on prices originating from the currency depreciation.

Over the course of the first quarter, this shock causes a rise in inflation of about 3-4 percentage points (except in the QPM and MULTIMOD models) and a fall in production (only M1-VECM yields a short-term increase in output). Monetary authorities react by boosting interest rates as of the first quarter between 50 to 275 basis points depending on the models. In the M1-VECM and MULTIMOD models, interest rates remain practically unchanged over the horizon of the simulation, though they increase considerably in QPM and WEFA. The rise in interest rates puts downward pressure on production and inflation. The falloff in real GDP from its potential level and the decline in inflation result in interest rate reductions as of the fourth quarter. Eight quarters after the beginning of the shock, inflation and interest rates have practically returned to their control levels, while production fluctuates somewhat before returning to its initial path.

v.) A temporary shock to wage growth

The fifth shock that we introduce into the model is a temporary shock to the growth rate of nominal wages. During the first quarter, these increase by 1 percentage point, during the second by 0.75 percentage point, during the third by 0.50 percentage point, and during the fourth by 0.25 percentage point. The figures labelled 1.5 illustrate the response functions for real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate in the nine models for 24 quarters following the beginning of the dynamic simulation.[25]

This shock can be interpreted as a standard Keynesian “wage-push” shock, in which wage dynamics generate inflation. We assume that this shock raises real wages beyond the marginal productivity of labour, i.e., the equilibrium condition. After a short lag, this “wage-push” generates an increase in inflation and a fall in production in all models. Monetary authorities react by boosting the short-term interest rate, which increases by about 25 basis points during the first quarter and continues to rise thereafter. It eventually peaks 75 basis points above the control value in the case of MULTIMOD and 225 basis points higher in QPM. Over the course of the second year of the simulation, the interest rate gradually begins to converge towards its starting value, to the extent that inflation and production return to their initial paths. After 24 quarters, inflation returns to the steady-state value in nearly all models. We observe, however, that the response of inflation is particularly persistent in FOCUS, QPM and MTFM, while production continues to move away from the steady-state solution in WEFA.

vi.) A temporary shock to short-term interest rates[26]

This shock is a temporary increase in short-term interest rates, which increase by 100 basis points, 75 basis points, 50 basis points, and 25 basis points, respectively, during the first four quarters. Figures 1.6a to 1.6d illustrate the response functions for real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate for the 11 models over a 24-quarter time horizon.

This shock can be interpreted as a temporary reduction of the inflation-control target. The increase in the real interest rate has a negative impact on the output gap and on inflation. The reaction of monetary authorities, which is artificially maintained over the course of the first year, accelerates the decline in production and in inflation. After approximately four quarters, the fall in inflation, combined with the excess supply generated by the initial shock, prompts monetary authorities to reduce interest rates below their initial level. Between the twelfth and sixteenth quarters, output, inflation, and the interest rate have returned to their control values on average, except in the INTERLINK and M1-VECM models. These models yield response functions whose

25. The response functions for the NAOMI and the M1-VECM models are omitted because they do not include wages.

26. Table 3 presents a summary of the first four quarters response of real GDP, CPI inflation and the exchange rate following a temporary increase in short-term interest rates.

magnitudes are greater than those of the other models, which could be partly attributable to the higher sensitivity of output to interest rate movements in these models.

vii.) A permanent shock to long-term interest rates

This shock is a permanent increase of 100 basis points in the long-term interest rate for the duration of the simulation. Figures 1.7a to 1.7d illustrate the response functions for real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate for the nine models.[27]

This shock can be interpreted as a permanent increase in the term premium, attributable to an increased risk of inflation that may result from uncertainty surrounding the probability that a part of the central government's debt will be monetized.[28]

Despite the fact that short-term interest rates are below their control values, the increase in the long-term interest rate causes real GDP to fall in all models. After a short delay, inflation drops relative to its steady-state value, except in the case of MTFM. Monetary authorities react with additional reductions to interest rates. The most pronounced declines are observed in models that show the greatest fall in real GDP and in inflation, such as QPM and the two versions of FOCUS. It is of interest to note that the substantial appreciation of the Canadian currency within QPM and FOCUS after the eighth quarter of simulation explains a large share of the apparently permanent decline in real GDP in QPM and in inflation in the two versions of FOCUS. After a 24-quarter simulation, the output gap has practically closed in all models except QPM and CEFM. The fall in inflation relative to its control value does not seem to be absorbed within the simulation time frame in the FOCUS models.

viii.) A temporary shock to the nominal exchange rate[29]

The final shock we introduce into the 11 models is a temporary exchange rate shock over four quarters. During the first quarter, the Canadian currency depreciates by 1 per cent relative to

27. The response functions of the NAOMI and MI-VECM models are omitted because they do not include long-term interest rates.

28. The reasons why the term premium may increase are complex. According to liquidity-preference theory, the term premium primarily reflects inflationary risks (second moment).

29. Table 4 presents a summary of the first four quarters response of real GDP and CPI inflation following a temporary depreciation in the exchange rate.

that of the United States, during the second by 0.75 per cent, during the third by 0.50 per cent, and during the fourth by 0.25 per cent, finally returning to its original value during the fifth quarter. Figures 1.8a to 1.8d present the response of real GDP, inflation, the nominal short-term interest rate, and the nominal exchange rate, within the 1l models for 24 quarters following the beginning of the dynamic simulation.

This shock may be interpreted as a temporary loss of confidence by investors in assets denominated in Canadian dollars. Since this shock represents a change in investors' preferences, and not a change in economic fundamentals, monetary authorities need to increase the interest rate to counter the effects of the depreciation of the Canadian dollar on domestic prices. Depending on the model, the impact of the depreciation will most likely be transmitted over two channels. The first, a direct effect, works over an increase in import prices and the second, indirect, over an increase in net exports.

In all models, the temporary depreciation of the Canadian currency induces an increase in real GDP and inflation over the course of the first year of the simulation. As monetary authorities raise the interest rate, the stimulative impact of the depreciation is quickly dampened. During the first year, interest rates rise by less than 25 basis points, diluting the stimulus imparted by the shock. Between the twelfth and sixteenth quarters, production, inflation, interest rates, and exchange rates nearly regain their initial level, except in the case of production and exchange rate in the WEFA model and in the case of all variables in QPM.

This temporary exchange rate shock suggests production and inflation responses that vary widely between the models. The response of the exchange rate, in particular, is higher in the QPM and WEFA models. However, these two models show very different responses for production, inflation, and the interest rate. The direct and indirect impact of the currency depreciation on prices appears particularly pronounced in QPM relative to the other models. In this model, non-linearity and forward-looking expectations in the Phillips curve may partially explain these results.

4. Comparison of Several of the Models' Impulse Response Functions with those of a Vector-Autoregressive Model

Ideally, we would be able to subject each model to a rigorous and detailed examination to determine how much weight to assign the information it yields for our evaluation of monetary policy rules. However, the number and diversity of the models impose severe constraints on what is achievable in that area. Nevertheless, to explore the extent to which the models under consideration reflect some of the characteristics of the Canadian economy, a measure of distance is computed by comparing some of their response functions with those generated by a simple vector-autoregressive (VAR) model (explained in more details later in this section). We then use this distance measure to rank the various models. This ranking is used to perform a robustness check on the results of our evaluation of the monetary policy rules.[30]

We use this VAR to estimate the historical response of CPI inflation, real Canadian GDP and the Canada/U.S. exchange rate to two types of shock: a shock to real U.S. GDP and a shock to commodity prices. We select these shocks because their identification is relatively uncontroversial. It is generally acknowledged that these variables can be assumed exogenous with respect to the Canadian economy. This is the hypothesis we retain for identification.[31] The advantage of using a VAR model as the benchmark for comparison is that this type of model is relatively unconstrained and can thus better reflect the characteristics of the data than models that have more structure built in so as to yield more theoretical interpretations.

To facilitate comparison of the VAR model's responses with those of the other models, we assume, as in the previous section, that short-term interest rates are determined by the original Taylor rule.

Vectors of the model's endogenous and exogenous variables (respectively) can be written as follows:

---

30. For more information and details, see "The Performance and Robustness of Simple Monetary Policy Rules in Models of the Canadian Economy", by Côté, Kuszczak, Lam, Liu and Saint-Amant, available from the Bank of Canada website at http://www.bankofcanada.ca/workshop2001/.

31. The results of exogeneity tests we ran confirm this assumption. These results are available on demand.

$$Z_t = \begin{bmatrix} \Delta r_t \\ \Delta y_t \\ \Delta p_t \\ \Delta e_t \end{bmatrix} \text{ and } X_t = \begin{bmatrix} \Delta y_t^{US} \\ \Delta pcom_t \end{bmatrix} ,$$

where r_t is the real interest rate on 90-day commercial paper rate, y_t is the logarithm of real Canadian GDP, p_t is the log of the Canadian consumer price index, e_t is the log of the Canada-U.S. exchange rate (Canadian dollars per U.S. dollar), y_t^{US} is the log of real U.S. GDP, and $pcom_t$ is the log of the Bank of Canada's commodity price index. Δ indicates that we have taken the first differences of these variables. The data are from Statistics Canada, the OECD, and the Bank of Canada. Our sample ends in the fourth quarter of 2000 and starts in the first quarter of 1965, the beginning of the period covering our series of commodity prices.[32]

The model we estimate can be written as follows:

$$Z_t = cst + \sum_i^q \beta_i Z_{t-i} + \sum_i^q \Pi_i X_{t-i+1} + \varepsilon_t, \tag{3}$$

where cst is a vector of constants, β_i and Π_i are vectors of coefficients, and ε_t is a vector of error terms. The number of lags, q, is determined using a likelihood-ratio test applied to a model having a long-lag structure (maximum of eight lags) from which we eliminate one lag at a time, retaining four lags.

We simulate the response of Z_t to shocks to the variables contained in X_t. The variables y_t^{US} and $pcom_t$ are assumed exogenous with respect to Z_t, but cannot be assumed to be independent of each other. For example, we expect that, typically, an increase in real U.S. GDP will put upward pressure on commodity prices. To account for the endogenous response of one of these variables to a shock affecting the other, we run simulations on the VAR described in the preceding section.

32. The frequency of this series is annual for the period from 1965 to 1973. We transformed it to quarterly for purposes of this exercise (to complete).

These shocks are of the same magnitude as those run with the other models. We generate a 95 per cent confidence interval around the VAR's estimated responses to the various shocks using bootstrapping-type simulations.

Our approach has several limitations. First, even though the VAR is a good approximation of the historical relationship between the affected variables, it remains an approximation. Second, while it is calibrated to closely reflect the importance of the various components to the Canadian economy, the commodity price index we use may differ from that used in the other models. For example, some models contain several prices, or price indexes, for commodities, which may appear in different equations. The shock to real U.S. GDP is not affected by this problem. Third, we consider only two types of shocks and three variables in the comparison. It is possible that our conclusions would have been different had other shocks and other variables been examined.

Figures 1.2 and 1.3 in Appendix 2 compare the impact of real U.S. GDP and commodity price shocks on real GDP, CPI inflation, and the Canadian exchange rate in the various participating models with the corresponding responses from the VAR model. The confidence interval associated with the VAR is indicated with the red dashed line.[33]

i.) A temporary external shock

Real U.S. GDP is increased by 1 per cent, 0.75 per cent, 0.50 per cent, and 0.25 per cent, respectively, during the first four quarters. Note that this temporary shock incorporates the endogenous response of some U.S. macroeconomic variables, such as inflation and short-term interest rates, as well as the endogenous response of commodity prices.

An interesting observation about the reaction of real Canadian GDP to a shock to real U.S. GDP is that it is often smaller, in the very short run, in the participating models than in the VAR model. In particular, it responds much less in the CEFM, M1-VECM, MULTIMOD, FOCUS, FOCUS-CE and WEFA models than in the VAR model. It is also interesting to note that the NAOMI model (in the short term) and QPM overestimate the response of inflation compared to the VAR and other models.

---

33. In the case of a positive shock, the QPM model response is biased upwards compared to the linear VAR because QPM has a non-linear aggregate supply curve. We therefore took an average of the model's response to the positive and negative shocks.

In several models the response of the exchange rate is quite different from, indeed often outside the (wide) estimated confidence interval of, that yielded by the VAR. Incidentally, several models yield responses of a different sign from the VAR, especially in the short term. For example, while in the VAR, an increase in real U.S. GDP causes a short-term depreciation of the Canadian dollar, the MTFM, MULTIMOD, both versions of the FOCUS model, WEFA, M1-VECM, and INTERLINK models, predict the opposite. It is also interesting to note that, while the VAR predicts that the exchange rate should return to control following the temporary U.S. GDP shock, NAOMI, QPM, and INTERLINK predict large long-run depreciations of the Canadian dollar in response to that shock.

ii.) A temporary shock to commodity prices

During the first quarter, commodity prices increase 4 per cent, during the second 3.5 per cent, during the third 3 per cent, … , and during the eighth 0.5 per cent, returning to their initial value by the ninth quarter. Note that this shock incorporates the endogenous response of several U.S. macroeconomic variables, such as production, inflation, and short-term interest rates.

In several models, the response of real GDP to commodity price shocks resembles that generated by the VAR. MTFM, in the short term, and INTERLINK, in the short and the longer term, differ most from the VAR. These two models are, in fact, the only ones to predict a short-term fall in production in Canada in response to a positive commodity price shock.

As in the case of the VAR, the response of inflation to this shock is generally positive in the short term, except in the FOCUS model with backward-looking expectations. However, several models, especially the WEFA model and FOCUS-CE, show much stronger reactions than the VAR in the short term. The MULTIMOD and MTFM models are the closest to the VAR in this respect.

We find great variation in our results for the exchange rate. DRI, INTERLINK, and QPM are particularly divergent from the VAR (and the other models) with respect to this variable. While the VAR predicts that an increase in commodity prices causes a short-term appreciation of the Canadian dollar, the DRI, MULTIMOD, CEFM, INTERLINK, and QPM models forecast a depreciation.[34]

We calculate a measure of the distance between the results generated by the response functions of the participating models and the VAR—computed as the sum of squares of the differences between the impulse response functions of the models and the VAR. We then use this distance measure to rank the various models in terms of the distance of their response functions from those of the VAR. Table 5 presents the results of these calculations for the two types of shocks and the three variables we consider, along with an aggregate ranking over all shocks and variables. A minimum-rank criterion is used here. The model that is ranked first is the one, on average, with the lowest score for the two shocks and three variables (each ranked equally). The last-ranked model, i.e., that differing most from the VAR overall, is the one with the highest score.

Our comparison of the models' impulse response functions with those of a VAR comprising very little economic structure reveal that the MTFM, NAOMI and CEFM models reflect relatively better than the other models the typical response of the Canadian economy to shocks to real U.S. GDP and to commodity prices. Some models yield results that are close to the VAR for certain variables and certain shocks, but are much further in other cases. No model is among the closest to the VAR for all variables and all shocks. Every model contains at least a few impulse response functions that diverge significantly from those estimated by the VAR. However, the responses generated by the MTFM are closest to the VAR overall followed by NAOMI and CEFM, and those from M1-VECM, DRI and INTERLINK, the furthest.

5. Conclusions

This report examines and compares twelve models of the Canadian economy with respect to their paradigm, structure and dynamics properties. Although they are all "open economy models", the models are nevertheless quite different. The twelve models can be classified under two economic paradigms. The first one is the "conventional" paradigm (or the Phillips-curve paradigm) and the second one is the "money matters" paradigm.

---

34. The endogenous response of real U.S. GDP to the commodity price shock partially explains the effect it has on the exchange rate in QPM. Indeed, real U.S. GDP tends to decline in response to this shock, reducing our exports to the United States. To maintain foreign debt at a constant level, the Canadian dollar must depreciate in order to stimulate Canadian exports.

Under the conventional paradigm, inflation is determined by price adjustments in reaction to inflation expectations and by factor disequilibrium in various markets (e.g., the labour market, the goods market). While most models fall under the "conventional" paradigm, there are nevertheless important differences within this paradigm.

Under the "money matters" paradigm, inflation is mostly determined by movements in monetary aggregates. Two models fall under this category: the M1-VECM in which the money gap - the disequilibrium between money supply and estimated long-term money demand - influences inflation, while still allowing a role for the output gap, and the Limited Participation Model (henceforth LPM), in which rigidities in adjusting money balances are the main source of the short-run non-neutrality of monetary policy.

Within the conventional paradigm, inflation is determined by a linear Phillips curve in six participating models: CEFM, DRI, FOCUS, INTERLINK, WEFA and NAOMI. While the M1-VECM falls under the "money matters" paradigm, the disequilibrium in the product market plays also a role in the adjustment of prices. Asymmetries in the inflation process are introduced in the models of FOCUS-CE, MULTIMOD and QPM. On the other hand, the MTFM model of the Conference Board uses a very disaggregated approach to determining the adjustment of prices.

Eight out of twelve models assume purely backward-looking inflation expectations: CEFM, DRI, FOCUS, INTERLINK, MTFM, WEFA, M1-VECM, and NAOMI while the following three models also impute model-consistent inflation expectations to them: FOCUS-CE, MULTIMOD, and QPM. In QPM and MULTIMOD, in particular, the hybrid Phillips curve assigns more weight to backward-looking inflation expectations as compared to model-consistent inflation expectations. The LPM is the only calibrated model which incorporates purely model-consistent behaviour and is optimally derived from microfoundations.

The models can be also differentiated based on the channels through which monetary policy actions affect the economy. In most participating models, monetary policy actions affect the economy through the level of short-term interest rates. This is the case of the following models: CEFM, DRI, FOCUS, FOCUS-CE, INTERLINK, MTFM, WEFA, LPM and MULTIMOD. In other models, such in the M1-VECM, NAOMI and QPM, the monetary policy transmission mechanism works through the slope of the yield curve. In all models, monetary policy affects the real economy in the short run because of nominal rigidities (wages and/or prices, money balances for LPM), but remains neutral in the long term owing to price flexibility. Finally, there are also differences in estimation techniques and sizes. For example, NAOMI is a small estimated model while QPM is a large-scale calibrated model. MTFM, on the other hand is a fairly disaggregated model compared to most of the other models.

To further understand the structure and properties of the twelve models of the Canadian economy, i.e., the way the various models respond to different macroeconomic shocks, we have examined the dynamic properties of the various models when they use a simple monetary authority reaction function, such as the one proposed by John Taylor (1993). The eight standard shocks considered in our study reveal significant differences in the dynamic properties of the participating models. However, we would expect that similar models react to the same shock in the same way—yet that is not what we observe.

Finally, our comparison of the models' reaction functions with those of a VAR comprising very little economic structure suggests that some models, especially MTFM, NAOMI and CEFM do a slightly better job than the others of reflecting the typical response of the Canadian economy to shocks to real U.S. GDP and to commodity prices. Nonetheless, every model contains at least a few reaction functions that diverge significantly from those estimated by the VAR.

References

Adam, C. and S. Hendry. 2000. “The M1 Vector-Error-Correction Model: Some Extensions and Applications.” In *Money, Monetary Policy, and Transmission Mechanisms*, 151–80. Proceedings of a conference held by the Bank of Canada, November 1999. Ottawa: Bank of Canada

Armour, J, B. Fung and D. Maclean, “Taylor Rules in the Quarterly Projection Model,” Working Paper 2002-1, Bank of Canada.

Black, R, D. Laxton, D. Rose, and R. Tetlow. 1994. “The Steady-State Model: SSQPM, The Bank of Canada’s New Quarterly Projection Model, Part 1,” Technical Report No. 72. Ottawa: Bank of Canada.

Blanchard, O.J. 1985. “Debt, Deficits, and Finite Horizons.” *Journal of Political Economy* 93 (April): 223–46.

Buiter, W.H. 1988. “Death, Birth, Productivity Growth and Debt Neutrality.” *The Economic Journal* 98 (391): 279–93.

Coletti, D., B. Hunt, D. Rose, and R. Tetlow. 1996. “The Dynamic Model: QPM, The Bank of Canada’s New Quarterly Projection Model, Part 3,” Bank of Canada Technical Report No. 75.

Côté, D. and J.-P. Lam, “Simple Rules in the M1-VECM,” presented at the 2001 meeting of the Canadian Economic Association at McGill University (Montréal).

DeSerres, A., B. Robidoux, and B.-S. Wong. 1998. “CEFM96: Part 2, Dynamic Forecasting and Simulation Properties,” July 1998, Working Paper 98–06.

Dungan, P. 1997. “Quarterly Forecasting and User Simulation Model of the Canadian Economy.” Institute for Policy Analysis, University of Toronto.

Hendry, Scott (1995) "Long-Run Demand for M1", Bank of Canada Working Paper 95-11.

Hendry, Ho, and Moran, 2001. “Policy Rules in Limited Participation Models.”

Hetzel, Robert L., The Taylor Rule: Is it a Useful Guide to Understanding Monetary Policy?” Federal Reserve Bank of Richmond *Quarterly Review*, Volume 86, No. 2, 2000.

Kozicki, Sharon, “How Useful are Taylor Rules for Monetary Policy,” Federal Reserve Bank of Kansas City *Economic Review*, 1999.

Laxton Douglas, Peter Isard, Hamid Faruqee, Eswar Prasad and Bart Turtelboom, “MULTIMOD Mark III: The Core Dynamic and Steady-State Models,” 1998, International Monetary Fund Occasional Paper No. 164.

Levin, Andrew, Volker Wieland and John C. Williams, “Robustness of Simple Monetary Policy Rules under Model Uncertainty,” in John B. Taylor, ed., *Monetary Policy Rules*, Chicago: University of Chicago Press, 1999, 263-299.

Murchison, Stephen, “A New Quarterly Forecasting Model Part II: A Guide to Canadian NAOMI,” Dept of Finance Working Paper 2001-25.

O’Reilly, B, G. Paulin, and P. Smith. 1983. “Responses of Various Econometric Models to Selected Policy Shocks,” Technical Report No. 38. Ottawa: Bank of Canada.

Richardson, P. 1988. “The Structure and Simulation Properties of OECD’s INTERLINK Model.” OECD Economic Studies, No. 10, Spring.

Robidoux, B. and B.-S. Wong. 1998. “CEFM96: Part 1, Model Structure,” July 1998. Working Paper 98–05.

Sims Chris, “A Review of Monetary Policy Rules, John B. Taylor, ed.,” *Journal of Economic Literature*, 2001, 39(2).

Svensson, Lars E.O., “Inflation Targeting as a Monetary Policy Rule,” *Journal of Monetary Economics*, 1999, 43, 607-654

Svensson, Lars E.O., “Open-Economy Inflation Targeting,” *Journal of International Economics*, 2000, 50(1), 155-83.

Svensson, Lars E.O, “What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy Through Targeting Rules,” 2001, Working Paper.

Taylor, John B., “Discretion Versus Policy Rules in Practice,” *Canergie-Rochester Conference Series on Public Policy*, 1993, 39, 195-214.

Taylor, John B, “The Robustness and Efficiency of Monetary Policy Rules as Guidelines for the ECB,” *European Economic Review*, 1999.

Taylor, John B, “The Role of the Exchange Rate in Monetary Policy Rules,” *The American Economic Review*, 2001, 91.

Weil, P. 1989. “Overlapping Families of Infinitely-Lived Agents.” *Journal of Public Economics* 38 (March): 183–98.

Table 1: Summary of Questionnaire

	FOCUS	WEFA	DRI	MTFM	NAOMI	INTERLINK	CEFM	MULTIMOD	LPM	M1VECM	QPM
Expectations											
1. Backward-looking, i.e., depend on only lagged values of variables.	Y	Y	Y	Y	Y	Y	Y	N	N	Y[a]	N
2. Combination of forward and backward components.	N[b]	N	N	N	N[c]	N	N	Y	Y[d]	N	Y
3. Do the inflation targets have any explicit role in the expectation process? If yes, describe how.	N	N	N	N	N	N	N	Y[e]	N	N	Y[f]
Asymmetry or non-linearities											
4. Interest rate increases have the same effect on demand as decreases.	Y	Y	Y[g]	Y	Y	Y	Y	Y	Y	Y	Y
5. Exchange rate appreciations have the same effect on demand as depreciations.	Y	Y	Y[h]	Y	Y	Y	Y	Y	Y	Y	Y
6. Exchange rate responds to the size of the interest rate differential and not how quickly it opens.	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	N[i]
7. Inflation responds differently to excess supply than excess demand.	Y[j]	N/A	Y	N/A	N	N	N	Y[k]	N	N	Y
Other											
8. Do you have estimates of potential output?	N	N	Y	N	Y[l]	Y	Y	Y	Y[m]	N	Y
9. Do you have estimates of equilibrium interest rate?	N	N	N	N	N	N	N	Y	Y	N[n]	Y
10. Do you have estimates of equilibrium exchange rate?	N	N	N	N	Y	N	N	Y	Y	N	Y
11. What is the sample period of the model?	65	66	61		73		70	70		56	65
12. Frequency of data.	Q	Q	Q	Q	Q	SA	Q	A	Q	Q	Q

a. The money gap, however, is supposed to bring some forward-looking dimension to the model.
b. Consistent expectation version.
c. Monetary policy is forward-looking.
d. Model-consistent expectations only.
e. No explicit role in expectations other than through the weight on the model-consistent lead of inflation in the expectation process.
f. Forward-looking expectations are model-consistent. If monetary authorities achieve target, then this is taken into account.
g. Small changes have similar effects. The biggest non-linearities occur when economy is in an excess demand position.
h. Small changes have similar effects. The biggest non-linearities occur when economy is in an excess demand position.
i. Uncovered interest differential is the driving effect, so persistence of gap matters. Also, expected future *pfx* has a backward-looking component. $pfx_t - E_t(pfx_{t+1}) = 0.25[i^* - i]$
j. Phillips curve is non-linear in usual form.
k. Phillips curve is non-linear.
l. Potential output is exogenous to model.
m. For 8, 9, and 10, use either steady-state values or values arising in a model with no nominal rigidities.
n. A steady-state value is imposed.

Table 2: Model Shocks

Shock	Description	Details
1. Domestic Demand	A 4-quarter transitory increase to the levels of consumption and investment at the same time.	Shock on consumption and investment: Q1: 1.00% Q2: 0.75% Q3: 0.50% Q4: 0.25% i.e. the levels of consumption and investment increase by one percent at the 1-quarter horizon and then progressively come back to control (there is no permanent increase in the level of output).
2. External Demand	A 4-quarter transitory increase in the level of real U.S. output with endogenous responses of U.S. inflation and interest rate, and world commodity prices.	Shock on U.S. GDP: Q1: 1.00% Q2: 0.75% Q3: 0.50% Q4: 0.25% Endogenous response of U.S. inflation Endogenous response of U.S. short-term interest rate Endogenous response of world commodity prices
3. Commodity prices	A 8-quarter transitory increase in the level of real commodity prices with endogenous responses of U.S. output, inflation and interest rate.	Shock on commodity prices: Q1: 4.00% Q2: 3.50% Q3: 3.00% Q4: 2.50% Q5: 2.00% Q6: 1.50% Q7: 1.00% Q8: 0.50% Endogenous response of U.S. output Endogenous response of U.S. inflation Endogenous response of U.S. short-term interest rate
4. Consumer Price	A 4-quarter transitory increase to the level of CPI excluding food, energy and indirect taxes.	Shock on CPI: Q1: 1.00% Q2: 0.75% Q3: 0.50% Q4: 0.25%

Shock	Description	Details
5. Wage growth	A 4-quarter transitory increase to nominal wage growth.	Shock on wage growth: Q1: 1.00 percentage point Q2: 0.75 of a percentage point Q3: 0.50 of a percentage point Q4: 0.25 of a percentage point
6. Short-term interest rate	A 4-quarter transitory increase in short-term interest rate.	Shock on short-term interest rate: Q1: 100 basis points Q2: 75 basis points Q3: 50 basis points Q4: 25 basis points
7. Long-term interest rate	A permanent change in the term premium.	Shock on long-term interest rate: Permanent increase of 100 basis points
8. Nominal exchange rate shock	A 4-quarter temporary increase to the risk premium on the exchange rate (a depreciation).	Shock on exchange rate: Q1: 1.00% Q2: 0.75% Q3: 0.50% Q4: 0.25%

Table 3: Peak Response to a Temporary Change in Short-Term Interest Rates

	Least Sensitive peak response in the first four quarters is < 0.25 per cent	Moderately Sensitive peak response in the first four quarters is between 0.25 and 0.5 per cent	Most Sensitive peak response in the first four quarters is > 0.5 per cent
GDP	CEFM FOCUS-CE WEFA	DRI INTERLINK MULTIMOD NAOMI QPM M1-VECM	FOCUS MTFM
Inflation	CEFM DRI INTERLINK MTFM MULTIMOD QPM WEFA	FOCUS FOCUS-CE NAOMI	M1-VECM
Exchange rate	CEFM DRI	QPM WEFA	FOCUS FOCUS-CE INTERLINK MTFM MULTIMOD NAOMI M1-VECM

* Short-term interest rates are increased by 100 basis points, 75 basis points, 50 basis points, and 25 basis points, respectively, during the first four quarters. Results for the LPM model were not available.

Table 4: Peak Response to a Temporary Change in the Exchange Rate

	Least Sensitive peak response in the first four quarters is < 0.25 per cent	Moderately Sensitive peak response in the first four quarters is between 0.25 and 0.5 per cent	Most Sensitive peak response in the first four quarters is > 0.5 per cent
GDP	CEFM DRI FOCUS FOCUS-CE INTERLINK MULTIMOD NAOMI WEFA	MTFM	QPM M1-VECM
Inflation	DRI FOCUS INTERLINK MTFM MULTIMOD NAOMI M1-VECM	CEFM FOCUS-CE WEFA	QPM

* The Canadian currency relative to that of the United States depreciates by 1 per cent in the first quarter, by 0.75 per cent in the second, 0.50 per cent in the third and 0.25 during the fourth. Results for the LPM model were not available.

Table 5: Distance of the Models from the VAR

	Shock to real U.S. GDP			Shock to commodity prices			
Models	Real GDP	CPI Inflation	Exchange Rate	Real GDP	CPI Inflation	Exchange Rate	Aggregate Measure
DRI	5	8	8	4	8	11	10
FOCUS	6	1	4	9	9	2	5
FOCUS-CE	4	4	3	6	11	8	6
INTERLINK	11	11	11	11	6	10	11
CEFM	9	3	2	1	5	7	3
MTFM	1	2	5	10	2	3	1
MULTIMOD	7	6	6	5	1	5	4
M1-VECM	8	7	10	8	4	6	9
NAOMI	3	9	7	2	3	1	2
QPM	2	10	9	3	7	9	8
WEFA	10	5	1	7	10	4	7

Appendix 1

For the two external shocks, a shock to the real U.S. output and a shock to commodity prices, we also calculate endogenous responses of some U.S. macroeconomic variables, as they are likely to play an important role in models of the Canadian economy. We derive these endogenous responses from a VAR model consisting of four variables, the U.S. real output, U.S. inflation, U.S. short-term interest rate and world commodity prices.

The vector containing these variables can be written as follows:

$$Z_t = \begin{bmatrix} \Delta \mathrm{y}^{\mathrm{us}}{}_t \\ \Delta \mathrm{pcom}_t \\ \Delta p^{\mathrm{us}}{}_t \\ \Delta \mathrm{r}^{\mathrm{us}}{}_t \end{bmatrix} ,$$

where $\mathrm{y}^{\mathrm{us}}{}_t$ is the log of real U.S. GDP, $pcom_t$ is the log of the Bank of Canada's commodity price index, $p^{\mathrm{us}}{}_t$ is the log of the U.S. consumer price index, and r^{us} is the U.S. 90-day commercial paper rate. The data are from Statistics Canada, the OECD, and the Bank of Canada. Our sample starts in the first quarter of 1970 and ends in the fourth quarter of 2000.[35]

The model we estimate can be written as follows:

$$Z_t = cst + \sum_i^q \beta_i Z_{t-i} + \varepsilon_t \quad , \tag{3}$$

where cst is a vector of constants, β_i is a vector of coefficients, and ε_t is a vector of error terms. The number of lags, q, determined using a likelihood-ratio test, is equal to six.

We simulate the response of Z_t to the shocks to $\mathrm{y}^{\mathrm{us}}{}_t$ and $pcom_t$ outlined in Table 2. Because the variables in Z_t are likely to be serially correlated with each other, ε_t is orthogonalized using the Choleski factorization method. The order of the variables is as shown in vector Z_t. Here we assume that a shock to the real U.S. output affects all other variables in the

35. The frequency of this series is annual for the period from 1970 to 1973. We transformed it to quarterly for purposes of this exercise.

same period, while a shock to the world commodity prices only affects the U.S. output in the next period.