Inflation, Nominal Debt, Housing, and Welfare*

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PRELIMINARY AND INCOMPLETE

Abstract

This paper evaluates the welfare effects of lowering the long-run inflation target in a life-cycle, heterogeneous-agent model of housing, nominal debts, and money (i.e., a liquid asset). In the presence of transactions costs and borrowing constraints, agents make portfolio choices in order to smooth out idiosyncratic earnings risk. We find that lowering inflation from 2% to 1% improves welfare and that the welfare gain is even larger when the reform achieves full price stability (zero inflation).

Keywords: Welfare cost of inflation, Distribution, Heterogeneity, Nominal assets and debts, Housing, Transactions costs.

*The views expressed in this paper are those of the authors. No responsibility or imputation of the views should be attributed to the Bank of Canada, the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

The widespread move towards lower inflation around the globe and the successful adoption of formal inflation targets by many industrialized countries such as Canada over the last two decades have generated a considerable interest in the welfare benefits of price stability. For example, the Bank of Canada is actively researching the possibility of lowering the inflation target below two percent in anticipation of its 2011 inflation-control policy agreement with the Government of Canada. In this paper, we contribute to this policy debate by quantitatively evaluating the welfare effects of reducing long-run inflation through changes in the value of nominal assets and by assessing the potential political support for such a reform.

To address this question, we develop a life-cycle model of housing, nominal bonds, and money with uninsurable idiosyncratic earnings risk.[1] To smooth out income uncertainty, households make portfolio decisions in terms of housing, nominal bonds, and money. Housing plays a dual role; it is both a durable consumption good and an asset. Housing investment is subject to nonconvex transactions costs, and hence housing is a lumpy investment. Specifically, because of the nonconvex transactions cost, households will infrequently adjust their housing size and will adjust by big amounts when they choose to do so. In addition, housing can be used as collateral for nominal debts but borrowing against a house requires a down payment. Borrowing and lending through private nominal bonds is also subject to transactions costs. Data show that about 80% of mortgages in Canada are in the form of a fixed long-term contract. Since refinancing these contracts before the maturity date is not costless, we introduce the transactions costs to emulate the nature of these fixed long-term nominal contracts. In contrast to housing and nominal bonds, money holdings do not face transactions costs and as a result, in our model money has an advantage over bonds and house as a vehicle for self-insurance. Households derive utility from real money balances and this is to proxy for the services that money provide in facilitating transactions. In each period, households also decide what fraction of their time to allocate to working.

In our model, inflation can affect welfare through several channels. We will start first by discussing how lower inflation affects welfare in the station-

[1] In addition to the heterogeneity in age, the uninsurable earnings risk will generate within-generation heterogeneity.

ary equilibrium and then subsequently discuss the transition path between steady states that differ in their rates of inflation. First, a decrease in inflation increases the real value of money holdings. This generates both substitution and wealth effects. On one hand, the substitution effects induce households to reallocate their portfolios by increasing their real money holdings, and by decreasing their holdings of other assets. On the other hand, the wealth effects lead to an increase in the holdings all assets. The extent of these portfolio adjustments is limited by transaction costs. In addition, the increase in the real value of money enhances the ability of households to self-insure against idiosyncratic shocks in the presence of borrowing constraints and transactions costs. Second, inflation acts as a distortionary tax and hence a reduction in inflation improves welfare. Third, higher inflation can have a positive effect on welfare through the redistributional channel. Inflation redistributes wealth and this operates through the lump-sum injections of money. This lump sum transfer relaxes the borrowing constraints of young and poor households. Thus, this effect improves lifetime utility by flattening the utility age-profile. As a result, lower inflation reduces this positive effect.

With respect to transition paths, a decrease in inflation will have some effects in the short-run and these effects depend on the agent's age and the distribution of wealth. Because the nominal interest rate was fixed based on expected inflation before the policy change, a decrease in inflation leads to an increase in the real payments on bonds and therefore there is a wealth transfer from borrowers to lenders. Lenders win out and borrowers will lose from disinflation. The strength of this channel depends on the extent to which households can adjust their portfolio. Given that borrowers in nominal bonds lose wealth from lower inflation, this negative wealth effect could lead to further adjustment of asset portfolios and labour supply. Housing may be downsized and money holdings may be lowered while the labour supply would increase to compensate for the wealth loss. For lenders, the effects would be the opposite. However, these portfolio adjustments are limited by the transactions costs in housing and bonds.

A parameterized version of the model based on Canadian cross-sectional and aggregate data is employed to evaluate the welfare effects of lowering the long-run inflation rate. The social welfare criterion used to conduct our policy experiment is the ex-ante lifetime utility of a newborn in a stationary equilibrium. When we take into account the transition path from the initial steady state to the new steady state with a lower inflation rate, we account for all households alive at the time of the reform. Doing so will provide insights into potential political support for the low inflation target reform.

Our main finding is that lowering inflation improves welfare. For example, lowering the long-run inflation target from 2% to 1% generates a long-run welfare gain of about 0.32%. Moreover moving to full price stability with zero long-run inflation leads to a larger welfare gain of about 0.85%.

Our paper is related to papers on the redistributional effects of inflation through the revaluation of nominal wealth. These include Doepke & Schneider (2006a), Doepke & Schneider (2006b), Meh & Terajima (2008), and Meh, Ríos-Rull, & Terajima (2008). These papers, however, do not analyze the welfare cost of anticipated inflation. Our work also relates to the literature on portfolio choices in the presence of transaction costs. Aiyagari & Gertler (1991) studied the equity premium puzzle in a Bewly-type economy with transaction costs on trading equity, but do not analyze the welfare cost of inflation. Similarly, Heaton & Lucas (1996) evaluate the economic effects of incomplete markets on risk sharing and asset pricing when agents can trade in securities, but are subject to borrowing constraints and transaction costs. Our work is also related to the literature on welfare cost of inflation in a heterogenous agent model. See Chiu & Molico (2007), İmrohoroğlu (1992), and Erosa & Ventura (2002).

The plan of the paper is as follows. In the next section we describe the model. Section 3 presents the parameterization of the model. Section 4 presents the findings. Section 5 concludes.

2 The model

We consider a small open economy with a given world real interest rate $\overline{r}$. The economy is populated by overlapping generations of individuals who live for a maximum of J periods. In each period, a continuum of newborn households enters the economy. Each agent retires at an exogenous age jr and during retirement ($j \geq jr$) receives a retirement benefit $Tr_{j,t}$ that is financed with a proportional payroll tax rate θ_t. Workers do not receive retirement benefits, that is, $Tr_{j,t} = 0$ for $j < jr$.

Preferences Each individual of age j maximizes his expected discounted lifetime utility,

$$E_0 \sum_{j=1}^{J} \beta^{j-1} U(c_{j,t}, 1 - l_{j,t}, h_{j,t}, M_{j,t}/P_{t-1}), \tag{1}$$

where $c_{j,t}$ is nondurable consumption at age j in period t. $1 - l_{j,t}$ is the fraction of hours spent on leisure ($l_{j,t}$ is for labour hours). $h_{j,t}$ is the housing service. $M_{j,t}$ is current money holdings of age-j households in period t and is carried from the previous period. P_t is the aggregate price level at time t. β is the subjective discount factor. We assume that one unit of the housing stock is transformed into one unit of housing service and that all households are home owners.

Endowments In each period, households are endowed with one unit of time which can be supplied to the labor market at a competitive wage rate w_t. Agents differ in their labor productivity due to differences in age and realizations of idiosyncratic uncertainty. The labor productivity of an individual of age j is given by $\varepsilon_j z$; where $\{\varepsilon_j\}_{j=1}^{J}$ denotes the age profile of average labor productivity. Retired households ($j \geq jr$) are not productive and therefore $\varepsilon_j = 0$. The stochastic component $z \in \mathcal{Z}$ follows a first-order finite state Markov process with a transition probability[2]

$$Q(z, z') = \Pr(z_{t+1} = z', z_t = z).$$

The shock received by age-1 agents are drawn from the stationary distribution $Q_z^*(z)$. Productivity shocks are assumed to be independently distributed across agents, and the law of large numbers is assumed to hold. This determines that no uncertainty will exist in the aggregate, even though uncertainty over the market return to labor supplied will prevail at the individual level.

Asset market structure Households make portfolio decisions to buffer their idiosyncratic earnings shocks, but the extent of this is limited by transactions costs and borrowing constraints. We consider three sets of assets: a house, nominal bonds, and money.

[2]All variables denoted by prime (’) refer to next period variables.

In addition to providing housing services, a house is an investment good. Furthermore, a house also serves as a collateral when borrowing. The purchase or sale of a house is subject to a nonconvex transactions cost function:

$$\Phi(h_{t+1}, h_t) = \phi h_t \ \text{ if } \ |h_{t+1} - h_t| > 0, \tag{2}$$

where the cost is proportional to the current stock of the house and the parameter ϕ controls the magnitude of the cost. This transaction cost function captures the common view in the housing market that fees paid to realtors correspond to a fraction of the value of the house being sold. Nonconvex transactions costs make housing a lumpy investment that is infrequently adjusted.

The household of age j can also borrow or save through a nominal bond $B_{j+1,t+1}$ at time t. The nominal interest rate of the bonds between periods t and $t+1$ and set at time t is denoted by R_{t+1}. Both buyers and sellers of bonds are subject to a quadratic transactions cost function:

$$\Psi(b_{t+1}, b_t) = \left(\frac{\psi}{2}\right)(b_{t+1} - b_t)^2, \tag{3}$$

where $b_{t+1} = B_{t+1}/P_t$ is the real value of the nominal bonds in terms of current consumption and the parameter ψ governs the size of the transactions cost. Quadratic transactions costs imply that the marginal cost of trading in the bond market increases with the size of trade and therefore households will have an incentive to adjust their bonds holdings by smaller amounts. As we interpret these nominal debts to be mortgages, these transactions costs are imposed to emulate the nature of fixed long-term mortgage contracts. Due to the presence of the costs, households would more likely adjust the debt size less, which is a feature of fixed long-term mortgage contracts. Borrowing is also subject to a collateral constraint where households can borrow up to a fraction of their house value. The collateral constraint (in nominal terms) at time t faced by age-j households, who are choosing to borrow $B_{j+1,t+1}$ and hold housing stock $h_{j+1,t+1}$ is defined as follows:

$$(1 + R_{t+1})B_{j+1,t+1} \geq -\xi E_t\Big[P_{t+1}(1 - \delta^h)h_{j+1,t+1}\Big], \tag{4}$$

where ξ is the loan-to-value ratio ($1 - \xi$ is the downpayment requirement) and δ^h is the depreciation rate of housing stock.

Households can also choose to hold money $M_{j+1,t+1}$ for the following period. Contrary to housing and bonds, the adjustment of money holdings is not subject to transactions costs. In the presence of transactions costs and uninsurable idiosyncratic risks, agents will choose to hold money for precautionary motives since it is the most *liquid asset*. However, money is a dominated asset as its nominal interest rate is zero. In addition, households choose to hold money for transactions. This transaction demand is captured by assuming that households derive utility from real money holdings. Unlike most papers in the literature, we do not assume additive separability of the function between real money balances and nondurable consumption; this formulation allows a further channel through which money can affect real activity, namely an effect of real money balances on the current marginal utility of nondurable consumption. Age-1 households start with zero bond holdings ($B_{1,t} = 0$), with a small amount of nominal money holdings $M_{1,t} = M_0$ and a small amount of housing $h_{1,t} = h_0$.

The budget constraint in nominal terms is defined as follows:

$$P_t c_{j,t} + B_{j+1,t+1} + M_{j+1,t+1} + P_t h_{j+1,t+1} + P_t \Phi(h_{j,t}, h_{j+1,t+1}) + P_t \Psi(b_{j,t}, b_{j+1,t+1}) = \\ (1-\theta_t) P_t w_t \varepsilon_j z_t l_t + P_t(1-\delta^h) h_{j,t} + R_t B_{j,t} + M_{j,t} + P_t Tr_{j,t} + \tau_t \overline{M}_t, \quad (5)$$

where $\tau_t \overline{M}_t$ is the per-capita (age-independent) lump-sum transfer to households with τ_t being the growth rate of the per-capita money supply $\overline{M}_t$. Note that in the stationary equilibrium, τ is equal to the long-run inflation rate.

It is convenient to express the budget constraint in real terms; by dividing equation (5) by P_t we have the following:

$$c_{j,t} + h_{j+1,t+1} + b_{j+1,t+1} + m_{j+1,t+1} + \Phi(h_{j,t}, h_{j+1,t+1}) + \Psi\Big(b_{j,t}, b_{j+1,t+1}\Big) = \\ (1-\theta_t) w_t \varepsilon_j z_t l_t + (1-\delta^h) h_{j,t} + \Big(\tfrac{1+R_t}{1+\pi_t}\Big) b_{j,t} + \tfrac{m_{j,t}}{1+\pi_t} + Tr_{j,t} + \tau\Big(\tfrac{\overline{m}_t}{1+\pi_t}\Big), \quad (6)$$

where we define $b_{j+1,t+1} \equiv B_{j+1,t+1}/P_t$, $m_{j+1,t+1} \equiv M_{j+1,t+1}/P_t$ and $\overline{m}_t \equiv \overline{M}_t/P_{t-1}$. According to these definitions, $b_{j+1,t+1}$ and $m_{j+1,t+1}$ are the real values of next period bonds and money holdings, respectively, in terms of current consumption. The nominal interest R_t is the nominal interest rate between periods $t-1$ and t and was set in period $t-1$.

Similarly, the collateral constraint in real terms is given by:

$$(1+R_{t+1}) b_{j+1,t+1} \geq -\xi E\Big[(1+\pi_{t+1})(1-\delta^h) h_{j+1,t+1}\Big]. \quad (7)$$

The nominal interest rate, R_{t+1}, is derived from the Fisher equation: $1 + R_{t+1} = (1+\overline{r}) E_t[1+\pi_{t+1}]$.

Production Competitive firms produce output with a Cobb-Douglas production function $F(K_t, N_t) = AK_t^{\alpha} N_t^{1-\alpha}$, where K_t and N_t are capital and labor inputs, respectively, at time t, and α is the capital income share. Given prices, firms maximize profits and therefore we have the following: $\overline{r} + \delta^k = A\alpha \left(\frac{K_t}{N_t}\right)^{\alpha-1}$ and $w = A(1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha}$, where δ^k is the depreciation rate of capital. Given that the world real interest rate $\overline{r}$ is fixed, the capital labor ratio is constant.

The central bank and the government The money supply follows the law of motion:

$$\overline{M}_{t+1} = (1+\tau)\overline{M}_t, \tag{8}$$

where $\overline{M}_t$ is the per-capita nominal money supply at time t. Dividing equation (8) by P_t,we have the following dynamics for the real money supply:

$$\overline{m}_{t+1} = \left(\frac{1+\tau}{1+\pi_t}\right)\overline{m}_t. \tag{9}$$

In addition, the government levies taxes on labor income to finance retirement benefits under a balanced budget every period.

Foreigners There are foreigners that can participate in the domestic asset market. We denote their nominal position at the end of the period t by B_{t+1}^F and their real asset by a_{t+1}^F. In this model, net exports are equal to interest payments to foreigners minus new foreign investment in the countries. Let us denote $b_{t+1}^F = B_{t+1}^F / P_t$ as the real value of nominal position in terms of current consumption.

2.1 Optimization problem of households

In each period, agents are characterized by their holdings of housing (h), bonds (b), and money (m) as well as their idiosyncratic earnings shock (z) and age (j). We define $V_t(h, b, m, z, j)$ as the value function in period t with individual state (h, b, m, z, j). Note that the value function is defined not only for the stationary equilibrium but also for the transition dynamics. In a stationary equilibrium this function will be independent of time. We also assume households do not die with assets or debts so that $V_t(\cdot, J+1) = 0$ in each period t. We denote $\Gamma_t(h, b, m, z, j)$ as the measure of agents of type

(h, b, m, z, j) in period t. The optimization problem of the household can be formulated recursively as follows:

$$V_t(h, b, m, z, j) = \max_{h', b', m', c, l} \left\{ U(c, l, h, m) + \beta E_t \left[V_{t+1}(h', b', m', z', j+1) \right] \right\} \quad (10)$$

subject to

$$c + h' + b' + m' + \Phi(h, h') + \Psi(b, b') = (1 - \theta_t) w_t \varepsilon_j z l +$$
$$(1 - \delta^h) h + \left(\frac{1 + R_t}{1 + \pi_t} \right) b + \frac{m}{1 + \pi_t} + \tau \left(\frac{\overline{m}_t}{1 + \pi_t} \right) + Tr_{j,t} \quad (11)$$

$$b' \geq -\xi E_t \left[\left(\frac{1 + \pi_{t+1}}{1 + R_{t+1}} \right) (1 - \delta^h) h' \right] \quad (12)$$

$$m' \geq 0,\ h' \geq 0, c \geq 0,\ 0 \leq l \leq 1. \quad (13)$$

2.2 Definition of the equilibrium

We are now ready to define the competitive equilibrium of our small open economy. Denote the endogenous individual state by $x = (h, b, m)$. Let $\Gamma_t(x, z, j)$ denote the measure of households of type (x, z, j).

Definition 1 Given the world risk-free rate, $\overline{r}$, a money supply growth rate τ, a sequence of real per-capita money stock $\{\overline{m}_t\}_{t=0}^{\infty}$, and initial conditions H_0, b_0, K_0, $\overline{m}_0$, and Γ_0, a recursive competitive equilibrium is a sequence of functions of households $\{V_t, c_t, l_t, h'_t, b'_t, m'_t\}_{t=0}^{\infty}$, a production plan for the firm $\{K_t, N_t\}_{t=0}^{\infty}$, aggregate housing stocks $\{H_t\}_{t=0}^{\infty}$, social security taxes $\{\theta_t\}_{t=0}^{\infty}$ and benefits $\{Tr_t\}_{t=0}^{\infty}$, foreigners' nominal and real assets $\{b_t^F, a_t^F\}_{t=0}^{\infty}$, prices $\{w_t, R_t\}_{t=0}^{\infty}$, inflation rates $\{\pi_t\}_{t=0}^{\infty}$ and measures $\{\Gamma_t\}_{t=0}^{\infty}$ such that:

1. given prices, government policies, transfers and initial conditions, for each t, V_t solves the functional equation (10) where c_t, l_t, h'_t, b'_t, and m'_t are the associated policy functions;

2. prices $\overline{r}$, w_t, and R_t satisfy:

$$\begin{aligned}\overline{r} &= \alpha A\Big(\frac{K_t}{N_t}\Big)^{\alpha-1} - \delta^k \\ w_t &= (1-\alpha)A\Big(\frac{K_t}{N_t}\Big)^{\alpha} \\ 1+R_{t+1} &= (1+\overline{r})E_t[1+\pi_{t+1}]\end{aligned}$$

3. The social security benefits equal total taxes

$$\theta_t \int w_t \varepsilon_j z_t l_t d\Gamma_t = Tr_t \int d\Gamma_t \tag{14}$$

4. Market clearing conditions:

$$\int \varepsilon_j z_t l_t(x,z,j) d\Gamma_t = N_t \tag{15}$$

$$\int m_t'(x,z,j) d\Gamma_t = \left(\frac{1+\tau}{1+\pi_t}\right)\overline{m}_t \tag{16}$$

$$\int b\, d\Gamma_t + b_t^F + a_t^F = K_t + H_t \tag{17}$$

$$\begin{aligned}&\int c_t(x,z,j) d\Gamma_t + H_{t+1} + (1-\delta^h)H_t + \\ &K_{t+1} + (1-\delta^k)K_t + \Omega_t + NX_t = F(K_t, N_t),\end{aligned} \tag{18}$$

where $\Omega_t = \int \Big(\Phi(h, h_t'(x,z,j)) + \Psi(b, b_t'(x,z,j))\Big)\, d\Gamma_t$ is the total transactions cost, the net exports are given by $NX_t = b_{t+1}^F - \left(\frac{1+R_t}{1+\pi_t}\right) b_t^F + a_{t+1}^F - (1+\overline{r})\, a_t^F$, and $H_t = \int h\, d\Gamma_t$ is the aggregate housing stock.

5. Law of Motion: [TO BE COMPLETED]

$$\Gamma_{t+1} = H_t(\Gamma_t). \tag{19}$$

3 Parameterization

This section characterizes the properties of the economy numerically with a parameterized version of the model. Although we do not conduct a formal calibration exercise at this point, the quantitative analysis provides results that seem to be robust to alternative parametrization values. In this section we describe how we set the parameters of the model. The model is calibrated to Canada and the model period is two-year. The world annual risk-free rate $\overline{r}$ is set at 4%.

Demography and preferences. In our model households are born at age twenty-one (model age $j = 1$). They retire at model age $jr = 23$ (age 65 in real time) and die with certainty at model age $J = 30$ (age 79 in the real world). The instantaneous utility function is given by

$$U(c, l, h, m) = \frac{\left[c^{\alpha_1} m^{1-\alpha_1}\right]^{1-\sigma} - 1}{1-\sigma} + \zeta \ln(1-l) + \omega \ln h, \tag{20}$$

where α_1 determines the relative importance of nondurable consumption, σ determines the risk aversion of the household, ζ determines the weight on leisure, and ω determines the weight on housing services. We set the coefficient of relative risk aversion $\sigma = 2$ and the discount factor $\beta = 0.96$. The parameter α_1 is chosen in order to replicate the average velocity of money (PY/M). During the period 1992-2008, the velocity of $M1$ was 4.5. We set $\alpha_1 = 0.99$. The weight on leisure is set to $\zeta = 2.5$ which implies the fraction of time devoted to work is 0.40. We choose the weight on housing services $\omega = 0.45$ and the depreciation rate $\delta^h = 0.05$. These two parameters lead to average housing stock to output ratios of around 1.25, and average housing investment to private output ratios of around 6% on an annual basis. These values are in accordance with the National Income and Product Accounts and the Fixed Assets Tables.

Average age labor productivity and earnings stochastic process. The average age profile of labor productivity $\{\varepsilon_j\}$ is taken from the estimates by Meh, Ríos-Rull, and Terajima (2008). The shock to earning ability evolves as follows:

$$\ln e_{j,t} = a_1 \cdot j + a_2 \cdot j^2 + a_3 \cdot EXPER + \eta_{j,t}, \tag{21}$$

where $EXPER$ is the full-time work experience, $\eta_{j,t} = \rho\eta_{j-1,t-1} + \varepsilon_{j,t}$, with $\rho < 1$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. We set $a_1 = 0.076012$, $a_2 = -0.0008465$, $\rho = 0.64$ and $\sigma_\varepsilon = 0.157$. The shock process is discretized with the Tauchen method in a finite Markov chain with three states.

House size. We assume house sizes to be discrete. Specifically, we assume three different sizes of 0.5, 1.0 and 2.0. The minimal house size is chosen to be equal to the average before-tax labor earning. The maximum house size is chosen so that the average debt-income ratio to be 0.9 (see Meh, Terajima, Chen and Carter (2009)).

Borrowing constraints and transactions costs. The loan-to-value ratio ξ is set to 0.95 which is the maximum loan-to-value ratio in Canadian mortgage markets. This corresponds to a 5% down payment requirement for purchasing a house. The household incurs a proportional cost equal to $\phi = 2.5\%$ of the current housing stock if its net housing investment changes. We interpret this cost as a low-range estimate of the actual costs of moving and changing a house: our model does not allow small adjustments to housing consumption (such as improvements and failure to maintain), so in absence of this margin we choose to be conservative on this value. The parameter, ψ, for the transactions cost in the bond market is set to $\psi = 0.01$ in the baseline. We will consider alternative values for the robustness check.

Technology The capital income share is set to $\alpha = 0.35$ and the annual depreciation rate to $\delta^k = 0.07$.

Government In the steady state, inflation is equal to the money supply growth rate. We set $\tau = 2\%$ which is consistent to the average long-run inflation target in Canada during the period 1991-2005. With respect to the retirement benefit we set the replacement rate to be 0.40 of the average labour earnings, implying that the payroll tax rate will be $\theta = 0.145$.

4 Findings

In this section, we present the results of changing the long-run inflation rate (or the money growth rate τ) on aggregates and welfare.

4.1 Steady state

Three columns in Table 1 present the steady state results for different inflation rates equal to 2%, 1%, and 0%, respectively. The benchmark economy has the 2% inflation rate. We can see that under lower inflation rates, aggregate output increases compared to the benchmark economy. This is a consequence of the increase in labor supply. We also observe that real cash balances increase since the real value of money rises with lower inflation rates. House size also increases as well as the amount of debt.

The main channel is through the real value of money. When the long-run inflation rate decreases, money becomes less costly to hold, and hence households increase their holdings. In turn, this increases current consumption as well as the holdings of other assets (i.e., future consumption) through the positive wealth effect of money.

Table 1: Steady state economies with different inflation rates

	$\pi = 2\%$ *Baseline*	$\pi = 1\%$	$\pi = 0\%$
Output	1.953	1.976	1.982
Consumption	0.734	0.741	0.747
House	1.221	1.239	1.256
Bonds	-0.851	-0.867	-0.887
Real money holdings	0.519	0.613	0.719
House to output ratio	0.625	0.627	0.634
Bond to output ratio	-0.436	-0.438	-0.447
Real money holdings to output ratio	0.266	0.310	0.363
Welfare Gain (ECV %)	—	0.32	0.85

Life-cycle profiles. Figure 1 shows average life-cycle patterns for different assets. First, we observe a hump-shape in house size and nominal debt over the life cycle. All households start with the smallest house type. After accumulating enough savings for the downpayment on a larger house, some households begin moving into the larger house. The timing of the move depends on the realization of labour productivity shocks. If a series of positive shocks occur, the household trades up in house size more quickly. Almost proportionally to the house size change, the nominal debt increases, which implies that most households are borrowing up to the collateral limit. Starting from around age 50, the nominal debt starts to decrease even though the house size remains constant. This occurs as households expect to downsize the house around retirement, and hence begin to slowly reduce their mortgage balance to avoid large transactions costs associated with a sudden change in the nominal debt. With respect to money holdings, they stay relatively low until near retirement as money is an undesirable asset to hold and the preference weight on money is relatively low. When retirement nears, some households move down in the house size. With this change, a large housing equity is freed up and saved as money. Even though money does not earn interest, retired households hold them for transaction purposes to buy consumption goods in a short period of time (i.e., before the end of life). As a result, accumulation of money is high at retirement age and runs down for consumption gradually over the rest of the life span. Finally, net worth is the sum of all three assets. The life-cycle pattern shows a typical shape observed in the overlapping generations model. It increases until retirement age and then gradually decreases over the rest of the lifetime.

Figure 2 shows the average consumption profile and the hours worked over the life cycle. The consumption profile is smooth and hump-shaped. The hours worked show that households work harder when young in order to accumulate a downpayment to buy a larger house. Hours worked are reduced somewhat after buying the larger house but they pick up again closer to retirement as labor productivity continues to increase with age.

How do these profiles change after the monetary reform? The age-profiles of the variables after lowering the long-run inflation rate are qualitatively similar to those before the policy change. With respect to the portfolio (see Figure 1), one can see that the amounts of real money balances are higher at all ages when inflation is low. With lower inflation and the associated wealth effect, households start buying a larger house earlier around their mid 40s instead of at 50 with higher inflation. They also keep the larger house longer; we observe the average house size after retirement becomes larger with lower inflation. The amount of debt moves proportionally with house size. Consumption still display a hump-shape but one can observe from Figure 2 that consumption is lower during the work life but higher after retirement under low inflation regime than in the baseline. This is due to the substitution effect with respect to the value of money. With lower inflation,

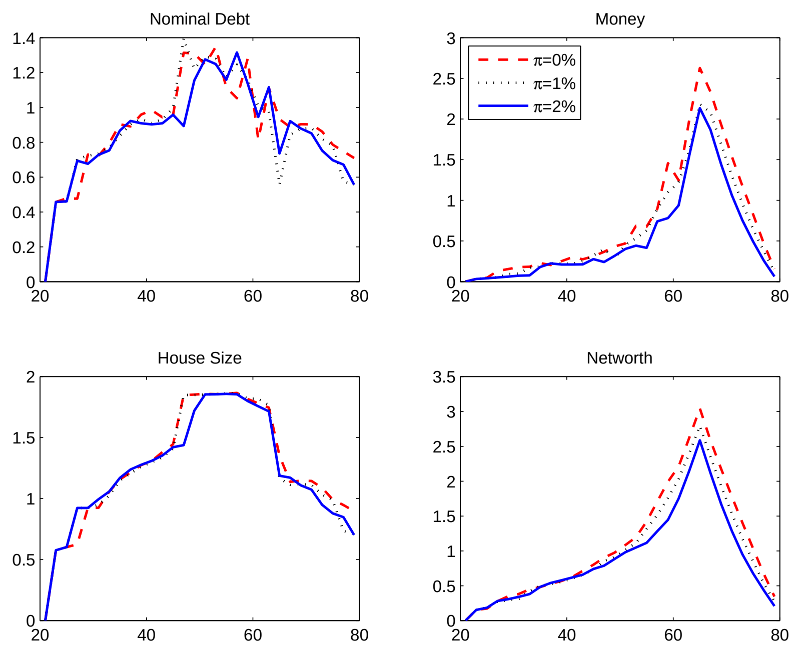

Figure 1: Life-cycle profile of debt, money, house size, and net worth in economies with different inflation rates.

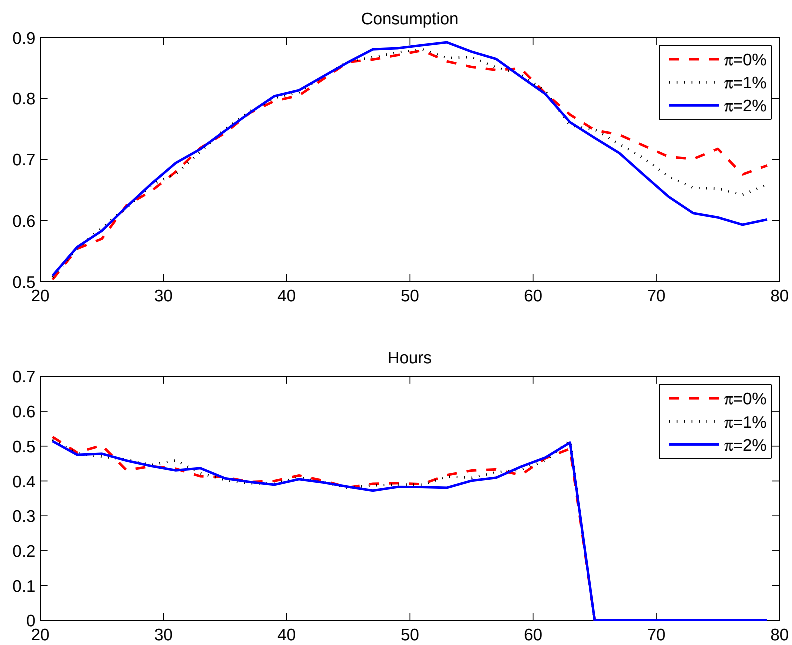

Figure 2: Life-cycle profile of consumption and hours worked in economies with different inflation rates.

money becomes a better store of value for future consumption. Hence, the consumption profile shifted more towards older ages.

Welfare In order to evaluate alternative monetary policy arrangements, we need a measure of social welfare. The social welfare criterion that we adopt is the steady-state ex-ante expected lifetime utility of a newborn. Given, a monetary policy regime with a long-run inflation rate, τ^0, we define the social welfare $W(\tau^0)$ as

$$W(\tau^0) = \sum_{j=1}^{J} \sum_{z,x} \beta^{j-1} \Gamma(x,z,j) U(c^0(x,z,j), 1 - l^0(x,z,j), h^0(x,z,j), m^0(x,z,j)), \quad (22)$$

where $W(\tau^0)$ is the expected lifetime utility that a newborn derives from consumption, labour supply/leisure, housing, and real money balance policy functions c^0, l^0, h^0, m^0 under the monetary regime τ^0. To compare the welfare gains (or costs) of alternative monetary regimes, we calculate a consumption equivalent variation CEV as a uniform change in nondurable consumption in each period and state that is necessary to make an individual indifferent between being born into the steady state with a new monetary regime τ^1 and being born into the initial steady state with the initial monetary regime τ^0. Note that labor-leisure, housing services, and real money balances allocations are held fixed to pre-reform values. The policy reform leads a welfare gain if this measure, CEV, is negative, and a welfare loss otherwise.[3]

We now turn to the steady state welfare effects for lowering the long-run inflation rate. The lower panel of Table 1 reports the steady state welfare implications of moving from 2% to 1% and zero. As be seen from the table, lowering inflation leads to a welfare gain in the long-run. Lowering long-run inflation rate to 1% generates a welfare gain of 0.32% while the gain is 0.85% when we move to full price stability (zero inflation).

4.2 Transition to lower inflation and welfare

In this section, we compute the aggregate effects and welfare implications by taking into account the transition path from the initial steady state with a long-run inflation rate of 2% to the new steady state with lower inflation rates. Accounting for the transition costs/gains allows us to determine the potential

[3]Note that in the next section when we discuss the transition to the new steady state, we will also take into account all households alive at the time of the reform.

winners and losers from the low inflation regime, thus shedding some light on the potential support for such a reform. Our policy experiment could be viewed as a surprise permanent change in the long-run inflation target. At the time of the policy change, households that have nominal bonds will be affected as the nominal interest rate was set based on expected inflation in the initial steady state. For example, nominal borrowers will experience an increase in the real burden of debt since the nominal interest rate is fixed. The extent of adjustment by the household will be limited by the transactions costs in the bond market. After the policy change (ie., lower inflation rate), the real value of nominal debt interest payments increases due to debt being predetermined in nominal terms. Hence, there are more households who will require a portfolio adjustment. However, the presence of transactions costs partially limits this process. This captures the idea that mortgage contracts are in general fixed nominal payments contracts and involve substantial renegotiation costs in order to change them.

Welfare. Accounting explicitly for the transitional dynamics generated by the monetary policy reform allows us to assess the welfare consequences of such a reform for all individuals alive at the time of the reform is put in place. Such an exercise also sheds some light on the political feasibility of any reform.

[TO BE COMPLETED]

5 Conclusion

This paper studies the welfare implications as well as the aggregate and redistributional effects of lowering the inflation rate from 2% to 1%. To do this we construct a heterogeneous-agent, life-cycle model of housing, nominal debt, and money with uninsurable earnings risk. To smooth out the idiosyncratic risk, households make portfolio choices with respect to housing, nominal bonds, and money, but the extent to which they can do this is limited by the presence of borrowing constraints and transactions costs. In particular, housing and bonds are subject to transactions costs while money can be exchanged freely. In the model, housing also serves as collateral for borrowing. In the presence of transactions costs and borrowing constraints, a liquid asset such as money has an edge over housing and bonds as a vehicle for self-insurance. Parameterizing the model to Canadian data, we show that lowering the long-run inflation rate below 2% improves welfare and increases aggregate output.

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