Financial Intermediation and Credit Policy in Business Cycle Analysis

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Abstract

We develop a canonical framework to help organize thinking about credit market frictions and aggregate economic activity in the context of the current crisis. We use the framework to focus on two issues in particular: first, how disruptions in financial intermediation can induce a crisis that affects real activity; and second, to illustrate how various credit market interventions by the central bank and/or the Treasury of the type we have seen recently, might work to mitigate the crisis. We make use of earlier literature to develop the particular framework we offer. We also try to characterize how very recent literature is incorporating insights from the crisis.
1 Introduction

To motivate interest in a paper on financial factors in business fluctuations it use to be necessary to appeal either to the Great Depression or to the experiences of many emerging market economies. This is no longer necessary. Over the past few years the U.S. and much of the industrialized world have experienced the worst financial crisis of the post-war. The global recession that has followed also appears to have been the most severe of this era. At the time of this writing there is evidence that the financial sector has stabilized and the real economy has stopped contracting. The path to recovery, however, remains highly uncertain.

The timing of recent events, though, poses a challenge for writing a Handbook chapter on credit market frictions and aggregate economic activity. It is true that over the last several decades there has been a robust literature in this area. Bernanke, Gertler and Gilchrist (BGG, 1999) surveyed much of the earlier work a decade ago in the Handbook of Macroeconomics. Since the time of that survey, the literature has continued to grow. While much of this work is relevant to the current situation, this literature obviously did not anticipate all the key empirical phenomena that have played out during the crisis. A new literature that builds on the earlier work is rapidly cropping up to address these issues. Most of these papers, though, are in preliminary working paper form.

Our plan in this chapter is to look both forward and backward. We look forward in the sense that we offer a canonical framework to help organize thinking about credit market frictions and aggregate economic activity in the context of the current crisis. The framework is not meant as comprehensive description of recent events but rather as a first pass at characterizing some of the key aspects and at laying out issues for future research. We look backward by making use of earlier literature to develop the particular framework we offer. In doing so, we address how this literature may be relevant to the new issues that have arisen. We also, as best we can, characterize how very recent literature is incorporating insights from the crisis.

From our vantage, there are two broad aspects of the crisis that have not been fully captured in work on financial factors in business cycles. First, by all accounts, the current crisis has featured a significant disruption of financial
intermediation.\footnote{For a description of the disruption of financial intermediation during the current recession, see Brunnermeier (2008), Gorton (2009) and Bernanke (2009).} Much of the earlier macroeconomics literature with financial frictions emphasized credit market constraints on non-financial borrowers and treated intermediaries largely as a veil (see, e.g. BGG). Second, to combat the crisis, both the monetary and fiscal authorities in many countries including the US. have employed various unconventional policy measures that in involve some form of direct lending in credit markets.

From the standpoint of the Federal Reserve, these "credit" policies represent a significant break from tradition. In the post war era, the Fed scrupulously avoided any exposure to private sector credit risk. However, in the current crisis the central bank has acted to offset the disruption of intermediation by making imperfectly secured loans to financial institutions and by lending directly to high grade non-financial borrowers. In addition, the fiscal authority acting in conjunction with the central bank injected equity into the major banks with the similar objective of improving credit flows. Though the issue is not without considerable controversy, many observers argue that these interventions helped stabilize financial markets and, as consequence, helped limit the decline in real activity. Since these policies are relatively new, much of the existing literature is silent about them.

With this background in mind, we begin in the next section by developing a baseline model that incorporates financial intermediation into an otherwise frictionless business cycle framework. Our goal is twofold: first to illustrate how disruptions in financial intermediation can induce a crisis that affects real activity; and second, to illustrate how various credit market interventions by the central bank and/or the Treasury of the type we have seen recently, might work to mitigate the crisis.

As in Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and others, we endogenize financial market frictions by introducing an agency problem between borrowers and lenders. The agency problem works to introduce a wedge between the cost of external finance and the opportunity cost of internal finance, which adds to the overall cost of credit that a borrower faces. The size of the external finance premium, further, depends on the condition of borrower balance sheets. Roughly speaking, as a borrower’s percentage stake in the outcome of an investment project increases, his or her incentive to deviate from the interests of lenders’ declines. The external finance premium then declines as a result.
In general equilibrium, a "financial accelerator" emerges. As balance sheets strengthen with improved economics conditions, the external finance problem declines, which works to enhance borrower spending, thus enhancing the boom. Along the way, there is mutual feedback between the financial and real sectors. In this framework, a crisis is a situation where balance sheets of borrowers deteriorate sharply, possibly due to a sharp deterioration in asset prices, causing the external finance premium to jump. The impact of the financial distress on the cost of credit then depresses real activity.

Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and others focus on credit constraints faced by non-financial borrowers. As we noted earlier, however, the evidence suggests that disruption of financial intermediation is a key feature of both recent and historical crises. Thus we focus our attention here on financial intermediation.

We begin by supposing that financial intermediaries have skills in evaluating and monitoring borrowers, which makes it efficient for credit to flow from lenders to non-financial borrowers through these institutions. In particular, we assume that households deposit funds in financial intermediaries that in turn lend funds to non-financial firms. We then introduce an agency problem that potentially constrains the ability of intermediaries to obtain funds from depositors. When the constraint is binding (or there is some chance it may bind), the intermediary’s balance sheet limits its ability to obtain deposits. In this instance, the constraint effectively introduces a wedge between the loan and deposit rates. During a crisis, this spread widens substantially, which in turn sharply raises the cost of credit that non-financial borrowers face.

As recent events suggest, however, in a crisis, financial institutions face difficulty not only in obtaining depositor funds but also in obtaining funds from one another in wholesale ("inter-bank") markets. Indeed, the first signals of a crisis are often strains in the interbank market. We capture this phenomenon by subjecting financial institutions to idiosyncratic "liquidity" shocks, which have the effect of creating surplus and deficits of funds across financial institutions. If the interbank market works perfectly, then funds flow smoothly from institutions with surplus funds to those in need. In this case, loan rates are thus equalized across different financial institutions. Aggregate behavior in this instance resembles the case of homogeneous intermediaries.

However, to the extent that the agency problem that limits an intermediary’s ability to obtain funds from depositors also limits its ability to obtain funds from other financial institutions and to the extent that nonfinancial
firms can obtain funds only from a limited set of financial intermediaries, disruptions of inter-bank markets are possible that can affect real activity. In this instance, intermediaries with deficit funds offer higher loan rates to nonfinancial firms than intermediaries with surplus funds. In a crisis this gap widens. Financial markets effectively become segmented and sclerotic. As we show, the inefficient allocation of funds across intermediaries can further depress aggregate activity.

In section 3 we incorporate credit policies within the formal framework. In practice the central bank employed three broad types of policies. The first, which was introduced early in the crisis, was to permit discount window lending to banks secured by private credit. The second, introduced in the wake of the Lehmann default was to lend directly in relatively high grade credit markets, including markets in commercial paper, agency debt and mortgage-backed securities. The third (and most controversial) involved direct assistance to large financial institutions, including the equity injections and debt guarantees under the Troubled Assets Relief Program (TARP) as well as the emergency loans to Bear Stearns and AIG.

In section 4, we use the model to simulate numerically a crisis that has some key features of the current crisis. Absent credit market frictions, the disturbance initiating the crisis induces only a mild recession. With credit frictions, however an endogenous disruption of financial intermediation works to magnify the downturn. We then explore how various credit policies can help mitigate the situation.

Our baseline model is quite parsimonious and meant mainly to exposit the key issues. In section 5, we discuss a number of questions and possible extensions. In some cases, there is relevant literature that we discuss. We conclude in section 6 with a brief overview of the literature, stressing the implications of this literature for going forward.

2 A Canonical Model of Financial Intermediation and Business Fluctuations

Overall, the specific business cycle model is a hybrid of Gertler and Karadi’s (2009) framework that allows for financial intermediation and Kiyotaki and Moore’s (2008) framework that allows for liquidity risk. We keep the core macro model simple in order to see clearly the role of intermediation and
liquidity. On the other hand, we also allow for some features prevalent in conventional quantitative macro models (such as Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007)) in order to get rough sense of the importance of the factors we introduce.

For simplicity we restrict attention to a purely real model and only credit policies, as opposed to conventional monetary models. Extending the model to allow for nominal rigidities is straightforward (see, e.g., Gertler and Karadi, 2009), and permits studying conventional monetary policy along with unconventional policies. However, because much of the insight into how the credit market frictions may affect real activity and how the various credit policies may work can be obtained from studying a purely real model, we abstract from nominal frictions.

2.1 Physical Setup

Before describing our economy with financial frictions, we present the physical environment. We begin with technology and resource constraints.

There are a continuum of firms of mass unity located on a continuum of islands. Each firm produces output using an identical constant returns to scale Cobb-Douglas production function with capital and labor as inputs. Because labor is perfectly mobile across firms and islands, we can express aggregate output $Y_t$ as a function of aggregate capital $K_t$ and aggregate labor hours $L_t$ as:

$$Y_t = A_t K_t ^\alpha L_t ^{1-\alpha}, \quad 0 < \alpha < 1,$$

where $A_t$ is aggregate productivity which follows a Markov process.

Each period investment opportunities arrive randomly to a fraction $\pi^i$ of islands. On a fraction $\pi^n = 1 - \pi^i$ of islands, there are no investment opportunities. Only firms on islands with investment opportunities can acquire new capital. The arrival of investment opportunities is i.i.d. across time and across islands. The structure of this idiosyncratic risk provides a simple way to introduce liquidity needs by firms, following Kiyotaki and Moore (2008). Let $I_t$ denote aggregate investment, $\delta$ the rate of physical depreciation and $\psi_{t+1}$ a shock to the quality of capital. Then the law of motion for capital is given by:

$$K_{t+1} = \psi_{t+1} [I_t + \pi^i (1 - \delta) K_t] + \psi_{t+1} \pi^n (1 - \delta) K_t$$

$$= \psi_{t+1} [I_t + (1 - \delta) K_t].$$
The first term of the right reflects capital accumulated by firms on investing islands and the second is capital remained on non-investing islands. Summing across islands yields a conventional aggregate relation for the evolution of capital, except for the presence of the disturbance $\psi_{t+1}$. Following the finance literature (e.g., Merton (19??)), we introduce the capital quality shock as a simple way to introduce an exogenous source of variation in the value of capital. (The random variable $\psi_{t+1}$ is best thought of as capturing some form of economic obsolescence, as opposed to physical depreciation).\footnote{One way to motivate this disturbance is to assume that final output is a C.E.S. composite of a continuum of intermediate goods that are in turn produced by employing capital and labor in a Cobb-Douglas production technology. Suppose that capital is good-specific and that each period a random fraction of goods become obsolete and are replaced by new goods. The capital used to produce the obsolete goods is now worthless and the capital for the new goods is not fully on line. The aggregate capital stock will then evolve according to equation (2).}

We assume the capital quality shock $\psi_{t+1}$ also follows a Markov process.

Firms on investing islands acquire capital from capital producers who operate in a national market. There are convex adjustment costs in the gross rate of change in investment. Aggregate output is divided between household consumption $C_t$, investment expenditures, and government consumption $G_t$,

$$Y_t = C_t + [1 + f(\frac{I_t}{I_{t-1}})]I_t + G_t$$  \hspace{1cm} (3)

where $f(\frac{I_t}{I_{t-1}})I_t$ reflects physical adjustment costs, with $f(1) = f'(1) = 0$ and $f''(1) > 0$.

Next we turn to preferences:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{X}{1 + \varphi} L_{t+i}^{1+\varphi} \right]$$  \hspace{1cm} (4)

where $E_t$ is the expectation operator conditional on date $t$ information. We abstract from many frictions in the conventional DSGE framework (e.g. nominal price and wage rigidities, variable capital utilization, etc.). However, we allow both the habit formation of consumption and the adjustment costs of investment because, as the DSGE literature has found, these features are helpful for reasonable quantitative performance and because they can be kept in the model at minimal cost of additional complexity.

If there were no financial frictions, the competitive equilibrium would correspond to a solution of the planner’s problem that involves choosing
aggregate quantities \((Y_t, L_t, C_t, I_t, K_{t+1})\) as a function of the aggregate state \((C_{t-1}, I_{t-1}, K_t, A_t, \psi_t)\) in order to maximize the expected discounted utility of the representative household subject to the resource constraints. This frictionless economy will serve as a benchmark to which we may compare the implications of the financial frictions.

In what follows we will introduce banks that intermediate funds between households and non-financial in a retail financial market. In addition, we will allow for a wholesale inter-bank market, where banks with surplus funds on non-investment islands lend to banks in need of funds on investing islands. We will also introduce financial frictions that may impede credit flows in both the retail and wholesale financial markets and then study the consequences for real activity.

2.2 Households

In our economy with credit frictions, households lend to non-financial firms via financial intermediaries. Following Gertler and Karadi (2009), we formulate the household sector in a way that permits maintaining the tractability of the representative agent approach.

In particular, there is a representative household with a continuum of members of measure unity. Within the household there are \(1 - f\) "workers" and \(f\) "bankers". Workers supply labor and return their wages to the household. Each banker manages a financial intermediary (which we will call a "bank") and also transfers dividends back to household. Within the family there is perfect consumption insurance.

Households do not hold capital directly. Rather, they deposit funds in banks. (It may be best to think of them as depositing funds in banks other than the ones they own). In our model, bank deposits are riskless one period securities (though banks may be restricted in the quantity they can issue). Households may also hold riskless one period government debt which is a perfect substitute for bank deposits.

Let \(W_t\) denote the wage rate, \(T_t\) lump sum taxes, \(R_t\) the gross return on riskless debt from \(t - 1\) to \(t\), \(D_t\) the quantity of riskless debt held, and \(\Pi_t\) net distributions from ownership of both banks and non-financial firms. Then the household chooses consumption, labor supply and deposits \((C_t, L_t, D_{t+1})\) to maximize expected discounted utility (4) subject to the flow of funds constraint,
\[ C_t = W_tL_t + \Pi_t + T_t + R_tD_t - D_{t+1}. \]  

(5)

where a significant component of \( \Pi_t \) is from dividends paid by banks.

Let \( u_{Ct} \) denote the marginal utility of consumption and \( \Lambda_{t,t+1} \) the household’s stochastic discount factor. Then the household’s first order conditions for labor supply and consumption/saving are given by

\[ E_t(u_{Ct})W_t = \chi L_t^\varphi, \]  

(6)

\[ E_t\Lambda_{t,t+1}R_{t+1} = 1, \]  

(7)

with

\[ u_{Ct} \equiv (C_t - \gamma C_{t-1})^{-1} - \beta \gamma (C_{t+1} - \gamma C_t)^{-1} \]  
and

\[ \Lambda_{t,t+1} \equiv \beta \frac{u_{Ct+1}}{u_{Ct}}. \]

Because banks may be financially constrained, bankers will retain earning to accumulate assets. Absent some motive for paying dividends, they may find it optimal to accumulate to the point where the financial constraint they face is no longer binding. In order to limit bankers’ ability to save to overcome financial constraints, we allow for turnover between bankers and workers. In particular, we assume that with i.i.d. probability \( 1 - \sigma \), a banker exits next period, (which gives an average survival time = \( \frac{1}{1-\sigma} \)). Upon exiting, a banker transfers retained earnings to the household and becomes a worker. Note that the expected survival time may be quite long (in our baseline calibration it is ten years.) It is critical, however, that the expected horizon is finite, in order to motivate payouts while the financial constraints are still binding.

Each period, \( (1 - \sigma)f \) workers randomly become bankers, keeping the number in each occupation constant. Finally, because in equilibrium bankers will not be able to operate without any financial resources, each new banker receives a "start up" transfer from the family as a small constant fraction of the total assets of entrepreneurs. Accordingly, \( \Pi_t \) is net the funds transferred to the household - funds transferred from exiting bankers minus the fund transferred to new bankers.

An alternative to our approach of having a consolidated family of workers and bankers would be to have the two groups as distinct sets of agents, without any consumption insurance between the two groups. It is unlikely, however, that the key results of our paper would change qualitatively. By
sticking with complete consumption insurance, we are able to have lending and borrowing in equilibrium and still maintain the tractability of the representative agent approach.

2.3 Banks

As we noted earlier, new investment opportunities randomly arrive to many islands according to a Poisson process that is independent across islands.

At the beginning of any period, before the realization of any uncertainty, banks choose an island to operate. During the period, a bank can only make loans to nonfinancial firms located on the same island. After the period ends, though, the bank is free to move to another islands. The free mobility of banks between periods ensures that returns to banking are the same across islands ex ante.

To finance lending in each period, banks raise funds in a national financial market. Within the national financial market, there is a retail market, where banks obtain deposit from households; and a wholesale market, where banks borrow and lend amongst one and another.

At the beginning of the period each bank raises deposits $d_t$ from households in the retail financial market at the deposit rate $R_{t+1}$. After the retail financial market closes, investment opportunities for nonfinancial firms arrive randomly to different islands. As we stated earlier, for a fraction $\pi^i$ of locations, new investment opportunities are available to finance as well as existing projects. Conversely, for a fraction $\pi^n = 1 - \pi^i$, no new investments are available to finance, only existing ones. On the interbank market, banks on islands with new lending opportunities will borrow funds from those on islands with no new project arrivals.

Financial frictions affect real activity in our framework via the impact on funds available to banks. For simplicity, however, there is no friction in transferring funds between a bank and non-financial firms in the same island. In particular, we suppose that the bank is efficient at evaluating and monitoring non-financial firms (of the same island), and also at enforcing contractual obligations with these borrowers (even after the bank moves to a new island). For simplicity we assume the costs to a bank of performing these activities are negligible. Accordingly, given its supply of available funds, a bank can lend frictionlessly to non-financial firms of the same island against their future profits. In this regard, firms are able to offer banks per-
fectly state-contingent debt. It is simplest to think of the bank’s claim on nonfinancial firms as equity.

After learning about its lending opportunities, a bank decides the volume of loans $s_t^h$ to make to non-financial firms and the volume of interbank borrowing $b_t^h$ where the superscript $h = i, n$ denotes the island type ($i$ for investing and $n$ for non-investing) on which the bank is located during the period. Let $Q_t^h$ be price of a loan (or "asset") - i.e. the market price of the bank’s claim on the future returns from one unit of present capital of non-financial firm at the end of period. We index the asset price by $h$ because, owing to temporal market segmentation, the $Q_t^h$ may depend on the volume of opportunities that the bank faces.

For an individual bank, the flow-of-funds constraint implies the value of loans funded within a given period, $Q_t^h s_t^h$, must equal the sum of the bank net worth $n_t^h$, its borrowings on the interbank market $b_t^h$ and deposits $d_t$:

$$ Q_t^h s_t^h = n_t^h + b_t^h + d_t. \tag{8} $$

Note that $d_t$, which is obtained at the beginning of the period, does not depend upon the volume of the lending opportunities, which is not realized until later in period.

Let $R_{it}$ be the interbank interest rate from periods $t-1$ to period $t$. Then net worth at $t$ is the gross payoff from assets funded at $t-1$, net borrowing costs, as follows:

$$ n_t^h = [Z_t + (1 - \delta)Q_t^h] \psi_t s_{t-1} - R_{it} b_{t-1} - R_t d_{t-1}, \tag{9} $$

where $Z_t$ is the dividend payment at $t$ on the loans the bank funds at $t-1$. (Recall that the exogenous variable $\psi_t$ is an aggregate shock to the gross return on asset). Observe that the gross payoff from assets depends on the location specific asset price $Q_t^h$, which is the reason $n_t^h$ depends on the realization of the location specific shock at $t$.

Given the constant exit rate of the bank, the objective of the bank at the end of period $t$ is given by

$$ V_t = E_t \sum_{i=1}^{\infty} (1 - \sigma) \sigma^{i-1} \Lambda_{t,t+i} n_{t+i}^h, \tag{10} $$

where $\Lambda_{t,t+i}$ is the stochastic discount factor, which is equal to the marginal rate of substitution between consumption of date $t+i$ and date $t$ of the representative household.
To motivate an endogenous constraint on the bank’s ability to obtain funds in either the retail or wholesale financial markets, we introduce the following simple agency problem: We assume that after an bank obtains funds, the banker managing the bank may transfer a fraction $\theta$ of "divertable" assets to his or her family. Divertable assets consists of total gross assets $Q^h_t s^h_t$ net a fraction $\omega$ of interbank borrowing $b^h_t$. If a bank diverts assets for its personal gain, it defaults on its debt and is shut down. The creditors may re-claim the remaining fraction $1 - \theta$ of funds. Because its creditors recognize the bank’s incentive to divert funds, they will restrict the amount they lend. In this way a borrowing constraint may arise.

We allow for the possibility that bank may be constrained not only in obtaining funds from depositors but also in obtaining funds from other banks. Though we permit the tightness of the constraint faced in each market to differ. In particular, the parameter $\omega$ indexes (inversely) the relative degree of friction in the interbank market:

With $\omega = 1$, the interbank market operates frictionlessly. Banks cannot divert assets financed by borrowing from other banks: Lending banks are able to perfectly recover the assets that underlie the loans they make. In such case, banks are not constrained in borrowing from one another. They may only be constrained in obtaining funds from depositors.

In contrast, with $\omega = 0$, lending banks are no more efficient than depositors in recovering assets from borrowing banks. In this case, the friction that constrains a banks ability to obtaining funds on the interbank market is the same as for the retail financial market. In general, we can allow parameter $\omega$ to differ for borrowing versus lending banks. However, maintaining symmetry simplifies the analysis without affecting the main results.

We assume that the banker’s decision over whether to divert funds must be made at the end of the period after the realization of the idiosyncratic uncertainty that determines its type, but before the realization of aggregate uncertainty in the following period. Here the idea is that if the banker is going to divert funds, it takes time to position assets and this must be done between the periods (i.e., during the night). Let $V(s^h_t, b^h_t, d_t)$ be the maximized value of $V_t$, given an asset and liability configuration $(s^h_t, b^h_t, d_t)$ at the end of period $t$. Then in order to ensure the bank does not divert funds, the following incentive constraint must hold for each bank type:

$$V(s^h_t, b^h_t, d_t) \geq \theta (Q^h_t s^h_t - \omega b^h_t).$$  \hspace{1cm} (11)$$

In general the value of the bank at the end of period $t - 1$ satisfies the
Bellman equation

\[ V(s_{t-1}, b_{t-1}, d_{t-1}) = E_{t-1} \Lambda_{t-1, t} \sum_{h=i,n} \pi^h \{ (1 - \sigma)n_t^h + \sigma \max_{d_t} \max_{s^h_t, b^h_t} V(s^h_t, b^h_t, d_t) \} \]  

(12)

Note that the loans and interbank borrowing are chosen after a shock to the loan opportunity is realized while deposits are chosen before.

To solve the decision problem, we first guess that the value function is linear:

\[ V(s^h_t, b^h_t, d_t) = \nu_{st} s^h_t + \nu_{bt} b^h_t - \nu_t d_t \]  

(13)

where \( \nu_{st}, \nu_{bt} \) and \( \nu_t \) are time varying parameters, and verify this guess later.

Note that \( \nu_{st} \) is value to the bank at the end of period \( t \) of an additional unit of assets; \( \nu_{bt} \) is the marginal cost of interbank debt; and \( \nu_t \) is the marginal cost of deposits.\(^3\)

Let \( \lambda^h_t \) be the Lagrangian multiplier for the incentive constraint (11) faced by bank of type \( h \) and \( \bar{\lambda}_t \equiv \sum_{h=i,n} \pi^h \lambda^h_t \) the average of this multiplier across states. Then given the conjectured form of the values function, we may expressed the first order conditions for \( d_t, s^h_t, \) and \( \lambda^h_t \), as follows:

\[
(\nu_{bt} - \nu_t) (1 + \bar{\lambda}_t) = \theta \omega \bar{\lambda}_t, \tag{14}
\]

\[
\left( \frac{\nu_{st}}{Q^h_t} - \nu_{bt} \right) (1 + \lambda^h_t) = \lambda^h_t \theta (1 - \omega), \tag{15}
\]

\[
[\theta - (\frac{\nu_{st}}{Q^h_t} - \nu_t) Q^h_t s^h_t - [\theta \omega - (\nu_{bt} - \nu_t)] b^h_t] \leq \nu_t n^h_t. \tag{16}
\]

According to equation (14), the marginal cost of interbank borrowing exceeds the marginal cost of deposit if and only if the incentive constraint is expected to bind on average (\( \bar{\lambda}_t > 0 \)) and the inter-bank market operates more efficiently than the retail deposit market (i.e., \( \omega > 0 \), meaning that assets financed by interbank borrowing are harder to divert than those financed by deposits). Equation (15) states that the marginal value of assets in terms of goods \( \frac{s^h_t}{Q^h_t} \) exceeds the marginal cost of interbank borrowing by banks on

\(^3\)The parameters in the conjectured value function are independent of the individual bank’s type because the value function is measured after the bank finishes its transaction for the current this period and because the shock to the loan opportunity is i.i.d. across periods.
type $h$ island to the extent that the incentive constraint is binding ($\lambda^h > 0$) and there is a friction in interbank market ($\omega < 1$). Finally, equation (16) is the incentive constraint. It requires that the values of the bank’s net worth (or equity capital), $\nu_t n^h_t$, must be at least as large as weighted measure of assets $Q^h_t s^h_t$ net of interbank borrowing $b^h_t$ that a bank holds. In this way, the agency problem introduces an endogenous balance sheet constraint on banks.

The model for the general case is somewhat cumbersome to solve. There are, however, two interesting special cases that provide insight into the model’s workings. In case 1, there is a perfect interbank market, which arises when $\omega = 1$. In case 2, the frictions in the interbank market are of the same magnitude as in the retail financial market, which arises when $\omega = 0$. We next proceed to characterize each of the cases.

2.3.1 Case 1: Frictionless wholesale financial market ($\omega = 1$)

If banks cannot divert assets financed by inter-bank borrowing ($\omega = 1$), interbank lending is frictionless. As equation (15) suggests, perfect arbitrage in the interbank market equalizes the shadow values of assets in each market, implying $\frac{\nu^s}{Q^s_t} = \frac{\nu^l}{Q^l_t}$, which in turn implies $Q^h_t = Q^l_t = Q_t$. The perfect interbank market, further, implies that the marginal value of assets in terms of good $Q^s_t$ must equal the marginal cost of borrowing on the interbank market $\nu_{bt}$:

$$\frac{\nu^s_t}{Q^s_t} = \nu_{bt}.$$ (17)

Because asset prices are equal across island types, we can drop the $h$ superscript in this case. Accordingly, let $\mu_t$ denote the excess value of a unit of assets relative to deposits, i.e., the marginal value of holding assets $\frac{\nu^s_t}{Q^s_t}$ net the marginal cost of deposits $\nu_t$. Then, given that banks are constrained in the retail deposit market, equations (14) and (15) imply that the

$$\mu_t \equiv \frac{\nu^s_t}{Q^s_t} - \nu_t > 0.$$ (18)

It follows that the incentive constraint (16) in this case may expressed as

$$Q_t s_t - b_t = \phi_t n_t$$ (19)
with
\[ \phi_t = \frac{\nu_t}{\theta - \mu_t}. \]  
(20)

Note that since interbank borrowing is frictionless, the constraint applies to assets intermediated minus interbank borrowing. How tightly the constraint binds depends positively on the fraction of net assets the bank can divert and negatively on the excess value of bank assets, given by \( \mu_t \). The higher the excess value is, the greater is the franchise value of the bank and the less likely it is to divert funds.

Let \( \Omega_{t+1} \) be the marginal value of net worth at date \( t+1 \) and let \( R_{kt+1} \) is the gross rate of return on bank assets. Then after combining the conjectured value function with the Bellman equation, we can verify the value function is linear in \( (s^h_t, b^h_t, d_t) \) if \( \mu_t \) and \( \nu_t \) satisfy:

\[ \nu_t = E_t \Lambda_{t,t+1} R_{t+1} \Omega_{t+1} \]  
(21)

\[ \mu_t = E_t \Lambda_{t,t+1} (R_{kt+1} - R_{t+1}) \Omega_{t+1} \]  
(22)

with
\[ \Omega_{t+1} = 1 - \sigma + \sigma(\nu_{t+1} + \phi_{t+1} \mu_{t+1}), \] and
\[ R_{kt+1} = \psi_{t+1} \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t}. \]

The excess value of assets in (22) is the discounted excess return on bank assets over the deposit interest rate, weighted by the marginal value of next period’s net worth \( \Omega_{t+1} \). (The marginal value of net worth is a weighted average of marginal values for exiting and for continuing banks. If a continuing bank has an additional net worth, it can increase assets by the leverage ratio \( \phi_{t+1} \), where assets have an excess value equal to \( \mu_{t+1} \) per unit).

Since the leverage ratio net of interbank borrowing, \( \phi_t \), is independent of both bank-specific factors and island-specific factors, we can sum across individual banks to obtain the following relation for the demand for total bank assets \( Q_t S_t \) as a function of total net worth \( N_t \):

\[ Q_t S_t = \phi_t N_t \]  
(23)

where \( \phi_t \) is given by equation (20). Overall, a setting with a perfect interbank is isomorphic to one where banks do not face idiosyncratic liquidity risks. Aggregate bank lending is simply constrained by aggregate bank capital.
If the banks’ balance sheet constraints on binding in the retail financial market, there will be excess returns on assets over deposit. However, a perfect interbank market leads to arbitrage in returns to assets across market as follows:

\[ E_t \Lambda_{t,t+1}R_{bt,t+1} \Omega_{t+1} = E_t \Lambda_{t,t+1}R_{bt+1,t+1} \Omega_{t+1} > E_t \Lambda_{t,t+1}R_{t+1,t+1}. \]  

(24)

As will become clear, a crisis in such economy is associated with an increase in the excess return on assets for banks of all types.

2.3.2 Case 2: Symmetric frictions in wholesale and retail financial markets \((\omega = 0)\)

In this instance the bank’s ability to divert funds is independent of whether the funds are obtained in either the retail or wholesale financial markets. This effectively makes the borrowing constraint the bank faces symmetric in the two credit markets. As a consequence, interbank loans and deposits become perfect substitutes as sources of finance. Accordingly, equation (14) implies that the marginal cost of interbank borrowing is equal to the marginal cost of deposits

\[ \nu_{bt} = \nu_t. \]  

(25)

Here, even if banks on investing islands are financially constrained, banks on non-investing islands may or may not be. Roughly speaking, if the constraint on inter-bank borrowing binds tightly, banks in non-investing islands will be more inclined to use their funds to re-finance existing investments rather than lend them to banks on investing islands. This raises the likelihood that banks on non-investing islands will earn zero excess returns on their assets.

As we will verify later, because asset supply per unit of bank net worth is larger on investing islands than on non-investing islands, the asset price is lower, i.e., \(Q^i_t < Q^n_t\). In the previous case of a perfect interbank market, funds flow from non-investing to investing islands to equalize asset prices. Here, frictions in the inter-bank market limit the degree of arbitrage, keeping \(Q^i_t\) below \(Q^n_t\).

A lower asset price on the investing island, of course, means a higher expected return. Let \(\mu^h_t \equiv \nu_t - \nu_t\) be the excess value of assets on a type \(h\) island. Then we have:

\[ \mu^i_t > \mu^n_t \geq 0. \]  

(26)
The positive excess return implies that banks in the investing islands are finance constrained. Thus the leverage ratios for banks on each island type are given by:

\[
\frac{Q^i_t S^i_t}{n^i_t} = \phi^i_t = \frac{\nu_t}{\theta - \mu^i_t} \tag{27}
\]

\[
\frac{Q^n_t S^n_t}{n^n_t} \leq \phi^n_t = \frac{\nu_t}{\theta - \mu^n_t}, \text{ and } \left( \frac{Q^n_t S^n_t}{n^n_t} - \phi^n_t \right) \mu^n_t = 0. \tag{28}
\]

In this case the method of undetermined coefficients yields

\[
\nu_t = E_t \Lambda_{t,t+1} \sum_{h'=i,n} \pi^{h'} R_{t+1} \Omega^{h'}_{t+1} = E_t \Lambda_{t,t+1} R_{t+1} \Omega^{h'}_{t+1} \tag{29}
\]

\[
\mu^h_t = E_t \Lambda_{t,t+1} (R^{hh'}_{kt+1} - R_{t+1}) \Omega^{h'}_{t+1} \tag{30}
\]

with

\[
\Omega^{h'}_{t+1} = 1 - \sigma + \sigma (\nu_{t+1} + \phi^{h'}_{t+1} \mu^{h'}_{t+1}), \text{ and } \quad R^{hh'}_{kt+1} = \psi_{t+1} Z_{t+1} + (1 - \delta) Q^{h'}_{t+1}. \]

With an imperfect interbank market, both the marginal value of net worth \(\Omega^{h'}_{t+1}\) and the return on assets \(R^{hh'}_{kt+1}\) depend on which island type a bank enters in the subsequent period. Accordingly, we index each by \(h'\) and take expectations over \(h'\) conditional on date \(t\) information denoted as \(E_t\).

Because leverage ratios differ across islands, we aggregate separately across bank-types to obtain the aggregate relations:

\[
Q^i_t S^i_t = \phi^i_t N^i_t \tag{31}
\]

\[
Q^n_t S^n_t \leq \phi^n_t N^n_t, \text{ and } \left( Q^n_t S^n_t - \phi^n_t N^n_t \right) \mu^n_t = 0, \tag{32}
\]

where \(\phi^i_t\) and \(\phi^n_t\) are given by equations (27) and (28). As we will see, in the general equilibrium, investment will depend on the price of capital on "investing" islands, \(Q^i_t\). Accordingly, it is the aggregate balance sheet constraint on asset demand for banks on investing islands, given by equation (31) that becomes critical for real/financial interactions.

Finally, from (25, 26, 29, 30), we learn that the returns obey
\[ E_t \Lambda_{t,t+1} R_{h,t+1}^{hl'} \Omega_{t+1}^{hl'} > E_t \Lambda_{t,t+1} R_{h,t+1}^{hl'} \Omega_{t+1}^{h'} \]
\[ \geq E_t \Lambda_{t,t+1} R_{t+1}^{h} \Omega_{t+1}^{h} = E_t \Lambda_{t,t+1} R_{t+1}^{h} \Omega_{t+1}^{h'}.
\]

with \( \geq \) holds with strict inequality iff \( \mu^u_i > 0 \) and holds with equality iff \( \mu^u_i = 0 \). With an imperfect inter-bank market, a crisis is associated with both a rise in the excess return for banks on investing islands and increase in the dispersion of returns between island types.

### 2.4 Evolution of Bank Net Worth

Let total net worth for type h banks, \( N^h_t \), equal the sum of the net worth of existing entrepreneurs \( N^h_{o,t} \) (o for "old") and of entering entrepreneurs \( N^h_{y,t} \) (y for young):

\[ N^h_t = N^h_{o,t} + N^h_{y,t}. \tag{34} \]

Net worth of existing entrepreneurs equals earnings on assets net debt payments made in the previous period, multiplied by the fraction that survive until the current period, \( \sigma \):

\[ N^h_{o,t} = \sigma \pi^h \{ [Z_t + (1-\delta)Q_t^h] \psi_t S_{t-1} - R_tD_{t-1} \}. \tag{35} \]

We assume that the family transfers to each new entrepreneur the fraction \( \xi/(1-\sigma) \) of the total value assets of exiting entrepreneurs, implying:

\[ N^h_{y,t} = \xi [Z_t + (1-\delta)Q_t^h] \psi_t S_{t-1}. \tag{36} \]

Finally, by the balance-sheet of the entire banking sector, deposits equal the difference between total assets and bank net worth as follows:

\[ D_t = \sum_{h=i,n} (Q_t^h S_t^h - N^h_t). \tag{37} \]

Observe that fluctuations in the return to assets affect the evolution of net worth. Further, the higher the leverage of the bank is, the larger will be the percentage impact of return fluctuations on net worth. Note also that a deterioration of capital quality (a decline in \( \psi_t \)) directly reduces net worth. As we will show, there will also be a second round effect, as the decline in net worth induces a fire sale of assets, depressing asset prices and thus further depressing bank net worth.
2.5 Nonfinancial Firms

There are two types of non-financial firms: goods producers and capital producers.

2.5.1 Goods Producer

Competitive goods producers on different islands operate a constant returns to scale technology with capital and labor inputs, given by equation (38). Since labor is perfectly mobile across islands, firms choose labor to satisfy

\[ W_t = (1 - \alpha) \frac{Y_t}{L_t} \]  

(38)

It follows that we may express gross profits per unit of capital \( Z_t \) as follows:

\[ Z_t = \frac{Y_t - W_t L_t}{K_t} = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha} \]  

(39)

As we noted earlier, conditional on obtaining funds from a bank, a goods producer does not face any further financial frictions. A goods producer with an opportunity to invest obtains funds from an intermediary by issuing new state-contingent securities (equity) at the price \( Q_i^t \). The producer then uses the funds to buy new capital goods from capital goods producers. Each unit of equity is a state-contingent claim to the future returns from one unit of investment:

\[ \psi_{t+1}^1 Z_{t+1}, (1 - \delta) \psi_{t+1}^2 Z_{t+2}, (1 - \delta)^2 \psi_{t+1}^3 Z_{t+3}, \ldots \]

Through perfect competition, the price of new capital goods is equal to \( Q_i^t \), and goods producers earn zero profits state-by-state.

2.6 Capital Goods Producers

Capital producers operate in a national market. They make new capital using input of final output and subject to adjustment costs, as described in section 2.2. They sell new capital to firms on investing islands at the price \( Q_i^t \). Given that households own capital producers, the objective of a capital producer is to choose \( I_i \) to solve:
From profit maximization, the price of capital goods is equal to the marginal cost of investment goods production as follows,

$$Q^i_t = 1 + f \left( \frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - E_t \Lambda_{t,t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 f'(\frac{I_{t+1}}{I_t})$$  \hspace{1cm} (40)$$

Profits (which arise only outside of steady state), are redistributed lump sum to households.

### 2.7 Equilibrium

To close the model (in the case without government policy), we require market clearing in both the market for securities and the labor market. Total securities issued on investing and non-investing islands correspond to aggregate capital acquired by each type, as follows:

$$S^i_t = I_t + (1 - \delta) \pi^i K_t$$
$$S^n_t = (1 - \delta) \pi^n K_t.$$  \hspace{1cm} (41)

Note that demand for securities by banks is given by equation (23) in the case of a frictionless interbank market and by equations (31) and (32) in the case of an imperfect interbank market. Observe first that the market price of capital on each island type will in general depend on the financial condition of the associated banks. Second, with an imperfect interbank market, state-contingent loans rates offered by banks on investing islands will in general by higher than elsewhere.

Finally, the condition that labor demand equals labor supply requires that

$$(1 - \alpha) \frac{Y_t}{L_t} E_t(u_{Ct}) = \chi L_t^o$$  \hspace{1cm} (42)$$

This completes the description of the model.

Absent credit market frictions, the model reduces to a real business cycle framework modified with habit formation and flow investment adjustment costs. With the credit market frictions, however, balance sheet constraints
on banks ability to obtain funds in retail and wholesale market may limit real investment spending, affecting aggregate real activity. As we will show, a crisis is possible where weakening of bank balance sheets significantly disrupts credit flows, depressing real activity.

As we have discussed, one example of a factor that could weaken bank balance sheets is a deterioration of the underlying quality of capital. A negative quality shock directly reduces the value of bank net worth, forcing banks to reduce asset holdings. A second round effect on bank net worth arises as the fire sale of assets reduces the market price of capital. Further, the overall impact on bank equity of the decline in asset values is proportionate to the amount of bank leverage. With highly leveraged banks, a substantial percentage drop in bank equity may arise, leading to a significant disruption of credit flows. We illustrate this point clearly in section 4.

3 Credit Policies

During the crisis the various central banks, including the US. Federal Reserve, made use of their powers as a lender of last resort to facilitate credit flows. To justifying doing so, the Fed appealed to Section 13.3 of the Federal Reserve Act, which permits it in "unusual end exigent circumstances" to make loans to the private sector, so long as the loans are judged to be of sufficiently high grade. In practice, the Fed employed two general types of credit policies. First, early on it expanded discount window operations by permitting discount window loans to be collateralized by high grade private securities and also by extending the availability of the window to non-bank financial institutions. Second, the Fed lent directly in high grade credit markets, funding assets that included commercial paper, agency debt and mortgage back securities. In addition, the Treasury, acting in concert with the Fed, injected equity in the banking system along with supplying bank debt guarantees.

There is some evidence that these types of policies were effective in stabilizing the financial system. The expanded liquidity helped smoothed the flow of funds between financial institutions, effectively by dampening the turmoil-induced increases in the spread between the interbank lending rate (LIBOR) and the Treasury Bill rate. The enhanced financial distress following the Lehmann failure, however, proved to much for the liquidity facilities alone to handle. At this point, the Fed set up facilities to lend directly to the commercial paper market and a number of weeks later phased in programs
to purchase agency debt and mortgage-backed securities. Credit spreads in each these markets fell.

The equity injections also came soon after Lehmann. Though not without controversy, the equity injections appeared to reduce stress in banking markets. Upon the initial injection of equity in mid-October 2008, credit default swap rates of the major banks fell dramatically. At the time of this writing, the receiving banks have paid back a considerable portion of the funds. Further, though risks remain, the government appears to have made money on many of these programs.

In the sub-sections below, we take a first pass at analyzing how these policies work, using our baseline model as a laboratory. As we showed in the previous section, within the context of our model, the financial market frictions open the possibility of periods of distress where excess returns on assets are abnormally high. Because they are balance sheet constrained, private financial intermediaries cannot immediately arbitrage these returns. One can view the point of the Fed’s various credit programs as facilitating this arbitrage in times of crisis. In this regard, each of the various policies works somewhat differently, as we discuss below.

Because it is simplest to begin with a discussion of direct central bank lending. We next turn to discount window lending, and then conclude with equity injections.

### 3.1 Lending Facilities (Direct Lending)

What we mean by direct lending is meant to broadly characterize the facilities the Fed set up for direct acquisition of high quality private securities.

Lending facilities work as follows: We suppose that the central banks has both an advantage and a disadvantage relative to private lenders. The advantage is that unlike private intermediaries, the central bank is not balance sheet constraint (at least in the same way). Private citizens do not have to worry about the central bank defaulting. The liabilities it issues are government debt and it can credibly commit to honoring this debt (aside from inflation). Thus, in periods of distress where private intermediaries are unable to obtain additional funds, the central bank can obtain funds and then channel them to markets with abnormal excess returns.

In the current crisis, the Fed funded the initial expansion of its lending programs by issuing government debt (that it borrowed from the Treasury) and then later made use of interest bearing reserves. The latter are effectively
government debt. It is true that the interest rate on reserves fell to zero as the Funds rate reached its lower bound, giving these reserves the appearance of money. However, once the Fed moves the Funds rate above zero it will also raise the interest rate on reserves. In this regard, the Fed’s unconventional policies should be thought of as expanded central intermediation as opposed to expanding the money supply. In the case of lending facilities, a key advantage of the central bank is that it is not constrained in its ability to funds the same way as private intermediaries may be in time of financial distress.

At the same time, we suppose that the central bank is less efficient at intermediating funds. It faces an efficiency cost $\tau$ per unit, which may be thought of as a cost of evaluating and monitoring borrowers that is above and beyond what a private intermediary would pay.

To obtain funds, the central bank issues government debt to the private that is a perfect substitute for bank deposits, and pays the riskless real rate $R_{t+1}$. It lends the funds in market $h$ at the private loan rate $R_{kt+1}^{hh'}$ which depends upon the state of the next period $h'$. Observe that the central banks is not offering the funds at a subsidized rate. However, by expanding the supply of funds available in the market, it will reduce equilibrium lending rates.

Let $S^h_t$ be total securities of type $h$ intermediated, $S^h_{pt}$ total securities of type $h$ intermediated by private banks, and $S^h_{gt}$ total type $h$ securities intermediated by the central bank. Then total intermediation of type $h$ assets is given by:

$$Q^h_t S^h_t = Q^h_t (S^h_{pt} + S^h_{gt})$$  \hspace{1cm} (43)

We suppose the central bank chooses to intermediate the fraction $\varphi^h_t$ of total credit in market $h$:

$$S^h_{gt} = \varphi^h_t S^h_t$$  \hspace{1cm} (44)

where $\varphi^h_t$ may be thought of as an instrument of central bank credit policy.

Assuming that banks investing regions are constrained, lending facilities expand the total amount of assets intermediated in the market. Combining equations (31), (43) and (44), yields

$$Q^i_t S^i_t = \frac{1}{1 - \varphi^i_t} \phi^i_t N^i_t$$  \hspace{1cm} (45)

The effect on asset demand for non-investing regions depends on whether banks in these regions are balance sheet constrained (i.e., on whether the
excess return $\mu^n_t > 0$ is positive). If they are, then lending facilities affect asset demands similarly to the way they do in investing regions:

$$Q^n_t S^n_t = \frac{1}{1 - \varphi^n_t N^n_t} \text{ iff } \mu^n_t > 0.$$ \hspace{1cm} (46)

One other hand, if banks in non-investing regions are not constrained (i.e., $\mu^n_t = 0$), then central bank credit merely displaces private credit, leaving total asset demand in the sector unaffected. Let $S^n_{t*}$ be total asset demand consistent with a zero excess return on assets on non-investing islands in equilibrium. Then

$$Q^n_t S^n_{t*} = Q^n_t S^n_{pl} + \varphi^n_t Q^n_t S^n_{t*}, \text{ iff } \mu^n_t = 0.$$ \hspace{1cm} (47)

Here an increase in central credit provision crowds out private intermediation one for one. Only when private intermediaries are financially constrained does central bank intermediation expand the overall supply of credit.

### 3.2 Liquidity Facilities (Discount Window Lending)

With lending facilities, the central bank directly intermediates funds. With liquidity facilities, the central bank uses the discount window to lend funds to banks that in turn lend them out to nonfinancial borrowers. Typically, liquidity facilities are used to offset disruption of inter-bank markets. Such was the case in the current crisis.

Another distinguishing feature of liquidity facilities is that central bank lending is typically done at a penalty rate. This prescription dates back to Bagheot. The idea is that during a liquidity crises, it is the breakdown of markets for short term funds that is responsible for many borrowers having limited credit access, as opposed to lack of credit worthiness of individual borrowers. Because excess returns for these borrowers are abnormally high during the crisis, they are more than willing to borrow at penalty rates. Offering the funds at a penalty rate, further, discourages inefficient use of central bank credit by the private sector.

In this section we use our model to illustrate how discount window lending may facilitate the flow of inter-bank lending during a crisis. To do so, we restrict attention to the case ($\omega = 0$), where borrowers in the inter-bank market face symmetric constraints on obtaining funds in both the wholesale and retail markets. In this instance, banks with surplus funds face the same
risk as depositors that borrowing banks may divert a fraction of gross assets for their own purposes.

We suppose the central bank offers discount window credit at the non-contingent interest rate $R_{m,t+1}$ to banks borrowing on the inter-bank market. It funds this activity by issuing government debt that is a perfect substitute for household deposits. In this instance, though, the central bank may borrow from lending banks as well as households. For discount window lending to expand the supply of funds in the inter-bank market, however, the central bank must have an advantage over private lenders in supplying funds to borrowing banks. Otherwise discount window lending will simply supplant private inter-bank lending.

Here we suppose that the central bank is better able to enforce repayment than private lenders. In particular for any unit of discount window credit supplied, a borrowing bank can divert only the fraction $\theta(1 - \omega_g)$ of assets, with $0 < \omega_g < 1$. Recall that for credit supplied by a private lender, the borrowing bank can divert the fraction $\theta > \theta(1 - \omega_g)$. Here the idea is that the government may have additional means at its disposal (IRS records, access to credit records, legal punishments, etc.) to retrieve assets. We suppose, however, that after a certain level of discount window lending, the central bank’s ability to retrieve assets more efficiently than the private sector disappears. Think of this as reflecting some capacity constraint on the central bank’s ability to efficiently process discounted window loans secured by private credit.\footnote{Alternatively, if we had asset heterogeneity this constraint might reflect a limitation on the kind of bank assets that might be suitable collateral for discount window lending. For example, information-intensive commercial and industrial loans are not good collateral for discount window loans since they require expertise for monitoring and evaluation. On the other hand, agency debt or high grade securitized mortgage might be suitable, but banks might only have a limited fraction in their portfolios.}

Let $m_{h,t}$ be discount window borrowing for a bank of type $h$. The flow of funds constraint is now,

$$Q^h_{s_t^h} = n^h_t + b^h_t + m^h_t + d_t,$$

with $m^h_t \geq 0$. Let $V(s^h_{t}, b^h_{t}, m^h_{t}, d_t)$ be the value of a bank who holds assets and liabilities $(s^h_{t}, b^h_{t}, m^h_{t}, d_t)$ at the end of period $t$. For the bank to continue operating this value must not fall below the gain from diverting assets, taking into account the central bank’s advantage in retrieving assets. Accordingly,
in this case the incentive constraint is given by:

\[ V(s_t^h, b_t^h, m_t^h, d_t) \geq \theta \left( Q_t^h s_t^h - \omega_y m_t^h \right). \]  \hspace{1cm} (49)

Under this formulation, the bank may divert the fraction of assets \( \theta \) funded from private sources, but only the fraction \( \theta(1 - \omega_y) \) of assets funded by discount credit.

We defer the details of the bank’s decision problem for this case to the appendix. Accordingly, let \( \mu_{mt} \) be the excess cost to a bank of discount window credit relative to deposits

\[ \mu_{mt} = E_t \Lambda_{t,t+1}(R_{mt+1} - R_{t+1}) Q_{t+1}^h. \]  \hspace{1cm} (50)

Next note that, because we are restricting attention to the case of symmetric frictions in private interbank and retail financial markets \((\omega = 0)\), the interbank rate equals the deposit rate: \( R_{mt+1} = R_{t+1} \). Then from the first order conditions we learn that in order for both private interbank borrowing and discount window to be actively used, we need:

\[ \mu_{mt} = \omega^g \mu_t^i \]  \hspace{1cm} (51)

where \( \mu_t^i \) is the excess value of assets on investing islands, given by equation (30).

According to equation (51), to make borrowers indifferent between discount window and private credit at the margin, the central bank should set \( R_{mt+1} \) to make the excess cost of discount window credit equal to the fraction \( \omega^g \) of the excess value of assets. Intuitively, because a unit of discount window credit permits a borrowing bank to expand assets by a greater amount than a unit private credit, it is willing to pay a higher cost for this form of credit. In this way, the model generates an endogenously determined penalty rate for discount window lending.

Let \( M_t \) be the total supply of discount window credit offered to the market. Then one can show that the market demand for assets by investing banks is given by

\[ Q^i_j S^i_{\mu_t} = \phi^i_j N^i_t + \omega_y M_t. \]  \hspace{1cm} (52)

Thus, so long as \( \omega_y > 0 \), discount window lending can expand the total level of assets intermediated by banks on investing regions.

So long as the excess value of bank assets on non-investing islands is less than that on investing islands, i.e., \( \mu_t^s < \mu_t^i \), banks on non-investing islands
will not borrow from the discount window. Given that the discount rate is set to satisfy equation (51) discount window lending will be too expensive for banks who do not have new investment to finance. However, if excess returns are equal across different regions, then non-investing as well as investing banks may go to the window. This can happen if discount window lending for investing banks becomes sufficiently large to drive down the excess value of assets in investing regions to the corresponding value in non-investing regions. It could also happen in the case of a perfect inter-bank market.

The question then arises as to why the central bank does not simply expand discount lending to drive excess values of assets to zero. As we noted earlier, it reasonable to suppose that there are capacity constraints on the central bank’s ability to adequately monitor bank’s asset management activities. With a capacity constraint on discount window lending (secured by private credit) the central bank may need to use other tools such as direct lending or equity injections during crisis periods of high excess returns. While liquidity facilities may be useful for improving the flow of funds in inter-bank markets, in a major crisis other kinds of interventions may be necessary to stabilize financial markets.

### 3.3 Equity Injections.

With equity injections, the fiscal authority coordinates with the monetary authority to acquire ownership positions in banks. As with direct central bank lending we suppose that there are efficiency costs associated with government acquisition of equity. Let this cost be $\tau_e$ per unit of equity acquired. During a financial crisis, however, the net benefits from equity injections may positive and significant.

The effect of equity injections depends on three factors: (1) the payout rule for government equity; (2) the price at which the government acquires the equity relative to the market price; and (3) the advantage the government might have relative to private creditors in addressing the agency problem with banks.

For simplicity we restrict attention to the case with a perfect interbank market in which banks cannot divert their assets that is financed by inter-bank borrowing. Then the asset price is equal across regions with different investment opportunity.
We suppose that a unit of government equity has the same payout stream as a unit of private equity. The government may hold the equity stake until the bank exits and then receive the liquidation value of its assets, equal to $Z_t + (1 - \delta)Q_t$ per unit of capital times the number of units of capital its shares are worth. Alternatively it may sell off its holding at this value before the bank exits, assuming the crisis has passed.

Accordingly, one can effectively divide the total number of securities held by the bank at time $t$ between those privately owned, $s_{pt}$, and those publicly owned, $s_{get}$:

$$s_t = s_{pt} + s_{get} \tag{53}$$

Let $n_{gt}$ be the market value of government equity. The bank’s balance sheet identity then implies:

$$Q_t s_t = n_t^h + b_t^h + d_t + n_{gt} \tag{54}$$

where each security the government holds is valued at the market price $Q_t$, implying:

$$n_{gt} = Q_t s_{get} \tag{55}$$

To acquire equity, the government may pay a price $Q_{gt}$ that is above $Q_t$. One rationale for the government paying a premium is that the market price is below it frictionless value due to financial distress. For example, the government could pick $Q_{gt}$ so that the excess return on government equity, $\mu_{gt}$, equals zero, as follows:

$$\mu_{gt} = E_t \Lambda_{t,t+1} (R_{gtk+1} - R_{t+1})\Omega_{t+1} \tag{56}$$

where $R_{gtk+1}$ is the gross return on a unit of government equity injected at time $t$ is:

$$R_{gtk+1} = \psi_{t+1} \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_{gt}} \tag{57}$$

Since the excess return of private equity is positive (see equation (22)), $Q_{gt} > Q_t$.

The premium the government pays for equity is effectively a transfer to the bank that shows up in its net worth as follows:

$$n_t = [Z_t + (1 - \delta)Q_t] \psi_{st} s_{pt-1} - R_{bt} b_{t-1} - R_t d_{t-1} + (Q_{gt} - Q_t)[s_{get} - (1 - \delta)\psi_{st} s_{get-1}] \tag{58}$$
where \((Q_{gt} - Q_t)(s_{gt} - s_{gt-1})\) is the "gift" to the bank from new government equity purchases.

We suppose that the bank cannot divert assets financed by government equity. As with discount window lending, the government has an advantage relative to the private creditors in recovering assets. Accordingly, the incentive constraint becomes,

\[
V(s_t - s_{get}, b_t, d_t) \geq \theta(Q_t(s_t - s_{get}) - b_t).
\]

where as before \(b_t\) is interbank borrowing.

Let \(N_{gt}\) be total government equity in the banking system and \(S_{gt}\) total holdings of government equity; then we can aggregate to obtain the following expressions for aggregate asset demand and for the evolution of net worth:

\[
Q_tS_t = \phi_tN_t + N_{gt} \tag{59}
\]

\[
N_t = (\sigma + \xi)[Z_t + (1-\delta)Q_t\psi_tS_{gt-1} - \sigma R_tD_{t-1} + .(Q_{gt} - Q_t)[S_{get} - (1-\delta)\psi_tS_{get-1}] \tag{60}
\]

where \(\phi_t\) is the leverage ratio privately intermediated assets in the case of a perfect inter-bank market (see equation (20)), and with \(N_{gt} = Q_tS_{get}\). Thus, in this case equity injections expand the value of assets intermediated one-for-one, as equation (59) suggests. In addition, to the extent the government paying pays a premium over the market price (which is depressed due to the financial frictions), then the equity injection also expands private bank net worth, as equation (60) indicates. This is in turn expands asset demand by a multiple equal to the leverage ratio \(\phi_t\).

One additional important effect of the government equity injection is that it reduces the impact of unanticipated changes in asset values on private bank equity. Absent government equity, for example, the bank absorbs entirely the loss from an unanticipated decline in asset values, given that its obligations to outsiders are all in the form of non-contingent debt. With public equity, however, the government shares proportionately in the loss.

A key question now is what might determine the allocation of credit policy intervention between direct lending, discount window lending and equity injections. We argued earlier that in the context of our model, it might be natural to think of capacity constraints on discount window lending secured by private credit. So long as the efficiency costs of direct central bank lending are not large, extensive use of this intervention makes sense. For high grade instruments like commercial paper, agency debt and mortgage backed
securities it is reasonable to suppose the costs of central bank intermediation are not large. This might account for why direct central bank lending in the current crisis involved these kinds of assets. On the other hand, it is easy to imagine that other forms of bank lending, such as commercial and industrialized loans, which involve extensive evaluation and monitoring, would be quite costly for the central bank to intermediate. In this case, in a period of crisis, equity injections that enhance the ability of private banks to make these kinds of loans would seem desirable, (if the efficiency cost of government equity injection is not too large.) In our model, capital is homogeneous. Getting at this issue, accordingly, will involve extending our framework to allow for asset heterogeneity.

3.4 Government Expenditures and Budget Constraint

Here government consumption consists exclusively on intermediation expenditures. Let $S^h_{gt}$ be total securities of type $h = i, n$ acquired via direct central bank lending and $S^h_{get}$ securities acquired via equity injections. Then $G_t$ is given by

$$G_t = \sum_{h=i,n} \pi^h [\tau_t S^h_{gt} + \tau_e S^h_{get}]$$

Putting together fiscal and monetary authority, government expenditures are financed by lump sum taxes $T_t$ and net earnings from credit market interventions as,

$$G_t + \sum_{h=i,n} \pi^h \{Q^h_t [S^h_{gt} - (1 - \delta) \psi_t S_{gt-1}] + Q^h_{gt} [S^h_{get} - (1 - \delta) \psi_t S_{get-1}]\}$$

$$= T_t + Z_t (S^h_{gt-1} + S^h_{get-1}) + R_{mt} M_{t-1} - M_t + B_t - R_t B_{t-1}$$

where $M_t$ is total discount window lending and $B_t$ is government bond. As we discussed earlier, the price the government pays for equity, $Q^h_{gt}$, could exceed the market price.

Note that the during the crisis the government will earn an extra normal on its portfolio, since excess private returns in the market are positive, but private intermediaries are constrained from exploiting this. On the other
hand, the government may takes losses on its portfolio. Here we assume that lump sum taxes adjust to finance the losses. It would be interesting though to consider distortionary taxes to get a better sense of the costs faced in pursuing these policies.

4 Crisis Simulations and Policy Experiments

In this section we present some numerical experiments designed to illustrate how the model may capture some key features of a financial crisis and also how credit policy might work to mitigate the crisis. The analysis is meant only to be suggestive. In this regard, our aim is to show how vulnerability of the financial system might propagate the effects of a disturbance to asset values and aggregate production that might otherwise have a relatively modest effect on the economy. In addition to identifying the significance of balance sheet effects on intermediaries in the process, we also isolate the importance of an imperfect inter-bank market.

We start with the calibration then turn to a "crisis" simulation. Finally we analyze the impact of policy. We focus on direct lending since this policy is the simplest to present. Though, we do not report the results here, the other policies ultimately affect the economy in a similar fashion.

4.1 Calibration

There are eleven parameters for which we need to assign values. Seven are standard preference and technology parameters. These include the discount factor $\beta$, the habit parameter $\gamma$, the utility weight on labor $\chi$, the inverse of the Frisch elasticity of labor supply $\varphi$, the capital share parameter $\alpha$, the depreciation rate $\delta$ and the elasticity of the price of capital with respect to investment $\eta$. For these parameters we use reasonably conventional values, as reported in Table 1. The one exception involves the labor supply elasticity. To compensate partly for the absence of labor market frictions, we use a Frisch labor elasticity of ten, which is well above the range found in the business cycle literature, which typically lies between unity and three. We emphasize, though that this compensation is only partial: Had we instead incorporated the various key of quantitative DSGE models, including variable capital utilization and nominal price and wage rigidities, employment volatility in our framework would be much greater, even with a conventional
labor supply elasticity.

The four additional parameters are specific to our model. The first is the probability of an investment opportunity, \( \pi^i \). The last three are the financial sector parameters: \( \sigma \) the quarterly survival probability of bankers; \( \xi \) the transfer parameter for new bankers, and \( \theta \) the fraction of gross assets the banker can divert. We set \( \pi^i \) equal to 0.25, implying that new investment opportunities on a island arise once a year on average. We set \( \sigma = 0.975 \), implying that bankers survive for ten years on average.

Finally, we choose \( \xi \) and \( \theta \) to hit the following two targets: an average credit spread of one hundred basis points per year and an economy-wide leverage ratio of four. The choice of a leverage ratio of four reflects a crude first pass attempt to average across sectors with vastly different financial structures. For example, before the beginning of the crisis, most housing finance was intermediated by financial institutions with leverage ratios between twenty (commercial banks) and thirty (investment banks.) The total housing stock, however, was only about a third of the overall capital stock. Leverage ratios are clearly smaller in other sectors of the economy. We base the steady state target for the spread on the pre-2007 spreads as a rough average of the following spreads: mortgage rates versus government bond rates, BAA corporate bond rates versus government bonds, as commercial paper rates versus T-Bill rates.

We consider both the case of a perfect inter-bank market (\( \omega = 1 \)) and of an imperfect inter-bank market (\( \omega = 0 \)). As we noted earlier, with a perfect inter-bank market, the model economy behaves as if banks were homogenous and did not face an idiosyncratic arrival of lending opportunities. Under our calibration, within a local region of the steady state, all banks are symmetrically constrained, i.e., have similar excess returns on assets.

With an imperfect inter-bank market, under our calibration only banks on investing islands are constrained (within a local region of the steady-state). Those on non-investing islands have sufficient funds relative to lending opportunities to bid the price on assets to the point where the excess return over deposit costs is zero. They lend surplus funds to investing banks. For reasonable variations of our calibration, non-investment banks remain unconstrained and investing banks remain constrained.

Finally, we suppose that the capital quality shock obeys a first order autoregressive process.
4.2 Crisis Experiment

4.2.1 No Policy Response

We now turn to the crisis experiment. The initiating disturbance is an exogenous decline in capital quality. What we are trying to capture in a simple way is an exogenous force that triggers a decline in the value of intermediary assets. Within the model economy, the initial exogenous decline is then magnified in two ways. First, because are banks leveraged, the effect of decline in assets values on bank net worth is enhanced by a factor equal to the leverage ratio. Second, the drop in net worth tightens the banks’ borrowing constraint inducing effectively a fire sale of assets that further depresses asset values. The crisis then feeds into real activity as the decline in asset values leads to a fall in investment.

The initiating disturbance is a five percent unanticipated decline in capital quality with an autoregressive factor of 0.66. We fix the size of the shock simply to produce a downturn of roughly similar magnitude to the one observed over the past year.

We began by analyzing the performance of the model economy without credit policy and we start with the case of a perfect inter-bank market. Figure 1 reports the impulse responses of the key economic variables to a negative shock to capital quality. The dotted line is the model without financial frictions and the solid line is our baseline model with a perfect inter-bank market.

Note first that the negative disturbance produces only a modest downturn in the frictionless model. The loss of capital initially produces a drop in output and consumption. However, high returns to capital induce an increase in investment and employment.

With financial frictions the output decline at the trough is roughly twice as large as in the frictionless case. It is also significantly more protracted. The five percent decline in the quality of capital leads to a roughly fifty percent decline in bank net worth. The magnified effect, as we noted earlier, is due to bank leverage and to the fall in the market price capital, arising from the fire sale of assets induced by the tightening of bank borrowing constraints. The contraction in asset prices induces a decline in investment that is nearly double the output decline. Of course, it is the enhanced decline in investment that is ultimately responsible for the magnified drop in output, in the case with financial frictions. Finally, the employment drop, while nearly several
percentage points larger than in the frictionless case, is relatively modest. This simply reflects the absence of various standard labor market frictions that would enhance the response.

That financial factors are at work during the crisis is reflected in the behavior of the spread between the expected return to capital and the riskless interest rate. In the frictionless model this spread does not move (to a first order.) In the case with financial frictions, the spread rises on impact as a product of the decline in bank net worth. The increase in the cost of capital is responsible for the magnified drop in investment and output.

Financial factors also contribute to the slow recovery back to trend. To reduce the spread between the expected return to capital and the riskless rate remain, bank net worth must increase. But this process takes time, as the figure shows. So long as the spread is above trend, financial factors are a drag on the real economy. Note that throughout this convergence process, banks are effectively deleveraging since they are building up equity relative to debt. This, in a way, the model captures how the deleveraging process can slow down a recovery.

Next we turn to the case with the imperfect inter-bank market. Observe that frictions in the interbank market magnify the overall decline. The overall decline in investment is roughly a third larger relative to the perfect interbank market case, the output decline twenty percent larger, and the employment decline nearly double. Intuitively, in this case investing banks are limited in their ability to obtain funds on the inter-bank market once the crisis hits. Accordingly, asset prices on investing islands fall by more than they otherwise would, leading to an enhanced drop in overall investment. Symptomatic of the imperfect inter-bank market is the sharp rise in the spread between the return on capital and the riskless rate, which increases well above five percent, as compared to one percent in the case of a perfect interbank market.

4.2.2 Credit Policy Response

Here we analyze the impact of direct central bank lending as a means to mitigate the impact of the crisis. Symptomatic of the financial distress in the simulated crisis is a large increase in the spread between the expected return on capital on investing islands and the riskless interest rate. In practice, further, it was the appearance of abnormally large credit spreads in various markets that induced the Fed to intervene with credit policy. Accordingly we
suppose that the Fed adjusts the fraction of private credit it intermediates to the difference between spread on investing islands, \((E_t R_{kt+1}^{th'} - R_{t+1})\), and its steady state value \((ER_{k'} - R)\), as follows:

\[
\varphi_t = \nu_g \left[ (E_t R_{kt+1}^{th'} - R_{t+1}) - (ER_{k'} - R) \right]
\] (63)

To be clear, the rule applies only during a crisis, i.e., during "unusual and exigent" circumstances.

We begin with the case a perfect interbank market. In this case the return on assets is equalized across islands. It does not matter to which locale the central bank supplies credit. If it intermediates funds on non-investing islands, banks in these locations will lend any surplus funds to banks on investing islands to the point where the return on assets is equalized across locations.

We set the policy parameter \(\nu_g\) equal to 100. Figure 3 reports the impulses for this case. The policy intervention dampens the overall decline in output by nearly . The increase in central bank credit significantly reduces the rise in the spread, which in turn reduces the overall drop in investment. At its peaks, central bank credit increases to slightly over ten percent of the capital stock.

With an imperfect interbank market the central bank acquires assets on investing islands. What we have in mind here is that the central bank is targeting assets with high excess returns, i.e. assets that may be underfunded due to shortages of intermediary capital in the relevant market. Note that by charging the market rate to borrowers in these regions, the policy screens out borrowers on non-investing islands who earn lower returns.

Figure 4 reports the results for this case. The credit policy similarly works to dampen the output decline by mitigating the increase in the spread. Interestingly, the policy is more effective at containing the crisis in this case. What matters in this case are the leverage constraints on bank borrowing in investing locations, as opposed to leverage constraints economy-wide. By directly facilitating credit flows in these regions, a given level of central bank intermediation can be more effective in relaxing financial constraints. Note in this case that at the peak, central bank credit intermediation is only about five percent of total assets intermediated, which is less than half of what it was in the previous case. However, it is roughly twenty percent of assets intermediated in investing regions. The high percentage of central bank intermediation in this distressed region is what accounts for the effectiveness
of the policy. This occurs even though total central bank intermediation is smaller than in the case of the perfect-interbank market.

As we noted earlier, both discount window lending and equity injections work in a similar fashion to mitigate a crisis. It would be interesting to extend our framework to allow for features like asset heterogeneity and so on that would make it clearer how credit market interventions should be allocated between the three approaches.

Finally, though we do not do the exercise here, one can evaluate the net welfare benefits from the credit policy intervention, given different assumptions about the efficiency costs of direct central bank lending, following Gertler and Karadi (2009). As these authors show, however, under reasonable assumptions about these costs, the net benefits to the intervention are large and approximately equal to the gross benefits.

5 Issues and Extensions

6 Concluding Remarks
References


[38] Mendoza, Enrique, 2008, "Sudden Stops, Financial Crises and Leverage: A Fisherian Deflation of Tobin’s Q,"


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Figure 1. Crisis Experiment: Perfect Interbank Market

- $\psi$
- $r$
- $E(r_k) - r$
- $c$
- investment
- $k$
- labor
- $q$
- net worth

- Perfect Interbank Market
- RBC
Figure 2. Crisis Experiment: Imperfect Interbank Market

- **ψ**
- **r** (x 10^-3)
- **spread**
- **y**
- **c**
- **investment**
- **k**
- **labor**
- **q**
- **net worth**

**Legend:**
- Imperfect Interbank Market (π_i=0.25)
- RBC
- Perfect Interbank Market
Figure 3. Lending Facilities: Perfect Interbank Market

[Graphs showing various economic indicators such as \( \psi \), \( r \), \( E(r_k) - r \), \( y \), \( c \), \( k \), \( \text{labor} \), \( q \), \( \text{net worth} \), and \( \text{fraction of government assets} \) with different parameter values.]
Figure 4. Lending Facilities: Imperfect Interbank Market

- $\psi$
- $r$
- Spread
- Investment
- Labor
- Net worth
- Fraction of government assets

$v_g = 0$
RBC
$v_g = 100$