# Measuring and Modeling Default Correlation: Evidence from CDO, CDS and Equity Data

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# Motivation I

Credit correlation is critically important. Using a pricing model, it can be computed using different securities.

- Equity market (KMV)
- Credit Default Swap (CDS) market (Tarashev and Zhu 2007)
- Collateralized Debt Obligation (CDO) market

The evidence is limited. Even less is known about the co-movements of the correlation time series based on these securities

Empirical objective: compare three time series of correlations; characterize time variation

# Motivation II

The industry standard for CDO valuation is the Gaussian copula. In industry practice, implied correlations take center stage.

What does this implied correlation mean?

- Is it related to correlation in the underlying names in CDS and equity markets?
- Or does it reflect other determinants of prices in a segmented CDO market, notably liquidity?
- Is it meaningless as a correlation measure because of the inadequacy of the Gaussian copula?

# Motivation III

Non-standard (bespoke) CDOs are typically priced using market information on standard (index) products

Q: Can we learn something more by investigating the actual correlation structure of the underlying?
A: Perhaps if implied correlation and the correlation in the underlying are moving together

How to price a CDO in a market with low (or no) liquidity

# CDS Markets

A CDS is an insurance product

- The protection buyer pays a periodic spread
- The protection seller pays the default costs

The CDS premium equates the present value of both sides of the transaction or “legs”

$$V_{Fixed}(t,T,P) = P_t \left[ \sum_{i=1}^{N} D(t,T_i)\alpha(T_{i-1},T_i)q(t,T_i) + \int_t^{T_N} D(t,s)\alpha(T_{I(s)},s)f(s)ds \right]$$

$$V_{Floating}(t) = \int_t^{T_N} D(t,s)(1-\delta)f(s)ds.$$

$$P_0 = V_{Floating}(0)/V_{Fixed}(0,T,1).$$

Default intensities can be extracted given a default model using simple econometric techniques (NLS)

# CDO Markets

- A CDO is a multi-name credit derivative. The attachment and detachment points of the tranche indicate which parts of the portfolio losses are assigned to the tranche.
- Like a CDS, a CDO is an insurance contract. A protection buyer pays a periodic amount based on the spread and the remaining notional, the original notional adjusted for losses.
- The value of the tranche to the insurance seller is determined by the difference between the present value of these payments and the expected present value of the sum of loss changes. The par spread for a new tranche is such that this value is zero.
- Clearly changes in default probabilities will change the value of the tranche. Correlation impacts the volatility of the distribution of portfolio losses: the stronger the dependence, the more likely extreme scenarios become. Thus correlation also affects tranche value.

# CDO Markets: Base Correlation

- Implied correlation for a CDO tranche is the correlation between the underlying names that equates the theoretical price of the CDO tranche to the observed market price, conditional on a choice of pricing model

- The industry standard is the Gaussian Copula. See Li (2000), Andersen and Sidenius (2004), Hull and White (2004)

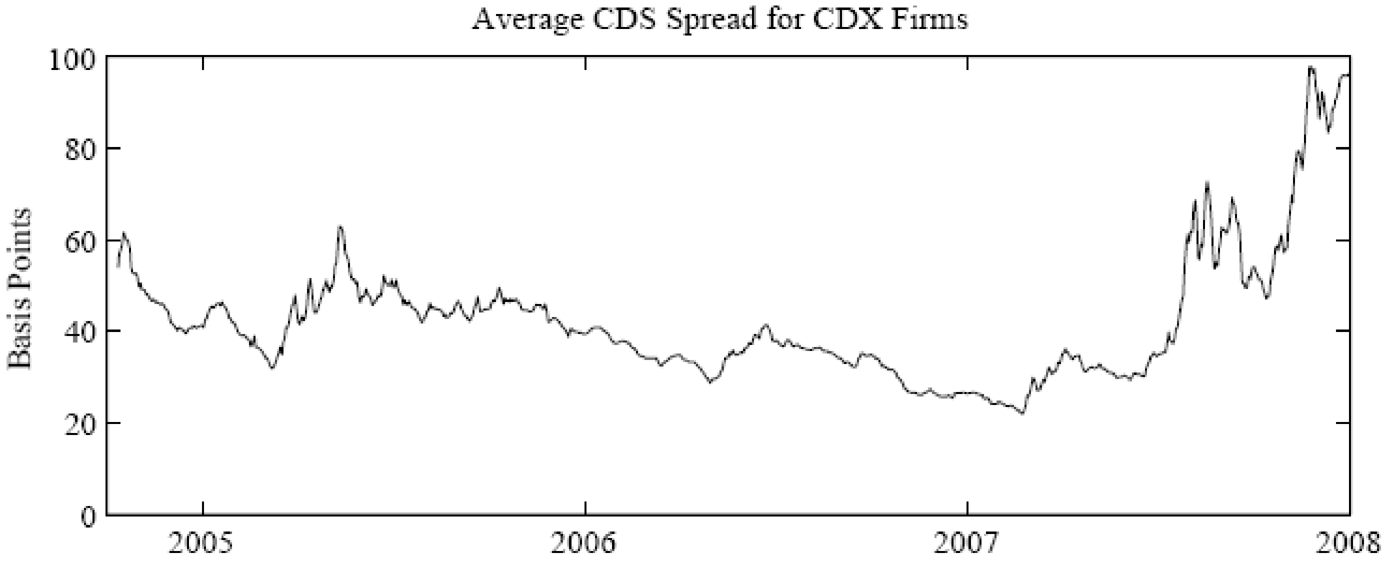
- Mostly a base correlation is used. If we have implied correlations for 0-3%, 3-7%, and 7-10% tranches, we can compute base correlations for 0-3%, 0-7%, and 0-10% tranches

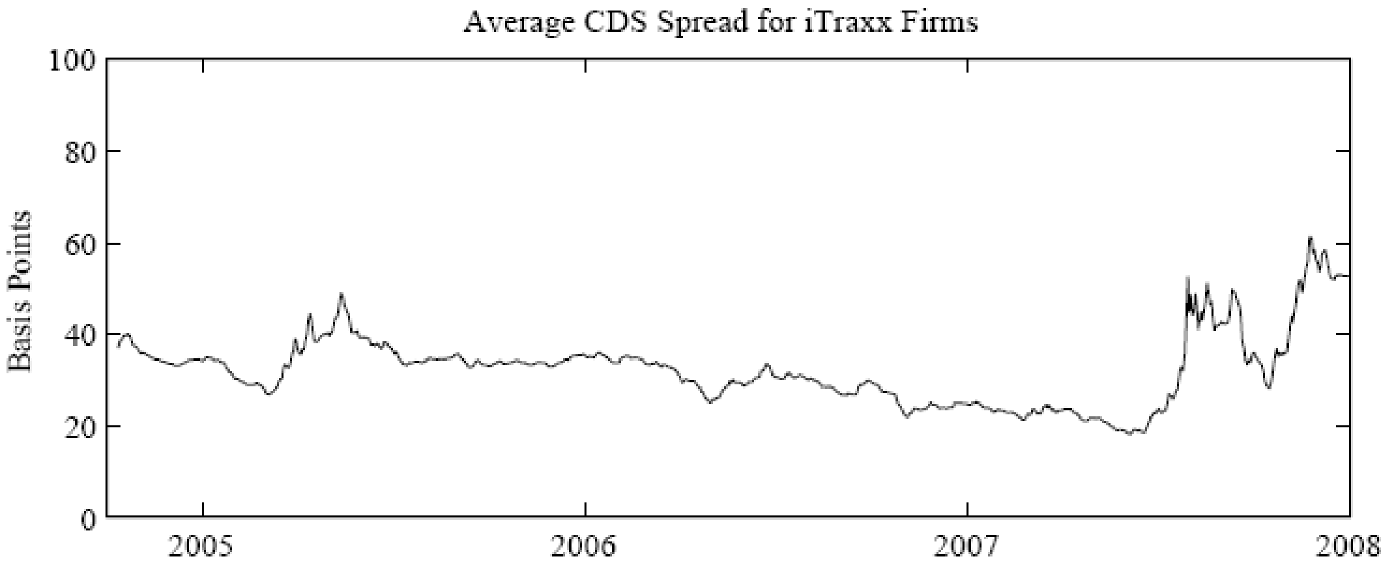
- Note analogy with implied volatility

# Data

- Our data choice is motivated by the CDO market, and organized around the CDX and iTraxx indices
- At any point in time, the CDX and iTraxx indices consists of 125 names. The composition changes every six months
- CDX contains North American names, the iTraxx European names
- Sample period is October 14, 2004 to December 31, 2007
- For CDX we have 61 names throughout the sample period without missing data. For the iTraxx 64 names
- Use 40% constant recovery throughout
- Use 5Y CDS spread
- Equity data standard

# Cross Sectional Averages of CDS Premia

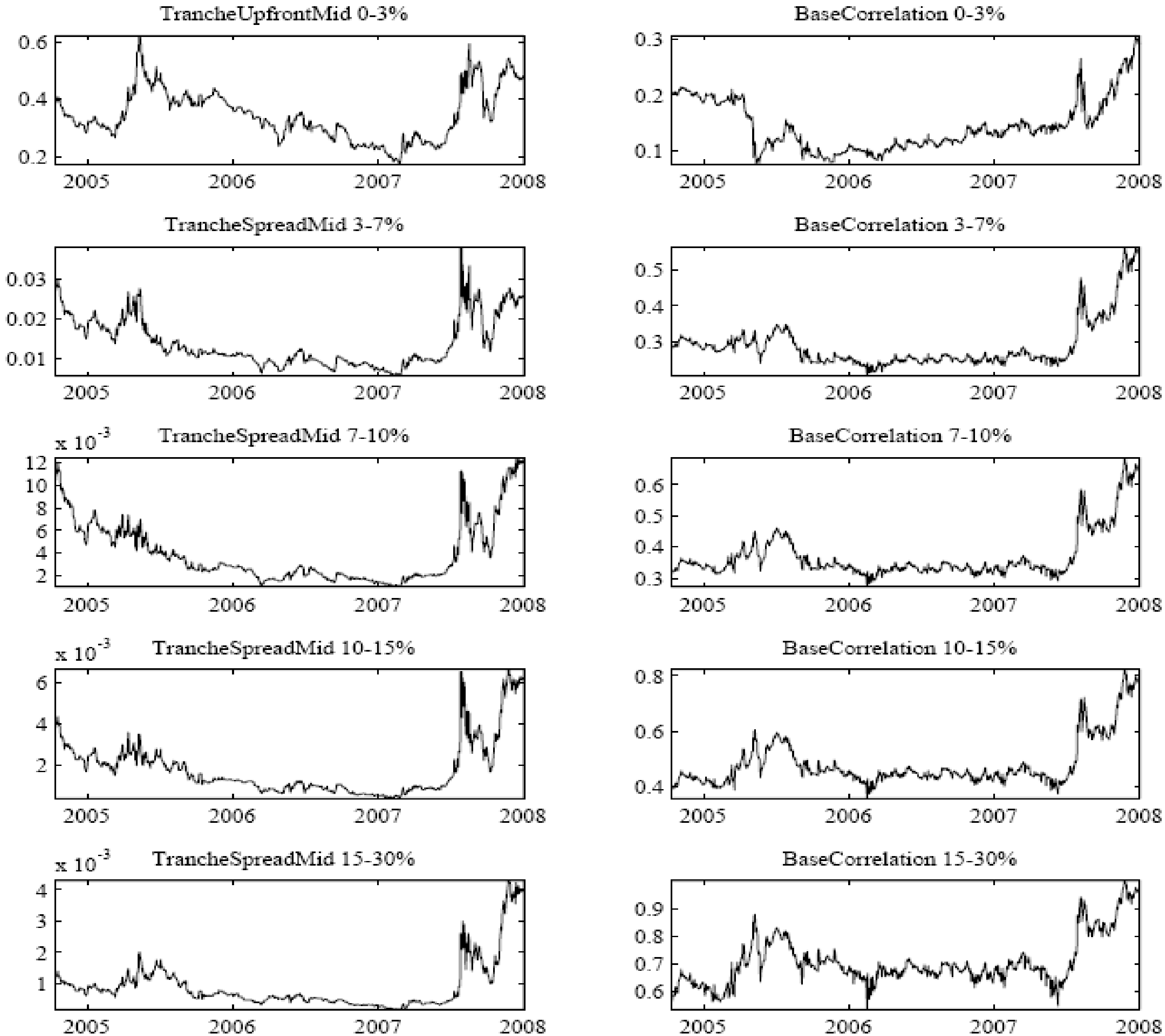


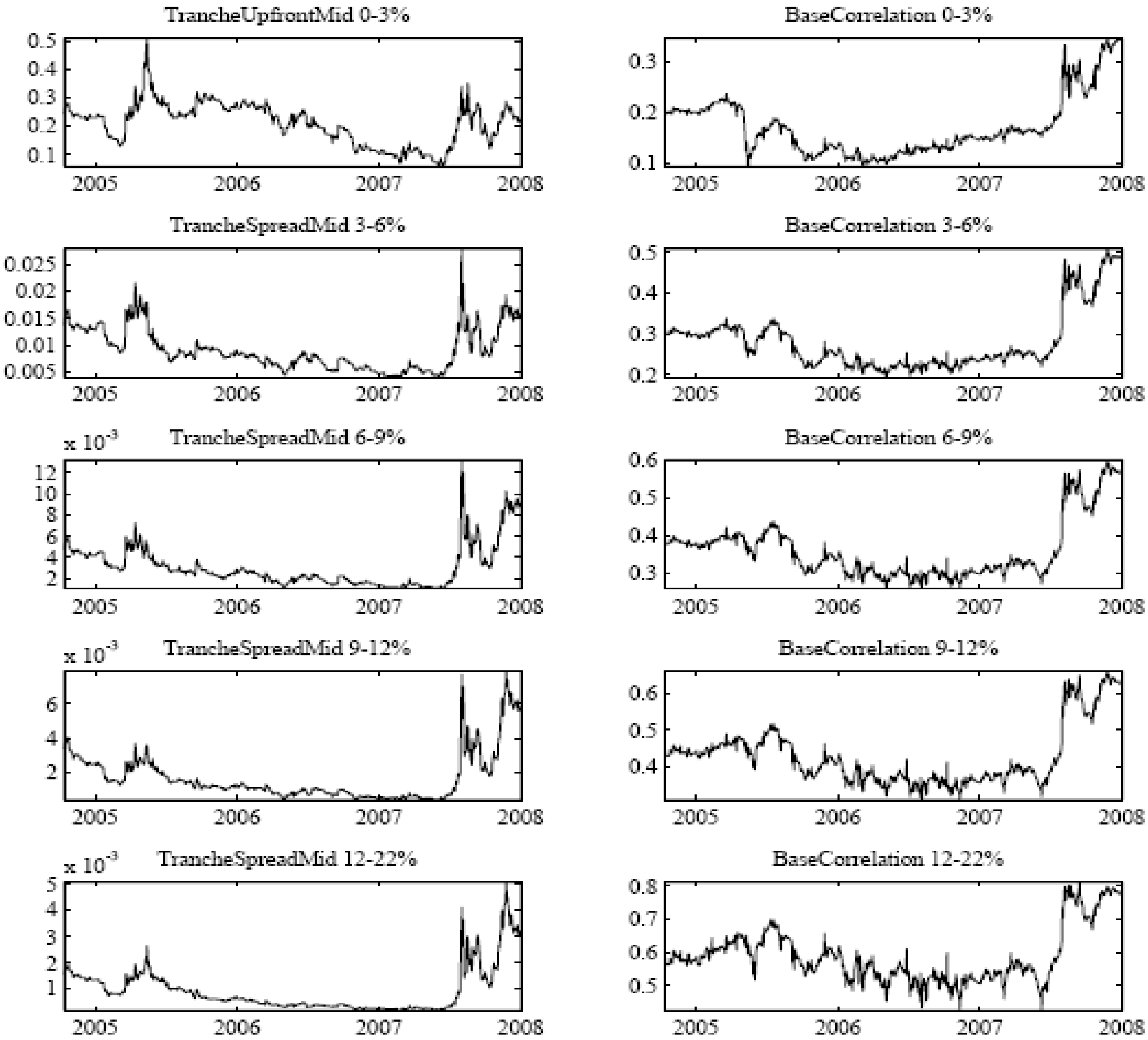
# Descriptive Statistics for CDX Spreads

| Firm Name | Ticker | Average | Std Dev | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Ace Limited | ACE | 37.14 | 14.05 | 0.59 | 2.73 |
| Alcan Inc. | AL | 29.50 | 8.32 | 1.45 | 5.33 |
| Alcoa Inc. | AA | 27.88 | 10.55 | 0.85 | 2.56 |
| Altria Group, Inc. | MO | 63.49 | 39.18 | 1.06 | 3.57 |
| American Express Company | AXP | 24.19 | 15.13 | 2.28 | 8.34 |
| American International Group, Inc. | AIG | 24.47 | 16.13 | 1.88 | 6.81 |
| Anadarko Petroleum Corporation | APC | 35.95 | 9.38 | 0.94 | 3.42 |
| Arrow Electronics, Inc. | ARW | 60.24 | 13.72 | 0.12 | 2.62 |
| AT&T Inc. | ATTINC | 38.01 | 35.82 | 2.49 | 9.45 |
| AutoZone, Inc. | AZO | 63.58 | 22.84 | 0.07 | 1.88 |
| Baxter International Inc. | BAX | 21.28 | 7.45 | -0.15 | 2.50 |
| Boeing Capital Corporation | BA-CapCorp | 16.78 | 7.00 | 0.43 | 2.33 |
| Bristol-Myers Squibb Company | BMY | 18.02 | 5.62 | -0.09 | 1.92 |
| Burlington Northern Santa Fe Corporation | BNI | 26.56 | 6.70 | 1.13 | 4.51 |
| Campbell Soup Company | CPB | 20.83 | 6.45 | 0.37 | 2.37 |
| Cardinal Health, Inc. | CAH | 42.04 | 22.83 | 2.92 | 16.17 |
| Carnival Corporation | CCL | 28.41 | 8.40 | 1.46 | 5.98 |
| CenturyTel, Inc. | CTL | 64.72 | 11.54 | -0.01 | 2.55 |
| Cigna Corporation | CI | 33.59 | 9.47 | 0.00 | 2.43 |
| CIT Group Inc. | CIT | 65.09 | 87.58 | 2.71 | 9.36 |
| Comcast Cable Communications, LLC | CMCSA-CableLLC | 38.77 | 12.32 | 0.04 | 2.23 |
| ConocoPhillips | COP | 21.70 | 6.05 | 0.32 | 3.41 |
| Constellation Energy Group, Inc. | CEG | 35.98 | 12.33 | 0.85 | 3.71 |
| Countrywide Home Loans, Inc. | CCR-HomeLoans | 108.36 | 187.52 | 3.56 | 15.37 |
| Cox Communications, Inc. | COX-CommInc | 55.79 | 18.04 | 0.05 | 4.04 |
| CSX Corporation | CSX | 38.27 | 12.59 | 0.73 | 2.99 |
| Devon Energy Corporation | DVN | 30.86 | 8.72 | 0.98 | 6.16 |
| Dominion Resources, Inc. | D | 35.47 | 9.71 | -0.66 | 2.41 |
| The Dow Chemical Company | DOW | 26.94 | 7.15 | 0.74 | 3.58 |
| Eastman Chemical Company | EMN | 46.45 | 8.59 | 0.60 | 2.82 |
| General Electric Capital Corporation | GE-CapCorp | 21.99 | 11.09 | 2.12 | 8.06 |
| Honeywell International Inc. | HON | 17.71 | 4.26 | -0.01 | 3.21 |
| IAC/InterActiveCorp | IACI | 99.11 | 23.21 | 0.72 | 4.24 |
| International Lease Finance Corporation | AIG-IntLeaseFin | 30.41 | 13.13 | 0.77 | 2.75 |
| Lennar Corporation | LEN | 115.32 | 124.14 | 2.82 | 10.38 |
| Loews Corporation | LTR | 24.61 | 9.04 | 0.28 | 2.53 |
| Marsh & McLennan, Inc. | MMC | 59.29 | 27.83 | 3.38 | 20.73 |
| National Rural Utilities Cooperative Finance Corporation | NRUC | 21.95 | 9.75 | 1.10 | 5.47 |
| News America Incorporated | NWS-AmInc | 39.55 | 13.91 | -0.06 | 1.88 |
| Omnicom Group Inc. | OMC | 26.30 | 7.64 | -0.25 | 2.01 |
| Progress Energy, Inc. | PGN | 34.79 | 15.15 | 0.13 | 3.01 |
| Pulte Homes, Inc. | PHM | 119.33 | 108.01 | 1.94 | 5.13 |
| Rohm and Haas Company | ROH | 26.59 | 6.66 | 0.61 | 3.71 |
| Safeway Inc. | SWY | 54.16 | 16.75 | 0.74 | 2.88 |
| Sempra Energy | SRE | 34.19 | 12.69 | 0.07 | 2.52 |
| Simon Property Group, L.P. | SPG-LP | 37.20 | 21.30 | 2.19 | 8.73 |
| Southwest Airlines Co. | LUV | 39.29 | 10.30 | 0.18 | 3.11 |
| Sprint Nextel Corporation | S | 50.01 | 22.11 | 2.54 | 10.20 |
| Starwood Hotels & Resorts Worldwide, Inc. | HOT | 112.96 | 25.79 | 0.46 | 2.84 |
| Textron Financial Corporation | TXT-FinCorp | 24.42 | 8.68 | 0.62 | 3.69 |
| Time Warner Inc. | TW | 44.53 | 12.47 | 0.40 | 2.68 |
| Transocean Inc. | RIG | 31.00 | 8.70 | 3.00 | 15.35 |
| Union Pacific Corporation | UNP | 33.46 | 8.49 | 0.79 | 4.47 |
| Valero Energy Corporation | VLOC | 40.84 | 11.97 | 1.88 | 9.46 |
| The Walt Disney Company | DIS | 24.33 | 9.54 | 0.27 | 2.32 |
| Washington Mutual, Inc. | WM | 60.27 | 78.24 | 3.60 | 15.42 |
| Wells Fargo & Company | WFC | 16.50 | 11.89 | 2.56 | 9.58 |
| Weyerhaeuser Company | WY | 53.80 | 17.74 | 1.00 | 4.76 |
| Whirlpool Corporation | WHR | 50.79 | 12.23 | 0.55 | 2.52 |
| Wyeth | WYE | 21.52 | 11.44 | 2.20 | 9.38 |
| XL Capital Ltd. | XL | 44.27 | 20.52 | 2.22 | 9.63 |

# CDX Tranche Spreads and Base Correlations



# iTraxx Tranche Spreads and Base Correlations



# Existing Approaches to Estimating Default Probabilities and Credit Correlation



- Historical default data allow us to compute default correlation directly

- Structural (Merton-type) models
- Reduced-form (intensity-based) models

To estimate credit correlation using structural and reduced-form models, we have to extract default intensities (or the relevant default measure) and subsequently use a correlation model

# Existing Approaches to Estimating Credit Correlation

Which correlation model to use?

- Factor models are convenient

- Can use simple rolling correlations
- To estimate time-varying correlations, multivariate GARCH is logical but problematic

  Recent advances in multivariate GARCH literature: DCC

  To ensure straightforward comparability with base correlations, we need an “average” time-varying correlation ⇨ DECO

# Dynamic Equicorrelation (DECO)
# Engle and Kelly (2008)

Dynamic equicorrelation matrix

$$R_t = (1-\rho_t)I_n + \rho_t J_{n\times n}$$

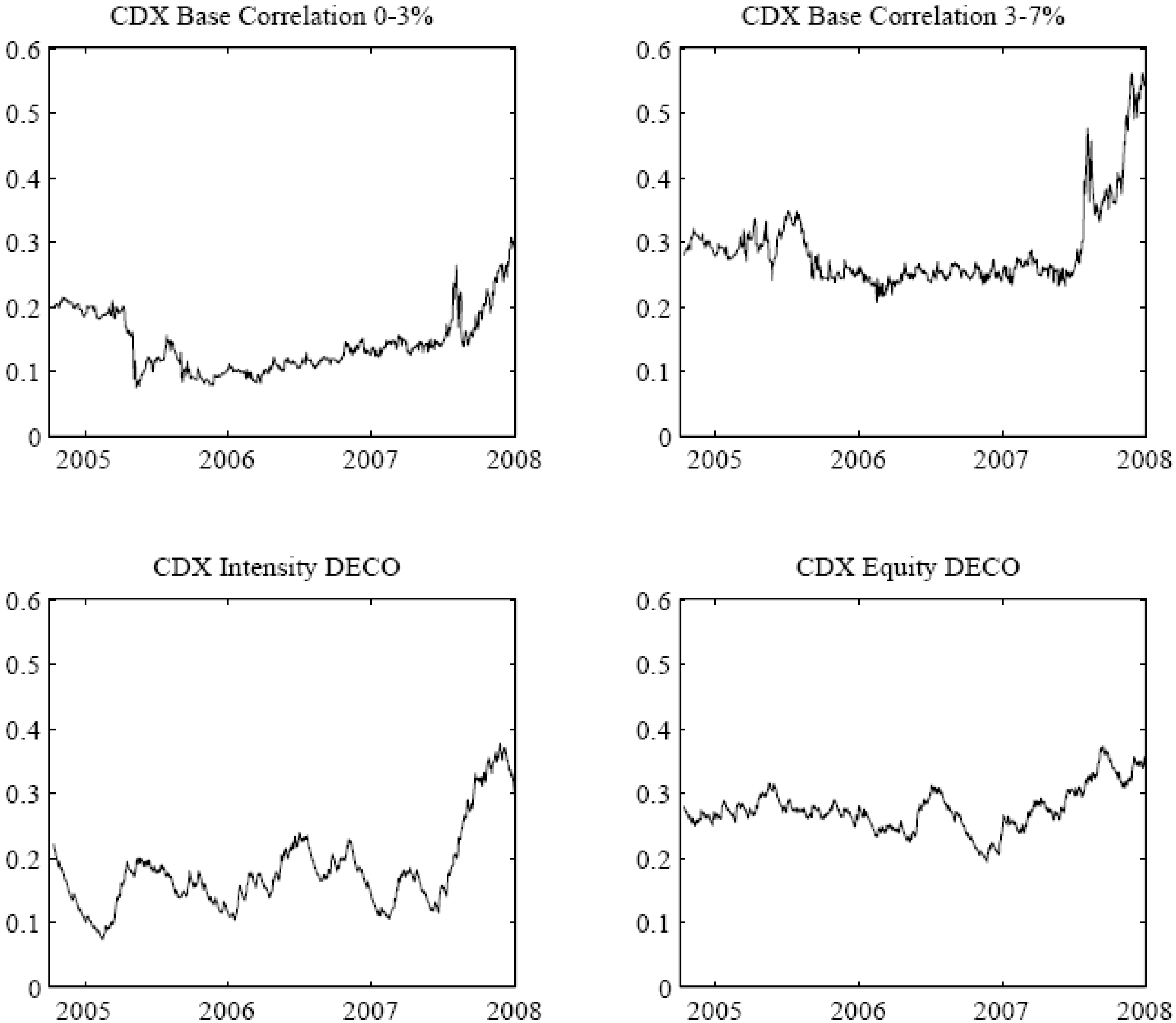
$$\rho_{t+1} = \omega + \alpha u_t + \beta \rho_t$$

$$u_t^{SS1} = \frac{1}{n(n-1)}\sum_{i\neq j}\varepsilon_{i,t}\varepsilon_{j,t} = \frac{1}{n(n-1)}\left[\left(\sum_i \varepsilon_{i,t}\right)^2 - \sum_i \varepsilon_{i,t}^2\right]$$

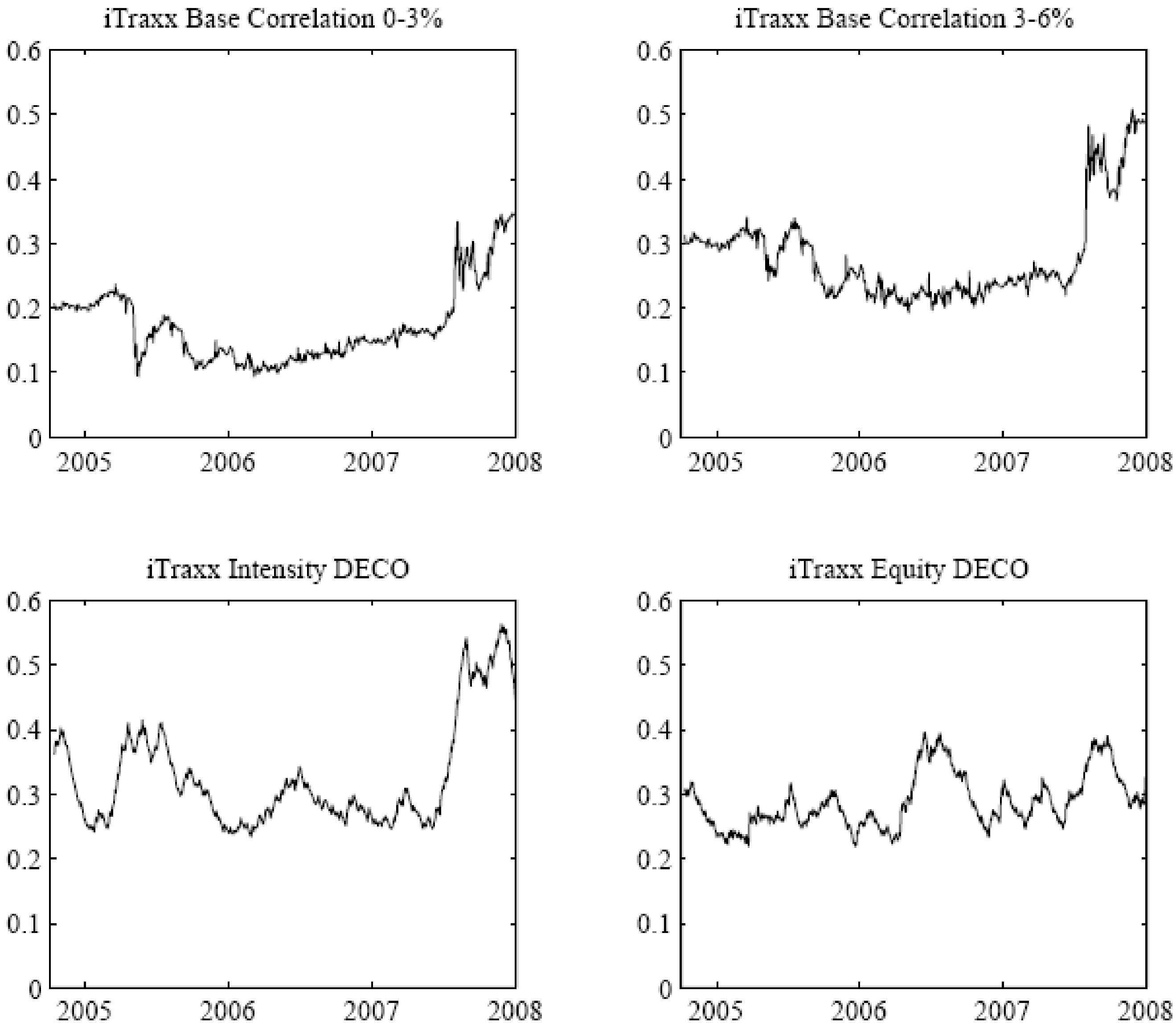
$$u_t^{SS2} = \frac{\sum_{i\neq j}\varepsilon_{i,t}\varepsilon_{j,t}}{(n-1)\sum_i \varepsilon_{i,t}^2} = \frac{\left(\sum_i \varepsilon_{i,t}\right)^2 - \sum_i \varepsilon_{i,t}^2}{(n-1)\sum_i \varepsilon_{i,t}^2}$$

Engle and Kelly find that $u^{SS2}$ is least sensitive to residual vol dynamics and extreme realizations

# DECOs and Base Correlations for CDX Companies



# DECOs and Base Correlations for iTraxx Companies



# How to Compare Correlations Across Markets? Beware of Apples and Oranges



- For CDS-based and equity-based correlations, which correlations do we use? Which ones to compare to CDO-implied correlation?

- One approach: use the Merton model for all markets to filter out the same object and compute its correlation

- What if I want to use another (more accurate) reduced-form model for CDS markets?

# Example: Reduced form CDS Model

- Extract a constant default intensity λ at each t
- Use the resulting time series of λ's to estimate the following model

$$\Delta\lambda_t = \mu_{\lambda,t} + z_t$$

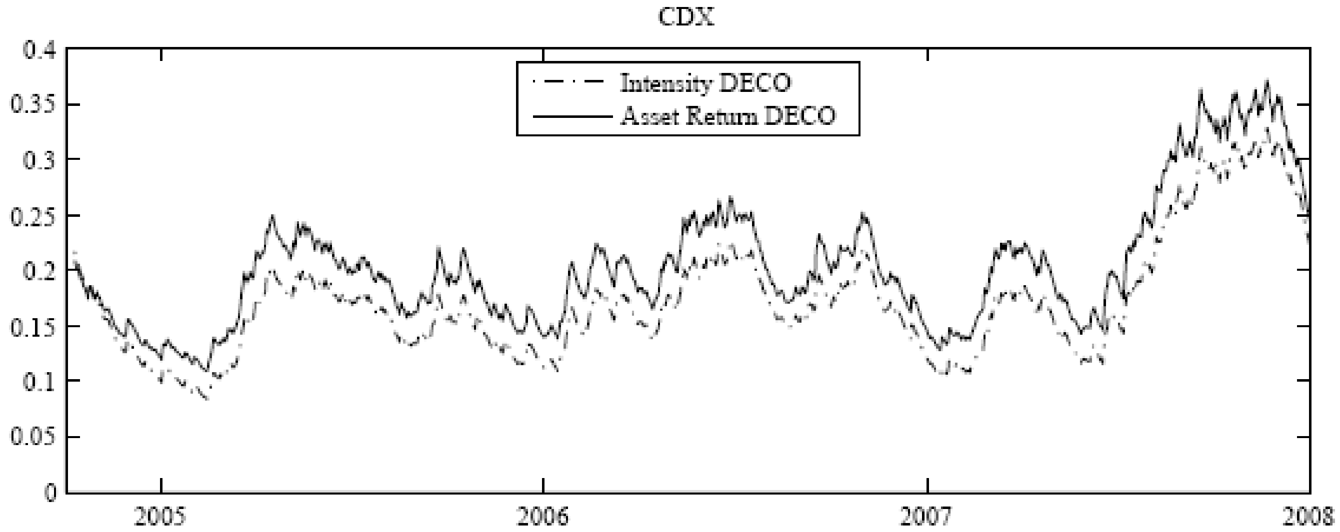
$$\mu_{\lambda,t} = \mu_\lambda + \sum_{i=1}^{p} \phi_i \Delta\lambda_{t-i} + \sum_{j=1}^{q} \theta_j z_{t-j}$$

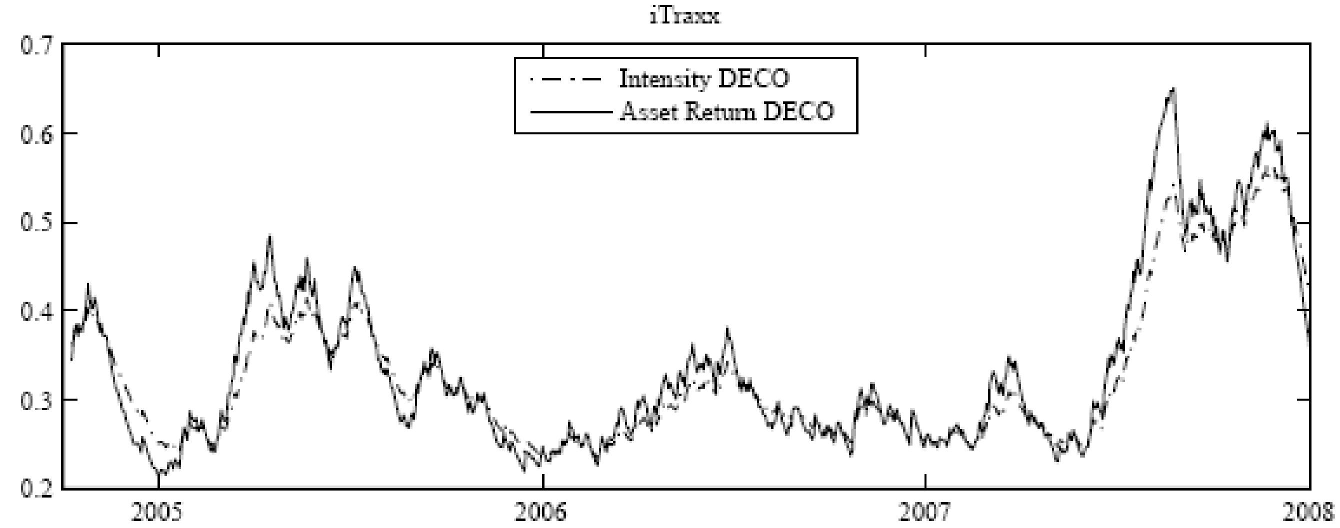
$$z_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \omega_\sigma + \alpha_\sigma z_{t-1}^2 + \beta_\sigma \sigma_{t-1}^2$$

$$\varepsilon_t \sim i.i.d.(0,1)$$

# Intensity DECOs and Asset Return DECOs



# Conclusion

- Implied correlations from CDOs co-move with correlation time series extracted from CDS and equity data.
  - CDO market is not completely segmented from markets for underlying
  - Can use underlying to learn about CDO pricing
  - Gaussian copula model may have some value
- Substantial time variation in correlation
- Unresolved issue: Extracting correlations from CDS data using reduced-form models that can be meaningfully compared with implied correlations

# Future Work

- Characterize cross-sectional variation in correlation dynamics (DCC)
- Use copula models on CDS data to characterize tails
- Price CDOs with model consistent with (time-varying) DECO or DCC
- Estimate time-varying correlation from CDO data