Sovereign Debt Without Default Penalties*

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In the process of revision

October 12, 2006

*We would like to thank conference and seminar participants at the FIRS conference Shanghai (2006), The Oxford Finance Summer Symposium (2006), ESSF Gerzensee, the Bank for International Settlements, the Federal Reserve Bank of Philadelphia, Toulouse University and at the Wharton School for helpful comments. We also benefited from discussions with Bruno Biais, Patrick Bolton, Christophe Chamley, Erik Feijen and Thomas Mariotti. All remaining errors are ours.

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Abstract

The basic question regarding sovereign debt is why sovereign borrowers ever repay, provided that penalties are largely ineffective. In this paper we suggest an answer: sovereign debt would be served as long as the median voter is a net loser from default. Crucially, the debt should be structured so that the interests of the median voter are more aligned with the foreign lenders than with the local taxpayers. We show how this theory can resolve several puzzles discovered by the literature, such as the greater frequency of default on bank debt relative to tradable bonds, the “sudden stop” phenomenon, or the recently observed phenomenon of sovereigns “renegotiating” debt with thousands of creditors. To the best of our knowledge, ours is the first attempt to integrate political-economy and market-microstructure modeling.

Keywords: Debt crisis, debt restructuring, political economy, median voter

JEL Classifications: D72, F32, G15, H63

1. Introduction

It is widely recognized that sovereign debt differs from corporate debt in that the debtor cannot credibly grant the debtor (conditional) property rights over fixed assets or cash flows.1 Hence, most of the literature assumes that sovereign debt is enforced under the threat penalties – such as trade sanctions. However, an important branch of the corporate-debt literature assumes that cash flows are not verifiable so that rights to fixed assets are actually used as threat-points for potential renegotiations; see Hart and Moore (1998). It thus follows that sovereign and corporate debt are quite similar. In both cases the debtor repays under threat, the difference being more in the ability to adjust and refine the penalty; see Eaton and Gersovitz (1981) or Bulow and Rogoff (1989a, b) for classic references. The conclusion that sovereign and corporate debt both have similar incentive

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1For an exceptional case where a creditor managed to threaten enforcement upon a sovereign lender see “Elliot Associates vs. the Republic of Peru”, discussed at length in IMF (2001). See also Zetelmeyer (2003), who estimates that, for a large pool of developing and emerging market countries, only 6.2% of outstanding debt is collateralised.
structure has quite important practical implications: c.f. Krueger’s (2002) proposal for a Sovereign Debt Restructuring Mechanism, which is modeled after Chapter 11 of the US Bankruptcy Code.

In this paper we consider an alternative (extreme) case where no penalty for default is implementable, so that sovereign debt must be supported by an entirely different incentive scheme. The main idea is that sovereign debt is structured so that it is in the best interest of the median voter to serve it. Two elements in this structure are critical; first, the debt should be tradable, so that in case of sanctions foreigners can sell the bonds to locals who would obtain repayment. Second, the debt should remain at the level at which the median voter still has an incentive to repay. Note that when domestic and foreign creditors hold identical instruments, default benefits domestic tax-payers but harms domestic bondholders, where the net effect depends on exact positions. The trick then is to find a level of debt where the interests of the median voter are more closely aligned with foreign bondholders than with local taxpayers.

This paper is partially motivated by a certain unease that the literature expresses about the current theory. While it is widely agreed that sovereign default may disrupt the debtor’s operations, there is a widespread feeling that the implied penalty is insufficient to support the level of activity that is observed in sovereign-debt markets. For example, Eichengreen (1988) finds no evidence for a negative relationship between pre-WWII default and post-war lending.² Bulow and Rogoff (1989a) comment that, “admittedly, there are many uncertainties surrounding the actual damage which a lender can inflict on an LDC” following default. They therefore dismiss reputational models – where sovereigns repay just in order to preserve the capacity of further borrowing.³ Instead, Bulow and Rogoff (1989b) suggest that sovereign debt is enforced by penalties that creditors can enforce within their own jurisdictions, like trade sanctions. However, Tirole (2002) points out that such sanctions suffer from similar weaknesses, particularly a severe free-rider problem amongst creditors, combined with a strong incentive to renegotiate ex-post-inefficient sanctions. Note that all these problems result from the sharp separation between domestic and foreign creditors, so that all domestic interests are unanimously aligned against serving the debt. We avoid the difficulty by assuming that some domest-

²Rose (2005) finds significant long-term decrease in bilateral trade flows following sovereign default. Martinez and Sandleris (2004), however, argue that this should not be interpreted as a penalty, because trade declined across all trading partners indicating that it might just be the result of the same crisis that caused the default.

³Whether reputational concerns are sufficient to enforce repayment hinges to some extent on the question whether a country can save the funds that were scheduled for repayment. Kletzer and Wright (2000) argue that default is made sufficiently costly to prevent default if a country cannot access capital markets as a lender and therefore cannot save. Amador (2003) shows that a government may save too little for political economy reasons, and this again makes default more costly and repayment sustainable.
tic agents hold “foreign” debt, which breaks the unanimity and gives the median voter a genuine interest in serving the debt.

Unfortunately, it is hard to motivate our theory by direct statistical evidence about domestic positions of sovereign debt; namely the “home bias”. Often, the debt is held by custodians, who will not reveal the identity of the ultimate creditor even during “renegotiations” (see Gray (2003)). According to our own theory, this is not surprising, because it is essential that the line between the foreign and domestic position is not clearly marked. Yet, there is a widespread feeling, that a “large fraction of the [foreign-currency denominated] government debt was issued domestically and purchased by domestic banks”; see Roubini (2002). Perhaps more revealing is some anecdotal evidence. Thus, for example, on May 7, 1999 the Russian government announced that it would pay $333m interest on five out of seven tranches of bonds. The announcement was a great surprise as – according to a contemporary analyst – “none expected to get any money in May”. A month later The Economist commented cynically that now that “a big chunk of ex-Soviet debt ... is held not by the original banks, but by hedge funds and other individuals” the repayment was actually “to the benefit of wealthy Russian individuals and institutions”. Then a year later, after reaching an agreement with the London Club to restructure $32bn of Soviet-era debt (on February 11, 2000) The Economist reported that “some observers ... note sourly that it has delivered a hefty profit to Russian banks which bought up the least popular category [of the debt]... at time its price had fallen following the leak of draft scheduling terms”.

Our paper attempts not only to provide an alternative enforcement mechanism, but also to resolve several puzzles pending in the sovereign-debt literature. There are four main results. First, we develop the basic setup. In a two-period economy the government offers sovereign debt for initial trading (IPO). We show that without any external penalty, service of that debt depends on the size of the debt in relation to the position of the median voter. The home bias emerges naturally as a condition for debt enforcement.

Second, through a modest extension of the assumptions, we show that the IPO may be re-interpreted as a secondary trading. This extension allows us to make the important

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4 One exception is IMF (2003) Figure 4.15, which reports a sharp increase in domestic positions among emerging markets, reaching 40% by 2003. The data is provided by PIMCO, a leading fixed-income firm.
5 Often, the debt is held by custodians, who will not reveal the identity of the ultimate creditor even during “renegotiations”; see Gray (2003).
6 See also Cline (2002) who stipulates that the primary subscribers to Argentina’s ‘megaswap’ of June 2001 were domestic pension funds.
7 See Reuters report by Jukie Talkacheva on May 7, 14:51.
8 See The Economist, June 3 1999.
9 The Economist, February 17, 2000.
10 We develop our model in the context where the political process is a simple majority vote. The model can easily be adapted to situations where political power is unequally distributed across the population.
distinction between non-tradable bank debt and tradeable bonds, and argue that it is much “easier” to enforce the latter. We relate this result to some historical observations that imply that the sovereign-debt crisis of the 1980s was essentially a bank-debt crisis; see more detailed discussion of the evidence next to the extension.

Third, we relax the assumption that domestic and foreign positions are common knowledge, making the bond-market “opaque” with respect to the identity of the traders. In such a setting, a Kyle-type market maker (see Kyle, 1985) uses his information of the order flow in order to update his belief about the position of the median voter. He then uses this “guess” in order to price the debt. An important implication is that random non-fundamental “noise” in international capital flows may cause erratic fluctuations in bond prices. Hence, a “sudden stop” phenomenon (Calvo, 1998) may occur. To the best of our knowledge, ours is the first attempt to integrate political-economy and market-microstructure modeling.

Fourth, we relax the assumption that the repayment of sovereign debt is funded by lump-sum taxes. In such a setting a non-trivial partial default may occur. We use this result in order to resolve an important puzzle in the sovereign-debt literature: the recently-observed phenomenon of sovereign borrowers “renegotiating” claims with a huge number of creditors, a phenomenon that does not exist in corporate borrowing (see evidence below). Our explanation is simple: these renegotiations are in fact unilateral write-offs down to the point that suits the interests of the median voter, but no further.

The paper is organized as follows. After a short review of the related literature in Section 2, Section 3 describes the basic set-up. We then proceed through three extensions of the basic setting. In Section 4 we highlight the important distinction between bank debt and tradable bonds. In Section 5 we introduce market opacity, which gives rise to the “sudden stop”. In Section 6 we provide a theory of partial default and debt “renegotiation”. Section 7 concludes.

2. Related literature

Our work is related to some of the Political Economy literature that explores the role of the political process in addressing commitment problems arising from inter-generational redistribution (see Persson and Tabellini, 2002). Tabellini (1991) shows that the intra-generational redistribution associated with (domestic) public debt repayment can break the inter-generational conflict that would otherwise render debt repayment non-credible.

\[\text{In addition our paper raises another important political economy question: how does securities trading prior to an election affect the median voter’s preferences and therefore the voting outcome? This question has been addressed by Musto and Yilmaz (2003) in the context of a security whose payoff is directly contingent on the identity of the winning party.}\]
The young generation holds no bonds and therefore suffers monetarily from repayment. Nevertheless, their altruism towards their parents’ welfare leads the children of the rich to vote in favor of repayment. Dixit and Londregan (2000) argue that repayment of public debt becomes credible when constituencies with a lot of political clout hold large amounts of the debt. Tabellini, and Dixit and Londregan only consider domestic public debt, while we focus specifically on a government’s ability to raise foreign debt. This allows us to derive new results regarding security design and debt renegotiation in the context of sovereign debt. Moreover, the inclusion of a market microstructure mechanism to price sovereign debt is novel to our model. This enables us to derive further results from the interaction between the voting and the pricing mechanisms.

Drazen (1996) analyzes the choice of foreign versus domestic debt in a political economy context. He assumes that a country can segment the market by issuing different instruments to domestic and foreign residents and thus charge different effective interest rates on its debt as well as default selectively. The political process determines the ex ante choice between domestic and foreign debt, but not, as in our paper, the repayment decision: Drazen (1996) assumes that the government can commit to repaying the domestic debt and that foreigners can impose a default penalty. The ex ante choice between domestic and foreign debt is then determined by domestic preferences over the rate of return on savings and government spending.

In a recent paper, Sandleris (2005) proposes a model of sovereign debt payment without default penalties. In his paper the government has private information about the fundamentals of the economy and can use its repayment behavior as a signal of such information. The mechanism is thus very different from the one put forward in our paper.

3. The basic model

In this section we describe the basic model. In a two-period economy, the government offers bonds for initial trading (IPO). The market is “transparent” so that the levels of local and foreign demand are both common knowledge. Ex post, voters decide whether to serve or default on the entire debt; lump-sum taxes are imposed later on to finance debt repayments. We present our “enforcement mechanism” in the simplest-possible manner, and discuss some of its implications. This simple formalization serves as a basis for subsequent extensions discussed below, where the policy implications become more apparent: the crucial importance of markets where domestic and foreign agents can trade sovereign debt, the potentially destabilizing effect of “noisy” capital flows when the level of demand is not common knowledge, and the scope for partial default and debt
“renegotiation” when repayments are funded by relative taxes.

3.1 The structure

In period $t = 0$ the government offers $B$ units of one-period-maturity bonds, each having a face value of one unit of the numeraire. That numeraire is essentially a unit of consumption good, though it might be convenient to normalize it to one “dollar”. On the demand side there are both domestic agents and foreigners. The domestic population is of measure one. Agents are ordered according to their period-0 income endowment, $w_i$, with the index $i \in [0,1]$ running across the entire population. Income is distributed as follows: agents $i \in [0, \mu_l]$, $\mu_l < 1/2$, have a zero income, while agents $i \in [\mu_h, 1]$, $\mu_h > 1/2$, have $W > 0$ income (in terms of the numeraire). The remaining agents, $i \in (\mu_l, \mu_h)$, have an income endowment of $w_i = \tilde{\delta}W$, where $\tilde{\delta}$ is the realization of a binary shock that equals $\delta \in [0,1)$ with probability $\gamma$ and 1 with a probability $1 - \gamma$. Domestic agents care only about period-1 consumption. To make things simple, we assume that government bonds are the only store of value so that

$$s_i = w_i,$$

where $s_i$ denotes agent $i$’s savings. The demand of the foreigners (not fully modelled) is $f$ units of the numeraire, which is the realization of a random variable with a density function $h(f)$. Note that both $f$ and $\tilde{\delta}$ are already realized at the time that the bond-market opens, and the realization is common knowledge to all market participants.

Given supply and demand, a risk-neutral market maker determines a fair price $P$ for the bond, and takes on the entire slack between supply and demand. A common justification of the fair-pricing assumption is that the market maker represents a reduced form of a competitive industry (or equivalently, an oligopolistic industry competing Bertrand); see Kyle (1985). If the market maker is a domestic agent, his weight within the voting population is still of measure zero. Without loss of generality – and more realistically – we may thus assume that the market maker is a foreigner, perhaps an investment bank. Obviously, the market maker will become a more interesting entity in Section 5 where we relax the assumption that the realization of both $f$ and $\tilde{\delta}$ is common knowledge at the time of trading.

At the first stage of period 1, the decision whether to serve the debt fully ($\tilde{\alpha} = 1$) or entirely default on it ($\tilde{\alpha} = 0$) is put forward for a vote; no generality is lost by ignoring – at this stage – the possibility of partial default or debt “renegotiation”. Each agent votes according to the personal benefit – and cost – that she draws from serving the debt. The cost is the bearing a lump-sum tax, $T$, which is levied in order to repay the debt; the
benefit is obtaining service on the bonds that she bought previously. According to the assumptions made so far, agent-$i$’s benefit from debt service equals to the face value of her position, namely the number of bonds she purchased at period 0, $\frac{s_i}{P}$. It follows that she votes in favor of repayment if

$$\frac{s_i}{P} > T.$$  

(2)

and against repayment if the inequality is reversed. In case of equality, we allow voters to play mixed strategies; moreover, we allow indifferent voters to correlate their voting, so that default actually occurs with the intended probability. As in any analysis of this sort, the median voter plays a critical role; we denote him by $m = 1/2$.\textsuperscript{12}

At the second stage of period 1 the voters’ decision is implemented and the government finances the bond repayment by a lump sum tax $T$ on domestic agents. We assume that by this time agents have enough additional income, on top of their period-0 endowment, to pay the tax

$$T = \tilde{\alpha}B.$$  

(3)

We relax the unrealistic assumption on lump-sum taxation in Section 6.

We do not explicitly model the expenditure side of the government’s budget, namely the good use to which the borrowing, $D = BP$, is put. Possibly, one may assume that the funds are invested in infrastructure, which bears a certain social rate of return $\rho > 0$ (a technological parameter) from which all domestic agents benefit equally at date 1. It follows that domestic agents are unanimously in favor of raising the maximum possible amount of funds.

3.1.1 Equilibrium

Given the realization of $\tilde{\delta}$ and thus the level of local demand, the equilibrium price $P$ implies a certain allocation of the bonds across the three classes of domestic agents. Given that allocation, agents vote according to (2), with an outcome that may be either perfectly anticipated, or random if agents decide to play mixed strategies. At the same time, the equilibrium price $P$ is set by the market maker in accordance with the fair-price assumption

$$P = E[\tilde{\alpha}|\tilde{\delta}].$$  

(4)

More formally,

\textsuperscript{12}The model can be easily adapted to incorporate other than a majority rule.
**Proposition 1.** There exists a mixed-strategy equilibrium where \( P \) is both the fair price of the bond and the probability of serving the debt, such that

\[
P = \begin{cases} 
1 & \text{if } B \leq w_m \\
\frac{w_m}{B} & \text{if } B > w_m 
\end{cases}
\]  

(5)

*Proof.* Consider the case where \( B \leq w_m \). Since \( \alpha \) is bounded (from above) by 1, its expected value \( P \) cannot exceed 1. Hence, condition (2) holds, repayment occurs with certainty and \( P = 1 \).

Now consider the case where \( B > w_m \). The price of the bond cannot be 1, for then condition (2) is reversed and default occurs with certainty, which is inconsistent with a fair price of 1. Similarly, the price of the bond cannot be zero, for then the median voter is allocated an infinite number of bonds and therefore votes for repayment. Hence, the price needs to be between zero and one, which implies that default is random; it follows that the middle-income group plays mixed strategies. For that to happen, the median voter must be indifferent between default and repayment, condition (2) should hold with equality and \( P = \frac{w_m}{B} \).

3.2 Debt Capacity and the home bias

It follows immediately from Proposition 1 that

**Corollary 1.** The maximum amount of funds \( \overline{D} \) that the government can raise is limited by the median voter’s wealth \( \overline{D} = w_m \).

If \( B < w_m \) and \( P = 1 \), then the government can increase \( B \) without affecting the price of the bond thereby increasing the dollar amount borrowed. If, however, \( B > w_m \) then any increase in \( B \) is followed by a proportional decrease in price without any change in revenue.

Hence, the government’s borrowing capacity is not affected by the usual macro-economic variables, e.g. aggregate income, but rather by the (period-0) income of the median voter. To see this more clearly, consider an economy where \( \tilde{\delta} \) has been realized at the level of \( \delta \). Now consider the following comparative statics: increase \( W \) and decrease \( \delta \) proportionately by the same factor of, say, two. The median voter’s income remains unchanged, but the richer agents \( i \in [\mu_h, 1] \) double their wealth. The whole economy grows by an additional \((1 - \mu_h)W\), but borrowing capacity is not affected at all.

In a sense, it is *income distribution* rather than aggregate income that determines borrowing capacity in our model, although the effect need not be monotonic. To see this
Figure 1: Shows the degree to which a country can leverage up its domestic debt capacity as a function of the fraction $\mu_l$ of domestic agents who are bond holders. Leverage is maximised when default generates a strong redistribution of wealth from domestic bondholders to domestic tax payers. This aligns the median voter’s interest with that of the foreign creditors and thereby makes repayment of larger amounts of external debt credible.

more clearly, define external borrowing capacity as

$$\bar{E} = w_m - \int_{i=0}^{1} \frac{w_i}{P} di,$$  \hspace{1cm} (6)$$

and consider the case where $\tilde{\delta} = 1$ and $\mu_h = 1$, so that there are only two income classes (in period 0) – with zero-income and the $W$-income. When $\mu_l$ increases above $1/2$ and the zero-income class becomes a majority, the median voter favors default, so that any attempt to raise debt (internal or external) is doomed to fail.\footnote{For the sake of this discussion and the diagram below we relax the assumption that $\mu_l < 1/2$.} At the other extreme, when $\mu_l = 0$ and society becomes completely egalitarian, the government cannot raise any external debt either. Between these two extremes, external-debt capacity increases linearly (see Figure 1).

This comparative statics highlights the basic mechanism of our model. On aggregate, serving the debt drains resources from the economy, which implies that at least some agents would vote for default. At the same time, debt is served when it is in the best interest of the median voter. It follows that sovereign debt can be “enforced” only thorough
a certain conflict of interests between local agents. Since in our model interests diverge as a result of differences of income, some inequality is essential in generating external debt capacity. At the same time, if income distribution is so un-equal that it is in the interest of only a small minority to serve the debt, capacity again falls to zero.

Lastly, Figure 1 implies a built-in home bias in sovereign debt: capacity is maximized when \( \mu_t \) approaches \( 1/2 \) from below. Hence, at least half of the debt must be held by domestic agents, otherwise the median voter would not be interested in serving it. We believe that our argument can be extended to equity, the more common object of the of the home-bias puzzle.\(^{14}\)

4. Extension: secondary trading, bonds and bank debt

Our first extension involves no formal elaboration of the model; we argue that the results of the previous section would still go through if the period-0 trading is interpreted as a reallocation of old debt via a secondary market rather than an IPO of new debt. Crucially, however, this reinterpretation allows us to make the important distinction between tradable and non-tradable debt – primarily bank debt.

The extension is motivated by the observation that tradable sovereign debt is “easier” to enforce relative to bank debt. Indeed, Beers and Chambers (2003) point out that the primary channel of sovereign lending during the 19th century was through tradable bonds. Banks entered the market during the second half of the 20th century, only to suffer the effect of the 1980s debt crisis. Interestingly, while default rates on bank lending increased sharply, default rates on bonds remained low. Moreover, there were quite a few cases where sovereign debtors defaulted on their bank debt, but served their bonds. Recently, maybe as a reaction to that experience, the structure of the sovereign-debt market has change significantly. An IMF (2003) publication reports that “the traditional syndicated loan market shrank in the second half of the 1990’s ... [and] the role of the lead bank has shifted ... from that of the agent of the lending group to the underwriter of the deal”. At the same time, there was a sharp fall in interest-rate spreads, which is consistent with a lower probability of default.\(^{15}\)

Suppose that at time \( t = -1 \) a certain amount of sovereign debt, \( B < w_m \), was issued. The debt had two components: non-tradable foreign bank debt, \( B_{BK} > 0 \), and tradable

\(^{14}\)Kremer and Mehta (2000) identify a related reason for home bias: since repayment and the amount of tax are negatively correlated for locals, their risk exposure from holding the domestic bond is smaller than for foreigners, who are not ‘hedged’ through tax savings in the case of default. Hence, home bias may also result from risk sharing. In the context of equity, Biais and Perotti (2002) investigate the political economy considerations of share allocations in privatizations.

\(^{15}\)See also Bolton and Jeanne (2005) .
bonds $B_{TR} > 0$, so that $B_{BK} + B_{TR} = B$. Most of the rest remains as in the previous section. Note, particularly that the domestic population receives its income endowment at $t = 0$, so that at the time of the IPO (i.e. at $t = -1$) the debt had to be taken by either the foreigners or the market maker. Note also that political decisions still take place at $t = 1$, which captures the reality of a much higher frequency of trading relative to political decisions. However, voters would have to make two separate decisions, one with respect to the bank debt and the other with respect to the bonds.

It thus follows that when period-0 market opens, the domestic agents would spend their endowment buying sovereign debt – the only store of value available to them. By construction, the banks cannot sell their debt, but the foreigners would (either the whole lot or part of it). Doing so, they will find themselves in exactly the same position as the government of the previous section. Since $B_{TR} < B < w_m$, the debt would be traded at a price of 1 in (perfect) foresight that it would be served at $t = 1$. Hence, the bond-holders would be fully served, either by holding the bonds to maturity or by selling them at face value at $t = 0$. At the same time, it is not in the interest of any domestic agent to serve the bank debt. Note, however, that had the foreign banks held their debt in the form of bonds they could have participated in the interim trade and obtain full service.\footnote{We ignore the question why the banks agreed to lend at $t = -1$; either another mechanism, e.g. the threat of sending “gun boats”, existed then but vanished later, or the analysis may be viewed as an off-the-equilibrium-path analysis of the impossibility of sovereign bank lending.}

Note that at period 0 it is already in the best interest of local agents to collude against the foreigners, abstain from trade and default on the debt ex post. Hence, this extension of the model highlights another important aspect of our mechanism, which is the role of open markets in breaking potential collusion. From a practical point of view, the question is how realistic is the assumption that the government cannot effectively prohibit such trade. Perhaps the many failures to impose currency controls are a testimony that it is. Note also that domestic traders need not hold their bonds directly; it is enough that a custodian holds (and redeems) them on their behalf for them to vote against default. From an ex-ante (period $-1$) point of view, the sovereign’s inability to impose restrictions on bond trading is a commitment device from which the economy clearly benefits (remember that the borrowing is used in order to fund welfare-improving investments in infrastructure).

5. Extension: market opacity and the “sudden stop”

In this Section we maintain the assumption that $f$ and $\delta$ are realized before period-0 trading starts, but relax the assumption that the realization is common knowledge. Under conditions of such “opacity” the market maker can no longer distinguish foreign
from domestic demand. Clearly, his “guess” regarding the size of the domestic position still plays a critical role in his pricing decision, for it is directly related the likelihood of default.

The extension is pursued not just for the sake of realism, but mainly because of its relation to the popular conjecture that random non-fundamental “noise” in international capital flows may cause substantial disruption in economic activity and sharp fluctuations in asset prices; c.f. Calvo’s (1998) analysis of the “sudden stop” in that year’s financial crisis.\(^\text{17}\) In our model, upon a random drop in foreign demand the market maker observes a low level of the order flow and assigns a lower probability to the possibility that the median voter has strong demand for the bond. As a result, he also assigns a higher probability to the event of default. We develop this idea further and derive results about the relationship between foreign-demand volatility on debt capacity.

Suppose that upon the realization of \(\delta\) and \(f\), agents make their investment decisions (which in this simple setting are independent of their inferences regarding other agents’ decisions) and submit their orders to the market maker, who observes the entire order flow

\[
d = f + l,
\]

where \(l\) denotes local demand. As before, the market maker prices the bond and takes on the slack between \(B\) and \(d\). Although the market maker cannot observe \(f\) and \(l\) directly, he can form expectations, conditional upon \(d\), using information about the unconditional distribution of these variables, which is still assumed to be common knowledge.

Denote by \(g(d)\) the market-maker’s conditional probability that \(\tilde{\delta} = 1\). If \(\tilde{\delta} = 1\), then local demand is

\[
l_1 = W (1 - \mu_l)
\]

and if \(\tilde{\delta} = \delta\) then local demand is

\[
l_\delta = W [1 - \mu_h + \delta (\mu_h - \mu_l)].
\]

The market maker observes (7) and uses Bayes’ law in order to infer the conditional probability

\[
g(d) = \frac{(1 - \gamma) h (d - l_1)}{(1 - \gamma) h (d - l_1) + \gamma h (d - l_\delta)}.
\]

This allows us a characterization of equilibrium prices when \(B = W\).\(^\text{18}\)

\(\text{17}\) See also Kaminsky and Reinhart (1999) exploration into the interrelations between balance-of-payment and banking crisis; see also Detragiach and Spilimbergo (2001).

\(\text{18}\) It is straightforward to complete the characterisation of equilibrium for any level of \(B\). For expositional clarity we focus on the case \(B = W\) which maximises the expected amount of funds raised ex ante.
Proposition 2 If $B = W$ then there exists a unique equilibrium with price

$$P = \max \{ \delta, g(d) \}. \quad (9)$$

Proof. Suppose $g(d) > \delta$. With probability $g(d)$, $\tilde{\delta} = 1$ and the median voter’s position is $\frac{W}{P} \geq B$, so that repayment occurs. With complementary probability the median voter’s position is $\frac{\delta W}{P}$. When $P = g(d) > \delta$ then $\frac{\delta W}{P} < B$ so default occurs. Hence, repayment occurs with probability $g(d)$ and the corresponding fair price is then $P = g(d)$.

Suppose $g(d) \leq \delta$. Therefore, the probability that $\tilde{\delta} = 1$ is smaller than $\delta$. If the median voter were to default for sure when $\tilde{\delta} = \delta$, then the price would drop below $\delta$. This, however, cannot be an equilibrium, because the median voter’s position in that case would be larger than $B$ and he would therefore vote in favor of repayment for sure. If the median voter always voted in favor of repayment, then the equilibrium price would have to be $P = 1$, which implies that the median voter has a strict preference for defaulting after $\tilde{\delta} = \delta$. The only equilibrium is therefore in mixed strategies over the default decision. The median voter is only indifferent between repayment and default after shock $\tilde{\delta} = \delta$ if $P = \delta$. Denote by $\theta$ the probability with which the median voter after shock $\tilde{\delta} = \delta$ votes in favor of repayment when he is indifferent. Then $\theta$ is determined such that the price is informationally efficient:

$$\delta = g(d) + \theta(1 - g(d)). \quad (10)$$

Since $\delta \geq g(d)$ it is clear that a $\theta \in [0, 1]$ exists such that (10) can be satisfied for any $\delta$.

5.1 The sudden-stop phenomenon

Consider the case where foreign demand $f$ is normally distributed with mean zero and variance $\sigma^2$. In this case, the market-maker’s Bayesian update (8) can be calculated explicitly by substituting $h(f)$ with the normal density. Figure 2 gives the resulting price function.

From Figure 2 we can see that the bond price is a non-linear function of total demand $d$. At high realizations of $d$ it is relatively certain that local demand is sufficiently high to guarantee repayment. Therefore, the price is relatively insensitive to changes in $d$ - the bond market is stable. For lower realizations of $d$ price can become extremely sensitive to changes in $d$. This occurs in the region where the market maker’s inference over local demand is very noisy. The market maker therefore responds erratically to variations in underlying demand. The price of debt drops sharply in response to a decline in demand - the bond market is unstable. This can be understood as a crisis phenomenon: a small shock is enough to push the country over the brink.
Figure 2: Gives the bond price as a function of underlying demand (solid line) and the density of demand (dashed line). Parameter values are $W = 1$, $\sigma^2 = 0.2$, $\gamma = 0.05$, $\delta = 0.1$, $\mu_l = 0.1$, and $\mu_h = 0.9$. A small probability of a shock to local demand can give rise to a strong drop in price for a small reduction in overall demand.

Two points are noteworthy about Figure 2. Firstly, the price of the bond falls when foreigners sell it, even though their demand has no impact on the bond’s intrinsic value. When foreigners sell, the realization of total demand is low. Due to market opacity, the demand shock cannot be identified as non-fundamental and drives down the bond price. As the picture shows, a non-fundamental outflow of capital can thus have a substantial effect on the market price of the bond.

Secondly, our model provides an interesting variation on existing market microstructure models. In that type of setting price effects are commonly generated by the presence of traders who are privately informed about an asset’s fundamental value. The market maker can then infer some of that information from the order flow and adjusts prices in response. In our setting there are no privately informed traders. Instead, demand itself determines the bond’s fundamental value via the political economy considerations that underlie the governments default decision. The information contained in total demand about its composition then generates a link between price and demand.

Note that more standard market microstructure models tend to generate a linear relationship between equilibrium price and demand (for an overview see O’Hara (1995)). An exception is Germain and Dridri (2001) who present a model where privately informed traders receive a signal with binary distribution (‘buy’ or ‘sell’), and normally distributed
noise. The resulting price function corresponds to our function \( g(d) \) and thus also features regions of relative price stability and a region where the price changes sharply in response to demand. An important difference between that model and our analysis is that in Germain and Dridri (2001) the price function is driven by the signal distribution which is essentially arbitrary. By contrast the price function in our model is driven by the binary payoff distribution, which is endogenous and a robust feature of the political economy set-up that determines default.

5.2 Debt capacity

Once we know the price function (9) we can calculate the expected amount of money \( K \) that the government can raise when it issues \( B \) bonds. Denote by \( H(f) \) the distribution function of \( f \), i.e., \( H(f) = \int_{-\infty}^{f} h(s)ds \). Moreover, define \( d^* \) by \( \delta = g(d^*) \). We can then calculate the expected price at which the government can issue the bond.

**Lemma 1** If the government issues \( B = W \) bonds, their expected price will be

\[
E(P) = (1 - \gamma)[1 - (1 - \delta) H(d^* - l_1)] + \gamma \delta H(d^* - l_\delta).
\]

Proof see Appendix.

Using (11) it is straightforward to calculate the country’s debt capacity \( K = B \cdot E(P) \) as a function of the underlying distribution of foreign demand.\(^{19}\) Going back to our earlier example of normally distributed foreign demand, the expected price can be calculated for different levels of foreign demand volatility \( \sigma \). The result is provided in the next Proposition and illustrated in Figure 3.

**Proposition 3** An increase in the volatility of foreign demand reduces debt capacity.

Proof see Appendix.

The expected price of debt and therefore debt capacity, falls when foreign demand is more volatile. Given that all agents in the economy are risk neutral and that debt is fairly priced this result is non trivial. In order to understand it, we need to consider how the market mechanism allocates bonds across the population of locals. In the extreme case where foreign demand is certain, the equilibrium price fully reflects the underlying realization of local demand. If local demand is high (\( \delta = 1 \)) then the equilibrium price

\(^{19}\) Again it is straightforward to show that debt capacity is maximised if the government issues \( B = W \).
Figure 3: Shows the expected bond price (and debt capacity) as a function of the volatility $\sigma$ of foreign demand. Parameter values are $\gamma = 0.4$, $\delta = 0.3$ and $W = 1$. The solid, dashed and dotted line have the following values for $(\mu_l, \mu_h)$, respectively: $(0,1)$, $(0.2,0.8)$, and $(0.3,0.7)$.

is $P = 1$ and debt capacity is fully utilized at $W = B$. If local demand is low ($\tilde{\delta} = \delta$), then the price drops to $P = \delta$. The price reduction is important, because it allows more bonds to be allocated to the median voter compared to if the price had stayed high at $P = 1$. This is crucial, because it stabilizes the underlying bond issue: even when local (and therefore the median voter’s) demand is low, prices adjust so as to allow the median voter to hold enough bonds to repay with positive probability.

The allocation of bonds changes when foreign demand is volatile. The price now reflects underlying local demand with noise and therefore distorts the bond allocation. Distortions are possible in two directions. The price can be too high or too low, compared to the benchmark price that would obtain if local demand were publicly observable. If foreign demand is high and local demand is low, the price will end up too high. This reduces the number of bonds that will be allocated to locals: foreign demand crowds out local demand. The bond will then not be repaid and anticipating this possibility the bond price will be lower than 1 even when overall demand is high. This has a negative effect on the amount of funds that can be raised. Conversely, if local demand is high and foreign demand is low the bond price may be too low. In this case, the median voter gets a large allocation of bonds. However, this increased allocation does not improve the bond price since local demand was high and repayment therefore occurs anyway. The pricing error introduced by the combination of foreign demand volatility and market opacity therefore

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has an asymmetric effect on debt capacity: overpricing crowds out the already low local demand and reduces the repayment probability, while underpricing has no effect on the repayment probability. On balance this reduces debt capacity.

The impact of volatility on debt capacity depends on the variability of local demand. Since a small variability in local demand can have a significant effect on the stability of sovereign debt, a correspondingly small amount of foreign noise can therefore have a large impact on debt capacity. This can also be seen from Figure 3, which provides the relationship between \( E(P) \) and \( \sigma \) for different values of \( \mu_l \) and \( \mu_h \). The variability of local demand can be calculated as

\[
l_1 - l_\delta = W (1 - \delta) (\mu_h - \mu_l).
\]

When \( \mu_h - \mu_l \) is large local demand variability is large and therefore aggregate demand is relatively informative, even when foreign demand is somewhat volatile. As \( \mu_h - \mu_l \) becomes smaller the inferences about local demand that can be drawn from total demand become more noisy. An increase in foreign demand volatility therefore has a stronger negative impact on debt capacity. In the example a 20% volatility in foreign demand (measured in fraction of the outstanding face value of debt) reduces the country’s overall debt capacity by 13%.

5.3 Book building: conditioning \( B \) on the order flow

The assumption that the government makes a decision about the volume of the IPO regardless of the “strength” of the market and the expected price may seem a little contrived. Hence, we develop in this section a variation of the opaque-market model where the government is allowed to fix \( B \) only after it has observed the order flow \( d \), capturing the flavor of a book-building process.

We start with an analysis of the revenue-maximizing issuing policy:

**Lemma 2** Debt capacity is maximized at \( W \) when the face value of bonds issued is

\[
B = \frac{W}{g(d)}.
\]

**Proof.** If the government issues \( B = \frac{W}{g(d)} \) bonds, then repayment occurs when local demand is high \( \delta = 1 \), but not otherwise. This follows directly from the repayment condition on the median voter. The fair price of the bond is then \( P = g(d) \) and the revenue raised is exactly \( W \). If \( B > \frac{W}{g(d)} \) then the price has to fall until the median voter is willing to repay when \( \delta = 1 \) (otherwise repayment would never occur, which cannot be an equilibrium). Willingness to repay requires \( \frac{W}{P} \geq B \) and from this it follows directly
that $B \cdot P \leq W$. Hence, $W$ is indeed the maximum debt capacity. Moreover, reducing $B$ to a level that would be compatible with repayment when $\tilde{\delta} = \delta$ reduces debt capacity, because it would require $\frac{\delta W}{P} \geq B$ and therefore $B \cdot P \leq \delta W$. ■

It is clear from the above that debt capacity is bigger when the government can condition on the observation of aggregate demand before setting $B$. This is not surprising, since aggregate demand is informative about the likelihood that the median voter has high demand, and therefore prices and $B$ can be adjusted to ensure that a constant amount $W$ can be raised. Alternatively, one could think about this mechanism in terms of a slightly different contract between the government and the issuing intermediary. The government makes a request to the intermediary to raise $W$ worth of debt and determine the required face value of debt after having observed aggregate demand. Note that in this case foreign demand volatility has no impact on debt capacity.

This is not to say, however, that foreign demand volatility has no impact on the quality of the debt that is issued. According to the issuance policy the face value and the price of debt do depend on the realization of aggregate demand. In particular, if demand $d$ falls, the government issues more and lower quality debt. If the government is concerned about the quality of its debt per se and not just revenues raised, then demand volatility does have an impact on debt capacity.

To see this consider a government that imposes a lower bound on the quality of its debt. Since the equilibrium price of debt exactly reflects the quality of debt, the government can simply impose a lower bound on the price of debt that it is willing to issue. Suppose the government has thus chosen a lower bound, or a price floor $P_{\text{floor}}$ for its debt. In that case it has to modify its issuance rule as follows. If $g(d) \geq P_{\text{floor}}$ then the government issues $B = \frac{W}{g(d)}$ like before. When $g(d) < P_{\text{floor}}$ the government reduces $B$ to $\frac{\delta W}{P_{\text{floor}}}$ so as to maintain a constant quality of debt. The government thus loses revenue whenever demand drops below the point where $g(d) = P_{\text{floor}}$.

An increase in foreign demand volatility now will have an impact on the expected revenue raised. This is because with higher demand volatility, the likelihood that demand shocks will force the government to reduce the size of the bond issue increases. In expectation debt capacity therefore falls when foreign demand volatility increases. Figure 4 illustrates this effect. As foreign demand volatility increases the expected amount of debt raised falls. This effect is more pronounced when the government is more concerned with debt quality. The higher the lower bound on quality (price floor), the stronger is the impact that volatility has on debt capacity.
Figure 4: Illustrates the impact of foreign demand volatility on debt capacity, when the government imposes a lower bound on debt quality (and therefore price). Parameter values are: $W = 1$, $\mu_h = 0.6$, $\mu_l = 0.4$, $\delta = 0.5$ and $\gamma = 0.2$.

6. Extension: partial default

In our initial set-up of Section 2, we ignored without loss of generality the possibility of partial default, i.e., we restricted attention to repayments $\tilde{\alpha} \in \{0, 1\}$. This resulted in mixed repayment strategies. It is actually possible to reinterpret the mixed-strategy equilibrium of Section 2 as a partial default. Suppose that $B > w_m$ so that $P < 1$. Since the median voter is risk neutral, he is indifferent between mixing $\tilde{\alpha} = 1$ and $\tilde{\alpha} = 0$ with a probability of $P$ and serving a fraction $P$ of the debt with certainty. For similar reasons, the market maker would equally price sovereign debt with the two voting patterns above at $P$. It follows that any mixed-strategy equilibrium can be re-interpreted as a partial-default equilibrium. Such an interpretation, however, would be awkward due to the high level of indeterminacy in the voting. The object of this extension is to develop a variation of the basic model where in equilibrium voters strictly prefer a particular level of partial default relative to any other level of service. Essentially, debt is written down to the point that serves the interests of the median voter but no further. An additional bonus of the extension is the substitution of the lump-sum tax with a more realistic relative consumption tax.\footnote{Another interesting question is what kind of tax regime would make sovereign debt most stable. We will not attempt to answer this question here.}
Again, the extension is motivated by an unresolved puzzle in the sovereign debt literature. It is a well-established wisdom that corporate debt cannot be renegotiated with a multitude of creditors; see Bolton and Jeanne (2005) for a reinterpretation of this observation within the context of sovereign debt. Surprisingly, however, Roubini (2002) points out a number of “recent cases where there were thousands of bondholders (Ukraine, Pakistan, Ecuador, Russia) [but nevertheless] unilateral exchange offers have had overwhelming success with 99% plus creditors accepting the offer”\textsuperscript{21}. We suggest, in line with the citation, that it is more appropriate to regard these “renegotiations” as “confiscatory restructuring scheme”\textsuperscript{22} rather than an outcome of a bargaining process in the usual sense of the word. Our interpretation is that the creditors’ status-quo point – in case they reject the offer – is to get nothing,\textsuperscript{23} while on the debtor’s side, it is actually in the best interest of the median voter to write-down the debt, but not all the way down to zero. No wonder that the exchange offer “succeeds” without any coordination mechanism on the creditors’ side.

Consider therefore the following modification to our basic model, restoring the common-knowledge assumption. Firstly, assume that $\delta = 1$, i.e., all agents $i \in (\mu_l, 1]$ have period-0 income $W$, while the other agents $i \in [0, \mu_l]$ have zero initial income. Assume also that those agents who have no initial income have a positive second period income $y_i = Y > 0$. By contrast, those agents who have initial income $W$ have no second period income, i.e.,

$$w_i = \begin{cases} 0 & \text{for } i \in [0, \mu_l] \\ W & \text{for } i \in (\mu_l, 1] \end{cases},$$

$$y_i = \begin{cases} Y & \text{for } i \in [0, \mu_l] \\ 0 & \text{for } i \in (\mu_l, 1] \end{cases}.$$

This structure lends itself to a generational interpretation; the $i \in l = [0, \mu_l]$ income class (the “$l$-class”) as the young generation who earn no income at period zero ($w_i = 0$) but will have high earnings in the next period, while group $i \in h = (\mu_l, 1]$ (the “$h$-class”) corresponds to an old generation who earns income at period 0 but nothing later. Denote domestic per capita income at dates 0 and 1 by

$$\overline{w} = (1 - \mu_l) W,$$

$$\overline{y} = \mu_l Y.$$

As before assume that all agents care only about date 1 consumption and the only store of value is the government bond. It therefore follows that (1) holds.

\textsuperscript{21}Sturzenegger and Zettelmeyer (2005) estimate that the (discounted) write-downs “clustered in the 25-35 percent range”.

\textsuperscript{22}The Economist, June 3, 1999.

\textsuperscript{23}See Gray (2003).
Suppose that the government finances the bond repayment through a consumption tax at rate $\tau$. If the government proposes (and the voters approve) to repay a fraction $\alpha$ of its liabilities, it needs to raise tax income $T = \alpha B$ and set the tax rate $\tau$ such that

$$\tau \left( \frac{y}{y} + \alpha \frac{w}{P} \right) = \alpha B.$$  

We can then determine the tax rate as a fraction of $\alpha$:

$$\tau(\alpha) = \frac{\alpha B}{y + \alpha \frac{w}{P}}. \quad (12)$$

If the government proposes to repay a fraction $\alpha$ of its debt, agent $i$ will end up with date 1 consumption given by

$$c_i(\alpha) = \left( y_i + \alpha \frac{w_i}{P} \right) [1 - \tau(\alpha)]. \quad (13)$$

Different income classes now have differing preferences over $\alpha$. This can be seen by taking the first derivative of $c_i(\alpha)$ with respect to $\alpha$ for agents belonging to each class. Taking the first derivative of (13) for the $l$-class yields

$$\frac{dc_l(\alpha)}{d\alpha} = -\frac{\tau_l(\alpha)}{y + \alpha \frac{w}{P}}.$$  

Since $\frac{d\tau_l(\alpha)}{d\alpha} < 0$ it follows that $\frac{dc_l(\alpha)}{d\alpha} < 0$. Hence, the $l$-class clearly prefers the lowest possible amount of repayment. This is obvious: since they hold no bonds but pay taxes, $l$-class agents always benefit from full default.

The $h$-class on the other hand may not necessarily favor a corner solution. This is captured in the following result. Denote by $\alpha_h^* = \arg \max c_h(\alpha)$.

**Lemma 3** The $h$-class’ consumption $c_h(\alpha)$ is maximized at an interior solution $0 < \alpha_h^* < 1$ if

$$B > \frac{(y + w)^2}{2y + \bar{w}}. \quad (14)$$

**Proof.** We first check that $c_h(\alpha)$ is concave. Setting $y_i = 0$ and $w_i = W$ in (13) we can calculate first and second derivatives as

$$\frac{dc_h(\alpha)}{d\alpha} = \frac{W}{P} (1 - \tau(\alpha)) - \alpha \frac{W d\tau(\alpha)}{P d\alpha},$$

$$\frac{d^2c_h(\alpha)}{d\alpha^2} = -\frac{2W d\tau(\alpha)}{P d\alpha} - \alpha \frac{W d^2\tau(\alpha)}{P d\alpha^2}.$$  

We then get

$$\frac{d^2c_h(\alpha)}{d\alpha^2} < 0 \iff -\alpha \frac{d^2\tau(\alpha)}{d\alpha^2} < 2 \frac{d\tau(\alpha)}{d\alpha}.$$
Using the first and second derivatives of (12) it is straightforward to see that this is always true.

Next solve for \( \alpha^*_h \) by setting \( \frac{d c_h(\alpha)}{d\alpha} = 0 \). This yields

\[
1 - \frac{\alpha B}{\bar{y} + \alpha \bar{w}} - \frac{\alpha B\bar{y}}{(\bar{y} + \alpha \bar{w})^2} = 0. \tag{15}
\]

Moreover, since there is no uncertainty, we must have that in equilibrium \( P = \alpha \). Using this in (15) and solving for \( \alpha \) yields

\[\alpha = \frac{(\bar{y} + \bar{w})^2}{B (2\bar{y} + \bar{w})}.\]

Setting \( \alpha < 1 \) yields condition (14).

For low values of \( \alpha \) the \( h \)-class benefits from an increase in \( \alpha \). Since they hold some bonds, they benefit directly from a higher repayment rate. Moreover, since their consumption depends on the default rate, they have little taxable income when \( \alpha \) is small, and the increase in the tax burden therefore falls most heavily on the \( l \)-class. When \( \alpha \) increases, the share of total taxable income that the \( h \)-class contributes also increases and at some point this effect more than outweighs the gains from higher repayment on the bond. When the debt burden \( B \) is high, there may therefore be an interior \( 0 < \alpha^*_h < 1 \) at which \( c_h(\alpha) \) is maximized. Suppose for the remainder of this Section that condition (14) is satisfied.

Consider then the following (standard) set-up of the electoral process (see Persson and Tabellini, 2002). Two parties \( a \) and \( b \) compete in the election by simultaneously proposing their electoral platform \( \alpha_a \) and \( \alpha_b \), respectively. Each party chooses their platform so as to maximize their probability of winning the election given the other party’s platform. The election outcome is given in the following proposition.

**Proposition 4** The winning proposal is \( \alpha^*_h \).

**Proof.** The two cohorts of the electorate can be divided according to their preferences over \( \alpha \). Since the \( h \)-class has a majority the unique sub-game perfect equilibrium of electoral competition yields \( \alpha_a = \alpha_b = \alpha^*_h \).

In our modified set-up debt “renegotiation” is simple. The policy maker implements the repayment decision which is preferred by the median voter. The latter now has a strict preference over repayment fractions \( \alpha \). This follows from (15) which implies that there is at most one positive and real solution to the first-order condition for any price \( P \). It follows that the \( h \)-class either strictly prefers a specific interior repayment amount, or has a strict preference over a corner solution. A mixed strategy over the repayment amount can therefore not be an equilibrium.
7. Conclusions

The preceding analysis shows how political economy considerations can help understand sovereign borrowers apparent ability to access international debt markets. It provides a framework that combines explicitly elements from political economy modeling with aspects of price formation in a market microstructure setting. A number of results have been derived regarding liquidity and financial crises, debt capacity and debt restructuring. Beyond the interest of the results themselves, the paper hopes to illustrate the power of the underlying approach to a wider range of problems.

One example is the ‘home bias’ puzzle, which is a direct corollary of our central hypothesis. Applied to equity markets, one could think of a situation in which companies may benefit from local political support, for example by receiving public contracts, subsidies etc. The degree to which policy makers yield to a company’s lobbying pressure may well be influenced by the fraction of the electorate amongst its shareholders. As a result companies with a local shareholder base may perform better than companies without it.

Another application concerns the currency denomination of corporate foreign borrowing. As Calvo and Guidotti (1990) pointed out, a government may have an incentive to devalue its exchange rate if corporations have all their foreign debt denominated in local currency. The approach espoused in this paper may help throw light on exchange rate volatility and currency crises in emerging markets.

8. Appendix

Proof of Lemma 1. Denote by \( k(d) \) the density of \( d \). We can write

\[
k(d) = (1 - \gamma) h(d - W(1 - \mu_l)) + \\
\gamma h(d - W(1 - \mu_h + \delta (\mu_h - \mu_l))).
\]  

(16)

On the interval \( d \leq d^* \), the price is \( \delta \) and for \( d > d^* \) it is \( g(d) \). The probability that \( d \leq d^* \) is given by

\[
\text{prob}(d \leq d^*) = (1 - \gamma) H(d^* - W(1 - \mu_l)) \\
+ \gamma H(d^* - W(1 - \mu_h + \delta (\mu_h - \mu_l))).
\]

For \( d > d^* \) we can calculate the expected price from

\[
\int_{d^*}^\infty k(s)g(s)ds.
\]

Using (8) and (16) this simplifies to

\[
\int_{d^*}^\infty (1 - \gamma) h(s - W(1 - \mu_l))ds.
\]
This expression can be rearranged to yield
\[(1 - \gamma) (1 - H (d^* - W(1 - \mu_1)))\].
Adding up \(\delta \text{prob}(d \leq d^*)\) and \((1 - \gamma) (1 - H (d^* - W(1 - \mu_1)))\) yields \(E(P)\).

**Proof of Proposition 3.**

We can calculate debt capacity \(K\) as a function of the variance \(\sigma^2\) by using our result from (11).
\[K(\sigma^2) = \int_{-\infty}^{d^*} \gamma \delta h(s - l_\delta) - (1 - \gamma)(1 - \delta)h(s - l_\delta)ds\]
We can then take the first derivative with respect to \(\sigma^2\) which is
\[
\frac{dK(\sigma^2)}{d\sigma^2} = (\gamma \delta h(d^* - l_\delta) - (1 - \gamma)(1 - \delta)h(d^* - l_\delta)) \frac{\partial d^*}{\partial \sigma^2} + \int_{-\infty}^{d^*} \frac{\gamma \delta}{\sigma^2} \frac{\partial h(s - l_\delta)}{\partial \sigma^2} - (1 - \gamma)(1 - \delta) \frac{\partial h(s - l_\delta)}{\partial \sigma^2} ds.
\]
Using the definition of \(d^*\) it follows that \(\gamma \delta h(d^* - l_\delta) - (1 - \gamma)(1 - \delta)h(d^* - l_\delta) = 0\). Taking the normal density for \(h\), we can calculate explicitly the derivative of \(h\) with respect to \(\sigma^2\) and therefore re-write the integral as
\[
\frac{1}{\sqrt{2\pi \sigma^2}} \int_{-\infty}^{d^*} \gamma \delta e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \left( \frac{(s - l_\delta)^2}{\sigma^2} - 1 \right) - (1 - \gamma)(1 - \delta)e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \left( \frac{(s - l_\delta)^2}{\sigma^2} - 1 \right) ds.
\]
In the next step we can calculate the integral \(\int_{-\infty}^{d^*} e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} ds\) by making the following substitution: Let \(u'(s) = e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \frac{(s - l_\delta)}{\sigma^2}\) and \(v(s) = s - l_\delta\). Using this substitution we know that \(u(s) = -e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}}\) and \(v'(s) = 1\). Integration by parts then yields
\[
\int_{-\infty}^{d^*} e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \frac{(s - l_\delta)^2}{\sigma^2} ds = \left[ -e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \frac{(s - l_\delta)}{\sigma^2} \right]_{-\infty}^{d^*} + \int_{-\infty}^{d^*} e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} ds.
\]
If we substitute this expression into (17) we can write the condition that \(\frac{dK}{d\sigma^2} < 0\) as
\[
\gamma \delta \left[ -e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \frac{(s - l_\delta)}{\sigma^2} \right]_{-\infty}^{d^*} - (1 - \gamma)(1 - \delta) \left[ -e^{-\frac{1}{2} \frac{(s - l_\delta)^2}{\sigma^2}} \frac{(s - l_\delta)}{\sigma^2} \right]_{-\infty}^{d^*} < 0.
\]
This is the same as

\[ \gamma \delta e^{-\frac{1}{2} \left( \frac{(d^* - l_\delta)^2}{\sigma^2} \right)} (d^* - l_\delta) - (1 - \gamma)(1 - \delta) e^{-\frac{1}{2} \left( \frac{(d^* - l_1)^2}{\sigma^2} \right)} (d^* - l_1) > 0. \]

From the definition of \( d^* \) and from \( l_\delta < l_1 \) it follows that the above inequality holds.

References


