Term Premium Dynamics and the Taylor Rule

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Bank of Canada Conference on Fixed Income Markets

Motivation

- Empirical success of NA affine term-structure models.
  - Essentially vs. completely affine: Essentially more flexible.
  - Limited economic interpretations of these models.
- Ideal to link back to the macroeconomy.
  - Identify the latent state variables.
    - Macro aggregates.
    - Monetary policy state variables.
  - Determine the pricing kernel through g.e. restrictions.
  - Model monetary authority setting a short-term nominal rate,

$$i_t^{(1)} = f(\text{macro variables}),$$

    imposes additional restrictions.

Questions

- Can we provide an economic interpretation in conjunction with an interest rate policy rule to an essentially affine model?
- What can we learn about term premiums when inflation is determined by an interest rate policy rule?
- Is monetary policy an important source of long-term interest rate variability?
- Can we learn about policy regimes from long-term rates?

Approach and Findings

Endowment economy with:

- preference shocks,
- an interest rate policy rule to pin down inflation,

Leads to an essentially affine equilibrium model for yields.

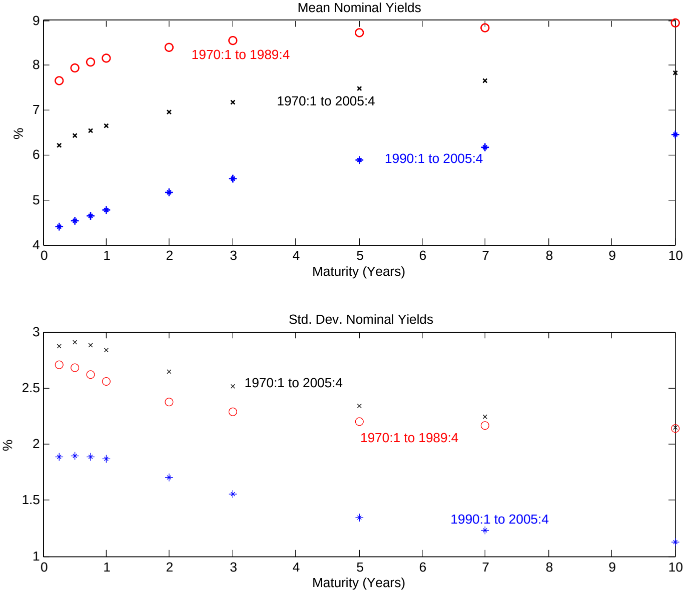
- The interest rate rule helps capture an upward-sloping yield curve, volatile long-term yields, & macroeconomic dynamics.

- Recent features of interest rates are consistent with a more aggressive response to inflation in monetary policy.

Related literature

- **Wachter (2006)** - Campbell-Cochrane habits. Exogenous inflation.
- **Piazzesi & Schneider (2006)** - Recursive utility & learning. Exogenous inflation. Inflation is bad news for consumption.
- **Buraschi & Jiltsov (2007)** - Campbell-Cochrane habits. Money supply determines inflation.
- **Gallmeyer, Hollifield, & Zin (2005) & Palomino (2007)** - "New Keynesian" macro model with an affine term structure. Inflation determined by monetary policy & firms' staggered price setting.
- **Gallmeyer et al. (2007)** - Recursive utility & stochastic volatility. An interest-rate policy rule determines inflation. Endogenous negative correlation between inflation & consumption.

Nominal Yields Across Maturity



Completely vs. Essentially Affine Models

- Completely affine pricing kernel:

$$-\log M_{t+1} = \Gamma_0 + \Gamma_1^\top \mathbf{s}_t + \lambda \Sigma(\mathbf{s}_t)^{1/2} \varepsilon_{t+1}.$$

- Essentially affine pricing kernel:

$$-\log M_{t+1} = \Gamma_0 + \Gamma_1^\top \mathbf{s}_t + \frac{1}{2}\lambda(\mathbf{s}_t)^\top \Sigma \lambda(\mathbf{s}_t) + \lambda(\mathbf{s}_t)^\top \Sigma^{1/2} \varepsilon_{t+1}$$

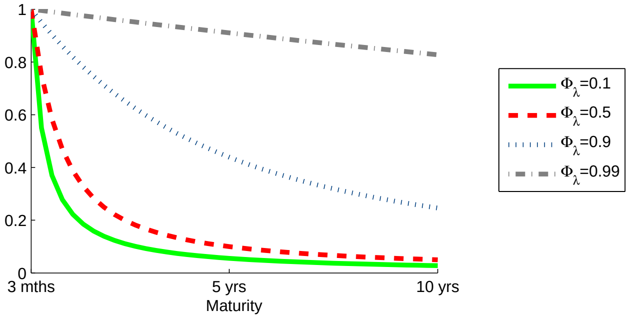
with $\lambda(\mathbf{s}_t) = \lambda_0 + \lambda_1 \mathbf{s}_t$.

- Interest rates:

$$e^{-n i_t^{(n)}} = \mathbb{E}_t \left[M_{t+n} \right] \quad \Rightarrow \quad i_t^{(n)} = \frac{1}{n} \left[\mathcal{A}_n + \mathcal{B}_n^\top \mathbf{s}_t \right].$$

Long Rate Volatility in Essentially Affine Models

$$\frac{\sigma\left(i_t^{(n)}\right)}{\sigma\left(i_t^{(1)}\right)} = \frac{1}{n}\frac{1-\Phi_\lambda^n}{1-\Phi_\lambda}, \qquad \Phi_\lambda = \left[\Phi - \Sigma\lambda_1\right].$$



Φ: Autocorrelation of state variables.

λ_1: Price-of-risk sensitivity to state variables.

Essentially Affine Economic Model - Real Part

- Utility: $\mathbb{E}\left[\sum_{t=0}^{\infty} e^{-\delta t} \frac{C_t^{1-\gamma}}{1-\gamma} Q_t\right]$.

- Consumption Growth ($c \equiv \log C$):

$$\Delta c_{t+1} = (1-\phi_c)\theta_c + \phi_c \Delta c_t + \sigma_c \varepsilon_{c,t+1}.$$

- Preference Shock ($q \equiv \log Q$):

$$-\Delta q_{t+1} = \frac{1}{2}\left(\eta_c \Delta c_t + \eta_\nu \nu_t\right)^2 \sigma_c^2 + \left(\eta_c \Delta c_t + \eta_\nu \nu_t\right) \sigma_c \varepsilon_{c,t+1}.$$

- Essentially Affine Pricing Kernel:

$$-\log M_{t+1} = \delta + \gamma \Delta c_{t+1} - \Delta q_{t+1}.$$

Essentially Affine Economic Model - Nominal

Nominal Pricing Kernel:

$$\log(M^{\$}_{t+1}) = \log(M_{t+1}) - \pi_{t+1}$$

- Exogenous inflation - a benchmark:

$$\pi_{t+1} = (1-\phi_\pi)\theta_\pi + \phi_\pi \pi_t + \sigma_\pi \varepsilon_{\pi,t+1}, \quad \varepsilon_{\pi,t+1} \perp \text{ other shocks.}$$

$$\Rightarrow i^{(n)}_t = \mathcal{A}^{\$}_n + \mathcal{B}^{\$}_{n,c}\Delta c_t + \mathcal{B}^{\$}_{n,\nu}\nu_t + \mathcal{B}^{\$}_{n,\pi}\pi_t.$$

- Endogenous inflation via a "Taylor Rule."

Economic Model - Endogenous Inflation via "Taylor Rule"

Monetary policy sets the 1-period nominal yield:

$$i_t = \bar{\imath} + \imath_c \Delta c_t + \imath_\pi \pi_t + u_t$$

with the "monetary policy shock" given by

$$u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t}.$$

π_t must simultaneously satisfy:

1. the "Taylor Rule,"
2. the NA bond pricing equation.

Equilibrium Inflation Process: "Guess and Verify"

$$\overbrace{\bar{\imath} + \imath_c \Delta c_t + \imath_\pi \underbrace{\left(\bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t\right)}_{\text{guess for } \pi_t} + u_t}^{i_t}$$

$$= -\log E_t[\exp\{\overbrace{\log M_{t+1} - \underbrace{\left(\bar{\pi} + \pi_c \Delta c_{t+1} + \pi_\nu \nu_{t+1} + \pi_u u_{t+1}\right)}_{\text{guess for } \pi_{t+1}}}^{\log M^{\$}_{t+1}}\}]$$

$$\pi_c = \frac{\gamma(\phi_c - \sigma_c^2 \eta_c) - \imath_c}{\imath_\pi - (\phi_c - \sigma_c^2 \eta_c)}, \qquad \pi_\nu = -\frac{(\gamma + \pi_c)\sigma_c^2 \eta_\nu}{\imath_\pi - \phi_\nu}, \quad \pi_u = -\frac{1}{\imath_\pi - \phi_u}.$$

$$\Rightarrow i_t^{(n)} = \mathcal{A}_n^{\$} + \mathcal{B}_{n,c}^{\$} \Delta c_t + \mathcal{B}_{n,\nu}^{\$} \nu_t + \mathcal{B}_{n,u}^{\$} u_t.$$

Prices of Risk

Shocks: $\varepsilon = (\varepsilon_c, \varepsilon_\nu, \varepsilon_u \text{ or } \varepsilon_\pi)$.

- Real

$$\lambda(\mathbf{s}_t) = (\gamma + \eta_c \Delta c_t + \eta_\nu \nu_t, 0, 0)^\top.$$

- Nominal - exogenous π

$$\lambda^\$(\mathbf{s}_t) = \lambda(\mathbf{s}_t) + (0, 0, 1)^\top.$$

- Nominal - endogenous $\pi_t = \bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t$

$$\lambda^\$(\mathbf{s}_t) = \lambda(\mathbf{s}_t) + (\pi_c, \pi_\nu, \pi_u)^\top.$$

Inflation & Term Premiums Driven by Monetary Policy

$$\mathbb{E}[i_t - r_t] = ... + \mathbb{E}[\mathrm{cov}_t(\log M_{t+1}, \pi_{t+1})],\ \text{where}$$
$$\mathbb{E}[\mathrm{cov}_t(\log M_{t+1}, \pi_{t+1})] = -\pi_c(\gamma + \eta_c\theta_c)\sigma_c^2$$

$$\mathbb{E}[i_t^{(2)} - i_t] = ... + \frac{1}{2}\mathbb{E}[\mathrm{cov}_t(\log M^{\$}_{t,t+1}, i_{t+1})],\ \text{where}$$
$$\mathbb{E}[\mathrm{cov}_t(\log M^{\$}_{t,t+1}, i_{t+1})] = -(\gamma+\pi_c)(\gamma+\pi_c+\eta_c\theta_c)(\phi_c-\eta_c\sigma_c^2)\sigma_c^2 + \text{(-) Term.}$$

- $\pi_c = \frac{\gamma(\phi_c - \sigma_c^2\eta_c) - \imath_c}{\imath_\pi - (\phi_c - \sigma_c^2\eta_c)} < 0$ if
  - A weak response to inflation or
  - A strong response to consumption growth.
- An upward sloping nominal curve is driven by π_c.

Calibration

- Calibrate the exogenous & endogenous inflation models to quarterly U.S. data (1971:3 to 2005:4).
  - Zero coupon yields (3 months - 10 years).
  - Per capita consumption of nondurables & services.
  - Inflation from methodology in Piazzesi & Schneider (2006).
- Both models calibrated to share the same real dynamics.

Calibration - Fitted Policy Rule Parameters

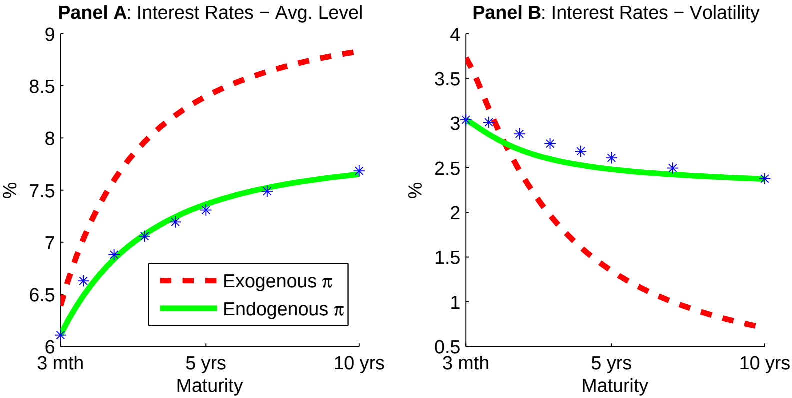
- Policy rule responds positively to consumption and inflation.
- Endogenous $\text{corr}(\Delta c_t, \pi_t) < 0$.
- Highly persistent policy shock captures long bond volatility.

Calibration - Fitted Preference Parameters

- Habit $\eta_c < 0$:
  - Upward-sloping yield curve,
  - Countercyclical price of risk.
- Taste shock v_t:
  - Short rate volatility through η_v,
  - Intermediate maturity volatilities through ϕ_v.
- No external habit model interpretation though:
  - Affine-class restriction invokes tensions on parameters to achieve upward sloping yield curves.
  - Model does not deliver countercyclical real yields.
  - Model requires a taste shock to fit volatilities.

Nominal Yield Curve

Highly autocorrelated policy shocks explain long rate volatility.



∗: 1971-2005

Two Policy Experiments

Increase the reaction coefficients to (1) inflation & (2) consumption growth to match the average short-term rate (1987-2005).

Baseline: $i_t = -0.007 + 0.79\Delta c_t + 1.68\pi_t + u_t.$

$\Delta\imath_\pi$: $i_t = -0.007 + 0.79\Delta c_t + 2.14\pi_t + u_t.$

$\Delta\imath_c$: $i_t = -0.007 + 1.07\Delta c_t + 1.68\pi_t + u_t.$

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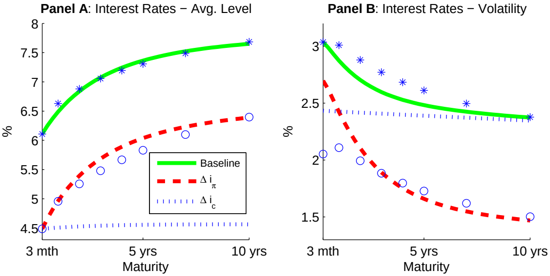

$*$: 1971-2005, $\circ$: 1987-2005

Policy Experiment

Changes in the dynamics of inflation are consistent with a more aggressive reaction to inflation.

	Data		Policy Experiment		
	(1971-2005)	(1987-2005)	Baseline	$\Delta\imath_\pi$	$\Delta\imath_c$
$\mathbb{E}\left[\Delta c_t\right] \times 4$ (%)	1.98	1.83	1.98	1.98	1.98
$\mathbb{E}\left[\pi_t\right] \times 4$ (%)	4.46	2.95	4.42	2.71	3.11
$\sigma\left(\Delta c_t\right) \times 4$ (%)	1.74	1.35	1.74	1.74	1.74
$\sigma\left(\pi_t\right) \times 4$ (%)	2.66	1.26	2.69	1.80	2.67
corr $\left(\Delta c_t, \Delta c_{t-1}\right)$	0.41	0.28	0.41	0.41	0.41
corr $\left(\pi_t, \pi_{t-1}\right)$	0.84	0.54	0.85	0.70	0.90
corr $\left(\Delta c_t, \pi_t\right)$	-0.33	-0.17	-0.18	-0.17	-0.41

Conclusions

- A policy rule aids a consumption-based bond pricing model.
  - Highly autocorrelated policy shocks needed.
  - Negative correlation between inflation & real activity.
  - Term structure information can help identify the policy regime.
- Future Work:
  - Role of endogenous inflation a general N.A. affine model.
    - The monetary policy rule still tractable in the exact discrete-time affine setting of Dai, Le, & Singleton (2006).
    - Jointly capture real & nominal term structures.
  - Source of the policy shock?
  - Inflation & the real side of the economy?