Exploring Dynamic Default Dependence*

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Abstract

Characterizing the dependence between companies' defaults is a central problem in the credit risk literature, and the dependence structure is a first order determinant of the value of credit portfolios and structured credit products such as collateralized debt obligations (CDO), as well as the relative values of CDO tranches. We compare correlation measures implied by CDO prices with time-varying correlations implied by equity returns and CDS spreads. We use flexible dynamic equicorrelation techniques introduced by Engle and Kelly (2008) to capture time variation in CDS-implied and equity return-implied correlations. We perform this analysis using North American firms from the CDX index, as well as European firms from the iTraxx index. All correlation time series are highly time-varying and persistent, and correlations extracted from CDSs and CDOs increased significantly in European and North American markets during the turbulent second half of 2007. Interestingly, we find that the correlation time-series implied by CDO prices co-moves very strongly with the correlation time-series extracted from CDS spreads, but somewhat less strongly with the correlations between equity returns. These findings suggest that the cross-sectional dependence in these complex structured products is fairly well measured. However, changes in CDO prices may be due to changes in correlation, and more sophisticated models with time-varying correlations are thus needed to value CDOs.

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1 Introduction

The valuation of structured finance products has come under scrutiny lately, because the fluctuations in the valuation of these products over the last two years are among the main causes of many banks' current difficulties. Interestingly though, we know relatively little about the determinants of the prices of structured finance products and the recent dramatic changes in price.

This paper investigates the valuation of structured credit products, in particular synthetic credit collateralized debt obligations (CDOs). These synthetic CDOs are typically defined in reference to an underlying pool of credit default swaps, and contain multiple tranches providing insurance against different types of credit events. For example, the equity tranche provides insurance against the first few defaults, while the most senior tranche is only called upon after the capital in the other tranches is exhausted.

For a given model, there are two main determinants of the price assigned to a CDO at any point in time. These can be thought of as model inputs, and it is of great interest to investigate whether these determinants might be responsible for recent price fluctuations or for potential valuation errors in these markets.[1] The first set of model inputs consists of the default probabilities of the underlying names. For default probabilities to explain recent price changes, market participants must have grossly underestimated future default probabilities prior to 2007, and subsequently updated their beliefs.[2] This paper investigates the second set of model inputs, which is the dependence between the underlying names' default events. Our knowledge of default dependence is limited, and it is a first order determinant of CDO prices. It is therefore not inconceivable that market participants misjudged either the magnitude of the default dependence or failed to anticipate the amount of time-variation in default dependence, and that this subsequently led to large price adjustments as they updated their beliefs.

This suggests two interesting research questions, which we address in this paper. First, is there evidence that the default dependence used by market participants to value CDOs dramatically differs from the default dependence extracted from the underlying securities? Second, what is the evidence on the time variation in default dependence? While the literature contains some evidence on the implications of default dependence for CDOs, the issue has not received a lot of attention.[3]

[1] There are of course interesting alternative explanations for recent price changes, which we do not pursue here. It is highly unlikely that recent price changes are driven by a change in modeling technique. A more likely explanation is that current prices are driven by market sentiment and/or liquidity problems.

[2] Coval, Jurek, and Stafford (2008), using 2004-2007 data, present evidence that market participants dramatically underestimated the tail risk of structured portfolios.

[3] See McGinty, Beinstein, Ahluwalia and Watts (2004) for an interesting case-based discussion that uses default data, equity prices and credit spreads to compute default correlation. A number of papers estimate default correlation from credit risky securities or default data and use these estimates to price CDOs. See for example Azizpour and Giesecke (2008), Akhavein, Kocagil and Neugebauer (2005), and Tarashev and Zhu (2007).

A systematic analysis of default dependence estimates from different sources, and an assessment of whether these measures are consistent with those used for the valuation of CDOs, therefore seems of substantial interest.

Such an analysis is not necessarily straightforward. The main problem is that measures of default dependence can be obtained using different types of data, and using different types of statistical and economic models. The conceptually most straightforward approach to estimate default dependence is to use historical default data, but it is widely accepted that the available time series of default data are not sufficiently long. Several methods have therefore been developed that estimate and model correlations implied by the prices of traded securities.

Methods that estimate default dependence using prices of traded securities themselves differ along three dimensions. First, they are applied to different types of securities data, including equity returns, corporate bonds, and credit default swaps; second, security prices can be filtered through different models to generate default probabilities or asset returns before correlation techniques are applied, with many variations of structural and reduced-form models in use;[4] and third, different statistical techniques can be used to model correlations after extracting default probabilities and/or asset returns from security data.

This paper provides evidence on the two research questions above by comparing correlation measures obtained from three different data sources: CDO prices, CDS spreads, and equity returns. We compare time-varying correlations implied by CDO tranche spreads with correlation paths extracted from CDS spreads and equity returns. Given the plethora of available modeling approaches, we are forced to make some choices, which on the one hand are motivated by ensuring that correlations can be meaningfully compared across asset classes, and on the other hand are constrained by computational complexity. Most importantly, we decided to limit ourselves to the use of prices of traded securities to extract default correlation. This not only eliminates default data as a source of information, it also implies that we do not make use of accounting data, which can be used to implement structural default models. Using only traded security prices, ideally one would like to use the same default and correlation models for the three types of securities to ensure comparability, but this proved not feasible because of computational constraints and implementation problems. We therefore opted for what we view as the most straightforward approach within each asset class among the possible configurations of methods available to estimate default probabilities.[5]

[4] The structural approach goes back to Merton (1974). See Black and Cox (1976), Leland (1994) and Leland and Toft (1996) for extensions. For recent applications of the structural approach see Ericsson and Renault (2006), Garlappi, Shu and Yan (2008), Schaefer and Strebulaev (2008), and Tarashev and Zhu (2007). See Jarrow and Turnbull (1995) and Duffie and Singleton (1999) for early examples of the reduced form approach, and Lando (2004) and Duffie and Singleton (2003) for surveys. Duffie and Garleanu (2001) use intensity-based models to value CDOs.

[5] The advantage of this approach is its transparency. Below we further discuss its limitations, as well as potential extensions that could be used to improve comparability of the correlation estimates across asset classes.

To extract correlation measures from CDO data, we use the standard Gaussian copula.[6] We follow the most commonly used implementation of the Gaussian copula, using a pairwise correlation which is assumed constant across time and firms.

To characterize correlation between default intensities from CDSs, we proceed in two steps. The first step involves the extraction of a time series of fitted default probabilities for each of the underlying companies. This step is strictly univariate and relatively straightforward: we fit a simple intensity based model to the time series of CDS spreads for each of the underlying names.

The second step is multivariate and involves the modeling of the correlation between default intensities. This step is conceptually obvious, but less straightforward to implement because our application involves a large cross-section of firms. Factor models are often used to model correlations for large portfolios, regardless of whether default probabilities are extracted using structural or reduced-form models. In many cases simple rolling correlations or exponential smoothers are used. Multivariate GARCH techniques have been developed over the past two decades, and it is well-recognized that the explicit modeling of the second moment in GARCH models may provide benefits for correlation modeling, but it has proven difficult to estimate multivariate GARCH models for large numbers of underlying securities.[7]

Given the implementation problems with more traditional approaches, we use the dynamic equicorrelation (DECO) framework of Engle and Kelly (2008) in order to model the default dependence structure between the index constituents. We also apply DECO techniques to equity returns. As the name suggests, DECO allows for correlation dynamics but assumes that the cross-sectional correlations are equal across all pairs of assets. It is therefore appropriate to compare DECO to the correlation implied by the Gaussian copula in the sense that both are assumed to be the same across firms.[8]

We perform our empirical analysis using the CDX and iTraxx indexes using data from October 2004 to December 2007. We obtain two important conclusions, which we discuss mainly from the perspective of CDO valuation, but which obviously have implications for portfolio credit risk

[6] On copulas see for example Li (2000), Andersen and Sidenius (2004), and Hull and White (2004). Schönbucher and Schubert (2001) incorporate copulas in an intensity-based model of default.

[7] The GARCH approach goes back to Engle (1982) and Bollerslev (1986). For early examples of multivariate GARCH models see Bollerslev, Engle and Kroner (1988) and Engle and Kroner (1995). Recently, a new and more flexible generation of multivariate models has been developed by Engle (2002), Tse and Tsui (2002), and Franses and Hafner (2003) as well as by Ledoit, Santa-Clara and Wolf (2003), and Ledoit and Wolf (2003). For overviews see Andersen, Bollerslev, Christoffersen, and Diebold (2006) and Bauwens, Laurent and Rombouts (2006).

[8] On the other hand, the comparison is obviously hampered by the fact that the DECO estimates originate from an explicitly time-varying model, while the implied correlations from CDOs are extracted using the Gaussian Copula one period at a time, assuming the correlation is constant over the maturity of the CDO. This is similar to comparing GARCH estimates of historical volatility with a time series of implied volatilities extracted using the Black–Scholes model.

more in general. First, for both indexes, the correlations extracted from CDS spreads and equity data exhibit very similar time series variation to that those implied by CDO spreads. This is perhaps somewhat surprising, because it is often argued that the Gaussian copula is such a stylized representation of reality that correlations implied by the model cannot usefully be interpreted as such. Our results suggest the opposite, even if the co-movement with the correlations extracted from equity returns is somewhat lower than for the case of CDS-implied correlations.[9] Our second conclusion is that default intensity correlations are highly time-varying. The DECOs, which can usefully be thought of as average pairwise correlations, can easily change by 30% over short periods of time. If time variation in correlation is ignored or not properly taken into account then CDOs will be mispriced by the conventional models.

The paper proceeds as follows. In Section 2 we discuss the importance of correlation for the valuation of credit risky securities such as CDSs and CDOs, and we also summarize existing methods for measuring default correlation. In Section 3 we extract implied correlations from CDO prices. Section 4 first presents the CDS spreads, then shows how we extract default intensities from the spreads, and finally discusses how we model the dynamics of the default intensities. Section 5 estimates dynamic correlations for default intensities as well as equity returns and compares these with the implied correlations from CDOs. Section 6 concludes.

2 Credit Markets

In this section we discuss a number of recent developments in credit markets, with an emphasis on Credit Default Swap (CDS) markets and Collateralized Debt Obligations (CDOs). We also discuss existing methods for computing default dependence, and the relevance of default dependence for the valuation of CDOs.

2.1 CDS and CDO Markets

The last decade has seen the emergence of a large variety of new portfolio credit products. In this paper we focus on collateralized debt obligations (CDOs). In essence, a CDO is a portfolio of credit risky exposures, acquired in cash markets (e.g. bonds or loans) or synthetically in derivative markets (using credit derivatives). These credit risky securities may or may not trade independently in separate markets. The underlying securities' payoffs are allocated to issued notes that differ in their seniority. These notes are typically referred to as the tranches of the CDO, and their riskiness

[9]The association between the CDO-implied correlations on the one hand and the CDS- and equity-implied correlations on the other hand would presumably be even stronger if we used a more sophisticated CDO pricing model.

differs because of the differences in seniority. The most risky tranche of the CDO absorbs the first x% of portfolio losses. It is usually referred to as the equity or first-loss tranche. The safest tranche only absorbs losses if they can no longer be covered by the more risky tranches. It is usually referred to as a senior or super-senior tranche. The intermediate tranches are often known as mezzanine tranches. For a more thorough discussion of CDOs, including some stylized examples, see Duffie and Garleanu (2001) and Longstaff and Rajan (2008).

The CDO market has experienced very rapid growth, followed by an almost complete stop in new issuance following the price declines in 2007 and 2008. Early on, the market consisted exclusively of cash CDOs. In a cash CDO, the underlying securities are assets such as bank loans, investment-grade and high-yield bonds, and commercial mortgages. In many cases, the motivation for the CDO was securitization. For instance, banks were interested in securitizing illiquid loan portfolios. Gradually, interest in the CDO market shifted away from securitization and synthetic CDOs became highly popular. A typical synthetic CDO consists of a portfolio of credit default swaps (CDS) rather than cash securities.

Synthetic CDOs include index products as well as "bespoke" or "single-tranche" CDOs. An index tranche is not backed by actual exposures but is referenced by an index of key names. The two most important examples are the CDX family, lead by the NA.IG grouping of the 125 most actively traded North-American investment grade reference entities, and the iTraxx family, which contains European reference entities. A bespoke CDO is created when a dealer sells a tailor-made tranche demanded by an investor. This tranche is not backed by the cash flows of existing assets or synthetic exposures. Instead the position is dynamically hedged by the CDO dealer who relies on the market for index tranches as well as single-name default swap contracts to manage his exposure.

Since a thorough understanding of CDSs is also important for our empirical work, we discuss this security now in more detail. A CDS is essentially an insurance contract where the insurance event is defined as default by an underlying entity such as a corporation or a sovereign country. Which events constitute default is a matter of some debate, but for the purpose of this paper it is not of great importance. The insurance buyer pays the insurance provider a fixed periodical amount, expressed as a "spread" which is converted into dollar payments using the notional principal–the size of the contract. In case of default, the insurance provider compensates the buyer for his loss. Synthetic CDOs collect these CDS contracts in portfolios. While the nature of the contracts is somewhat different than in a cash CDO, it can be easily seen that defaults in the underlying securities affect the value of the CDO and the value of the CDOs tranches in a similar way. In particular, changes in default dependence affect the value of the CDO tranches as a function of their respective seniorities.

The CDS market has exploded in recent years. One element of its success has been the creation of market indexes consisting of CDSs, the CDX index in North America and the iTraxx index in

Europe. In our empirical work, we use constituents of both indices, which consist of 125 underlying investment grade actively traded corporate reference entities. The list of firms we use is provided in Table 1.[10]

In 1998 the credit derivatives market size stood at 350 billion $US, but over the next decade the market grew very fast. According to the International Swaps and Derivatives Association 2007 mid-year survey, the size of the credit derivatives market as a whole reached a staggering 45 Trillions $US in notional principal. Based on this metric, the credit derivatives market was four times larger than the equity derivatives markets. Single-name credit default swaps and index credit derivatives each represented approximately one third of the overall market, while synthetic CDO tranches constituted more than one fifth of the market. This implies that multi-name credit derivatives, which traded in very low volumes before 2004, constituted the largest market segment in 2007.

Starting in the summer of 2007, the market for CDOs began experiencing severe problems, and in this paper we document a number of stylized facts in both CDS and CDO markets during this period. Interestingly, while the market's appetite for complex CDO products is virtually nonexistent for the time being as a result of these market fluctuations, the CDS market is still quite active, highlighting the importance of a market for single-name default insurance. The purpose of our paper is to better understand correlation between credit names, either underlying a CDO product or as part of any portfolio consisting of CDS or other credit-risky securities.

2.2 Methods for Measuring and Modeling Default Dependence

Measuring default dependence has always been a problem of interest in the credit risk literature. For instance, a bank that manages a portfolio of loans is interested in how the borrowers' credit-worthiness fluctuates with the business cycle. While the change in the probability of default for an individual borrower is of interest, the most important question is how the business cycle affects the value of the overall portfolio, and this depends on default dependence. An investment company or hedge fund that invests in a portfolio of corporate bonds faces a similar problem. Over the last decade, the measurement of default dependence has taken on added significance because of the emergence of new portfolio credit products, and as a result new methods to measure correlation have been developed.

We now discuss different available techniques for estimating default correlation. First, default correlation can be computed using historical data. Second, the Merton (1974) model–or any of its offspring–can be used in conjunction with a factor model to model default dependence using equity

[10] The composition of the index changes every six months. Our sample consists of those firms which remain in the index throughout the sample period.

prices. Third, there are different ways to estimate and model default correlation in the context of intensity-based credit risk models. Finally, we discuss the modeling of default correlation using copula-based models.

The oldest and most obvious way to estimate default correlation is the use of historical default data. This can simply be thought of as an application of the CreditMetrics methodology to estimate ratings transitions, where default is one particular rating. The analogy with CreditMetrics also immediately clarifies the weakness of this approach for the purpose of estimation default correlations. In order to reliably estimate ratings transitions or default probabilities for an individual firm, typically a large number of historical observations are needed. This problem is obviously compounded when using historical data to estimate default correlations. Nevertheless, historical data on default are a rich and indispensable source of information. See for instance deServigny and Renault (2002).

For publicly traded corporates, a second source of data on default correlation is the use of Merton (1974) type structural models model that link equity returns or the prices of credit-risky securities to the underlying asset returns.[11] This approach is used for instance by KMV corporation. The use of a factor model for the underlying equity return implies a factor model for the value of the credit risky securities, and it also determines the default dependence. Clearly the reliability of the default dependence estimate is determined by the quality of the factor model.

A third way to estimate default dependence is in the context of intensity-based models, which have become very popular in the academic credit risk literature over the last decade.[12] This approach typically models the default intensity using a jump diffusion, and is also sometimes referred to as the reduced-form approach. Within this class of models, there are different approaches to modeling default dependence. One class of models, referred to as conditionally independent models or doubly stochastic models, assumes that cross-firm default dependence associated with observable factors determining conditional default probabilities is sufficient for characterizing the clustering in defaults. See Duffee (1999) for an example of this approach. Das, Duffie, Kapadia and Saita (2007) provide a test of this approach and find that this assumption is violated. Other intensity-based models consider joint credit events that can cause multiple issuers to default simultaneously, or they model contagion or learning effects, whereby default of one entity affects the defaults of others. See for example Davis and Lo (2001) and Jarrow and Yu (2001). Jorion and Zhang (2007) investigate contagion using CDS data.

Finally, modeling default correlation using copula methods has become extremely popular, es-

[11] The structural approach goes back to Merton (1974). See Black and Cox (1976), Leland (1994) and Leland and Toft (1996) for extensions. See Zhou (2001) for a discussion of default correlation in the context of the Merton model.

[12] See Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Duffee (1999), and Duffie and Singleton (1999) for early examples of the reduced form approach. See Lando (2004) and Duffie and Singleton (2003) for surveys.

pecially among practitioners and for the purpose of CDO modeling. The advantage of the copula approach is its flexibility, because the parameters characterizing the multivariate default distribution, and hence the correlation between the default probabilities, can be modeled in a second stage, after the univariate distributions have been calibrated. In many cases the copulas are also relatively parsimoniously parameterized, which facilitates calibration. The most commonly used model is the Gaussian copula, and calibration of the correlation structure is mostly performed using CDO data.

Copula modeling is sometimes interpreted as an alternative to the structural and reduced-form approaches. Strictly speaking, this is not the case. Copulas are in fact more usefully interpreted in a narrow sense as a tool for modeling dependence. Of course, within the context of a given copula model, assumptions about default are usually made in order to price CDOs. For example, the popular Gaussian copula can be thought of as a multivariate structural Merton (1974) model because of the normality assumption (see Hull, Predescu and White (2006)). For our purpose, it is important to remember that for most applications of the copula approach to CDO pricing, the parameterization of the correlation structure is low-dimensional, thus facilitating calibration. The Gaussian copula is typically implemented with constant correlations.

3 Implied Correlations from CDOs

In this section we first describe how we derive implied correlations from CDO price data. We then briefly discuss the CDO data we use. Finally we report on the extracted CDO-implied correlations.

3.1 Extracting Implied Correlations from CDO Data

Motivated by the volatility modeling literature which has found it fruitful to consider option implied volatilities, we want to assess credit correlations implied from multi-name credit derivatives. There is an interesting analogy between the correlation parameter in the market-standard Gaussian Copula model for valuing structured credit products such as CDOs and the volatility parameter in the more traditional equity option pricing models. First, the correlation can loosely be thought of as the volatility of the distribution of portfolio losses for a CDO: the stronger the dependence, the more likely are scenarios with either very few or with many aggregate losses; for lower levels of dependence the distribution of potential losses is narrower. Second, it is used in very much the same way for both asset classes. It is common in both equity option and structured credit markets to imply parameters from market prices of benchmark instruments–option implied volatilities and tranche implied correlations. A correlation implied from CDO prices can best be understood by analogy to implied Black-Scholes volatilities: it is the correlation between the CDS names underlying the

CDO that makes the price of the CDO tranche equal to the observed market price, conditional on the Gaussian copula method being the correct pricing method.

A typical CDO tranche consists of a stream of coupon payments which are made in exchange of payments that compensate for losses as they accrue to that tranche. To be more precise, let $L_{a,d}$ denote the risk-adjusted present value of the "loss leg" for a tranche with attachment and detachment points a and d respectively. As an example, consider the loss leg of a 5 year 0-3% first-loss or equity tranche with 20 quarterly payments

$$L_{0,3} = \sum_{t=1}^{20} \Delta EL_t D(t),$$

where $\Delta EL_t = EL_t - EL_{t-1}$ denotes the incremental expected loss during the time interval $[t-1, t]$ and $D(t)$ is a discount factor with maturity t. Now consider the present value of an annuity that pays 1 dollar per dollar of remaining notional on each payment date. The original notional principal is denoted N. When defaults arrive the size of the payment is scaled down by the proportional total losses. Hence

$$A_{0,3} = \sum_{t=1}^{20} (N - EL_t)\, D(t)$$

can be thought of as the present value of a stream of payments equal to the expected remaining notional over the lifetime of the tranche. For a given spread $S_{0,3}$, the value of the tranche to the insurance seller is then[13]

$$\frac{S_{0,3}}{4} A_{0,3} - L_{0,3} \tag{3.1}$$

The par spread for a newly struck tranche is such that both parties view the transaction as a zero NPV deal. Given an observed market par spread $S_{0,3}^{mkt}$, one can solve for the correlation parameter such that the model value (3.1) of the tranche is zero–that is the implied correlation $\widehat{\rho}_{0,3}$ solves

$$\frac{S_{0,3}^{mkt}}{4} = \frac{L_{0,3}\left(\widehat{\rho}_{0,3}\right)}{A_{0,3}\left(\widehat{\rho}_{0,3}\right)}$$

In the early days of the CDO market it was common to extract an implied correlation parameter for each tranche of a CDO. Since, the standard that has evolved to what is known as an implied base correlation. These are defined for hypothetical equity or first-loss tranches and have the desirable feature of avoiding multiple solutions in the inversion of tranche values for the correlation parameter. So instead of extracting correlations from e.g. 0-3%, 3-7% or 7-10% tranches, base correlations will

[13] For convenience we assume that all tranches are quoted without upfront payment.

be associated with 0-3%, 0-7% and 0-10% equity tranches, although only the 0-3% tranche will actually be traded in CDO markets. The remaining hypothetical tranches are constructed using a bootstrapping procedure similar to that used to compute implied zero-coupon bond prices from coupon bonds.[14]

To see how this works in practice, consider the on-the-run 3-7% tranche for which we know

$$L_{3,7} = \frac{S_{3,7}^{mkt}}{4} A_{3,7}$$

Losses and payment streams are additive so that

$$L_{3,7} = L_{0,7} - L_{0,3}$$

and

$$A_{3,7} = A_{0,7} - A_{0,3}$$

Then, given a value for $\widehat{\rho}_{0,3}$ extracted from the trade 0-3% tranche, we can let $\widehat{\rho}_{0,7}$ solve

$$L_{0,7}\left(\widehat{\rho}_{0,7}\right) - L_{0,3}\left(\widehat{\rho}_{0,3}\right) = \frac{S_{3,7}^{mkt}}{4} A_{0,7}\left(\widehat{\rho}_{0,7}\right) - A_{0,3}\left(\widehat{\rho}_{0,3}\right)$$

Now, given the base correlations $\widehat{\rho}_{0,3}$ and $\widehat{\rho}_{0,7}$ we can together with the market spread for the traded 7-10% tranche compute $\widehat{\rho}_{0,10}$, the implied base correlation for the hypothetical 0-10% equity tranche and so on.

We extract base correlations from CDO data using the Gaussian copula framework to estimate values for the loss and premium legs L and A in the above. This method is discussed in more detail in Li (2000), Andersen and Sidenius (2004) and Hull and White (2004), and we refer the interested reader to those papers for a more detailed discussion.

There is of course a fundamental inconsistency in extracting correlations from CDO tranches using the Gaussian copula at each point in time, and subsequently comparing the resulting correlation path with the time-varying DECOs extracted from CDSs and equity returns. Unfortunately, estimating a model with time varying-correlation that imposes consistency between the correlation path used in pricing at different dates is computationally very complex, and we leave it for future work. In a sense, this paper can be thought of as equivalent to the first generation of studies that compared Black-Scholes implied volatility with historical GARCH volatility, before the next generation of models filtered time-varying volatility paths from options using stochastic volatility models.

[14] See McGinty, Beinstein, Ahluwalia and Watts (2004) for a detailed discussion.

3.2 CDO Data

We use the time series for the on-the-run investment grade CDX and Itraxx index spread with 5 years to maturity, from October 14, 2004 to December 31, 2007, to coincide with the CDS samples. For each index, we have five time series. For the CDX index depicted in Figure 1, we have tranche spreads for attachment and detachment points 0%-3%, 3%-7%, 7%-10%, 10%-15% and 15%-30%. For the iTraxx index depicted in Figure 2, we have tranche spreads for attachment and detachment points 0%-3%, 3%-6%, 6%-9%, 9%-12% and 12%-22%.

Figures 1 and 2 depict tranche spreads, and Table 2 presents descriptive statistics. The patterns are very similar for the CDX and the iTraxx tranche spreads. As a function of the tranches' attachment points, the spread and the spread volatility decrease, but the kurtosis and the (positive) skewness increase.

3.3 Implied Correlations Extracted from CDO Index Spreads

Figures 1 and 2 plot the time series of implied base correlations extracted from iTraxx index spreads, one for each combination of available attachment-detachment points. The implied base correlations are significantly higher for higher attachment points, which is a well-known stylized fact known as the correlation skew. It could be argued that this finding in itself is evidence of the inadequacy of the Gaussian copula model used to extract the correlations, and that this indicates that the implied base correlations should not be interpreted as correlations, but rather as a rest category that captures the model's inadequacies.[15] We instead note that while the levels of the implied correlations strongly depend on the attachment points, the five CDX time series in Figure 2 seem highly correlated. The correlation matrix in Table 3 indicates that many series have correlations of over 85%. The base correlation for the equity tranche is the least related to the four others, but even the lowest of the correlations, between the equity tranche and the most senior tranche, is 40%. Figure 2 and Table 3 clearly indicate that the same is true, a fortiori, for the iTraxx base correlations. The lowest correlation between any of these two time series is 83%.

The time horizon of our sample is somewhat limited, because of the fact that credit derivatives markets are a relatively new phenomenon. However, because of the very nature of these markets, the data contain a lot of variation. Most interestingly, our sample contains two periods of significant stress in credit markets. The first stress period occurs in April-May 2005 after the GM and Ford

[15] These level differences can be reduced by building in higher tail dependence and/or random recovery. See for example Andersen and Sidenius (2004) and Burtschell, Gregory and Laurent (2009). Even if the Gaussian copula is the correct model, differences in the level of the base correlations do not necessarily indicate problems with the correlation estimates. For example, if we use biased estimates of default probabilities to extract correlations from data generated with the Gaussian copula and a constant correlation, we will obtain a correlation skew.

downgrades. Interestingly, while Figure 3 indicates that CDS spreads on average increase in this period for the CDX and iTraxx constituents (and inspection of the time series for individual spreads indicates that most of them increase in this period), Figures 1 and 2 indicate that base correlations decrease, significantly so for the equity tranche. We also note that the co-movements between the different time series of base correlations for the CDX decrease during this period. The second period of market stress begins in the summer of 2007. In this episode, CDS spreads as well as correlations go up, and the variations in base correlations are very significant across the different tranches.

Interestingly, while there is substantial time variation in the base correlations, this variability is very similar across tranches, as can be seen from the standard deviations in Table 2. The standard deviations for iTraxx tranches are also very similar to those of CDX tranches. Moreover, the third and fourth moments for the iTraxx tranches are also not dramatically different from those of the CDX tranches. Finally, we note that whereas skewness and kurtosis of the spreads monotonically increase as a function of the attachment points, this is not the case for the skewness and kurtosis of the base correlation time series.

We must of course be careful when comparing CDX and iTraxx base correlations directly. As mentioned before, it is a robust finding that base correlations increase as a function of the attachment point. It is therefore not very instructive to compare the base correlation for the CDX senior tranche with that of the iTraxx senior tranche, because the attachment points are very different. The most obvious comparison is between the equity tranches for both indexes, which are both 0-3% tranches. For these tranches, the average iTraxx base correlation is 17%, compared to 14% for the CDX. Table 3 indicates that the correlation between the series is 90%.

4 Default Intensity Dynamics

Our empirical strategy based on CDS data proceeds in three steps. In the first step, we use an intensity model to extract default intensities from CDS spreads. In the second step, we use ARMA-GARCH models to capture the dynamics in the default intensities. In the third step, we perform a DECO analysis on the standardized residuals from the second step in order to determine their co-variation across time. The first two steps are presented below and the third step in the subsequent section.

4.1 CDS Spreads

We conduct our empirical investigation using daily CDS premia on American corporates provided by CDX, and on European corporates provided by iTraxx. At any point in time, the CDX and iTraxx indexes consists of 125 components. The composition of the index is changed every six months. We use CDS data from October 14, 2004 to December 31, 2007. From the 125 corporates that constitute the CDX index on October 14, 2004, 61 corporates stay in the index over the whole sample period. For the iTraxx index, this number is 64. Our empirical analysis uses these 61 respectively 64 corporates, which are listed in Table 1. Descriptive statistics for the CDS premia of the 61 CDX components and the CDS premia of the 64 iTraxx components are provided in Table 4. Panel A of Figure 3 depicts the cross-sectional average of the spreads for the 61 CDX components, and Panel B does the same for the iTraxx components. Figure 3 indicates that the average premia are highly persistent over time. The same is true for the spreads of individual companies, which are omitted because of space constraints. They also contain occasional large shifts, as evidenced by skewness and kurtosis statistics. Most of the companies display positive skewness, and for some of the lower rated companies we occasionally observe substantial excess kurtosis (not reported).

The time series of spreads for individual companies (not reported) also suggest significant commonality in CDS premia across companies: For example, the spike in the CDS premia in early May 2005, which is associated with the General Motors and Ford downgrades, is shared by many (but not all) of the companies and so is evident in Figure 3. Even more importantly, the spreads significantly increase in the second half of 2007 for almost all companies. Table 4 and Figure 3 indicate that compared to the European names, the American names tend to have a higher average spreads and on average higher spread standard deviations.

4.2 Extracting Default Intensities from CDS Spreads

The valuation of CDS contracts, and the estimation of default intensities that relies on this valuation, has been studied in several papers.[16] Consider a given risk-neutral survival probability $q(t,T)$. The premium on an CDS is the spread paid by the protection buyer that equates the expected present value of default costs to be borne by the protection seller ("floating leg") to the expected present value of investing in the CDS ("fixed leg"). The value of the fixed leg is the present value of the spread payments the protection seller receives from the protection buyer, while the unknown floating leg comprises the potential payment by the protection seller to the buyer.

Consider now a CDS contract with payment dates $T = (T_1, ..., T_N)$, maturity T_N, premium P and notional 1. Denote the value of the fixed leg by $V_{Fixed}(t,T,P)$, the value of the floating leg by $V_{Floating}(t)$, and the discount factors by $D(t,T_i)$. At each payment date T_i, the buyer has to pay

[16] The literature on CDS contracts has expanded rapidly. For theoretical work, see Das (1995), Das and Sundaram (1998) and Hull and White (2000). For empirical studies, see Berndt, Douglas, Duffie, Ferguson, and Schranz (2004), Blanco, Brennan, and Marsh (2005), Ericsson, Jacobs and Oviedo (2007), Houweling and Vorst (2005), Hull, Predescu and White (2004), Longstaff, Mithal and Neiss (2004), and Zhang, Zhou and Zhu (2006).

$a(T_{i-1}, T_i)P$ to the seller, where $a(T_{i-1}, T_i)$ represents the time period between T_{i-1} and T_i (T_0 is equal to t). If the reference entity does not default during the life of the contract, the buyer makes all payments. However, if default occurs at time $s \leq T_N$, the buyer has made $I(s)$ payments, where $I(s) = \max(i = 0, ..., N : T_i < s)$, and has to pay an accrual payment of $\alpha(T_I(s), s)P$ at time s. Denote the probability density function associated with the default intensity process λ_t by $f(t)$, such that $f(t) = \frac{dq(t)}{dt}$, and let the recovery rate be δ, then

$$V_{Fixed}(t, T, P) = P_t \left[\sum_{i=1}^{N} D(t, T_i)\alpha(T_{i-1}, T_i)q(t, T_i) + \int_t^{T_N} D(t, s)\alpha(T_{I(s)}, s)f(s)ds \right]$$

$$V_{Floating}(t) = \int_t^{T_N} D(t, s)(1 - \delta)f(s)ds.$$

At initiation of the contract, the premium P_0 is chosen in such a way that the value of the default swap is equal to zero. Since the value of the fixed leg is homogeneous of degree one in P, the premium should be chosen as $P_0 = V_{Floating}(0)/V_{Fixed}(0, T, 1)$.

In our empirical application, we compute the integrals by numerical approximations. We define a monthly grid of maturities $s_0, ..., s_m$, where $s_0 = t$ and $s_m = T_N$ and set

$$\int_t^{T_N} D(t, s)(1 - \delta)f(s)ds \approx \sum_{i=1}^{m} D(t, s_i)(1 - \delta)(q(t, s_{i-1}) - q(t, s_i))$$

$$\int_t^{T_N} D(t, s)\alpha(T_{I(s)}, s)f(s)ds \approx \sum_{i=1}^{m} D(t, s_i)\alpha(T_{I(s_i)}, s_i)(q(t, s_{i-1}) - q(t, s_i)$$

We use this valuation framework to back out time paths of default intensities that are subsequently used as inputs in an econometric analysis of the correlation of default intensities across companies. We proceed with the simplest possible approach. We back out a time path of default intensities assuming that the default intensity is constant at every time t. The link between the risk-neutral survival probability $q(t, T)$ and the default intensity is then simply

$$q(t, T) = \exp(-\lambda_t(T - t))$$

The time-subscript on the λ indicates that a new value for λ in the constant-intensity model is extracted on each day. This method gives economically very plausible results for all companies.

Panels A and B of Figure 4 plot the time path of the average (annualized) implied risk-neutral default intensities for the CDX and iTraxx respectively. Clearly these probabilities are closely related to the average CDS premia in Figure 3. This is also the case for default intensities for individual companies and individual spreads, which are not depicted because of space constraints. Table 5 reports descriptive statistics for the annualized default intensities. The results indicate substantial cross-sectional heterogeneity in default probabilities. For the CDX, the highest average annualized default probability is 1.97% and the lowest one is 0.27%. For the iTraxx companies, those numbers are 1.18% and 0.22%. Moreover, there are substantial differences between companies in the second, third and fourth moments. Many of these cross-sectional differences can be directly related to the spread data in Table 4. Note also from Table 5 that the default intensity is highly persistent in all firms. The first-order autocorrelation is virtually one in all cases.

Our constant-intensity assumption could be criticized as overly simplistic. We investigated the robustness of our results by specifying dynamic alternatives. First, we estimated a simple square root (CIR) model for the default intensity λ_t. This analysis largely confirmed the results obtained with the assumption of a constant default intensity. However, in approximately 5% of the cases, we obtained local optima that were economically unrealistic, in the sense that they implied implausible average default intensities. Attempts to address these deficiencies with multifactor models were not successful. We prefer the economically more plausible results obtained using the constant intensity model, and use these results throughout.

Another possible approach is to back out default intensities from CDS spreads using a structural model. This approach is interesting when comparing correlations to CDO implied correlations obtained using a Gaussian copula, because the normality assumption used in the Gaussian copula means that it can be interpreted as a multivariate structural Merton model. However, many applications of structural models use accounting data, and we exclusively want to rely on market data. Some implementations of structural models that only rely on market data are feasible in principle. In a structural model, stock prices and risk-adjusted default probabilities depend (in addition to asset risk) on the level of a firm's asset value relative to some default threshold driven by the level of the firm's book liabilities. In some situations, for example when computing default probabilities in the Merton (1974) model, it is sufficient to know the ratio of asset value to the threshold. This in turn would allow the model to be estimated without using observed book levels of debt.[17] We considered such an estimation strategy using the Black and Cox (1976) model, but found it computationally cumbersome as well as highly unstable. We therefore rely on the simple reduced form model described above.

[17] This is not be the case when estimating the Merton model using equity returns, because the value of equity depends separately on the ratio and the levels of asset value and book debt.

4.3 Modeling Default Intensity Dynamics

We are ultimately interested in modeling the conditional correlation of default intensities across firms, defined as

$$Corr_t\left(\lambda_{i,t+1}, \lambda_{j,t+1}\right) = \frac{E_t\left\{\left[\lambda_{i,t+1} - E_t\left(\lambda_{i,t+1}\right)\right]\left[\lambda_{j,t+1} - E_t\left(\lambda_{j,t+1}\right)\right]\right\}}{\sqrt{E_t\left\{\left[\lambda_{i,t+1} - E_t\left(\lambda_{i,t+1}\right)\right]^2\right\} E_t\left\{\left[\lambda_{j,t+1} - E_t\left(\lambda_{j,t+1}\right)\right]^2\right\}}}$$

So in order to model the dynamic correlations we first need to model the conditional mean and the conditional variance of the default intensity for each firm.

Statistical tests indicate that the constant intensity time series extracted from the CDS data are non-stationary. We therefore start by first-differencing the default intensities. Before proceeding we note that because $\lambda_{i,t}$ is in the time t information set, we have

$$\Delta\lambda_{i,t+1} - E_t\left(\Delta\lambda_{i,t+1}\right) = \lambda_{i,t+1} - E_t\left(\lambda_{i,t+1}\right)$$

so that

$$Corr_t\left(\Delta\lambda_{i,t+1}, \Delta\lambda_{j,t+1}\right) = Corr_t\left(\lambda_{i,t+1}, \lambda_{j,t+1}\right)$$

Thus modeling the conditional mean and variance of the first-difference of default intensities suffices for our ultimate goal of modeling conditional default correlations, because the innovations to the first-differences are the same as the innovations to the levels.

The default intensities contain short-run dynamics as well. Therefore, it is not sufficient to treat the first-differenced default intensities as white noise. Instead, in order to obtain white-noise innovations required for econometric correlation modeling from the intensities, we fit univariate $ARIMA(p,1,q) - GARCH(1,1)$ models to the intensity time series.

To be specific, we estimate the following ARMA model for each firm

$$\Delta\lambda_t = \mu_\lambda + \sum_{i=1}^{p} \phi_i \Delta\lambda_{t-i} + \sum_{j=1}^{q} \theta_j z_{t-j} + z_t$$

where z_t is uncorrelated with $\Delta\lambda_s$ for $s < t$.

We fit all possible $ARMA$ models with $p \leq 4, q \leq 4$, with and without intercepts, and choose the model with the lowest $AICC$ value defined by

$$AICC = -2LLF + 2(p+q+c+1)T/(T-p-q-c-2)$$

where LLF is the log likelihood value, $c = 0$ if the $ARMA$ model does not have an intercept, and

$c = 1$ if the $ARMA$ model has an intercept, and T is the number of observations.

In a second step we fit a $GARCH(1,1)$ on the $ARMA$ filtered residuals z_t

$$\begin{aligned} z_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \omega_\sigma + \alpha_\sigma z_{t-1}^2 + \beta_\sigma \sigma_{t-1}^2 \\ \varepsilon_t &\sim i.i.d.D(0,1) \end{aligned}$$

where $\omega_\sigma > 0$, $\alpha_\sigma \geq 0$, $\beta_\sigma \geq 0$. Note that the distribution of the shocks is not assumed to be normal.

Table 6 reports the descriptive statistics for the first-difference intensity residuals from the ARMA-GARCH models. Note that the first-order autocorrelations are very small now. In general the ARMA-GARCH model has been successful in removing the dynamics in both the first and second moments of the residuals from the first-differenced intensities.

5 Dynamic Default Intensity and Equity Correlations

In this section we first briefly describe the DECO approach to dynamic correlation modeling. We then apply DECO to the default intensity innovations extracted above as well as to daily equity returns. Finally we compare these dynamic default and equity correlations with the implied correlations from CDOs derived earlier.

5.1 Modeling Dynamic Equicorrelation

Engle (2002) proposes a new class of models, named Dynamic Conditional Correlation Multivariate GARCH (DCC), which preserves the convenience of Bollerslev's (1990) constant correlation model, while allowing correlations to change over time. The DCC approach allows each of the companies in the analysis to have its separate dynamic for the marginal distribution. The DCC approach puts a dynamic multivariate distribution on top of the dynamic marginal distributions and can therefore be viewed as a dynamic copula approach.

The dynamic equicorrelation (DECO) model in Engle and Kelly (2008) is essentially an extreme case of a DCC model in which the correlations are equal across all pairs of companies but where this common equicorrelation is changing over time. The resulting dynamic correlation can be thought of as an average dynamic correlation between the companies included in the analysis.

In the standard DCC analysis, the correlation matrix R_t must be inverted for each iteration required in the numerical optimization procedure. This is costly for small cross-sections, and potentially infeasible for larger ones. In the DECO approach which has compact forms for the determinant and inverse of the correlation matrix, the problem is reduced to the optimization of a function whose arguments are all scalars with no matrix inversion or determinant computation required. It is therefore relatively straightforward to generate results for arbitrarily large cross-sections of companies. We will rely solely on the DECO approach in the empirical analysis below.

Following Engle and Kelly (2008), we parameterize the dynamic equicorrelation matrix as

$$R_t = (1-\rho_t)I_n + \rho_t J_{n\times n}$$

where I_n denotes the n-dimensional identity matrix and $J_{n\times n}$ is an $n \times n$ matrix of ones. The inverse and determinants of the equicorrelation matrix, R_t, are given by

$$\begin{aligned} R_t^{-1} &= \frac{1}{(1-\rho_t)}[I_n - \frac{\rho_t}{1+(n-1)\rho_t}J_{n\times n}] \text{ and} \\ \det(R_t) &= (1-\rho_t)^{n-1}[1+(n-1)\rho_t] \end{aligned}$$

Note that R_t^{-1} exists if and only if $\rho_t \neq 1$ and $\rho_t \neq \frac{-1}{n-1}$, and R_t is positive definite if and only if $\rho_t \in \left(\frac{-1}{n-1}, 1\right)$.

The time-varying equicorrelation parameter, ρ_t is assumed to follow the simple dynamic

$$\rho_{t+1} = \omega + \alpha u_t + \beta \rho_t$$

where u_t represents the equicorrelation update. We considered different updating rules proposed by Engle and Kelly (2008). Consistent with their results, we obtain the best results for the following updating rule

$$u_t = \frac{\sum_{i\neq j} \varepsilon_{i,t}\varepsilon_{j,t}}{(n-1)\sum_i \varepsilon_{i,t}^2} = \frac{\left(\sum_i \varepsilon_{i,t}\right)^2 - \sum_i \varepsilon_{i,t}^2}{(n-1)\sum_i \varepsilon_{i,t}^2}$$

Note that u_t lies within the positive definite range $\left(\frac{-1}{n-1}, 1\right)$.

The correlation matrices R_t are guaranteed to be positive definite if the parameters satisfy $\omega/(1-\alpha-\beta) \in (-1/(n-1), 1)$, $u_t \in (-1/(n-1), 1)$, and $\alpha + \beta < 1$, $\alpha > 0$, $\beta > 0$.

Engle and Kelly (2008) report that the u_t form above will be the least sensitive to residual volatility dynamics and extreme realizations, due to the use of a normalization that uses the mean cross sectional variance. However it can be subject to downward bias because it is a ratio of correlated random variables. One way to address this bias is to alter the implementation to ensure that the fitted equicorrelation process obeys the bounds $\left(\frac{-1}{n-1}, 1\right)$ without imposing $\alpha + \beta < 1$. We found that the intensity DECO for the CDX companies is indeed somewhat higher when removing this restriction. For the iTraxx companies, removing this restriction did not change the estimates. The estimates for equity returns were also not affected by this constraint.

5.2 Correlations Between CDS-Implied Default Intensities

We perform a dynamic equicorrelation (DECO) analysis on the 61 components of the CDX index that are part of the index for the entire sample period, and for the 64 components of the iTraxx index that are part of the index for the entire sample period. The results of the DECO analysis are reported in Figures 5 and 6, and the parameter estimates are reported in Table 7. The correlations between the DECO dynamic correlations and the implied correlations from CDOs are reported in Table 8. Table 8 also reports the sample moments of the dynamic correlations.

Figure 5 presents results for CDX data, and Figure 6 for iTraxx data. Consider first the CDX data in Figure 5, which contains the time series for the correlations implied by the equity tranche (top-left panel) and a mezzanine tranche (top-right panel), as well as the CDS-based DECO correlations (bottom left). The correlation between the CDS-based DECO correlation with the base correlation for the mezzanine tranche is very high at 68%, as indicated by Table 8. The correlation with the equity tranche is lower at 37%. Figure 6 shows that for the iTraxx, the CDS-based and CDO-implied correlations move together even more strongly.

The overall conclusion from Figures 5 and 6 will of course depend on one's prior, but in our opinion the evidence in favor of co-movement between these different correlation measures is arguably unexpectedly strong. The Gaussian copula is widely acknowledged to be overly simplistic, as evidenced for instance by the different correlation levels implied by different tranches. In light of this, the correlations in Table 8 are surprisingly high. This high degree of similarity between the time series of CDO-implied correlations and their CDS-implied counterparts is surprising not only as the implied correlation acts as a catch all for obvious misspecifications in the modeling of default risk, but also because it will capture any CDO market specific risk premia, illiquidity premia and demand/supply effects.

We therefore arrive at two main conclusions. First, there is a large amount of co-variation between the DECO-implied correlations from CDSs and the base correlations from CDOs, for the CDX as well as the iTraxx. The correlation matrix in Table 8 confirms this. Second, there is a substantial amount of time variation in these DECO correlations. The intensity-based correlations vary more than the base correlations, as confirmed by the second moments in Table 8.

A number of other conclusions obtain. Interestingly, there is a large common component to the intensity-based correlations for the CDX and the iTraxx. Table 8 indicates that the correlation between the two time series is 85%. However, the levels are dramatically different, as confirmed by the averages in Table 8: the average CDX intensity DECO is 17%, and the average iTraxx intensity DECO is 33%. Standard deviation, skewness and kurtosis are not dramatically different. The co-movements as well as the level differences can clearly be seen from Figures 5 and 6.

5.3 Correlations Between Equity Returns

We also apply DECO techniques to equity returns, to verify whether this source of information on correlation yields results that are comparable to the base correlations from CDOs. To estimate correlations between equity returns, we estimate DECO models for the sample period January 1, 2000 to December 31, 2007.[18] We use all of the 61 CDX constituents and 64 iTraxx constituents used in the analysis of the CDS data for which we are able to obtain equity returns over this samples. This amounts to 54 CDX constituents and 58 iTraxx constituents. Because of space constraints, we do not present descriptive statistics for these equity return samples. They are entirely standard and are available upon request.

The bottom-right panels of Figures 5 and 6 report the resulting correlation paths for the October 14, 2004 to December 31, 2007 period. Parameter estimates are reported in Table 7. For the CDX in Figure 5, there is a substantial amount of co-movement between the equity-based DECO correlations and the base correlations. Table 8 indicates that the correlations are between 44% and 70%. For the iTraxx in Figure 6, this is not the case: Table 8 indicates that the correlations are between 15% and 19%. The correlation between the equity DECO correlation and the CDS-implied DECO correlation is 61% in the case of the CDX, and 45% for the iTraxx. Interestingly however, when we look at the average correlations in Table 8, the average level of the equity-based CDX correlation is much closer to the average intensity-based correlation for the iTraxx than for the CDX. In fact, the iTraxx and CDX equity-based correlations are very similar on average (29% and 28% respectively) and the correlation between the two series is 50%. It is clear from Figures 5 and 6 that the equity-implied correlations do not increase as much as the CDO-implied and CDS-implied correlations in the turbulent second half of 2007.

5.4 Discussion

The main purpose of this paper is to provide a description of a set of stylized facts regarding default dependence. Although correlation between different companies can be measured using a variety of

[18] When using the time series of equity returns for the period October 14, 2004 to December 31, 2007, which coincides with the CDS sample period, estimation results are less precise and the resulting time series of correlations is more variable. This indicates that the CDS-implied default intensities are a richer source of information on correlations than equity returns.

different securities, the literature does not contain a great deal of systematic comparisons between these different correlation measures. This is perhaps not surprising: the comparison between different correlation measures is fraught with difficulties because the correlations are obtained using different models, and the choice of model is often motivated by the complexity of the credit risky security. For example, when extracting correlations from CDOs, most often a Gaussian copula with constant correlation is used because of the complexity of the CDO valuation problem.

The CDO valuation problem is complex, and extracting time-varying correlation from CDO prices is computationally extremely expensive. We therefore use the workhorse Gaussian copula to extract a time series of correlations based on CDO tranche spreads. Even though the time variation in the resulting correlation time series is somewhat hard to interpret, because each estimate is obtained assuming constant correlation, we compare this somewhat ad hoc time variation in implied correlation with time varying correlations from CDS spreads and equity returns which are captured using an explicitly time-varying DECO correlation model.

It is instructive to keep these limitations in mind when comparing the correlation measures. However, to some extent the value of our results depends on how strongly the resulting time series of correlations move together. If the correlations implied by equity returns, CDS premia, and CDO tranche spreads are very different, this might be due to differences in the underlying correlation and default intensity models. Because we find that the resulting correlation series move together in many instances, we are able to draw a fairly strong conclusion; presumably if we could correct for the differences in the underlying models, the correlation time series would co-vary even more.

Besides the strong co-movements between the different correlation series, our most important finding is probably the significant amount of time-variation in the three correlation time series. This has important implications for the valuation of portfolio credit products.

Our empirical findings can be interpreted in various alternative ways. Most importantly, following the reasoning in Tarashev and Zhu (2007), we could compare the correlations based on CDS data with the correlations based on CDO tranche spreads to draw conclusions about correlation risk. The underlying logic is that while the default probabilities implied by CDS data are risk neutral default probabilities, the default dependences are physical because the underlying securities are univariate and therefore do not incorporate a risk premium for correlation risk. This contrasts with CDO tranche spreads, which do incorporate such a risk premium. A comparison of these two correlation time series is therefore thought to be instructive about correlation risk.

There are two potential problems with this reasoning. First, as can be clearly seen from Figures 1 and 2, while the base correlations for different tranches are highly correlated, the levels of the correlation time series increase with the attachment points. Any inference regarding the risk premium will therefore depend on which tranche spread one chooses as a reference point. In Tarashev and

Zhu (2007), this point is somewhat obscured because the comparison is made using tranche spreads rather than implied correlations, but the exercise is similarly constrained by the fact that the CDO valuation models are not rich enough. A second, more subtle, limitation of this type of analysis is that while credit default swaps are univariate credit products, they are priced in equilibrium, and therefore it is not clear that they do not contain a correlation risk premium. By analogy, we do not need portfolio equity products to unearth market risk: it is priced in the equity returns of individual companies. Therefore, while we could interpret the ratio between the CDS and CDO-implied correlations in Figures 5 and 6, or similar ratios using the base correlations in Figures 1 and 2, as correlation risk premia, in our opinion this type of conclusions are better addressed within the context of an explicit general equilibrium model. We prefer to focus on the similarities and differences in the correlations, without necessarily interpreting the differences as risk premia, and we prefer to focus on co-movements rather than level differences.

While overall we emphasize the similarities between the three types of correlations, there are some interesting differences. First, the overall level of the CDX intensity correlations is low in comparison with the iTraxx intensity correlations and the equity correlations. We carefully verified the robustness of this empirical finding. Second, following the downgrade of Ford and GM in the credit crisis of May 2005, implied correlations decrease significantly for the equity tranche, and much less for the more senior tranches, while equity-based and CDS-based correlations do not decrease. It may be possible that this finding is due to market mispricing, with the prices of equity tranches overreacting to expectations of higher default rates. Third, correlations implied by equity returns do not increase as much as CDS-implied correlations in the second half of 2007.

6 Conclusion

This paper systematically compares correlation measures implied by three different types of securities. We compare base correlations implied by CDOs with correlations implied by equity returns and correlations implied by default intensities. We perform this analysis using both North American data, using the CDX index, and European data, using the iTraxx index. Our results are largely complementary to existing findings in the intensity-based literature. Existing studies attempt to characterize observable macro variables that induce realistic correlation patterns in default probabilities (see Duffee (1999) and Duffie, Saita and Wang (2007)). Our approach does not attempt to characterize the dependence of the default intensities on the observables. Instead, we keep the common component in the default intensities in the first stage, and subsequently characterize it in the second stage.

We obtain base correlations from CDO data using the standard Gaussian copula. To characterize the correlation of default intensities, we use a two-step procedure, where we first extract default intensities from CDS data using a pricing model. To characterize time variation in equity return and default dependence, we use flexible dynamic equicorrelation techniques. The empirical results for time-varying intensity-based correlations provide valuable information regarding default dependence from market prices that is different from existing estimates of default dependence, which typically use either historical default rates or equity returns.

We obtain two main findings. Default intensity correlations are substantially time-varying. Neglecting the time variation in correlations may induce substantial errors in the pricing of structured credit products, in particular the relative pricing of CDO tranches with different seniority levels. Correlations can easily drop or increase by 30% over short periods of time. Another important finding is that the correlation measures obtained from different data sources are in most cases highly correlated. Base correlations obtained using the Gaussian copula are strongly related to default intensity correlations estimated from CDS data. We find this result surprising given that CDO market implied base correlations will include structured credit market specific effects as well as being a catch-all for any consequences of model misspecification. It is equally interesting that correlations extracted from equity returns also positively co-vary with the base correlations, but less so than CDS-implied correlations.

Our results suggest a number of extensions. First, given the richness and complexity of the estimated correlation structures, it will prove interesting to explore the implications of equity-based and intensity-based correlation dynamics for CDO valuation. See Berd, Engle and Voronov (2007) for such an approach. A second potentially interesting extension is to extract correlation risk premia from estimates such as the ones in this paper, but this will necessitate richer models for CDO valuation. Third, a more extensive exploration of the cross-section of correlation dynamics seems warranted.

In this paper, we use very straightforward approaches to modeling default dependence in each asset class–a basic intensity model for CDS prices, raw equity returns for stock based correlations and the market workhorse Gaussian Copula for CDO base correlations. We chose to do this with a view to keeping our methodology as simple and transparent as possible. Alternatively, one could require that the models used for various asset classes are as similar as possible. For example we could use Merton's model to extract asset values from stock prices as well as CDS prices and also interpret the Gaussian Copula model in the Merton context, or we could compute implied correlations from CDOs based on a reduced-form model. Finally, models with time-varying correlation estimated directly from CDO data would be of significant interest.

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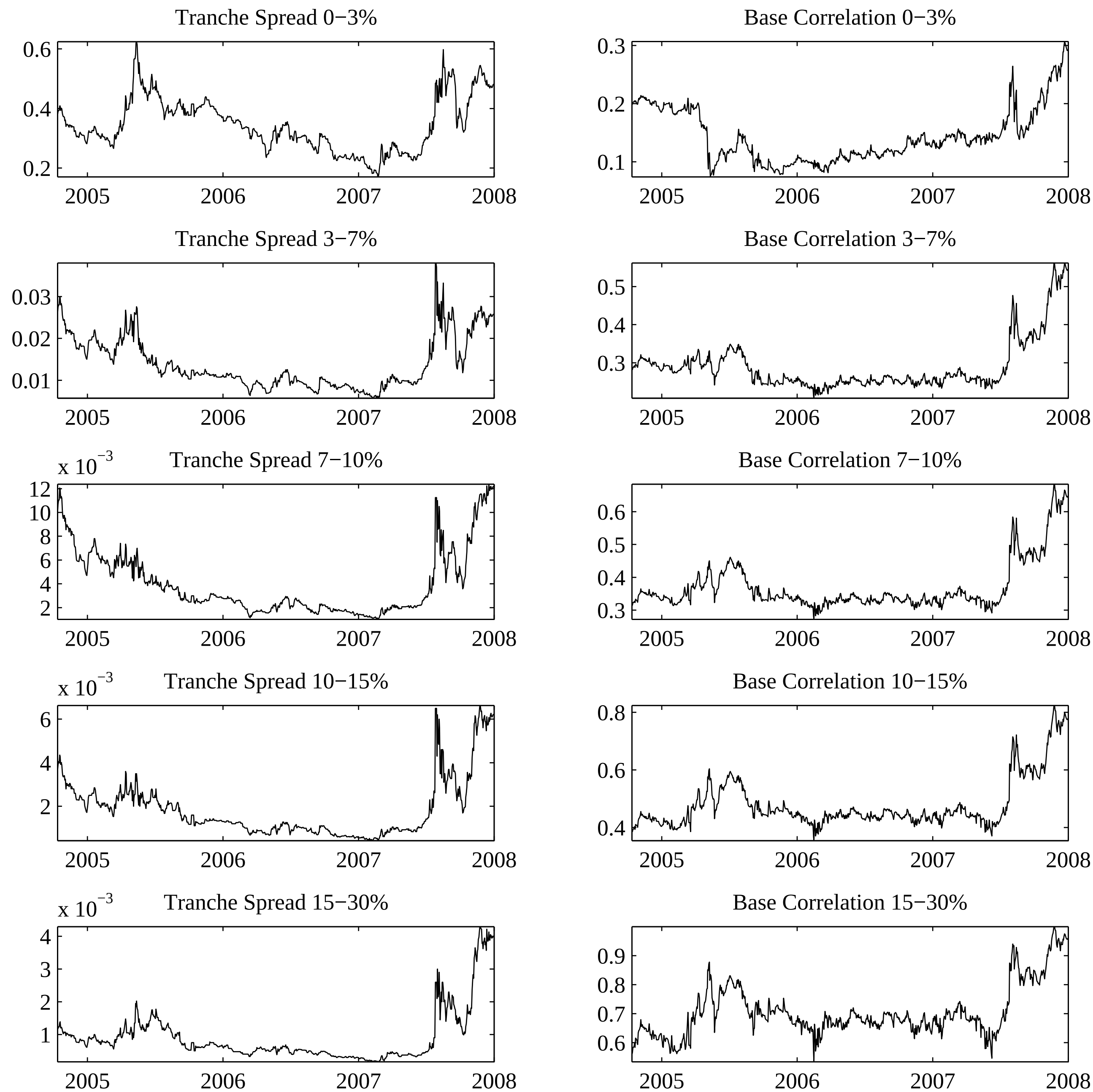
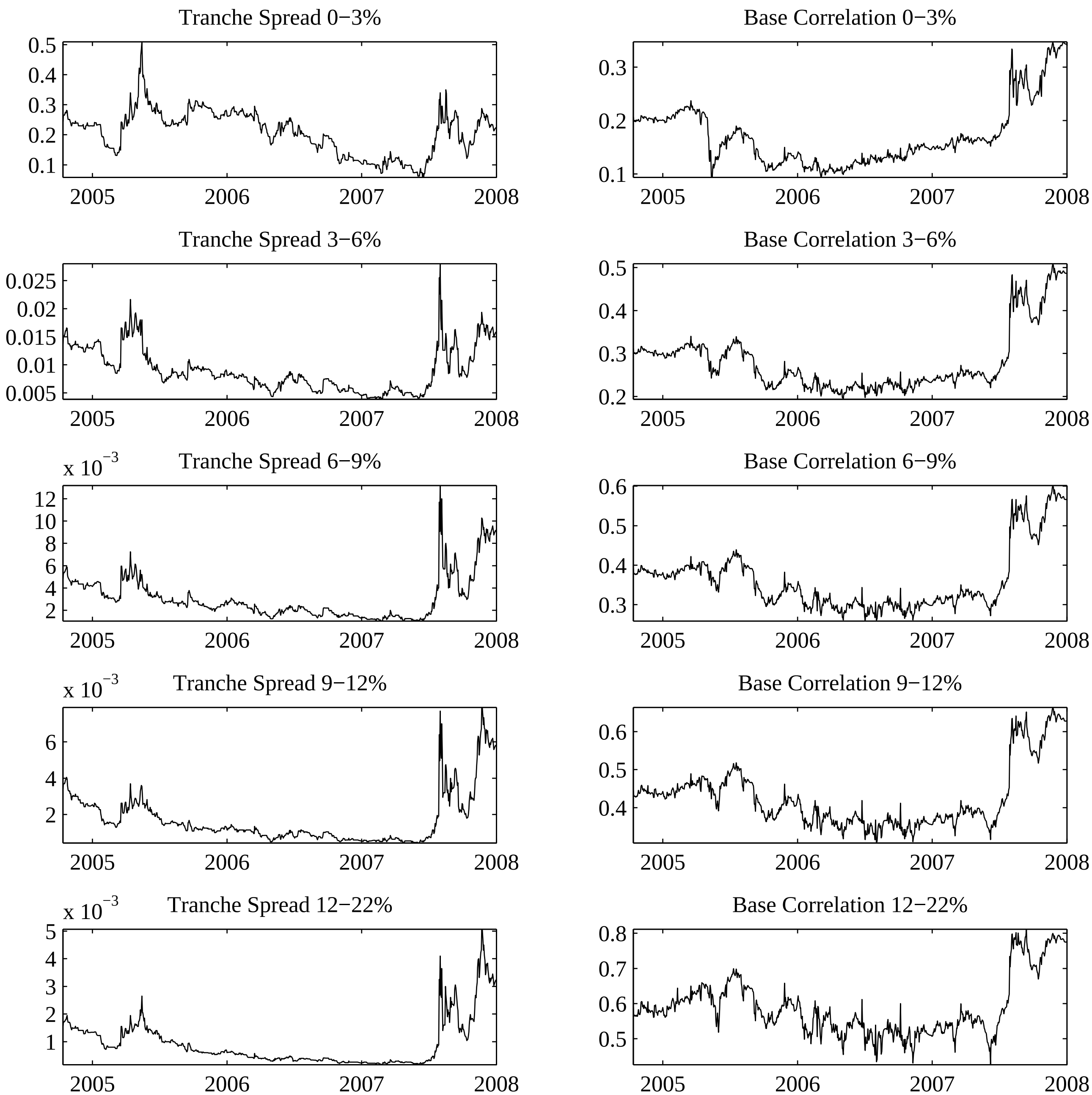
Figure 1: CDX Tranche Spreads and Base Correlations. October 14, 2004 - December 31, 2007.



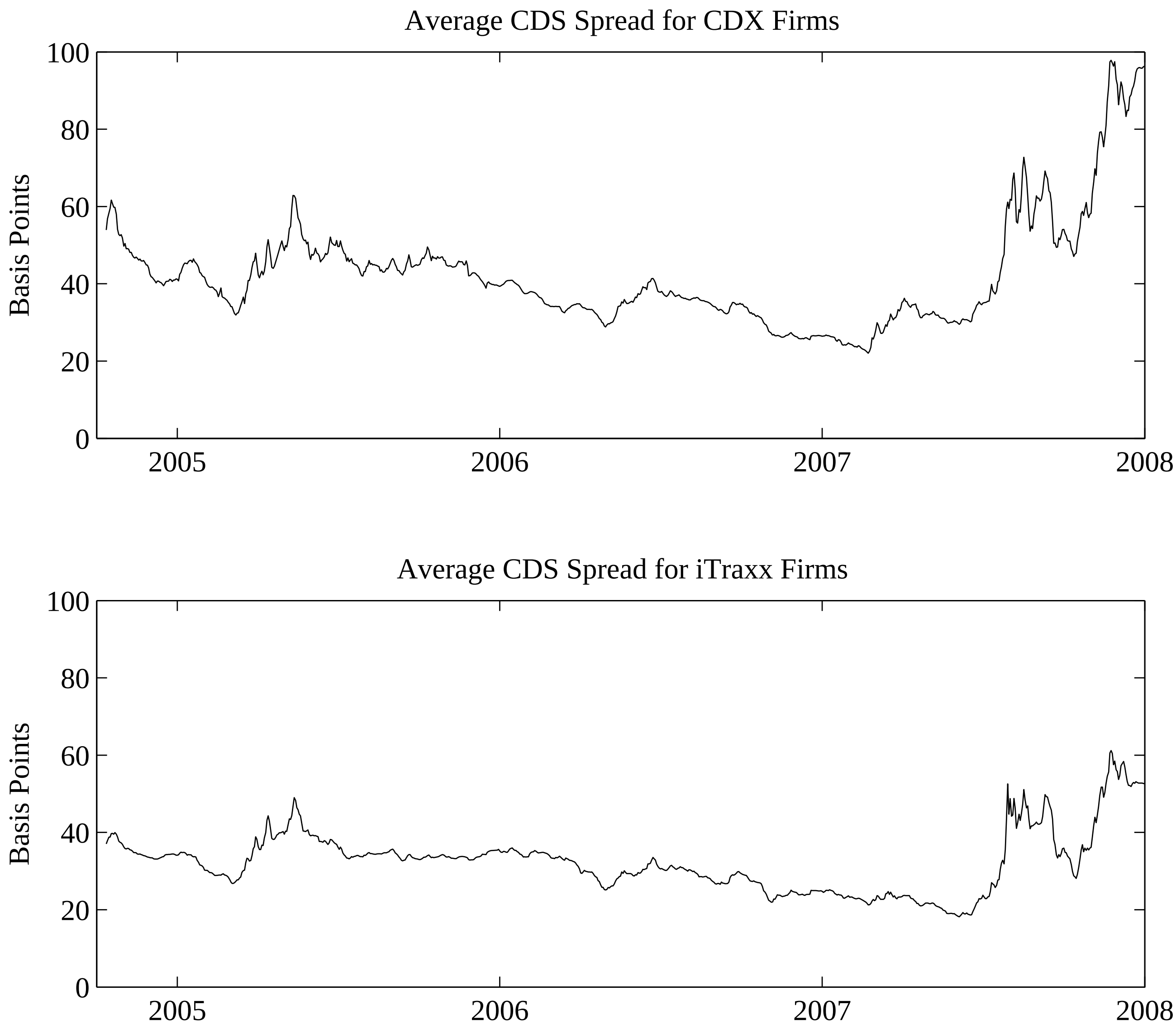
Notes to Figure: We plot tranche spreads and base correlations for all CDX tranches using data from October 14, 2004 through December 31, 2007.

Figure 2: iTraxx Tranche Spreads and Base Correlations. October 14, 2004 - December 31, 2007.



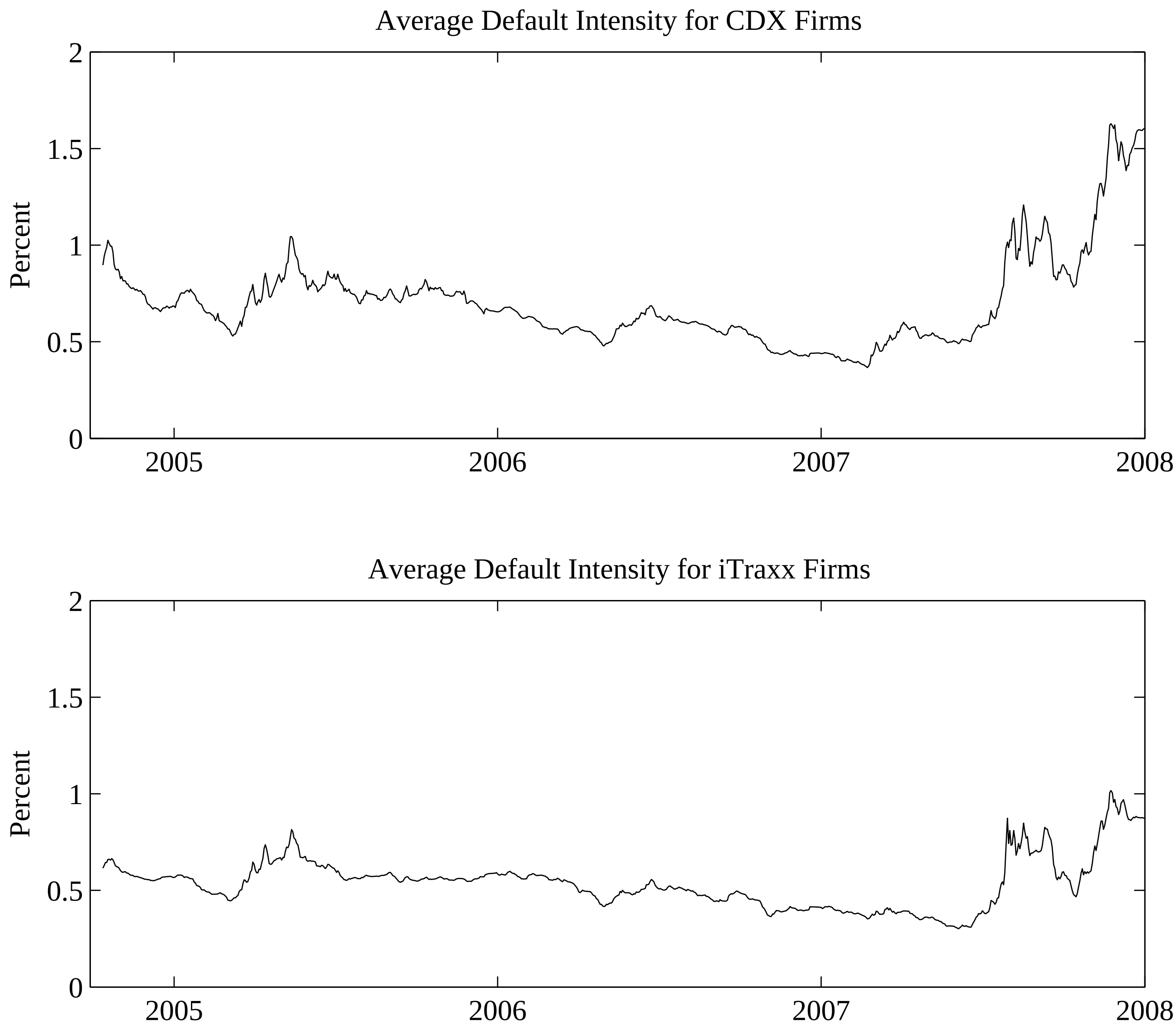
Notes to Figure: We plot tranche spreads and base correlation for all iTraxx tranches using data from October 14, 2004 through December 31 2007.

Figure 3: Cross Sectional Averages of CDS Premia.
October 14, 2004 - December 31, 2007.



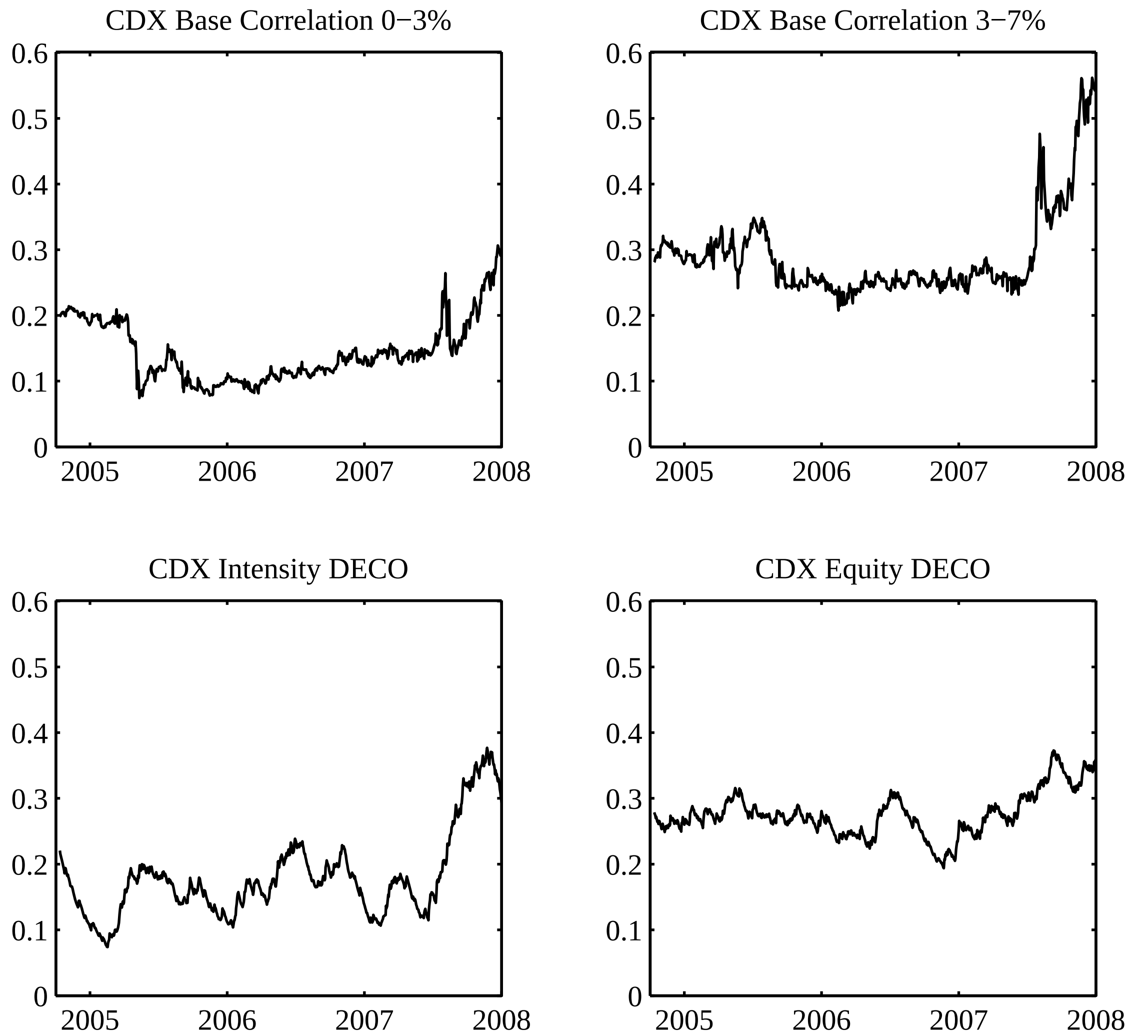
Notes to Figure: We plot cross sectional averages of CDS premia for CDX and iTraxx companies using October 14, 2004 through December 31, 2007 data.

Figure 4: Cross Sectional Average Default Intensities.
October 14, 2004 - December 31, 2007.



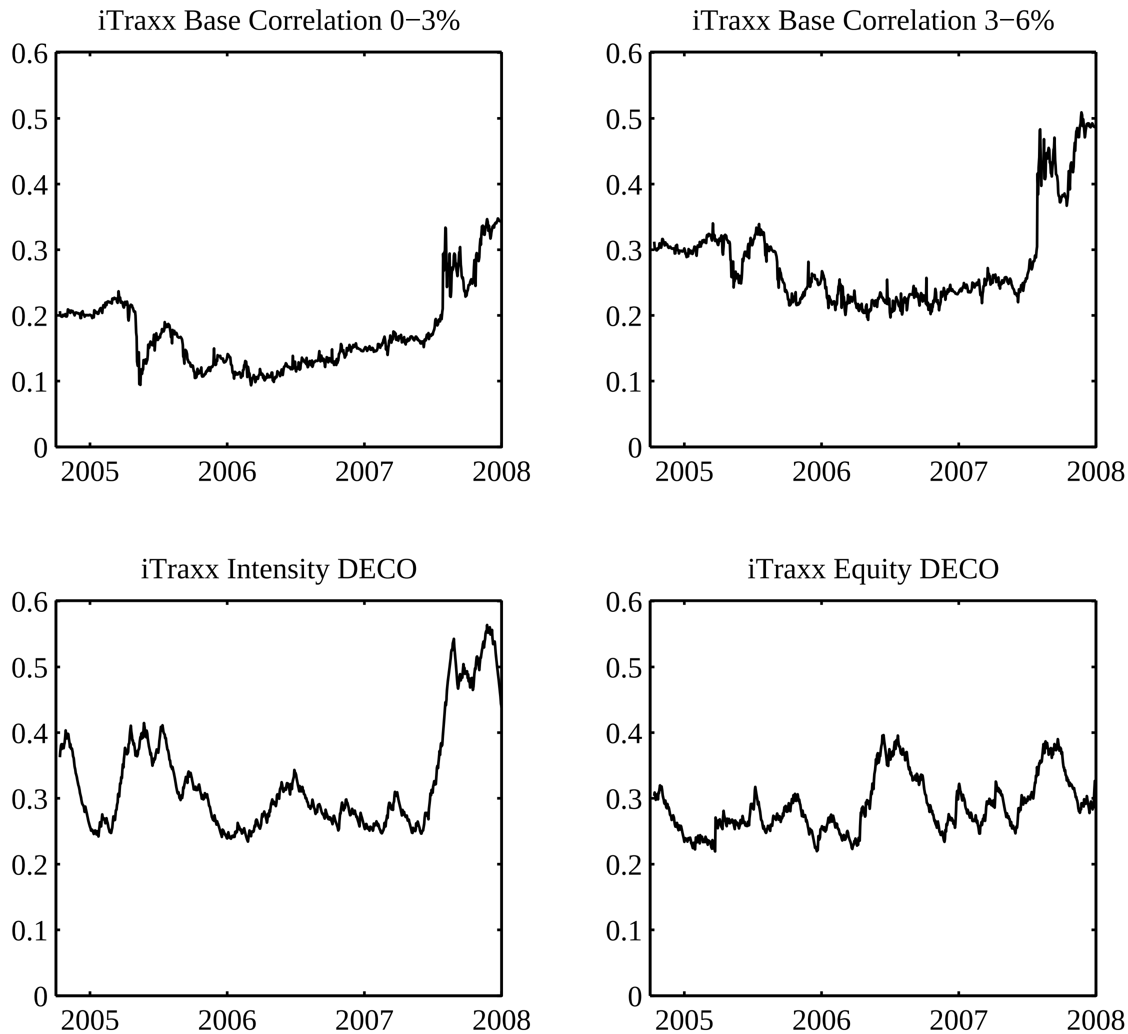
Notes to Figure: We plot cross sectional average default intensities for CDX and iTraxx companies using October 14, 2004 through December 31, 2007 data.

Figure 5: DECOs and Base Correlations for CDX Companies
October 14, 2004 - December 31, 2007



Notes to Figure: We plot equity and intensity dynamic equicorrelations (DECO) for CDX companies, and base correlations for the 0-3% and 3-7% CDX tranches, using data from October 14, 2004 through December 31, 2007.

Figure 6: DECOs and Base Correlations for iTraxx Companies
October 14, 2004 - December 31, 2007



Notes to Figure: We plot equity and intensity dynamic equicorrelations (DECO) for iTraxx companies, and base correlations for the 0-3% and 3-6% iTraxx tranches, using data from October 14, 2004 through December 31, 2007.

Table 1: Firm Names and Tickers. CDX and iTraxx

CDX Firm Name	Ticker	iTraxx Firm Name	Ticker
Ace Limited	ACE	Aktiebolaget Volvo	VLVY
Alcan Inc.	AL	Bayerische Motoren Werke AG	BMW
Alcoa Inc.	AA	Compagnie Financiere Michelin	MICH
Altria Group, Inc.	MO	Continental Aktiengesellschaft	CONTI
American Express Company	AXP	GKN Holdings PLC	GKNLN
American International Group, Inc.	AIG	Peugeot SA	PEUGOT
Anadarko Petroleum Corporation	APC	Renault	RENAUL
Arrow Electronics, Inc.	ARW	Valeo	VLOF
AT&T Inc.	ATTINC	Volkswagen Aktiengesellschaft	VW
AutoZone, Inc.	AZO	Accor	ACCOR
Baxter International Inc.	BAX	British American Tobacco PLC	BATSLN
Boeing Capital Corporation	BA	Carrefour	CARR
Bristol-Myers Squibb Company	BMY	Deutsche Lufthansa AG	LUFTHA
Burlington Northern Santa Fe Corp	BNI	Koninklijke Philips Electronics N.V.	PHG
Campbell Soup Company	CPB	LVMH Moet Hennessy Louis Vuitton	MOET
Cardinal Health, Inc.	CAH	Marks and Spencer PLC	MKS
Carnival Corporation	CCL	Metro AG	METFNL
CenturyTel, Inc.	CTL	PPR	PPR
Cigna Corporation	CI	Sodexho Alliance	EXHO
CIT Group Inc.	CIT	Unilever N.V.	ULVR
Comcast Cable Communications, LLC	CMCSA	E.ON AG	EON
ConocoPhillips	COP	Edison SPA	FERRUZ
Constellation Energy Group, Inc.	CEG	EDP - Energias de Portugal, SA	EDP
Countrywide Home Loans, Inc.	CCR	Electricite de France	EDF
Cox Communications, Inc.	COX	EnBW Energie Baden-Wuerttemberg AG	BAD
CSX Corporation	CSX	Endesa, SA	ELESM
Devon Energy Corporation	DVN	Enel SPA	ENEL
Dominion Resources, Inc.	D	Fortum Oyj	FORTUM
The Dow Chemical Company	DOW	Iberdrola, SA	IBERDU
Eastman Chemical Company	EMN	Repsol YPF SA	REP
General Electric Capital Corporation	GE	RWE Aktiengesellschaft	RWE
Honeywell International Inc.	HON	Suez	LYOE
IAC/InterActiveCorp	IACI	Union Fenosa SA	UNFSM
International Lease Finance Corporation	AIG	Vattenfall Aktiebolag	VATFAL
Lennar Corporation	LEN	Veolia Environnement	VEOLIA
Loews Corporation	LTR	ABN AMRO Bank NV	AAB
Marsh & McLennan, Inc.	MMC	Aegon NV	AEGON
National Rural Utilities	NRUC	AXA	AXAF
News America Incorporated	NWS	Banca Monte Dei Paschi Di Siena Spa	MONTE
Omnicom Group Inc.	OMC	Barclays Bank PLC	BACR
Progress Energy, Inc.	PGN	Commerzbank Aktiengesellschaft	CMZB
Pulte Homes, Inc.	PHM	Deutsche Bank Aktiengesellschaft	DB
Rohm and Haas Company	ROH	Hannover Rueckversicherung AG	HANRUE
Safeway Inc.	SWY	Muenchener Rueckversicherung	MUNRE
Sempra Energy	SRE	Swiss Reinsurance Company	SCHREI
Simon Property Group, L.P.	SPG-LP	Zurich Insurance Company	VERSIC
Southwest Airlines Co.	LUV	Adecco S.A.	ADO
Sprint Nextel Corporation	S	AKZO Nobel N.V.	AKZO
Starwood Hotels & Resorts	HOT	Bayer Aktiengesellschaft	BYIF
Textron Financial Corporation	TXT	Compagnie de Saint-Gobain	STGOBN
Time Warner Inc.	TW	EADS N.V.	EAD
Transocean Inc.	RIG	Lafarge	LAFCP
Union Pacific Corporation	UNP	Siemens Aktiengesellschaft	SIEM
Valero Energy Corporation	VLOC	UPM-Kymmene Oyj	UPMKYM
The Walt Disney Company	DIS	Bertelsmann AG	BERTEL
Washington Mutual, Inc.	WM	Deutsche Telekom AG	DT
Wells Fargo & Company	WFC	France Telecom	FRTEL
Weyerhaeuser Company	WY	Hellenic Telecommunications	OTE
Whirlpool Corporation	WHR	Koninklijke KPN N.V.	KPN
Wyeth	WYE	Reuters Group PLC	RTRGRP
XL Capital Ltd.	XL	Telecom Italia SPA	TIIMN
		Telefonica, SA	TELEFO
		Vodafone Group PLC	VOD

Table 2: Moments of CDO Tranche Spreads and Base Correlations

Panel A: CDX Firms	Average	Std Dev	Skewness	Kurtosis
TrancheUpfrontMid 0-3%	3446.73	894.47	0.49	2.63
TrancheSpreadMid 3-7%	139.52	61.93	0.98	3.15
TrancheSpreadMid 7-10%	40.29	27.89	1.31	3.92
TrancheSpreadMid 10-15%	18.58	13.71	1.70	5.70
TrancheSpreadMid 15-30%	9.19	8.30	2.40	8.76
Base Correlation 0-3%	0.14	0.05	1.00	3.58
Base Correlation 3-7%	0.29	0.07	2.23	7.92
Base Correlation 7-10%	0.37	0.08	2.15	7.26
Base Correlation 10-15%	0.48	0.09	1.91	6.25
Base Correlation 15-30%	0.71	0.09	1.21	4.16

Panel B: iTraxx Firms	Average	Std Dev	Skewness	Kurtosis
TrancheUpfrontMid 0-3%	2087.69	725.68	0.01	2.98
TrancheSpreadMid 3-6%	90.64	39.17	0.98	3.60
TrancheSpreadMid 6-9%	31.00	19.98	1.72	6.15
TrancheSpreadMid 9-12%	17.24	14.20	1.94	6.81
TrancheSpreadMid 12-22%	9.38	8.70	1.91	6.85
Base Correlation 0-3%	0.17	0.06	1.22	3.99
Base Correlation 3-6%	0.28	0.07	1.48	4.46
Base Correlation 6-9%	0.36	0.08	1.34	4.04
Base Correlation 9-12%	0.43	0.08	1.22	3.72
Base Correlation 12-22%	0.59	0.08	0.97	3.28

Notes to Table: We present the first four moments for the tranche spreads and base correlations based on the CDX Industrials and the iTraxx Europe indexes. The sample is October 14, 2004, through December 31, 2007.

Table 3: Unconditional Pairwise Correlations Between Implied Base Correlations (in percent)

	CDX BaseCorrelation 0-3%	CDX BaseCorrelation 3-7%	CDX BaseCorrelation 7-10%	CDX BaseCorrelation 10-15%	CDX BaseCorrelation 15-30%	iTraxx BaseCorrelation 0-3%	iTraxx BaseCorrelation 3-6%	iTraxx BaseCorrelation 6-9%	iTraxx BaseCorrelation 9-12%	iTraxx BaseCorrelation 12-22%
CDX BaseCorrelation 0-3%	100.00									
CDX BaseCorrelation 3-7%	78.91	100.00								
CDX BaseCorrelation 7-10%	65.13	97.67	100.00							
CDX BaseCorrelation 10-15%	54.87	93.88	98.99	100.00						
CDX BaseCorrelation 15-30%	40.08	84.86	93.60	97.31	100.00					
iTraxx BaseCorrelation 0-3%	89.96	89.23	81.42	74.16	61.68	100.00				
iTraxx BaseCorrelation 3-6%	78.41	93.32	89.62	84.77	74.47	96.02	100.00			
iTraxx BaseCorrelation 6-9%	72.52	92.04	89.77	85.86	76.72	92.56	99.32	100.00		
iTraxx BaseCorrelation 9-12%	68.15	90.39	89.16	85.92	77.70	89.87	98.18	99.68	100.00	
iTraxx BaseCorrelation 12-22%	59.03	86.02	86.94	85.11	79.07	83.50	94.31	97.15	98.62	100.00

Notes to Table: We compute unconditional correlations for the implied base correlations for iTraxx and CDX indexes. The sample period is from October, 14, 2004, through December 31, 2007.

Table 4: Moments of CDS Spreads (in Basis Points)
October 14, 2004 - December 31, 2007

	CDS Spreads for CDX Firms			
Moments Across Firms	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**
Mean	42.47	21.90	1.09	5.51
Median	35.95	12.23	0.74	3.58
Mininum	16.50	4.26	-0.66	1.88
Maximum	119.33	187.52	3.60	20.73

	CDS Spreads for iTraxx Firms			
Moments Across Firms	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**
Mean	32.44	10.90	1.04	4.66
Median	28.86	9.70	0.91	3.78
Mininum	13.11	3.96	-0.44	2.06
Maximum	71.28	35.67	2.58	9.59

Notes to Tables: We first compute the first four sample moments of CDS spreads across time for each of the 61 CDX and 64 iTraxx firms. We then report the mean, median, minimum and maximum of these moments across firms. The underlying sample consists of daily CDS premia for the period October 14, 2004, through December 31, 2007. The sample exclusively consists of firms who were part of the index over the entire sample period.

Table 5: Moments of Default Intensities from CDS Spreads (in Percent)
October 14, 2004 - December 31, 2007

	Default Intensities for CDX Firms				
Moments Across Firms	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**	**AutoCorr**
Mean	0.70	0.36	1.09	5.56	0.99
Median	0.60	0.20	0.74	3.61	0.99
Mininum	0.27	0.07	-0.66	1.88	0.96
Maximum	1.97	3.05	3.67	20.75	1.00

	Default Intensities for iTraxx Firms				
Moments Across Firms	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**	**AutoCorr**
Mean	0.54	0.18	1.04	4.71	0.99
Median	0.48	0.16	0.91	3.80	0.99
Mininum	0.22	0.07	-0.43	2.06	0.98
Maximum	1.18	0.59	2.62	9.85	1.00

Notes to Tables: We first compute the first four sample moments and first-order autocorrelations of the default intensities across time for each of the 61 CDX and 64 iTraxx firms. We then report the mean, median, minimum and maximum of these moments across firms. The underlying sample consists of daily default intensities extracted from CDS premia for the period October 14, 2004, through December 31, 2007. The sample exclusively consists of firms who were part of the index over the entire sample period.

Table 6: Moments of Standardized ARIMA-GARCH Intensity Residuals
October 14, 2004 - December 31, 2007

Default Intensity Residuals for CDX Firms

Statistics Across Firms	Mean	Std Dev	Skewness	Kurtosis	AutoCorr	LB(20) P-Value	LB(20) P-Val on Abs Residuals
Mean	0.01	1.05	1.76	28.63	0.04	0.29	0.31
Median	0.01	1.04	1.39	20.09	0.05	0.18	0.21
Mininum	-0.09	0.99	-2.87	5.67	-0.09	0.00	0.00
Maximum	0.06	1.22	8.21	141.86	0.19	1.00	0.99

Default Intensity Residuals for iTraxx Firms

Statistics Across Firms	Mean	Std Dev	Skewness	Kurtosis	AutoCorr	LB(20) P-Value	LB(20) P-Val on Abs Residuals
Mean	0.00	1.02	1.56	23.30	0.09	0.12	0.32
Median	0.01	1.01	1.02	12.02	0.09	0.01	0.25
Mininum	-0.06	1.00	-1.12	6.56	-0.05	0.00	0.00
Maximum	0.05	1.13	8.38	154.39	0.27	0.67	0.92

Notes to Tables: We first compute the first four sample moments and first-order autocorrelations of the default intensity residuals across time for each of the 61 CDX and 64 iTraxx firms. We also compute the P-values from the Ljung-Box tests that the first 20 autocorrelations of the residuals, and then of the absolute residuals, are zero. We then report the mean, median, minimum and maximum of these statistics across firms. The underlying sample consists of daily default intensity residuals from the ARMA-GARCH models estimated firm-by-firm for the period October 14, 2004, through December 31, 2007. The sample exclusively consists of firms who were part of the index over the entire sample period.

Table 7: Parameter Estimates for the DECO Model

Panel A: Intensity Innovations

	CDX Firms			iTraxx Firms		
	ω	**α**	**β**	**ω**	**α**	**β**
Estimate	0.0012	0.0307	0.9680	0.0044	0.0251	0.9701
Standard Error	3.16E-05	3.55E-03	3.04E-03	4.46E-04	1.20E-02	9.36E-03

Panel B: Equity Returns

	CDX Firms			iTraxx Firms		
	ω	**α**	**β**	**ω**	**α**	**β**
Estimate	0.0017	0.0166	0.9802	0.0027	0.0257	0.9710
Standard Error	9.17E-04	4.71E-03	5.08E-04	1.94E-04	3.74E-03	3.22E-03

Notes to Table: We report parameter estimates for the dynamic equicorrelation models. The sample period for equity returns is October 11, 2000 through December 31, 2007. The sample period for default intensities is October 14, 2004, through December 31, 2007.

Table 8: Correlation Moments

	Correlation with DECO Dynamic Correlations				Correlation Sample Moments			
	CDX Intensity	iTraxx Intensity	CDX Equity	iTraxx Equity	Average	Std Dev	Skewness	Kurtosis
CDX Equity DECO	61.47	76.04	100.00	49.71	0.17	0.05	1.05	3.81
CDX Intensity DECO	100.00	85.08	61.47	56.42	0.28	0.03	0.50	3.40
CDX BaseCorrelation 0-3%	37.26	56.21	44.33	4.80	0.14	0.05	1.00	3.58
CDX BaseCorrelation 3-7%	67.79	84.80	67.58	20.68	0.29	0.07	2.23	7.92
CDX BaseCorrelation 7-10%	72.93	86.81	69.51	26.26	0.37	0.08	2.15	7.26
CDX BaseCorrelation 10-15%	75.05	86.43	68.68	29.76	0.48	0.09	1.91	6.25
CDX BaseCorrelation 15-30%	75.37	82.87	65.77	35.76	0.71	0.09	1.21	4.16
iTraxx Equity DECO	56.42	44.53	49.71	100.00	0.33	0.08	1.27	3.66
iTraxx Intensity DECO	85.08	100.00	76.04	44.53	0.29	0.04	0.76	2.77
iTraxx BaseCorrelation 0-3%	54.42	76.13	65.38	15.32	0.17	0.06	1.22	3.99
iTraxx BaseCorrelation 3-6%	62.38	85.35	71.92	18.60	0.28	0.07	1.48	4.46
iTraxx BaseCorrelation 6-9%	62.49	86.59	73.09	17.88	0.36	0.08	1.34	4.04
iTraxx BaseCorrelation 9-12%	61.71	86.26	73.09	17.00	0.43	0.08	1.22	3.72
iTraxx BaseCorrelation 12-22%	59.74	83.33	71.77	16.60	0.59	0.08	0.97	3.28

Notes to Table: We compute unconditional correlations between base correlations and intensity and equity dynamic equicorrelations. We also report the first four sample moments of the correlation series. The sample period is from October, 14, 2004, through December 31, 2007.