Price-Level Targeting and Risk Management in a Low-Inflation Economy[1]

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Abstract

With inflation and policy interest rates at historically low levels, policymakers show great concern about "downside tail risks" due to a zero lower bound on nominal interest rates. Low probability or tail events, such as sustained deflation or recession, are disruptive for the economy and can be difficult to resolve. This paper shows that price-level targeting mitigates downside tail risks respect to inflation targeting when policy is conducted through a simple interest-rate rule subject to a zero lower bound. Thus, price-level targeting is a more effective policy framework than inflation targeting for the management of downside tail risks in a low-inflation economy. At the same time, the average performance of the economy is not very different if policy implements price-level targeting instead of inflation targeting through a simple interest-rate rule. Price-level targeting may imply less variability of inflation than inflation targeting because policymakers can shape private-sector expectations about future inflation more effectively by targeting directly the price level path rather than inflation.

Keywords: kurtosis, liquidity trap, long-run tradeoffs, monetary policy design, nonlinear, simple optimal rules, skewness, long-run stationary distribution

JEL Classification: C63, E31, E52

1 Introduction

The public, policymakers, and economists agree that a return to the high rates of inflation experienced in the 1970s and 1980s must be avoided because high inflation is detrimental to the economic well-being of the public.[1] As a result, central banks have adopted policies to keep inflation low in recent decades.

The Federal Reserve's preferred measure of inflation is the personal consumption expenditure (PCE) price index. This paper shows that in a small New-Keynesian sticky-price model, private-sector consumption is as much as 0.5 percentage point higher if PCE-price inflation in the long run is 1.5 percent per year rather than 3.5 percent per year.[2] Thus, the Federal Reserve must keep inflation low because reverting back to moderate or high rates of inflation would hamper the economic well-being of the public over the long run.

When inflation is low and expected to remain low, then nominal interest rates tend to be low. But nominal interest rates cannot fall below zero under normal circumstances.[3] Since central banks counteract slowing economic activity by lowering short-term interest rates, the extent to which policymakers can respond to an economic slowdown is limited in a low-inflation economy. Once short-term nominal interest rates fall to zero, conventional monetary policy tools no longer work to stimulate economic activity. As a consequence, downside risks to the economy, such as deflation or recession, are greater when inflation is low.

With inflation and policy interest rates at historically low levels, policymakers show concern about downside risks due to a zero lower bound on nominal interest rates. Policymakers are particularly concerned about downside tail risks, such as sustained deflation or recession, which are disruptive for the economy and can be difficult to resolve. Moreover, models are typically better approximations of how the economy functions on average rather than in extreme circumstances. As a result, policymakers have an incentive to embrace 'risk management' and make economic decisions that are robust to the occurrence of low probability catastrophes or tail risks.

The economics literature provides policymakers intuition about the effectiveness of some monetary policy frameworks for the management of downside risks due to a zero lower bound on nominal interest rates, such as deflation or recession. However, the economics literature does not provide intuition about the effectiveness of monetary policy frameworks for the management

[1] See Fischer (1996) for a discussion of the costs of inflation.

[2] Available measures of inflation tend to be biased upward. As former Federal Reserve Governor Gramlich (2003) discusses, recent estimates place the measurement bias in the PCE price index at about 0.5 percentage point per year. Such bias has to be added to the model-based inflation rate to obtain an actual, measured inflation rate.

[3] In theory, achieving negative nominal interest rates is feasible by levying a tax on money holdings or giving up free convertibility of financial assets into cash—Buiter and Panigirtzoglou (2003), and Goodfriend (2000) discuss this idea.

Policy Framework[a]	Loss%	s.d.(i)%	Fr($i = 0$)%	s.d.(π)%	Skew.(π)	Kurt.(π)
Optimal Policy	-0.13	1.6	0.4	0.91	0.0	3.0
Optimal Simple IT Rule[b]	-0.17	1.2	0.0	0.80	0.0	3.0
Optimal Simple PLT Rule[c]	-0.18	1.0	0.0	0.77	0.0	3.0
Aggress. Simple IT Rule[d]	-0.18	1.9	1.0	0.65	-0.3	3.8
Aggress. Simple PLT Rule[e]	-0.19	1.4	0.0	0.65	0.0	3.0

[a]PCE-price inflation in the long run is 1.5 percent per year under each policy framework. The simple policy rules change the nominal interest rate subject to a zero lower bound in response to inflation deviations from the inflation target, or price-level deviations from the target path, but not output deviations.

[b]$\phi_\pi = 1.0$; [c]$\phi_p = 0.4$; [d]$\phi_\pi = 2.5$; [e]$\phi_p = 1.0$.

Table 1: Low Inflation and the Zero Lower Bound in the small New-Keynesian Model

of downside tail risks, such as sustained deflation or recession.

For instance, Coenen and Wieland (2004), Eggertsson and Woodford (2003), Gaspar, Smets and Vestin (2007), McCallum (2000), Nakov (2008), Svensson (2003), Wolman (2005), and others show how price-level targeting can mitigate the zero-lower-bound constraint on the policy interest rate and help manage downside risks. The private sector anticipates that the central bank will undo any price changes under price-level targeting, and thus a central bank is more effective at shaping private-sector expectations about future inflation by targeting directly the price level path rather than inflation. Such studies, however, do not show how price-level targeting can help manage downside tail risks.

This paper shows the effectiveness of price-level targeting, as an alternative to inflation targeting, for the management of downside tail risks in a small New-Keynesian sticky-price model when policy is conducted through a simple interest-rate rule subject to a zero lower bound on nominal interest rates. The performance of the economy under simple policy rules, which implement price-level targeting or inflation targeting, is compared to the welfare-maximizing performance that would be achieved under time-zero optimal policy subject to a zero lower bound.

Table 1 summarizes the main findings. First, when policy is conducted through an aggressive simple policy rule the variability of the nominal interest rate rises, thus the likelihood of hitting the zero lower bound rises, and the performance of the economy deteriorates. Assuming PCE-price inflation in the long run is 1.5 percent per year, which is in line with the findings of Billi (2007) regarding the optimal long-run rate of inflation, private-sector consumption is roughly 0.06 percentage point lower under an aggressive simple price-level targeting rule than time-zero optimal policy. Thus, the policymaker can sustain a level of consumption for the private sector which is close to fully optimal even when policy is conducted through a aggressive simple policy rule.

Second, table 1 shows also that the effects of the zero lower bound are less severe if policy is conducted through a simple price-level-targeting rule, since the policymaker can shape private-sector expectations more effectively under price-level targeting than inflation targeting. Inflation in the long run has a longer left tail and fatter tails—negative skewness and higher kurtosis—if policy is conducted through an aggressive simple inflation-targeting rule.[4] Thus, price-level targeting provides 'insurance' against downside tail risks. The cost of such insurance is the loss in performance of the economy bared on average to avoid tail risks. Since private-sector consumption is roughly 0.01 percentage point lower under price-level targeting than inflation targeting, the cost of insurance is not large.[5] Price-level targeting implies less variability of inflation than inflation targeting because the policymaker can shape private-sector expectations about future inflation more effectively by targeting directly the price level path rather than inflation.

The remainder of this paper is structured as follows. Section 2 introduces the model, then Section 3 explains the equilibrium definition. In Section 4, the model is calibrated to recent U.S. data. Section 5 shows that moderate inflation imparts substantial costs respect to low inflation. Sections 6 and 7 show that price-level targeting is a more effective policy framework than inflation targeting for the management of tail risks of deflation or recession. Section 8 shows the robustness of the findings to a wide range of calibrations, and Section 9 briefly concludes.

2 Model

The setting adopts the well-known sticky-price version of the small New-Keynesian model, which is discussed in-depth by Clarida, Galí and Gertler (1999), Woodford (2003a), Galí (2008), and others.[6]

2.1 Private Sector

The private sector consists of a representative consumer and firms in monopolistic competition facing restrictions on the frequency of price adjustments à la Calvo (1983). Thus, the behavior of the private sector is described by

[4] Also the output gap in the long run has a longer and fatter left tail when policy is conducted through a simple inflation-targeting rule that prescribes an aggressive response to output deviations from an output target.

[5] The approximated welfare-theoretic objective function in the small New-Keynesian model is quadratic in deviations of inflation from zero and deviations of output from the socially efficient level. The level of welfare is not very different if policy is conducted through a simple price-level-targeting rule or a simple inflation-targeting rule, because the variability of inflation and the variability of the output gap are not very different if policy implements price-level-targeting or inflation-targeting through a simple rule.

[6] To save space, the complete derivation of the small New-Keynesian model is not shown here.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

$$x_t = E_t x_{t+1} - \varphi \left(i_t - E_t \pi_{t+1} - r_t^n \right) \tag{2}$$

$$u_t = \rho_u u_{t-1} + \sigma_{\varepsilon u} \varepsilon_{ut} \tag{3}$$

$$r_t^n = \rho_r r_{t-1}^n + \left(1 - \rho_r\right) \bar{r} + \sigma_{\varepsilon r} \varepsilon_{rt} \tag{4}$$

where E_t denotes the rational expectations operator conditional on all information available at time t. π_t is the inflation rate, and x_t is the output gap or the deviation of output from its flexible-price equilibrium.[7] Monetary policy controls the nominal interest rate i_t.[8]

Equation (1) is a log-linear approximation to the aggregate-supply relation, which describes the optimal price-setting behavior of firms under staggered price setting. The slope parameter

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\varphi^{-1} + \omega}{1 + \omega\theta} > 0 \tag{5}$$

is a function of the structure of the model economy. $\beta \in (0,1)$ is the subjective discount factor. $\theta > 1$ is the price elasticity of demand substitution among differentiated goods produced by firms operating in monopolistic competition. $\omega > 0$ is the elasticity of a firm's real marginal cost with respect to its own output level. Each period, a share $\alpha \in (0,1)$ of randomly picked firms cannot adjust their prices and the remaining $(1-\alpha)$ firms get to choose prices optimally. The shifter of the aggregate-supply curve, u_t, is interpreted as a 'mark-up' shock or the variation over time in the degree of monopolistic competition between firms.

Equation (2) is a log-linear approximation to the intertemporal Euler equation describing the representative consumer's private expenditure decisions. $\varphi > 0$ is the intertemporal elasticity of substitution or the real-rate elasticity of output. Shifting the Euler equation is the 'natural' real-rate of interest shock r_t^n.[9]

Equations (3) and (4) describe the evolution of the exogenous mark-up shock (u_t) and the real-rate shock (r_t^n). The shocks follow AR(1) stochastic processes with autoregressive coefficients $\rho_j \in (-1,1)$ for $j = u, r$. The deterministic steady state of the real interest rate is $r_{ss} \equiv 1/\beta - 1$, such that $r_{ss} \in (0, +\infty)$. The innovations ($\sigma_{\varepsilon j} \varepsilon_{jt}$ for $j = u, r$) are independent both across time and cross-sectionally, and normally distributed with mean zero and standard deviations $\sigma_{\varepsilon j} \geq 0$ for $j = u, r$.

[7] Output is efficient at its deterministic steady state level due to an output subsidy that neutralizes the distortions from monopolistic competition.

[8] By abstracting from money-demand distortions associated with positive nominal interest rates, the model can be interpreted as the 'cashless limit' of a model with money holdings.

[9] The real-rate shock summarizes all shocks that under flexible prices generate variation in the real interest rate; it captures the combined effects of preference shocks, productivity shocks, and exogenous changes in government expenditure.

2.2 Policy

Mainly for reasons of analytical tractability, the economics literature typically studies policies which do not rule out negative nominal interest rates.[10] This paper, instead, contrasts the performance of two policy rules within the family of implementable, simple interest-rate rules. The rules are implementable because policy ensures the zero lower bound on nominal interest rates is never violated ($i_t \geq 0$ for *all* t and *each* state of the economy). The rules are simple because policy responds to readily observable macroeconomic variables rather than the shocks buffeting the economy.

Simple Inflation-Targeting Rule. Under the first policy rule, when the zero lower bound is not binding ($i_t > 0$) the change in the nominal interest rate responds to deviations of inflation from an inflation target π^* and deviations of output from an output target x^*according to

$$i_t = \max\left[0, i_{t-1} + \phi_\pi \left(\pi_t - \pi^*\right) + \phi_x \left(x_t - x^*\right)\right] \tag{6}$$

where $\pi_t \equiv p_t - p_{t-1}$ and p_t is the price level in period t. The output target is the steady-state value of output consistent with the inflation target, where $x^* \equiv (1-\beta)\,\kappa^{-1}\pi^*$ solves equation (1) in steady state, so that the change in the nominal interest rate is on average equal to zero in an equilibrium in which the inflation target is achieved on average.

Simple Price–Level-Targeting Rule. The second policy rule is given by

$$i_t = \max\left[0, i_{t-1} + \phi_p \left(p_t - \bar{p}_t\right) + \phi_x \left(x_t - x^*\right)\right] \tag{7}$$

where the change in the nominal interest rate responds to deviations of output from an output target x^* as before, but responds also to deviations of the price level p_t from a target path for the price level $\{\bar{p}_t\}$, which grows deterministically at a rate π^*.

The simple targeting rules (6) and (7) are 'first-difference' interest-rate rules since they set the change in the nominal interest rate from its past level. In contrast, 'partial-adjustment' interest-rate rules set the current level of the nominal interest rate putting less or no weight on its past level.[11] Why policymakers should employ a first-difference rule rather than a partial-adjustment rule? Under a first-difference interest-rate rule, the policymaker does not need to know the equilibrium value of the interest rate, since the change in the nominal interest rate is zero when price changes are at the target rate in equilibrium. Thus, policy is less difficult to implement or communicate to the public with a first-difference rule because policy requires

[10] See for example Woodford (2003a) or Galí (2008), and references therein.

[11] With partial-adjustment of the nominal interest rate from its past level, the simple inflation-targeting rule (6) for instance has the more general representation $i_t = \max\left[0, (1-\phi_i)\,\bar{\imath} + \phi_i i_{t-1} + \phi_\pi \left(\pi_t - \pi^*\right) + \phi_x \left(x_t - x^*\right)\right]$, where $\bar{\imath} = r_{ss} + \pi^*$ is the equilibrium value of the nominal interest rate. However, $\bar{\imath}$ is irrelevant for the simple targeting rules studied in this paper since $\phi_i = 1$.

less information about the economy.[12]

Woodford (2003a) explains the properties of the simple targeting rules (6) and (7) in the theoretical case where nominal interest rates are allowed to be negative. When the policymaker uses an interest-rate rule of the form (6), but in addition can set nominal interest rates to negative values, then equilibrium is determinate if and only if the policy response coefficients satisfy $\phi_\pi > -(1-\beta)\,\kappa^{-1}\phi_x$. When the policymaker uses instead a rule of the form (7), but can set negative nominal interest rates, then equilibrium is necessarily determinate if the policy response coefficients satisfy $\phi_p > 0$ and $\phi_x \geq 0$.

By not taking into account the zero lower bound on nominal interest rates, the rational-expectations equilibrium in the small New-Keynesian model is necessarily determinate regardless of how small the policy response by the policymaker if policy responds to price deviations from target ($\phi_\pi > 0$ or $\phi_p > 0$), but not output deviations ($\phi_x = 0$). Intuitively, no matter how small the policy response may be, a sustained increase in inflation in excess of the target eventually results in the nominal interest rate being permanently raised by more than the amount of the excess inflation. In other words, the Taylor principle is satisfied by any such rule regardless of the strength of the policy response.

Once the zero lower bound on nominal interest rates is correctly taken into account, however, the policymaker cannot respond too aggressively to price deviations from target to ensure determinacy of equilibrium. Intuitively, monetary policy cannot stabilize the economy if nominal interest rates are excessively variable and thereby the zero lower bound is encountered too frequently. Thus, the policy response coefficients have an upper bound, $\bar{\phi}_j \subset (0, +\infty)$ for $j = \pi, p, x$, beyond which interest-rate policy cannot ensure determinacy of equilibrium.[13]

The small New-Keynesian model is developed from explicit micro-foundations. As a result, a welfare-theoretic objective function can be derived by taking a second-order Taylor series approximation to the expected life-time utility of the consumer. Woodford (2003a) shows that the resulting objective function is quadratic in deviations of output from the socially efficient level and deviations of inflation from zero. Thus, the policymaker could chose the response coefficients of the simple policy rules to maximize welfare for the representative consumer.

In the case of 'optimal' simple policy rules, the optimal policy response coefficients, ϕ_π^{op} or ϕ_p^{op}, maximize the welfare-theoretic objective function

[12] In addition, numerical simulations show that the policymaker can sustain a level of private-sector consumption that is closer to the fully optimal level, achieved under time-zero optimal policy, if the policymaker uses a first-difference rule rather than a partial-adjustment rule. Intuitively, more dependence of current policy actions on past policy allows the policymaker to steer private-sector expectations of future policy more effectively. Woodford's (2003b) argument that policymakers should embrace interest-rate smoothing to mimic optimal policy is even stronger when the policy interest rate approaches the zero lower bound.

[13] A practical example of the upper bound on the policy response coefficients is given in Section 5.

$$-E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right] \tag{8}$$

subject to the model equations (1)-(4) and the policy rule (6) or (7), where the weight assigned to the goal of output stability, relative to price stability,

$$\lambda \equiv \frac{\kappa}{\theta} > 0 \tag{9}$$

is a function of the structure of the model economy.

3 Equilibrium

Equations (1), (2) and (6) or (7) form a nonlinear system of three equations with three unknowns, which must be satisfied by policy in equilibrium. Solving the system delivers a three-dimensional nonlinear equilibrium response function

$$y(s_t) \equiv (\pi_t, x_t, i_t \geq 0) \subset R^3$$

over a four-dimensional state space

$$s_t \equiv (u_t, r_t^n, i_{t-1} \geq 0, \hat{p}_{t-1}) \subset R^4$$

where $\hat{p}_{t-1}$ is the deviation of the price level from its target path in period $t-1$ ($\hat{p}_{t-1} \equiv p_{t-1} - \bar{p}_{t-1}$).

When the policymaker uses the simple price-level-targeting rule (7), it needs to know the price level in period $t-1$ to conduct policy in period t. When the policymaker uses the simple inflation-targeting rule (6), however, $\hat{p}_{t-1}$ is not a state variable of the policy problem because the price level is irrelevant for the equilibrium of the economy under a policy regime of inflation targeting.

The state in period $t+1$ depends on the state and equilibrium response in period t and the shock innovations that are unknown in period t,

$$s_{t+1} = g(s_t, y(s_t), \varepsilon_{t+1}) \tag{10}$$

Associated with the equilibrium response function, the expectations function is

$$E_t y_{t+1}(s_t) = \int y(g(s_t, y(s_t), \varepsilon_{t+1})) f(\varepsilon_{jt+1}) d(\varepsilon_{t+1}) \tag{11}$$

where $f(\cdot)$ is the probability density function of the shock innovations, $\varepsilon_t \equiv (\varepsilon_{ut}, \varepsilon_{rt}) \in R^2$.[14] The following definition of a stochastic rational expectations equilibrium is proposed.

Definition 1 (SREE) *Assume $\sigma_{\varepsilon j} \geq 0$ for $j = u, r$. A 'stochastic rational expectations equilibrium' of the model is a nonlinear response function $y(s_t)$, over the state s_t with law of motion (10), such that the nonlinear system of equilibrium conditions (1), (2) and (6) or (7) is satisfied.*

Importantly, the nonlinear system in Definition 1 does not have a closed-form solution. A numerical procedure must be used to find a fixed-point in the space of nonlinear response functions. Since the number of state variables is unusually high for a model with an occasionally-binding constraint on policy, the algorithm must be highly efficient. Billi (2007) explains the numerical procedure.

4 Calibration

The model is calibrated to the U.S. economy and the time period is one quarter. Table 2 summarizes the baseline parameter values, which are expressed in quarters unless otherwise noted. The values for the main structural parameters ($\varphi, \alpha, \theta, \omega$, and the resulting κ, λ) are taken from tables 5.1 and 6.1 of Woodford (2003a).

The parameters describing the shock processes ($r_{ss}, \sigma_{\varepsilon r}, \rho_r$, and $\sigma_{\varepsilon u}, \rho_u$) are estimated over the period 1983:1−2007:4, with the same approach of Rotemberg and Woodford (1997) and Adam and Billi (2006).[15] The predictions of an unconstrained VAR in inflation, the output gap, and the nominal interest rate are used to construct the expectations of inflation and the output gap.[16] These estimated expectations and the actual data are then plugged into equations (1) and (2). The equation residuals identify the historical shock processes u_t and r_t^n. Fitting AR(1) processes to the historical shocks justifies the shock processes in table 2.

The quarterly subjective discount factor is $\beta = (1 + r_{ss})^{-\frac{1}{4}} \approx 0.9926$, as implied by the estimate for the deterministic steady state of the real interest rate $r_{ss} = 3.0$ percent per year.

[14] When agents have 'perfect foresight' ($\sigma_{\varepsilon j} \to 0$ for $j = u, r$), however, the state in period $t+1$ is completely described by the state and equilibrium response in period t, $s_{t+1} = g(s_t, y(s_t))$. Since agents can anticipate future variables with certainty, the expectations function (11) is not integrated over the probability density function of the shock innovations, $E_t y_{t+1}(s_t) = y(g(s_t, y(s_t)))$.

[15] Adam and Billi (2006) estimate the historical shocks over the shorter period 1983:1−2002:4. The steady state real interest rate is lower and the mark-up shock is more variable, and thereby the effects of the zero lower bound are more severe, when the historical shocks are estimated over the longer period 1983:1−2007:4.

[16] *Inflation* is measured as the continuously compounded rate of change in the GDP Chain-type Price Index, from the Bureau of Economic Analysis. The *output gap* is measured as the difference between Real GDP, from the Bureau of Economic Analysis, and Real Potential GDP, from the Congressional Budget Office. The *nominal interest rate* is measured as the average effective federal funds rate, from the Federal Reserve Board of Governors.

Parameter Definition	Assigned Value
Subjective discount factor	$\beta = 0.9926$
Real-rate elasticity of output	$\varphi = 6.25$
Share of firms keeping prices fixed	$\alpha = 0.66$
Price elasticity of demand	$\theta = 7.66$
Elasticity of firms' marginal cost	$\omega = 0.47$
Slope of the Phillips Curve	$\kappa = 0.024$
Weight on output gap in the utility function	$\lambda = 0.003$
Steady state real interest rate	$r_{ss} = 3.0\%$ per year
s.d. real-rate shock innovation	$\sigma_{\varepsilon r} = 0.24\%$
s.d. mark-up shock innovation	$\sigma_{\varepsilon u} = 0.30\%$
AR(1)-coefficient of real-rate shock	$\rho_r = 0.8$
AR(1)-coefficient of mark-up shock	$\rho_u = 0.1$
Inflation Target	$\pi^* = 1.0\%$ per year
Interest-rate response to inflation, or	$\phi_\pi = 1.0$
Interest-rate response to the price level	$\phi_p = 0.4$
Interest-rate response to the output gap	$\phi_x = 0.0$

Table 2: Baseline Calibration (Quarterly Model)

In line with the findings of Billi (2007) regarding the optimal long-run rate of inflation, the inflation target is $\pi^* = 1.0$ percent per year, which corresponds to an inflation target of 1.5 percent per year for the PCE price index after accounting for 0.5 percentage point per year inflation measurement bias. The policy response coefficients are such that welfare for the representative consumer is maximized when the policymaker responds optimally to price deviations from target ($\phi_\pi^{op} = 1$ or $\phi_p^{op} = 0.4$), but not output deviations ($\phi_x = 0$).

5 Why Low Inflation?

Figure 1 shows the representative consumer's welfare using the baseline calibration in table 2 under the different simple policy rules. Accordingly, the policymaker uses a simple policy rule that responds to price deviations from target ($\phi_\pi > 0$ or $\phi_p > 0$), but not output deviations ($\phi_x = 0$). The representative consumer's welfare is measured in terms of its permanent consumption loss due to business cycle fluctuations, which is derived via a transformation of the unconditional loss in the welfare-theoretic objective function (8).[17] Appendix A.1 explains the computation of the permanent consumption loss.

[Figure 1 about here]

[17] The unconditional loss is computed as the average discounted loss across 10^4 stochastic simulations, each 10^3 periods long after discarding several pre-simulated periods in order to ascertain that the distribution did reach its stationary configuration prior to the computation of the loss.

The left-hand panel of figure 1 shows the permanent consumption loss if the policymaker uses a simple inflation-targeting (IT) rule. The permanent consumption loss depends on the strength of the policy response to inflation deviations from target. For a range of inflation targets, it is optimal for the policymaker to change the nominal interest rate one-to-one in response to deviations of inflation from target. The various lines appear rather flat, but they all peak at $\phi_\pi = 1$. At the same time, moderate inflation imparts substantial costs on consumers respect to low inflation. The level of permanent consumption is as much as 1.0 percentage point higher if inflation is on average 1.0 percent per year rather than 4.0 percent per year (difference between the line with circles and the line with triangles) when inflation measurement bias is not considered.

Although moderate inflation is costly, inflation can be too low. When the policymaker aims for low inflation, a policy response that is stronger than optimal is more likely to be excessive for the determination of equilibrium. Excessive easing of policy causes too frequent encounters of the policy interest rate with the zero lower bound thus monetary policy fails to stabilize the economy. If the policymaker aims at zero inflation correctly measured (line with crosses in the top-left panel of figure 1), then $\bar{\phi}_\pi = 1.5$ is the strongest policy response for which the numerical algorithm can identify an equilibrium.

As a point of comparison, the fully optimal equilibrium is attainable in theory if the policymaker implements time-zero optimal policy. The bottom-left panel of figure 1 compares welfare under a simple IT rule (solid line with circles) and the time-zero optimal policy (dotted line with triangles).[18] When the inflation target is 1.0 percent per year, if inflation measurement bias is not considered, a simple IT rule prescribing a one-to-one change in the nominal interest rate attains a level of permanent consumption roughly 0.04 percentage point less than time-zero optimal policy. Thus, an optimal simple IT rule attains a level of welfare for the representative consumer that is close to fully optimal.

The right-hand panel of figure 1 shows the permanent consumption loss if the policymaker follows a simple price-level-targeting (PLT) rule. The permanent consumption loss is a function of the intensity of the policy response to price-level deviations from the target path. For a range of inflation targets, it is optimal for the policymaker to change the nominal interest rate 0.4-to-1 in response to deviations of the price level from the target path. The various lines appear rather flat, but they all peak at $\phi_p = 0.4$. At the same time, moderate inflation entails substantial costs on consumers when compared to low inflation, similarly to the case of the policymaker using a simple IT rule.

[18] Billi (2007) explains the solution of the time-zero optimal policy problem. The policymaker selects the equilibrium paths of inflation, the output gap, and the nominal interest rate $\{\pi_t, x_t, i_t\}_{t=0}^{\infty}$ to maximize $-E_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi^*)^2 + \lambda x_t^2 \right]$ and achieve the inflation target π^*. Welfare is evaluated on objective (8).

In contrast to the case of the simple IT rule, however, a simple PLT rule with a stronger-than-optimal response ensures determination of equilibrium over the range of policy responses considered, even when the policymaker aims at zero inflation correctly measured (line with crosses in the top-right panel of figure 1). Intuitively, price-level targeting stabilizes the economy even when the policymaker is too aggressive because price-level targeting is more effective at shaping private-sector expectations than inflation targeting, and thereby the zero lower bound is not encountered as frequently.

The bottom-right panel of figure 1 shows the welfare comparison between the simple PLT rule (solid line with circles) and the time-zero optimal policy (dotted line with triangles). When the policymaker aims at a rate of inflation of 1.0 percent per year, if inflation measurement bias is not taken into account, the simple PLT rule that prescribes a 0.4-to-1 change in the nominal interest rate attains a level of permanent consumption about 0.05 percentage point less than time-zero optimal policy. Thus, the optimal simple PLT rule attains a level of welfare for the representative consumer that is not as high as the optimal simple IT rule, but still close to fully optimal.

6 Price-Level Targeting Mitigates the Tail Risk of Deflation

Figure 2 shows the standard deviations of inflation and the nominal interest rate, and the frequency of zero nominal interest rates, when the policymaker uses a simple policy rule that responds to price deviations from target ($\phi_\pi > 0$ or $\phi_p > 0$), but not output deviations ($\phi_x = 0$). In the left-hand panel the policymaker uses the simple IT rule, while in the right-hand panel the policymaker uses the simple PLT rule. Independent of which simple policy rule the policymaker uses, a more aggressive response to price deviations from target gives rise to lower variability of inflation (top panel). At the same time, the better performance of the economy on the inflation front is attained through higher variability of the nominal interest rate (middle panel). As a result, the likelihood of the nominal interest rate hitting the zero lower bound rises if the policymaker is more aggressive fighting prices deviations from target (bottom panel).

[Figure 2 about here]

Figure 2 shows also that the variability of inflation (top panel) is not very different if the policymaker uses the simple PLT rule or the simple IT rule. At the same time, the nominal interest rate is less variable (middle panel) and the zero lower bound is encountered less frequently (bottom panel) if the policymaker uses the simple PLT rule rather than the simple IT rule. Intuitively, price-level targeting protects the economy against hitting the zero lower bound more frequently because the policymaker is more effective at shaping private-sector expectations about future inflation by targeting directly the price-level path rather than inflation.

Figure 3 shows the long-run stationary distribution of inflation. The distribution is presented in terms of probability density.[19] In the various panels, the dashed-vertical lines indicate the unconditional mean. In the top panel the policymaker uses a simple policy rule that responds optimally to price deviations from target ($\phi_\pi = 1$ or $\phi_p = 0.4$), but does not respond to output deviations ($\phi_x = 0$). The variability of inflation is not very different if the policymaker uses the optimal simple PLT rule or the optimal simple IT rule. When the policymaker uses an optimal simple rule, the long-run stationary distribution of inflation is symmetric and normally distributed around the unconditional mean because the nominal interest rate does not hit the zero lower bound too frequently.[20]

[Figure 3 about here]

The bottom panel of figure 3 shows the long-run stationary distribution of inflation when the policymaker uses a simple policy rule with a response to price deviations that is more aggressive than optimal ($\phi_\pi = 2.5$ or $\phi_p = 1$). Compared to an optimal simple rule, the long-run stationary distribution of inflation has a longer left tail (skewness is -0.3) and fatter tails (kurtosis is 3.8) than a normal distribution if the policymaker reacts too aggressively to inflation based on a simple IT rule.[21] The variability of inflation is not very different if the policymaker uses the aggressive simple PLT rule or the aggressive simple IT rule. At the same time, the long-run stationary distribution of inflation remains symmetric and normally distributed when the policymaker uses an aggressive simple PLT rule. Thus, price-level targeting protects the economy against the tail risk of deflation respect to inflation targeting.

7 Price-Level Targeting Mitigates the Tail Risk of Recession

Figure 4 shows the implications of the policymaker responding to output deviations from target ($\phi_x > 0$), in addition to responding optimally to price deviations from target ($\phi_\pi = 1$ or

[19] The distribution is computed by assembling 10^5 stochastic simulations at a specific time period. The simulations are initialized to the deterministic steady state of the model. By tracking the time-evolution of the mean, standard deviation, skewness, and kurtosis, it is ascertained that the distribution did reach its long-run stationary configuration.

[20] The zero lower bound on the nominal interest rate is the only nonlinearity in the model. In a linearized model the endogenous variables inherit the properties of the exogenous shock processes. The mark-up shock and real-rate shock are normally distributed. Thus, also the inflation rate and the output gap are normally distributed if the nominal interest rate does not hit the zero lower bound too frequently.

[21] The coefficient of skewness of a normal distribution is 0, while negative (positive) skewness indicates a longer left (right) tail. The kurtosis of a normal distribution is 3, while higher (lower) kurtosis indicates a sharper peak and fatter tails (smaller peak and thinner tails).

$\phi_p = 0.4$). The top panel compares welfare under the different simple policy rules (solid lines with circles) and the time-zero optimal policy (dotted line with triangles). When the policymaker uses a simple IT rule (left panel) or a simple PLT rule (right panel), a small positive response to output deviations from target is beneficial on welfare grounds. It is optimal for the policymaker to change the nominal interest rate 0.2-to-1 in response to output deviations from target ($\phi_x = 0.2$).[22] However, the welfare gain of responding to output deviations is marginal if the policymaker already responds optimally to price deviations from target.[23]

[Figure 4 about here]

Figure 4 shows also that the variability of the output gap (middle panel) is not very different if the policymaker uses the simple PLT rule or the simple IT rule. However, responding strongly to output deviations from target reduces significantly the variability of the output gap. At the same time, the improvement in terms of greater stability of the output of the economy is traded off with higher variability of the nominal interest rate. Thus, responding strongly to output deviations from target ultimately makes policy less flexible and leads to a higher likelihood of the nominal interest rate hitting the zero lower bound (bottom panel).

[Figure 5 about here]

The top panel of figure 5 shows that the long-run stationary distribution of the output gap is symmetric and normally distributed if the policymaker uses a simple policy rule that responds optimally to price deviations from target ($\phi_\pi = 1$ or $\phi_p = 0.4$) and has a muted response to output deviations from target ($\phi_x = 0.2$). The bottom panel shows the long-run stationary distribution of the output gap if the policymaker uses a simple policy rule that instead has a stronger-than-optimal responses to output deviations from target ($\phi_x = 0.6$). The variability of the output gap is not very different if the policymaker uses a simple PLT rule or a simple IT rule. When the policymaker uses a simple IT rule, the long-run stationary distribution of the output gap has a longer left tail (skewness is -0.5) and fatter tails (kurtosis is 6.9) than a normal distribution. When the policymaker uses a simple PLT rule, however, the long-run

[22] As can be seen in the top panel of figure 4, numerical simulations show that the policymaker can sustain a level of permanent consumption that is closer to the fully optimal level if the policymaker uses a simple PLT rule including a small positive response to output deviations from target rather than a simple IT rule. Intuitively, some dependence of current policy actions on the past state of the economy allows the policymaker to steer private-sector expectations even more effectively under price-level targeting than inflation targeting.

[23] Also Schmitt-Grohé and Uribe (2007) show that a very small positive response to output fluctuations is optimal if the policymaker uses a simple policy rule that embraces interest-rate smoothing. In addition, they show that responding to output fluctuations leads to a significant welfare loss if the policymaker uses a simple policy rule that does not embrace interest-rate smoothing. Schmitt-Grohé and Uribe's (2007) argument that policymakers should not respond to output fluctuations is even stronger when the policy interest rate approaches the zero lower bound.

Calibration	Aggress. Simple IT Rule[a]			Aggress. Simple PLT Rule[b]		
	Loss%	Skew.(π)	Kurt.(π)	Loss%	Skew.(π)	Kurt.(π)
Baseline	-0.18	-0.3	3.8	-0.19	0.0	3.0
More variable mark-up shock[c]	-0.22	-0.4	4.2	-0.23	0.0	3.0
More variable real-rate shock[d]	-0.18	-0.9	7.0	-0.19	0.0	3.0
Both shocks more variable[e]	-0.23	-1.2	8.2	-0.24	0.0	3.0

[a]$\phi_\pi = 2.5$; [b]$\phi_p = 1.0$; [c]$1.2 \cdot \sigma_{\varepsilon u}$; [d]$1.2 \cdot \sigma_{\varepsilon r}$; [e]$1.2 \cdot \sigma_{\varepsilon}$.

Table 3: Robustness of Simple Price-Level-Targeting Rule to More Variable Shocks

stationary distribution of the output gap is closer to normal (skewness is -0.4 and kurtosis is 6.2). Thus, price-level targeting protects the economy against the tail risk of recession respect to inflation targeting.

8 Robustness of Price-Level Targeting

Table 3 compares the findings for the baseline level of uncertainty to alternative scenarios of greater uncertainty about the future state of the economy. When the policymaker sets the policy interest rate using an aggressive simple IT rule, the long-run stationary distribution of inflation has skewness -0.3 for the baseline, -0.4 if the mark-up shock is 20 percent more variable than the baseline, and -0.9 if instead the real-rate shock is 20 percent more variable than the baseline. The skewness rises to as much as -1.2 for the scenario of both type of shocks 20 percent more variable than the baseline. At the same time, the kurtosis rises to 3.8 when both type of shocks are 20 percent more variable than the baseline. Thus, the more variable the shocks buffeting the economy the longer the left tail (negative skewness) and the fatter the tails (higher kurtosis) of the long-run stationary distribution of inflation, since the risk of the nominal interest rate hitting the zero lower bound is greater.

Table 3 shows also that, if the policymaker sets the policy interest rate using an aggressive simple PLT rule rather than an aggressive simple IT rule, the long-run stationary distribution of inflation does not have a longer left tail (there is no skewness) and the tails are not fatter (kurtosis is not higher) than a normal distribution, even when both type of shocks are 20 percent more variable than the baseline. Thus, a simple PLT rule protects the economy against the tail risk of deflation respect to a simple IT rule, even more so when there is greater uncertainty about the future state of the economy and thereby the risk of the nominal interest rate hitting the zero lower bound is greater. Moreover, the variability of inflation and the variability of the output gap are not very different if the policymaker uses an aggressive simple IT rule or an aggressive simple PLT rule. As a result, the cost of protection against tail risks (difference in loss between the two policy regimes) is not large, since it is roughly 0.01 percentage point of permanent consumption.

Calibration	Aggress. Simple IT Rule[a]			Aggress. Simple PLT Rule[b]		
	Loss%	Skew.(π)	Kurt.(π)	Loss%	Skew.(π)	Kurt.(π)
Baseline	−0.18	−0.3	3.8	−0.19	0.0	3.0
Low real-rate elasticity of output[c]	−0.11	−0.8	8.3	−0.12	0.0	3.0
High real-rate elasticity of output[d]	−0.21	−0.2	3.5	−0.22	0.0	3.0
More flexible prices[e]	−0.08	−0.7	7.3	−0.08	0.0	3.0
Less flexible prices[f]	−0.45	−0.2	3.4	−0.52	0.0	3.0
Low competition[g]	−0.06	−0.2	3.4	−0.06	0.0	3.0
High competition[h]	−0.49	−0.4	4.3	−0.53	0.0	3.0

[a] $\phi_\pi = 2.5$; [b] $\phi_p = 1.0$; [c] $\varphi = 1$; [d] $\varphi = 10$; [e] $\alpha = 0.56$; [f] $\alpha = 0.76$; [g] $\theta = 3$; [h] $\theta = 15$.

Table 4: Robustness of Simple Price-Level-Targeting Rule to Extreme Calibrations

Table 4 shows the results for a wide range of changes to each structural parameter of the model. The alternative calibrations include low or high real-rate elasticity of output ($\varphi = 1$ or 10), more or less flexible prices ($\alpha = 0.56$ or 0.76), as well as low or high competition among firms ($\theta = 3$ or 15). In all the alternative calibrations investigated, the long-run stationary distribution of inflation has a longer left tail (more negative skewness) and fatter tails (higher kurtosis) than a normal distribution when the policymaker sets the policy interest rate using an aggressive simple IT rule.

Table 4 shows also that for the alternative calibrations the long-run stationary distribution of inflation does not have a longer left tail (there is no skewness) and the tails are not fatter (kurtosis is not higher) than a normal distribution when the policymaker sets the policy interest rate using an aggressive simple PLT rule rather than an aggressive simple IT rule. The variability of inflation and the variability of the output gap are not very different if the policymaker uses an aggressive simple IT rule or an aggressive simple PLT rule. As a result, the cost of protection against tail risks (difference in loss between the two policy regimes) is not large. Among the alternative calibrations investigated, the cost of protection against tail risks is largest when prices are less flexible than the baseline, but still less than 0.07 percentage point of permanent consumption.

9 Conclusions

The economics literature suggests that price-level targeting is a potential solution to downside risks in a low-inflation economy due to the zero lower bound on nominal interest rates, such as deflation or recession. Since the private sector anticipates that a central bank with a price-level target will undo any price changes, a central bank is more effective at shaping private-sector expectations if it targets the price level path rather than inflation.

This paper shows that price-level targeting is a solution to downside tail risks, such as sustained deflation or recession, when policy is conducted through a simple interest-rate rule subject to a zero lower bound. At the same time, the average performance of the economy is not very different if policy implements price-level targeting instead of inflation targeting. Price-level targeting may imply less variability of inflation than inflation targeting, since policymakers can shape private-sector expectations about future inflation more effectively by targeting directly the price level path rather than inflation.

A few caveats must be kept in mind in interpreting the findings. The model focuses on the effects of the zero lower bound on nominal interest rates and ignores other reasons for which policymakers show concern about downside risks to the economy. These other reasons include downward wage rigidity, as argued by Tobin (1972), and debt-deflation, as argued by Fisher (1933). Thus, price-level targeting may offer even greater protection against downside tail risks in a low-inflation economy than is shown in this paper.

A Appendix

A.1 Permanent Consumption Loss

The expected life-time utility of the representative consumer, as shown in Chapter 6 of Woodford (2003a), is validly approximated by

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{U_c \overline{C}}{2} \frac{\alpha\theta\,(1+\omega\theta)}{(1-\alpha)\,(1-\alpha\beta)} L \tag{12}$$

where $\overline{C}$ is steady state consumption, $U_c > 0$ is steady state marginal utility of consumption and

$$L \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right] \leq 0$$

is the welfare-theoretic objective function (8) which the policymaker maximizes.

At the same time, the utility loss from a permanent consumption loss $\mu \leq 0$ is

$$E_0 \sum_{t=0}^{\infty} \beta^t U_c \overline{C} \mu = \frac{1}{1-\beta} U_c \overline{C} \mu \tag{13}$$

Equating the right-hand sides of (12) and (13), then the permanent consumption loss is

$$\mu = \frac{1-\beta}{2} \frac{\alpha\theta\,(1+\omega\theta)}{(1-\alpha)\,(1-\alpha\beta)} L \tag{14}$$

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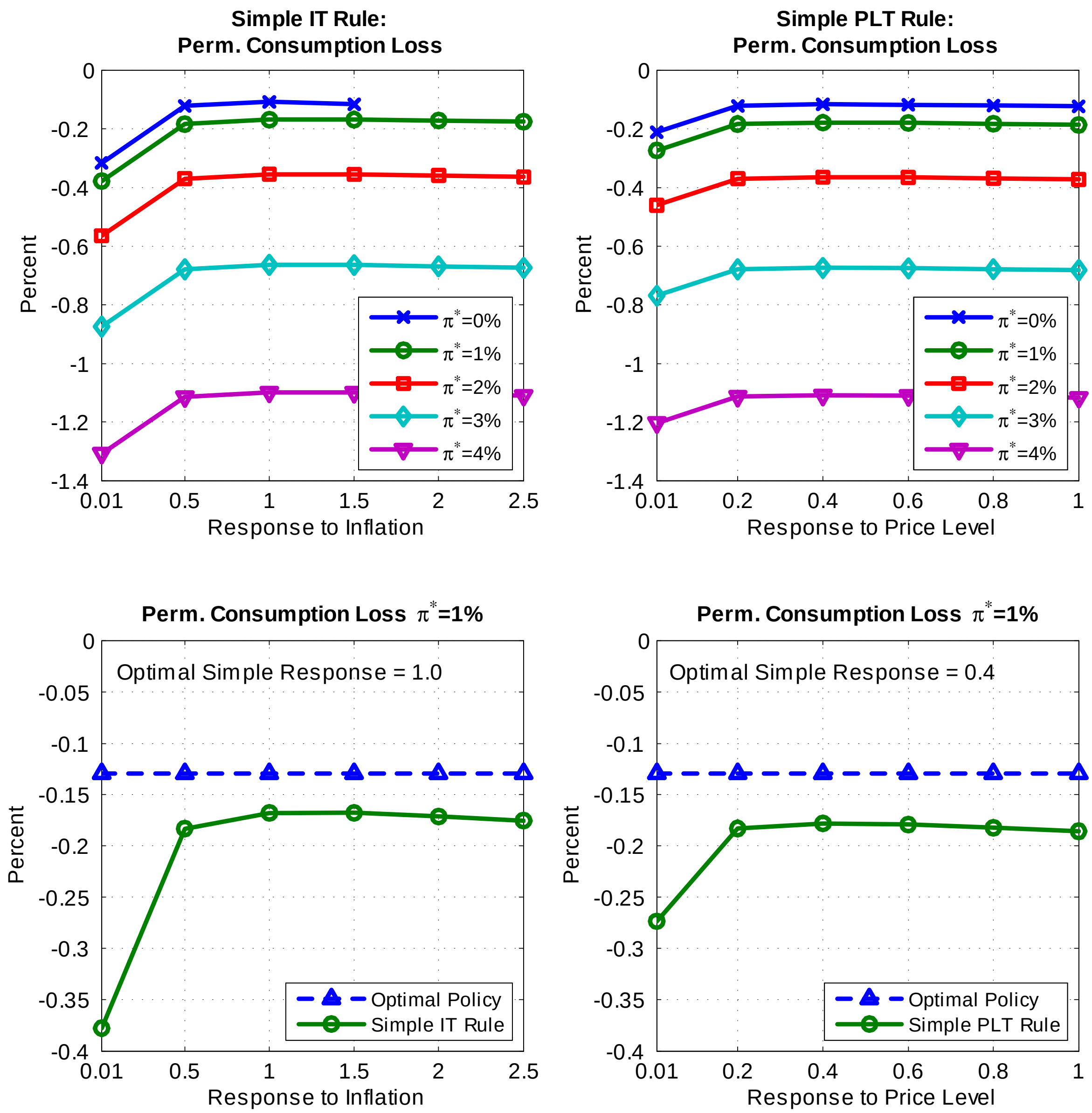

Figure 1: Moderate Inflation Is Very Costly Compared to Low Inflation

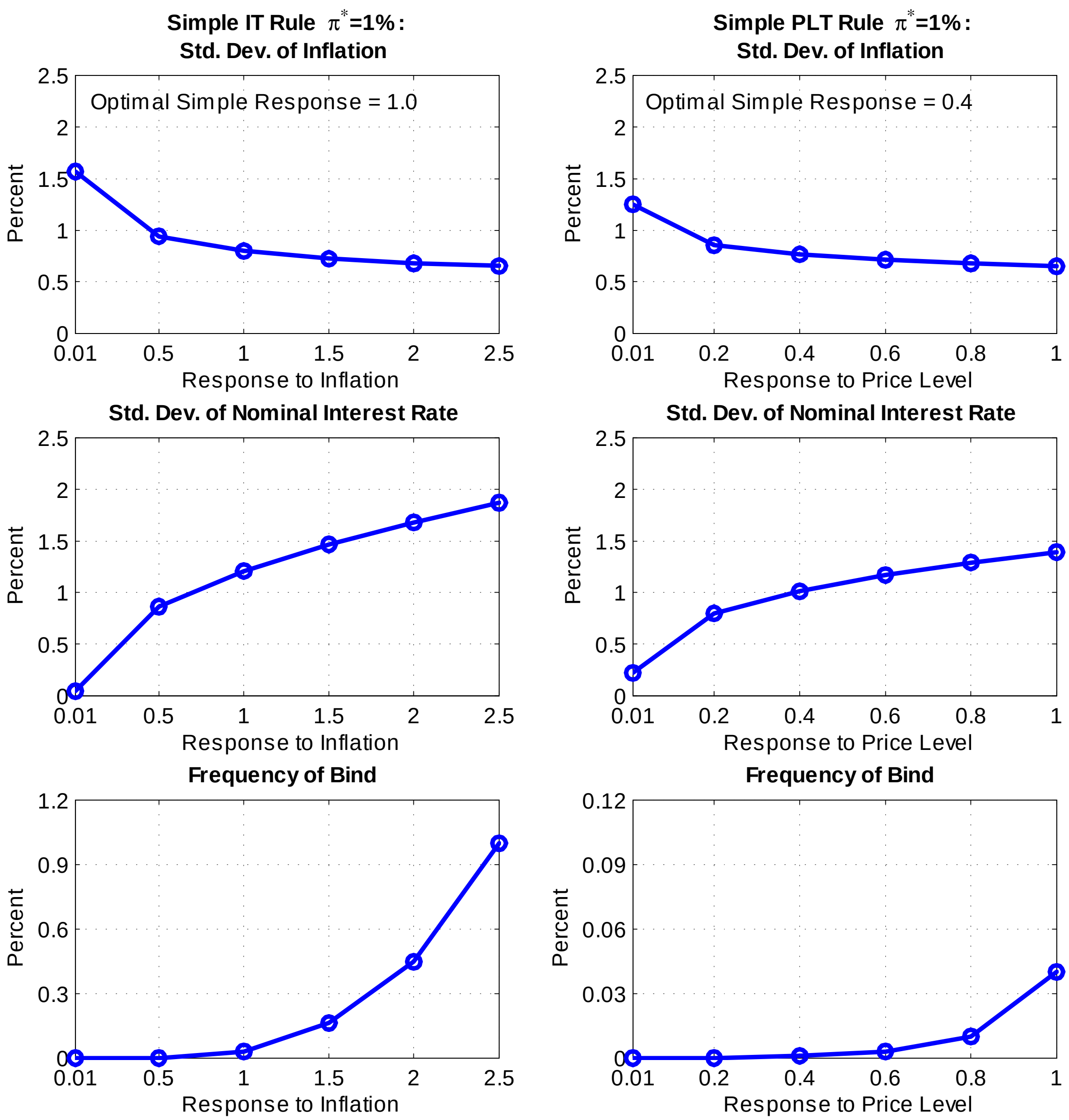

Figure 2: Price-Level Targeting Protects Against Hitting the Zero Lower Bound

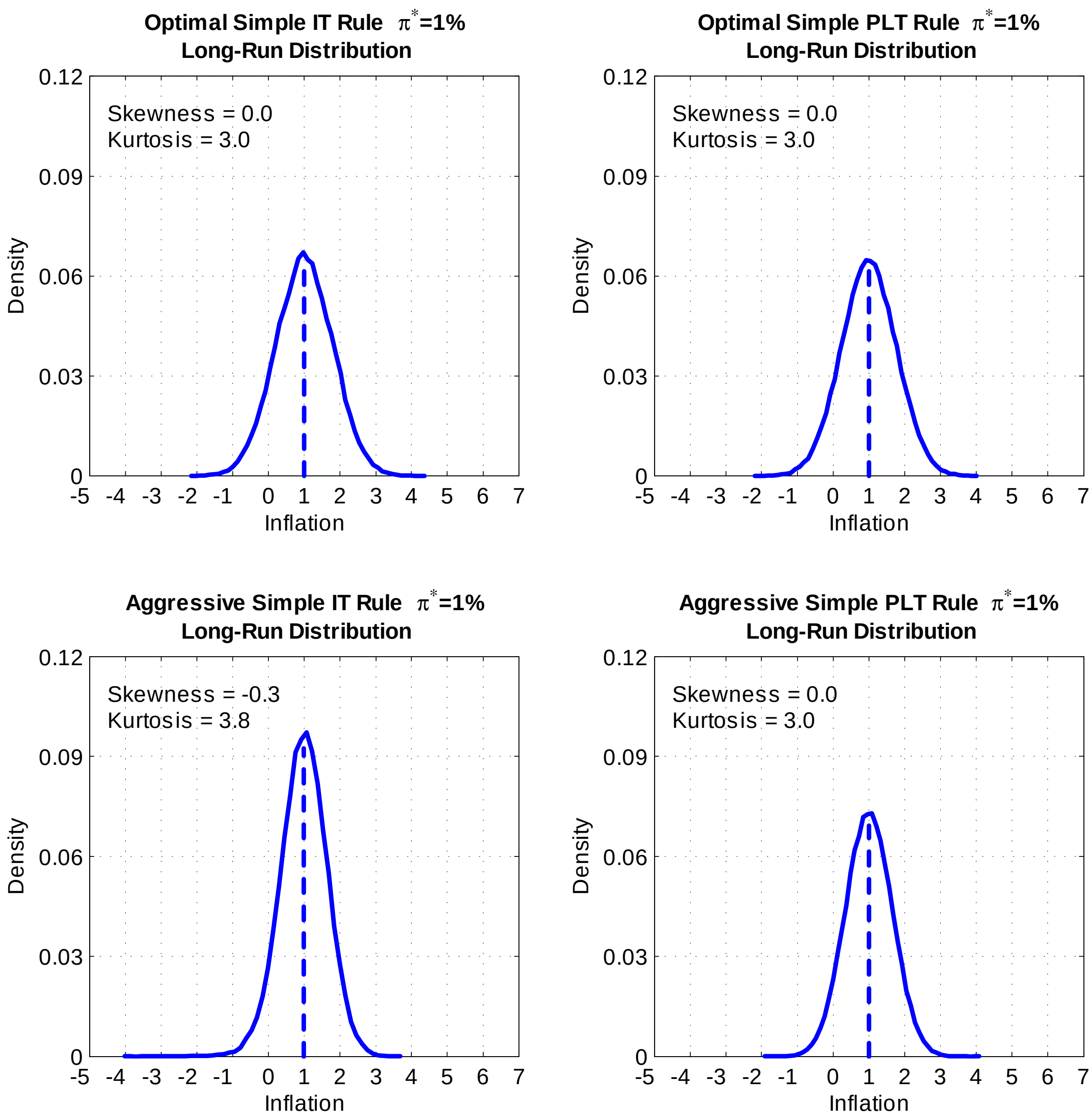

Figure 3: Price-Level Targeting Protects Against the Tail Risk of Deflation

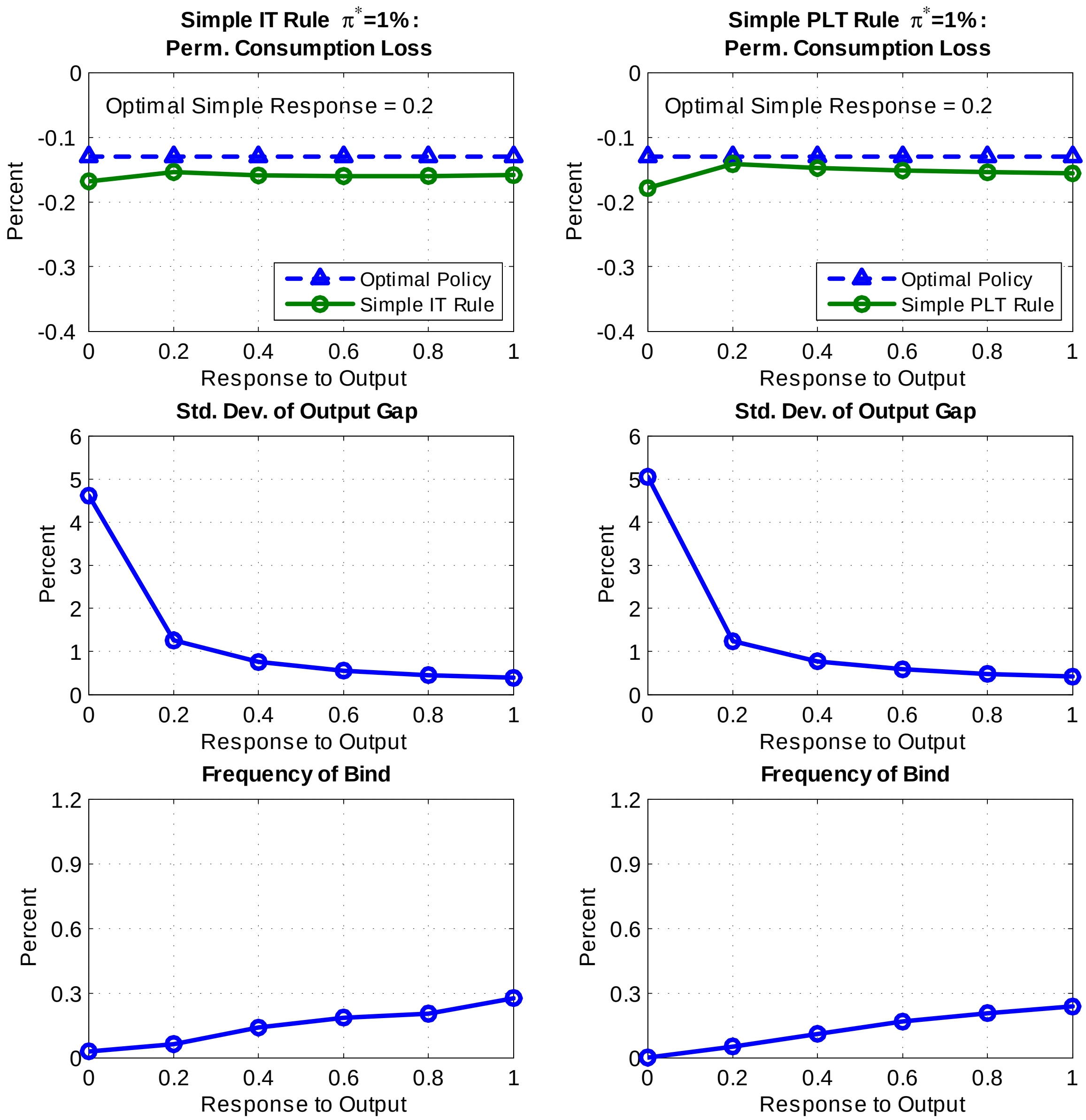

Figure 4: Responding to Output Improves Welfare Little and Makes Policy Less Flexible

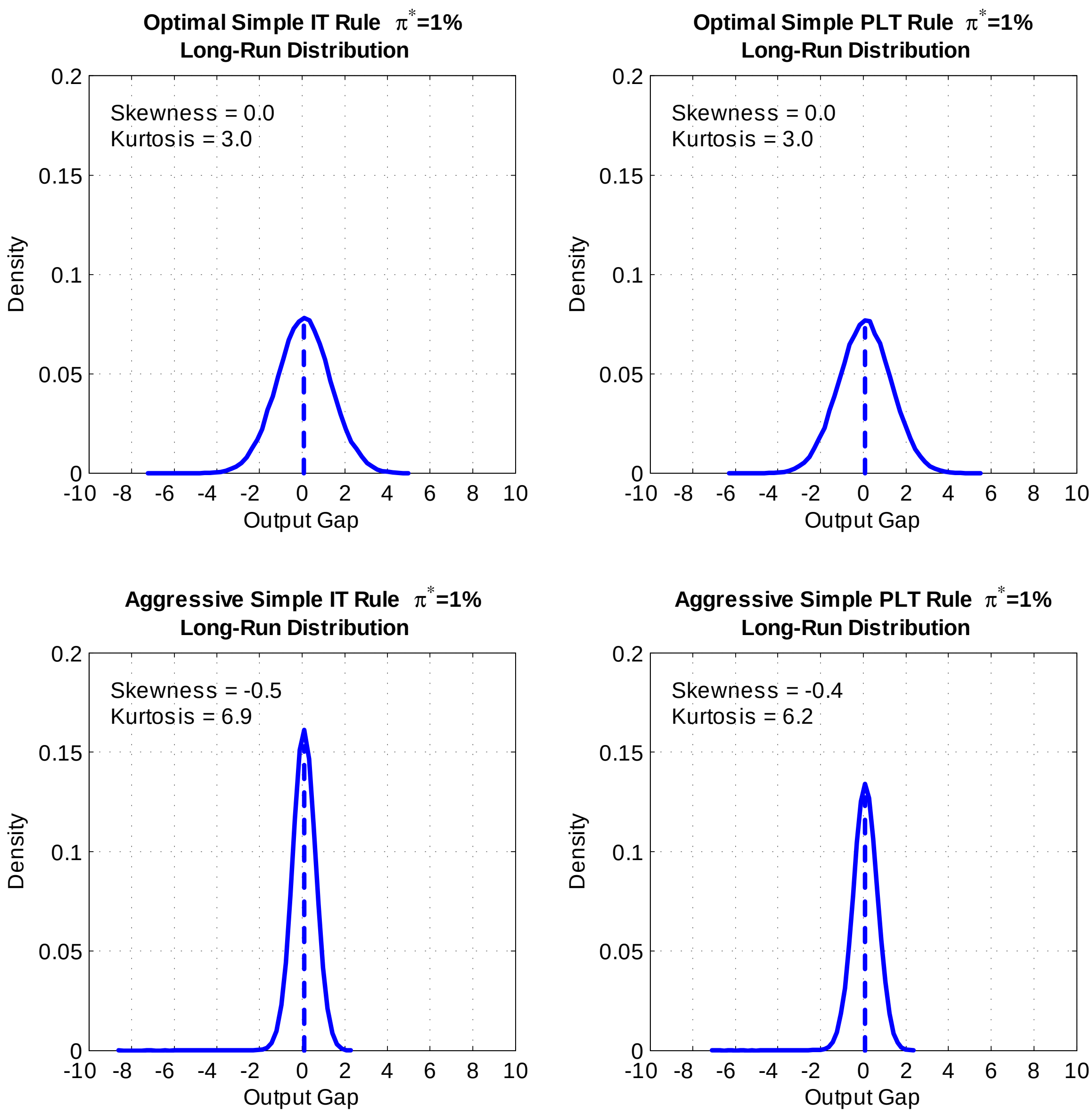

Figure 5: Price-Level Targeting Protects Against the Tail Risk of Recession