Bank Capital and Monetary Policy *

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Abstract

This paper develops a quantitative, monetary model in which agency problems affect both the relationship between banks and firms as well as that linking banks to their depositors. As a result, bank capital and entrepreneurial net worth jointly determine aggregate investment, and help propagate over time shocks affecting the economy. We find that the effects of monetary policy shocks depend on the capitalization of the banking system. More specifically, in an environment with a low average capital asset-ratio, monetary contractions lead to more substantial declines in economic activity than they do in an economy with highly capitalized banks.

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1 Introduction

In addition to its much-discussed implications for the general safety of financial systems, the evolution of bank net worth (bank capital) appears to have significant repercussions on the cyclical features of the real economy. Evidence from the late 1980s and early 1990s experiences in the U.S. and Japan, a time where many banks faced capital erosion from loan losses, regulatory changes, or equity price declines, suggests that poorly-capitalized banks reduced lending more significantly than their better-capitalized counterparts.\(^1\) Further, cross-sectional differences in bank capital may significantly affect the rates at which bank clients can borrow (Hubbard et al., 2002). Finally, evidence obtained using bank-level or state-level data suggests that monetary policy contractions will depress lending and real activity more significantly when bank capital is low.\(^2\)

Despite this evidence, bank capital remains largely absent from the quantitative-theoretic literature that examines the links between financial factors, monetary policy, and economic activity. This literature has concentrated on analyzing informational frictions that affect the relationship between firms and their lenders and that lead the balance sheet of firms to play a key role in the monetary transmission mechanism.\(^3\) Contributors to this line of research have observed, however, that an environment where the relationship between banks and their depositors is itself affected by informational frictions would lead to a similar role for bank capital in the transmission of monetary shocks.\(^4\)

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\(^1\)The perception that the U.S. economy suffered from such a ‘capital crunch’ in the early 1990s is discussed in Bernanke and Lown (1991) and further assessed by Peek and Rosengren (1995) and Brinkmann and Horvitz (1995). Evidence that shocks to the capital position of Japanese banks resulting from the late 1980s crash in the Nikkei had negative effects on their lending activities in the United States is contained in (Peek and Rosengren, 1997, 2000). Note that all these studies face the difficult challenge of disentangling the effects of changes in the demand for loans from the supply shocks they seek to identify.

\(^2\)See Van den Heuvel (2002c) for the link between the capital position of a state’s banking system and the subsequent reaction to that state’s output following monetary policy shocks. Kishan and Opiela (2000) describe the connection between bank capital and the amplitude in the decline of that bank’s lending following monetary contractions. In related results, Kashyap and Stein (2000) show that banks holding more liquid securities are able to limit the reductions in lending following similar contractions.

\(^3\)This ‘balance sheet channel’, first modelled in Bernanke and Gertler (1989), has been introduced into the standard quantitative RBC framework by (Carlstrom and Fuerst, 1997, 2001), Cooley and Nam (1998) and Bernanke et al. (1999).

\(^4\)For example, “There are several ways to incorporate a nontrivial role for banks in our framework; one possibility is to allow the financial intermediaries which lend to entrepreneurs to face financial frictions in raising funds themselves. In this case, the net worth of the banking sector... will matter for the model’s dynamics.” (Bernanke et al., 1998, pg. 45). They also state (page 41) that “...the incorporation of a banking sector into our model would be a highly worthwhile exercise.” See also Bernanke and Gertler (1985) for an early effort at including bank capital in a quantitative model of banking.
In this context, we develop a quantitative model that is used to study the link between the evolution of bank capital and entrepreneurial net worth, on the one hand, and monetary policy and economic activity, on the other. The framework we employ is a monetary, dynamic general equilibrium version of the environment in Holmstrom and Tirole (1997), which features two layers of moral hazard, the first one affecting the relationship between banks and their borrowers (entrepreneurs), and the second influencing the link between banks and their depositors (households). The first source of moral hazard arises because entrepreneurs can (privately) choose to undertake riskier projects in order enjoy private benefits. To mitigate this problem, banks require entrepreneurs to invest their own net worth in the projects. The second source of moral hazard stems from the fact that banks, to whom depositors delegate the monitoring of entrepreneurs, may not adequately do so in order to lower their costs. In response, depositors will demand that banks engage their own net worth in the financing of entrepreneurial projects. Consequently, this twin moral hazard problem implies that the amount of external financing entrepreneurs can raise (and thus the scale of the investment projects they can undertake) depends on the joint evolution of entrepreneurial net worth and bank capital. This dependence plays a key part in the propagation of shocks in our economy.

In our environment, a contractionary monetary policy shock, represented by a rise in the interest rate on deposits, increases the costs of funds for banks. This leads to a decrease in their demand for deposits, which in turn reduces bank lending to entrepreneurs and aggregate investment in the current period. The reduction in investment leads to declines in current earnings of banks and entrepreneurs, and thus also in future bank capital and entrepreneurial net worth. These reduced levels of bank capital and net worth contribute to propagate the shock over time, after the initial impulse to the interest rate has dissipated. We find that the effects of monetary policy shocks depend on the capitalization of the banking system. More specifically, in an environment where the extent of the double moral hazard problem is reduced and the resulting capital-asset ratio of banks is therefore low, monetary contractions lead to deeper declines in economic activity than they do in an economy with highly capitalized banks. This result arises because in the high capital-asset ratio economy, external financing relies more heavily on bank capital, so that banks are able to better shield their lending from the increase in the cost of deposits. In other terms, the increase in deposit rates worsens the moral hazard problem affecting the relationship between banks and households, but worsens it less in the high capital-asset ratio economy, in which banks hold more capital on average.

Our goal of analyzing the interaction between bank capital and monetary policy is shared by Van den Heuvel (2002a). Our modelling strategy differs from his, however, along several dimensions: in Van den Heuvel (2002a), regulation plays an important part in the determination of bank capital and the production, savings and monetary sides of the model are not fully developed; by comparison, the capital-asset ratio of banks in our model is market-determined and we present a detailed
general-equilibrium economy. Our modelling environment is more closely related to the one in Chen (2001), who also constructs a dynamic version of Holmstrom and Tirole (1997) and uses the model to study the propagation of adverse technology shocks. Compared to Chen (2001), the present paper broadens the analysis by embedding the twin moral hazard environment in a standard monetary version of the neo-classical model, with money introduced via a cash-in-advance constraint and monetary non-neutrality generated by the assumption of limited participation. Other recent papers considering bank capital in dynamic frameworks include Smith and Wang (2000) and Berka and Zimmermann (2002). The role assigned to bank capital in both of these contributions differs, however, from the one it plays in the present paper.

The remainder of this paper is organized as follows. Section 2 describes the basic structure of the model. In order to focus on the core element of our analysis—the financial contract linking banks, entrepreneurs, and households that makes the production of capital goods possible—that structure assumes that households are risk-neutral and that only entrepreneurs require external financing. The model is then calibrated in Section 3 and results drawn from its implications are described in Section 4. Section 5 extends the model and introduces risk-aversion in household preferences as well as bank financing for both sectors of the economy. Section 6 reports that the main qualitative features of the results identified using the basic model are not affected by extending the model. Section 7 concludes.

2 The Model

2.1 The environment

A continuum of risk-neutral agents inhabits the economy. There are three classes of agents: households, entrepreneurs, and bankers, with population mass \( \eta^h \), \( \eta^e \), and \( \eta^b \), respectively, where \( \eta^h + \eta^e + \eta^b = 1 \). In addition, there is a monetary authority which conducts monetary policy by targeting interest rates.

There are two distinct sectors of production. In the first, many competitive firms produce the economy’s final good, using a standard constant-returns-to-scale technology that employs physical capital and labour services as inputs. Production in this sector is not affected by any financial frictions.

In the second sector, entrepreneurs produce a capital good which will serve to
augment the economy’s stock of physical capital. In contrast to the situation in
the first sector, the production environment in the capital good sector is character-
ized by two distinct sources of moral hazard, with the resulting agency problems
limiting the extent to which entrepreneurs can receive external funding to finance
their production. First, the technology available to entrepreneurs is characterized by
idiosyncratic risk that is partially under the (private) control of the entrepreneur.
Monitoring entrepreneurs is thus necessary to limit the riskiness of the projects
they engage in. Second, the monitoring activities performed by the agents capable
of undertaking them, the bankers, are themselves not publicly observable, creating
a second source of moral hazard originating within the banking system. Moreover,
asset returns within a given bank are not perfectly diversified, thus implying that a
bank can fail.

In order to limit the impact of these financial imperfections, the households –
the ultimate lenders in this economy– thus require that both entrepreneurial net
worth and bank capital be sufficiently high when discussing the financial contracts
that channel funding to the entrepreneurs’ projects. The evolution of aggregate
entrepreneurial net worth and bank capital, as well as their dynamic interactions,
thus become an important determinant in the reaction of the economy to the shocks
affecting it.

Households are infinitely-lived; they save by holding physical capital and money.
They then divide their money holdings between what they send to banking institu-
tions and what they keep as cash; a cash-in-advance constraint for consumption ra-
tionalizes their demand for that latter asset. They cannot monitor entrepreneurs or
enforce financial contracts and will therefore not lend directly to them. Bankers will
act as delegate monitors of households. Bankers and entrepreneurs face a constant
probability of exiting the economy; surviving individuals save by holding capital
whereas those who receive the signal to exit the economy consume their accumulated
wealth. Exiting entrepreneurs and bankers are replaced by newly-born individual,
so that the population masses of the three classes of agents does not change. Figure
1 illustrates the timing of events that unfold each period in our artificial economy:
next, we proceed to describe in greater detail these events, the optimizing behaviour
of each type of agents and the connections between them.

2.2 Households

Each household enters period $t$ with a stock $M_t$ of money and a stock $k^h_t$ of physi-
cal capital. The household is also endowed with one unit of time which is divided
between leisure, work, and the time cost of adjusting the household’s financial port-
folio (see below). At the beginning of the period the current value of the aggregate
technology and monetary shocks are revealed.

The household then separates into three different agents with specific tasks.
The household \textit{shopper} takes an amount $M^c_t$ of the household’s money balances and travels to a retail market where it purchases consumption goods ($c^h_t$) for the household. The \textit{financier} gets the remaining amount of money balances $M_t - M^c_t$, which, along with $X_t$ (the household’s share of the current period injection of new money from the central bank) will serve as the household’s contribution to the financing of entrepreneurial projects. Note that the return from this financing is risky: entrepreneurs financed with the help of the household’s funds could fail, in which case those funds are lost completely; the probability that this happens is denoted by $\alpha^g$. The household’s \textit{worker} is given the household’s stock of capital $k^h_t$ and travels to the final good sector, to rent the household’s labour services at a real wage $w^h_t$ and the households’s physical capital, which carry a (real) rental rate of $r^k_t$.

Note that we have assumed that the current period’s monetary injection is distributed to the households’ financiers rather than to the shoppers. The monetary injection therefore enters the economy through the financial markets, creating an imbalance between the amount of liquidity present in financial markets and what is available in the final good market. In principle, households could correct this imbalance by reducing the amount of liquidity they send to financial markets (i.e. increasing $M^c_t$) but the presence of costs inherent to adjusting financial portfolios limits the extent to which they are prepared to do so. As a consequence, some of the imbalance remains, leading to a reduction in the opportunity cost of funds in the financial market. This \textit{limited participation} assumption is used in several recent quantitative models of monetary policy, such as Dotsey and Ireland (1995), Christiano and Gust (1999) and Cooley and Quadrini (1999).

The maximization problem of a representative household is the following:

$$\max \{c^h_t, M_{t+1}, M^c_t, h_t, k^h_t \} \sum_{t=0}^{\infty} \beta^t \left[ c^h_t - \chi \frac{(h_t + v_t)\gamma}{\gamma} \right].$$

where $\beta$ is the time discount of households, and the expectation is taken over uncertainty about the two aggregate shocks and the idiosyncratic shock affecting each household (the success or failure of the projects that the household will indirectly finance through his association with a banker). The term $v_t$ is defined as $v_t = \frac{\phi}{2} \left( \frac{M^c_t}{M^c_{t-1}} - \varphi \right)^2$ and expresses the (time) cost of adjusting the household financial portfolio.\footnote{We follow Christiano and Gust (1999) in expressing the costs of adjusting financial portfolios in units of time.} The risk neutrality behaviour characterizing this utility function implies that households only care about expected returns and do not value smooth consumption patterns.\footnote{The assumption of risk neutrality is important for the financial contract between households, banks, and entrepreneurs discussed in Section 2.5}
and the budget constraint:

$$\frac{M_{t+1}^c}{P_t} + q_t k_{t+1}^h = \frac{M_t}{P_t} - c_t^h + s_t \frac{r^d_t}{\alpha^y} \left( \frac{M_t - M_t^c + X_t}{P_t} \right) + w_t^h h_t + (r_t^k + q_t (1 - \delta)) k_{t+1}^h. \tag{3}$$

The cash-in-advance constraint (2) states that the real value of the shopper’s cash position ($\frac{M_t}{P_t}$) must be sufficient to cover planned expenditures of consumption goods ($c_t^h$). The budget constraint (3) expresses the evolution of the household’s assets: at the end of the period, any leftover currency from the shopper’s activities ($\frac{M_t}{P_t} - c_t^h$) is added to the real return from the deposits the financier was given. This return is $\frac{r^d_t}{\alpha^y} \left( \frac{M_t - M_t^c + X_t}{P_t} \right)$ if the projects financed by the household’s funds have been successful (an outcome indicated by $s_t = 1$) but zero if the projects failed ($s_t = 0$). Because the probability that the project succeeds is $\alpha^y$, the gross nominal expected return on household’s deposits is actually $r^d_t$. This financial income is combined with the labour and capital rental income brought back by the worker ($w_t^h h_t + r_t^k k_{t+1}^h$), and the real value of the undepreciated stock of capital $q_t (1 - \delta) k_{t+1}^h$, where $q_t$ is the value of capital at the end of the period, in terms of final goods. Total income is then transferred into financial assets (end-of-period real money balances $M_{t+1}/P_t$) or holdings of physical capital ($k_{t+1}^h$).

The first-order conditions of the problem with respect to $c_t^h$, $M_{t+1}$, $M_t^c$, $h_t$, and $k_{t+1}^h$ are the following:

$$1 = \lambda_{1t} + \lambda_{2t}; \tag{4}$$

$$\frac{\lambda_{2t}}{P_t} = \beta E_t \left[ \frac{\lambda_{2,t+1} r^d_{t+1}}{P_{t+1}} \right]; \tag{5}$$

$$\frac{\lambda_{2t} r^d_t}{P_t} + \chi(h_t + v_t)^{\gamma - 1} v_t(\gamma) = \frac{\lambda_{1t} + \lambda_{2t}}{P_t} - \beta^h E_t \left[ \chi(h_{t+1} + v_{t+1})^{\gamma - 1} v_{t+1}(\gamma + 1) \right]; \tag{6}$$

$$\chi(h_t + v_t)^{\gamma - 1} = \lambda_{2t} w_t^h; \tag{7}$$

$$\lambda_{2t} q_t = \beta^h E_t \left[ \lambda_{2,t+1} (r_{t+1}^k + q_{t+1} (1 - \delta)) \right]. \tag{8}$$

In these expressions, $\lambda_{1t}$ represents the Lagrange multiplier of the cash-in-advance constraint (2) and $\lambda_{2t}$ a similar multiplier for the budget constraint (3).

Equation (4), equating the sum of the two Lagrange multipliers to 1, reflects the fact that the marginal utility of consumption is constant for the risk-neutral household. Equation (5) states that by choosing an extra unit of currency as a saving vehicle, the household is foregoing a utility value of $\frac{\lambda_{2t}}{P_t}$; the household is compensated, in the next period, with the return from holding this extra unit of currency (the gross nominal interest rate $r_{t+1}^d$) a return which, when properly deflated, discounted and expressed in utility terms, is valued at $\beta E_t \left[ \frac{\lambda_{2,t+1} r^d_{t+1}}{P_{t+1}} \right]$. Equation (6)
states that by choosing to keep an extra unit of currency for use in the final good sector, the household foregoes the return associated with this extra unit if it had been sent to the financial sector \( (r_d t) \) and must also pay adjustment costs valued at \( \chi(h_t + v_t)\gamma^{-1}v_1 (\gamma t) \). In return, the household receives the current period utility value of this extra liquidity \( (\lambda_1t + \lambda_2t) \) and relaxes next period’s expected portfolio adjustment costs by an amount valued at \( \beta E_t [\chi(h_{t+1} + v_{t+1})\gamma^{-1}v_1 (\gamma_{t+1})] \). Equations (7) and (8) are standard; notice, however, that because \( \lambda_2 < 1 \), inflation introduces a distortion in labour supply decisions.

### 2.3 Final Good Production

The final good sector features perfectly competitive producers that transform physical capital and labour inputs into the economy’s final good. The production function they employ exhibits constant returns to scale and is affected by serially correlated technology shocks. Aggregate output \( Y_t \) is given by:

\[
Y_t = z_t F(K_t, H^h_t, H^e_t, H^b_t),
\]

where \( z_t \) is the technology shock, \( K_t \) is the aggregate stock of physical capital, and \( H^h_t, H^e_t, \text{ and } H^b_t \) are the aggregate labour inputs from households, entrepreneurs, and bankers, respectively. No financial frictions are present in this sector, so that the usual first-order conditions for profit maximization apply and aggregate profits of final good producers are zero. The constant-returns-to-scale feature of the production function implies that we can concentrate on economy-wide relations, which will coincide with the firm-level ones.

We assume that the technology shock evolves according to a standard AR(1) process, so that:

\[
z_t = \rho z_{t-1} + \epsilon_t, \quad \epsilon_t \sim (0, \sigma^2). \tag{10}
\]

The competitive nature of this sector implies that the rental rates of capital, as well as the various wages, are equal to their respective marginal products:

\[
r^k_t = z_t F_1(K_t, H^h_t, H^e_t, H^b_t); \tag{11}
\]

\[
w^h_t = z_t F_2(K_t, H^h_t, H^e_t, H^b_t); \tag{12}
\]

\[
w^e_t = z_t F_3(K_t, H^h_t, H^e_t, H^b_t); \tag{13}
\]

\[
w^b_t = z_t F_4(K_t, H^h_t, H^e_t, H^b_t); \tag{14}
\]

where \( w^e_t \) and \( w^b_t \) denote the (real) wage rates of entrepreneurial labor and a banker’s labor. The assumption that entrepreneurs and bankers receive a wage income from final good producers is necessary to ensure that all representatives of these two classes of agents can always pledge a positive (but possibly very small) amount of net worth in the financial contract negotiations. Below, when calibrating the production
function $F(K_t, H_t^b, H_t^e, H_t^f)$, we set the marginal products of labour inputs in such a way that these wages only represent a small fraction of the net worth of entrepreneurs and bankers.\footnote{These wages could alternatively be interpreted as payments for managerial (from entrepreneurs) and financial (from bankers) services rendered to the final good producers. Carlstrom and Fuerst (1997) assume the presence of similar wages for their entrepreneurs, to ensure that all these agents always have positive net worth. Similarly, Chen (2001) assumes that entrepreneurs and bankers are entitled to modest levels of endowment each period.}

### 2.4 Capital good production and the financial contract

Each entrepreneur has access to a (risky) production technology that takes units of the final good as input and delivers capital goods if successful. Specifically, an investment of size $i_t$ units of final goods will contemporaneously yield a (publicly observable) return of $R_i$ units of capital if the project succeeds, but zero units if it fails: note that the investment size $i_t$ is jointly chosen by the entrepreneur and his financial backers.

Entrepreneurs can influence the riskiness of the projects they undertake; they may choose to pursue a project with low probability of success because of the existence of private benefits that stem from such a project and which accrue solely to them. Specifically, we follow the formulation of Holmstrom and Tirole (1997) and Chen (2001) and posit the existence of three types of projects, each carrying a different mix of public return and private benefits.\footnote{We introduce three projects (or two levels of shirking) in order to have a sufficiently rich modelling of monitoring.}

- First, the \textit{good} project involves a high probability of success (denoted $\alpha^g$) and zero private benefits. Second, the \textit{average} project, while associated with a lower probability of success $\alpha^b$ ($\alpha^b < \alpha^g$), is associated with private benefits proportional to the investment size and equal to $bi_t$.
- Finally, the \textit{bad} project, while also associated with the low probability of success $\alpha^b$, brings to the entrepreneurs even higher private benefits $B_i$, with $B > b$. Given that the average and bad projects have the same probability of success but different levels of private benefits, entrepreneurs would prefer the bad project (which has a higher private benefit) over the average project regardless of the financial contract.

We further assume that only the good project is socially desirable, that is

\begin{equation}
q\alpha^b R + B - (1 + \mu) < 0 < q\alpha^g R - (1 + \mu), \tag{15}
\end{equation}

where $\mu$ is the monitoring cost of banks.

The bankers have access to a monitoring technology that can limit the extent to which entrepreneurs are able to engage in risky projects. Specifically, monitoring entrepreneurs can detect whether they have undertaken the bad project, but cannot distinguish between the good and the average project.\footnote{Following Holmstrom and Tirole (1997) and Chen (2001), we interpret the monitoring activities} This implies that if banks
monitor, the entrepreneur can only choose the average or the good project. Monitoring costs are assumed to be proportional to the size of the project so that $\mu_i t$ units of final good are spent on monitoring when a project of size $i_t$ is financed. The monitoring activities of bankers are not, however, publicly observable. This creates an additional source of moral hazard, affecting the relationship between bankers and their depositors (the households). Because banks act as delegates of households to monitor entrepreneurs, households entrust their funds only to banks which are well-capitalized and have a lot to lose in case of loan default.

The nature of the monitoring technology is assumed to imply that all projects funded by a given bank either succeed together or fail together. This perfect correlation across the project returns implies that each bank faces an idiosyncratic risk of failure that cannot be diversified away.\(^{12}\) Note that this stark assumption is not necessary for the bank’s capital position to matter; what is necessary is that the correlation not be zero.\(^{13}\)

An entrepreneur with net worth $n_t$ undertaking a project of size $i_t > n_t$ will rely on external financing worth $l_t = i_t - n_t$ from banks. This funding is arranged by the banker, who collects deposits $d_t$ from the household’s financier. To mitigate the moral hazard problem affecting their relationship with depositors, bankers pledge some of their own capital $a_t$ towards the entrepreneur’s project, such that $a_t = l_t + \mu_i t - d_t$. Engaging some of their own funds implies that bankers have a personal incentive to monitor entrepreneurs, in order to limit erosion to their capital position. This reassures depositors, who can then provide more of their own funds towards the financing package.

### 2.5 Financial contract

We concentrate on financial contracts that lead entrepreneurs to only undertake the good project; under assumption (15), this project is socially preferable. We also assume the presence of inter-period anonymity, which implies that only one-period contracts are feasible.\(^{14}\) This allows us to abstract from the complexities that arise of bankers as inspecting cash flows, balance sheets, etc. or verifying that firm managers conform with the covenants of a loan. Note that this interpretation is different from the one that is assigned to monitoring costs in the costly-state verification (CSV) literature, where they are associated with bankruptcy-related activities.

\(^{12}\)The assumption of perfect correlation in the returns of bank assets is the opposite of the extreme assumption in Diamond (1984) and Williamson (1987) where bank assets are perfectly diversified so that banks do not fail and can be encouraged to monitor without their own capital.

\(^{13}\)The assumption that a given banker cannot diversify perfectly across all his lines of business can be interpreted as a situation where a given banker has specialized its activities within a given sector of the economy, or a given geographical area; in such a situation, the risk of failure will naturally be positively correlated across all projects.

\(^{14}\)This assumption is also made by Carlstrom and Fuerst (1997) and Bernanke et al. (1999).
from dynamic contracting. To undertake a project, the entrepreneur uses his own funds as well as external financing obtained from banks (and thus, indirectly, from households). A contract specifies how much each side should invest in the project and how much it should be paid as a function of the project outcome. One optimal contract will have the following structure: (i) the entrepreneur invests all his net worth, while the bank and the households put up the balance $i_t - n_t$, (ii) if the project succeeds, the unit return $R$ is distributed among the entrepreneur ($R_e > 0$), the banker ($R_b > 0$) and the households ($R_h > 0$), and (iii) all agents receive nothing from if the project fails.

Recall that an investment of size $i_t$ returns $Ri_t$ units of capital good if it is successful, and nothing if it fails. The expected return, in terms of final goods, going to the entrepreneur is thus $q_t\alpha^g R^e_t i_t$ when the good project is chosen, where recall that $q_t$ is the relative price of capital goods in terms of final goods. The financial contract linking the entrepreneur, the banker (and, implicitly, the household) seeks to maximize the entrepreneur’s expected return subject to a number of constraints that ensure entrepreneurs and bankers behave as agreed and that the funds contributed by the banker and the household earn (market-determined) required rates of return. More precisely, an optimal contract is given by the solution to the following optimization program:

$$\max_{\{i_t, R^e_t, R^h_t, R^b_t, a_t, d_t\}} q_t\alpha^g R^e_t i_t, \tag{16}$$

subject to

$$R = R^e_t + R^h_t + R^b_t \tag{17}$$

$$q_t\alpha^g R^h_t i_t - \mu i_t \geq q_t\alpha^b R^h_t i_t \tag{18}$$

$$q_t\alpha^g R^e_t i_t \geq q_t\alpha^b R^e_t i_t + q_t b_i t \tag{19}$$

$$q_t\alpha^g R^b_t i_t \geq r^a_t a_t \tag{20}$$

$$q_t\alpha^g R^h_t i_t \geq r^d_t d_t \tag{21}$$

$$i_t - \mu i_t - n_t = a_t + d_t \tag{22}$$

Equation (18) is the incentive compatibility constraint for bankers, which must be satisfied in order for monitoring to occur. It states that the expected return from monitoring, net of the monitoring costs themselves, must be at least as high as the expected return from not monitoring, a situation in which entrepreneurs would choose the bad project, leading to a low probability of success. Given that bankers monitor, entrepreneurs cannot choose the bad project. The incentive compatibility of entrepreneurs, equation (19), induces them to choose the good project, by promising them an expected return that is at least as high as the expected return they would get, inclusive of private benefits, if they were to choose the average project.

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15 General-equilibrium environments that pay explicit attention to dynamic contracting are found in Gertler (1992), Cooley et al. (2000), and Smith and Wang (2000).
The (market-determined) required rates of return on bank capital and household deposits are \( r^a_t \) and \( r^d_t \), respectively. Equations (20) and (21), the participation constraints of bankers and households, ensure that these agents, when engaging bank capital and deposits \( a_t \) and \( d_t \), respectively, are promised shares of the project’s return that are sufficient to attain these required rates. Equation (17) simply states that the shares promised to the three different agents must add-up to the total return; finally, equation (22) indicates that the amounts lent by the banker, net of the monitoring costs, come from their own capital and from the deposits they have attracted.

In equilibrium, the constraints (18)-(21) hold with equality, so that the shares are given by:

\[
\begin{align*}
R^e_t &= \frac{b}{\Delta \alpha}; \\
R^b_t &= \frac{\mu}{q_t \Delta \alpha}; \\
R^h_t &= R - \frac{b}{\Delta \alpha} - \frac{\mu}{q_t \Delta \alpha};
\end{align*}
\]

where \( \Delta \alpha = \alpha^g - \alpha^b > 0 \) and \( R^j_t > 0 \) for \( j = e, b, h \).

Note from (23) and (24) that were the private benefits \( b \) and the monitoring costs \( \mu \) to increase, the per-unit share of project return allocated to entrepreneurs and bankers must increase in order to give these agents the incentive to behave as agreed. In turn, (25) implies that this reduces the per-unit share of project return that can be credibly promised to households as payments for their deposits. Introducing (25) in the participation constraint of households (21) holding with equality, leads to the following:

\[
r^d_t d_t = q_t \alpha^g \left( R - \frac{b}{\Delta \alpha} - \frac{\mu}{q_t \Delta \alpha} \right) i_t.
\]

Next, eliminating \( d_t \) from (26) using the resource constraint (22) leads to this rewriting of the participation constraint of depositors:

\[
r^d_t [(1 + \mu) i_t - a_t - n_t] = q_t \alpha^g \left( R - \frac{b}{\Delta \alpha} - \frac{\mu}{q_t \Delta \alpha} \right) i_t.
\]

This equation illustrates the mechanism that will lead monetary policy shocks to have an effect on the leverage of the economy. All things equal, an increase in the required rate on deposits \( r^d_t \) must be compensated for by increases in the contribution of bank capital \( a_t \) and entrepreneurial net worth \( n_t \) in the financing of a given-size project.

Finally, solving for \( i_t \) in the preceding equation leads to the following relation between the size of the project undertaken, entrepreneurial net worth, and the bank’s
capital position:

\[ i_t = \frac{n_t + a_t}{G_t}, \]  

(28)

where \( G_t \) is as follows:

\[ G_t = 1 + \frac{q_t \alpha^g}{r^d_t} \left( R - \frac{b}{\Delta \alpha} - \frac{\mu}{\Delta \alpha q_t} \right). \]  

(29)

In equilibrium, the investment \( i_t \) must be positive, so \( G_t \) must be positive (since \( a_t \) and \( n_t \) are positive). Therefore in equilibrium, rates of return and prices should be such that:

\[ q_t \alpha^g (b + \mu / q_t) / \Delta \alpha > q_t \alpha^g R - r^d_t (1 + \mu), \]  

(30)

where condition (30) says that the sum of expected shares paid to the entrepreneur and banker is higher than the expected unit surplus of the good project.

An immediate implication of equation (28) is that an increase in either entrepreneurial net worth \( n_t \) or bank capital \( a_t \) increases the project size \( i_t \). Further, note that \( \frac{\partial i_t}{\partial q_t} = -\left( \frac{n_t + a_t}{G_t^2} \right) \frac{\partial G_t}{\partial q_t} \), while \( \frac{\partial G_t}{\partial q_t} = \left( \frac{\alpha^g (b - R)}{r^d_t \Delta \alpha} \right) \). From assumption (15) we have \( \alpha^g R > \alpha^b R + b \) so that \( \frac{\partial i_t}{\partial q_t} > 0 \). An increase in the price of capital leads to increases in the size of the investment projects undertaken by the entrepreneurs. Notice also that investment is a decreasing function of the interest rate \( r^d_t \): monetary policy tightenings, by increasing \( r^d_t \), will thus lead to reductions in the scale of investment projects.

2.6 Entrepreneurs

Entrepreneurs seek to maximize the expected value of their lifetime utility. In addition to managing their investment projects, they are endowed each period with one unit of (working) time that they inelastically supply to the final good producers. They are risk-neutral and are thus willing to accept very low or zero consumption for many periods in return for relatively high consumption in the future (they care only about expected returns). They face a constant probability of exiting the economy; denote this probability by \( 1 - \tau^e \), so that \( \tau^e \) is the probability of surviving until the next period. The assumption of finite horizons for entrepreneurs is one way to guarantee that entrepreneurs will never become sufficiently wealthy to overcome financial constraints.\(^{16}\) We calibrate \( \tau^e \) such that in the steady state, entrepreneurs continue to rely on external financing for their activities. Expected lifetime utility is thus the following:

\[ E_0 \sum_{t=0}^{\infty} (\beta \tau^e)^t c^e_t, \]  

(31)

\(^{16}\)Another way to guarantee that entrepreneurs do not become self-financed is to assume that entrepreneurs are infinitely-lived but discount the future more heavily than households do. This is the approach used by Carlstrom and Fuerst (1997).
where \( c_e^t \) denotes entrepreneurial consumption.

Entrepreneurs that must exit the economy receive the signal to do so at the end of the period. Thus, surviving and exiting entrepreneurs both participate similarly in the period’s activities (financial contract, capital good production, etc.). They do differ in their saving decisions however: exiting entrepreneurs consume all available income, while surviving ones save for the future. Finally, exiting entrepreneurs are replaced, at the beginning of the following period, by newborn agents; in this manner, the measure of entrepreneurs within the total population remains constant at \( \eta^e \).

At the beginning of period \( t \), a fraction \( \tau^e \) of the total number of entrepreneurs present are therefore agents having survived from the preceding period, possibly carrying with them accumulated assets: denote by \( k_e^t \) the stock of physical capital that such a surviving entrepreneur holds. The remaining fraction \( (1-\tau^e) \) of entrepreneurs are newborn agents, who begin the period with one unit of time endowment and no assets.

During the early part of the period, each entrepreneur travels to the final good sector, where he rents out any physical capital holdings and labour services. These sources of income, plus the value of the undepreciated part of the physical capital, form the net worth that entrepreneurs can pledge towards the financing of the investment projects. Thus entrepreneurial net worth is given by:

\[
n_t = w_t^e + r_t^ek_t^e + q_t(1-\delta)k_t^e;
\]

In the second part of the period, after meeting with a banker and (implicitly) the household’s financier, each entrepreneur engages in an investment project of size \( i_t \), the maximum that financial backers will allow; recall from (28) that the size of the project is related to net worth \( n_t \) by \( i_t = \frac{n_t + a_t}{G_t} \). As the spot market for capital now opens, this entrepreneur can now sell some of this capital to purchase consumption, or save it for the next period. Recall that a failed project returns nothing. The following accumulation equation emerges:

\[
c_t^e + q_t k_{t+1}^e \leq s_t q_t R_t^e i_t(n_t, a_t; G_t),
\]

where \( s_t \) is the indicator function that takes a value of 1 if the entrepreneur’s project was a success and 0 if it failed.

Successful, surviving entrepreneurs could, in principle, allocate part of their income to consumption, and part to saving. However, the risk-neutrality feature of their preferences, as well as the high (expected) internal return from their assets lead them to postpone consumption and save all of their available income. Successful, exiting entrepreneurs, on the other hand, do not wish to save any capital but simply consume all proceeds from their activities before exiting. The upshot of this optimizing behaviour is found in the following set of consumption and savings...
decisions:

\[
c_t^e = \begin{cases} 
q_t R_t^i i_t(n_t, a_t; z_t) & \text{if exiting and successful} \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (34)

\[
k_{t+1}^e = \begin{cases} 
R_t^i i_t(n_t, a_t; z_t) & \text{if surviving and successful,} \\
0 & \text{otherwise.}
\end{cases}
\]  \hspace{1cm} (35)

2.7 Bankers

The banker’s problem is similar to the entrepreneur’s problem, so we will be brief in the description. Bankers are risk-neutral agents facing a constant probability of exit from the economy \((1 - \tau^b)\). Exiting bankers are replaced by newborn agents entering the economy with one unit of time and no assets. All banking agents inelastically supply their entire time endowment to final good producers. The entering rate of new bankers is such that their population is constant over time. They seek to maximize the expected value of their lifetime utility, which is as follows:

\[
E_0 \sum_{t=0}^{\infty} (\beta \tau^b)^t c_t^b,
\]  \hspace{1cm} (36)

where \(c_t^b\) denotes bank consumption. As in the case of the entrepreneur’s problem, the finite horizon assumption of bankers also assures that bankers do not become too wealthy and financially unconstrained.\(^{17}\)

The banker’s specificity arises from a technology with which they are endowed and that allows them to monitor entrepreneurs and thus acts as a delegated monitor of households (the ultimate lenders).

Once again, at the beginning of period \(t\), a fraction \(\tau^b\) of the existing bankers are agents having survived from the preceding period, with \(k_t^b\) in accumulated assets; the remaining fraction \((1 - \tau^b)\) are newborn agents with no assets. Bank capital in terms of final goods is given by:

\[
a_t = w_t^b + r_t^k k_t^b + q_t (1 - \delta) k_t^b.
\]  \hspace{1cm} (37)

In the second part of the period, a banker having succeeded in attracting deposits \(d_t\) in terms of final goods and pledging \(a_t\) of his own capital can finance a project of size \(i_t\), where as before from (28), the size of the project is related to bank capital \(a_t\) by \(i_t = \frac{a_t}{G_t}\). His share of the return from a successful project consists of \(R_t^b i_t\) units of the capital good, which can be used to buy consumption or can be saved, according to the accumulation equation:

\[
c_t^b + q_t k_{t+1}^b \leq s_t q_t R_t^b i_t(n_t, a_t; G_t),
\]  \hspace{1cm} (38)

\(^{17}\)A small \(\tau^b\) will guarantee that the bank net worth or capital remains scarce.
where \( s_t \) now indicates whether the projects funded by the banker where all successful \((s_t = 1)\) or whether they all failed \((s_t = 0)\); recall our assumption of perfect correlation across the outcomes of all projects funded by a given banker.

Bankers face incentives to save and postpone consumption that are very similar to those experienced by the entrepreneurs; therefore the following decision rules for consumption and savings emerge:

\[
e^b_t = \begin{cases} 
q_t R^b_{it}(n_t, a_t; G_t) & \text{if exiting and successful} \\
0 & \text{otherwise}
\end{cases} 
\quad (39)
\]

\[
k^b_{t+1} = \begin{cases} 
R^b_{it}(n_t, a_t; G_t) & \text{if surviving and successful} \\
0 & \text{otherwise}
\end{cases} 
\quad (40)
\]

### 2.8 Aggregation

The linear nature of the capital goods, private benefits and monitoring technologies lead us to obtain the aggregate expected production of capital goods by simply adding all the investment policies of each entrepreneur (The same aggregation procedure applies to all the other variables except prices). We denote all aggregate variables by uppercase and individual variables by lowercase. Notice because of the linearity in the model, only the first moments of the distributions of entrepreneurial net worth \( n_t \) and bank capital \( a_t \) matter for the aggregate economy, thus allowing us to avoid keeping track of the entire cross-section distributions of entrepreneurial net worth and bank capital across entrepreneurs and bankers. For example, \( I_t \) denotes aggregate investment while \( i_t \) denotes the investment policy a given entrepreneur.

\[
I_t = \frac{N_t + A_t}{G_t}, 
\]

(41)

where \( N_t \) and \( A_t \) denote aggregate entrepreneurial net worth and aggregate bank capital, respectively. \( G_t \) is defined in equation (29). Notice that a fall in either \( A_t \) or \( N_t \) leads a decrease in current investment.

The aggregation of (32) and (33), as well as (35) and (40) yields aggregate entrepreneurial net worth and banking capital and laws of motion \( K^e_{t+1} \) and \( K^b_{t+1} \):

\[
N_t = \eta^e w^e_t + (r^k_t + q_t(1 - \delta)) K^e_t; 
\]

(42)

\[
K^e_{t+1} = \tau^e \alpha^e R^e_t I_t; 
\]

(43)

\[
A_t = \eta^e w^b_t + (r^k_t + q_t(1 - \delta)) K^b_t; 
\]

(44)

\[
K^b_{t+1} = \tau^b \alpha^b R^b_t I_t. 
\]

(45)
It is useful to provide the laws of motion of aggregate entrepreneurial net worth $N_{t+1}$ and aggregate bank capital $A_{t+1}$. To do so we combine equations (41)-(45) which hold for all time $t$ and we thus find the following expressions:

\[ N_{t+1} = \eta^e w_{t+1}^e + \left( r_{t+1}^k + q_{t+1}(1 - \delta) \right) \tau^e \alpha^g R_t^e \left( \frac{A_t + N_t}{G_t} \right); \quad (46) \]

\[ A_{t+1} = \eta^b w_{t+1}^b + \left( r_{t+1}^k + q_{t+1}(1 - \delta) \right) \tau^b \alpha^g R_t^b \left( \frac{A_t + N_t}{G_t} \right). \quad (47) \]

Equations (46) and (47) show that banking capital and entrepreneurial net worth are interrelated. More precisely, aggregate bank capital at time $t+1$ depends not only upon on its own date $t$ stock, but also upon the ratio of aggregate entrepreneurial net worth and bank capital. Therefore a shock to the banking sector at time $t$ will be transmitted to the capital good sector which will be propagated into subsequent periods.

Finally, the aggregation of (34) and (39) across all entrepreneurs and bankers yield the following expressions for aggregate consumption by these agents:

\[ C_t^e = (1 - \tau^e) q_t \alpha^g R_t^e I_t(N_t, A_t); \quad (48) \]

\[ C_t^b = (1 - \tau^b) q_t \alpha^g R_t^b I_t(N_t, A_t). \quad (49) \]

### 2.9 Monetary Policy

Denote the aggregate supply of money in the economy at the beginning of period $t$ as $M_t$, and the aggregate injection of new money during period $t$ as $X_t$: we thus have $M_{t+1} = M_t + X_t$.

As in Christiano and Gust (1999), monetary policy is interpreted as targeting a given value for the nominal deposit rate $r^d_t$, and adjusting money supply in a manner that is consistent with this target. This interest rate targeting is represented by the following expression, or rule:

\[ r^d_t = \frac{r^d}{y} \left( \frac{\pi_t}{\pi} \right)^{\rho_y} \left( \frac{\pi_t}{\pi} \right)^{\rho_\pi} \epsilon_{t}^{mp}, \quad \epsilon_{t}^{mp} \sim (0, \sigma_{mp}). \quad (50) \]

where $r^d$, $y$, and $\pi$ are the steady-state values of $r^d_t$, $y_t$, and $\pi_t$, respectively, and $\epsilon_{t}^{mp}$ is an i.i.d monetary policy shock, that is instances where monetary authorities depart from the systematic portion of their rule (50).\(^{18}\)

When $\rho_y > 0$, and $\rho_\pi > 0$, monetary policy follows a Taylor (1993) rule in which the central bank increases the nominal interest rate in response to deviations of output and inflation from their steady-state values.\(^{19}\)

\(^{18}\)Taking logs of the rule in (50) leads to a form more familiar in the literature:

\[ \log(R_t^d/R^d) = \rho_y \log(y_t/y) + \rho_\pi \log(\pi_t/\pi) + \epsilon_{t}^{mp}. \]

\(^{19}\)Many authors have stressed that a unique equilibrium may not exist unless $\rho_\pi$ is greater than
2.10 Competitive Equilibrium

To close the model we present the following market clearing conditions:

1. Three labor markets:

\[ H_t^h = \eta^h h_t; \quad H_t^e = \eta^e; \quad H_t^b = \eta^b. \]  \quad (51)

2. The final good market:

\[ Y_t = C_t^h + C_t^e + C_t^b + (1 + \mu)I_t; \]  \quad (52)

where \( C^h \) denote aggregate household’s consumption.

3. The capital good market:

\[ K_{t+1} = (1 - \delta) K_t + \alpha^g R I_t; \]  \quad (53)

where \( K_t \) is aggregate (inclusive of households, entrepreneurs and bankers) capital.

4. The market for deposits:

\[ \frac{q_t \alpha^g [R - b/\Delta \alpha - \mu/q_t \Delta \alpha] I_t}{r_t^d} = \frac{M_t - M_t^c + X_t}{P_t}; \]  \quad (54)

where the left hand side is aggregate demand of deposits by bankers and the right hand side is the supply of deposits of households plus the monetary injections engineered by monetary authorities.

The equilibrium rate return on bank capital is given by the following equation:

\[ r_t^a = \frac{\alpha^g \mu (1 + N_t/A_t)}{G_t \Delta \alpha}. \]  \quad (55)

3 Calibration

The model’s parameters are calibrated in a manner that ensures certain features of the non-stochastic steady state approximately match their empirical counterparts. Further, whenever possible, we follow the calibration procedures of recent contributions to the agency problems literature (Carlstrom and Fuerst, 1997; Bernanke

1. When \( \rho_\tau > 1 \), an increase in the inflation rate of 1 per cent generates an increase in the nominal interest rate of more than 1 per cent which, in turn, increases the real interest rate. In many types of models, this negative reaction of real short term interest rates to upwards pressures in inflation acts as a stabilizer for the economy, ensuring that a unique, stable equilibrium exists.
et al., 1999; Cooley and Quadrini, 1999), in order to facilitate the comparison of our results with those featured in these models.

The discount factor $\beta$ is set at 0.99, so that the average real rate of return on deposits is around 4 percent.\footnote{Recall our interpretation of deposits not as literal bank deposits but rather as relatively illiquid assets that provide a higher return than the most liquid assets like cash.} We set $\gamma$, the curvature parameter on labour effort in the utility function, to a value of 1.5; this implies that the steady-state wage elasticity of labour supply is 2. The scaling parameter $\chi$ is determined by the requirement that steady-state labour effort be 0.3.

The production technology in the final good sector is assumed to take the Cobb-Douglas form

$$Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} H_t^{\theta_e} H_t^{\theta_b}, \quad (56)$$

where recall that the aggregate technology shock, $z_t$, follows a standard AR(1) process:

$$z_t = \rho_z z_{t-1} + \epsilon_t^z, \quad \epsilon_t^z \sim (0, \sigma^z). \quad (57)$$

We set $\theta_k$ to 0.36, $\theta_h$ to 0.639, and $\theta_e = \theta_b = 0.00005$. The autocorrelation parameter $\rho_z$ is 0.95 while $\sigma^z$, the standard deviation of the innovations to $z_t$, is fixed at 0.01.

Monetary policy is assumed to take the form of the original Taylor (1993) rule, so that $\pi = 1.5$ and $\pi_y = 0.5$. The average rate of money growth (and thus the steady-state inflation rate) is set at 5 percent on an annualized basis, a value close to post-war averages in many industrialized countries.

The parameters that remain to be calibrated ($g, b, R, \tau^c, \tau^b$) are linked more specifically to the capital good production and the financial relationship linking entrepreneurs to banks and households. We set $\alpha^{g}$ to 0.97, so that the (quarterly) failure rate is around 3 percent. In our benchmark calibration, we set the remaining parameters in order for the steady-state properties of the model to display the following characteristics: 1) a capital-asset ratio of around 12 percent; 2) a leverage ratio (size of entrepreneurial projects to their accumulated net worth close to 2.0; 3) a ratio of bank operating costs to bank revenues around 5 percent; 4) a ratio of bank revenues to the economy’s GDP of 2 percent; 5) a net return on bank capital (bank equity) equal to 20 percent on an annualized basis.

A capital-asset ratio of 12 percent is on the high end of the empirical distribution of such ratios for US banks reported in Van den Heuvel (2002b); we thus interpret our benchmark calibration as corresponding to a well-capitalized economy. We also experiment with parameter values that lead to the economy’s steady state displaying a low capital-asset ratio. Specifically, we construct an alternative economy where $b$ the parameter governing the importance of the private benefits available to entrepreneurs, is lowered from its benchmark value of 0.09 to 0.06. This reduction in the severity of the moral hazard leads this alternative economy to feature a lower bank capital-asset ratio (8%) as well as a higher leverage In Section 4.3, we show
that the effects of monetary policy tightenings will differ in the two economies.

Once all parameter values are chosen, an approximate solution to the model’s dynamics is found by linearizing all relevant equations around the steady state; we use the methodology described in King and Watson (1998) to do so.

4 Quantitative Findings

4.1 Shock to Bank Capital

In order to build intuition for the mechanism by which bank capital helps propagate shocks through time, Figure 2 reports the impulse responses from a ‘bank capital crunch’. This shock consists in a wealth transfer that shifts some of the banks’ asset holdings to the household sector. Specifically, bank asset holdings \( K^B_t \) are reduced by 10%.

As a result from this shock, bank capital declines immediately (recall equation (44) linking bank capital to their asset holdings). Bank capital is thus scarcer at the aggregate level and the required return on bank capital rises accordingly. This leads the financial contract to rely less on bank capital and more on entrepreneurial net worth in financing projects: consequently, both the capital-asset ratio of the banking sector and entrepreneurial leverage decline. Because the later effect is more important (recall that in steady-state, entrepreneurial net worth is much larger than bank capital) bank lending and thus aggregate investment falls. Because bank capital and entrepreneurial net worth depend on lagged aggregate investment (through retained earnings), the fall in aggregate investment leads to prolonged periods of depressed levels of bank capital and entrepreneurial net worth, and further periods of low bank lending and investment, in the interrelated manner described by equations (41)- (45). Note that these negative effects of shocks to the capital position of banks on bank lending and investment accords well with the evidence presented in Peek and Rosengren (1995) (for American banks) or Peek and Rosengren (1997, 2000) (for branches of Japanese banks operating in the United States).

4.2 Monetary Policy Tightening

Next, Figure 3 presents the basic model’s response to a one percentage point contractionary monetary policy shock \( \epsilon^{mp}_t = -0.01 \). This shock increases the cost of the deposits that banks rely on when organizing the external financing of the entrepreneurs. Banks thus react to this increase in the cost of deposits by tightening lending, which in turn causes a fall in the scale of the investment projects entrepreneurs are able to undertake. This reduction in project scales means that both entrepreneurs and banks cannot leverage their net worth as much as they could before: this is reflected in the fall of the leverage ratio \( I_t/N_t \) and in the increase in
the capital-asset ratio of banks. Note that the counter-cyclical movement in the capital-asset ratio is market-determined.

The intuition for this result is the following. Recall equations (27), repeated here for convenience:

\[ r_t^d ((1 + \mu)i_t - a_t - n_t) = q_t \alpha^a \left( R - \frac{b}{\Delta \alpha} - \frac{\mu}{q_t \Delta \alpha} \right) i_t. \]

This equation states that the per-unit share of project return that can be credibly promised to households for deposit repayments (the right-hand side of the equation) is limited by the double moral hazard problem. This limitation on payments to households means that the increase in \( r_t^d \) must be met with a reduction in the reliance on deposits (a decrease in \( d_t \)) for the financing of a project of given size. In turn this must mean that banks and entrepreneurs will be required to invest more of their own net worth in the financing of that given size project. Said otherwise, the ratios \( a_t/i_t \) and \( n_t/i_t \) must increase: the capital-asset ratio of banks increases while entrepreneurial net worth decreases. As the levels of entrepreneurial net worth \( n_t \) and bank capital \( a_t \) are mostly fixed (they consist of accumulated, retained earnings from past periods), most of the adjustment will have to be borne by a decrease in the size of investment projects bankers can finance, i.e. decreases in lending and in project scale \( i_t \).

Another way to interpret this result is to notice that the increase in the deposit rate \( r_t^d \) worsens the moral hazard problem affecting the relationship between banks and households: as depositors now need to better remunerated for their deposits, it becomes harder for the contract to satisfy their participation constraint will keeping the contract incentive-compatible. To alleviate this worsening of moral hazard, banks are lead to pledge more of their own capital in the financial contract.

Aggregate investment thus falls on impact, while the price of new capital increases slightly, as it would following a standard adverse supply shock. Earnings of banks and entrepreneurs also fall, following the reduced scale of investment projects. Because entrepreneurial net worth and bank capital consists of past retained earnings, which in turn depend directly on the scale of past investment projects, the initial fall in investment leads to extended declines in the stock of entrepreneurial net worth and bank capital. These declines are responsible for helping propagating the shock over time, as the interest rate returns to its steady-state level immediately after the impact period. Low net worth and bank capital continue to restrict the scale of investment projects for several periods, which leads to persistent declines in aggregate physical capital and thus also in aggregate output.
4.3 Monetary Policy Tightening: High versus Low Capital-Asset Ratio Economies

In order to better assess the influence that bank capital holds over the transmission of monetary policy shocks, Figure 4 compares the responses of two economies following the same contractionary shock. First, the responses displayed in Figure 3 for the benchmark economy are repeated (the full lines in Figure 4). The second set of responses (the dashed lines) reflect those of an economy where banks have, on average, a lower capital-asset ratio.

This alternative economy arises from a recalibration of the model that features less severe agency problems. Specifically, the parameter $b$, governing the extent of private benefits available to entrepreneurs, is lower. This result in an economy with high leverage, where the steady-state capital-asset ratio of banks is now 8% percent rather than the higher rate of 12% in the benchmark.\(^{21}\)

The responses of the low capital-asset ratio economy, while possessing approximately the same persistence as those from the benchmark, now exhibit much more substantial amplitude. As mentioned above, the lower extent of the asymmetric information problem results in a highly-leveraged economy, relative to the benchmark. A given increase in the cost of deposits $r^d_t$, through this higher leverage, tightens bank lending and the market-generated capital-asset ratio more substantially, leading to more significant declines in the scale of projects and thus aggregate investment.

To build intuition for the differentiated responses following monetary policy shocks, consider once more equation (27). The lower value of $b$ means that in the steady state, the per-unit share of investment projects that can be allocated to households is higher. For a given (steady-state) value of the nominal deposit rate, banks thus attract more deposits and rely less on their own capital to finance given-size projects. The steady-state value of $a_t/i_t$ is thus smaller, a fact reflected in the lower capital-asset ratio. This relatively small contribution of bank capital to external financing makes it difficult for banks to replace deposits by bank capital when the costs of the former increase following the monetary tightening. These relatively low levels of bank capital thus lead to larger increases in the capital asset-ratio of banks, as well as larger decreases in project size. Another way to interpret this result is that the increase in deposit rates worsens the moral hazard problem affecting the relationship between banks and households, but worsens it less in the high capital-asset ratio economy, in which banks hold more capital on average than in the low capital-asset ratio economy. In that economy pledging more capital per unit of investment project to replace household deposits is more difficult due to the relative scarcity of accumulated bank capital.

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\(^{21}\)The remaining steady-state characteristics resulting from this alternative calibration of the model are presented in Table 2.
A given increase in \( r^d_t \) thus leads to more substantial decreases in aggregate investment in the low capital-asset ratio economy. In turn, this deeper decline in investment leads to similar, deeper declines in future entrepreneurial net worth and bank capital (through the retained earnings effect), which continue to propagate the shock in subsequent periods, after the initial effects of the rate increase have dissipated.

5 The Extended Model

The financial contract linking banks, entrepreneurs, and households, which makes the production of capital goods possible, represents the essential component of the present paper’s explanation of the importance of bank capital in the propagation mechanism of monetary policy. In this section, we extend the model in which this contract is embedded, in order to make our analysis easily comparable to those contained in standard monetary versions of the real-business cycle model. Specifically, we assume that households are risk-averse, that the cash-in-advance constraint faced by households is richer than the one we have described so far, and, finally, that financing from banks is required not only for entrepreneurs but also for final good producers.\(^{22}\) We now discuss the modifications to the model’s equations these extensions require.

First, the introduction of risk-aversion in the utility of households implies that their intertemporal maximization problem is now the following:

\[
\max_{\{c^i_t, M_{t+1}, M^e_t, h_t, k^h_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(c^i_t) - \chi \log(1 - h_t - v_t) \right],
\]

with \( h_t \) hours worked and \( v_t \) the time costs of adjusting financial portfolios. The presence of risk-aversion means that households now seek to smooth their consumption paths, an objective that was absent in the basic model. Consumption smoothing implies that households are now less ready to experience big swings in consumption in order to take full advantage of low price of investment goods, for example, which will have an impact on the economy’s response to monetary shocks.

The assumption of risk-aversion, however, does not lend itself well to the definition of the financial contract in equations (16) to (22) which depended on risk-neutrality of all three parties to the contract. We thus introduce an insurance scheme that allows households to insure themselves perfectly against all idiosyncratic risk related to the financial contract (this follows Andolfatto (1996) and Cooley and Quadrini (1999)). The return on their deposits is now supplemented by the (net) receipts from this insurance, so that the (now risk-free) rate of return on financial

\(^{22}\) These features appear in the limited-participation models of Christiano and Gust (1999) and Cooley and Quadrini (1999).
assets is \( r^d_t \). This effectively renders the households risk-neutral with respect to the financial contract because idiosyncratic risk has been diversified away and the production of capital good does not feature any aggregate risk.\(^{23}\) For the other household decisions (on labour supply, physical capital holdings, etc.) the insurance scheme allows us to treat the model as a representative agent one. Further details are available in Appendix A.

The second additional feature that we introduce in the optimization problem of households is to make the cash-in-advance constraint more comparable to those used by recent monetary models (such as Cooley and Quadrini (1999)). In order to do so, we assume that the current wage income of households is paid to them in time to be available for purchasing consumption in the current period. This implies that the distortion in labour supply that the basic model featured is now eliminated. Further, we also assume that the (net) purchases of physical capital undertaken by households must be made with cash. Inflation, which in the basic model introduced a distortion in the labour supply decisions of households, now distorts their investment demand.

The combination of risk aversion (along with perfect insurance) and the inclusion of wage income and physical capital purchases in the cash-in-advance constraint leads us to rewrite equations (2) and (3) so that the new cash-in-advance constraint is

\[
 c^h_t + q_t \left( k^h_{t+1} - (1 - \delta) k^h_t \right) \leq \frac{M^c_t}{P_t} + w^h_t h_t; \tag{59}
\]

while the budget constraint is now:

\[
 \frac{M_{t+1}^c}{P_t} = \frac{M_t^c}{P_t} + w^h_t h_t - c_t^h - q_t \left( k^h_{t+1} - (1 - \delta) k^h_t \right) + r^d_t \left( \frac{M_t^c - M^c_t + X_t}{P_t} \right) + r^d_t k^h_t. \tag{60}
\]

The assumption that wage income is present in the cash-in-advance constraint begs the question of how can final good producers pay the households’ wage income in cash. To resolve this issue, we are lead to the third extension of the basic structure. We postulate that final good producers also borrow funds from banking institutions, in order to pay for their wage bill.\(^{24}\) As was expressed above, there is no informational asymmetry in this sector, so that this type of borrowing can proceed without any agency problems. Correspondingly, bank capital is not necessary to conduct this type of lending because moral hazard and thus monitoring is not an issue. While we could envision banks in our basic model engaging in the two types of lending, we instead posit the existence of two types of financial intermediaries. On the one hand, banks, who lend to entrepreneurs and use their monitoring technology to resolve the moral hazard affecting production in that sector; the private nature of

\(^{23}\)See (Carlstrom and Fuerst, 1998, pg. 587) for further discussion.

\(^{24}\)In the literature, such loans are often thought to correspond to the ‘working capital’ or ‘lines of credit’ of big firms.
this monitoring giving rise to the need for bank capital explained in the preceding sections of the paper. On the other hand, banking agents (or ‘brokers’) that simply transfer funds from households to final good producers. Our use of these two types of lending and financial intermediaries is reminiscent of the modelling framework of Bernanke and Gertler (1985).\textsuperscript{25} Note that each financial intermediary must offer households the same rate of return on deposits for the two types of lending to coexist in equilibrium. Further, because the second type of lending is costless, brokers make zero profits.

Consequently, the market clearing condition for deposits now reflects the fact that total supply, which arises from households’ savings decision and monetary injections (represented by $\frac{\bar{M}_t - \bar{M}_t^c + X_t}{P_t}$) must now be divided by the two different classes of lending; equation (54) thus becomes:

$$\frac{[R - b/\Delta \alpha - \mu/q_t \Delta \alpha] I_t}{r_t^d} + w_t^h H_t = \frac{\bar{M}_t - \bar{M}_t^c + X_t}{P_t}.$$  \hspace{0.5cm} (61)

Finally, note that the market-clearing wage rate for households must now reflect the fact that wage costs are borrowed, making the nominal interest rate a distortion that affects labour demand. Consequently, the complete model adds another dimension along which monetary policy contractions affect the economy, by reducing the demand for labour stemming from the activities of final good producers. Equation (12) is now replaced by

$$w_t^h = z_t F_2(K_t, H_t, H_t^e, H_t^b)/r_t^l,$$ \hspace{0.5cm} (62)

where $r_t^l$ is the rate at which final good producers are able to get funding from the financial ‘brokers’. Perfect competition and the fact that the activities of the brokers are costless ensure that $r_t^l = r_t^d$ in equilibrium.

6 The Extended Model: Results

The calibration of the complete model follows the steps detailed in Section 3. Because the consumption-smoothing motive only affects the dynamic responses of the economy, and not the features of the non-stochastic steady state, it does not impinge on the calibration.

A natural question to ask is whether the differentiated effects of monetary policy tightenings discussed in Section 4.3 remain a feature of the extended model. To this end, Figure 5 reports the result of a similar experiment to the one conducted in that section. The figure graphs the responses of two economies to the standard monetary

\textsuperscript{25}Further, note that our definition of the role played by these ‘brokers’ is similar to the one played by banks in the standard monetary models such as Christiano and Gust (1999).
policy tightening: the full lines depict the responses of the benchmark, high capital-asset ratio economy, while the dashed lines illustrates how the low capital-asset ratio economy reacts to the shock.

First, notice that the responses of the benchmark economy (the full lines), while qualitatively similar to the corresponding one in the basic model (Figure 3) exhibit smoother paths. The limited intertemporal elasticity of substitution (compared to the risk-neutral case of the basic model) now leads the economy to converge back to initial steady-state values much faster following the shock. Further, compared to Figure 3, the responses of investment are now characterized by a hump-shape response.\(^{26}\)

The comparison between the two cases of Figure 5 shows that the effects identified in Section 4.3 are present in the extended model. The increase in deposit rates leads to a more substantial decline in aggregate investment in an economy with a low capital-asset ratio. As was the case in the basic model, while the increase in \(r_t^d\) worsens the moral hazard problem affecting the relationship between banks and households, it worsens it less in the high capital-asset ratio economy, in which banks hold more capital on average. The limited intertemporal elasticity of substitution characterizing the extended model leads to the declines in output being much less extensive and persistent in both economies (compare with Figure 4). For that reason, the differences in the output responses are less pronounced that they were in the basic model.

Figure 6 illustrates that the ability of bank capital to amplify shocks affecting the economy is also present in the case of technology shocks, although not to the same extent. The graphs depict the responses of the two economies following a negative innovation to the productive capacities of the final good producers (\(\epsilon_t^z = -0.01\)). The high capital-asset ratio economy is reflected by the full lines while dashed lines refer to the low capital asset-ratio environment. The reduction in the productive capacities implies that the rental rate on physical capital will be low for several periods in the future (recall that the technology shocks are very persistent). This lowers the demand for physical capital, which results in sharp drops in \(q_t\), the price of newly created capital goods and aggregate investment. The drop in \(q_t\) also leads to immediate decreases in the net worth of entrepreneurs and banks, decreases which are further compounded in the following periods by the decreases in earnings that the low levels of aggregate investment entail. The initial decline in aggregate investment is much more pronounced in the economy with the low capital-asset ratio; while this differentiated response leads to similar difference in the paths of entrepreneurial net worth and bank capital, they do not affect the response of output significantly, which is dominated by the direct effect of the lower productive capacities.

\(^{26}\)On that dimension, our model is thus able to replicate the hump shape in the response of investment that Carlstrom and Fuerst (2001) report. However, note that we are able to generate this hump shape in an environment with finite-lived agents, whereas the found that only their framework with infinitely-lived, impatient entrepreneurs generated a hump shape in investment.
7 Conclusion

This paper presents a monetary, quantitative, dynamic model of the interrelations between bank capital and entrepreneurial net worth, on the one hand, and monetary policy and economic activity, on the other. The model features two distinct sources of moral hazard. The first, arising because entrepreneurs can privately influence the probability of success of the projects they engage in even in the presence of bank monitoring, leads banks to require that entrepreneurs invest their own net worth in the projects they undertake. The second, which takes its source in the fact that the monitoring activities of banks are themselves not publicly observable, induces households to require that banks invest their own capital in entrepreneurial projects before depositing funds at banks. Entrepreneurial net worth and bank capital thus key determinants of the propagation over time of shocks affecting the economy, even after the initial, direct impact of the original disturbances have faded away.

Quantitative simulations conducted with the model show that bank capital can have a significant impact on the amplitude of the effects of monetary policy contractions on the economy. Specifically, highly-leveraged economies will experience deeper recessions following contractionary shocks than those affecting economies relying on more substantially capitalized banking systems. In all of our simulations, the (market-determined) capital-asset ratio of banks reacts counter-cyclically to shocks, increasing as adverse shocks affect the economy; this countercyclical behaviour is a result of the reduced general levels of leverage that the economy can sustain following these shocks.

In future work, we plan to experiment with a version of the model that would position the double incidence of moral hazard in the sector producing the final good, rather than the present situation where it is the creation of new capital goods that is affected by the agency problems. Contrasting these two frameworks would allow us to link our results better to those in Carlstrom and Fuerst (1998, 2001) and the comparisons between the ‘output’ and ‘investment’ model they discuss. Further, it would be useful to analyze environments where the distribution of entrepreneurial net worth and bank capital matters for the aggregate implications of the model.
References


Timing of Events

- **1. Aggregate shocks are realized**
- **2. Time**
- **3. Households make consumption and deposit decisions**
- **4. Final good production takes place**
- **5. Time**
- **6. Households, banks, and entrepreneurs agree to finance projects**
- **7. Time**
- **8. Returns are realized (public) and shared between the 3 agents**
- **9. Time**
- **t+1**
- **Surviving agents buy capital for future periods; exiting agents sell their capital and consume**
- **t+1**
- **- Households choose next period’s money and capital holdings**
- **- Newborn banks and entrepreneurs enter the economy**

1. Timing of Events

<table>
<thead>
<tr>
<th>Event</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Aggregate shocks are realized</td>
<td>t-1</td>
</tr>
<tr>
<td>2. Final good production takes place</td>
<td>t</td>
</tr>
<tr>
<td>3. Households make consumption and deposit decisions</td>
<td>t+1</td>
</tr>
<tr>
<td>4. Households, banks and entrepreneurs agree to finance projects</td>
<td>t+2</td>
</tr>
<tr>
<td>5. Returns are realized (public) and shared between the 3 agents</td>
<td>t+3</td>
</tr>
<tr>
<td>6. Surviving agents buy capital for future periods; exiting agents sell their capital and consume</td>
<td>t+4</td>
</tr>
<tr>
<td>7. Households choose next period’s money and capital holdings</td>
<td>t+5</td>
</tr>
<tr>
<td>8. Newborn banks and entrepreneurs enter the economy</td>
<td>t+6</td>
</tr>
</tbody>
</table>
Figure 2. A ‘Capital Crunch’: Basic Model
Figure 3. Contractionary Monetary Policy Shock: Basic Model
Figure 4. Contractionary Monetary Policy Shock: Basic Model
High versus Low Capital Asset Ratio Economies
Figure 5. Contractionary Monetary Policy Shock: Extended Model
High versus Low Capital Asset Ratio Economies

[Graphs illustrating the impact of contractionary monetary policy shock on various economic indicators such as aggregate output, aggregate investment, price of capital, entrepreneurial net worth, bank capital, capital-asset ratio, entrepreneurial leverage, bank deposit rate, and inflation, comparing high and low capital-asset ratio economies.]
Figure 6. Adverse Technology Shock: Extended Model
High versus Low Capital Asset Ratio Economies
Appendix A: Insurance within Risk-averse Households in the Extended Model

Following Andolfatto (1996) and Cooley and Quadrini (1999), we assume the existence of an (actuarially fair) insurance market that allows households to eliminate the idiosyncratic risk inherent to the financial contracts. Specifically, the household can purchase \( y_t \) real units of insurance, at the price \( j_t \). These units are paid to the household in the event that he receives a zero return from his bank deposits. The budget constraint (3) can now be rewritten as follows:

\[
\frac{M_{t+1}}{P_t} + q_t k_{t+1}^h + j_t y_t = \frac{M^c_t}{P_t} - c^h_t + s_t \left( \frac{r^d_t}{\alpha^g} \left( \frac{M_t - M^c_t + X_t}{P_t} \right) \right) + w^h_t h_t + (r^k_t + q_t (1 - \delta)) k^h_t + (1 - s_t) y_t; \tag{A.1}
\]

where recall that \( s_t \) is an indicator function that takes a value of 1 if the bank deposits are repaid (and insurance payments are not necessary) and a value of 0 if deposits have a zero return and insurance payments are made. Notice that we have written the constraint in a manner that removes any dependence in the choices made by households over the value of \( s_t \); we are anticipating the result that this is their optimal response.

Note that the first-order condition for the choice of \( y_t \) is the following:

\[
j_t = E_t[1 - s_t] = 1 - \alpha^g, \tag{A.2}
\]

which repeats the statement that the insurance market is actuarially fair. That feature, as well as the strict concavity of the utility function in consumption, implies that households will seek to remove any risk to their financial income flows. This would require financial revenues (including net insurance revenues) when deposits bring no returns should equal revenues when deposits pay their promised returns (net of insurance premiums). We thus have:

\[
y_t - j_t y_t = \frac{r^d_t}{\alpha^g} \left( \frac{M_t - M^c_t + X_t}{P_t} \right) - j_t y_t, \tag{A.3}
\]

which simplifies to

\[
y_t = \frac{r^d_t}{\alpha^g} \left( \frac{M_t - M^c_t + X_t}{P_t} \right). \tag{A.4}
\]

Inserting this result back into (A.1) makes clear that the budget constraint is now similar to the one in a representative-agent economy with no idiosyncratic risk to household deposits:

\[
\frac{M_{t+1}}{P_t} + q_t k_{t+1}^h = \frac{M^c_t}{P_t} - c^h_t + r^d_t \left( \frac{M_t - M^c_t + X_t}{P_t} \right) + w^h_t h_t + (r^k_t + q_t (1 - \delta)) k^h_t. \tag{A.4}
\]