Predictability of Interest Rates and Interest-Rate Portfolios*

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ABSTRACT

Due to the near unit-root behavior of interest rates, the movements of individual interest-rate series are inherently difficult to forecast. In this paper, we propose an innovative way of applying dynamic term structure models to forecast interest-rate movements. Instead of directly forecasting the movements based on the estimated factor dynamics, we use the dynamic term structure model as a decomposition tool and decompose each interest-rate series into two components: a persistent component captured by the dynamic factors, and a strongly mean-reverting component given by the pricing residuals of the model. With this decomposition, we form interest-rate portfolios that are first-order neutral to the persistent dynamic factors, but are fully exposed to the strongly mean-reverting residuals. We show that the predictability of these interest-rate portfolios is significant both statistically and economically, both in sample and out of sample.

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Predictability of Interest Rates and Interest-Rate Portfolios

Forecasting interest-rate movements attracts great attention from both academics and practitioners. A central theme underlying the traditional literature is to exploit the information content in the current term structure to forecast the future movement of interest rates. These studies formulate forecasting relations based on various forms of the expectation hypothesis.[1] More recently, several researchers apply the theory of dynamic term structure models in further understanding the links between the cross-sectional behavior (term structure) of interest rates and their time-series dynamics. They propose and test model specifications that can best explain the empirical evidence on the expectation hypothesis and the properties of excess bond returns.[2] In this paper, we propose a new way of applying multivariate dynamic term structure models in forecasting interest-rate movements.

Modern dynamic term structure models accommodate multiple dynamic factors in governing the interest-rate movements. Several empirical studies also identify nonlinearity in the interest-rate dynamics.[3] Thus, if the focus is on forecasting, a better formulation should be in a multivariate framework, and potentially in nonlinear forms, rather than in the form of simple linear regressions designed to verify the expectation hypothesis and the behavior of market risk premium. Furthermore, if interest rates are composed of multiple factors, do these factors exhibit the same predictability? If not, can we separately identify these factors from the interest-rate series and forecast only the more predictable components while hedging away the less predictable ones?

We address these questions based on weekly data on 12 eurodollar LIBOR and swap rate series from May 11, 1994 to December 10, 2003 at maturities from one month to 30 years. We perform our analysis within the framework of three-factor affine dynamic term structure models. We apply the unscented Kalman filter to estimate the term structure models and to extract the interest-rate factors and the pricing errors. Similar to earlier findings in the literature, we observe that the estimated three interest-rate factors are

[1] Prominent examples include Roll (1970), Fama (1984), Fama and Bliss (1987), Mishkin (1988), Fama and French (1989, 1993), Campbell and Shiller (1991), Evans and Lewis (1994), Hardouvelis (1994), Campbell (1995), Bekaert, Hodrick, and Marshall (1997, 2001), Longstaff (2000), Bekaert and Hodrick (2001), and Cochrane and Piazzesi (2005).

[2] See, for example, Backus, Foresi, Mozumdar, and Wu (2001), Dai and Singleton (2002), Duffee (2002), and Roberds and Whiteman (1999).

[3] Examples include Aït-Sahalia (1996a,b), Stanton (1997), Chapman and Pearson (2000), Jones (2003), and Hong and Li (2005).

highly persistent. When we try to forecast four-week ahead interest-rate changes based on the estimated factor dynamics, the performance is no better than the basic assumption of random walk.

In pricing 12 interest rate series with three factors, we will observe pricing errors. The unscented Kalman filter estimation technique accommodates these pricing errors in the form of measurement errors. Thus, the estimation procedure decomposes each interest-rate series into two components: the model-implied fair value as a function of the three factors, and the pricing error that captures the idiosyncratic movement of each interest-rate series. We can think of the pricing errors as the higher dimensional dynamics that are not captured by the three factors. Compared to the highly persistent interest-rate factors, we find that the pricing errors on the interest-rate series are strongly mean reverting.

Based on this observation, we propose a new way of applying the dynamic term structure models in forecasting interest-rate movements. Instead of using the estimated dynamic factors to forecast the movements of each individual interest-rate series, we use the model as a decomposition tool. We form interest-rate portfolios that are first-order neutral to the persistent interest-rate factors, but are fully exposed to the more mean-reverting idiosyncratic components. To illustrate the idea, we use swap rates at two-, five-, ten-, and 30-year maturities to form such a portfolio. We find that, in contrast to the low predictability of the individual swap rate series, the portfolio shows strong predictability. For example, in forecasting interest-rate changes over a four-week horizon based on an AR(1) specification, we obtain R-squares less than 2% for all 12 individual interest-rate series. In contrast, the forecasting regression on the swap-rate portfolio generates an R-square of 14%.

To generalize, we use the 12 interest-rate series to form 495 different combinations of four-instrument interest-rate portfolios that are hedged with respect to the three persistent interest-rate factors. We find that all these 495 portfolios show strong predictability. The R-squares from the AR(1) forecasting regression range from 7.84% to 55.72%, with an average of 19.32%, illustrating the robustness of the portfolio-construction strategy in enhancing the predictability of the portfolio return.

We use the same idea to form two-instrument interest-rate portfolios to hedge away the most persistent interest-rate factor, and three-instrument interest rate portfolios to hedge away the first two factors. We find that the predictability of most of the two- and three-instrument portfolios remains weak, indicating that we must hedge away all the first three factors to generate portfolios with strong predictability.

To investigate the economic significance of the predictability of the four-instrument interest-rate portfolios, we follow the practice of Kandel and Stambaugh (1996) and devise a simple mean-variance investment strategy on the four-instrument portfolios over a four-week horizon that exploits the portfolios' strong predictability. During our sample period, the investment exercise generates high premiums with low standard deviation. The annualized information ratio estimates range from 0.36 to 0.94, with an average of 0.7, illustrating the strong economic significance of the predictability of the four-instrument portfolios. Furthermore, the excess returns from the investment exercise show positive skewness, and the average positive premiums cannot be fully explained by systematic factors in the stock, corporate bond, and interest-rate options markets.[4] Given their independence of systematic market factors, we hypothesize that the average positive excess returns are premiums to bearing short-term liquidity shocks to individual interest-rate swap contracts. We construct measures that proxy the absolute magnitudes of contract-specific liquidity shocks in the swap market and find that larger liquidity shocks at a given date often lead to more positive excess returns ex post for investments put on that day.

The literature has linked the three persistent interest-rate factors to systematic movements in macroeconomic variables such as the long-run expected inflation rate, the output gap, and the short-run Fed policy shocks. Within a short investment horizon, e.g., four weeks, these systematic movements are difficult to predict. The four-instrument interest-rate portfolios that we have constructed in this paper are relatively immune to these persistent and systematic movements, but are exposed to more transient shocks due to temporary supply-demand variations. Trading against these shocks amounts to providing short-term liquidity to the market and hence bearing a transient liquidity risk. Our analysis shows that historically the swap market has assigned a relatively high average premium to the liquidity risk, potentially due to high barriers to entry, limits of arbitrage, and compensation for intellectual capital (Duarte, Longstaff, and Yu (2005)). Our subsample analysis further shows that the risk premium has declined over the past few years, a sign of increasing liquidity and efficiency in the interest-rate swap market. Nevertheless, the predictability of our four-instrument interest-rate portfolios remains much stronger than that of individual interest-rate series

[4]The high information ratios and positive skewness are also observed in excess returns on popular fixed income arbitrage strategies (Duarte, Longstaff, and Yu (2005)). In contrast, excess returns from other high-information-ratio investment strategies reported in the literature often exhibit negatively skewed distributions, e.g., selling out-of-the-money put options (Coval and Shumway (2001), Goetzmann, Ingersoll, Spiegel, and Welch (2002)), shorting variance swap contracts (Carr and Wu (2004)), and merger arbitrage (Mitchell and Pulvino (2001)).

across different sample periods, both in sample and out of sample. Furthermore, our results are robust to different model specifications within the three-factor affine class.

Our new application of the dynamic term structure models sheds new insights for future interest-rate modeling. The statistical and economic significance of the predictability of the interest-rate portfolios point to a dimension of deficiency in three-factor dynamic term structure models. These models capture the persistent movements in interest rates, but discard the transient interest-rate movements. Yet, not only can these transient movements be exploited in investment decisions to generate economically significant premiums, as we have shown in this paper, but they can also play important roles in valuing interest-rate options (Collin-Dufresne and Goldstein (2002) and Heidari and Wu (2002, 2003)) and in generating the observed low correlations between non-overlapping forward interest rates (Dai and Singleton (2003)).

The remainder of this paper is structured as follows. Section 1 describes the specification and estimation of the three-factor affine dynamic term structure models that underlie our analysis. We also describe the data and estimation methodology, and summarize the estimation relevant results in this section. Section 2 investigates the statistical significance of the predictability of interest rates and interest-rate portfolios. Section 3 studies the economic significance of the interest-rate portfolios from an asset-allocation perspective. Section 4 performs robustness analysis by examining the predictability variation across different subsamples, in sample and out of sample, and under different model specifications. Section 5 concludes.

1. Specification and Estimation of Affine Dynamic Term Structure Models

We perform our analysis based on affine dynamic term structure models (Duffie and Kan (1996) and Duffie, Pan, and Singleton (2000)). The specification and estimation of affine term structure models have been studied extensively in the literature, e.g., Backus, Foresi, Mozumdar, and Wu (2001), Dai and Singleton (2000, 2002), Duffee (2002), and Duffee and Stanton (2000). We follow these works in specifying and estimating a series of standard three-factor models in this section. However, our proposed applications of the estimated models in the subsequent sections are completely different from the previous studies.

1.1. Model specification

To fix notation, we consider a filtered complete probability space $\{\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{0 \le t \le \mathcal{T}}\}$ that satisfies the usual technical conditions with $\mathcal{T}$ being some finite, fixed time. We use $X \in \mathbb{R}^n$ to denote an n-dimensional vector Markov process that represents the systematic state of the economy. We assume that for any time $t \in [0, \mathcal{T}]$ and maturing date $T \in [t, \mathcal{T}]$, the fair value at time t of a zero-coupon bond with time-to-maturity $\tau = T - t$ is fully characterized by $P(X_t, \tau)$ and that the instantaneous interest rate r is defined by continuity:

$$r(X_t) \equiv \lim_{\tau \downarrow 0} \frac{-\ln P(X_t, \tau)}{\tau}. \tag{1}$$

We further assume that there exists a risk-neutral measure $\mathbb{P}^*$ such that the fair values of the zero-coupon bonds and future instantaneous interest rates are linked as follows,

$$P(X_t, \tau) = \mathbb{E}_t^* \left[\exp\left(-\int_t^{t+\tau} r(X_s) ds \right) \right], \tag{2}$$

where $\mathbb{E}_t^*[\cdot]$ denotes the expectation operator under measure $\mathbb{P}^*$ conditional on the filtration $\mathcal{F}_t$.

Under the affine class, the instantaneous interest rate is an affine function of the state vector,

$$r(X_t) = a_r + b_r^\top X_t, \tag{3}$$

and the state vector follows affine dynamics under the risk-neutral measure $\mathbb{P}^*$,

$$dX_t = \kappa^* (\theta^* - X_t) dt + \sqrt{S_t} dW_t^*, \tag{4}$$

where S_t is a diagonal matrix with its ith element given by

$$[S_t]_{ii} = \alpha_i + \beta_i^\top X_t, \tag{5}$$

with α_i being a scalar and β_i an n-dimensional vector. Under these specifications, the fair-values of the zero-coupon bonds are exponential affine in the state vector X_t,

$$P(X_t, \tau) = \exp\left(-a(\tau) - b(\tau)^\top X_t\right), \tag{6}$$

where the coefficients can be solved from a set of ordinary differential equations (Duffie and Kan (1996)).

Dai and Singleton (2000) classify the affine models into a canonical $A_m(n)$ structure such that

$$[S_t]_{ii} = \begin{cases} X_{t,i} & i = 1, \cdots, m; \\ 1 + \beta_i^\top X_t, & i = m+1, \cdots, n, \end{cases} \tag{7}$$

$$\beta_i = [\beta_{i1}, \cdots, \beta_{im}, 0, \cdots, 0]^\top. \tag{8}$$

The normalization amounts to setting $\alpha_i = 0$ and $\beta_i = 1_i$ for $i \leq m$, where 1_i denotes a vector with its ith element being one and other elements being zero. In essence, the first m factors follow square-root dynamics.

To derive the state dynamics under the physical measure $\mathbb{P}$, Dai and Singleton (2000) assume that the market price of risk is proportional to $\sqrt{S_t}$, $\gamma(X_t) = \sqrt{S_t}\lambda_1$, where λ_1 is an $n \times 1$ vector of constants. Duffee (2002) proposes a more general specification,

$$\gamma(X_t) = \sqrt{S_t}\lambda_1 + \sqrt{S_t^-}\lambda_2 X_t, \tag{9}$$

where λ_2 is an $n \times n$ matrix of constants and S_t^- is a diagonal matrix with its ith diagonal element given by

$$[S_t^-]_{ii} = \begin{cases} 0, & i = 1, \cdots, m; \\ \left(1 + \beta_i^\top X_t\right)^{-1}, & i = m+1, \cdots, n. \end{cases} \tag{10}$$

Under the general market price of risk specification in (9), the $\mathbb{P}$-dynamics of the state vector remains affine,

$$dX_t = \kappa(\theta - X_t)\,dt + \sqrt{S_t}dW_t, \tag{11}$$

with

$$[\kappa\theta]_i = [\kappa^*\theta^*]_i + \begin{cases} 0, & i = 1, \cdots, m; \\ \lambda_{1,i} & i = m+1, \cdots, n, \end{cases} \tag{12}$$

$$\kappa_{i,\cdot} = \kappa^*_{i,\cdot} - \begin{cases} \lambda_{1,i} 1_i^\top, & i = 1, \cdots, m; \\ \left(\lambda_{1,i}\beta_i^\top + \lambda_{2,i}\right) & i = m+1, \cdots, n, \end{cases} \tag{13}$$

where $\kappa_{i,\cdot}$ and $\lambda_{2,i}$ denote the ith row of κ and λ_2, respectively, and $\lambda_{1,i}$ denotes the ith element of the vector. Since the first m rows of λ_2 do not enter the dynamics of X, we normalize $\lambda_{2,i} = 0$ for $i = 1, \cdots, m$.

For our analysis, we follow the common practice in the literature in focusing on three-factor models. We estimate four generic $A_m(3)$ models with $m = 0, 1, 2, 3$ and with the general market price of risk specification in equation (9). In the case of $m = 0$, S_t and S_t^- become identity matrices. The three factors follow a multivariate Ornstein-Uhlenbeck process,

$$dX_t = -\kappa X_t dt + dW_t, \tag{14}$$

where we normalize the long-run mean $\theta = 0$. For identification purpose, we restrict the κ matrix to be lower triangular. In this case, the essentially affine market price of risk specification in equation (9) becomes

$$\gamma(X_t) = \lambda_1 + \lambda_2 X_t, \tag{15}$$

so that the risk-neutral state dynamics becomes

$$dX_t = \kappa^* \left(\theta^* - X_t\right) dt + dW_t^*, \quad \kappa^*\theta^* = -\lambda_1, \quad \kappa^* = \kappa + \lambda_2. \tag{16}$$

We also confine λ_2 and hence κ^* to be lower triangular. For $A_m(3)$ models with $m = 1, 2, 3$, we normalize $\theta_i^* = 0$ for $i = m+1, c \ldots, n$. We also normalize κ^* and κ to be lower triangular matrices.

1.2. Data and estimation

We estimate the four affine dynamic term structure models and analyze the predictability of interest rates based on five eurodollar LIBOR and seven swap rate series. The LIBOR rates have maturities at one, two, three, six and 12 months, and the swap rates have maturities at two, three, five, seven, ten, 15, and 30 years. For each rate, the Bloomberg system computes a composite quote based on quotes from several broker dealers. We use the mid quotes of the Bloomberg composite for model estimation. The data are sampled weekly (every Wednesday) from May 11, 1994 to December 10, 2003, 501 observations for each series.

LIBOR rates are simply compounded interest rates, relating to the values of the zero-coupon bonds by,

$$LIBOR(X_t, \tau) = \frac{100}{\tau}\left(\frac{1}{P(X_t,\tau)} - 1\right), \tag{17}$$

where the time-to-maturity τ is computed based on actual over 360 convention, starting two business days forward. The swap rates relate to the zero-coupon bond prices by,

$$SWAP(X_t, \tau) = 100h \times \frac{1 - P(X_t,\tau)}{\sum_{i=1}^{h\tau} P(X_t, i/h)}, \tag{18}$$

where τ denotes the maturity of the swap and h denotes the number of payments in each year. For the eurodollar swap rates that we use, the number of payments is twice per year, $h = 2$, and the day counting convention is 30/360.

Table 1 reports the summary statistics of the 12 LIBOR and swap rates. The average interest rates have an upward sloping term structure. The standard deviation of the interest rates shows a hump-shaped term structure that reaches its plateau at one-year maturity. All interest-rate series show small estimates for skewness and excess kurtosis.

The interest rates are highly persistent. The first-order weekly autocorrelation ranges from 0.985 to 0.995, with an average of 0.991. An AR(1) dynamics approximates well the autocorrelation function at higher orders. If we assume an AR(1) dynamics for interest rates, an average weekly autocorrelation estimate of 0.991 implies a half life of 78 weeks.[5] Therefore, if we make forecasting and investment decisions

[5]We define the half life as the number of weeks for the weekly autocorrelation (ϕ) to decay to half of its first-order value: Half-life (in weeks) $= \ln(\phi/2)/\ln(\phi)$.

based on the mean-reverting properties of interest rates, we need a very long investment horizon for the mean reversion to actually materialize.

We cast the four dynamic term structure models into state-space forms and estimate the model parameters using the quasi-maximum likelihood method based on observations on the 12 interest-rate series. Under this estimation technique, we regard the three interest-rate factors as unobservable states and the LIBOR and swap rates as observations. The state propagation equation follows a discrete-time version of equation (11),

$$X_{t+1} = A + \Phi X_t + \sqrt{Q_t}\varepsilon_{t+1}, \tag{19}$$

where $\varepsilon \sim IIN(0, I)$, $\Phi = \exp(-\kappa\Delta t)$ with $\Delta t = 1/52$ as the discrete-time (weekly) interval, $A = \theta(I - \Phi)$, and $Q_t = S_t \Delta t$.

We define the measurement equation using the 12 LIBOR and swap rates, assuming additive and normally-distributed measurement errors,

$$y_t = \begin{bmatrix} LIBOR(X_t, i) \\ SWAP(X_t, j) \end{bmatrix} + e_t, \quad cov(e_t) = \mathcal{R}, \qquad \begin{array}{l} i = 1, 2, 3, 6, 12 \text{ months} \\ j = 2, 3, 5, 7, 10, 15, 30 \text{ years}. \end{array} \tag{20}$$

For the estimation, we assume that the measurement errors on each series are independent but with distinct variance: $\mathcal{R}_{ii} = \sigma_i^2$ and $\mathcal{R}_{ij} = 0$ for $i \neq j$.

When both the state propagation equation and the measurement equations are Gaussian and linear, the Kalman (1960) filter generates efficient forecasts and updates on the conditional mean and covariance of the state vector and the measurement series. In our application, the state propagation equation in (19) is Gaussian and linear, but the measurement equation in (20) is nonlinear. We use the unscented Kalman filter (Wan and van der Merwe (2001)) to handle the nonlinearity. The unscented Kalman filter directly approximates the posterior state density using a set of deterministically chosen sample points (sigma points). These sample points completely capture the true mean and covariance of the Gaussian state variables, and when propagated through the nonlinear functions of LIBOR and swap rates, capture the posterior mean and covariance accurately to the second order for any nonlinearity.

Let $\bar{y}_{t+1}$ and $\overline{A}_{t+1}$ denote the time-t ex ante forecasts of time-$(t+1)$ values of the measurement series and the covariance of the measurement series obtained from the unscented Kalman filter, we construct the log-likelihood value assuming normally distributed forecasting errors,

$$l_{t+1}(\Theta) = -\frac{1}{2}\log\left|\overline{A}_t\right| - \frac{1}{2}\left((y_{t+1}-\bar{y}_{t+1})^\top \left(\overline{A}_{t+1}\right)^{-1}(y_{t+1}-\bar{y}_{t+1})\right). \tag{21}$$

The model parameters are chosen to maximize the log likelihood of the data series,

$$\Theta \equiv \arg\max_{\Theta} \mathcal{L}(\Theta, \{y_t\}_{t=1}^N), \quad \texttt{with} \quad \mathcal{L}(\Theta, \{y_t\}_{t=1}^N) = \sum_{t=0}^{N-1} l_{t+1}(\Theta), \tag{22}$$

where $N = 501$ denotes the number of weeks in our sample of estimation.

1.3. The dynamics of interest-rate factors and pricing errors

The model specifications and estimations are relatively standard, and our results are also similar to those reported in the literature. Since all four models generate similar performance, our conclusions are not particularly sensitive to the exact model choice. For expositional clarity, we henceforth focus our discussions on the $A_0(3)$ model, and address the similarities and differences of the other three models, $A_m(3), m = 1,2,3$, in a separate section. From the estimated models, we analyze the dynamics of the interest-rate factors and the behavior of the pricing errors, both of which are important for our subsequent analysis on the predictability of interest rates and interest-rate portfolios.

1.3.1. Factor dynamics

Table 2 reports the parameter estimates and the absolute magnitudes of the t-values for the $A_0(3)$ model. The parameter estimates on κ control the mean-reverting feature of the time-series dynamics of the three Gaussian factors. For the factor dynamics to be stationary, the real parts of the eigenvalues of the κ matrix must be positive. Under the lower triangular matrix assumption, the eigenvalues of the κ matrix coincide with the diagonal elements of the matrix.

The estimate for the first diagonal element is very small at 0.002. Its t-value is also very small, implying that the estimate is not statistically different from zero. Hence, the first factor is close to being nonstationary. The estimate for the second eigenvalue is 0.48, with a t-value of 1.19, and hence not significantly different from zero. The estimate for the third eigenvalue of the κ matrix is significantly different from zero, but the magnitude remains small at 0.586, indicating that all three factors are highly persistent. The largest eigenvalue of 0.586 corresponds to a weekly autocorrelation of 0.989, and a half life of 62 weeks.

The κ^* matrix represents the counterpart of κ under the risk-neutral measure. The estimates for κ^* are close to the corresponding estimates for κ, indicating that the three interest-rate factors also show high persistence under the risk-neutral measure. Compared to the κ matrix, which controls the time-series dynamics of the interest rates, the risk-neutral counterpart κ^* controls the cross-sectional behavior (term structure) of interest rates. The t-values for κ^* are much larger than the t-values for the corresponding elements of κ. Thus, by estimating the dynamic term structure model on the interest-rate data, we can identify the risk-neutral dynamics and hence capture the term structure behavior of interest rates much more accurately than capturing the time-series dynamics.

The difference in t-values between κ and κ^* also implies that from the perspective of a dynamic term structure model, forecasting future interest-rate movements is more difficult than fitting the observed term structure of interest rates. This difficulty is closely linked to the near unit-root behavior of interest rates. The difficulty in forecasting persists even if we perform the estimation on the panel data of interest rates across different maturities and hence exploit the full information content of the term structure.

1.3.2. Properties of pricing errors

In using a three-factor model to fit the term structure of 12 interest rates, we will see discrepancies between the observed interest rates and the model-implied values. In the language of the state-space model, the differences between the two are called measurement errors. They can also be regarded as the model pricing errors. The unscented Kalman filter minimizes the pricing errors in a least square sense.

Table 3 reports the sample properties of the pricing errors. The sample mean shows the average bias between the observed rates and the model-implied rates. The largest biases come from the six- and 12-month

LIBOR rates, potentially due to margin differences and quoting non-synchronousness between LIBOR and swap rates (James and Webber (2000)). The root mean squared pricing error (rmse) measures the relative goodness-of-fit on each series. The largest root mean squared error comes from the 12-month LIBOR rate at 14.145 basis points. The maximum absolute pricing error is 60.495 basis points on the one-month LIBOR rate. The skewness and excess kurtosis estimates are much larger than the corresponding estimates on the original interest rates, especially for the short-term LIBOR rates, reflecting the occasionally large mismatches between the model and the market at the short end of the yield curve (Heidari and Wu (2003b)). Overall, the model captures the main features of the term structure well. The last column reports the explained percentage variation (VR) on each series, defined as one minus the ratio of pricing error variance to the variance of the original interest-rate series, in percentage points. The estimates suggest that the model can explain over 99% of the variation for 11 of the 12 interest-rate series.

We also report the weekly autocorrelation for the pricing errors. The autocorrelation is smaller for the better-fitted series. The average weekly autocorrelation for the pricing errors is at 0.69, much smaller than the average of 0.991 for the original interest-rate series. Based on an AR(1) structure, a weekly autocorrelation of 0.69 corresponds to a half life of less than three weeks, much shorter than the average half life for the original series. Thus, if we can make an investment on the pricing errors instead of on the original interest-rate series, our ability to forecast will become much stronger and convergence through mean-reversion will become much faster.

2. Predictability of Interest-Rate Portfolios

Given the estimated dynamic term structure models, a traditional approach is to directly predict future interest-rate movements based on the estimated factor dynamics, e.g., Duffee (2002) and Duffee and Stanton (2000). We start this section by repeating a similar exercise as a benchmark for comparison. We then propose a new, innovative application of the estimated dynamic term structure models to enhance the predictability. In this new application, we do not use the estimated factor dynamics to directly predict interest-rate movements, but use the model as a decomposition tool and form interest-rate portfolios that are significantly more predictable than are the individual interest-rate series.

2.1. Forecasting interest rates based on estimated factor dynamics: A benchmark

As a benchmark for our subsequent analysis, we forecast each LIBOR and swap rate series using the estimated $A_0(3)$ model via the following procedure. At each date, based on the updates on the three factors, we forecast the values of the three factors four weeks ahead according to the state propagation equation in (19) and with the time horizon $\Delta t = 4/52$. The choice of a four-week forecasting horizon is a compromise between the weekly data used for model estimation and a reasonably long horizon for forecasting. Given the high persistence in interest rates, the investment horizon is usually one month or longer.

Using the forecasts on the three factors, we compute the forecasted values of zero-coupon bond prices according to equation (6) and the forecasted LIBOR and swap rates according to equations (17) and (18). Then, we compute the forecasting error as the difference in basis points between the realized LIBOR and swap rates four weeks later and the forecasted values.

We compare the forecasting performance of the $A_0(3)$ model with two alternative strategies: the random walk hypothesis (RW), under which the four-week ahead forecast of the LIBOR and swap rate is the same as the current rate, and a first-order autoregressive regression (OLS) on the LIBOR or swap rate over a four-week horizon.

Table 4 reports the sample properties of the four-week ahead forecasting error from the three forecasting strategies. By design, the in-sample forecasting error from the regression is always smaller in the least square sense than that from the random walk hypothesis. However, due to the high persistence of interest rates, the differences between the sample properties of the forecasting errors from RW and OLS are very small. The root mean squared forecasting errors on each series from the two strategies are less than half a basis point apart. In the last column in each panel, we report the explained percentage variation, defined as one minus the variance of the forecasting errors over the variance of the four-week changes in the interest rate series. By definition, the random walk strategy has zero explanatory power on the changes in LIBOR and swap rates. The OLS strategy generates positive results, but the outperformance is very small, with the highest percentage being 1.528% for the 30-year swap rates. Hence, for short-term investment over a horizon of four weeks, the gain from exploiting the mean-reverting property of individual interest-rate series is negligible, even for in-sample analysis.

The last panel reports the properties of the forecasting errors from the $A_0(3)$ dynamic term structure model. The model's forecasting performance is not significantly better than the simple random walk hypothesis. In fact, the root mean squared error from the model is larger than the mean absolute forecasting error from the random walk hypothesis for seven of the 12 series, and the explained variation estimates are negative for eight of the 12 series. Therefore, the dynamic term structure model delivers poor forecasting performance. Duffee (2002) and Duffee and Stanton (2000) observe similar performance comparisons for a number of different dynamic term structure models, reflecting the inherent difficulty in forecasting interest-rate movements using multi-factor dynamic term structure models.

2.2. Forming interest-rate portfolios that are strongly predictable

Given the near unit-root behavior of interest rates, neither dynamic term structure models nor autoregressive regressions can do much better than a simple random walk assumption in predicting future changes in the individual interest-rate series. However, the pricing errors from the dynamic term structure models show much smaller persistence than both the interest-rate factors and the original interest-rate series. As a result, an autoregressive regression can predict future changes in the pricing errors much better than does the random walk hypothesis. Therefore, the predictable component in the interest-rate movements is not in the estimated dynamic factors, but in the pricing errors. Based on this observation, we propose a new way of applying the term structure model in forecasting interest rates.

Instead of using the term structure model to directly forecast movements in the individual interest-rate series, we use the model as a decomposition tool, which decomposes each interest-rate series into two components, a very persistent component as a function of the three interest-rate factors, and a relatively transient component that constitutes the pricing error of the model. We think of the pricing errors as reflecting higher dimensional dynamics of the interest rates that are not captured by the three factors.

With this decomposition, we can use the model to form interest-rate portfolios that have minimal exposure to the three persistent factors, and hence magnified exposure to the more transient pricing errors. We expect that future movements in these interest-rate portfolios are more predictable than movements in each individual interest-rate series, given the portfolios' magnified exposure to the more predictable component in interest rates.

In principle, when dealing with a portfolio of bonds, we can use two different interest-rate series to hedge away its first-order dependence on one factor, and three series to hedge away its first-order dependence on two factors. To hedge away a portfolio's first-order dependence on three factors, we need four interest-rate series in the portfolio. To illustrate the idea, we use an example of four swap rates at maturities of two, five, ten, and 30 years to form such a portfolio. Formally, we let $\mathbf{H} \in \mathbb{R}^{3\times 4}$ denote the matrix formed by the partial derivatives of the four swap rates with respect to the three interest-rate factors,

$$\mathbf{H}(X_t) \equiv \left[\frac{\partial SWAP(X_t,\tau)}{\partial X_t}\right], \quad \tau = 2,5,10,30. \tag{23}$$

We use $\mathbf{m} = [m(\tau)]$, with $\tau = 2,5,10,30$, to denote the (4×1) portfolio weight vector on the four swap rates. To minimize the sensitivity of the portfolio to the three factors, we require that

$$\mathbf{Hm} = 0, \tag{24}$$

which is a system of three linear equations that set the linear dependence of the portfolio on the three factors to zero, respectively.

The three equations in (24) determine the relative proportion of the four swap rates. We need one more condition to determine the size or scale of the portfolio. There are many ways to perform this relatively arbitrary normalization. For this specific example, we set the portfolio weight on the ten-year swap rate to one. We can interpret this normalization as being long one unit of the ten-year swap contract, and then using (fractional units of) the other three swap contracts (two-, five-, and 30-year swaps) to hedge away its dependence on the three factors.

Based on the parameter estimates in Table 2, we first evaluate the partial derivative matrix $\mathbf{H}$ at the sample mean of X_t and solve for the portfolio weight as,

$$\mathbf{m} = \left[\begin{array}{cccc} 0.0277, & -0.4276, & 1.0000, & -0.6388 \end{array}\right]^\top. \tag{25}$$

In theory, the partial derivative matrix $\mathbf{H}$ depends on the value of the state vector X_t, but under the affine models, the relation between swap rates and the state vector is well approximated by a linear relation. Hence, the derivative is close to a constant. Our experiments also indicate that within a reasonable range, the partial derivative matrix is not sensitive to the choice of the level of the factors X_t. Thus, we evaluate the partial derivative at the sample mean and hold the portfolio weights fixed over time.

Figure 1 plots the time series of this swap-rate portfolio in the left panel. The solid line denotes the market value based on the observed swap rates and the dashed line denotes the model-implied fair value as a function of the three interest-rate factors. We observe a very flat dashed line in the left panel of Figure 1, indicating that the fixed-weight portfolio is not sensitive to changes in the interest-rate factors over the whole sample period. The flatness of this dashed line also confirms that the partial derivative matrix in equation (23) is relatively invariant to changes in the interest-rate factors.

[Figure 1 about here.]

The market value (solid line) of the portfolio shows significant variation and strong mean reversion around the model-implied value (dashed line). The weekly autocorrelation of this four-swap rate portfolio is 0.816, corresponding to a half life of about a month. For comparison, we also plot the time series of the unhedged ten-year swap rate series in the right panel, which shows much less mean reversion than the hedged swap rate portfolio. The weekly autocorrelation estimate for the ten-year swap rate is 0.987, corresponding to a half life of about one year, in contrast to a half life of one month for the hedged swap-rate portfolio.

In the right panel, we also plot the model-implied value of the unhedged ten-year swap rate in dashed line, but the differences between the market quotes (solid line) and the model values (dashed line) are so small that we cannot visually distinguish the two lines. Therefore, from the perspective of fitting individual interest-rate series, the $A_0(3)$ model performs very well and the pricing errors from the model are very small. However, by forming a four-instrument portfolio in the left panel, we magnify the significance of the pricing errors by hedging away the variation in the three interest-rate factors.

To investigate the predictability of this interest-rate portfolio, we employ the OLS forecasting strategy on this portfolio. The AR(1) regression generates the following result:

$$\begin{array}{rcccccl} \Delta R_{t+1} & = & -0.0849 & - & 0.2754 R_t & + & e_{t+1}, \quad R^2 = 14\% \\ & & (0.0096) & & (0.0306) & & \end{array} \tag{26}$$

where R_t denotes the portfolio of the four swap rates and ΔR_{t+1} denotes the changes in the portfolio value over a four-week horizon. We estimate the regression parameters by using the generalized methods of moments (GMM) with overlapping data. We compute the standard errors (in parentheses) of the estimates following Newey and West (1987) with eight lags.

The explained variation (VR) in the second panel of Table 4 corresponds to the R-squares of a similar AR(1) regression on individual LIBOR and swap rate series. The VR estimate on the unhedged ten-year swap rate is 1.068%. In contrast, by hedging away its dependence on the three persistent factors, the hedged ten-year swap rate has an R-square of 14%, a dramatic increase in predictability.

Equation (26) reflects the predictability of the swap rate portfolio based on the OLS strategy. However, we construct the portfolio based on the estimates of the $A_0(3)$ dynamic term structure model. Therefore, the strong predictability in equation (26) represents the combined power of the dynamic term structure model and the AR(1) regression. In this application, we do not use the dynamic term structure model to directly forecast future interest-rate movements, but rather use it to form an interest-rate portfolio that is more predictable. The portfolio weights are a function of the partial derivatives matrix $\mathbf{H}(X_t)$, which is determined by the risk-neutral dynamics of the interest-rate factors and the short-rate function, both of which we can estimate accurately.

When a time series is close to a random walk, forecasting becomes difficult irrespective of the forecasting methodology. Individual interest-rate series provide such an example. Table 4 shows that using the estimated factor dynamics generates forecasting results no better than the random walk assumption. Nevertheless, we show that the dynamic term structure model can still be useful. The model captures the cross-sectional (term structure) properties of the interest rates well. We use this strength of the dynamic term structure model to form an interest-rate portfolio that minimizes its dependence on the persistent interest-rate factors. As a result, the portfolio's exposure to the more transient interest-rate movements is magnified. The portfolio becomes more predictable, even when the prediction is based on a simple AR(1) regression.

The idea of using four interest-rate series to form the portfolio is to achieve first-order neutrality to the three persistent factors. In principle, any four interest-rate series should be able to achieve this neutrality. With 12 interest-rate series, we can construct 495 distinct four-instrument portfolios. To investigate the sensitivity of the predictability to the choice of the specific interest-rate series, we exhaust the 495 combinations of portfolios and run the AR(1) regression in equation (26) on each portfolio. For each portfolio, we normalize the holding on the interest-rate series by setting the largest portfolio weight to one.

Table 5 reports in the first panel the summary statistics on the parameter estimates, t-statistics, and the R-squares from the 495 regressions on the four-instrument portfolios. The slope estimates are all statistically significant, with the minimum absolute t-statistic at 6.504. The minimum R-square is 7.844%, the maximum is 55.724%, and the median is 15.68%. Even in the worst case, the predictability of the four-instrument portfolio is much stronger than the predictability of the individual interest-rate series.

In principle, we can use any four distinct interest-rate series to neutralize the impact of the three persistent factors, but practical considerations could favor one portfolio over another. First, when the maturities of the interest rates in the portfolio are too close to one another, the derivative matrix $\mathbf{H}$ could approach singularity, and the portfolio weights could become unstable. Second, we see from Table 3 that the pricing errors from the better fitted interest rates series show smaller serial dependence. Thus, a portfolio composed of better-fitted interest-rate series should show stronger predictability. These considerations lead to sample variation in the R-squares for different portfolios. However, the fact that the predictability of even the worst-performing portfolio is better than that of the best-performing individual interest-rate series shows the robustness of our portfolio construction strategy.

So far, we have been using four interest-rate series to neutralize the effect of all three interest-rate factors. However, the idea is not limited to forming four-instrument portfolios. For example, we can use two interest-rate series to form a portfolio that is immune to the first, and also the most persistent, factor. We can also form three-instrument portfolios to neutralize the impact of the first two factors. Finally, we can estimate an even higher dimensional model, and form portfolios with even more interest-rate series.

Based on the 12 interest-rate series, we form 66 two-instrument portfolios that have minimal exposures to the first factor. We also form 220 three-instrument portfolios that have minimal exposures to the first two interest-rate factors. For each portfolio, we run the AR(1) regression as in equation (26). The second and third panels in Table 5 report the summary statistics of these regressions on two- and three-instrument portfolios, respectively. The predictability of the two-instrument portfolios is not much different from the predictability of the individual interest rate series. The average R-square for the two-instrument portfolios is merely 0.42%, not much better than the random walk hypothesis. The maximum R-square is only 5.47%.

Therefore, hedging away the first factor is not enough to improve interest-rate predictability significantly over a four-week horizon.

By hedging away the first two persistent factors, some of the three-instrument portfolios show markedly higher predictability. The maximum R-square is as high as 27.42%. Furthermore, about 10% of the three-instrument portfolios generate R-squares greater than 5%. Nevertheless, the median R-square is only 0.242%, and the R-square at the 75-percentile remains below 1%. Thus, improved predictability only happens on a selective number of three-instrument portfolios. We need to hedge away the first three factors to obtain universally strong predictability over a four-week horizon.

3. The Economic Significance of Portfolio Predictability

The predictability of a time series does not always lead to economic gains. In this section, we investigate the economic significance of the predictability of the four-instrument portfolios from an asset-allocation perspective, an approach popularized by Kandel and Stambaugh (1996). We then analyze the risk and return characteristics of the excess returns from the asset-allocation exercise, and discuss the economic and theoretical implications of our results.

3.1. A simple buy and hold strategy based on AR(1) forecasts

We assume that an investor exploits the mean-reverting property of the interest-rate portfolios and makes capital allocation decisions based on the current deviation of the portfolio from its model-implied mean value. Since floating rate loans underlying the LIBOR rates have low interest-rate sensitivities, we focus our investment decisions on swap contracts of different maturities.

Following industry practice, we regard each swap contract as a par bond with the coupon rate equal to the swap rate. We regard the floating leg of the swap contract (three-month LIBOR) as short-term financing for the par bond. Hence, forecasting the swap rates amounts to forecasting the coupon rates of the portfolio of par bonds. When the current portfolio of swap rates is higher than the model value, the mean-reverting property of the portfolio predicts that the portfolio of swap rates will decline in the future and move toward the model value. Then, it can be beneficial to go long the portfolio and receive the current fixed swap rates as coupon payments. However, as time goes by, the maturities of the invested portfolios also decline. Thus, only when the maturity effect is small compared to the forecasted movements of the fixed-maturity swap rates, does the investment lead to economic gains. The shortest maturity of the swap contracts under consideration is two years. An investment horizon of four weeks is short relative to the swap maturity. Hence, strong predictability on the swap rates can potentially lead to economic gains for investing in swap contracts.

At each time t, we determine the allocation weight to a portfolio based on a mean-variance criterion:

$$w_t = \frac{ER_t}{Var(ER)} \tag{27}$$

where ER denotes the deviation of the portfolio from its model-implied fair value. The term $Var(ER)$ denotes its sample variance estimate and serves as a scaling factor in our application.

We consider a four-week investment horizon for a simple buy and hold strategy based on the above mean-variance criterion. At the end of the investment horizon, we close our position and compute the profit and loss based on the market value of each coupon bond. Since we only observe LIBOR and swap rates at fixed maturities, not the whole spot-rate curve, we linearly interpolate the swap rate curve and bootstrap the spot-rate curve. We then compute the monthly excess capital gains based on the market value of the investment portfolio at the end of the four-week horizon and the financing cost of the initial investment. We compute the financing cost based on the floating leg rate, which is the three-month LIBOR. Since the initial investment is a very small number, we report the excess capital gain, which we regard as excess returns over the ten-year par bond. First, we use the portfolio composed of two-, five-, ten-, and 30-year swap rates as an illustrative example. Then, we report the summary results on other four-instrument portfolios.

Based on the decision rule in equation (27), the allocation weight on this specific portfolio (w_t) is between -0.5578 and 0.7283 during our sample period. The portfolio weight $\mathbf{m}$ sums to a very small number -0.0388. If we buy \$1,000 par notional value of the ten-year par bond and form the corresponding hedged portfolio, we will have a small net sales revenue of \$38.8. We use this \$1,000 par notional value on the ten-year par bond as a base position and multiply this position by w_t at each date t. The excess capital gains from this investment can be regarded as excess returns on a $1000 investment in the ten-year par bond, hedged to be factor-neutral using two-, five-, and 30-year par bonds.

The left panel of Figure 2 plots the time-series of the excess returns for each weekly investment. The right panel plots the cumulative wealth. To make full use of the weekly sample, we make investments every week. We hold each investment for a four-week horizon to compute the excess returns.

[Figure 2 about here.]

The excess returns during each investment period are predominantly positive. The right panel shows a fast cumulation of wealth from this exercise. The average excess return over the four-week investment horizon is 0.2494, and the standard deviation is 1.2831, resulting in an annualized information ratio of 0.701, defined as the ratio of the mean to the standard deviation, multiplied by $\sqrt{52/4}$. An annualized information ratio of 0.701 is comparable to that from popular fixed income arbitrage strategies (Duarte, Longstaff, and Yu (2005)). Thus, the predictability of the swap portfolio formed according to the dynamic term structure model is not only statistically strong and significant based on AR(1) regressions, but also economically pronounced from the perspective of a simple mean-variance investor. Furthermore, the skewness estimate for the excess return is strongly positive at 5.4293, adding a second layer of attraction in addition to the high information ratio. Over our sample period, the maximum loss for the investments is $2.8293, but the maximum gain $17.3718.

To investigate how the profitability varies with different choices of swap rates in the portfolio formulation, we use the seven swap rates to form 35 four-instrument portfolios. We then perform the same investment exercise on the 35 portfolios. The left panel of Figure 3 plots the cumulative gains from investing in each of the swap portfolios. Investing in different portfolios accumulates wealth at different rates, but the sample-path variations are small for all portfolios and we make profits on all portfolios. The right panel of Figure 3 plots the histogram of the annualized information ratios from investing in each of the 35 swap portfolios. The predictability is economically significant for most four-instrument portfolios.

[Figure 3 about here.]

Table 6 reports more detailed summary statistics of the excess returns on investing in each of the 35 swap portfolios. All investments generate positive mean and median excess returns. For all investments, the maximum losses are smaller than the maximum gains, and the excess return distribution shows large kurtosis and positive skewness. The returns on non-overlapping sample periods show little autocorrelation. The information ratio estimates range from 0.366 to 0.941, with an average of 0.696. Duarte, Longstaff, and Yu (2005) observe similar high information ratios and positive skewness from several popular fixed income arbitrage strategies. In contrast, other high-information-ratio investment strategies reported in the literature often generate excess returns with negatively skewed distributions. Examples include selling out-of-the-money put options (Coval and Shumway (2001), Goetzmann, Ingersoll, Spiegel, and Welch (2002)), shorting variance swap contracts (Carr and Wu (2004)), and merger arbitrage (Mitchell and Pulvino (2001)).

3.2. Risk and return characteristics for the swap portfolio investment

By design, the four-instrument swap portfolios are orthogonal to the three interest-rate factors identified from the dynamic term structure model. Hence, the excess returns from investing in the four-instrument portfolios are not due to their exposures to the three interest-rate factors. However, if the residual risks are correlated with other market factors, the positive average excess returns shown in Table 6 may represent compensation for the investment's exposure to these market factors.

To better understand the risk and return characteristics of the swap portfolio investments, we regress the excess returns from each investment on systematic factors in the stock market, the corporate bond market, and the interest-rate options market:

- **Stock market**: We follow Fama and French (1993) and Carhart (1997) and use the excess returns on the market portfolio ($R_m - R_f$), the small-minus-big size portfolio (SMB), the high-minus-low book-to-market equity portfolio (HML), and the up-minus-down momentum portfolio (UMD). All these excess returns series are available on Ken French's online data library. To match the excess returns on the swap portfolios, we first download the daily excess returns and then cumulate the excess return over the past four weeks at each Wednesday to generate a weekly series of overlapping four-week returns.

- **Corporate bond market**: We download the corporate bond yields from the Federal Reserve Statistical Release at the Aaa and Baa rating groups. Then, we construct a weekly series of four-week changes over the same sample period on the credit spreads between the two credit rating groups (CS). We use this series to proxy the excess returns for the credit risk exposure.

- **Interest-rate options market**: We obtain from Bloomberg at-the-money cap implied volatility quotes during the same sample period. From these quotes, we compute the excess returns from investing in a five-year straddle and holding it for four weeks. We use this excess return series as a proxy for the compensation to interest-rate volatility risk exposure. If our estimated dynamic term structure models could price both the yield curve and the options well, the interest-rate volatility risk would also be spanned by the three interest-rate factors. Nevertheless, there is evidence that the interest-rate volatility risks observed from the interest-rate caps and swaptions market are not spanned by the risk factors identified from the yield curve (Collin-Dufresne and Goldstein (2002) and Heidari and Wu (2003a)). Hence, we include this excess return series to investigate whether the excess returns to the swap portfolios is due to their exposures to the unspanned volatility risk (USV).

Formally, for excess returns on each swap portfolio investment, we run the following regression,

$$R_t = \alpha + \beta_1 (R_{m,t} - R_{f,t}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \beta_5 CS_t + \beta_6 USV_t + e_t. \tag{28}$$

We estimate the relation using generalized methods of moments, with the weighting matrix constructed according to Newey and West (1987) using eight lags. The intercept estimate α represents the excess return to the swap portfolio investment after accounting for its risk exposures to the stock market, the corporate credit market, and the unspanned interest-rate volatility. We scale each excess return series by $\sqrt{52/4}/\mathrm{std}(R_t)$ so that the α estimate is comparable to the annualized information ratio estimates (IR=$\sqrt{52/4}\mathrm{mean}(R_t)/\mathrm{std}(R_t)$) reported in Table 6 before we adjust for these risk exposures. Table 7 reports the intercept estimates, the t-statistics for all parameter estimates, and the R-squares of the regressions for each of the 35 portfolio series. The last column reports the skewness estimates for the risk-adjusted excess return (e_t).

After accounting for the risk exposures, the average excess returns (α) range from 0.03 to 0.93, with an average of 0.66, which is not much smaller than the raw information ratios without adjusting for these risk exposures. The t-statistics indicate that 23 out of the 35 intercepts are statistically significant at 95% confidence level. The t-statistics on the loading coefficients of the market factors show that the excess returns may have some exposure on the SMB risk factor, with 14 of the 35 estimates significantly negative at 95% confidence level. The loading estimates on other market factors are mostly insignificant, and the R-squares of the regressions are low, ranging from 0.23% to 3.49%. The last column shows that the risk-adjusted excess returns remain positively skewed. Thus, the positive excess returns and positive skewness from investing in the four-instrument swap portfolios cannot be fully explained by the systematic factors in the stock, corporate bond, interest-rate, and interest-rate options markets.

3.3. Economic interpretations and theoretical implications

The literature regards three-factor dynamic term structure models as sufficient in capturing the interest-rate movements. This conclusion holds from the perspective of fitting individual interest-rate series since three factors explain over 99% of the variation in the interest-rate movements (Table 3). However, by forming four-instrument portfolios that are relatively immune to the variation of the three interest-rate factors, we expose the deficiency of a three-factor model. We show that the pricing errors can be economically significant for short-term investments, although they are small relative to the main variation of interest rates.

A large body of the literature assigns statistical and economic interpretations to the first three interest-rate factors. Based on the statistical factor analysis, Litterman and Scheinkman (1991) label the first three interest-rate factors as the level, the slope, and the curvature factors. A series of recent papers (e.g., Ang and Piazzesi (2003), Lu and Wu (2004), and Wu (2005)) link the first three factors to macroeconomic variables such as the long-run inflation rate, the output gap, and shocks to the short-term central bank policy. By contrast, we can think of the higher-order dynamics captured by the pricing errors of a three-factor model as mainly due to short-term supply and demand shocks to the specific interest-rate swap contracts. To investigate whether the excess returns from our investment exercise are correlated with contract-specific liquidity shocks, we construct three measures that proxy the absolute magnitude of liquidity shocks in the interest-rate swap market:

- **L1**: We define the first liquidity measure based on the absolute daily swap rate changes during the past week. First, we measure the absolute daily changes on each swap rate series. Second, we take the sample average of the absolute changes over the past week (from the current Wednesday to the last Thursday) on each series. Then, we define **L1** as the median value of the estimates on the seven swap rate series. This measure is similar to Amihud (2002)'s illiquidity measure for the stock market, except that Amihud normalizes the absolute price changes by the trading volume. We do not perform the normalization as we do not have the trading volume data available. The measure captures the average median variation of the swap rates, which can be caused by either aggregate liquidity or economic shocks to the swap market.

- **L2**: We define the second liquidity measure **L2** as the difference between the largest positive daily move and the largest negative daily move among the seven swap rate series. For example, if the largest upward movement is 12 basis points on the five-year swap rate and the largest downward movement is seven basis points on the 30-year swap rate, then $\mathbf{L2} = 12 - (-7) = 19$ basis points. It is a measure of maximum nonparallel "twist" of the swap rate curve. If we regard parallel interest-rate shifts as due to aggregate economic shocks, the "twists" of the swap rate curve can be caused by liquidity shocks on a specific swap contract. Hence, **L2** is a better measure than **L1** for contract-specific liquidity shocks. Nevertheless, it is important to realize that slope and curvature changes on the yield curve that are due to systematic economic shocks (such as real output gap and Fed policy shocks) can also induce a large estimate for **L2**.

- **L3**: To obtain a cleaner measure of contract-specific liquidity shock that does not respond to systematic slope and curvature changes, we define the third liquidity measure **L3** based on the pricing errors, defined as the difference between the market quotes and the values computed from the estimated $A_0(3)$ model. Analogous to **L2**, we define **L3** as the difference between the largest positive pricing error and the largest negative pricing error. Under this measure, an interest-rate movement is not classified as a liquidity shock as long as the three dynamic factors can account for it.

The three panels in Figure 4 plot the cross-correlation estimates at different leads and lags between the average excess returns from investing in the 35 swap portfolios and the three liquidity measures, with each panel representing the correlation with one liquidity measure. The two dash-dotted lines in each panel denote the 95% confidence bands. The first liquidity measure **L1**, which is based on the average median absolute daily changes, does not show a significant correlation with the average excess returns. In contrast, both **L2** and **L3** show strongly positive correlations with the excess returns at the four-week lag point.

Since the excess returns are from investments put on four weeks ago, the positive correlations indicate that investments made on days with large liquidity shocks captured by **L2** and **L3** are more likely to generate large and positive ex post returns. Therefore, although the excess returns from our investment exercise are not highly correlated with the average absolute daily changes in swap rates (**L1**), nor are highly correlated with the cap implied volatilities and other stock and corporate bond market systematic factors (Table 7), they show strongly positive correlation with **L2** and **L3**, which measure the non-systematic part of the swap rate movements that we deem as caused by liquidity shocks.

[Figure 4 about here.]

Our investment analysis shows that bearing the risk from the liquidity shocks can lead to a substantially positive average risk premium. We attribute this high premium to several facets of market frictions. First, the liquidity shock induces less than 1% of the variation in each interest-rate series. Therefore, only institutions with large amounts of capital and small costs of funds can exploit the high premium of the liquidity risk. Second, to hedge against the first three factors and to expose the liquidity risk, an investor needs to form four-instrument portfolios that involve both long and short positions. Many large institutions such as mutual funds cannot initiate short positions. Thus, even if they meet the capital requirement, institutional constraints on their investment styles prevent them from exploiting the profits. Third, the four-instrument portfolios are formed according to a three-factor dynamic term structure model. The specification and estimation of three-factor dynamic term structure models are only recent endeavors in the academia. Hence, implementing such a strategy requires significant investment in intellectual capital.

In a recent working paper, Duarte, Longstaff, and Yu (2005) analyze the risk and return characteristics of several popular fixed income arbitrage strategies. They find that the annualized information ratios for these strategies range from 0.3 to 0.9, similar to that from our simple investment exercise. Furthermore, they find that after controlling for market factors, the mean excess returns on simple strategies become insignificant, but the mean excess returns on strategies that require more intellectual capital remain significant.

Our analysis also has important implications for future interest-rate modeling. The analysis reveals the key reasons behind the poor forecasting performance of traditional dynamic term structure models. By capturing only the most persistent movement in interest rates, a three-factor dynamic term structure model misses the most predictable component in interest rates. Therefore, to improve the model performance in forecasting, it is important to account for the higher-order dynamics, or the liquidity risk, in the interest-rate movements.

In a survey analysis, Dai and Singleton (2003) identify two important features of the interest rate data that the existing dynamic term structure models fail to capture. First, although dynamic term structure models can explain over 99% of the variation in interest rates, they perform very poorly in explaining the variation in interest-rate option implied volatilities (Collin-Dufresne and Goldstein (2002) and Heidari and Wu (2003a)). Second, non-overlapping forward interest rates show very low, and sometimes negative, cross-correlations, but almost all existing estimated dynamic term structure models generate strongly positive cross-correlations. Accounting for the small pricing errors in the interest rates from a dynamic term structure model can go a long way in explaining both puzzles. Although persistent factors dominate the movements of the interest-rate series, the higher-order dynamics revealed in the pricing errors can have strongly significant impact on the variation of interest-rate options (Heidari and Wu (2003c)). Furthermore, it is well known in the dynamic control literature that a small "wavy noise" can dramatically alter the correlation pattern of two persistent series. Therefore, for future research, successfully modeling and identifying these higher-order interest-rate dynamics could prove fruitful in improving the model's performance in forecasting, derivative pricing, and in capturing the co-variation of different interest-rate series.

4. Robustness Analysis

In this section, we analyze the robustness of the interest-rate portfolio predictability by comparing the portfolio behaviors across different subsamples, between in-sample and out-of-sample, and across different dynamic term structure model specifications.

4.1. Subsample analysis

To study the time-variation of the swap portfolio predictability, we divide the whole sample period into two subsamples, with the first subsample spanning the first four years from May 11, 1994 to May 6, 1998, and the second subsample spanning the remaining sample from May 13, 1998 to December 10, 2003.

First, we investigate whether the statistical predictability of the interest-rate portfolios varies across the two subsample periods. For this purpose, we run the AR(1) regression on the four-instrument portfolios for the two subsample periods separately. The left panel of Figure 5 plots the histogram of the explained percentage variation estimates from the regressions, with the dark blue bar denoting the first subsample and the light yellow bar denoting the second subsample. We observe that the explained variations for the two subsample periods stay in the same range. The minimum and maximum explained variations during the first subsample are 7.3% and 28.1%, respectively. The minimum and maximum for the second subsample are 8.5% and 56.9%, respectively. Thus, the predictability of the interest-rate portfolios is strong in both subsamples as well as in the whole sample. The difference between the two subsamples only lies in the distribution of the explained variation estimates. Due to the distributional differences, the sample mean of the explained variation estimates during the first subsample is 28.1%, higher than the average during the second subsample at 19%.

[Figure 5 about here.]

Second, we study how the economic significance of the predictability varies across the two subsample periods. For each portfolio investment exercise, we compute the sample mean and standard deviation of the excess returns separately for the two subsample periods. The sample average of the mean excess return is 0.4335 for the first subsample, but lower at 0.1427 for the second subsample. The sample averages of the standard deviation estimates for the first and second subsamples are 1.5988 and 1.0755, respectively, showing that the risk also declines in the second subsample.

The right panel of Figure 5 plots the histogram of the annualized information ratios for the two subsample periods, again with the dark blue bar denoting the first subsample and the light yellow bar denoting the second subsample. It is evident that the informational ratio is higher during the first subsample than during the second subsample. During the first subsample, the minimum information ratio is 0.6957, the maximum is 1.3094, with a mean of 1.1069. During the second subsample, the minimum information ratio is 0.1357, the maximum is 0.9906, with a mean of 0.4881, less than half of the mean information ratio in the first subsample. Therefore, although the statistical significance of predictability remains high over the whole sample period, the economic significance has declined over time.

We have argued that the positive average premiums to the four-instrument portfolio investments are compensation for bearing short-term liquidity risks and for investing in intellectual capital. Thus, we expect the premium to be higher when the interest-rate swap market is more susceptible to supply and demand shocks. The interest-rate swap market started active trading in the early 1990s. The market has expanded tremendously since then. According to surveys at the International Swaps and Derivatives Association, Inc (ISDA) and the Bank for International Settlements (BIS), the notional amount of outstanding interest-rate swaps at the end of 1994 was 8.82 trillion US dollars. The notional amount increased to 111.21 trillion by the end of 2003. We expect that such tremendous growth has made the interest-rate swap market more liquid and deep and less likely to be significantly moved by small supply and demand shocks. Furthermore, given the rapid progress in theory and estimation of dynamic term structure models during the past decade, we also expect the risk premiums to decline over time as the human capital investment needed to implement dynamic term structure models declines.

4.2. Out-of-sample analysis

All the above results are based on in-sample analysis. The model estimation, the portfolio construction, the forecasting regression, and the investment decision are all based on the common sample period from May 11, 1994 to December 10, 2003. To investigate how robust the results are out of sample, we re-estimate the model using the first four years of data from May 11, 1994 to May 6, 1998. Then we perform out-of-sample analysis on the remaining sample from May 13, 1998 to December 10, 2003.

First, the robustness of the predictability depends on the stability of the portfolio weights, which in turn depends on the stability of the (risk-neutral) parameter estimates of the dynamic term structure models. In Table 8, we report the parameter estimates of the $A_0(3)$ model using the two subsamples. Compared to the full-sample estimates in Table 2, the t-statistics for κ become smaller. Other than that, the parameter estimates are relatively stable over the two subsamples and also not substantially different from the full-sample estimates in Table 2.

Second, we use the parameter estimates from the first subsample to price the LIBOR and swap rates in the second subsample. Table 9 reports the summary statistics of the out-of-sample pricing errors. Comparing to the in-sample pricing errors in Table 3, we observe a slight decline in the average explained percentage variance from 99.71% in sample to 99.53% out of sample. The average autocorrelation is also higher at 0.77 (as compared to 0.69 for the full sample). Nevertheless, 0.77 corresponds to a half life of less than four weeks, still much shorter than the half life of the raw interest-rate series.

Third, we compute the portfolio weights based on the model parameter estimates from the first subsample. We then analyze the out-of-sample predictability of the four-instrument portfolios during the second subsample. The left panel of Figure 6 plots the histogram of the R-squares from the AR(1) regression over four-week horizons on the out-of-sample four-instrument portfolios. The R-squares range from 6.79% to 55.58%, similar to those obtained from in-sample regressions in Figure 5. Therefore, the strong predictability remains out of sample.

[Figure 6 about here.]

Finally, we perform the investment exercise for this out-of-sample period, using the same procedure as described in the previous subsection. To remain truly out of sample, we compute $Var(ER)$ based on the first subsample in determining the allocation weight w_t according to equation (27). The right panel of Figure 6 plots the histogram of the annualized information ratios of the investment strategies. Comparing to the in-sample case during the same sample period (yellow bar in the right panel of Figure 5), the histogram for the out-of-sample information ratios shows slightly larger variation across different portfolios. The sample mean of the information ratios is 0.4723, close to the in-sample mean of 0.4881 over the same sample period. Overall, the out-of-sample analysis shows that our estimation strategy is robust and the estimated factor dynamics are stable over time.

4.3. Robustness with respect to model specifications

So far, the analysis has been based solely on the $A_0(3)$ model. We also estimate $A_m(3)$ models with $m = 1, 2, 3$, respectively. Table 10 summarizes the estimation results on these three models. To save space, we only report the parameter estimates and t-statistics on κ and κ^*, which control the factor drift dynamics under the two measures, respectively. For the properties of the pricing errors, we report the sample average of the weekly autocorrelation (Auto) of the pricing errors and the explained percentage variation (VR). The last column reports the maximized log likelihood value ($\mathcal{L}$).

Under all three models, the κ and κ^* estimates indicate high persistence for the interest-rate factors. The minimum eigenvalue is close to zero. The largest eigenvalue estimate is 1.269 for κ under the $A_3(3)$ model. Even this largest estimate corresponds to a weekly autocorrelation of 0.976 and half life of 29 weeks. In contrast, the average weekly autocorrelations for the pricing errors range from 0.69 under the $A_0(3)$ model to 0.77 under the $A_1(3)$ model. In all cases, the implied half life for the pricing errors averages below four weeks, considerably shorter than the half lives of the raw interest-rate series or the interest-rate factors.

Comparing the average explained percentage variations and the maximized log likelihood values of the four three-factor affine models, we find that the performance differences between the four models are very small. All four models explain over 99% of the variation in the interest rates. The log likelihood values from different models only differ slightly. The likelihood value estimate is the highest at 12,966 for the $A_0(3)$ model and the lowest at $12,034$ for the $A_1(3)$ model. Bikbov and Chernov (2004) find that different three-factor affine specifications all perform well in capturing the term structure behavior of eurodollar futures. Our estimation and investment exercise suggest the same for the U.S. dollar LIBOR and swap market.

Given the similar fitting performance from four models, we expect to obtain similar predictability for interest-rate portfolios formed based on these four models. To save space, we only report the histograms of the R-squares from the AR(1) forecasting regressions on four-instrument portfolios formed using different models. Figure 7 reports the four histograms from the four models estimated using the full sample period. We observe some mild variations in predictability from portfolios formed using the four models. In particular, we observe a larger proportion of low R-squares from the $A_1(3)$ model than from the other three models. Recall that the $A_1(3)$ model also has the lowest log likelihood value. Thus, when a dynamic term structure model fits the dynamics and the term structure of interest rates better, the model can also be used to form interest-rate portfolios that show higher predictability.

[Figure 7 about here.]

5. Conclusion

Due to the near unit-root behavior of interest rates, it is well-known that the movements of individual interest-rate series are inherently difficult to forecast, even under a multivariate dynamic term structure framework. In this paper, we explore an innovative way of applying multivariate dynamic term structure models to forecast interest-rate movements. Instead of directly forecasting interest-rate movements based on the estimated factor dynamics, we use the dynamic term structure model as a decomposition tool. We decompose each interest-rate series into two components: a persistent component captured by the dynamic factors, and a strongly mean-reverting component given by the pricing residuals of the model. Given this decomposition, we form interest-rate portfolios that are first-order neutral to the persistent factor dynamics, but are fully exposed to the strongly mean-reverting movements. We show that the predictability of these interest-rate portfolios is significant both statistically and economically, both in sample and out of sample.

Our new application of the dynamic term structure models has important implications for future interest-rate modeling. The statistical and economic significance of the interest-rate portfolios reveal an important reason behind the limited success of finite-dimensional dynamic term structure models in pricing interest-rate options. The finite-dimensional dynamic term structure models capture the persistent factors well, but often discard the transient interest-rate movements. Yet, not only can these transient components be exploited in investment decisions, as we have shown in this paper, but they can also have potentially important impacts on the valuation of interest-rate options. An important line for future research is to theoretically model and econometrically identify the dynamics of these transient components and to investigate the impact of these dynamics on different aspects of interest rates and interest-rate options behavior.

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Table 1

Summary statistics of interest rates

Entries are summary statistics of interest rates. Mean, Std, Skew, Kurt, and Autocorrelation denote the sample estimates of the mean, standard deviation, skewness, excess kurtosis, and the autocorrelation of order one, five, ten, and 20, respectively. In the maturity column, m denotes months and y denotes years. The data are weekly (Wednesday) closing midquotes downloaded from Bloomberg, from May 11, 1994 to December 10, 2003, 501 observations for each series.

Maturity	Mean	Std	Skew	Kurt	Autocorrelation			
					1	5	10	20
1 m	4.617	1.772	-0.985	-0.594	0.994	0.969	0.935	0.848
2 m	4.650	1.789	-0.987	-0.603	0.995	0.972	0.937	0.851
3 m	4.684	1.809	-0.981	-0.615	0.995	0.973	0.938	0.851
6 m	4.761	1.838	-0.962	-0.615	0.995	0.973	0.937	0.849
12 m	4.957	1.853	-0.910	-0.557	0.994	0.970	0.930	0.835
2 y	5.271	1.676	-0.882	-0.330	0.993	0.963	0.917	0.810
3 y	5.524	1.510	-0.852	-0.184	0.992	0.958	0.907	0.793
5 y	5.856	1.283	-0.732	-0.143	0.990	0.950	0.891	0.768
7 y	6.069	1.145	-0.594	-0.207	0.988	0.943	0.881	0.751
10 y	6.276	1.028	-0.427	-0.324	0.987	0.937	0.872	0.736
15 y	6.491	0.916	-0.221	-0.429	0.985	0.931	0.863	0.723
30 y	6.618	0.852	0.024	-0.405	0.986	0.933	0.865	0.726
Average	5.481	1.456	-0.709	-0.417	0.991	0.956	0.906	0.795

Table 2

Parameter estimates of the $A_0(3)$ model

Entries report the parameter estimates (and the absolute magnitude of the t-values in parentheses) of the $A_0(3)$ model based on the eurodollar LIBOR and swap rates. The data consist of weekly observations on LIBOR at maturities of one, two, three, six, and 12 months and swap rates at maturities of two, three, five, seven, ten, 15, and 30 years. For each series, the data are closing Wednesday midquotes from May 11, 1994 to December 10, 2003, 501 observations for each series. The model is estimated using a quasi-maximum likelihood method jointly with unscented Kalman filter. The last column reports the maximized log likelihood value ($\mathcal{L}$).

Parameters	κ			κ^*			b_r	λ_1	a_r	$\mathcal{L}$
Estimates (t-values)	0.002	0	0	0.014	0	0	0.000	−0.117	0.044 (2.94)	12,966
	(0.02)	−−	−−	(11.6)	−−	−−	(0.00)	(4.30)		
	−0.186	0.480	0	0.068	0.707	0	0.000	−0.376		
	(0.42)	(1.19)	−−	(1.92)	(20.0)	−−	(0.00)	(0.75)		
	−0.749	−2.628	0.586	−2.418	−3.544	1.110	0.005	−4.959		
	(1.80)	(3.40)	(2.55)	(10.7)	(12.0)	(20.0)	(13.3)	(1.96)		

Table 3
Summary statistics of pricing errors on the $A_0(3)$ model
Entries report the summary statistics of the pricing errors on the LIBOR and swap rates under the estimated $A_0(3)$ model. We define pricing errors as the basis-point difference between the observed LIBOR/swap rates and the model-implied values. The data consist of weekly observations on LIBOR at maturities of one, two, three, six, and 12 months and swap rates at maturities of two, three, five, seven, ten, 15, and 30 years. For each series, the data are closing Wednesday midquotes from May 11, 1994 to December 10, 2003 (501 observations). We estimate the model using a quasi-maximum likelihood method jointly with unscented Kalman filter. The columns titled Mean, Rmse, Skew, Kurt, Max, and Auto denote the mean, the root mean squared error, the skewness, the excess kurtosis, the maximum absolute error, and the first-order weekly autocorrelation of the measurement errors at each maturity, respectively. The last column (VR) reports the percentage variance explained for each series by the three factors, defined as one minus the ratio of pricing error variance to the variance original interest-rate series, in percentage points. The last row reports average statistics.

Maturity	Mean	Rmse	Skew	Kurt	Max	Auto	VR
1 m	1.823	10.679	-1.018	7.254	60.495	0.802	99.647
2 m	1.065	4.983	-3.407	25.636	37.142	0.703	99.926
3 m	0.349	3.717	4.598	34.115	31.963	0.730	99.958
6 m	-4.389	9.326	-0.237	1.472	33.532	0.881	99.799
12 m	-9.791	14.145	-1.063	2.615	55.118	0.789	99.696
2 y	-0.895	4.251	-1.062	2.698	23.028	0.872	99.938
3 y	0.428	1.061	0.410	1.058	3.999	0.394	99.996
5 y	0.197	1.811	0.330	2.953	10.116	0.558	99.980
7 y	0.001	0.013	-1.202	11.880	0.090	0.118	99.999
10 y	0.068	3.113	0.115	0.695	12.333	0.704	99.908
15 y	2.162	7.387	0.335	-0.139	22.292	0.853	99.404
30 y	-0.529	11.070	-0.012	-0.315	34.579	0.901	98.314
Average	-0.793	5.963	-0.184	7.493	27.057	0.692	99.714

Table 4
Predictability of individual LIBOR and swap rate series

Entries report the sample mean (Mean), root mean squared error (Rmse), skewness (Skew), excess kurtosis (Kurt), maximum absolute error (Max), weekly autocorrelation (Auto), and explained percentage variation (VR) of the four-week ahead forecasting errors from the random-walk hypothesis (first panel), AR(1) regression (second panel), and the $A_0(3)$ dynamic term structure model. The weekly autocorrelation is computed on the overlapping sample of the four-week ahead forecasting errors.

Maturity	Mean	Rmse	Skew	Kurt	Max	Auto	VR
			Forecasting Error Assuming Random-Walk				
1 m	-1.954	19.882	-0.309	7.926	107.875	0.748	0.000
2 m	-2.032	16.676	-0.813	8.184	93.000	0.771	-0.000
3 m	-2.117	16.329	-0.788	7.480	92.875	0.798	0.000
6 m	-2.297	17.190	-0.785	5.335	99.875	0.783	0.000
12 m	-2.460	20.969	-0.195	1.863	103.750	0.738	-0.000
2 y	-2.344	24.133	0.187	0.458	93.200	0.714	0.000
3 y	-2.202	24.724	0.249	0.339	92.300	0.707	0.000
5 y	-1.999	24.564	0.368	0.432	93.600	0.687	0.000
7 y	-1.854	23.910	0.403	0.496	91.000	0.681	0.000
10 y	-1.710	23.250	0.426	0.500	88.300	0.677	0.000
15 y	-1.591	21.831	0.383	0.350	80.300	0.664	-0.000
30 y	-1.514	19.763	0.342	0.272	68.100	0.668	0.000
			Forecasting Error From AR(1) Regression				
1 m	-0.000	19.763	-0.352	7.940	109.417	0.746	0.230
2 m	-0.000	16.501	-0.862	8.213	90.148	0.768	0.613
3 m	-0.000	16.135	-0.832	7.493	89.864	0.795	0.697
6 m	0.000	16.991	-0.816	5.263	96.739	0.780	0.529
12 m	0.000	20.813	-0.203	1.820	100.832	0.737	0.103
2 y	-0.000	24.017	0.182	0.452	91.037	0.715	0.015
3 y	-0.000	24.610	0.223	0.307	93.195	0.708	0.128
5 y	0.000	24.429	0.305	0.344	93.291	0.690	0.438
7 y	-0.000	23.753	0.326	0.372	89.970	0.684	0.713
10 y	0.000	23.063	0.342	0.351	86.461	0.682	1.068
15 y	-0.000	21.617	0.314	0.192	78.042	0.669	1.424
30 y	0.000	19.554	0.288	0.122	66.116	0.673	1.528
			Forecasting Error from $A_0(3)$ Factor Dynamics				
1 m	3.863	15.440	-0.990	4.802	73.725	0.693	42.917
2 m	3.129	14.580	-1.417	6.793	82.468	0.747	25.987
3 m	2.295	16.290	-0.806	5.943	89.991	0.813	0.780
6 m	-2.652	19.730	-0.862	3.183	108.317	0.836	-31.711
12 m	-8.082	24.962	-0.577	1.814	124.598	0.795	-28.623
2 y	0.255	24.947	-0.007	0.343	99.638	0.734	-7.872
3 y	1.159	24.760	0.234	0.335	96.000	0.703	-0.878
5 y	0.328	24.386	0.352	0.509	99.756	0.684	0.809
7 y	-0.199	23.889	0.378	0.460	92.050	0.679	-0.415
10 y	-0.397	23.635	0.415	0.425	82.167	0.681	-3.866
15 y	1.494	23.479	0.450	0.426	84.567	0.697	-15.811
30 y	-1.352	23.073	0.317	0.138	74.146	0.726	-36.639

Table 5
Predictability of interest-rate portfolios
Entries report the sample properties of the estimates, t-statistics in parentheses, and the R-squares on the following AR(1) regression:

$$\Delta R_{t+1} = a + bR_t + e_{t+1},$$

where ΔR_{t+1} denotes the changes in the portfolio value over a four-week horizon. We estimate the relation using the generalized methods of moments on overlapping data. We compute t-statistics of the estimates following Newey and West (1987) with eight lags. Regressions in the first panel are on the 495 four-instrument portfolios that are first-order neutral to the three interest-rate factors. Regressions in the second panel are on the 66 two-instrument portfolios that are first-order neutral to the first interest-rate factor. The last panel reports results on the 220 three-instrument portfolios that are first-order neutral to the first two interest-rate factors.

Percentile	Intercept		Slope		R-square, %
			495 four-instrument interest rate portfolios		
Min	-0.441	(-15.071)	-1.115	(-25.010)	7.844
10	-0.101	(-11.114)	-0.717	(-16.664)	9.117
25	-0.070	(-8.662)	-0.493	(-12.716)	11.115
50	-0.044	(-6.629)	-0.311	(-9.614)	15.680
75	-0.019	(-4.052)	-0.224	(-7.887)	24.525
90	0.010	(2.002)	-0.179	(-7.061)	35.846
Max	0.073	(12.669)	-0.155	(-6.504)	55.724
			66 two-instrument interest rate portfolios		
Min	0.002	(0.205)	-0.111	(-5.361)	0.000
10	0.007	(1.402)	-0.013	(-1.352)	0.011
25	0.014	(1.626)	0.000	(0.029)	0.090
50	0.046	(1.924)	0.005	(0.888)	0.198
75	0.255	(2.503)	0.007	(1.597)	0.528
90	0.388	(2.706)	0.007	(1.649)	0.611
Max	0.646	(5.221)	0.007	(1.763)	5.467
			220 three-instrument interest rate portfolios		
Min	-5.853	(-2.342)	-0.548	(-13.703)	0.000
10	-1.706	(-2.232)	-0.105	(-5.164)	0.038
25	-0.540	(-1.976)	-0.008	(-0.887)	0.128
50	-0.111	(-1.437)	0.004	(0.796)	0.242
75	0.006	(0.321)	0.006	(1.257)	0.445
90	0.061	(4.880)	0.007	(1.446)	5.006
Max	0.174	(13.332)	0.007	(1.570)	27.420

Table 6

Excess returns to swap portfolio investments

Entries report the summary statistics of the excess returns on investing in each of the 35 four-instrument swap portfolios: the sample mean, median, minimum (Min), maximum (Max), skewness (Skew), excess kurtosis (Kurt), average first-order autocorrelations on four-week apart non-overlapping data (Auto), and the annualized information ratio (IR). The last row reports the average statistics over the 35 portfolios.

Mean	Median	Min	Max	Skew	Kurt	Auto	IR
0.281	0.035	-1.623	12.280	6.762	59.943	-0.034	0.879
0.228	0.062	-4.121	8.861	2.600	18.124	0.008	0.756
0.114	0.026	-5.306	7.357	-0.037	11.375	-0.056	0.405
0.124	0.032	-5.534	7.544	0.402	7.312	0.022	0.366
0.329	0.097	-4.140	14.161	4.068	33.731	0.065	0.862
0.147	0.022	-5.236	10.496	1.154	16.694	-0.045	0.453
0.155	0.033	-4.294	8.773	0.935	7.059	0.023	0.447
0.270	0.028	-3.468	20.474	6.322	74.032	-0.040	0.652
0.203	0.050	-3.455	12.644	2.759	23.734	0.016	0.605
0.195	0.085	-4.358	6.873	1.116	6.997	-0.021	0.659
0.442	0.083	-1.991	34.631	11.269	171.441	0.067	0.786
0.247	0.049	-3.651	23.929	8.389	113.679	-0.035	0.559
0.228	0.026	-3.516	18.959	5.393	63.419	0.006	0.577
0.343	0.027	-2.883	27.214	8.885	120.344	-0.011	0.707
0.249	0.065	-2.829	17.372	5.429	65.002	0.020	0.701
0.199	0.069	-4.368	7.014	1.192	7.522	-0.032	0.667
0.426	0.048	-2.721	23.304	7.953	83.726	0.027	0.811
0.299	0.074	-2.194	13.818	4.674	41.222	0.042	0.873
0.199	0.041	-4.274	7.330	1.486	9.719	-0.029	0.672
0.228	0.064	-3.661	9.534	2.403	18.121	0.006	0.752
0.456	0.084	-1.643	37.827	12.288	198.482	0.067	0.774
0.277	0.054	-3.239	27.120	9.915	140.714	-0.035	0.580
0.250	0.041	-3.441	21.536	6.842	88.062	0.001	0.603
0.360	0.032	-2.952	28.661	9.347	129.479	-0.003	0.718
0.260	0.072	-2.725	18.395	6.063	76.337	0.021	0.719
0.199	0.065	-4.353	7.020	1.206	7.633	-0.034	0.669
0.436	0.055	-2.692	23.316	8.021	84.407	0.032	0.819
0.306	0.076	-2.212	13.790	4.790	42.234	0.046	0.888
0.199	0.040	-4.256	7.364	1.503	9.839	-0.029	0.673
0.228	0.063	-3.657	9.538	2.406	18.149	0.006	0.752
0.463	0.067	-2.708	25.599	8.090	84.666	0.050	0.847
0.326	0.074	-2.339	13.360	4.890	41.444	0.064	0.941
0.200	0.042	-4.187	7.510	1.590	10.462	-0.028	0.677
0.229	0.062	-3.638	9.558	2.421	18.280	0.006	0.755
0.231	0.066	-3.595	9.627	2.456	18.591	0.008	0.761
0.267	0.055	-3.464	15.794	4.714	54.914	0.005	0.696

Table 7

Explaining excess returns to swap portfolio using market factors

Entries report the intercept estimates, the t-statistics on the intercept and slope estimates, and R-squares of the following regression,

$$R_t = \alpha + \beta_1(R_{mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 UMD_t + \beta_5 CS_t + \beta_6 USV_t + e_t.$$

The t-statistics are computed according to Newey and West (1987) with eight lags. The intercept estimates are multiplied by $\sqrt{52/4}/\mathsf{std}(R_t)$ so that they are comparable to the annualized information ratio estimates (IR= $\sqrt{52/4}\mathsf{mean}(R_t)/\mathsf{std}(R_t)$) in Table 6. The last column reports the skewness of the regression residuals (e_t). The last row reports the average statistics over the 35 portfolios.

Estimates	t-statistics							R^2	Skew
α	α	$R_m - R_f$	SMB	HML	UMD	CS	USV	%	(e_t)
0.80	3.02	1.07	-1.40	-1.76	1.10	-0.04	0.28	1.96	6.61
0.54	1.35	0.48	-0.06	-0.79	-0.78	0.24	0.93	1.50	2.50
0.03	0.06	0.25	0.07	-0.37	-0.33	0.37	1.11	1.88	0.20
0.11	0.22	0.40	-0.42	-0.39	-0.75	0.66	0.85	2.02	0.46
0.71	1.85	0.74	0.62	0.24	-1.69	0.17	0.81	1.03	4.15
0.15	0.29	0.37	-0.05	0.01	-0.34	0.08	0.96	1.21	1.36
0.27	0.63	0.53	-0.88	-0.42	-0.80	0.78	0.68	2.02	0.97
0.45	1.32	0.35	-0.49	-0.75	0.62	-0.47	0.96	0.93	6.37
0.49	1.55	0.46	-1.57	-0.93	-0.34	0.49	0.56	1.76	2.78
0.83	2.35	-0.77	-3.01	-1.82	-0.88	1.29	-0.32	3.49	1.04
0.76	2.81	1.23	-0.04	0.45	-0.11	-0.22	-0.00	0.23	11.26
0.43	1.19	0.75	-0.54	-0.08	0.14	-0.38	0.58	0.50	8.38
0.52	1.65	0.81	-1.56	-0.76	-0.43	0.70	0.32	1.42	5.43
0.57	1.98	0.44	-0.71	-1.05	0.80	-0.82	0.83	0.79	8.89
0.65	2.48	0.49	-2.04	-1.31	-0.18	0.19	0.37	1.59	5.48
0.86	2.21	-0.95	-2.67	-1.88	-0.74	1.10	-0.40	3.29	1.09
0.70	2.70	0.43	-0.68	-1.22	0.93	-1.02	0.82	0.75	7.96
0.84	3.23	0.39	-2.04	-1.49	0.02	-0.41	0.34	1.42	4.71
0.87	2.12	-1.16	-2.45	-1.82	-0.64	0.90	-0.42	2.92	1.40
0.93	2.23	-1.26	-1.99	-1.58	-0.31	0.82	-0.37	2.35	2.35
0.77	2.93	1.28	-0.23	0.33	0.28	-0.32	-0.29	0.24	12.26
0.49	1.46	0.85	-0.69	-0.19	0.32	-0.50	0.44	0.48	9.86
0.57	1.95	0.89	-1.70	-0.91	-0.26	0.61	0.17	1.30	6.87
0.59	2.13	0.46	-0.76	-1.12	0.84	-0.89	0.79	0.77	9.34
0.69	2.68	0.50	-2.13	-1.42	-0.13	0.10	0.31	1.55	6.11
0.87	2.18	-0.99	-2.62	-1.89	-0.71	1.06	-0.41	3.24	1.10
0.71	2.77	0.44	-0.68	-1.23	0.93	-1.04	0.81	0.74	8.03
0.86	3.30	0.39	-2.04	-1.52	0.04	-0.47	0.32	1.40	4.83
0.87	2.11	-1.17	-2.43	-1.82	-0.63	0.89	-0.42	2.90	1.42
0.93	2.23	-1.26	-1.99	-1.58	-0.31	0.82	-0.37	2.34	2.35
0.75	2.96	0.45	-0.62	-1.21	0.94	-1.07	0.77	0.70	8.11
0.92	3.47	0.37	-1.96	-1.50	0.11	-0.71	0.31	1.28	4.94
0.88	2.11	-1.21	-2.39	-1.79	-0.61	0.84	-0.42	2.81	1.51
0.93	2.24	-1.26	-1.99	-1.58	-0.30	0.82	-0.37	2.33	2.37
0.93	2.25	-1.26	-1.97	-1.57	-0.28	0.82	-0.36	2.30	2.40
0.66	2.06	0.10	-1.32	-1.05	-0.13	0.15	0.29	1.64	4.71

Table 8

Subsample parameter estimates of the $A_0(3)$ model

Entries report the parameter estimates (and the absolute magnitude of the t-values in parentheses) of the $A_0(3)$ model based on the eurodollar LIBOR and swap rates for two subsamples. The data consist of weekly observations on LIBOR at maturities of one, two, three, six, and 12 months and swap rates at maturities of two, three, five, seven, ten, 15, and 30 years. Data are closing Wednesday midquotes. The first subsample is from May 11, 1994 to May 6, 1998. The second subsample is from May 13, 1998 to December 10, 2003. The model is estimated using a quasi-maximum likelihood method jointly with unscented Kalman filter.

Sample Period	κ			κ^*			b_r	λ_1	a_r
	0.002	0	0	0.003	0	0	0.000	−0.128	
	(0.01)	−−	−−	(20.0)	−−	−−	(0.00)	(7.34)	
1994−	0.064	0.451	0	−0.051	0.512	0	0.000	−0.673	0.044
1999	(0.13)	(0.61)	−−	(1.83)	(20.0)	−−	(0.00)	(0.84)	(0.65)
	−0.866	−2.610	0.559	−2.260	−4.171	1.650	0.005	−5.308	
	(1.32)	(3.10)	(0.76)	(8.24)	(11.8)	(20.0)	(15.6)	(0.96)	
	0.002	0	0	0.060	0	0	0.000	−0.149	
	(0.01)	−−	−−	(19.8)	−−	−−	(0.00)	(0.88)	
1999−	−0.263	0.415	0	−0.076	0.935	0	0.000	−0.745	0.048
2003	(0.17)	(0.54)	−−	(0.40)	(4.42)	−−	(0.00)	(0.46)	(1.82)
	−0.760	−2.272	0.740	−2.480	−3.693	1.049	0.006	−4.450	
	(0.45)	(1.91)	(2.06)	(5.31)	(7.29)	(3.96)	(7.72)	(1.09)	

Table 9

Summary statistics of out-of-sample pricing errors on the LIBOR and swap rates

Entries report the summary statistics of the out-of-sample pricing errors in basis points on the LIBOR and swap rates. We define pricing errors as the basis-point difference between the observed LIBOR/swap rates and the model-implied values. The sample used for model estimation consists of LIBOR rates at maturities of one, two, three, six, and 12 months and swap rates at maturities of two, three, five, seven, ten, 15, and 30 years, from May 11, 1994 to May 6 24, 1998. The pricing errors underlying this table are on LIBOR and swap rates from May 13, 1998 to December 10, 2003. The columns titled Mean, MAE, Std, Max, and Auto denote the mean, the mean absolute error, the standard deviation, the maximum in absolute magnitude, and the first order autocorrelation of the pricing errors at each maturity, respectively. The last column (VR) reports the percentage variance explained for each series by the three factors. The last row reports average statistics.

Maturity	Mean	MAE	Std	Max	Auto	VR
1 m	2.30	7.70	11.40	64.93	0.79	99.69
2 m	1.04	2.38	3.74	20.95	0.72	99.97
3 m	0.13	2.95	6.10	42.47	0.81	99.91
6 m	−5.55	9.61	10.77	35.21	0.91	99.73
12 m	−11.71	13.12	12.01	53.33	0.81	99.66
2 y	−1.62	2.37	3.09	16.58	0.89	99.97
3 y	1.07	1.58	1.89	9.78	0.85	99.99
5 y	0.83	1.95	2.59	9.94	0.80	99.96
7 y	0.00	0.01	0.01	0.09	0.06	99.99
10 y	−1.15	3.16	3.94	11.46	0.81	99.85
15 y	−0.38	7.39	9.09	24.09	0.91	98.86
30 y	−9.13	13.05	13.19	49.04	0.91	96.71
Average	−2.01	5.44	6.49	28.16	0.77	99.53

Table 10

Factor dynamics and pricing errors under $A_m(3)$ models

Entries report factor dynamics (κ and κ^*) estimates (and the absolute magnitude of the t-values in parentheses) of $A_m(3)$ models, with $m = 1,2,3$. We also report the average weekly autocorrelation (Auto) of the pricing errors and the average explained percentage variation (VR) for each model. The last column reports the maximized log likelihood value ($\mathcal{L}$).

Model	κ	κ^*	Auto	VR	$\mathcal{L}$
$A_1(3)$	$\begin{bmatrix} 0.049 & 0 & 0 \\ (0.22) & -- & -- \\ -0.007 & 0.048 & 0 \\ (0.00) & (0.01) & -- \\ 0.020 & -2.413 & 0.306 \\ (0.95) & (0.24) & (0.92) \end{bmatrix}$	$\begin{bmatrix} 0.000 & 0 & 0 \\ (0.00) & -- & -- \\ -0.354 & 1.126 & 0 \\ (0.20) & (13.1) & -- \\ -0.371 & -4.295 & 0.858 \\ (0.12) & (0.24) & (15.6) \end{bmatrix}$	0.77	99.49	12,034
$A_2(3)$	$\begin{bmatrix} 0.560 & 0 & 0 \\ (10.3) & -- & -- \\ -3.379 & 0.952 & 0 \\ (9.61) & (21.5) & -- \\ 7.423 & -1.609 & 0.048 \\ (0.58) & (0.82) & (1.02) \end{bmatrix}$	$\begin{bmatrix} 0.647 & 0 & 0 \\ (34.7) & -- & -- \\ -3.379 & 1.120 & 0 \\ (9.61) & (28.4) & -- \\ 11.765 & -3.771 & 0.014 \\ (0.78) & (0.77) & (12.0) \end{bmatrix}$	0.74	99.75	12,916
$A_3(3)$	$\begin{bmatrix} 0.013 & 0 & 0 \\ (6.81) & -- & -- \\ -0.089 & 0.874 & 0 \\ (6.15) & (4.94) & -- \\ 0.457 & -14.00 & 1.269 \\ (4.23) & (8.25) & (16.4) \end{bmatrix}$	$\begin{bmatrix} 0.000 & 0 & 0 \\ (0.00) & -- & -- \\ -0.089 & 0.684 & 0 \\ (6.15) & (26.2) & -- \\ 0.457 & -14.00 & 0.960 \\ (4.23) & (8.25) & (22.8) \end{bmatrix}$	0.76	99.78	12,730

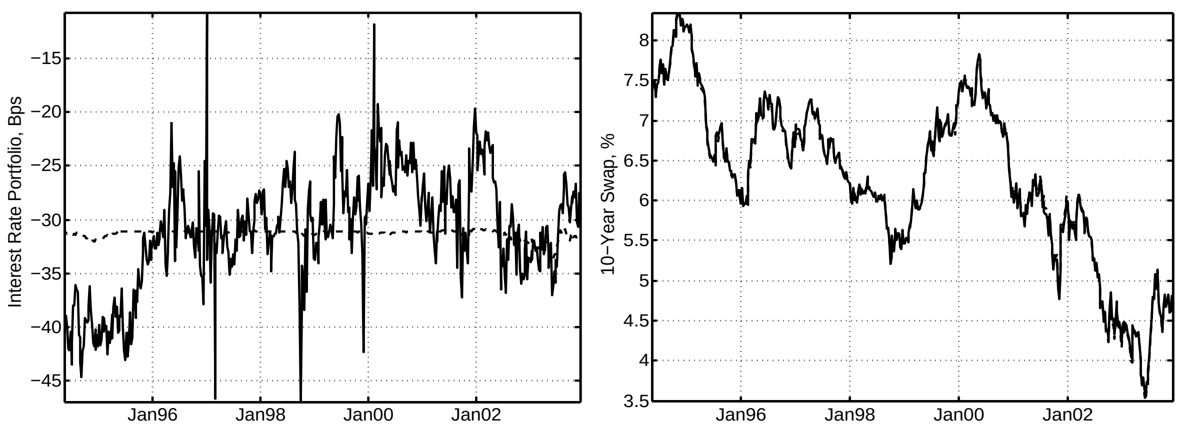

Figure 1
The time series of hedged and unhedged ten-year swap rates.
The left panel plots the time series of the ten-year swap rate hedged against the three interest-rate factors using two-, five-, and 30-year swap rates. The right panel plots the time series of the unhedged ten-year swap rate series. In both panels, the solid lines denote the market value and the dashed lines denote the model-implied value. In the right panel, the market value and the model-implied value for the unhedged ten-year swap rates are not visually distinguishable.

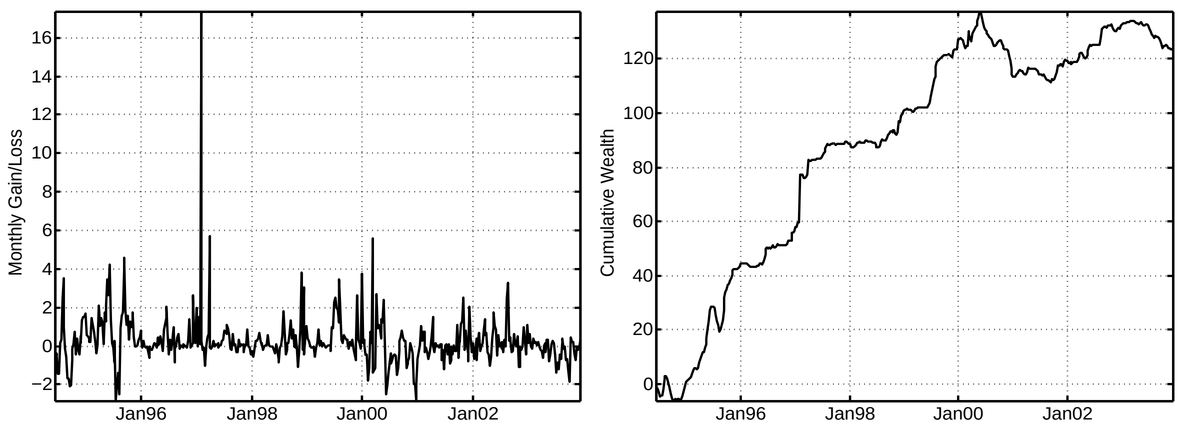

Figure 2

Capital gains from the portfolio investment.

The left panel plots the four-week excess returns from the weekly investment in the portfolio with a $1000 notional amount on the 10-year par bond. The right panel plots the cumulative wealth as a result of this investment exercise. The portfolio is made of two, five, ten, and 30-year swap rates, designed to achieve first-order neutrality to the three interest-rate factors.

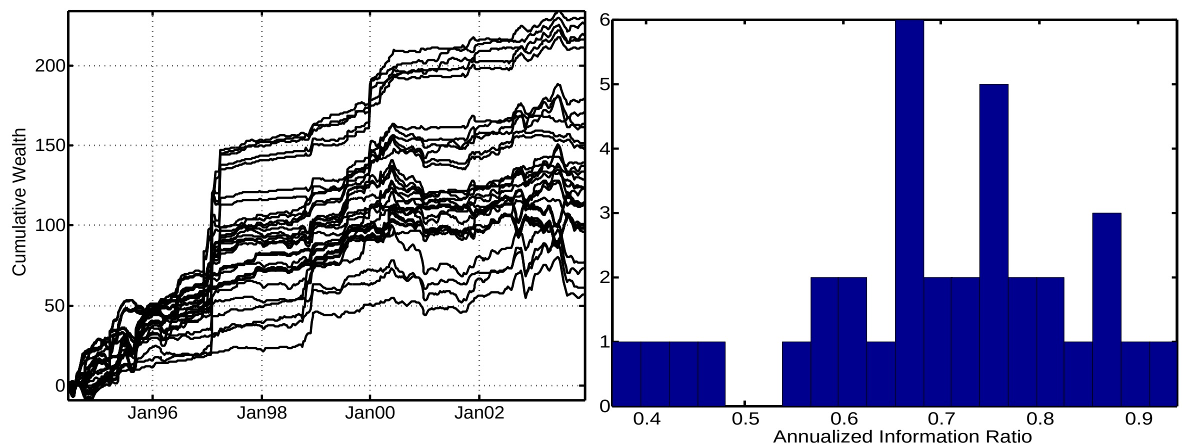

Figure 3
Profitability of investing in four-instrument interest-rate portfolios.
The left panel plots the cumulative wealth paths of investing in each of the 35 different combinations of the four-instrument swap portfolios. The right panel plots the histogram of the annualized information ratios from investing in each portfolio.

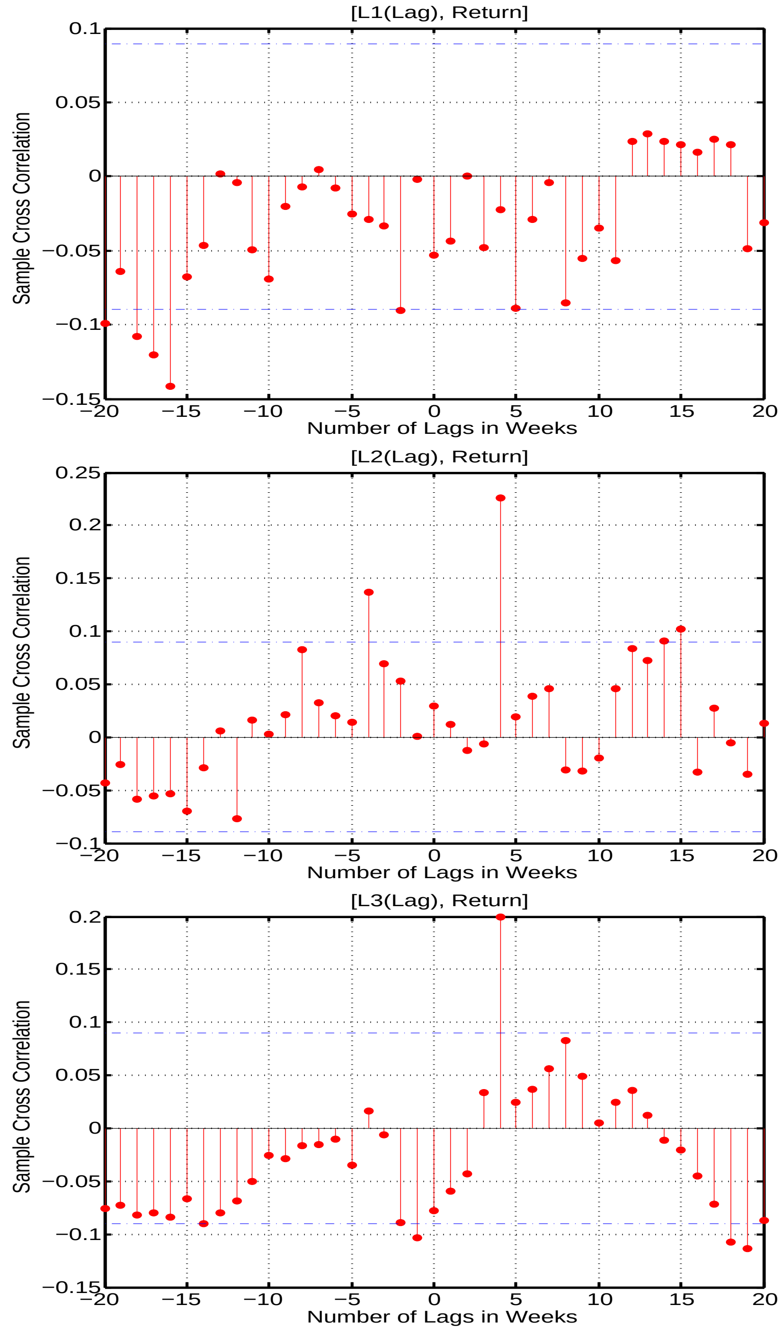

Figure 4

Cross-correlation between liquidity measures and swap portfolio investment returns.

The bars in the three panels represent the cross-correlation estimates at different leads and lags between the three liquidity measures and the average excess returns from the weekly investment in the 35 four-instrument swap portfolios. Each panel reports the correlation with one liquidity measure.

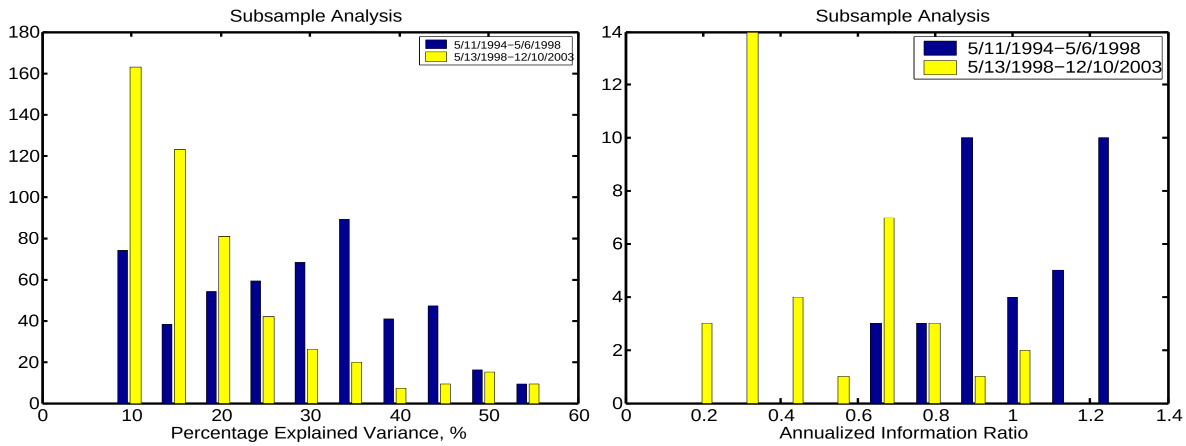

Figure 5

Subsample analysis on the statistical and economic significance of portfolio predictability.

The left panel plots the histogram of the R-squares of the AR(1) regressions over four-week horizons on the 495 combinations of four-instrument portfolios during the first (dark blue bars) and second (light yellow bars) subsamples. The right panel plots the histogram of the annualized information ratios from investing in each of 53 different combinations of the four-instrument swap portfolios during the first (dark blue bars) and second subsamples (light yellow bars). The first subsample spans the first four years from May 11, 1994 to May 6, 1998, and the second subsample is from May 13, 1998 to December 10, 2003.

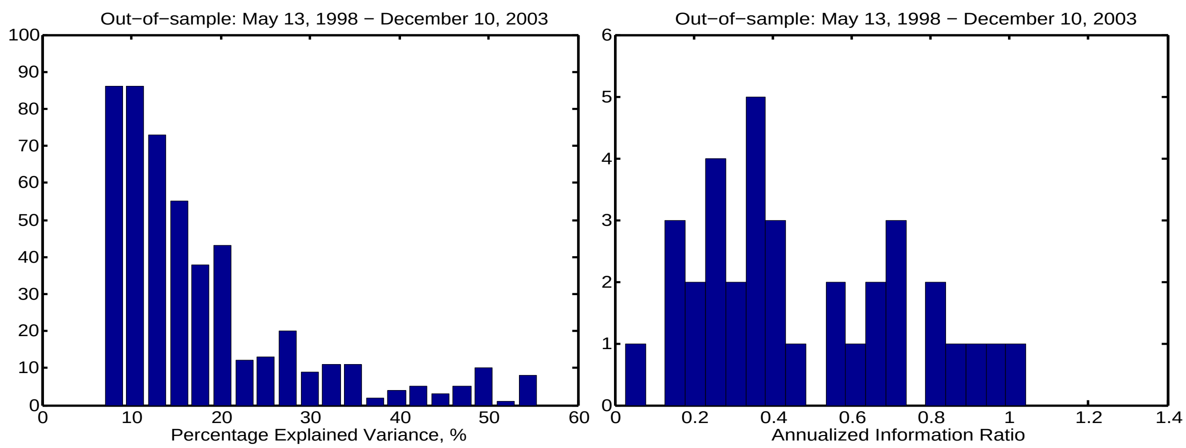

Figure 6

Out-of-sample predictability of four-instrument interest-rate portfolios.

The left panel shows the histogram of the R-squares of the AR(1) regressions over four-week horizons on the 495 combinations of four-instrument portfolios. The right panel plots the histogram of the annualized information ratios from investing in the four-instrument swap portfolios. We determine the portfolio weight based on the model estimates and data on LIBOR and swap rates from the first subsample from May 11, 1994 to May 6, 1998. Then we run the AR(1) regression (the left panel) and perform the investment exercise (the right panel) out of sample on data from May 13, 1998 to December 10, 2003.

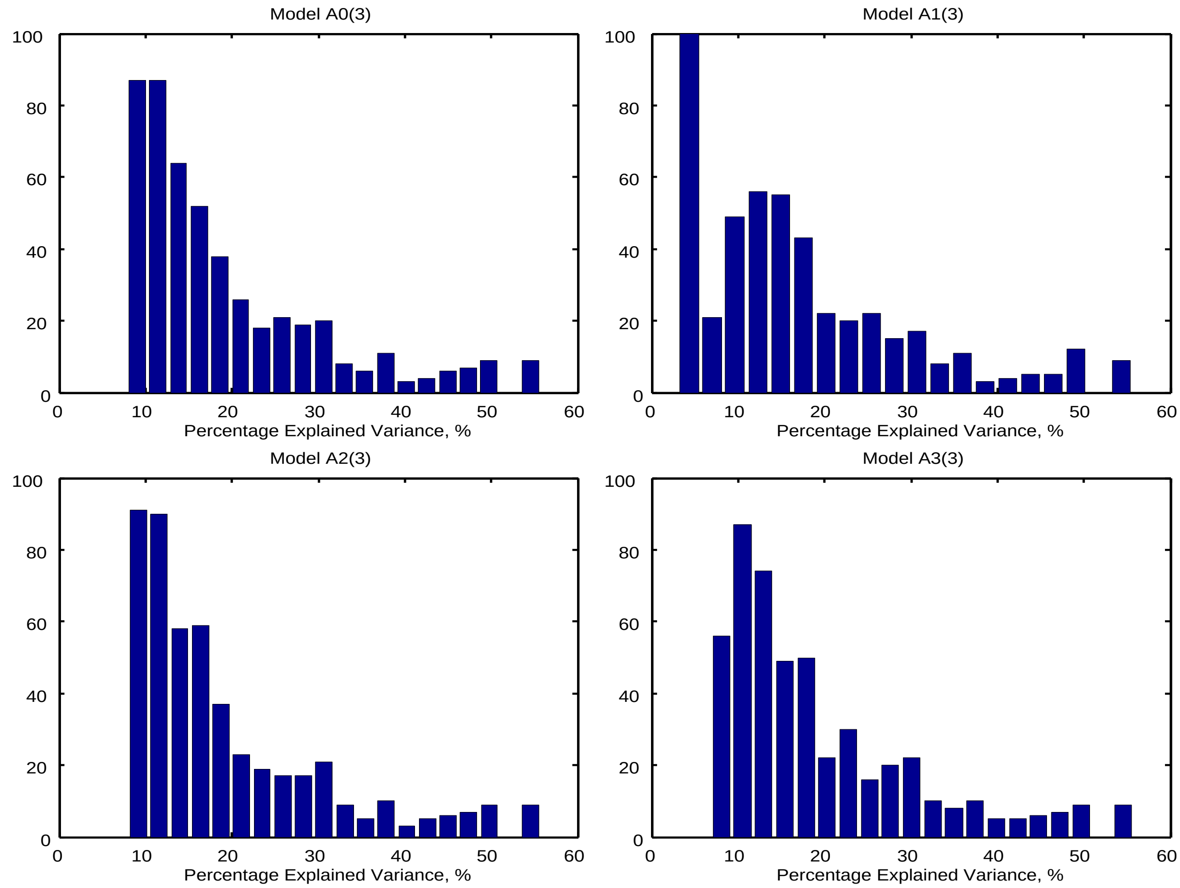

Figure 7
Predictability of four-instrument interest-rate portfolios formed using $A_m(3)$ models.
The four panels show the histograms of the R-squares of the AR(1) regressions over four-week horizons on the 495 combinations of four-instrument portfolios, formed by using four affine three-factor dynamic term structure models, $A_m(3)$ with $m = 0, 1, 2, 3$.