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2010-22

# Liquidity Transformation and Bank Capital Requirements

by Hajime Tomura

Bank of Canada Working Paper 2010-22

August 2010

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## **Acknowledgements**

I thank Jason Allen, David Andolfatto, Jonathan Chiu, Ian Christensen, Paul Gomme, Toni Gravelle, Zhiguo He, Nobuhiro Kiyotaki, Cyril Monnet (discussant), Shouyong Shi, Yaz Terajima, Randy Wright, Mark Zelmer, and seminar participants at the Bank of Canada “Liquidity and the Financial System” workshop, CEA 2010, Midwest Macro 2010, SED 2010, Financial Services Agency (Japan), GRIPS, Hitotsubashi U, Nihon U, and U of Ottawa for their comments on earlier versions of the paper.

## Abstract

This paper presents a dynamic general equilibrium model where asymmetric information about asset quality leads to asset illiquidity. Banking arises endogenously in this environment as banks can pool illiquid assets to average out their idiosyncratic qualities and issue liquid liabilities backed by pooled assets whose total quality is public information. Moreover, the liquidity mismatch in banks' balance sheets leads to endogenous bank capital (outside equity) requirements for preventing bank runs. The model indicates that banking has both positive and negative effects on long-run economic growth and that business-cycle dynamics of asset prices, asset illiquidity and bank capital requirements are interconnected.

*JEL classification: E44, G21, D82*

*Bank classification: Financial stability; Financial system regulation and policies*

## Résumé

L'auteur présente un modèle d'équilibre général dynamique dans lequel l'asymétrie de l'information relative à la qualité des actifs entraîne leur illiquidité. L'activité bancaire est générée de façon endogène : les banques peuvent regrouper les actifs illiquides de façon à ce que leurs caractéristiques idiosyncrasiques se neutralisent et émettre des passifs illiquides adossés au bloc d'actifs constitué, dont la qualité globale est connue de tous. Dans le modèle, la disparité qui existe en matière de liquidité entre les actifs et les passifs figurant au bilan des banques impose la détention de fonds propres (l'apport de capitaux extérieurs) afin de prévenir des retraits massifs de dépôts. Le modèle indique que l'activité bancaire a des effets tant positifs que négatifs sur la croissance économique à long terme et que la dynamique du cycle des prix d'actifs, l'illiquidité des actifs et le niveau requis de fonds propres sont interdépendants.

*Classification JEL : E44, G21, D82*

*Classification de la Banque : Stabilité financière; Réglementation et politiques relatives au système financier*

## 1 Introduction

This paper presents a dynamic general equilibrium model of banking where asymmetric information about asset quality leads to illiquidity of real assets, liquidity transformation by banks, and bank capital requirements endogenously. The model provides explanations as to why banks can issue liquid liabilities while other assets are illiquid, and why part of bank liabilities must be outside equity, i.e., bank capital. Using this model, this paper analyzes the long-run effects of banking on economic growth as well as business-cycle dynamics of asset prices, asset illiquidity and bank capital requirements in response to productivity shocks and changes in the degree of asymmetric information. This paper also discusses the implications of the model for dynamic bank capital requirements recently discussed in policy forums.<sup>1</sup>

The model is a version of the AK model, where goods are produced from productive real assets (physical capital) and new real assets are produced from goods. In the model, the fraction of agents who can produce new real assets, which is determined by idiosyncratic shocks, is so small that income from these agents' real assets is not enough to achieve the efficient level of aggregate investment in new real assets. Agents who can produce new real assets can obtain goods from other agents by selling their existing real assets in a competitive secondary market. However, because the productivity of each real asset is private information for the seller in the secondary market, the secondary market price of real assets becomes identical for every real asset sold, undervaluing high-quality real assets. The market's undervaluation discourages agents who can produce new real assets from selling the high-quality fraction of their real assets, resulting in a decline in the transfer of goods to these agents, which reduces aggregate investment in new real assets. The market's undervaluation is the definition of illiquidity in this paper.

This basic feature of the model is similar to the findings of Eislefeldt (2004) on illiquidity

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<sup>1</sup>For example, see the reports by the Bank for International Settlements (2008), the Financial Stability Forum (2009), and the Committee of European Banking Supervisors (2009).

of real assets due to asymmetric information about asset quality.<sup>2</sup> It is also closely related to the results of Kiyotaki and Moore (2008), who introduce a constraint on the resaleable fraction of real assets in a dynamic general equilibrium model. This paper endogenizes the resaleability constraint in Kiyotaki and Moore's model as agents choose not to sell the undervalued fraction of their real assets in the secondary market.

The model shows that banking emerges endogenously in this environment. While the illiquidity of real assets leads to agents' demand for liquid assets, banks can meet this demand as they can pool illiquid assets to average out the assets' idiosyncratic qualities, which makes the total quality of bank assets public information. As a result, bank liabilities backed by pooled bank assets are priced fairly in the market, i.e., liquid. The model also explains existence of bank capital requirements as the liquidity mismatch in banks' balance sheets makes self-fulfilling bank runs possible if all bank liabilities are deposits. The holders of bank liabilities require part of bank liabilities to be outside equity (i.e., bank capital) to prevent bank runs.<sup>3</sup>

The comparative statics of the model indicate that banking has both positive and negative effects on long-run economic growth. The positive effect is a direct effect of supply of liquid liabilities by banks, which increases the transfer of goods to agents who can produce new real assets through sales of liquid assets, expanding aggregate investment in new real assets. The negative effect is an indirect general equilibrium effect, or externality, of supply of liquid liabilities by banks, which raises the required rate of returns for illiquid real assets and thus lowers their price. This effect reduces the transfer of goods to agents who can produce new real assets through sales of illiquid real assets. The numerical examples of the model show that the positive effect dominates the negative effect if there is no operational cost of

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<sup>2</sup>Both Eisfeldt (2004) and this paper impose Akerlof's (1970) lemon problem on the competitive secondary asset market. Kurlat (2009) analyzes the effect of learning in a similar model of illiquid assets. See the paper by Gale (1992) for a model of competitive markets with adverse selection in a more general setup.

<sup>3</sup>This is the same type of self-fulfilling bank run as analyzed by Diamond and Dybvig (1983).

banking, but that this is not the case if the cost is large.<sup>4</sup>

The dynamic analysis of the model shows that business-cycle dynamics of asset prices, asset illiquidity and bank capital requirements are interconnected. The model incorporates two types of business-cycle shocks: productivity shocks and changes in the degree of asymmetric information. Changes in the degree of asymmetric information cause fluctuations in the economic growth rate because resulting changes in asset illiquidity affect the transfer of goods to agents who can produce new real assets. The model shows that, for both types of shocks, higher secondary market prices of real assets during economic booms mitigate illiquidity of real assets because higher prices make agents who can produce new real assets willing to sell better real assets in the secondary market. As the average quality of real assets sold in the market improves, the market's undervaluation (illiquidity) of high-quality real assets becomes less. Also, less illiquidity of real assets leads to higher market prices of real assets as real assets become more convenient stores of wealth. Thus there are two-way interactions of asset prices and asset illiquidity in equilibrium dynamics.

The model identifies downside risk to the market value of real assets and expected illiquidity of real assets as crucial factors for bank capital requirements. As these factors fluctuate over the business cycle, bank capital requirements are also dynamic. The model shows that, when the aggregate productivity of real assets shifts between high and low states randomly, bank capital requirements are pro-cyclical as pro-cyclical downside risk to the market value of real assets becomes the dominant factor for bank capital requirements. However, when a deterioration in asymmetric information causes an economic downturn, an increase in expected illiquidity of real assets raises bank capital requirements. These results suggest that the so-called "counter-cyclical capital buffer" recommended by the Financial Stability Forum (2009) is effective in preventing self-fulfilling bank runs when downside risk to the

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<sup>4</sup>In the model, the operational cost of banking is a bank equity holding cost for agents, which generates an equity premium on bank equity.

market value of bank assets is the dominant concern regarding financial stability, but that the counter-cyclical capital buffer would not help to free up bank capital as designed in a liquidity crisis.<sup>5</sup>

The model of banking in this paper adds to the vast literature on asymmetric information and financial intermediation. Specifically, the model is related to the papers by Williamson (1988) and Gorton and Pennacchi (1990).<sup>6</sup> Related to their work, this paper analyzes the role of banks in providing bank liabilities free of asymmetric information that contaminates the secondary market for real assets. As in Gorton and Pennacchi's model, bank liabilities circulate among agents.

This paper is also related to the model of Holmström and Tirole (1998) as it focuses on the effect of asset pooling by banks. In Holmström and Tirole's model, banks pool short-term assets to provide liquidity insurance for firms that invest in long-term assets, i.e., funding liquidity. In contrast, in this paper, asset pooling by banks creates liquid bank liabilities, i.e., market liquidity.<sup>7</sup>

The analysis of bank capital requirements is related to the findings of Diamond and Rajan (2000, 2001), who analyze the role of outside bank equity as a buffer to volatile bank asset value in a model where both the role of banking and bank fragility arise endogenously from costly enforcement of debt repayment. Also, this paper's model incorporates equilibrium pricing of liquid bank liabilities on the basis of agents' demand for liquid assets as in Holmström and Tirole (2001). This paper incorporates these features of banking in a unified

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<sup>5</sup>The counter-cyclical capital buffer requires banks to increase bank capital during booms to absorb losses in downturns.

<sup>6</sup>Williamson models a bank as a coalition of agents that internalizes the externality of adverse selection in the asset market. Modelling banks as a coalition is similar to the model of Boyd and Prescott (1986). Gorton and Pennacchi analyze the role of banks in providing information-insensitive riskless bank debt that circulates among uninformed agents who avoid trading risky assets with informed agents.

<sup>7</sup>In fact, Kiyotaki and Moore (2005) foresee this result by interpreting their resaleability constraint as a reduced-form representation of the effectiveness of liquidity creation by banks through asset pooling when asymmetric information about asset quality exists. This paper confirms their insight.



framework, adding to the literature on dynamic general equilibrium models of banking.<sup>8</sup>

In addition, the key feature of banks in the models of Diamond and Rajan (2000, 2001) is that banks have higher collateral value of borrowers' assets than other agents. This paper derives this feature of banks endogenously from the ability for banks to conduct liquidity transformation, showing that banks intermediate collateralized lending.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 solves the model. Section 4 analyzes the effects of asset illiquidity and liquidity transformation by banks on aggregate investment. Section 5 investigates the dynamics of asset illiquidity, asset prices and bank capital requirements. Section 6 discusses why banks intermediate collateralized lending. Section 7 analyzes the sensitivity of bank capital requirements to the bank liquidation procedure. Section 8 concludes.

## 2 The model

### 2.1 Agents

*Time and utility.*—Time is discrete and there is a continuum of infinite-lived agents who gain utility from consumption of goods. The utility function for each agent is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{i,t}, \quad (1)$$

where  $c_{i,t}$  is the consumption of goods,  $i$  is the index for each agent,  $t$  denotes the time period, and  $\beta$  ( $\in (0, 1)$ ) is the time discount rate.

*Production of goods.*—Agents can produce homogeneous goods using trees (physical cap-

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<sup>8</sup>For examples of dynamic general equilibrium models of banking in the literature, see the papers by Williamson (1987), Chen (2001) and Kato (2006). The last two papers extend the models of Holmström and Tirole (1997, 1998), respectively, to dynamic general equilibrium models.

ital) they own at the beginning of each period:

$$y_{i,t} = \alpha_t k_{i,t-1}, \quad (2)$$

where  $y_{i,t}$  is output,  $k_{i,t-1}$  is the quantity of trees held at the beginning of period  $t$ , and  $\alpha_t$  is an aggregate productivity shock.

*Depreciation of trees.*—After production, each infinitesimal unit of trees, which are divisible, depreciates at its own rate. The distribution of depreciation rates is i.i.d. uniform such that:

$$f_{i,\delta,t} = \frac{k_{i,t-1}}{2\Delta_t} \quad \text{for } \delta \in [\bar{\delta} - \Delta_t, \bar{\delta} + \Delta_t], \quad (3)$$

where  $f_{i,\delta,t}$  is the density of agent  $i$ 's trees that depreciate at rate  $\delta$  in period  $t$ ,  $\bar{\delta}$  ( $\in (0, 1)$ ) is the average depreciation rate, and  $\Delta_t$  ( $\in (0, 1 - \bar{\delta})$ ) is a stochastic mean-preserving spread to the range of depreciation rates of trees.

Depreciation in the model represents permanent shocks to the individual productivity levels of real assets in general. Given that private information about productivity of real assets exists widely in reality, assume that the depreciation rate of each tree is only observed by the agent who owns the tree at the beginning of the period.<sup>9</sup> Also assume that the current depreciation rate of each tree becomes public information at the beginning of the next period.<sup>10</sup> This assumption keeps the information dynamics in the model simple and tractable. Given these assumptions,  $\Delta_t$  becomes a shock to the degree of asymmetric information in

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<sup>9</sup>Physical depreciation of consumer durables such as cars and houses is a good example of private information about productivity of real assets. For empirical analysis of business capital, see Eisfeldt and Rampini (2006).

<sup>10</sup>Assume that all agents can learn the previous depreciation rate of each tree by observing the amount of goods produced by each tree. After the revelation of depreciation rates, trees net of depreciation become homogeneous once again and then each infinitesimal unit of homogeneous trees depreciates at its own rate.

the economy in this paper.<sup>11</sup>

*The secondary market for trees.*—After depreciation of trees, agents can trade trees in a competitive secondary market. The depreciation rate of each tree sold is private information for the seller, given the assumption in the previous paragraph. Assume that agents are anonymous so that the price of each tree in the market cannot be contingent on the characteristics of the buyer or the seller, including the volume of sales by the seller. As a consequence, every tree is traded at an identical price in each period.<sup>12</sup> At the same time with the secondary market for trees, agents can also trade bank liabilities (demand deposits and bank equity) in competitive markets. The details on banks will be described in the next subsection.

*Investment in new trees.*—After asset market transactions, a fraction of agents can invest goods in production of new trees:  $n_{i,t} = \phi_{i,t}x_{i,t}$ , where  $n_{i,t}$  is the quantity of new trees,  $\phi_{i,t} \in \{0, \phi\}$  ( $\phi > 0$ ), and  $x_{i,t}$  is the amount of goods invested in new trees. Thus, only agents with  $\phi_{i,t} = \phi$  have investment opportunity. The value of  $\phi_{i,t}$  is determined by an idiosyncratic Markov process with a transition probability function,  $P$ , such that  $P(\phi_{i,t+1} = \phi \mid \phi_{i,t} = \phi) = \rho_P$  and  $P(\phi_{i,t+1} = 0 \mid \phi_{i,t} = 0) = \rho_U$  for all  $i$  and  $t$ . Each agent learns the value of  $\phi_{i,t}$  at the beginning of period  $t$ .

*The maximization problem for each agent.*—Each agent maximizes the utility function (1) subject to the following constraints in each period, which are implied by the assumptions

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<sup>11</sup>Williamson (1987) analyzes the effects of a mean-preserving spread to the distribution of investment returns. In his model, the mean-preserving spread worsens credit rationing due to costly state verification. In this paper, the mean-preserving spread worsens adverse selection in the competitive secondary market for trees as described below.

<sup>12</sup>This anonymous feature of the market is similar to centralized asset markets in reality, such as stock exchanges. If there are multiple competitive markets sorted by the quantity of trees sold by each seller, then the quantity could signal the average depreciation rate of trees sold in each market. Even in this case, anonymity of sellers would let each seller split her trees into multiple lots and sell them in different markets to maximize the total revenue from the sales. This paper abstracts from the interaction between competitive market prices and this type of strategic seller behaviour.

set so far:

$$\begin{aligned} c_{i,t} + x_{i,t} + Q_t h_{i,t} + b_{i,t} + (1 + \zeta) V_t s_{i,t} \\ = \alpha_t k_{i,t-1} + Q_t \int_{\hat{\delta}-\Delta_t}^{\hat{\delta}+\Delta_t} l_{i,\delta,t} d\delta + \tilde{R}_t b_{i,t-1} + (D_t + V_t) s_{i,t-1}, \end{aligned} \quad (4)$$

$$k_{i,t} = \phi_{i,t} x_{i,t} + (1 - \hat{\delta}_t) h_{i,t} + \int_{\hat{\delta}-\Delta_t}^{\hat{\delta}+\Delta_t} (1 - \delta) \left( \frac{k_{i,t-1}}{2\Delta_t} - l_{i,\delta,t} \right) d\delta, \quad (5)$$

$$l_{i,\delta,t} \in \left[ 0, \frac{k_{i,t-1}}{2\Delta_t} \right], \quad c_{i,t} \geq 0, \quad x_{i,t} \geq 0, \quad h_{i,t} \geq 0, \quad b_{i,t} \geq 0, \quad s_{i,t} \geq 0, \quad (6)$$

where  $h_{i,t}$  is the quantity of trees gross of depreciation bought in the secondary market,  $l_{i,\delta,t}$  is the density of trees gross of depreciation with depreciation rate  $\delta$  sold by the agent,  $Q_t$  is the secondary market price of trees,  $\hat{\delta}_t$  is the average depreciation rate of trees sold in the secondary market,  $b_{i,t-1}$  is the amount of demand deposits held at the beginning of period  $t$ ,  $s_{i,t-1}$  represents the units of bank equity held at the beginning of period  $t$ ,  $\tilde{R}_t$  is the ex-post deposit interest rate,  $D_t$  is the amount of bank dividends per unit of equity,  $V_t$  is the ex-dividend price of bank equity, and  $\zeta (> 0)$  is an exogenous marginal cost of holding bank equity, which is a reduced-form representation of equity management costs, such as transaction costs and monitoring costs.<sup>13</sup> This cost leads to an equity premium on bank equity as described later. Each agent chooses  $\{c_{i,t}, x_{i,t}, h_{i,t}, l_{i,\delta,t}, b_{i,t}, s_{i,t}\}_{t=0}^{\infty}$  taking as given the probability distribution of  $\{Q_t, \hat{\delta}_t, \tilde{R}_t, D_t, V_t, \alpha_t, \Delta_t, \phi_{i,t}\}_{t=0}^{\infty}$ .<sup>14</sup>

Equations (4) and (5) are the flow-of-funds constraint and the law of motion for trees net of depreciation (i.e.,  $k_{i,t}$ ), respectively, and Constraints (6) are a short-sale constraint on trees and non-negativity constraints on the choice variables. Note that the market price of trees,  $Q_t$ , is identical irrespective of the value of  $\delta$  in  $l_{i,\delta,t}$  in Equation (4) because the depreciation rate of each tree sold is private information for the seller. Also, because every

<sup>13</sup>The face value of deposits is protected by the court. In contrast, equity holders need to identify the cash flow for each bank and negotiate with the bank on the amount of dividends in each period.

<sup>14</sup>The choice variables are state-contingent. The notation of state contingency is omitted here.

unit of trees is infinitesimal, the average depreciation rate of trees bought by each buyer equals the average depreciation rate of trees sold in the market,  $\hat{\delta}_t$ , by the law of large numbers.<sup>15</sup> Thus,  $(1 - \hat{\delta}_t)h_{i,t}$  in Equation (5) is the total quantity of trees net of depreciation that the agent obtains through the secondary market with certainty. In Equations (5) and (6),  $k_{i,t-1}(2\Delta_t)^{-1}$  is the density of the agent's trees with depreciation rate  $\delta$  as specified by Equation (3).

Equation (4) and Constraints (6) imply that agents cannot borrow due to their anonymity, which makes it difficult to enforce their intertemporal commitments. Assume that new trees cannot be collateral when agents invest in them because new trees materialize only at the beginning of the next period. The assumption of no borrowing lets the model have a closed-form solution to dynamic equilibrium equations when there is no bank, which enables the basic features of the model to be clarified analytically. Section 6 will extend the model by allowing agents to borrow against new trees. The section will show that the ability of banks in pooling illiquid assets and providing liquid liabilities, which will be described in the next subsection, increases the collateral value of new trees for banks, inducing banks to intermediate collateralized lending.

## 2.2 Banks

There are many small homogeneous banks that buy trees in the secondary market by financing the cost through issuing demand deposits and bank equity to agents in competitive markets. In contrast to agents, banks are not anonymous. Banks can commit to redeeming deposits and paying dividends by goods produced from their trees in the future with no agency problem. This paper considers only deposit and equity contracts, assuming that con-

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<sup>15</sup>This is a common feature of competitive equilibrium models with adverse selection. See Gale (1992) and Eisfeldt (2004) for example.

tingent contracts are not verifiable.<sup>16</sup> The production function for goods and the distribution of depreciation rates of trees for banks are the same as in Equations (2) and (3) for agents.

*The flow of funds and the law of motion for trees for each bank.*—Because banks are homogeneous, consider a representative bank to simplify the notation. The flow-of-funds constraint on the representative bank and the law of motion for its trees are:

$$D_t S_{B,t-1} + \tilde{R}_t B_{B,t-1} + Q_t(H_{B,t} - L_{B,t}) = \alpha_t K_{B,t-1} + B_{B,t} + V_t(S_{B,t} - S_{B,t-1}), \quad (7)$$

$$K_{B,t} = (1 - \hat{\delta}_t)H_{B,t} + (1 - \bar{\delta})(K_{B,t-1} - L_{B,t}), \quad (8)$$

where  $S_{B,t-1}$  represents the units of bank equity outstanding at the beginning of period  $t$ ,  $B_{B,t-1}$  is the amount of demand deposits outstanding at the beginning of period  $t$ ,  $H_{B,t}$  is the amount of trees gross of depreciation bought by the bank in the secondary market,  $L_{B,t}$  is the amount of trees gross of depreciation sold by the bank in the secondary market, and  $K_{B,t-1}$  is the amount of trees held at the beginning of period  $t$ .<sup>17</sup> Equation (8) implies that banks, like agents, do not know the depreciation rate of each tree they buy in the secondary market, so the average depreciation rate of trees bought by each bank in the market equals  $\hat{\delta}_t$  by the law of large numbers, as in Equation (5) for agents. Also, note that  $L_{B,t}$  is not specific to the depreciation rate of each tree sold. It is assumed that banks do not know the depreciation rate of each tree they have, so they cannot sell their trees selectively. As a result, in the last term on the right-hand side of Equation (8), the average depreciation rate of trees sold by each bank equals the average depreciation rate of all of its trees,  $\bar{\delta}$ , by

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<sup>16</sup>Thus the analysis of the (non) existence of bank runs under the optimal contingent contract, such as the work by Green and Lin (2003), Peck and Shell (2003), Andolfatto and Nosal (2008) and Ennis and Keister (2009), is beyond the scope of this paper. Also, note that equity is not contingent contracts that specify contingent returns ex-ante. Instead, ex-post negotiation of dividends must take place as if default on debt occurs every period. See Hart and Moore (1994) for more details on the feature of equity as a financial contract.

<sup>17</sup>The last term on the right-hand side of Equation (7) is the revenue from newly issued equity if it is positive or the expenditure on equity repurchases if it is negative.

the law of large numbers. This assumption makes banks keep holding trees in equilibrium as shown by Proposition 1 below.<sup>18</sup> Overall, banks do not have any informational advantage over agents in the secondary market for trees and the only advantage of banks over agents is the ability of banks to issue deposits and equity against the trees they hold.

*Bank runs.*—The ex-post deposit interest rate,  $\tilde{R}_t$ , equals the non-contingent ex-ante deposit contract rate denoted by  $\bar{R}_{t-1}$ , which is determined in period  $t-1$ , if the representative bank does not default. But assume that a self-fulfilling bank run occurs if the face value of deposits,  $\bar{R}_{t-1}B_{B,t-1}$ , exceeds the liquidation value of trees held by the bank at the beginning of the period,  $(\alpha_t + Q_t)K_{B,t-1}$ .<sup>19</sup> In this case, the bank cannot roll over its deposits and must maximize the repayment to depositors by liquidating all of the trees it owns. Because the liquidation value of the bank's trees is less than the face value of deposits, the bank must default. Bank equity thus loses value and no dividend is paid on equity. Hence:

$$\tilde{R}_t = \begin{cases} \bar{R}_{t-1}, & \text{if } \bar{R}_{t-1}B_{B,t-1} \leq (\alpha_t + Q_t)K_{B,t-1}, \\ \frac{(\alpha_t + Q_t)K_{B,t-1}}{B_{B,t-1}}, & \text{if } \bar{R}_{t-1}B_{B,t-1} > (\alpha_t + Q_t)K_{B,t-1}, \end{cases} \quad (9)$$

$$L_{B,t} = K_{B,t-1}, H_{B,t} = V_t = D_t = 0, \text{ if } \bar{R}_{t-1}B_{B,t-1} > (\alpha_t + Q_t)K_{B,t-1}. \quad (10)$$

The recovery rate of deposits in the second line of Equation (9) is determined by the flow-of-funds constraint (7), given Equation (10).

Note that the liquidation value of the representative bank's trees,  $(\alpha_t + Q_t)K_{B,t-1}$ , is eval-

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<sup>18</sup>If banks had private information about the depreciation rate of each tree they owned, then in equilibrium the existence of banks would worsen the adverse selection problem in the secondary market for trees because banks never have opportunity to invest in new trees and would sell only a low-quality fraction of their trees. Even in this case, the average depreciation rate of trees held by each bank would be public information through rational expectations of bank behaviour, which would make deposits and bank equity liquid as explained below.

<sup>19</sup>As shown below, the present discounted value of future income generated by the bank's trees exceeds the liquidation value of trees in equilibrium. Thus, if the bank can roll over deposits, then the bank can avoid default. However, if all of the depositors expect that the bank cannot roll over deposits, then their expectations become self-fulfilling.

uated by the competitive secondary market price of trees,  $Q_t$ . The underlying assumptions are that the only channel for banks to sell their trees is the competitive secondary market and that each bank is so small that a failure of a bank does not affect the market price. Thus agents who run to a bank take  $Q_t$  as given. Section 7 will extend the model to discuss the effect of an alternative bank liquidation procedure in which a bank hit by a bank run can set up a market for liquidating the bank's trees separately from the secondary market for trees.

The model assumes no deposit insurance or suspension of convertibility of deposits by the government, which would prevent self-fulfilling bank runs as shown by Diamond and Dybvig (1983).<sup>20</sup> This is a simplifying assumption, given that short-term funding not covered by deposit insurance, such as wholesale funding, is an important source of finance for banks and that suspension of convertibility of deposits is a drastic policy measure that is not often used. As will be shown below, this assumption lets the model incorporate bank capital requirements due to the risk of self-fulfilling bank runs, which enables this paper to analyze dynamics of bank capital requirements with endogenous fluctuations in illiquidity of bank assets.

*The maximization problem for each bank.*—Given Equations (7)-(10), the representative bank maximizes the value of the bank for equity holders,  $(D_t + V_t)S_{B,t-1}$ , in each period given the predetermined value of  $S_{B,t-1}$ .<sup>21</sup> In so doing, the bank internalizes the first-order conditions with respect to  $s_{i,t}$  and  $b_{i,t}$  in the maximization problem for agents defined by Equations (1) and (4)-(6), which represent the responses of the ex-ante deposit contract rate,

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<sup>20</sup>In this paper's model, limiting the repayment of deposits to the flow income from each bank's trees prevents bank runs because banks are not forced to sell their trees to redeem deposits. Even though the flow income from banks' trees is not enough to redeem all deposits held by depositors who need to convert deposits into goods (i.e., productive agents who were unproductive in the previous period), banks can redeem deposits held by these agents by issuing new deposits to other agents, if there is no stochastic shock to the economy.

<sup>21</sup>There is no disagreement between productive and unproductive equity holders, since the maximum of each agent's utility function increases in the agent's net worth regardless of the value of  $\phi_{i,t}$ , given the probability distribution of exogenous variables for agents.



$\bar{R}_t$ , and the equity price,  $V_t$ , to the bank's behaviour:

$$V_t = E_t [\Lambda_{V,t+1}(D_{t+1} + V_{t+1})], \quad (11)$$

$$1 = E_t [\Lambda_{R,t+1} \min \{ \bar{R}_t, (\alpha_{t+1} + Q_{t+1})K_{B,t}(B_{B,t})^{-1} \}], \quad (12)$$

where:

$$\Lambda_{V,t+1} \equiv \frac{\beta c_{i^*,t}}{(1 + \zeta)c_{i^*,t+1}}, \quad i^* \equiv \operatorname{argmax}_{i \in \mathcal{I}} E_t \left[ \frac{\beta c_{i,t}(D_{t+1} + V_{t+1})}{(1 + \zeta)c_{i,t+1}} \right], \quad (13)$$

$$\Lambda_{R,t+1} \equiv \frac{\beta c_{i^{**},t}}{c_{i^{**},t+1}}, \quad i^{**} \equiv \operatorname{argmax}_{i \in \mathcal{I}} E_t \left[ \frac{\beta c_{i,t} \min \{ \bar{R}_t, (\alpha_{t+1} + Q_{t+1})K_{B,t}(B_{B,t})^{-1} \}}{c_{i,t+1}} \right], \quad (14)$$

and  $\mathcal{I}$  denotes the set of all indices for agents. In Equations (12) and (14), Equation (9) is substituted into  $\tilde{R}_{t+1}$ , and  $K_{B,t}(B_{B,t})^{-1}$  is replaced with infinity if  $B_{B,t} = 0$ . The maximum operators in Equations (13) and (14) indicate that the buyers of the bank liabilities are the agents who value the liabilities the most.<sup>22</sup> Note that the bank equity holding cost,  $\zeta$ , makes agents require a higher rate of return on bank equity than on deposits. This is an equity premium.

Substituting Equation (11) into Equation (7) implies that  $(D_t + V_t)S_{B,t-1}$  is determined recursively. The maximization problem for the representative bank is defined as:

$$\begin{aligned} (D_t + V_t)S_{B,t-1} &= \Omega_t(K_{B,t-1}, B_{B,t-1}, \bar{R}_{t-1}) \equiv \\ &\max_{\{H_{B,t}, L_{B,t}, B_{B,t}, \bar{R}_t\}} \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + B_{B,t} \\ &\quad + E_t [\Lambda_{V,t+1} \Omega_{t+1}(K_{B,t}, B_{B,t}, \bar{R}_t)], \\ \text{s.t. Equations (8) - (10) and (12), } &L_{B,t} \in [0, K_{B,t-1}], H_{B,t} \geq 0, B_{B,t} \geq 0, \end{aligned} \quad (15)$$

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<sup>22</sup>If there is no supply of bank liabilities, then the left-hand sides of Equations (11) and (12) need only to be weakly greater than the right-hand sides. For this case, assume that the left-hand side equals the right-hand side in each equation in equilibrium without loss of generality.

where the last three constraints are a no short-sale constraint on trees in the secondary market and non-negativity constraints on choice variables. The bank takes as given the probability distribution of  $\{Q_t, \hat{\delta}_t, \alpha_t, \Lambda_{V,t}, \Lambda_{R,t}\}_{t=0}^{\infty}$ .<sup>23</sup>

Note that banks can choose to shut down their business by liquidating all of their trees if financial intermediation is not profitable. Thus, if banks conduct financial intermediation by buying trees and issuing liabilities, then it is an endogenous result in the model.

### 2.3 Shock processes

There are two types of aggregate shocks,  $\alpha_t$  and  $\Delta_t$ . Each type of shock follows a two-state Markov process. More specifically,  $\alpha_t \in \{\bar{\alpha}, \underline{\alpha}\}$ ,  $\Delta_t \in \{\bar{\Delta}, \underline{\Delta}\}$ , and the transition probability function denoted by  $P$  is such that  $P(\alpha_{t+1} = \bar{\alpha} \mid \alpha_t = \bar{\alpha}) = \bar{\eta}_\alpha$ ,  $P(\alpha_{t+1} = \underline{\alpha} \mid \alpha_t = \underline{\alpha}) = \underline{\eta}_\alpha$ ,  $P(\Delta_{t+1} = \bar{\Delta} \mid \Delta_t = \bar{\Delta}) = \bar{\eta}_\Delta$ , and  $P(\Delta_{t+1} = \underline{\Delta} \mid \Delta_t = \underline{\Delta}) = \underline{\eta}_\Delta$  for all  $t$ .

### 2.4 Equilibrium conditions

Market equilibrium conditions in the model are:

$$\hat{\delta}_t = \frac{\int_{\mathcal{I}} \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} \delta l_{i,\delta,t} d\delta d\mu + \bar{\delta} L_{B,t}}{\int_{\mathcal{I}} \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} l_{i,\delta,t} d\delta d\mu + L_{B,t}}, \quad (16)$$

$$\int_{\mathcal{I}} h_{i,t} d\mu + H_{B,t} = \int_{\mathcal{I}} \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} l_{i,\delta,t} d\delta d\mu + L_{B,t}, \quad (17)$$

$$\int_{\mathcal{I}} b_{i,t} d\mu = B_{B,t}, \quad (18)$$

$$\int_{\mathcal{I}} s_{i,t} d\mu = S_{B,t}, \quad (19)$$

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<sup>23</sup>Assume that if there is no existing equity holder for a bank (i.e.,  $S_{B,t-1} = 0$ ), then the bank maximizes the net profit from an initial public offering of its equity and consumes the profit right away. Because the net profit equals the value of  $\Omega_t$ , this case is covered by the maximization problem (15). It can be shown that the net profit from the initial public offering becomes zero in equilibrium.

where  $\mu$  is the measure of indices for agents on  $\mathcal{I}$ . These equations are, in order, the definition of  $\hat{\delta}_t$  and the market clearing conditions for trees, deposits, and bank equity. An equilibrium in the model is characterized by fulfillment of the following: the maximization problem for each agent defined by Equations (1) and (4)-(6) is solved for all  $i \in \mathcal{I}$ ; the maximization problem for the representative bank (15) is solved for  $t = 0, 1, 2, \dots$ ; the bank and agents hold rational expectations; and Equations (16)-(19) are satisfied for all  $t = 0, 1, 2, \dots$

### 3 Equilibrium behaviour of agents and banks

This section solves the model. Call agents with  $\phi_{i,t} = \phi$  “productive” and those with  $\phi_{i,t} = 0$  “unproductive”. Throughout the paper, suppose that the following conditions hold in equilibrium:

$$\phi > (1 - \hat{\delta}_t)Q_t^{-1}, \quad (20)$$

$$1 > E_t \left[ \frac{\beta c_{i,t} \tilde{R}_{t+1}}{c_{i,t+1}} \mid \phi_{i,t} = \phi \right], \quad (21)$$

$$(1 + \zeta)V_t > E_t \left[ \frac{\beta c_{i,t}(D_{t+1} + V_{t+1})}{c_{i,t+1}} \mid \phi_{i,t} = \phi \right]. \quad (22)$$

These conditions will be verified in the numerical examples of equilibria considered below. The first condition says that agents with one unit of goods can obtain a larger amount of trees net of depreciation by investing in new trees than by buying trees in the secondary market. Thus productive agents do not buy trees. The second and third conditions say that the rate of return on investing in new trees for productive agents dominates the rates of return on deposits and bank equity. Under these conditions, productive agents only invest in new trees (i.e.,  $x_{i,t} > 0$  and  $h_{i,t} = b_{i,t} = s_{i,t} = 0$ , if  $\phi_{i,t} = \phi$ ) and unproductive agents become the buyers of deposits and bank equity in equilibrium. Thus  $\Lambda_{V,t+1}$  and  $\Lambda_{R,t+1}$  are

determined by the stochastic discount factor,  $\beta c_{i,t}(c_{i,t+1})^{-1}$ , for unproductive agents.<sup>24</sup>

Also, hereafter, the number of exogenous states is limited to two by considering one of the two types of aggregate shocks,  $\alpha_t$  and  $\Delta_t$ , at a time. If  $\bar{\alpha} > \underline{\alpha}$ , so that  $\alpha_t$  fluctuates, then set  $\bar{\Delta} = \underline{\Delta}$ . Otherwise set  $\bar{\alpha} = \underline{\alpha}$  and  $\bar{\Delta} > \underline{\Delta}$ . This assumption simplifies the representative bank's problem about whether it should take the risk of a bank run, which makes the computation of equilibrium dynamics tractable.

### 3.1 Asset illiquidity and adverse selection by agents

The maximization problem for each agent defined by Equations (1) and (4)-(6) implies that each agent sells a tree if the marginal revenue from the sale,  $Q_t$ , is greater than the internal rate of return on keeping the tree until the next period, given the tree's depreciation rate:

$$l_{i,\delta,t} = \begin{cases} k_{i,t-1}(2\Delta_t)^{-1}, & \text{if } Q_t \geq \lambda_{i,t}(1 - \delta), \\ 0, & \text{otherwise,} \end{cases} \quad (23)$$

where  $\lambda_{i,t}$  is the shadow value of trees net of depreciation at the end of period  $t$  (i.e.,  $k_{i,t}$ ), so that  $\lambda_{i,t}(1 - \delta)$  is the shadow value of trees with depreciation rate  $\delta$ .<sup>25</sup>

In equilibrium, the shadow value of trees net of depreciation is less than or equal to the marginal cost of obtaining them for each agent because otherwise the agent would be better off cutting consumption to spend more on trees, which would contradict the definition of

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<sup>24</sup>As shown by Equation (34) below, unproductive agents have an identical stochastic discount factor in each period in equilibrium due to the log utility function.

<sup>25</sup>The shadow value of  $k_{i,t}$  is given by current consumption,  $c_{i,t}$ , multiplied by the Lagrange multiplier for the law of motion for trees net of depreciation (Equation (5)) in the maximization problem for each agent defined by Equations (1) and (4)-(6).

equilibrium. Given Conditions (20)-(22), it can be shown that:

$$\lambda_{i,t} = \begin{cases} \phi^{-1}, & \text{if } \phi_{i,t} = \phi, \\ \lambda_{U,t} \leq Q_t(1 - \hat{\delta}_t)^{-1}, & \text{if } \phi_{i,t} = 0, \end{cases} \quad (24)$$

where  $\lambda_{U,t}$  denotes the value of  $\lambda_{i,t}$  for unproductive agents. The value of  $\lambda_{i,t}$  for productive agents equals the marginal cost of producing new trees. The right-hand side of the weak inequality in Equation (24) is the marginal cost for unproductive agents to obtain trees net of depreciation through the secondary market. If the inequality holds strictly, then unproductive agents do not buy trees. Thus:

$$\begin{cases} \lambda_{U,t} = Q_t(1 - \hat{\delta}_t)^{-1}, & \text{if } h_{i,t} > 0 \text{ for all } i \text{ s.t. } \phi_{i,t} = 0, \\ h_{i,t} = 0 \text{ for all } i \text{ s.t. } \phi_{i,t} = 0, & \text{if } \lambda_{U,t} < Q_t(1 - \hat{\delta}_t)^{-1}. \end{cases} \quad (25)$$

Equations (23) and (24) imply that there exists a lower bound for the depreciation rates of trees sold by each agent,  $\delta_{i,t}$ , such that:<sup>26</sup>

$$\delta_{i,t} = \begin{cases} \delta_{P,t} \equiv \max \{ \bar{\delta} - \Delta_t, 1 - Q_t\phi \}, & \text{if } \phi_{i,t} = \phi, \\ \delta_{U,t} \equiv \max \{ \bar{\delta} - \Delta_t, 1 - Q_t(\lambda_{U,t})^{-1} \}, & \text{if } \phi_{i,t} = 0. \end{cases} \quad (26)$$

For each agent, trees whose depreciation rates are lower than  $\delta_{i,t}$  are illiquid in the sense that the secondary market price of trees,  $Q_t$ , is less than the internal value of the trees for the holder. As a result, agents do not sell these trees (i.e., adverse selection). Hereafter, consider  $\delta_{P,t}$  and  $\delta_{U,t}$  as the indicators of illiquidity of trees for productive agents and unproductive agents, respectively.

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<sup>26</sup>The maximum operator ensures that the value of  $\delta_{i,t}$  is within the range of the distribution of depreciation rates. Condition (20) and Equations (24) and (26) imply that  $\delta_{P,t} < \hat{\delta}_t$  and  $\delta_{U,t} \leq \hat{\delta}_t$ . Thus  $\delta_{i,t} \leq \bar{\delta} + \Delta_t$  for all  $i$ .

Equation (16) implies that the adverse selection leads to  $\hat{\delta}_t > \bar{\delta}$ , if there exist illiquid trees (i.e., if  $\delta_{P,t} > \bar{\delta} - \Delta_t$  or  $\delta_{U,t} > \bar{\delta} - \Delta_t$ ). On the other hand, Condition (20) and Equation (26) imply  $\delta_{P,t} < \hat{\delta}_t$ , which leads to  $\hat{\delta}_t < \bar{\delta} + \Delta_t$ , i.e., a positive volume of trade in the secondary market for trees, given Equation (16). Intuitively, productive agents sell some high-quality trees (whose depreciation rates are below  $\hat{\delta}_t$ ) despite the market's undervaluation as the return on investment in new trees exceeds the cost of the market's undervaluation for these trees. The supply of undervalued trees by productive agents saves the secondary market for trees from a complete shutdown ( $\hat{\delta}_t = \bar{\delta} + \Delta_t$ ).<sup>27</sup>

### 3.2 Liquidity transformation by banks

The solution to the maximization problem (15) leads to the following proposition.

**Proposition 1** *As assumed above, the number of exogenous states is two in each period. Given the values of period- $t$  variables, denote the smaller value of  $\alpha_{t+1} + Q_{t+1}$  by  $\underline{\omega}_{t+1}$ , the larger value by  $\bar{\omega}_{t+1}$ , and the conditional probability that  $\alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1}$  by  $P_t(\bar{\omega}_{t+1})$ . Suppose Conditions (20)-(22) hold in equilibrium. Then:*

$$\tilde{R}_t B_{B,t-1} + (D_t + V_t) S_{B,t-1} = \begin{cases} [\alpha_t + \lambda_{B,t}(1 - \bar{\delta})] K_{B,t-1}, & \text{if } \bar{R}_{t-1} B_{B,t-1} \leq (\alpha_t + Q_t) K_{B,t-1}, \\ (\alpha_t + Q_t) K_{B,t-1}, & \text{if } \bar{R}_{t-1} B_{B,t-1} > (\alpha_t + Q_t) K_{B,t-1}, \end{cases} \quad (27)$$

where  $\lambda_{B,t} \equiv \max\{\lambda'_{B,t}, \lambda''_{B,t}\}$  and  $\lambda'_{B,t}$  and  $\lambda''_{B,t}$ , respectively, are the present discounted values of the future marginal income from the representative bank's trees conditional on

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<sup>27</sup>Note that the measure of the trees whose depreciation rates equal  $\bar{\delta} + \Delta_t$  is zero in the economy.

$\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$  and  $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$  such that:

$$\lambda'_{B,t} \equiv E_t \left\{ \frac{\beta c_{i,t} [\alpha_{t+1} + \lambda_{B,t+1}(1 - \bar{\delta}) - \underline{\omega}_{t+1}]}{(1 + \zeta) c_{i,t+1}} + \frac{\beta c_{i,t} \underline{\omega}_{t+1}}{c_{i,t+1}} \middle| \phi_{i,t} = 0 \right\}, \quad (28)$$

$$\lambda''_{B,t} \equiv P_t(\bar{\omega}_{t+1}) E_t \left\{ \frac{\beta c_{i,t} [\alpha_{t+1} + \lambda_{B,t+1}(1 - \bar{\delta}) - \bar{\omega}_{t+1}]}{(1 + \zeta) c_{i,t+1}} \middle| \begin{array}{l} \phi_{i,t} = 0 \\ \alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1} \end{array} \right\} \\ + E_t \left[ \frac{\beta c_{i,t} (\alpha_{t+1} + Q_{t+1})}{c_{i,t+1}} \middle| \phi_{i,t} = 0 \right], \quad (29)$$

$$\bar{R}_t B_{B,t} = \begin{cases} \underline{\omega}_{t+1} K_{B,t}, & \text{if } \lambda'_{B,t} > \lambda''_{B,t}, \\ \bar{\omega}_{t+1} K_{B,t}, & \text{if } \lambda'_{B,t} < \lambda''_{B,t}. \end{cases} \quad (30)$$

Also, if banks buy trees in the secondary market, then the present discounted value of the future marginal income from the representative bank's trees equals the marginal acquisition cost of the trees and also banks keep holding their trees:

$$\lambda_{B,t} = \frac{Q_t}{1 - \hat{\delta}_t}, \quad \text{if } H_{B,t} > 0, \quad (31)$$

$$L_{B,t} = 0, \quad \text{if } \hat{\delta}_t > \bar{\delta} \text{ and } H_{B,t} > 0. \quad (32)$$

**Proof.** See Appendix A. ■

Equation (27) implies that, because each bank can commit to paying all of the current and future income from its trees to the holders of bank liabilities, the total market value of bank liabilities,  $\tilde{R}_t B_{B,t-1} + (D_t + V_t) S_{B,t-1}$ , equals the present discounted value of the current and future income from the bank's trees,  $[\alpha_t + \lambda_{B,t}(1 - \bar{\delta})] K_{B,t-1}$ , given no bank run in the current period. As shown by the flow-of-funds constraint (4), agents can obtain this value of goods when they transfer bank liabilities to other agents in the markets. Thus, bank liabilities are fairly priced in the markets, i.e., liquid.

Note that, if there were asymmetric information about the quality of each bank's trees, then bank liabilities would be illiquid like trees. This does not happen in the model because asset pooling by each bank averages out the idiosyncratic depreciation rates of the bank's trees, so that the total quantity of trees net of depreciation held by each bank (i.e.,  $(1 - \bar{\delta})K_{B,t-1}$ ) becomes public information.

If  $\hat{\delta}_t > \bar{\delta}$  due to adverse selection as described in Section 3.1, then substituting Equation (31) into Equation (27) and comparing the first and second lines of Equation (27) imply that agents can increase the market value of their assets by holding liquid bank liabilities instead of illiquid trees. On the flip side, liquidation of a bank's trees due to a bank run is costly for the holders of bank liabilities. Proposition 1 shows that, if  $\lambda'_{B,t} > \lambda''_{B,t}$ , then the cost of a bank run is so high that, despite an equity premium generated by the bank-equity holding cost,  $\zeta$ , each bank limits the face value of deposits,  $\bar{R}_t B_{B,t}$ , to  $\underline{\omega}_{t+1} K_{B,t}$ , which is the maximum face value of deposits without any possibility of a bank run in the next period. Given the endogenous limit on the face value of deposits, Equations (11), (27) and (31) imply that banks must finance by bank equity a part of the acquisition cost of banks' trees, which equals the present discounted value of the future income from the trees in equilibrium as shown by Equation (31):

$$V_t S_{B,t} = E_t \left\{ \frac{\beta c_{i,t} \left[ \alpha_{t+1} + Q_{t+1} (1 - \hat{\delta}_t)^{-1} (1 - \bar{\delta}) - \underline{\omega}_{t+1} \right] K_{B,t}}{(1 + \zeta) c_{i,t+1}} \Bigg| \phi_{i,t} = 0 \right\}, \quad (33)$$

which is positive, given  $\hat{\delta}_t > \bar{\delta}$ .



### 3.3 Equilibrium laws of motion for aggregate variables

Given the log utility function, each agent consumes a fraction  $1 - \beta$  of net worth and saves the rest in each period:

$$c_{i,t} = (1 - \beta)w_{i,t}, \quad (34)$$

$$\lambda_{i,t}k_{i,t} + b_{i,t} + (1 + \zeta)V_t s_{i,t} = \beta w_{i,t}, \quad (35)$$

where  $w_{i,t}$  is the agent's net worth defined by:

$$w_{i,t} \equiv \left( \alpha_t + \int_{\bar{\delta}-\Delta_t}^{\delta_{i,t}} \frac{\lambda_{i,t}(1-\delta)}{2\Delta_t} d\delta + \int_{\delta_{i,t}}^{\bar{\delta}+\Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) k_{i,t-1} + \tilde{R}_t b_{i,t-1} + (D_t + V_t) s_{i,t-1}. \quad (36)$$

In Equations (35) and (36), trees are evaluated by the shadow value of trees net of depreciation for each agent,  $\lambda_{i,t}$ , except the fraction of trees sold.<sup>28</sup>

Now suppose  $\lambda'_{B,t} > \lambda''_{B,t}$  and  $H_{B,t} > 0$  for all  $t$  so that there is no bank run and banks are always providing liquidity transformation services in equilibrium. Note that  $\hat{\delta}_t > \bar{\delta}$  is a necessary condition for  $\lambda'_{B,t} > \lambda''_{B,t}$ . Substituting Equations (17)-(19), (24), (27), (31) and (32) into Equations (5), (8), (16), (28) and (35) implies:

$$\frac{K_{P,t}}{\phi} = \beta \left\{ \left( \alpha_t + \int_{\bar{\delta}-\Delta_t}^{\delta_{P,t}} \frac{1-\delta}{\phi \cdot 2\Delta_t} d\delta + \int_{\delta_{P,t}}^{\bar{\delta}+\Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) [\rho_P K_{P,t-1} + (1 - \rho_U) K_{U,t-1}] \right. \\ \left. + (1 - \rho_U) \left[ \alpha_t + \frac{Q_t(1-\bar{\delta})}{1-\hat{\delta}_t} \right] K_{B,t-1} \right\}, \quad (37)$$

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<sup>28</sup>To confirm Equations (34)-(36), apply the envelope theorem to the first-order condition with respect to  $k_{i,t}$  in the maximization problem for each agent defined by Equations (1) and (4)-(6), which yields:

$$\lambda_{i,t} = E_t \left[ \frac{\beta c_{i,t}}{c_{i,t+1}} \left( \alpha_{t+1} + \int_{\bar{\delta}-\Delta_{t+1}}^{\delta_{i,t+1}} \frac{\lambda_{i,t+1}(1-\delta)}{2\Delta_{t+1}} d\delta + \int_{\delta_{i,t+1}}^{\bar{\delta}+\Delta_{t+1}} \frac{Q_{t+1}}{2\Delta_{t+1}} d\delta \right) \right].$$

$$\lambda_{U,t}K_{U,t} + \left[ \frac{(1+\zeta)Q_t}{1-\hat{\delta}_t} - \frac{\zeta\omega_{t+1}}{R_t} \right] K_{B,t} = \beta \left\{ \left[ \alpha_t + \int_{\bar{\delta}-\Delta_t}^{\delta_{U,t}} \frac{\lambda_{U,t}(1-\delta)}{2\Delta_t} d\delta + \int_{\delta_{U,t}}^{\bar{\delta}+\Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right] [(1-\rho_P)K_{P,t-1} + \rho_U K_{U,t-1}] + \rho_U \left[ \alpha_t + \frac{Q_t(1-\bar{\delta})}{1-\hat{\delta}_t} \right] K_{B,t-1} \right\}, \quad (38)$$

$$K_{P,t} = \phi X_{P,t} + \int_{\bar{\delta}-\Delta_t}^{\delta_{P,t}} \frac{1-\delta}{2\Delta_t} d\delta [\rho_P K_{P,t-1} + (1-\rho_U)K_{U,t-1}], \quad (39)$$

$$K_{U,t} = (1-\bar{\delta})H_{U,t} + \int_{\bar{\delta}-\Delta_t}^{\delta_{U,t}} \frac{1-\delta}{2\Delta_t} d\delta [(1-\rho_P)K_{P,t-1} + \rho_U K_{U,t-1}], \quad (40)$$

$$K_{P,t} + K_{U,t} + K_{B,t} = \phi X_{P,t} + (1-\bar{\delta})(K_{P,t-1} + K_{U,t-1} + K_{B,t-1}), \quad (41)$$

$$\hat{\delta}_t = \frac{\theta_{t-1} \int_{\delta_{P,t}}^{\bar{\delta}+\Delta_t} \delta d\delta + \int_{\delta_{U,t}}^{\bar{\delta}+\Delta_t} \delta d\delta}{\theta_{t-1}(\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \delta_{U,t}}, \quad (42)$$

$$\frac{Q_t}{1-\hat{\delta}_t} = E_t \left\{ \frac{\beta c_{i,t} \left[ \alpha_{t+1} + \frac{Q_{t+1}}{1-\hat{\delta}_{t+1}}(1-\bar{\delta}) - \underline{\omega}_{t+1} \right]}{(1+\zeta)c_{i,t+1}} + \frac{\beta c_{i,t} \underline{\omega}_{t+1}}{c_{i,t+1}} \middle| \phi_{i,t} = 0 \right\}, \quad (43)$$

where  $K_{P,t} \equiv \int_{\{i|\phi_{i,t}=\phi\}} k_{i,t} d\mu$ ,  $X_{P,t} \equiv \int_{\{i|\phi_{i,t}=\phi\}} x_{i,t} d\mu$ ,  $K_{U,t} \equiv \int_{\{i|\phi_{i,t}=0\}} k_{i,t} d\mu$ ,  $H_{U,t} \equiv \int_{\{i|\phi_{i,t}=0\}} h_{i,t} d\mu$ ,  $B_{U,t} \equiv \int_{\{i|\phi_{i,t}=0\}} b_{i,t} d\mu$ ,  $S_{U,t} \equiv \int_{\{i|\phi_{i,t}=0\}} s_{i,t} d\mu$ , and  $\theta_{t-1} \equiv [\rho_P K_{P,t-1} + (1-\rho_U)K_{U,t-1}][(1-\rho_P)K_{P,t-1} + \rho_U K_{U,t-1}]^{-1}$ .<sup>29</sup> Given Conditions (20)-(22),  $\lambda'_{B,t} > \lambda''_{B,t}$ , and  $H_{B,t} > 0$  for all  $t$ , these equations together with Equations (25) and (26) determine the equilibrium dynamics of the model recursively. The conditions are verified in all of the numerical examples of equilibria considered below.

#### 4 Effects of asset illiquidity and liquidity transformation by banks on aggregate investment

This section describes an analytical result regarding the negative effect of illiquidity of trees on aggregate investment in new trees in the baseline case of the model without banks, and

<sup>29</sup>Regarding the second term on the left-hand side of Equation (38), note that Equations (11), (12) and (43) imply  $B_{B,t} + (1+\zeta)V_t S_{B,t} = [(1+\zeta)Q_t(1-\hat{\delta}_t)^{-1} - \zeta\omega_{t+1}(\bar{R}_t)^{-1}]K_{B,t}$ .

then shows comparative statics analysis of how introduction of banking to the model economy changes aggregate investment in new trees.

#### 4.1 *The negative illiquidity effect on aggregate investment : Comparison with the complete information case*

To derive equilibrium in the baseline case of the model without banks, impose  $K_{B,t} = 0$  on Equations (37)-(42) for all  $t$  and ignore Equation (43) as this is derived from the maximization problem for banks (15).

The following proposition shows that aggregate investment in new trees in the baseline case without banks is lower than in the complete information case.<sup>30</sup>

**Proposition 2** *Suppose Condition (20) holds in equilibrium in the baseline case of the model without banks. Then:*

$$\begin{aligned} \frac{X_{P,t}}{Y_t} &= \frac{\theta_{t-1}}{(1 + \theta_{t-1})\phi\alpha_t} \left[ \phi\beta \left( \alpha_t + \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) - (1 - \beta) \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{1 - \delta}{2\Delta_t} d\delta \right] \\ &< \beta - \frac{(1 - \beta)(1 - \bar{\delta})}{\phi\alpha_t}, \end{aligned} \quad (44)$$

where  $Y_t \equiv \int_{\mathcal{I}} y_{i,t} d\mu$ , so that  $X_{P,t}(Y_t)^{-1}$  is the ratio of aggregate investment in new trees to aggregate output. The right-hand side of the inequality,  $\beta - (1 - \beta)(1 - \bar{\delta})(\phi\alpha_t)^{-1}$ , is the value of  $X_{P,t}(Y_t)^{-1}$  in the complete information case as well as in the representative agent case where  $\phi_{i,t} = \phi$  for all  $i$  and  $t$ , if and only if:

$$(1 - \bar{\delta})(1 + \theta_{t-1} - \beta) \geq \phi\beta\alpha_t. \quad (45)$$

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<sup>30</sup>In the complete information case, the secondary market price of trees,  $Q_t$ , and the amount of trees bought by each agent in the secondary market,  $h_{i,t}$ , become specific to the depreciation rate of each tree in the maximization problem for each agent defined by Equations (1) and (4)-(6). The market clearing condition for the trees with each depreciation rate must be satisfied separately in equilibrium. See Appendix B.1 for more details on the definition of the complete information case.

Also, if Conditions (20) and (45) hold in equilibrium in the baseline case of the model without banks, then  $\delta_{P,t} = 1 - Q_t\phi > \bar{\delta} - \Delta_t$ .

**Proof.** See Appendix B. ■

Given the predetermined value of  $\theta_{t-1}$ , all of the numerical examples considered below satisfy Condition (45).<sup>31</sup> The intuition for this proposition is as follows: In the complete information case, the competitive secondary market price of each tree reflects the depreciation rate of the tree, which makes all trees liquid in the secondary market. Given Condition (45), productive agents have enough liquid trees (i.e., high  $\theta_{t-1}$ ) to achieve the desired level of investment in new trees.<sup>32</sup> On the other hand, asymmetric information makes a fraction of trees illiquid in the baseline case of the model without banks. Illiquidity of trees reduces the sales of trees by productive agents, as confirmed by  $\delta_{P,t} > \bar{\delta} - \Delta_t$ , decreasing aggregate investment in new trees.

#### 4.2 General equilibrium effects of introduction of banking to the economy: Numerical analysis of comparative statics

To illustrate how introduction of banking to the model economy changes aggregate investment in new trees, Figure 1 compares the balanced growth paths in the baseline case of the model without banks and those in the full model with banks under different values of the bank equity holding cost,  $\zeta$ , fixing the values of the other parameters to their benchmark values. The benchmark parameter values are set to replicate post-war sample averages of U.S. data on the balanced growth path in the full model with banks.<sup>33</sup>

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<sup>31</sup>In the numerical examples below, the arrival of the opportunity to invest in new trees is i.i.d for each agent (i.e.,  $\rho_P = 1 - \rho_U$ ) so that  $\theta_{t-1} = \rho_P(1 - \rho_P)^{-1}$ . Thus, Condition (45) becomes a parameter restriction.

<sup>32</sup>In this case, the allocation of net worth among agents is irrelevant for aggregate investment in new trees and, as a result, the complete information case becomes identical to the representative agent case.

<sup>33</sup>The benchmark parameter values are  $(\bar{\delta}, \phi, \beta, \zeta, \rho_P, \rho_U) = (0.1, 4.75, 0.99, 0.02, 0.45, 0.55)$ ,  $\bar{\alpha} = \underline{\alpha} = 0.03$ , and  $\bar{\Delta} = \underline{\Delta} = 0.09$ . Suppose the length of a period in the model is a year. For 1948-2007 in the U.S., the average real GDP growth rate was 3.4%, the average real interest rate on three-month Treasury bills was 3.9%, and the average ratio of the bank credit of commercial banks to the fixed assets in the economy was

Figure 1 shows that, in the long run, introduction of banking to the model economy increases  $X_{P,t}/Y_t$  if the value of  $\zeta$  is small, but reduces  $X_{P,t}/Y_t$  otherwise. There are two opposite effects behind this result. First, the supply of liquid bank liabilities by banks increases the transfer of goods from unproductive agents to productive agents via sales of bank liabilities, which expands aggregate investment in new trees. This effect can be confirmed by the following proposition.

**Proposition 3** *If Condition (20) holds and  $\hat{\delta}_t > \bar{\delta}$ , then:*

$$\frac{Q_t(1 - \bar{\delta})}{1 - \hat{\delta}_t} > \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{1 - \delta}{\phi \cdot 2\Delta_t} d\delta + \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta. \quad (46)$$

**Proof.** See Appendix C. ■

Equation (46) implies that the coefficient of  $K_{B,t-1}$  is larger than that of  $K_{U,t-1}$  in the aggregate saving rule for productive agents specified by Equation (37). Thus, a shift in the saving portfolio of unproductive agents from trees to bank liabilities, which increases  $K_{B,t-1}$  and reduces  $K_{U,t-1}$ , expands investments in new trees by productive agents who were unproductive buying bank liabilities in the previous period.<sup>34</sup> This is a direct positive effect of introduction of banking on aggregate investment in new trees.

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15.0%. These numbers are approximately replicated by the growth rate of aggregate output ( $G_t - 1$ ),  $\bar{R}_t - 1$ , and  $K_{B,t}/(K_{P,t} + K_{U,t} + K_{B,t})$ , in order, on the balanced growth path in the model. (See Equation (47) for the definition of the variable  $G_t$ .) The capital-asset ratio of banks,  $V_t S_{B,t} (B_{B,t} + V_t S_{B,t})^{-1}$ , is around 8%, which is the minimum requirement by the Basel agreement. The 10% annual average depreciation rate of trees implied by  $\bar{\delta}$  is a standard assumption. Rouwenhoust (1995) reports that the equity premium on S&P 500 was 1.99% on average for 1948-1992. The equity premium on bank equity in the model takes a similar value. The data sources for the first three sample averages are NIPA data from the BEA and financial data from the Federal Reserve Board. Note that  $\rho_P = 1 - \rho_U$ , which implies that the arrival of the opportunity to produce new trees is i.i.d. for each agent. This assumption is set to reduce the dimension of the parameter space.

<sup>34</sup>Appendix C also shows that, if  $\hat{\delta}_t > \bar{\delta}$  and  $\lambda_{U,t}$  is sufficiently close to  $Q_t(1 - \hat{\delta}_t)^{-1}$ , then:

$$\frac{Q_t(1 - \bar{\delta})}{1 - \hat{\delta}_t} < \int_{\bar{\delta} - \Delta_t}^{\delta_{U,t}} \frac{\lambda_{U,t}(1 - \delta)}{2\Delta_t} d\delta + \int_{\delta_{U,t}}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta.$$

Second, the supply of liquid bank liabilities by banks also raises the expected future consumption of unproductive agents because they will be able to obtain more goods from the sales of their assets, and thus consume more, when they become productive in the next period.<sup>35</sup> A resulting decline in the stochastic discount factor,  $\beta c_{i,t}(c_{i,t+1})^{-1}$ , for unproductive agents, which can be confirmed by an increase in  $\bar{R}_t$  in the figure, leads to a drop in the secondary market price of trees,  $Q_t$ , through Equation (43).<sup>36</sup> Given  $\delta_{P,t} = 1 - Q_t\phi$  in the figure, a decline in  $Q_t$  increases the illiquid fraction of trees held by productive agents who were productive investing in new trees in the previous period, which reduces their investments in new trees. This is an indirect negative effect of introduction of banking on aggregate investment in new trees.

Figure 1 indicates that, if the bank equity holding cost,  $\zeta$ , is low, then the positive effect dominates the negative effect, increasing  $X_{P,t}/Y_t$  as well as the gross rate of growth of aggregate output,

$$G_t \equiv \frac{Y_t}{Y_{t-1}}. \quad (47)$$

In this case, introduction of banking to the model economy makes the value of  $X_{P,t}/Y_t$  approach, but remain less than, the value of  $X_{P,t}/Y_t$  in the complete information case,  $\beta - (1 - \beta)(1 - \bar{\delta})(\phi\alpha_t)^{-1}$  ( $= 0.9268$  under the benchmark parameter values). But if the bank equity holding cost is large, then the opposite result holds as holding bank equity consumes goods, adding to the indirect negative effect of introduction of banking to the

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Thus, if unproductive agents remain unproductive in the next period, then holding bank liabilities can be ex-post costly as they will lose the opportunity to sell trees with high depreciation rates at an overvalued secondary market price. Overall, unproductive agents hold bank liabilities if the benefit shown by Proposition 3 exceeds the opportunity cost.

<sup>35</sup>Equation (34) implies that consumption is increasing in the net worth of the agent defined by Equation (36).

<sup>36</sup>See Equation (12) for the relationship between  $\beta c_{i,t}(c_{i,t+1})^{-1}$  and  $\bar{R}_t$ . For the baseline case without banks,  $\bar{R}_t$  is the hypothetical deposit contract rate with no supply of deposits.

model economy.<sup>37</sup>

## 5 Dynamics of asset illiquidity, asset prices and bank capital requirements

This section describes the equilibrium dynamics of the model, especially highlighting the illiquidity of trees, the secondary market price of trees, and the capital-asset ratio of banks.

### 5.1 Dynamics of asset illiquidity and the asset price in the baseline case of the model without banks

The equilibrium dynamics can be clarified analytically in the baseline case of the model without banks. When there is no bank, nobody would buy trees in the secondary market unless unproductive agents do. Thus,  $h_{i,t} > 0$  if  $\phi_{i,t} = 0$ , which leads to  $\lambda_{U,t} = Q_t(1 - \hat{\delta}_t)^{-1}$  and  $\delta_{U,t} = \hat{\delta}_t$ , given Condition (20) and Equations (25) and (26).<sup>38</sup> Then, Equations (37)-(39) and (41) imply that the values of  $(Q_t, \hat{\delta}_t)$  in each period are determined by Equation (42) and:

$$\begin{aligned} \frac{Q_t}{1 - \hat{\delta}_t} \left\{ (1 - \bar{\delta})(1 + \theta_{t-1}) - \theta_{t-1} \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{1 - \delta}{2\Delta_t} d\delta \right\} \\ = \beta \left( \alpha_t + \frac{Q_t}{1 - \hat{\delta}_t} \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right), \quad (48) \end{aligned}$$

given the predetermined value of  $\theta_{t-1}$ , the exogenous values of  $\alpha_t$  and  $\Delta_t$ , and  $\delta_{P,t} = \max\{\bar{\delta} - \Delta_t, 1 - Q_t\phi\}$ . Equation (48) can be interpreted as the market clearing condition

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<sup>37</sup>This result crucially depends on the assumption that the bank equity holding cost is a physical cost. This result might not hold if the equity premium on bank equity is modeled as an endogenous spread due to some agency problem. Also, note that the indirect negative effect of banking is not internalized by any agent or bank. Even if the bank equity holding cost is large, agents and banks find banking profitable, taking as given the secondary market price of trees,  $Q_t$ .

<sup>38</sup>Condition (20) implies that  $1 - Q_t\phi < \hat{\delta}_t$ . Because  $\delta_{P,t} = \min\{\bar{\delta} - \Delta_t, 1 - Q_t\phi\}$  and  $\hat{\delta}_t \leq \bar{\delta} + \Delta_t$  by definition,  $[\delta_{P,t}, \bar{\delta} + \Delta_t]$  is not an empty set, so productive agents sell some fraction of trees in the market. Because  $h_{i,t} = 0$  if  $\phi_{i,t} = \phi$ , the buyers of trees must be unproductive when there is no bank. Thus  $h_{i,t} > 0$  if  $\phi_{i,t} = 0$ . Then Equations (25) and (26) imply  $\lambda_{U,t} = Q_t(1 - \hat{\delta}_t)^{-1}$  and  $\delta_{U,t} = \hat{\delta}_t$ .

for trees, where the left-hand side of the equation is the value of trees net of depreciation that unproductive agents must hold at the end of the period, and the right-hand side is the fraction of net worth that unproductive agents save. Both sides are normalized by  $(1 - \rho_P)K_{P,t-1} + \rho_U K_{U,t-1}$ .

Figure 2 draws Equations (42) and (48) on the  $(Q_t, \hat{\delta}_t)$  plane and shows how a decline in  $\alpha_t$  makes them shift. It is possible to show that the two equations are downward-sloping and that Equation (48) has a steeper slope than Equation (42) at the intersection of the two equations if  $\beta$  is sufficiently close to 1 and Condition (20) is satisfied at the intersection. See Appendix D for the proof. In the figure, Equation (48) shifts inward as a decline in  $\alpha_t$  lowers  $Q_t$  through decreased aggregate spending on trees due to decreased unproductive agents' income, given  $\hat{\delta}_t$ . Then, given  $\delta_{P,t} = 1 - \phi Q_t$ , a decline in the market price of trees,  $Q_t$ , discourages productive agents from selling high-quality trees in the secondary market, leading to an increase in the average depreciation rate of trees sold in the secondary market,  $\hat{\delta}_t$ . Thus, the illiquidity of trees for each type of agent indicated by  $\delta_{P,t}$  and  $\hat{\delta}_t (= \delta_{U,t})$  is negatively correlated with productivity shocks.

Figure 3 shows the effects of an increase in  $\Delta_t$  on the  $(Q_t, \hat{\delta}_t)$  plane, which increases the degree of asymmetric information in the economy. While Equation (42) shifts upward unambiguously, the direction of the shift in Equation (48) is ambiguous.<sup>39</sup> It can be shown that Equation (48) shifts inward if  $\delta_{P,t}$  is sufficiently close to  $\hat{\delta}_t$  and shifts outward if  $\delta_{P,t} \leq \bar{\delta}$ . See Appendix E for the proof.<sup>40</sup> The top panel of Figure 3 shows the first case.<sup>41</sup> In this case, a deterioration in asymmetric information increases illiquidity of trees, as indicated by

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<sup>39</sup>As agents sell only low-quality trees in the secondary market, an increase in low-quality trees due to a higher value of  $\Delta_t$  raises  $\hat{\delta}_t$ , given  $Q_t$ .

<sup>40</sup>The intuition for this result regarding Equation (48) is that an increase in  $\Delta_t$  expands both ends of the distribution of the depreciation rates so that whether the fraction of trees sold by productive agents increases or not depends on the level of  $\delta_{P,t}$ . If  $\delta_{P,t}$  is sufficiently close to  $\hat{\delta}_t$ , then the fraction of trees sold by productive agents increases, which in turn reduces  $Q_t$  through the market clearing condition, given  $\hat{\delta}_t$ . If  $\delta_{P,t} \leq \bar{\delta}$ , then the fraction of trees sold by productive agents declines, which in turn increases  $Q_t$ , given  $\hat{\delta}_t$ .

<sup>41</sup>Because Equation (42) implies that  $\hat{\delta}_t$  is close to  $\bar{\delta} + \Delta_t$  if  $\delta_{P,t}$  is close to  $\hat{\delta}_t$ , the top panel of Figure 3 is a case where severe adverse selection takes place in the secondary market for trees.



an upward shift in Equation (42), which in turn reduces the market price of trees,  $Q_t$ .

Overall, Figures 2 and 3 illustrate that both shocks to  $\alpha_t$  and  $\Delta_t$  can cause a negative correlation between  $Q_t$  and  $\hat{\delta}_t$ . The next subsection, however, shows that the two types of shocks have opposite effects on bank capital requirements.

## 5.2 Cyclicalities of bank capital requirements: Numerical analysis of equilibrium dynamics

Now investigate the equilibrium dynamics of the full model with banks, especially highlighting the dynamics of the capital-asset (equity-asset) ratio of banks. Equations (11), (12), (33) and (43) imply that the capital-asset ratio of banks satisfies:<sup>42</sup>

$$\frac{V_t S_{B,t}}{B_{B,t} + V_t S_{B,t}} = E_t \left\{ \frac{(1 - \hat{\delta}_t) \beta c_{i,t}}{Q_t (1 + \zeta) c_{i,t+1}} \left[ \frac{Q_{t+1} (\hat{\delta}_{t+1} - \bar{\delta})}{1 - \hat{\delta}_{t+1}} + (\alpha_{t+1} + Q_{t+1} - \underline{\omega}_{t+1}) \right] \middle| \phi_{i,t} = 0 \right\}. \quad (49)$$

Note that the total value of bank liabilities,  $B_{B,t} + V_t S_{B,t}$ , equals the value of bank assets in the balance sheets of banks.

Equation (49) indicates that the equilibrium capital-asset ratio of banks depends on two factors: the expected value of illiquidity of banks' trees,  $\hat{\delta}_{t+1} - \bar{\delta}$ , and the downside risk to the market value of banks' trees,  $\alpha_{t+1} + Q_{t+1} - \underline{\omega}_{t+1}$ . The first factor matters as higher expected illiquidity of bank assets lowers the limit on bank deposits, which increases the fraction of bank asset value that must be financed through bank equity, as described in Section 3.2. The second factor is relevant because a larger possible decline in the market value of bank assets requires more bank capital as a buffer for preventing bank runs in the next period. To illustrate that these two factors have opposite effects on the cyclicalities of the capital-asset ratio of banks, Figures 4 and 5 show sample paths of the model driven by

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<sup>42</sup>Equations (11), (12) and (43) imply  $B_{B,t} + V_t S_{B,t} = Q_t (1 - \hat{\delta}_t)^{-1} K_{B,t}$ .

periodic fluctuations in  $\alpha_t$  and  $\Delta_t$ , respectively.<sup>43</sup> See Appendix F for the numerical solution method.

Figure 4 indicates that the capital-asset ratio of banks is pro-cyclical when business cycles are driven by productivity shocks,  $\alpha_t$ . Note that  $Q_t$  and  $\hat{\delta}_t$  are, respectively, positively and negatively correlated with  $\alpha_t$ , as explained in Section 5.1. Given that the economic growth rate,  $G_t$ , is positively correlated with  $\alpha_t$ , the positive correlation between  $Q_t$  and  $G_t$  implies that the largest possible decline in the market value of banks' trees becomes greater during economic booms (i.e., when  $G_t$  is high) than during downturns (i.e., when  $G_t$  is low), which leads to a pro-cyclical capital-asset ratio of banks. Even though the negative correlation between  $\hat{\delta}_t$  and  $G_t$  implies that expected illiquidity of banks' trees is counter-cyclical given the persistence of shocks, the effect of this factor is dominated by the effect of the pro-cyclical downside risk to the market value of banks' trees in the figure.

In contrast, Figure 5 indicates that the capital-asset ratio of banks is counter-cyclical when business cycles are driven by changes in  $\Delta_t$ , i.e., changes in the degree of asymmetric information. In the figure,  $Q_t$  and  $\hat{\delta}_t$  are, respectively, negatively and positively correlated with  $\Delta_t$ . The underlying mechanism is the same as in the top panel of Figure 3 in Section 5.1. Also, the economic growth rate,  $G_t$ , is negatively correlated with  $\Delta_t$  because an increase in  $\delta_{P,t}$  due to a rise in  $\Delta_t$  implies higher illiquidity of trees for productive agents, which reduces aggregate investment in new trees by discouraging productive agents from selling their trees. While the positive correlation between  $Q_t$  and  $G_t$  and the negative correlation between  $\hat{\delta}_t$  and  $G_t$  have opposite implications for the cyclicity of the capital-asset ratio of banks, the effect of the counter-cyclical fluctuations in expected illiquidity of banks' trees dominates in

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<sup>43</sup>In Figure 4,  $(\bar{\alpha}, \underline{\alpha}, \bar{\eta}_\alpha, \underline{\eta}_\alpha) = (0.0306, 0.0294, 0.75, 0.75)$  and  $\bar{\Delta} = \underline{\Delta} = 0.09$ , so the growth rate of output is around 4% in booms and around 2% in downturns, on average, and the expected durations of booms and downturns are four years, given that the length of a period in the model is interpreted as a year. In Figure 5,  $(\bar{\Delta}, \underline{\Delta}, \bar{\eta}_\Delta, \underline{\eta}_\Delta) = (0.1, 0.08, 0.75, 0.75)$  and  $\bar{\alpha} = \underline{\alpha} = 0.03$ . Thus:  $\Delta_t$  fluctuates symmetrically around the benchmark value, 0.9;  $\bar{\delta} - \bar{\Delta} = 0$ ; and the expected durations of booms and downturns are four years. The other parameters take the benchmark values specified in Footnote 33.

Figure 5, which leads to a counter-cyclical capital-asset ratio of banks.

### *5.3 Implications of the model for the “counter-cyclical capital buffer” discussed in policy forums*

While bank capital requirements in the model are imposed by rational investors who dislike losing the internal value of bank assets due to a bank run, the equilibrium dynamics of bank capital requirements can be seen as a benchmark for regulators who act rationally on behalf of the public, taking the market prices as given.<sup>44</sup> The results suggest that the counter-cyclical capital buffer recommended by the Financial Stability Forum (2009), which requires banks to increase bank capital in booms to absorb losses in downturns, is sufficient to prevent self-fulfilling bank runs when downside risk to the market value of bank assets is the dominant concern regarding stability of banks. However, if a deterioration in asymmetric information in the asset market increases the illiquidity of bank assets significantly, then banks need to raise more bank capital during the liquidity crisis to prevent self-fulfilling bank runs. This result is consistent with the recent episode in which, amid a severe decline in market liquidity of asset-backed securities, short-term lending to Bear Stearns dried up despite satisfying the supervisory capital standard.<sup>45</sup> This result also implies that the counter-cyclical capital buffer will not help to free up bank capital as designed in a liquidity crisis.

## **6 Why do banks intermediate collateralized lending?**

This section extends the model to discuss why banks are engaged with both liquidity transformation and intermediation of collateralized lending in reality. Modifying the assumption of no borrowing, suppose that productive agents can sell a right to receive a fraction of new

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<sup>44</sup>Dewatripont and Tirole (1994) make a similar interpretation of their models, calling the interpretation “representative hypothesis”.

<sup>45</sup>See the letter from the SEC chairman to the Basel Committee of Banking Supervision on March 20, 2008 ([http://www.sec.gov/news/press/2008/2008-48\\_letter.pdf](http://www.sec.gov/news/press/2008/2008-48_letter.pdf)).

trees that they invest in up to a certain limit:

$$m_{i,t} \geq -\psi\phi_{i,t}x_{i,t}, \tag{50}$$

where  $m_{i,t}$  is the net balance of the right to receive new trees, which is positive if the agent buys the right or negative if the agent sells the right, and  $\psi$  ( $\in (0, 1)$ ) is the pledgeable fraction of investment in new trees. Assume that pledged new trees are delivered to buyers before production in the next period so that buyers can use the new trees for their own production. The price of the right to receive new trees, which is denoted by  $P_t$ , is determined in a competitive market. This price can be interpreted as the collateral value of pledged new trees as agents receive an amount  $P_t|m_{i,t}|$  of goods against a quantity  $|m_{i,t}|$  of new trees pledged as collateral.

The modified model, which is fully described in Appendix G, implies that productive agents sell the right to receive new trees up to the limit, given Condition (20). Also, it is possible to show that  $P_t$  equals the highest shadow value of trees net of depreciation among agents and banks (i.e.,  $P_t = \max\{\max_{i \in \mathcal{I}} \lambda_{i,t}, \lambda_{B,t}\}$ ) as those who have the highest shadow value become the buyers of the right to receive new trees. Equations (24) and (31) imply  $\lambda_{U,t} \leq \lambda_{B,t}$ . Thus banks become buyers if banks exist. Moreover, if  $\lambda_{U,t} < \lambda_{B,t}$ , which is the case in the numerical examples shown above, then unproductive agents do not buy the right to receive new trees because the price, which equals to  $\lambda_{B,t}$ , is too high. This result implies that banks intermediate collateralized lending without special ability in enforcing debt repayment or monitoring borrowers in the model because the ability of banks in pooling illiquid assets and providing liquid liabilities increases the collateral value of new trees for banks.

### 6.1 General equilibrium effects of banking on the collateral value of new real assets

If there is no bank, then unproductive agents buy the right to receive new trees because there are no other possible buyers. Thus  $P_t = \lambda_{U,t}$  in this case. Given  $P_t = \lambda_{U,t}$  without banks and  $P_t = \lambda_{B,t}$  with banks, Equations (25) and (31) imply that  $P_t = Q_t(1 - \hat{\delta})^{-1}$  regardless of the existence of banks. Assuming that  $\psi$  is arbitrarily close to 0, Figure 6 compares the steady state value of  $Q_t(1 - \hat{\delta})^{-1}$  with and without banks under the benchmark parameter values, as in Figure 1. It shows that introduction of banking to the model economy reduces the collateral value of new trees,  $Q_t(1 - \hat{\delta})^{-1}$ . Figure 1 indicates two opposite effects of introduction of banking. First, introduction of banking to the model economy reduces the secondary market price of trees,  $Q_t$ , by raising the internal rate of return for unproductive agents, which is represented by  $\bar{R}_t$ . This effect reduces the collateral value of new trees. Second, at the same time, a decline in  $Q_t$  increases the average depreciation rate of trees sold in the secondary market,  $\hat{\delta}_t$ , as productive agents are discouraged from selling high-quality trees, given  $\delta_{P,t} = 1 - Q_t\phi$  in Figures 1 and 6. This effect increases the collateral value of new trees because higher illiquidity of trees in the secondary market increases the attractiveness of new trees, which are free of asymmetric information. However, the first negative effect dominates the second positive effect under the benchmark parameter values.

## 7 The effect of an alternative bank liquidation procedure on bank capital requirements

So far it has been assumed that a bank hit by a bank run can only sell trees one by one in the competitive secondary market. Now suppose that, before a bank run starts, the bank or the government, foreseeing the bank run, can set up a competitive bank liquidation market to liquidate the bank's trees separately from the secondary market for trees. The comparison of the two markets for bank liquidation is useful to clarify the effect of the bank

liquidation procedure on bank capital requirements.

It can be shown that the bank or the government's commitment to setting up a bank liquidation market separately from the competitive secondary market for trees can prevent self-fulfilling bank runs, making bank capital requirements unnecessary. See Appendix H for details on the extension of the model and the proof of this result. To understand the intuition for this result, note that the whole bank's trees are sold in the bank liquidation market and that no other seller exists in the market, which prevents adverse selection that contaminates the secondary market for trees. As a result, the average depreciation rate of the trees sold in the bank liquidation market equals the average depreciation rate of the liquidated bank's trees. Thus the liquidated bank's trees are priced fairly on average in the bank liquidation market and the total liquidation value of the liquidated bank's trees equals their total fundamental value, i.e., the present discounted value of the current and future income from the liquidated bank's trees.<sup>46</sup> As rational agents expect this, self-fulfilling bank runs do not occur. Hence the holders of bank liabilities do not need to impose costly bank capital requirements on banks.

## 8 Conclusions

This paper has presented a dynamic competitive general equilibrium model of liquidity transformation by banks, where illiquidity of bank assets leads to endogenous bank capital requirements for preventing self-fulfilling bank runs. The model shows that bank capital requirements depend on two factors, downside risk to the market value of bank assets and expected illiquidity of bank assets. It is shown that these two factors fluctuate with shocks to the economy, making bank capital requirements dynamic. The model suggests that the counter-cyclical capital buffer recommended by the Financial Stability Forum (2009) is suf-

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<sup>46</sup>The market price of trees is determined by the average depreciation of trees sold in the bank liquidation market, as in the secondary market for trees.

efficient to prevent self-fulfilling bank runs when downside risk to the market value of bank assets is the dominant concern regarding stability of banks. This result regarding the relationship between volatile bank asset value and the counter-cyclical capital buffer adds to the findings of Meh and Moran (2010). In their model, banks need to finance part of lending to borrowers through their own capital to commit to efficient monitoring services. If banks suffer from unexpected loan losses, then bank capital requirements decline as the scarcity of bank capital gives greater incentive for banks to monitor borrowers during such an episode.

The model, however, also shows that if a deterioration in asymmetric information in the asset market increases the illiquidity of bank assets significantly, then banks need to raise more bank capital during the liquidity crisis to prevent self-fulfilling bank runs. This result implies that the counter-cyclical capital buffer will not help to free up bank capital as designed in a liquidity crisis.

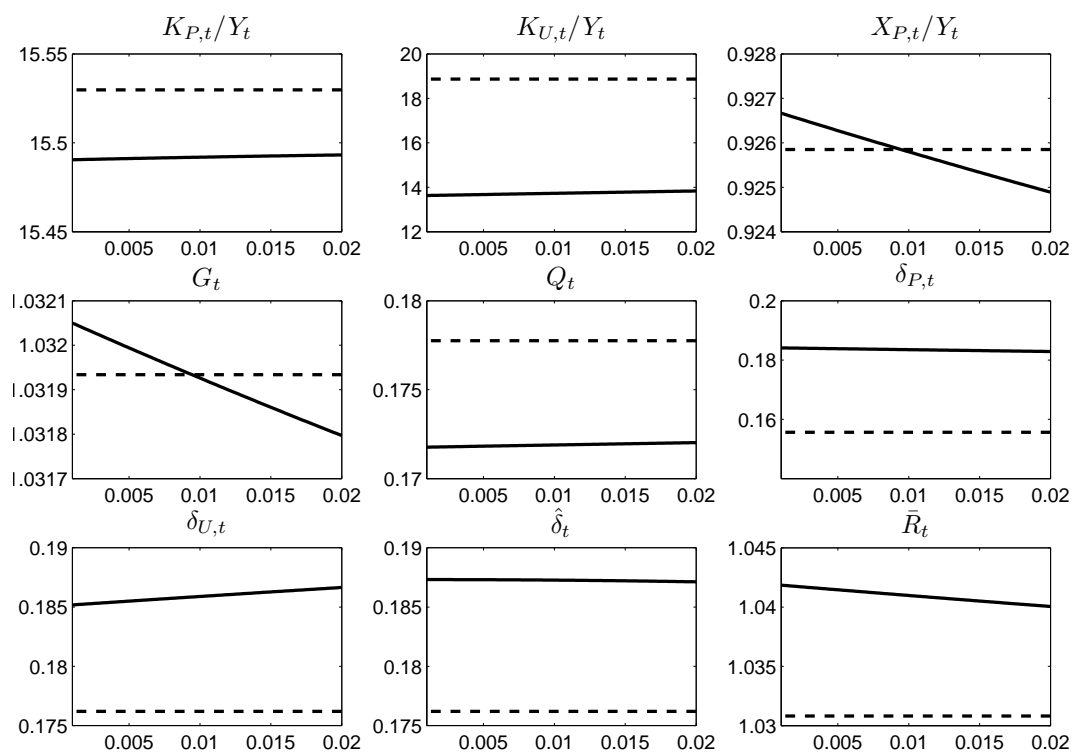
A question arises from this result regarding how a concern about a future liquidity crisis would modify bank capital requirements. Because it is difficult to raise bank capital in a liquidity crisis in reality, concern about a future liquidity crisis might increase desirable bank capital requirements during a boom, unless there is some insurance mechanism for the liquidity crisis, such as contingent capital. To answer this question, it would be necessary to incorporate the difficulty in raising bank capital in a liquidity crisis by introducing heterogeneous banks and asymmetric information about the quality of each bank's assets in the model. This issue is left for future research.

In addition, the model shows that the level of bank capital requirements declines if the bank liquidation procedure can avoid undervaluation of banks' illiquid assets. The same result would hold if the government or the central bank could commit to lending to banks hit by bank runs up to the fundamental value of bank assets, rather than the market value of bank assets at illiquid market prices. Thus, along with improving the effectiveness of bank capital regulations, it is important to design an efficient bank liquidation procedure and an

efficient emergency lending facility for distressed banks to minimize the necessity of bank capital requirements, given the cost of equity financing for banks.

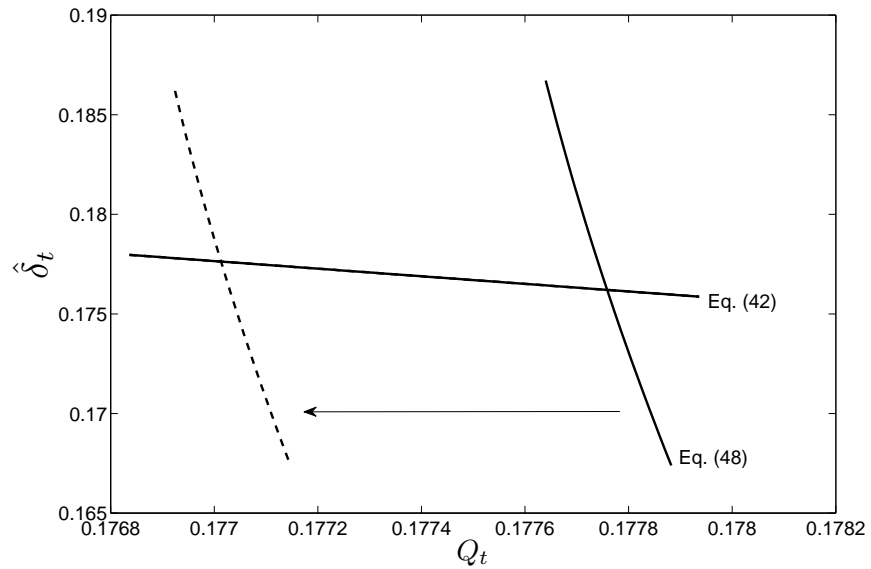


Figure 1: Comparative statics: balanced growth paths with and without banks



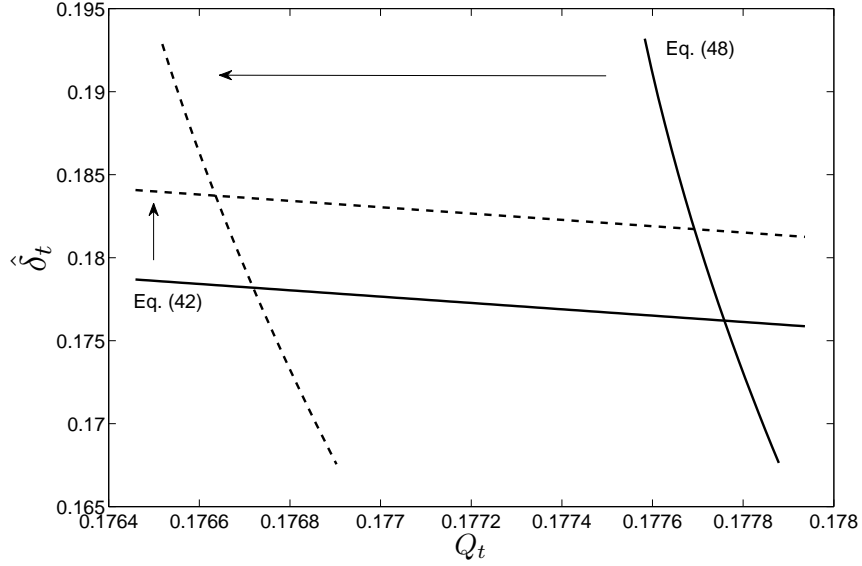
Notes: The horizontal axis is the value of  $\zeta$ . The other parameter values are  $(\bar{\delta}, \phi, \beta, \rho_P, \rho_U) = (0.1, 4.75, 0.99, 0.45, 0.55)$ ,  $\bar{\alpha} = \underline{\alpha} = 0.03$  and  $\bar{\Delta} = \underline{\Delta} = 0.09$ . The solid lines represent the model with banks and the dashed lines represent the model without banks.

Figure 2: Dynamic equilibrium without banks: the effect of a decline in  $\alpha_t$

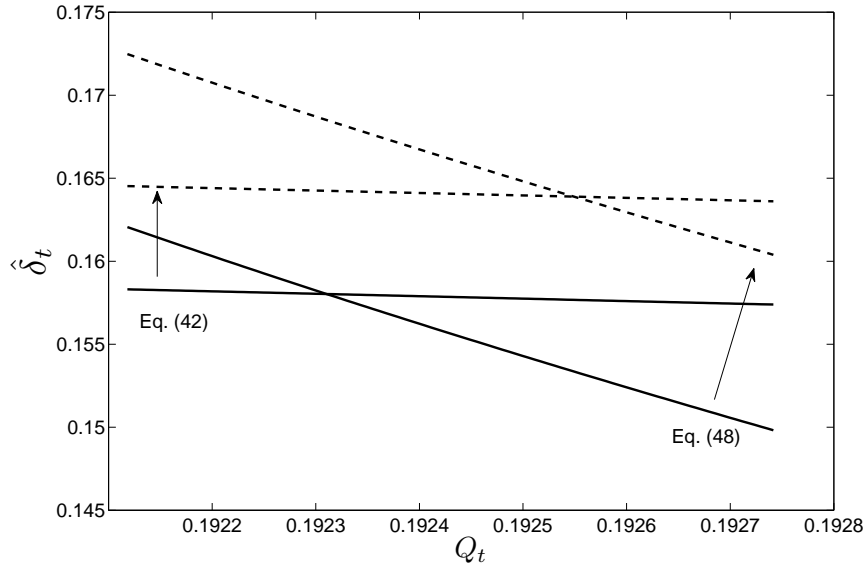


Notes: For each curve in the figure, parameter values used are  $(\bar{\delta}, \phi, \beta, \zeta, \rho_P, \rho_U) = (0.1, 4.75, 0.99, 0.02, 0.45, 0.55)$  and  $\bar{\Delta} = \underline{\Delta} = 0.09$ . The solid lines are Equations (42) and (48) with  $\alpha_t = 0.03$  and the dashed lines are these equations with  $\alpha_t = 0.027$ . Given  $\rho_P = 1 - \rho_U$ ,  $\theta_{t-1} = \rho_P(1 - \rho_P)^{-1}$  for all  $t$ , regardless of shocks.

Figure 3: Dynamic equilibrium without banks: the effect of an increase in  $\Delta_t$   
(a)

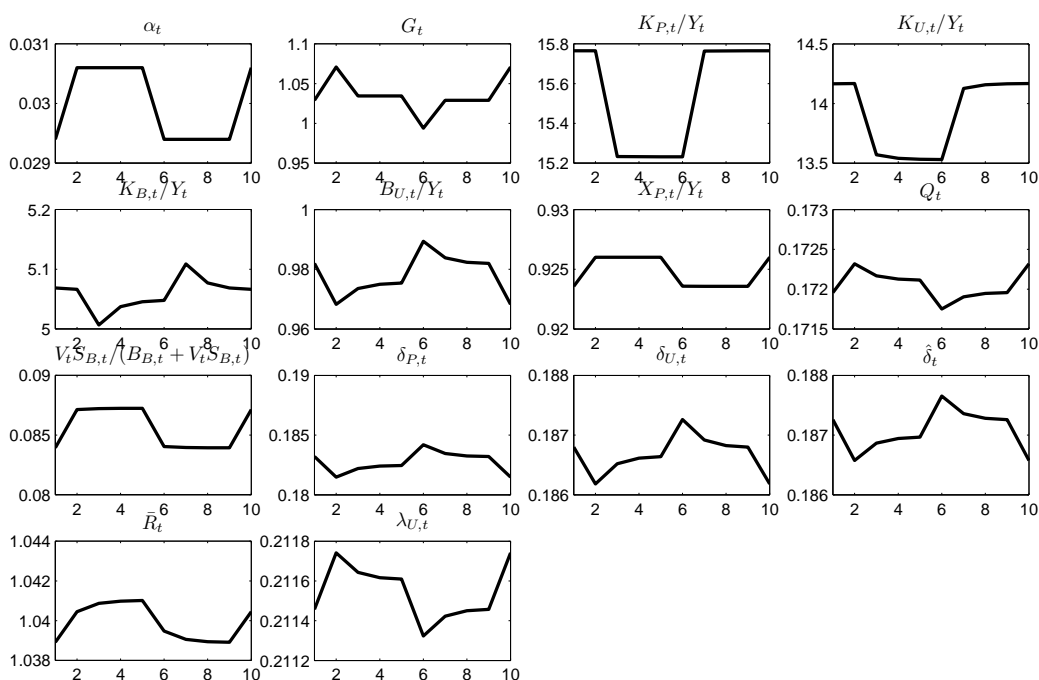


(b)



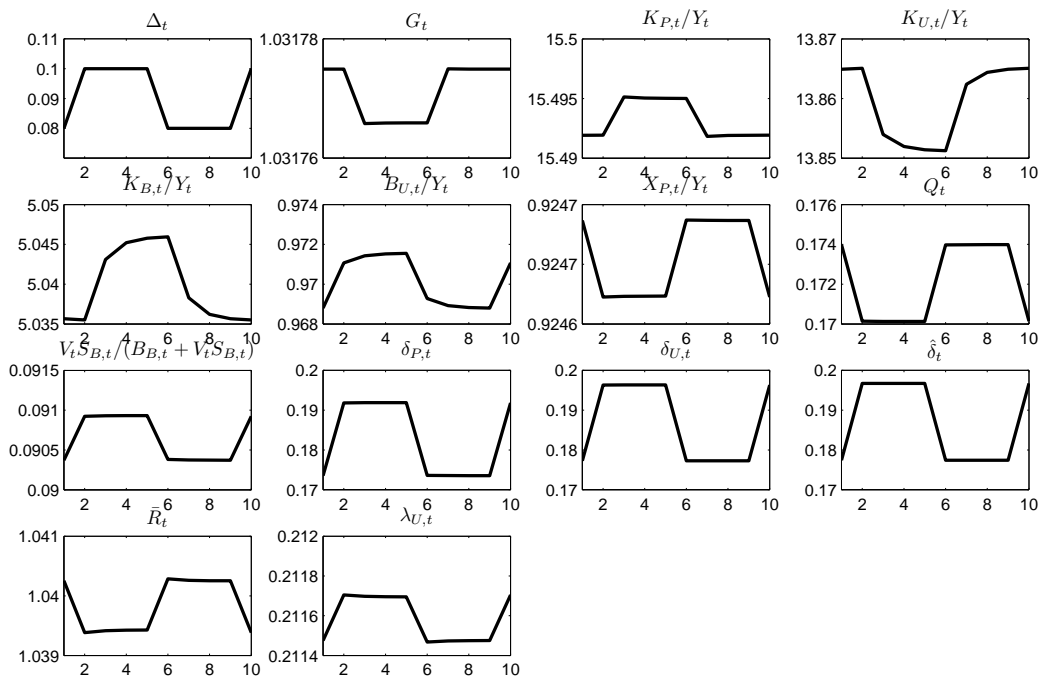
Notes: Parameter values used are:  $(\bar{\delta}, \phi, \beta, \zeta) = (0.1, 4.75, 0.99, 0.02)$ ,  $\bar{\alpha} = \underline{\alpha} = 0.03$  and  $\bar{\Delta} = \underline{\Delta} = 0.09$  for both panels;  $(\rho_P, \rho_U) = (0.45, 0.55)$  for the top panel; and  $(\rho_P, \rho_U) = (0.2, 0.8)$  for the bottom panel. In each panel, the solid lines are Equations (42) and (48) with  $\Delta_t = 0.09$  and the dashed lines are these equations with  $\Delta_t = 0.099$ . Given  $\rho_P = 1 - \rho_U$ ,  $\theta_{t-1} = \rho_P(1 - \rho_P)^{-1}$  for all  $t$ , regardless of shocks.

Figure 4: Dynamic equilibrium with banks: business cycles driven by  $\alpha_t$



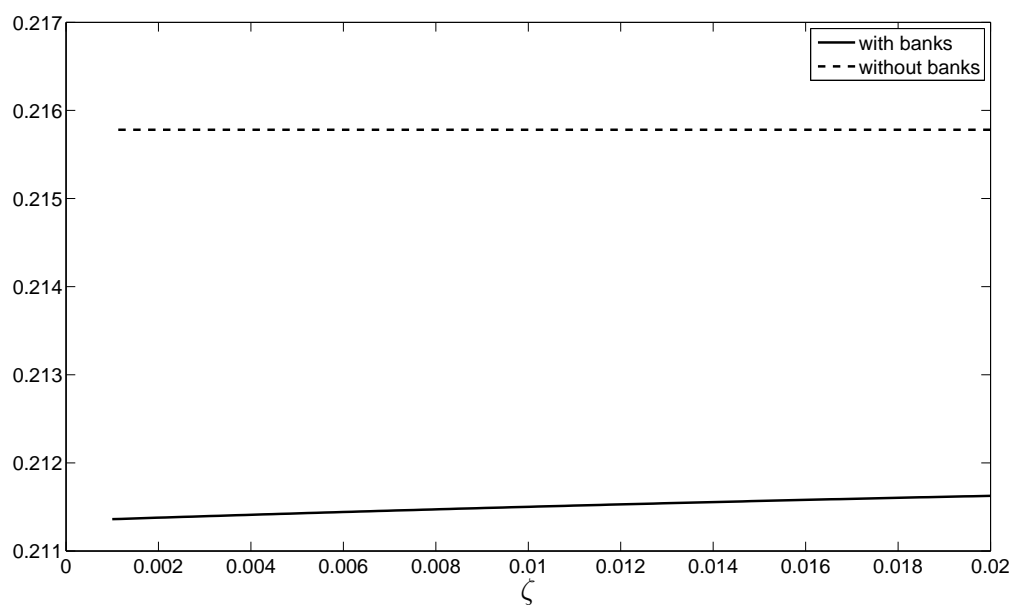
Notes: Parameter values are  $(\bar{\delta}, \phi, \beta, \zeta, \rho_P, \rho_U) = (0.1, 4.75, 0.99, 0.02, 0.45, 0.55)$ ,  $(\bar{\alpha}, \underline{\alpha}) = (0.0306, 0.0294)$ ,  $\bar{\eta}_\alpha = \underline{\eta}_\alpha = 0.75$ , and  $\bar{\Delta} = \underline{\Delta} = 0.09$ . The figure shows a sample path when  $\alpha_t$  keeps changing its value every 4 periods for a sufficiently long time.  $H_{U,t} = 0$  for all periods.

Figure 5: Dynamic equilibrium with banks: business cycles driven by  $\Delta_t$



Notes: Parameter values are  $(\bar{\delta}, \phi, \beta, \zeta, \rho_P, \rho_U) = (0.1, 4.75, 0.99, 0.02, 0.45, 0.55)$ ,  $\bar{\alpha} = \underline{\alpha} = 0.03$ ,  $(\bar{\Delta}, \underline{\Delta}) = (0.1, 0.08)$ , and  $\bar{\eta}_\Delta = \underline{\eta}_\Delta = 0.75$ . The figure shows a sample path when  $\Delta_t$  keeps changing its value every 4 periods for a sufficiently long time.  $H_{U,t} = 0$  for all periods.

Figure 6: Comparative statics: the collateral value of new trees  $(Q_t(1 - \hat{\delta}_t)^{-1})$  with and without banks



Notes: The horizontal axis is the value of  $\zeta$ . The other parameter values take the benchmark values specified in Section 4.2:  $(\bar{\delta}, \phi, \beta, \rho_P, \rho_U) = (0.1, 4.75, 0.99, 0.45, 0.55)$ ,  $\bar{\alpha} = \underline{\alpha} = 0.03$  and  $\bar{\Delta} = \underline{\Delta} = 0.09$ . The figure shows the limit case where  $\psi$  is arbitrarily close to 0, given the other parameter values. The solid line represents the model with banks and the dashed line represents the model without banks.

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## Appendices

### A Proof of Proposition 1

The following lemma will be used in the proof.

**Lemma 1** *Suppose that  $\Omega_{t+1}$  satisfies Equations (27)-(29) for period  $t + 1$ . Split the constraint set of the maximization problem (15) into three regions:  $\bar{R}_t B_{B,t} \leq \underline{\omega}_{t+1} K_{B,t}$ ;  $\bar{R}_t B_{B,t} \in (\underline{\omega}_{t+1} K_{B,t}, \bar{\omega}_{t+1} K_{B,t}]$ ; and  $\bar{R}_t B_{B,t} > \bar{\omega}_{t+1} K_{B,t}$ . Then, in equilibrium,  $\bar{R}_t B_{B,t}$  equals  $\underline{\omega}_{t+1} K_{B,t}$  at optimum in the first region and  $\bar{\omega}_{t+1} K_{B,t}$  at optimum in the second region.*

**Proof.** Use the Lagrange method to solve the maximization problem in the first and the second regions. For the second region, solve the maximization problem in the closure of the region and suppose that  $\Omega_{t+1}$  takes the limit value when  $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$ . This makes the function  $\Omega_{t+1}$  differentiable in each region. This expansion of the second region does not affect the solution to the maximization problem, since it will be shown that  $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$  at optimum in the second region.

In the first region,  $\bar{R}_t$  is determined solely by Equation (12) and can be taken as exogenous for the representative bank. Equation (12) implies that  $\bar{R}_t > 0$ , since agents never choose zero consumption with the time-separable log utility function in equilibrium. The first-order condition with respect to  $B_{B,t}$  is:

$$1 - \frac{1}{1 + \zeta} E_t \left[ \frac{\beta c_{i,t} \bar{R}_t}{c_{i,t+1}} \Big| \phi_{i,t} = 0 \right] - \bar{\theta}_{rgn1,t} \bar{R}_{t+1} = 0, \quad (51)$$

where  $\bar{\theta}_{rgn1,t}$  is the Lagrange multiplier for the upper bound of the first region ( $\bar{R}_t B_{B,t} \leq \underline{\omega}_{t+1} K_{B,t}$ ). Thus,  $\bar{\theta}_{rgn1,t} = \zeta(1 + \zeta)^{-1}(\bar{R}_t)^{-1} > 0$ , given  $\zeta > 0$  and  $\bar{R}_t > 0$ . Hence,  $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$  at optimum in the first region.

For the second region, if  $K_{B,t} = 0$ , then the claim is automatically satisfied since Equation (12) implies that  $B_{B,t}$  must be 0, given that  $K_{B,t}(B_{B,t})^{-1}$  in the equation is replaced with infinity if  $B_{B,t} = 0$ , as defined in the main text. Hereafter suppose  $K_{B,t} > 0$  in the second region. In equilibrium,  $Q_t$  is always positive and thus  $\underline{\omega}_t > 0$  for all  $t$ , since otherwise each agent would demand an infinite amount of trees in the secondary market, which would violate the market clearing condition for trees. In the second region,  $K_{B,t} > 0$  and  $\underline{\omega}_{t+1} > 0$  imply that  $B_{B,t} > 0$  and  $\bar{R}_t > 0$ , since  $B_{B,t}$  must be non-negative by the non-negativity constraint. The first-order conditions with respect to  $B_{B,t}$  and  $\bar{R}_t$  in the second region, respectively, are:

$$1 - \frac{P_t(\bar{\omega}_{t+1})}{1 + \zeta} E_t \left[ \frac{\beta c_{i,t} \bar{R}_t}{c_{i,t+1}} \middle| \phi_{i,t} = 0, \alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1} \right] + (\underline{\theta}_{rgn2,t} - \bar{\theta}_{rgn2,t}) \bar{R}_t - \theta_{PC,t} P_t(\underline{\omega}_{t+1}) E_t \left[ \frac{\beta c_{i,t} \underline{\omega}_{t+1} K_{B,t}}{c_{i,t+1} (B_{B,t})^2} \middle| \phi_{i,t} = 0, \alpha_{t+1} + Q_{t+1} = \underline{\omega}_{t+1} \right] = 0, \quad (52)$$

$$- \frac{P_t(\bar{\omega}_{t+1})}{1 + \zeta} E_t \left[ \frac{\beta c_{i,t} B_{B,t}}{c_{i,t+1}} \middle| \phi_{i,t} = 0, \alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1} \right] + (\underline{\theta}_{rgn2,t} - \bar{\theta}_{rgn2,t}) B_{B,t} + \theta_{PC,t} P_t(\bar{\omega}_{t+1}) E_t \left[ \frac{\beta c_{i,t}}{c_{i,t+1}} \middle| \phi_{i,t} = 0, \alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1} \right] = 0, \quad (53)$$

where  $\bar{\theta}_{rgn2,t}$  is the Lagrange multiplier for the upper bound of the closure of the second region ( $\bar{R}_t B_{B,t} \leq \bar{\omega}_{t+1} K_{B,t}$ ),  $\underline{\theta}_{rgn2,t}$  is the Lagrange multiplier for the lower bound of the closure of the second region ( $\bar{R}_t B_{B,t} \geq \underline{\omega}_{t+1} K_{B,t}$ ), and  $\bar{\theta}_{PC,t}$  is the Lagrange multiplier for Equation (12). Equations (52) and (53) imply that  $\theta_{PC,t} = B_{B,t}$ . Substituting this into Equation (52) leads to:

$$(\bar{\theta}_{rgn2,t} - \underline{\theta}_{rgn2,t}) \bar{R}_t = \frac{\zeta P_t(\bar{\omega}_{t+1})}{1 + \zeta} E_t \left[ \frac{\beta c_{i,t} \bar{R}_t}{c_{i,t+1}} \middle| \phi_{i,t} = 0, \alpha_{t+1} + Q_{t+1} = \bar{\omega}_{t+1} \right], \quad (54)$$

which in turn indicates that  $\bar{\theta}_{rgn2,t} > 0$  and  $\underline{\theta}_{rgn2,t} = 0$ , given  $\zeta > 0$  and  $\bar{R}_t > 0$ . Thus,  $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$  at optimum in the second region.

■

### Proof of Proposition 1:

Suppose that  $\Omega_{t+1}$  satisfies Equations (27)-(29) for period  $t + 1$ . Note that Equation (27) satisfies the bank-run conditions (9) and (10).

To verify Equation (27), split the constraint set of the maximization problem (15) into three regions:  $\bar{R}_t B_{B,t} \leq \underline{\omega}_{t+1} K_{B,t}$ ;  $\bar{R}_t B_{B,t} \in (\underline{\omega}_{t+1} K_{B,t}, \bar{\omega}_{t+1} K_{B,t}]$ ; and  $\bar{R}_t B_{B,t} > \bar{\omega}_{t+1} K_{B,t}$ . First of all, any point in the third region,  $\bar{R}_t B_{B,t} > \bar{\omega}_{t+1} K_{B,t}$ , is weakly dominated by  $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$ , since the feasible set of the choice variables is identical and the value of  $\Omega_{t+1}$  is always 0 in the third region while it can be positive with  $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$ . Thus, the third region can be ignored.

By Lemma 1,  $\bar{R}_t B_{B,t} = \underline{\omega}_{t+1} K_{B,t}$  and  $\bar{R}_t B_{B,t} = \bar{\omega}_{t+1} K_{B,t}$  at optimum in the first and the second regions, respectively. Denote the maximum values of the objective function of the maximization problem (15) in the first and the second regions by  $\Omega'_t$  and  $\Omega''_t$ , respectively. Given that  $\Omega_{t+1}$  satisfies Equations (27)-(29) for period  $t + 1$ , substituting the optimal values of  $\bar{R}_t B_{B,t}$  in the first and the second regions and Equations (9), (10) and (12) into the objective function of the maximization problem (15) yields:

$$\Omega'_t = \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + \lambda'_{B,t} K_{B,t}, \quad (55)$$

$$\Omega''_t = \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + \lambda''_{B,t} K_{B,t}. \quad (56)$$

The global solution to the maximization problem (15) can be obtained by maximizing the values of  $\Omega'_t$  and  $\Omega''_t$  with satisfying Equation (8),  $L_{B,t} \in [0, K_{B,t-1}]$  and  $H_{B,t} \geq 0$ . Since the first and the second regions have the same feasible set of  $H_{B,t}$  and  $L_{B,t}$ ,  $\Omega_t = \Omega'_t$  if  $\lambda'_{B,t} \geq \lambda''_{B,t}$  and  $\Omega_t = \Omega''_t$  if  $\lambda'_{B,t} \leq \lambda''_{B,t}$ . This result proves Equations (28)-(30).

Given this result, now prove Equations (31) and (32). The maximization problem (15) can be rewritten as:

$$\begin{aligned} \Omega_t = \max_{\{H_{B,t}, L_{B,t}\}} & \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} + \lambda_{B,t} K_{B,t}, \\ \text{s.t. Equations (8), (9) and (10), } & L_{B,t} \in [0, K_{B,t-1}], H_{B,t} \geq 0, \end{aligned} \quad (57)$$

where  $\lambda_{B,t} = \max\{\lambda'_{B,t}, \lambda''_{B,t}\}$ . Note that Equation (12) is already incorporated by the definitions of  $\lambda'_{B,t}$  and  $\lambda''_{B,t}$ . The maximization problem (57) implies that the equilibrium value of  $\lambda_{B,t}$  satisfies:

$$\lambda_{B,t} \begin{cases} = Q_t(1 - \hat{\delta}_t)^{-1}, & \text{if } H_{B,t} > 0, \\ = Q_t(1 - \bar{\delta})^{-1}, & \text{if } L_{B,t} \in (0, K_{B,t-1}), \\ \leq Q_t(1 - \bar{\delta})^{-1}, & \text{if } L_{B,t} = K_{B,t-1}, \\ \in [Q_t(1 - \bar{\delta})^{-1}, Q_t(1 - \hat{\delta}_t)^{-1}], & \text{if } H_{B,t} = 0 \text{ and } L_{B,t} = 0. \end{cases} \quad (58)$$

When  $\hat{\delta}_t > \bar{\delta}$ , Equation (58) implies that  $L_{B,t} = 0$  if  $H_{B,t} > 0$  and that  $H_{B,t} = 0$  if  $L_{B,t} > 0$ . Thus Equations (31) and (32) are proved. Substituting Equations (8) and (58) in the objection function in the maximization problem (57) proves Equation (27).

## B Proof of Proposition 2

### B.1 Part I: Definition of the complete information case of the model without banks

Suppose that the depreciation rate of each tree in each period is public information and that there exists a competitive secondary market for trees with each depreciation rate in the model without banks. The maximization problem for each agent defined by Equations (1)

and (4)-(6) is modified to:

$$\max_{\{c_{i,t}, x_{i,t}, l_{i,\delta,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{i,t} \quad (59)$$

$$\text{s.t. } c_{i,t} + x_{i,t} = \alpha_t k_{i,t-1} + \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} Q_{\delta,t} l_{i,\delta,t} d\delta, \quad (60)$$

$$k_{i,t} = \phi_{i,t} x_{i,t} + \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} (1-\delta) \left( \frac{k_{i,t-1}}{2\Delta_t} - l_{i,\delta,t} \right) d\delta, \quad (61)$$

$$l_{i,\delta,t} \leq \frac{k_{i,t-1}}{2\Delta_t}, \quad c_{i,t} \geq 0, \quad x_{i,t} \geq 0, \quad (62)$$

where  $Q_{\delta,t}$  is the secondary market price of trees with depreciation rate  $\delta$  and  $l_{i,\delta,t}$  is the net sales of trees in the secondary market if positive or the net purchase of trees in the secondary market if negative. The variable  $h_{i,t}$  and the non-negativity constraint on  $l_{i,\delta,t}$  are erased as they are not needed, given the change in the definition of  $l_{i,\delta,t}$ . The market clearing condition (17) is modified to  $\int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} l_{i,\delta,t} d\delta = 0$  for  $\delta \in [\bar{\delta} - \Delta_t, \bar{\delta} + \Delta_t]$ . The other equilibrium condition, Equation (16), does not exist in the complete information case, since the variable  $\hat{\delta}_t$  does not exist.

Suppose that  $Q_{\delta,t}(1-\delta)^{-1} = \phi^{-1}$  for all  $\delta \in [\bar{\delta} - \Delta_t, \bar{\delta} + \Delta_t]$ . The first-order condition with respect to  $k_{i,t}$  implies that:

$$c_{i,t} = (1-\beta) \left( \alpha + \frac{1-\bar{\delta}}{\phi} \right) k_{i,t-1}, \quad (63)$$

$$\frac{k_{i,t}}{\phi} = \beta \left( \alpha + \frac{1-\bar{\delta}}{\phi} \right) k_{i,t-1}, \quad (64)$$

for all  $i \in \mathcal{I}$ . Then aggregating (64) implies that  $K_{P,t} + K_{U,t} = \beta (\phi\alpha_t + 1 - \bar{\delta}) (K_{P,t-1} + K_{U,t-1})$ . Since aggregating Equation (61) and substituting the market clearing condition yield that  $\phi X_{P,t} = K_{P,t} + K_{U,t} - (1 - \bar{\delta})(K_{P,t-1} + K_{U,t-1})$ , the ratio of aggregate investment in new trees to aggregate output,  $X_{P,t}(Y_t)^{-1}$ , equals  $\beta - (1 - \beta)(1 - \bar{\delta})(\phi\alpha_t)^{-1}$ , given  $Y_t =$

$\alpha_t(K_{P,t-1} + K_{U,t-1})$ .

To verify  $Q_{\delta,t}(1 - \delta)^{-1} = \phi^{-1}$  for all  $\delta \in [\bar{\delta} - \Delta_t, \bar{\delta} + \Delta_t]$ , it is necessary and sufficient to check the remaining equilibrium condition,  $l_{i,\delta,t} \leq \frac{k_{i,t-1}}{2\Delta_t}$  for all  $\delta \in [\bar{\delta} - \Delta_t, \bar{\delta} + \Delta_t]$ . Given Equation (61), this is satisfied if and only if  $K_{P,t} \geq \phi X_{P,t}$ . This is equivalent to  $K_{U,t} \leq (1 - \bar{\delta})(K_{P,t-1} + K_{U,t-1})$  and substituting this condition in Equation (64) aggregated for unproductive agents (i.e.,  $K_{U,t}\phi^{-1} = \beta[\alpha + (1 - \bar{\delta})\phi^{-1}][(1 - \rho_P)K_{P,t-1} + \rho_U K_{U,t-1}]$ ) yields that this condition is in turn equivalent to  $(1 - \bar{\delta})(1 + \theta_{t-1} - \beta) \geq \phi\beta\alpha_t$ .

## B.2 Part II: Comparison between the baseline case and the complete information case

Suppose Condition (20) holds in equilibrium in the model without banks. Show that:

$$(1 + \theta_{t-1})[\phi\beta\alpha_t - (1 - \beta)(1 - \bar{\delta})] - \theta_{t-1} \left[ \phi\beta \left( \alpha_t + \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) - (1 - \beta) \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{1 - \delta}{2\Delta_t} d\delta \right] > 0. \quad (65)$$

The left-hand side is equivalent to:

$$\begin{aligned} & \phi\beta\alpha_t - (1 - \beta)\theta_{t-1} \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta - (1 - \beta)(1 - \bar{\delta}) \\ & \quad - \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{\theta_{t-1}\phi\beta Q_t}{2\Delta_t} d\delta \\ & = \phi\beta\alpha_t - (1 - \beta) \left[ \theta_{t-1} \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta \right] \\ & \quad - \frac{\theta_{t-1}\phi\beta Q_t}{1 - \hat{\delta}_t} \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta \\ & > \phi\beta\alpha_t - (1 - \beta) \left[ \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta \right] - \frac{\theta_{t-1}\phi Q_t}{1 - \hat{\delta}_t} \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta. \quad (66) \end{aligned}$$



The last inequality is obtained by Condition (20), which implies  $\phi Q_t(1 - \hat{\delta}_t)^{-1} > 1$ , and Equation (42), which implies:

$$\theta_{t-1} \int_{\hat{\delta}_{P,t}}^{\bar{\delta}+\Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta = \theta_{t-1} \int_{\hat{\delta}_{P,t}}^{\bar{\delta}+\Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta. \quad (67)$$

Equation (42), which is equivalent to Equation (67), and Equation (48) imply that:

$$\frac{Q_t}{1 - \hat{\delta}_t} = \beta\alpha_t \left[ \theta_{t-1} \int_{\hat{\delta}_{P,t}}^{\bar{\delta}+\Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta + (1 - \beta) \left( \int_{\bar{\delta}-\Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta \right) \right]^{-1}. \quad (68)$$

Substituting this into  $\beta\alpha_t$  on the right-hand side of Inequality (66) implies that the right-hand side is equivalent to:

$$(1 - \beta) \left( \frac{\phi Q_t}{1 - \hat{\delta}_t} - 1 \right) \left[ \int_{\bar{\delta}-\Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta}+\Delta_t} \frac{1 - \hat{\delta}_t}{2\Delta_t} d\delta \right]. \quad (69)$$

This is positive, given Condition (20).

Also, suppose that Conditions (20) and (45) hold and that  $\delta_{P,t} = \bar{\delta} - \Delta_t$  in equilibrium. Then Equations (67) and (68) imply that  $Q_t(1 - \hat{\delta}_t)^{-1} \leq \beta\alpha[(1 + \theta_{t-1} - \beta)(1 - \bar{\delta})]^{-1}$ . This contradicts Condition (45), given Condition (20). Thus, if Conditions (20) and (45) hold in equilibrium, then  $\delta_{P,t} > \bar{\delta} - \Delta_t$ .

### C Proof of Proposition 3

It is obvious that Inequality (46) holds when  $\delta_{P,t} = \bar{\delta} - \Delta$  or  $\delta_{P,t} = \bar{\delta} + \Delta$ , given  $\hat{\delta}_t > \bar{\delta}$  and Condition (20). When  $\delta_{P,t} = 1 - \phi Q_t$ :

$$\begin{aligned} \frac{Q_t(1 - \bar{\delta})}{1 - \hat{\delta}_t} - \left( \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{(1 - \delta)}{\phi \cdot 2\Delta_t} d\delta + \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) \\ = \frac{1}{\phi} \left\{ \frac{x(1 - \bar{\delta})}{1 - \hat{\delta}_t} - \frac{1 - x - y - \frac{(1-x)^2}{2} + \frac{y^2}{2}}{z - y} - \frac{x[z - (1 - x)]}{z - y} \right\}, \end{aligned} \quad (70)$$

where

$$x \equiv \phi Q_t, \quad y \equiv \bar{\delta} - \Delta, \quad z \equiv \bar{\delta} + \Delta. \quad (71)$$

Given the value of  $\hat{\delta}_t$ , the right-hand side of Equation (70) can be rewritten as a quadratic function of  $x$ . Note that  $x \in [1 - z, 1 - y]$  by the definition of  $\delta_{P,t}$  given by Equation (26). Since the coefficient of  $x^2$  is negative, the right-hand side takes the minimum value for  $x \in [1 - z, 1 - y]$  when  $x = 1 - z$  or  $x = 1 - y$ . In either case, the minimum value is positive, and so is the right-hand side of Equation (70) for  $x \in [1 - z, 1 - y]$ .

In addition, prove the inequality in Footnote 34. Suppose  $\lambda_{U,t} = Q_t(1 - \hat{\delta}_t)^{-1}$ . Then  $\delta_{U,t} = \hat{\delta}_t$  by Equation (26). Given  $\hat{\delta}_t > \bar{\delta}$ , it holds that:

$$\begin{aligned} \frac{Q_t(1 - \bar{\delta})}{1 - \hat{\delta}_t} - \left( \int_{\bar{\delta} - \Delta_t}^{\delta_{U,t}} \frac{\lambda_{U,t}(1 - \delta)}{2\Delta_t} d\delta + \int_{\delta_{U,t}}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) \\ = \frac{Q_t(1 - \bar{\delta})}{1 - \hat{\delta}_t} - \left( \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{Q_t(1 - \delta)}{(1 - \hat{\delta}_t)2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) \\ > \frac{Q_t(1 - \bar{\delta})}{1 - \hat{\delta}_t} - Q_t > 0. \end{aligned} \quad (72)$$

By continuity, the inequality in Footnote 34 holds if  $\lambda_{U,t}$  is sufficiently close to  $Q_t(1 - \hat{\delta}_t)^{-1}$ .

## D Slopes of Equations (42) and (48)

**Lemma 2** Equation (48) is a downward-sloping curve on the  $(Q_t, \hat{\delta}_t)$  plane, given the values of  $\alpha_t$ ,  $\Delta_t$  and  $\theta_{t-1}$ . Equation (42) is also downward-sloping, if  $\delta_{P,t} = 1 - \phi Q_t$ , and is a flat line, if  $\delta_{P,t} = \bar{\delta} - \Delta_t$ . Equation (48) has a steeper slope than Equation (42) at the intersection of the two curves, if  $\beta$  is sufficiently close to 1 and Condition (20) is satisfied at the intersection.

**Proof.** Denote the implicit functions for  $\hat{\delta}_t$  implied by Equations (42) and (48) by  $\hat{\delta}_t = h(Q_t, \alpha_t, \Delta_t, \theta_{t-1})$  and  $\hat{\delta}_t = \ell(Q_t, \alpha_t, \Delta_t, \theta_{t-1})$ , respectively.

There are two cases to consider. The first case is that  $\delta_{P,t} = 1 - \phi Q_t$  at the intersection of the two curves and the second case is that  $\delta_{P,t} = \bar{\delta} - \Delta_t$  at the intersection.

In the first case, it can be shown that:

$$\frac{\partial h(Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial Q_t} = -\frac{\theta_{t-1}\phi\hat{\delta}_t(1 - \delta_{P,t})}{\theta_{t-1}(\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \hat{\delta}_t}, \quad (73)$$

$$\frac{\partial \ell(Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial Q_t} = -\frac{\frac{\beta\alpha_t(1-\hat{\delta}_t)}{Q_t^2} + \frac{\theta_{t-1}\phi(1-\delta_{P,t})}{2\Delta_t}}{\beta\left(\frac{\alpha_t}{Q_t} + \frac{\bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t}\right)}, \quad (74)$$

which are always strictly negative. This result proves that the implicit functions  $h$  and  $\ell$  exist by the implicit function theorem and that Equations (42) and (48) are downward-sloping on the  $(Q_t, \hat{\delta}_t)$  plane.

It can be shown that  $\frac{\partial \ell(Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial Q_t} - \frac{\partial h(Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial Q_t}$  has the same sign with:

$$\begin{aligned}
& - \left[ \frac{\beta \alpha_t (1 - \hat{\delta}_t)}{Q_t^2} + \frac{\theta_{t-1} \phi(1 - \delta_{P,t})}{2\Delta_t} \right] \left[ \theta_{t-1} (\bar{\delta} + \Delta_t - \delta_{P,t}) \right. \\
& \quad \left. + \bar{\delta} + \Delta_t - \hat{\delta}_t \right] + \theta_{t-1} \phi \hat{\delta}_t (1 - \delta_{P,t}) \beta \left( \frac{\alpha_t}{Q_t} + \frac{\bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} \right) \\
& = - \frac{\beta \alpha_t (1 - \hat{\delta}_t)}{Q_t^2} \left[ \theta_{t-1} (\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \hat{\delta}_t \right] \\
& \quad - \theta_{t-1} \phi (1 - \delta_{P,t}) \left[ \frac{\theta_{t-1} (\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} \right. \\
& \quad \quad \left. - \hat{\delta}_t \beta \left( \frac{\alpha_t}{Q_t} + \frac{\bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} \right) \right]. \quad (75)
\end{aligned}$$

At the intersection of Equations (42) and (48), it holds that:

$$\begin{aligned}
& \frac{\theta_{t-1} (\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} - \hat{\delta}_t \beta \left( \frac{\alpha_t}{Q_t} + \frac{\bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} \right) \\
& = \frac{\theta_{t-1} (\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} \\
& \quad - \frac{\hat{\delta}_t}{1 - \hat{\delta}_t} \left[ \theta_{t-1} \int_{\delta_{P,t}}^{\bar{\delta} + \Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{1 - \delta}{2\Delta_t} d\delta \right. \\
& \quad \quad \left. + (1 - \beta) \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta \right] \\
& = \left[ \frac{\theta_{t-1} (\bar{\delta} + \Delta_t - \delta_{P,t}) + \bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t} \right] \left( 1 - \hat{\delta}_t \right) - \frac{\hat{\delta}_t (1 - \beta)}{1 - \hat{\delta}_t} \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta. \quad (76)
\end{aligned}$$

The first equality is obtained from Equation (48) and the second equality is obtained from

Equation (42). Note that:

$$\frac{\hat{\delta}_t}{1 - \hat{\delta}_t} \leq \frac{\bar{\delta} + \Delta_t}{1 - (\bar{\delta} + \Delta_t)}, \quad (77)$$

$$\int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta \leq 1 - \bar{\delta}, \quad (78)$$

since  $\hat{\delta}_t \leq \bar{\delta} + \Delta_t$ . Thus, the last term on the last line of Equation (76) goes to 0 as  $\beta$  goes to 1, if equilibrium continues to exist. Because Condition (20) and Equation (26) imply that  $\delta_{P,t} < \hat{\delta}_t$  in equilibrium, this result proves that  $\frac{\partial \ell(Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial Q_t} < \frac{\partial h(Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial Q_t}$  at the intersection of Equations (42) and (48) for the first case (i.e.,  $\delta_{P,t} = 1 - Q_t\phi$ ), if  $\beta$  is sufficiently close to 1.

For the second case (i.e.,  $\delta_{P,t} = \bar{\delta} - \Delta_t$ ), Equation (42) implies that  $\hat{\delta}_t$  is constant. Also, it can be shown that:

$$\frac{\partial \ell(Q_t, \alpha_t, \Delta_t)}{\partial Q_t} = -\frac{\frac{\alpha_t(1-\hat{\delta}_t)}{Q_t^2}}{\frac{\alpha_t}{Q_t} + \frac{\bar{\delta} + \Delta_t - \hat{\delta}_t}{2\Delta_t}} < 0. \quad (79)$$

Thus, the implicit functions  $h$  and  $\ell$  exist and Equation (48) has a steeper slope than Equation (42) on the  $(Q_t, \hat{\delta}_t)$  plane. ■

## E Shifts in Equation (48) in response to changes in $\Delta_t$

Rewrite Equation (48) to define a function  $g$ :

$$g(\hat{\delta}_t, Q_t, \alpha_t, \Delta_t, \theta_{t-1}) \equiv \frac{Q_t}{1 - \hat{\delta}_t} \left\{ (1 - \bar{\delta})(1 + \theta_{t-1}) - \theta_{t-1} \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{1 - \delta}{2\Delta_t} d\delta \right\} - \beta \left( \alpha_t + \frac{Q_t}{1 - \hat{\delta}_t} \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2\Delta_t} d\delta + \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{Q_t}{2\Delta_t} d\delta \right) = 0. \quad (80)$$

Since  $\frac{\partial g}{\partial Q_t} > 0$ , an increase in  $\Delta_t$  makes Equation (48) shift inward if  $\frac{\partial g}{\partial \Delta_t} > 0$  and outward if  $\frac{\partial g}{\partial \Delta_t} < 0$ . The following Lemma shows that both cases exist.

**Lemma 3**  $\frac{\partial g}{\partial \Delta_t} > 0$ , if  $\delta_{P,t}$  is sufficiently close to  $\hat{\delta}_t$ .  $\frac{\partial g}{\partial \Delta_t} < 0$ , if  $\delta_{P,t} \leq \bar{\delta}$ .

**Proof.** Take the partial derivative:

$$\begin{aligned} \frac{\partial g(\hat{\delta}_t, Q_t, \alpha_t, \Delta_t, \theta_{t-1})}{\partial \Delta_t} = & \frac{Q_t}{1 - \hat{\delta}_t} \left\{ \frac{-\theta_{t-1}[1 - (\bar{\delta} - \Delta_t)]}{2\Delta_t} + \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{\theta_{t-1}(1 - \delta)}{2(\Delta_t)^2} d\delta \right. \\ & \left. - \beta \left[ \frac{1 - (\bar{\delta} - \Delta_t)}{2\Delta_t} - \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{2(\Delta_t)^2} d\delta + \frac{1 - \hat{\delta}_t}{2\Delta_t} - \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{1 - \hat{\delta}_t}{2(\Delta_t)^2} d\delta \right] \right\}. \end{aligned} \quad (81)$$

It can be shown that:

$$\frac{-[1 - (\bar{\delta} - \Delta_t)]}{2\Delta_t} + \int_{\bar{\delta} - \Delta_t}^{\delta_{P,t}} \frac{1 - \delta}{2(\Delta_t)^2} d\delta \begin{cases} < 0, & \text{if } \delta_{P,t} \leq \bar{\delta}, \\ > 0, & \text{if } \delta_{P,t} \text{ is sufficiently close to } \bar{\delta} + \Delta_t, \end{cases} \quad (82)$$

given  $\Delta_t \in (0, 1 - \bar{\delta})$  and  $\bar{\delta} \in (0, 1)$ .

Also, given  $\hat{\delta}_t \in [\bar{\delta}, \bar{\delta} + \Delta_t]$  by Equation (42), it can be shown that:

$$- \left[ 1 - (\bar{\delta} - \Delta_t) - \int_{\bar{\delta} - \Delta_t}^{\hat{\delta}_t} \frac{1 - \delta}{\Delta_t} d\delta + 1 - \hat{\delta}_t - \int_{\hat{\delta}_t}^{\bar{\delta} + \Delta_t} \frac{1 - \hat{\delta}_t}{\Delta_t} d\delta \right] \leq 0, \quad (83)$$

where the equality holds if and only if  $\hat{\delta}_t = \bar{\delta} + \Delta_t$ . Note that Equation (42) implies that  $\hat{\delta}_t$  is sufficiently close to  $\bar{\delta} + \Delta_t$  if  $\delta_{P,t}$  is sufficiently close to  $\bar{\delta} + \Delta_t$ . Thus, substituting Inequalities (82) and (83) in Equation (81) proves the proposition. ■

## F The numerical solution method for the equilibrium dynamics of the model with banks

The dynamic equilibrium is approximated by the following projection method:

Step 0. It can be shown that the dynamic equilibrium in each period is homogeneous of degree 1 with respect to  $K_{P,t-1}$ ,  $K_{U,t-1}$  and  $K_{B,t-1}$ . Set grid points on the state space for  $K_{P,t-1}$ ,  $K_{U,t-1}$  and the shock parameter ( $\alpha_t$  or  $\Delta_t$ ). The value of  $K_{B,t-1}$  is set to  $1 - K_{P,t-1} - K_{U,t-1}$  on each grid point. Guess the equilibrium values of endogenous variables on each grid point, including  $\bar{\omega}_{t+1}$  and  $\underline{\omega}_{t+1}$ . Call this correspondence between state variables and endogenous variables as a ‘candidate array’.

Step 1. Suppose the candidate array returns equilibrium values in the next period for each set of  $K_{P,t}$ ,  $K_{U,t}$  and the shock parameter. The next-period equilibrium values on a point between the grid points in the state space are approximated by linear interpolation. Given this, derive the candidate array for the current period through the aggregate equilibrium conditions.

Step 2. Compare the candidate arrays for the current period and for the next period. If the ratio of each element between the two arrays becomes sufficiently close to 1, then take the candidate array as the equilibrium correspondence. Otherwise, update the candidate array for the next period by a linear combination of the two arrays and go back to Step 1.

In the numerical examples in this paper, I set grid points in the  $\pm 5\%$  range of the deterministic steady state values of  $K_{P,t-1}$  and  $K_{U,t-1}$ . The number of grid points are 20

for these endogenous state variables. Note that the shock parameter only takes two values by assumption. The convergence criterion in Step 2 is 1e-03. In updating the candidate array in Step 2, the weight on the candidate array for the current period is 0.001. The initial guess in Step 0 is obtained through homotopy starting from the parameter values with which deterministic steady state values given the value of the shock parameter are a successful initial guess of the candidate array that leads to convergence.

The equilibrium conditions are checked for each element of the converged candidate array. For each figure, random simulations of the dynamics for 5000 periods confirm that the equilibrium dynamics move within the grid points that satisfy the equilibrium conditions.

## G The modification of the model when agents can borrow against new trees

In addition to the short-sale constraint on new trees (50), the flow-of-funds constraint (4) and the law of motion for trees net of depreciation (5) for each agent are modified to:

$$\begin{aligned} P_t m_{i,t} + c_{i,t} + x_{i,t} + Q_t h_{i,t} + b_{i,t} + (1 + \zeta) V_t s_{i,t} \\ = \alpha_t k_{i,t-1} + Q_t \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} l_{i,\delta,t} d\delta + \tilde{R}_t b_{i,t-1} + (D_t + V_t) s_{i,t-1}, \end{aligned} \quad (84)$$

$$k_{i,t} = m_{i,t} + \phi_{i,t} x_{i,t} + (1 - \hat{\delta}_t) h_{i,t} + \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} (1 - \delta) \left( \frac{k_{i,t-1}}{2\Delta_t} - l_{i,\delta,t} \right) d\delta, \quad (85)$$

where  $P_t$  is the competitive market price of pledged new trees. Similarly, the flow-of-funds constraint and the maximization problem for the representative bank are modified to:

$$K_{B,t} = M_{B,t} + (1 - \hat{\delta}_t) H_{B,t} + (1 - \bar{\delta})(K_{B,t-1} - L_{B,t}), \quad (86)$$



$$\begin{aligned}
\Omega_t(K_{B,t-1}, B_{B,t-1}, \bar{R}_{t-1}) \equiv & \\
\max_{\{H_{B,t}, L_{B,t}, B_{B,t}, \bar{R}_t, M_{B,t}\}} & P_t M_{B,t} + \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - \tilde{R}_t B_{B,t-1} \\
& + B_{B,t} + E_t [\Lambda_{V,t+1} \Omega_{t+1}(K_{B,t}, B_{B,t}, \bar{R}_t)], \\
\text{s.t. Equations (9), (10), (12) and (86), } & L_{B,t} \in [0, K_{B,t-1}], M_{B,t}, H_{B,t}, B_{B,t} \geq 0, \quad (87)
\end{aligned}$$

where  $M_{B,t}$  is the amount of the right to receive new trees bought by the bank. The market clearing condition for the right to receive new trees is:

$$\int_{i \in \mathcal{I}} m_{i,t} d\mu + M_{B,t} = 0, \quad (88)$$

which is added to the equilibrium conditions listed in Section 2.4.

The first-order conditions with respect to  $x_{i,t}$  and  $m_{i,t}$  imply that:

$$P_t = \lambda_{i,t}, \quad \text{if } m_{i,t} > 0, \quad (89)$$

$$m_{i,t} = -\psi \phi_{i,t} x_{i,t}, \quad \text{if } P_t > \lambda_{i,t}, \quad (90)$$

$$\lambda_{i,t} \leq \frac{1 - P_t \psi \phi}{(1 - \psi) \phi}, \quad \text{if } \phi_{i,t} = \phi. \quad (91)$$

Similarly, the first-order condition with respect to  $M_{B,t}$  yields that:

$$P_t = \lambda_{B,t}, \quad \text{if } M_{B,t} > 0, \quad (92)$$

$$M_{B,t} = 0, \quad \text{if } P_t > \lambda_{B,t}. \quad (93)$$

Equations (89), (90), (92) and (93) imply that  $P_t = \max\{\max_{i \in \mathcal{I}} \lambda_{i,t}, \lambda_{B,t}\}$ , given that there exist sellers of the right to receive new trees (i.e., agents with  $m_{i,t} < 0$ ). Then Condition (20) implies that  $P_t$  is greater than  $\phi^{-1}$ , which is in turn greater than  $\lambda_{i,t}$  for productive agents, given Equation (91). Thus,  $m_{i,t} = -\psi \phi_{i,t} x_{i,t}$  if  $\phi_{i,t} = \phi$ . These results prove that,

given Condition (20), productive agents sell the right to receive new trees and unproductive agents buy the right to receive new trees ( $P_t = \lambda_{U,t}$  and  $m_{i,t} > 0$  if  $\phi_{i,t} = 0$ ) if there is no bank. If banks exist and Equation (31) hold, then Equation (24) implies that banks buy the right to receive new trees ( $P_t = \lambda_{B,t}$  and  $M_{B,t} > 0$ ), as described in Section 6.

## H The effect of an alternative bank liquidation procedure

Suppose that, if a bank run is expected to happen, then the bank or the government sets up a competitive bank liquidation market to sell the bank's trees separately from the secondary market for trees. Modify the maximization problem for each agent defined by Equations (1) and (4)-(6) as follows:

$$\max_{\{c_{i,t}, x_{i,t}, h_{i,t}, l_{i,\delta,t}, b_{i,t}, s_{i,t}, h_{i,t}^{LB}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{i,t} \quad (94)$$

s.t.

$$\begin{aligned} c_{i,t} + x_{i,t} + Q_t h_{i,t} + b_{i,t} + (1 + \zeta) V_t s_{i,t} + Q_t^{LB} h_{i,t}^{LB} \\ = \alpha_t k_{i,t-1} + Q_t \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} l_{i,\delta,t} d\delta + \tilde{R}_t b_{i,t-1} + (D_t + V_t) s_{i,t-1}, \end{aligned} \quad (95)$$

$$k_{i,t} = \phi_{i,t} x_{i,t} + (1 - \hat{\delta}_t) h_{i,t} + (1 - \bar{\delta}) h_{i,t}^{LB} + \int_{\bar{\delta}-\Delta_t}^{\bar{\delta}+\Delta_t} (1 - \delta) \left( \frac{k_{i,t-1}}{2\Delta_t} - l_{i,\delta,t} \right) d\delta, \quad (96)$$

$$l_{i,\delta,t} \in \left[ 0, \frac{k_{i,t-1}}{2\Delta_t} \right], \quad c_{i,t}, x_{i,t}, h_{i,t}, b_{i,t}, s_{i,t}, h_{i,t}^{LB} \geq 0, \quad (97)$$

where  $Q_t^{LB}$  is the competitive price of the liquidated bank's trees in the bank liquidation market and  $h_{i,t}^{LB}$  is the quantity of trees bought in the bank liquidation market. Note that the average depreciation rate of trees sold in the bank liquidation market equals  $\bar{\delta}$ , since all of the liquidated bank's trees are sold in the bank liquidation market and no agent can sell their trees there. Thus  $(1 - \bar{\delta}) h_{i,t}^{LB}$  in Equation (96) is the quantity of trees net of depreciation

obtained through the bank liquidation market.

Similarly, the maximization problem for banks that are not hit by bank runs is modified to:

$$\begin{aligned}
(D_t + V_t)S_{B,t-1} &= \Omega_t(K_{B,t-1}, B_{B,t-1}, \bar{R}_{t-1}) \equiv \\
&\max_{\{H_{B,t}, L_{B,t}, B_{B,t}, \bar{R}_t, H_{B,t}^{LB}\}} \alpha_t K_{B,t-1} - Q_t(H_{B,t} - L_{B,t}) - Q_t^{LB} H_{B,t}^{LB} - \tilde{R}_t B_{B,t-1} \\
&\quad + B_{B,t} + E_t [\Lambda_{V,t+1} \Omega_{t+1}(K_{B,t}, B_{B,t}, \bar{R}_t)], \quad (98)
\end{aligned}$$

s.t.

$$K_{B,t} = (1 - \hat{\delta}_t)H_{B,t} + (1 - \bar{\delta})H_{B,t}^{LB} + (1 - \bar{\delta})(K_{B,t-1} - L_{B,t}), \quad (99)$$

$$\tilde{R}_t = \begin{cases} \bar{R}_{t-1}, & \text{if } \bar{R}_{t-1} B_{B,t-1} \leq (\alpha_t + Q_t^{LB})K_{B,t-1}, \\ \frac{(\alpha_t + Q_t^{LB})K_{B,t-1}}{B_{B,t-1}}, & \text{if } \bar{R}_{t-1} B_{B,t-1} > (\alpha_t + Q_t^{LB})K_{B,t-1}, \end{cases} \quad (100)$$

$$\Omega_t = 0, \quad \text{if } \bar{R}_{t-1} B_{B,t-1} > (\alpha_t + Q_t^{LB})K_{B,t-1} \quad (101)$$

$$\text{Equations (12), } L_{B,t} \in [0, K_{B,t-1}], H_{B,t} \geq 0, B_{B,t} \geq 0, H_{B,t}^{LB} \geq 0, \quad (102)$$

where  $H_{B,t}^{LB}$  is the quantity of trees obtained through the bank liquidation market, given no bank run to the representative bank. Note that the second constraint implies that a bank run occurs if the face value of deposits exceeds the liquidation value of the bank evaluated by  $Q_t^{LB}$ . Also,  $\Omega_t = 0$  in the third constraint is equivalent to  $D_t = V_t = 0$  in Equation (10).

The marker clearing condition for the liquidated bank's trees is  $\int_{\mathcal{I}} h_{i,t}^{LB} d\mu + H_{B,t}^{LB} = L_t^{LB}$ , where  $L_t^{LB}$  is the total quantity of the liquidated bank's trees. In equilibrium, this condition and Equations (16)-(19) are satisfied in each period and the modified maximization problems above are solved with rational expectations.

It can be shown that a modified version of Proposition 1 holds if the definitions of  $\bar{\omega}_{t+1}$

and  $\underline{\omega}_{t+1}$  are modified to the high and the low values of  $\alpha_{t+1} + Q_{t+1}^{LB}$ , respectively, and  $Q_t$  and  $Q_{t+1}$  in Equations (27)-(30) are replaced with  $Q_t^{LB}$  and  $Q_{t+1}^{LB}$ , respectively. Equations (31) and (32) hold as they are.

Given this result, the first-order conditions with respect to  $h_{i,t}$  and  $H_{B,t}$  in the maximization problems for agents and banks above imply that  $Q_t - \lambda_{i,t}(1 - \hat{\delta}_t) \geq 0$  and  $Q_t - \lambda_{B,t}(1 - \hat{\delta}_t) \geq 0$ , where the equalities hold if  $h_{i,t} > 0$  and  $H_{B,t} > 0$ , respectively. Similarly, the first-order conditions with respect to  $h_{i,t}^{LB}$  and  $H_{B,t}^{LB}$  in the maximization problems for agents and banks yield that  $Q_t^{LB} - \lambda_{i,t}(1 - \bar{\delta}) \geq 0$  and  $Q_t^{LB} - \lambda_{B,t}(1 - \bar{\delta}) \geq 0$ , where the equalities hold if  $h_{i,t}^{LB} > 0$  and  $H_{B,t}^{LB} > 0$ , respectively. Without loss of generality, assume that  $Q_t^{LB} - \lambda_{i,t}(1 - \bar{\delta}) = 0$  for some  $i \in \mathcal{I}$  or  $Q_t^{LB} - \lambda_{B,t}(1 - \bar{\delta}) = 0$  in equilibrium if there is no liquidated bank.

Now suppose  $H_{B,t} > 0$  for all  $t$ , which implies that banks are always providing liquidity transformation services. Then the inequalities shown above indicate that  $\lambda_{B,t} = Q_t(1 - \hat{\delta}_t)^{-1} \geq \lambda_{i,t}$  for all  $i \in \mathcal{I}$ . Thus  $Q_t^{LB} = \lambda_{B,t}(1 - \bar{\delta}) = Q_t(1 - \hat{\delta}_t)^{-1}(1 - \bar{\delta})$ . Given Proposition 1 revised for the maximization problem (98)-(102), this result implies that  $\tilde{R}_t B_{B,t-1} + (D_t + V_t)S_{B,t-1} = [\alpha_t + Q_t(1 - \hat{\delta}_t)^{-1}(1 - \bar{\delta})]K_{B,t-1}$  for any value of  $\bar{R}_{t-1}B_{B,t-1}$ . Hence the bank liquidation market values the liquidated bank's trees fairly and the total value of bank liabilities always equals the present discounted value of the current and future income from the liquidated bank's tree. As rational agents expect this, they do not run to banks.

Also, it can be shown that  $\lambda''_{B,t} > \lambda'_{B,t}$  and  $\tilde{R}_t B_{B,t} = \bar{\omega}_{t+1}K_{B,t}$ , if  $\zeta > 0$ . This result implies that, since the bank liquidation market values the liquidated bank's trees fairly, the holders of bank liabilities are indifferent to bank failures, so that equity holders do not impose costly bank capital requirements. The contingent pay-off to deposits becomes equivalent to the pay-off to bank equity without the bank-equity holding cost in this case.