Can Affine Term Structure Models Help Us to Predict Exchange Rates? - Discussion

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Outline

Summary
  Objectives
  Framework
  Results

Comments
  Interpretation
  Questions
  Suggestions

Conclusion
Forward Premium Puzzle

- Uncovered Interest Rate Parity:

\[ \Delta s_{t+1} = \alpha_0 + \alpha_1 [r_t - r_t^*] + \varepsilon_{t+1}, \]

where \( s_t \) is in $/units of foreign currency.

- UIP condition: \( \alpha_0 = 0 \) and \( \alpha_1 = 1 \).

- In the data: \( \alpha_1 < 1 \) and mostly \( \alpha_1 < 0 \).
Exchange rates are hard to predict.


This Paper: An Internationally Affine Model

- State vector: \( X_t = \begin{bmatrix} r_t \\ r_t^* \end{bmatrix} \).

- \( X_t \) follows an Orstein-Uhlenbeck process.

- Market price of risk \( \Lambda_t \) affine in \( X_t \):

\[
\begin{align*}
\text{d}X_t &= \Phi(\Theta - X_t) + \Sigma^{1/2}\text{d}W_t \\
\Lambda_t &= \Lambda_0 + \Lambda_1 X_t \\
\Lambda_t^* &= \Lambda_0^* + \Lambda_1^* X_t.
\end{align*}
\]
For a foreign investor buying a bond from her country, the real return $R_{t,t+1}^*$ satisfies:

$$E_t(M_{t,t+1}^* R_{t,t+1}^*) = 1.$$ 

But a domestic investor can also buy a foreign bond:

$$E_t(M_{t,t+1}^* \frac{Q_{t+1}}{Q_t} R_{t,t+1}^*) = 1.$$ 

Thus, in complete markets, real exchange rate $Q$ is defined as:

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{t,t+1}^*}{M_{t,t+1}}.$$
Procedure

- No arbitrage in bond and FX markets.

- Interest rates:

\[
\begin{align*}
r_t^h &= A(h) + B(h)'X_t \\
\tilde{r}_t^{*,h} &= A^*(h) + B^*(h)'X_t, \text{ where } X_t = \begin{bmatrix} r_t \\ r_t^* \end{bmatrix}.
\end{align*}
\]

- Exchange rates:

\[
\Delta s_{t+1} = C(1)+D(1)\tilde{X}_t+v_{t+1}, \text{ where } \tilde{X}_t = [X_t', vech(X_tX_t')]'.
\]

- MLE.
Results

▶ Forward bias:
  - US-Canada, 3-month horizon:
    \[ \alpha_1 = -0.5 \text{ in the model} \]
    \[ = -0.8 \text{ in the data.} \]
  - US-UK, 3-month horizon:
    \[ \alpha_1 = -1.9 \text{ in the model} \]
    \[ = -1.5 \text{ in the data.} \]

▶ Exchange rate predictability out-of-sample:
  - US-Canada, 9% lower RMSE (vs random walk and VAR) at 12-month
  - US-UK: 36% lower.
Two possible mechanisms

- Complete markets, no inflation risk.
- Log currency risk premium:

\[ std_t m_{t+1} \left[ std_t m_{t+1} - \rho_t (m_{t+1}, m^*_{t+1}) std_t m^*_{t+1} \right]. \]

- Two possible mechanisms: lower foreign interest rate means
  - Heteroskedasticity: \( std_t m^*_{t+1} \uparrow \)
  - Time-varying correlation: \( \rho_t (m_{t+1}, m^*_{t+1}) \uparrow \)

\[ \Rightarrow \] Both mechanisms are playing here (see signs of estimated \( \lambda_{i,j} \))
$M_{t,t+1} = e^{-r_t} e^{-\lambda_t X_{t+1}} / \phi^P (-\lambda_t, X_t)$.

- $std_t m_{t+1}$ is prop. to $-\lambda_t$.

- As a first approximation, take $dr_t \perp dr_t^*$.

- Estimation (Table 3 in the paper):

  \[
  \Lambda_t = 5.7 - 2.1 r_t - 0.6 r_t^* \\
  \Lambda_t^* = -5.9 + 2.6 r_t + 0.3 r_t^*
  \]

- $r_t^* \downarrow \iff \Lambda_t^* \downarrow \iff std_t m_{t+1}^* \uparrow$

- $r_t^* \downarrow \iff cov_t (m_{t+1}, m_{t+1}^*) \uparrow$
Two issues


- Example 2: negative factors $\Rightarrow$ negative interest rates (but low prob.);

- Example 4: interdependent factor model $\Rightarrow$ asymmetry: one shock impacts more the foreign interest rate than the other shock, but impacts less the foreign pricing kernel.
Procedure

- Quasi-MLE, using interest rate and exchange rate data.

- Paper uses two assumptions:
  - Exchange rate innovations homoskedastic: \( \nu_{t+1} \sim N(0, \sigma_v^2) \).
  - Exchange rate innovations \( \nu_{t+1} \) uncorrelated to interest rate residuals.

- Justifications?
  - Heteroskedasticity in FX (Andersen, Bollerslev, 1998)?
  - Currency risk premia only explained by interest rates?
What drives the predictability results?

- Reference point: Clarida, Sarno, Taylor and Valente (2003): MS-VECM beats VECM by 40%.

- Standard errors?

- Comparing to VARs or VECMs:
  - Is this about adding non-linearities \((r_t)^2, (r_t^*)^2, \ldots\) ?
  - Or about restrictions on estimated coefficients implied by no-arbitrage?
  - \(\Rightarrow\) Compare to VAR or VECM with higher moments?
More moments

- Variance of changes in exchange rates:

  \[ \frac{Q_{t+1}}{Q_t} = \frac{M^*_{t,t+1}}{M_{t,t+1}}. \]

  \[ \sigma^2_{\Delta q} = \sigma^2_{m^*} + \sigma^2_m - 2\rho_{m^*,m}\sigma_{m^*}\sigma_m. \]

- Campbell-Shiller tests of the EH:

  \[ y_{n-1}^{n} - y_{t}^{n} = \alpha + \beta_n\left(\frac{y_t^n - y_t^1}{n-1}\right) + \varepsilon_{t+1}. \]

- Does FX data and no-arbitrage condition across countries lead to better yield predictability?
Combining bond and FX data gives much better FX out-of-sample predictability.

Very exciting results!